Lista 7 — Exercício 1

Qual a forma do termo de armazenamento de energia da equação da condução de calor em regime transiente?

$$Arm = \rho c \Delta V \cdot \frac{\partial T}{\partial t} \Rightarrow \frac{Arm}{\ell} = \frac{\rho c \Delta x \Delta y}{\Delta t}(T_p^n - T_p^{n-1}).$$

Lista 7 — Exercício 2

Há duas opções para resolver a equação da condução de calor em regime transiente, os Métodos Explícito e Implícito. Apresente cada um e destaque as diferenças entre eles.

Explícito: Para quatro conduções, a equação é: $\frac{k\Delta y}{\Delta x}(T_w^{n-1} - T_p^{n-1}) + \frac{k\Delta y}{\Delta x}(T_e^{n-1} - T_p^{n-1}) + \frac{k\Delta x}{\Delta y}(T_s^{n-1} - T_p^{n-1}) + \frac{k\Delta x}{\Delta y}(T_n^{n-1} - T_p^{n-1}) + S\Delta x\Delta y = \frac{\rho c\Delta x\Delta y}{\Delta t}(T_p^n - T_p^{n-1})$.

Nós determinamos T_p^n (variáveis, isoladas na forma $T^n = A \cdot T^{n-1} + B$) em função dos parâmetros T_p^{n-1}. Isolando, fica:

$$T_p^n = \left[\frac{k\Delta y}{\Delta x}(T_w^{n-1} - T_p^{n-1}) + \frac{k\Delta y}{\Delta x}(T_e^{n-1} - T_p^{n-1}) + \frac{k\Delta x}{\Delta y}(T_s^{n-1} - T_p^{n-1}) + \frac{k\Delta x}{\Delta y}(T_n^{n-1} - T_p^{n-1}) + S\Delta x\Delta y\right] \cdot \frac{\Delta t}{\rho c\Delta x\Delta y} + T_p^{n-1}.$$

Implícito: Para quatro conduções, a equação é: $\frac{k\Delta y}{\Delta x}(T_w^n - T_p^n) + \frac{k\Delta y}{\Delta x}(T_e^n - T_p^n) + \frac{k\Delta x}{\Delta y}(T_s^n - T_p^n) + \frac{k\Delta x}{\Delta y}(T_n^n - T_p^n) + S\Delta x\Delta y = \frac{\rho c\Delta x\Delta y}{\Delta t}(T_p^n - T_p^{n-1})$.

Nós determinamos T_p^n (variáveis, formando uma equação linear $A \cdot T = B$) em função dos parâmetros T_p^{n-1}.

A diferença é que no método explícito é feito o cálculo das temperaturas de cada ponto em cada instante a partir de todo o Domínio de Solução no instante anterior; além disso o Δt deve obedecer o critério de estabilidade da questão abaixo. Já no método implícito é feito o cálculo das temperaturas de todo o Domínio de Solução de uma só vez. É um sistema $A(t)T(t) = B(t)$ a cada intervalo de tempo.

Lista 7 — Exercício 3

Discuta o Critério de Estabilidade.

No método explícito, exibimos T_p^n como uma combinação linear de temperaturas, mais o termo de geração de calor. Espera-se que todos os coeficientes sejam não negativos. Por isso precisamos garantir que o de T_p^{n-1} também o seja.

Na equação de exemplo acima, o critério é: $\left[-\frac{k\Delta y}{\Delta x}-\frac{k\Delta y}{\Delta x}-\frac{k\Delta x}{\Delta y}-\frac{k\Delta x}{\Delta y}\right]\cdot\frac{\Delta t}{\rho c\Delta x\Delta y}+1\geq 0.$

Lista 7 — Exercício 4.a.1

Desenvolva as equações de discretização, para as malhas de uma placa de concreto conforme apresentada na figura. Determine os perfis de temperatura da placa (pontos 5 a 11) ao longo do tempo até que a placa atinja o equilíbrio (regime permanente) usando o método explícito.

