A very specific review and some coupled oscialltions

August 16, 2021

Things to recall:

- $E = T + U$, the total energy, is a constant of motion (in closed mechanical systems). The dynamical data under this constaint is encoded in $\dot{E} = 0$.

  Ex (Simple pendulum)**.** *We have the total energy* $\frac{1}{2}ml^2\dot{\theta}^2 - mgl\cos\theta$*. The constancy of this quantitiy implies*

  $$\dot{E} = 0 \implies ml^2\ddot{\theta}\dot{\theta} + mgl\sin\theta\dot{\theta} = 0$$

  which yields the familiar

  $$\ddot{\theta} + \frac{g}{l}\sin\theta = 0$$

  as the equation of motion. In this case the statement that total energy is conserved contains all the dynamical information of the system.

- A rolling object is said to not slip if its transnational velocity is matched by the tangential velocity of rolling: $\dot{q}_{center} = r\dot{\phi}$.

- What we mean when we say *solving* a classical mechanical system (in the Lagrangian formalism): For N degrees of freedom identify N generalized coordinates. Work in coordinate systems that are convenient for the problem. Once the coordinate system is identified, $\{q_i, \dot{q}_i\}_{i=1,\ldots,N}$, construct the Lagrangian

  $$L[q(t), \dot{q}(t); t] = T - U$$

  and solve

  $$\frac{\partial L}{\partial q_i} = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i}$$

  for each coordinate.

- When we say frequency of oscillation, we specifically mean the oscillation frequency of the harmonic oscillator. This is the ω that appears in

  $$L_{SHO} = \frac{1}{2}m\dot{q}^2 - \underbrace{\frac{1}{2}m\omega^2 q^2}_{U_{SHO}}$$

When you are asked to find the frequency of small oscillations, you need to figure out the parameters for which

$$U_{\text{your system}} = U_{SHO} + \mathcal{O}(q^3)$$