SERIES EXPANSIONS

$f(x) = \sum_{n=0}^{\infty} a_n x^n$ POWER SERIES

MAKES SENSE IF SUM CONVERGES

THM GIVEN SOME $f(x)$ REPRESENTED BY POWER SERIES,

THE CHOICE OF a_n IS UNIQUE.

$$a_n = \frac{f^{(n)}(0)}{n!}$$

SUCH $f(x)$ ARE SAID TO BE REAL ANALYTIC (ON SOME DOMAIN)

$\Longrightarrow$ ALL ANALYTIC FUNCTIONS ARE INTEGRABLE ON THEIR DOMAIN OF CONV.

$$\int_a^b f(x)\,dx = \int_a^b \sum_n \frac{f^{(n)}(0)}{n!} x^n$$

$$= \sum_n \frac{f^{(n)}(0)}{n!} \int_a^b x^n\,dx$$

$$= \sum_n \frac{f^{(n)}(0)}{n!} \frac{(b-a)^{n+1}}{n+1}$$

PRACTICALLY, YOU MAKE SURE x IS SMALL, TRUNCATE AT SOME ORDER, SAY $O(x^3)$

$$\int_0^a \frac{1-\cos x}{x^2}\,dx \qquad \cos x \sim 1 - \frac{x^2}{2} + \frac{x^4}{4!}$$

$$\sim \int_0^a \frac{1}{2} - \frac{x^2}{4!}\,dx = \frac{a}{2} - \frac{a^3}{3 \cdot 4!}$$

ERROR $\to 0$ AS YOU GO HIGHER

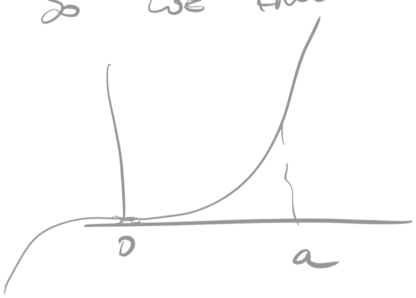
NOTE $\frac{1-\cos x}{x^2}$ IS ANALYTIC FOR $x \in [0,a]$

WHAT ABOUT

$\frac{x}{\cosh[1/x]}$, $x \sim 0$. THIS IS NOT AN ANALYTIC FUNCTION.

$$\frac{2x}{e^{1/x}+e^{-1/x}} \sim \frac{2x}{e^{1/x}}$$
$$= 2xe^{-1/x}$$

SO WE KNOW



LET $a = 1$

$$-Ei(x) = \int_x^\infty \frac{e^{-t}}{t}\,dt$$

EXPONENTIAL INTEGRAL

$$\int_0^1 2xe^{-1/x} = -Ei(-1)$$

$$\int_0^1 \frac{x}{\cosh\frac{1}{x}}\,dx \sim \underline{-Ei(-1)}$$

WHAT ABOUT ERROR? HARD IN THIS EXAMPLE (STABILITY ISSUES)

$$\frac{2x}{e^{1/x} + e^{-1/x}}$$

LOOK AT $e^{-1/x}$ AS $x \to \varepsilon$

$$e^{-1/x} \sim e^{-1/\varepsilon}\left(1 + \frac{x-\varepsilon}{\varepsilon^2}\right)$$

$$\sim e^{-1/\varepsilon}\left(\frac{x-\varepsilon}{\varepsilon^2}\right)$$

$$\sim e^{-1/\varepsilon}\,\frac{x}{\varepsilon^2}$$

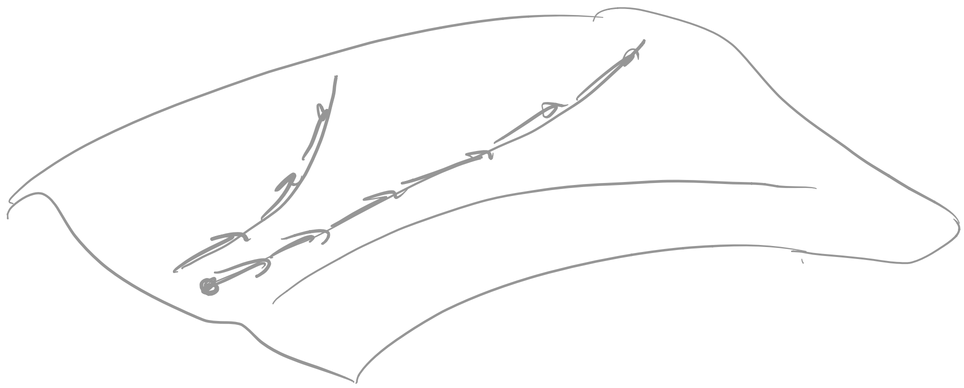
WHEN SERIES CONVERGENT,

USE IT TO SOLVE ODES

OTHERWISE, APPROXIMATIONS

WORK WITH $\vec{v} \in \mathbb{R}^3$ WITH CANONICAL BASIS.

$$\vec{v} = \sum_i \hat{e}_i \times v_i$$



$\vec{F} : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ VECTOR FIELD

$$\nabla \cdot \vec{F} = \partial_x F_x + \partial_y F_y + \partial_z F_z$$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ F_x & F_y & F_z \end{vmatrix}$$

WE'LL ALSO NEED CURVI-LINEAR COORDS.

GIVEN SCALAR FUNCTION, $U : \mathbb{R}^3 \to \mathbb{R}$

(CONSERVATIVE)

∇U IS A VECTOR FIELD

$$\partial_x U \hat{x} + \partial_y U \hat{y} + \partial_z U \hat{z}$$