Simplificamos $T_p^{n-1}=T_p$ (parâmetros). Vamos determinar T_p^n (variáveis) em função deles.

Malha 5: $T_5^n=\left[\frac{\alpha\Delta y}{\Delta x}(T_6-T_5)+\frac{\alpha\cdot 0.5\Delta x}{\Delta y}(T_9-T_5)+\frac{\alpha\cdot 0.5\Delta x}{\Delta y}(T_1-T_5)\right]\cdot\frac{\Delta t}{0.5\Delta x\Delta y}+T_5.$

Malha 6: $T_6^n=\left[\frac{\alpha\Delta y}{\Delta x}(T_5-T_6)+\frac{\alpha\Delta y}{\Delta x}(T_7-T_6)+\frac{\alpha\Delta x}{\Delta y}(T_{10}-T_6)+\frac{\alpha\Delta x}{\Delta y}(T_2-T_6)\right]\cdot\frac{\Delta t}{\Delta x\Delta y}+T_6.$

Malha 7: $T_7^n=\left[\frac{\alpha\Delta y}{\Delta x}(T_6-T_7)+\frac{\alpha\cdot 0.5\Delta y}{\Delta x}(T_8-T_7)+\frac{h}{\alpha k}\cdot 0.5\Delta y(T_\infty-T_7)+\frac{\alpha\cdot 0.5\Delta x}{\Delta y}(T_{11}-T_7)+\frac{h}{\alpha k}\cdot 0.5\Delta x(T_\infty-T_7)+\frac{\alpha\Delta x}{\Delta y}(T_3-T_7)\right]\cdot\frac{\Delta t}{0.75\Delta x\Delta y}+T_7.$

Malha 8: $T_8^n=\left[\frac{\alpha\cdot 0.5\Delta y}{\Delta x}(T_7-T_8)+\frac{h}{\alpha k}\cdot 0.5\Delta x(T_\infty-T_8)+\frac{\alpha\cdot 0.5\Delta x}{\Delta y}(T_4-T_8)\right]\cdot\frac{\Delta t}{0.25\Delta x\Delta y}+T_8.$

Malha 9: $T_9^n = \left[\dfrac{\alpha \cdot 0.5\Delta y}{\Delta x}(T_{10} - T_9) + \dfrac{q''}{\alpha k} \cdot 0.25\Delta x\Delta y + \dfrac{\alpha \cdot 0.5\Delta x}{\Delta y}(T_5 - T_9)\right] \cdot \dfrac{\Delta t}{0.25\Delta x\Delta y} + T_9.$

Malha 10: $T_{10}^n = \left[\dfrac{\alpha \cdot 0.5\Delta y}{\Delta x}(T_9 - T_{10}) + \dfrac{\alpha \cdot 0.5\Delta y}{\Delta x}(T_{11} - T_{10}) + \dfrac{q''}{\alpha k} \cdot 0.5\Delta x\Delta y + \dfrac{\alpha\Delta x}{\Delta y}(T_6 - T_{10})\right] \cdot \dfrac{\Delta t}{0.5\Delta x\Delta y} + T_{10}.$

Malha 11: $T_{11}^n = \left[\dfrac{\alpha \cdot 0.5\Delta y}{\Delta x}(T_{10} - T_{11}) + \dfrac{h}{\alpha k} \cdot 0.5\Delta y(T_\infty - T_{11}) + \dfrac{q''}{\alpha k} \cdot 0.25\Delta x\Delta y + \dfrac{\alpha \cdot 0.5\Delta x}{\Delta y}(T_7 - T_{11})\right] \cdot \dfrac{\Delta t}{0.25\Delta x\Delta y} + T_{11}.$

O critério de estabilidade inicial para a malha 5 é $\Delta t \leq 57692{,}32$.

