

Presentation title

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May 7, 2022

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OUTLINE

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Introduction

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• Pseudo-differential operators are generalized derivatives defined on $\Omega \subseteq X$ via an integration rule over Ω [They do not satisfy Leibniz]

DEF

$$(-\Delta)^s f(x) = C_{n,s} \mathsf{p.v.} \int_{\Omega} \frac{f(x) - f(\zeta)}{|x - \zeta|^{n+2s}} d\zeta.$$

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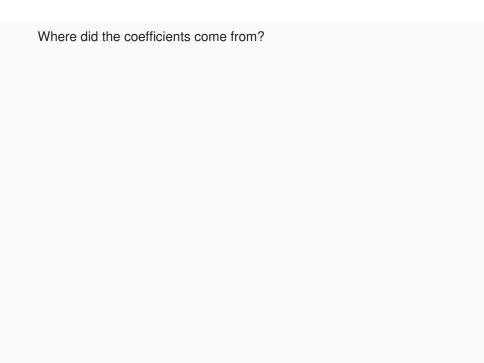
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· Let's call this classical nonlocality



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$$\theta$$
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$$\partial_z^{\gamma} z^{\kappa} = (\kappa - \gamma + 1) \frac{\Gamma(1+\kappa)}{\Gamma(1+1-\gamma+\kappa)} z^{\kappa-\gamma}$$
$$= \frac{\Gamma(1+\kappa)}{\Gamma(1+\kappa-\gamma)} z^{\kappa-\gamma}$$

• Choosing $\kappa = \gamma k$ gives the desired coefficients - which are now meromorphic functions.