Presentation title

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Outline

OUTLINE

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Introduction

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- For d a suitable first order derivative, the Lagrangian density with $\phi : (X, g_X) \to (M, g_M)$,

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- Pseudo-differential operators are generalized derivatives defined on $\Omega \subseteq X$ via an integration rule over Ω [They do not satisfy Leibniz]

Def

$$(-\Delta)^s f(x) = C_{n,s}\text{p.v.} \int_\Omega \frac{f(x) - f(\zeta)}{|x - \zeta|^{n+2s}} d\zeta.$$

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- Let's call this *classical nonlocality*

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- Choosing $\kappa = \gamma k$ gives the desired coefficients - which are now meromorphic functions.