Busquei na internet a densidade do concreto $\rho = 2{,}4\,kg/m^3$ e o calor específico do granito $c = 0{,}79\,J/(gK)$ e assim fiz $k = \dfrac{\rho c}{\alpha} = 1{,}264 \times 10^9\,W/(mK)$.

Com $\Delta t = 0{,}01$ s, entrei em loop, calculando a norma do vetor $T^n - T^{n-1}$ até que fosse inferior a 0,001.

$T^n := A \cdot T^{n-1} + B$; $J(n) = \|T^n - T^{n-1}\| \leq 0{,}001$. O loop executou 22551 vezes e o vetor encontrado foi:

$T_5 = 357{,}9608$; $T_6 = 357{,}7171$; $T_7 = 122{,}5382$; $T_8 = 293{,}4656$; $T_9 = 358{,}5942$; $T_{10} = 358{,}5184$; $T_{11} = 293{,}1317$ K.

Os resultados parciais (explícito) foram:

$t = 0\,; T_5 = 358\,; T_6 = 358\,; T_7 = 358\,; T_8 = 358\,; T_9 = 358\,; T_{10} = 358\,; T_{11} = 358$

$t = 8\,; T_5 = 357.9986\,; T_6 = 357.9978\,; T_7 = 326.4824\,; T_8 = 347.901\,; T_9 = 358.0211\,; T_{10} = 358.0207\,; T_{11} = 343.4791$

$t = 16\,; T_5 = 357.9972\,; T_6 = 357.9943\,; T_7 = 299.1036\,; T_8 = 339.3692\,; T_9 = 358.0422\,; T_{10} = 358.0408\,; T_{11} = 332.2068$

$t = 24\,; T_5 = 357.9958\,; T_6 = 357.9895\,; T_7 = 275.3204\,; T_8 = 332.1613\,; T_9 = 358.0633\,; T_{10} = 358.0605\,; T_{11} = 323.4563$

$t = 32\,; T_5 = 357.9945\,; T_6 = 357.9836\,; T_7 = 254.6604\,; T_8 = 326.0718\,; T_9 = 358.0844\,; T_{10} = 358.0797\,; T_{11} = 316.6633$

$t = 40\,; T_5 = 357.9931\,; T_6 = 357.9768\,; T_7 = 236.7135\,; T_8 = 320.927\,; T_9 = 358.1055\,; T_{10} = 358.0987\,; T_{11} = 311.3898$

$t = 48\,; T_5 = 357.9917\,; T_6 = 357.9692\,; T_7 = 221.1235\,; T_8 = 316.5805\,; T_9 = 358.1266\,; T_{10} = 358.1175\,; T_{11} = 307.2959$

$t = 56\,; T_5 = 357.9903\,; T_6 = 357.961\,; T_7 = 207.5808\,; T_8 = 312.9083\,; T_9 = 358.1477\,; T_{10} = 358.136\,; T_{11} = 304.1177$

$t = 64\,; T_5 = 357.9889\,; T_6 = 357.9521\,; T_7 = 195.8166\,; T_8 = 309.8058\,; T_9 = 358.1687\,; T_{10} = 358.1545\,; T_{11} = 301.6503$

$t = 72\,; T_5 = 357.9875\,; T_6 = 357.9427\,; T_7 = 185.5974\,; T_8 = 307.1845\,; T_9 = 358.1898\,; T_{10} = 358.1728\,; T_{11} = 299.7347$

$t = 80\,; T_5 = 357.9861\,; T_6 = 357.9328\,; T_7 = 176.7202\,; T_8 = 304.9699\,; T_9 = 358.2109\,; T_{10} = 358.191\,; T_{11} = 298.2474$

$t = 88\,; T_5 = 357.9848\,; T_6 = 357.9225\,; T_7 = 169.0089\,; T_8 = 303.0987\,; T_9 = 358.232\,; T_{10} = 358.2092\,; T_{11} = 297.0927$

$t = 96\,; T_5 = 357.9834\,; T_6 = 357.9119\,; T_7 = 162.3102\,; T_8 = 301.5178\,; T_9 = 358.2531\,; T_{10} = 358.2274\,; T_{11} = 296.1961$

$t = 104\,; T_5 = 357.982\,; T_6 = 357.901\,; T_7 = 156.4912\,; T_8 = 300.182\,; T_9 = 358.2742\,; T_{10} = 358.2454\,; T_{11} = 295.4999$

$t = 112\,; T_5 = 357.9806\,; T_6 = 357.8899\,; T_7 = 151.4365\,; T_8 = 299.0534\,; T_9 = 358.2952\,; T_{10} = 358.2635\,; T_{11} = 294.9594$

$t = 120\,; T_5 = 357.9792\,; T_6 = 357.8785\,; T_7 = 147.0456\,; T_8 = 298.0998\,; T_9 = 358.3163\,; T_{10} = 358.2815\,; T_{11} = 294.5396$

$t = 128\,; T_5 = 357.9778\,; T_6 = 357.8669\,; T_7 = 143.2313\,; T_8 = 297.294\,; T_9 = 358.3374\,; T_{10} = 358.2995\,; T_{11} = 294.2137$

$t = 136\,; T_5 = 357.9764\,; T_6 = 357.8551\,; T_7 = 139.9179\,; T_8 = 296.6132\,; T_9 = 358.3585\,; T_{10} = 358.3175\,; T_{11} = 293.9605$

$t = 144\,; T_5 = 357.975\,; T_6 = 357.8432\,; T_7 = 137.0397\,; T_8 = 296.0379\,; T_9 = 358.3796\,; T_{10} = 358.3355\,; T_{11} = 293.7639$

$t = 152\,; T_5 = 357.9736\,; T_6 = 357.8312\,; T_7 = 134.5395\,; T_8 = 295.5518\,; T_9 = 358.4006\,; T_{10} = 358.3535\,; T_{11} = 293.6112$

$t = 160\,; T_5 = 357.9723\,; T_6 = 357.8191\,; T_7 = 132.3676\,; T_8 = 295.1411\,; T_9 = 358.4217\,; T_{10} = 358.3715\,; T_{11} = 293.4926$

$t = 168\,; T_5 = 357.9709\,; T_6 = 357.8068\,; T_7 = 130.481\,; T_8 = 294.794\,; T_9 = 358.4428\,; T_{10} = 358.3894\,; T_{11} = 293.4004$

$t = 176\,; T_5 = 357.9695\,; T_6 = 357.7945\,; T_7 = 128.8421\,; T_8 = 294.5007\,; T_9 = 358.4639\,; T_{10} = 358.4074\,; T_{11} = 293.3288$

$t = 184\,; T_5 = 357.9681\,; T_6 = 357.7821\,; T_7 = 127.4185\,; T_8 = 294.2529\,; T_9 = 358.4849\,; T_{10} = 358.4253\,; T_{11} = 293.2732$

$t = 192\,; T_5 = 357.9667\,; T_6 = 357.7697\,; T_7 = 126.1818\,; T_8 = 294.0435\,; T_9 = 358.506\,; T_{10} = 358.4433\,; T_{11} = 293.23$

$t = 200\,; T_5 = 357.9653\,; T_6 = 357.7572\,; T_7 = 125.1075\,; T_8 = 293.8665\,; T_9 = 358.5271\,; T_{10} = 358.4612\,; T_{11} = 293.1964$

$t = 208\,; T_5 = 357.9639\,; T_6 = 357.7447\,; T_7 = 124.1743\,; T_8 = 293.7169\,; T_9 = 358.5481\,; T_{10} = 358.4791\,; T_{11} = 293.1703$

$t = 216\,; T_5 = 357.9625\,; T_6 = 357.7321\,; T_7 = 123.3637\,; T_8 = 293.5905\,; T_9 = 358.5692\,; T_{10} = 358.4971\,; T_{11} = 293.15$

$t = 224\,; T_5 = 357.9611\,; T_6 = 357.7195\,; T_7 = 122.6595\,; T_8 = 293.4837\,; T_9 = 358.5903\,; T_{10} = 358.515\,; T_{11} = 293.1342$

Lista 7 — Exercício 4.a.2

Determine os perfis de temperatura da placa (pontos 5 a 11) ao longo do tempo até que a placa atinja o equilíbrio (regime permanente) usando o método implícito.

Simplificamos $T_p^n = T_p$ (variáveis), as quais vamos determinar em função de T_p^{n-1} (parâmetros).

Malha 5: $\dfrac{\alpha\Delta y}{\Delta x}(T_6 - T_5) + \dfrac{\alpha \cdot 0.5\Delta x}{\Delta y}(T_9 - T_5) + \dfrac{\alpha \cdot 0.5\Delta x}{\Delta y}(T_1 - T_5) = \dfrac{0.5\Delta x\Delta y}{\Delta t}(T_5 - T_5^{n-1})$.

Malha 6: $\dfrac{\alpha\Delta y}{\Delta x}(T_5 - T_6) + \dfrac{\alpha\Delta y}{\Delta x}(T_7 - T_6) + \dfrac{\alpha\Delta x}{\Delta y}(T_{10} - T_6) + \dfrac{\alpha\Delta x}{\Delta y}(T_2 - T_6) = \dfrac{\Delta x\Delta y}{\Delta t}(T_6 - T_6^{n-1})$.

Malha 7: $\dfrac{\alpha\Delta y}{\Delta x}(T_6 - T_7) + \dfrac{\alpha \cdot 0.5\Delta y}{\Delta x}(T_8 - T_7) + \dfrac{h}{\alpha k} \cdot 0.5\Delta y(T_\infty - T_7) + \dfrac{\alpha \cdot 0.5\Delta x}{\Delta y}(T_{11} - T_7) + \dfrac{h}{\alpha k} \cdot 0.5\Delta x(T_\infty - T_7) + \dfrac{\alpha\Delta x}{\Delta y}(T_3 - T_7) = \dfrac{0.75\Delta x\Delta y}{\Delta t}(T_7 - T_7^{n-1})$.

Malha 8: $\dfrac{\alpha \cdot 0.5\Delta y}{\Delta x}(T_7 - T_8) + \dfrac{h}{\alpha k} \cdot 0.5\Delta x(T_\infty - T_8) + \dfrac{\alpha \cdot 0.5\Delta x}{\Delta y}(T_4 - T_8) = \dfrac{0.25\Delta x\Delta y}{\Delta t}(T_8 - T_8^{n-1})$.

Malha 9: $\dfrac{\alpha \cdot 0.5\Delta y}{\Delta x}(T_{10} - T_9) + \dfrac{q''}{\alpha k} \cdot 0.25\Delta x\Delta y + \dfrac{\alpha \cdot 0.5\Delta x}{\Delta y}(T_5 - T_9) = \dfrac{0.25\Delta x\Delta y}{\Delta t}(T_9 - T_9^{n-1})$.

Malha 10: $\dfrac{\alpha \cdot 0.5\Delta y}{\Delta x}(T_9 - T_{10}) + \dfrac{\alpha \cdot 0.5\Delta y}{\Delta x}(T_{11} - T_{10}) + \dfrac{q''}{\alpha k} \cdot 0.5\Delta x\Delta y + \dfrac{\alpha\Delta x}{\Delta y}(T_6 - T_{10}) = \dfrac{0.5\Delta x\Delta y}{\Delta t}(T_{10} - T_{10}^{n-1})$.

Malha 11: $\dfrac{\alpha \cdot 0.5\Delta y}{\Delta x}(T_{10} - T_{11}) + \dfrac{h}{\alpha k} \cdot 0.5\Delta y(T_\infty - T_{11}) + \dfrac{q''}{\alpha k} \cdot 0.25\Delta x\Delta y + \dfrac{\alpha \cdot 0.5\Delta x}{\Delta y}(T_7 - T_{11}) = \dfrac{0.25\Delta x\Delta y}{\Delta t}(T_{11} - T_{11}^{n-1})$.

Novamente, com $\Delta t = 0{,}01$ s, entrei em loop, calculando a norma do vetor $T^n - T^{n-1}$ até que fosse inferior a 0,001.

$T^n := A^{-1} \cdot B$; $J(n) = \|T^n - T^{n-1}\| \leq 0{,}001$. O loop executou 22554 vezes e o vetor encontrado foi:

$T_5 = 357{,}9608$; $T_6 = 357{,}7170$; $T_7 = 122{,}5390$; $T_8 = 293{,}4658$; $T_9 = 358{,}5943$; $T_{10} = 358{,}5184$; $T_{11} = 293{,}1317$ K.

Os resultados parciais (implícito) foram:

$t = 0\,; T_5 = 358\,; T_6 = 358\,; T_7 = 358\,; T_8 = 358\,; T_9 = 358\,; T_{10} = 358\,; T_{11} = 358$
$t = 8\,; T_5 = 357.9986\,; T_6 = 357.9978\,; T_7 = 326.4875\,; T_8 = 347.903\,; T_9 = 358.0211\,; T_{10} = 358.0207\,; T_{11} = 343.4831$
$t = 16\,; T_5 = 357.9972\,; T_6 = 357.9943\,; T_7 = 299.1126\,; T_8 = 339.3725\,; T_9 = 358.0422\,; T_{10} = 358.0408\,; T_{11} = 332.2131$
$t = 24\,; T_5 = 357.9958\,; T_6 = 357.9895\,; T_7 = 275.332\,; T_8 = 332.1655\,; T_9 = 358.0633\,; T_{10} = 358.0605\,; T_{11} = 323.4636$
$t = 32\,; T_5 = 357.9945\,; T_6 = 357.9836\,; T_7 = 254.6739\,; T_8 = 326.0765\,; T_9 = 358.0844\,; T_{10} = 358.0797\,; T_{11} = 316.6708$
$t = 40\,; T_5 = 357.9931\,; T_6 = 357.9768\,; T_7 = 236.7282\,; T_8 = 320.932\,; T_9 = 358.1055\,; T_{10} = 358.0987\,; T_{11} = 311.3972$
$t = 48\,; T_5 = 357.9917\,; T_6 = 357.9692\,; T_7 = 221.1388\,; T_8 = 316.5856\,; T_9 = 358.1266\,; T_{10} = 358.1175\,; T_{11} = 307.3028$
$t = 56\,; T_5 = 357.9903\,; T_6 = 357.961\,; T_7 = 207.5964\,; T_8 = 312.9133\,; T_9 = 358.1477\,; T_{10} = 358.136\,; T_{11} = 304.1239$
$t = 64\,; T_5 = 357.9889\,; T_6 = 357.9521\,; T_7 = 195.8321\,; T_8 = 309.8106\,; T_9 = 358.1687\,; T_{10} = 358.1545\,; T_{11} = 301.6558$
$t = 72\,; T_5 = 357.9875\,; T_6 = 357.9427\,; T_7 = 185.6125\,; T_8 = 307.1891\,; T_9 = 358.1898\,; T_{10} = 358.1728\,; T_{11} = 299.7395$
$t = 80\,; T_5 = 357.9861\,; T_6 = 357.9328\,; T_7 = 176.7348\,; T_8 = 304.9742\,; T_9 = 358.2109\,; T_{10} = 358.191\,; T_{11} = 298.2515$
$t = 88\,; T_5 = 357.9848\,; T_6 = 357.9225\,; T_7 = 169.0228\,; T_8 = 303.1027\,; T_9 = 358.232\,; T_{10} = 358.2092\,; T_{11} = 297.0962$
$t = 96\,; T_5 = 357.9834\,; T_6 = 357.9119\,; T_7 = 162.3234\,; T_8 = 301.5214\,; T_9 = 358.2531\,; T_{10} = 358.2274\,; T_{11} = 296.1991$
$t = 104\,; T_5 = 357.982\,; T_6 = 357.901\,; T_7 = 156.5036\,; T_8 = 300.1854\,; T_9 = 358.2742\,; T_{10} = 358.2454\,; T_{11} = 295.5025$
$t = 112\,; T_5 = 357.9806\,; T_6 = 357.8899\,; T_7 = 151.4481\,; T_8 = 299.0564\,; T_9 = 358.2952\,; T_{10} = 358.2635\,; T_{11} = 294.9615$
$t = 120\,; T_5 = 357.9792\,; T_6 = 357.8785\,; T_7 = 147.0563\,; T_8 = 298.1025\,; T_9 = 358.3163\,; T_{10} = 358.2815\,; T_{11} = 294.5414$
$t = 128\,; T_5 = 357.9778\,; T_6 = 357.8669\,; T_7 = 143.2413\,; T_8 = 297.2965\,; T_9 = 358.3374\,; T_{10} = 358.2995\,; T_{11} = 294.2151$
$t = 136\,; T_5 = 357.9764\,; T_6 = 357.8551\,; T_7 = 139.9272\,; T_8 = 296.6154\,; T_9 = 358.3585\,; T_{10} = 358.3175\,; T_{11} = 293.9617$
$t = 144\,; T_5 = 357.975\,; T_6 = 357.8432\,; T_7 = 137.0482\,; T_8 = 296.0399\,; T_9 = 358.3796\,; T_{10} = 358.3355\,; T_{11} = 293.7649$
$t = 152\,; T_5 = 357.9736\,; T_6 = 357.8312\,; T_7 = 134.5473\,; T_8 = 295.5536\,; T_9 = 358.4006\,; T_{10} = 358.3535\,; T_{11} = 293.612$
$t = 160\,; T_5 = 357.9723\,; T_6 = 357.8191\,; T_7 = 132.3747\,; T_8 = 295.1427\,; T_9 = 358.4217\,; T_{10} = 358.3715\,; T_{11} = 293.4932$
$t = 168\,; T_5 = 357.9709\,; T_6 = 357.8068\,; T_7 = 130.4875\,; T_8 = 294.7954\,; T_9 = 358.4428\,; T_{10} = 358.3894\,; T_{11} = 293.401$
$t = 176\,; T_5 = 357.9695\,; T_6 = 357.7945\,; T_7 = 128.848\,; T_8 = 294.502\,; T_9 = 358.4639\,; T_{10} = 358.4074\,; T_{11} = 293.3293$
$t = 184\,; T_5 = 357.9681\,; T_6 = 357.7821\,; T_7 = 127.4238\,; T_8 = 294.254\,; T_9 = 358.4849\,; T_{10} = 358.4253\,; T_{11} = 293.2736$
$t = 192\,; T_5 = 357.9667\,; T_6 = 357.7697\,; T_7 = 126.1866\,; T_8 = 294.0444\,; T_9 = 358.506\,; T_{10} = 358.4433\,; T_{11} = 293.2303$
$t = 200\,; T_5 = 357.9653\,; T_6 = 357.7572\,; T_7 = 125.1119\,; T_8 = 293.8673\,; T_9 = 358.5271\,; T_{10} = 358.4612\,; T_{11} = 293.1966$
$t = 208\,; T_5 = 357.9639\,; T_6 = 357.7447\,; T_7 = 124.1783\,; T_8 = 293.7177\,; T_9 = 358.5481\,; T_{10} = 358.4791\,; T_{11} = 293.1705$
$t = 216\,; T_5 = 357.9625\,; T_6 = 357.7321\,; T_7 = 123.3673\,; T_8 = 293.5912\,; T_9 = 358.5692\,; T_{10} = 358.4971\,; T_{11} = 293.1501$
$t = 224\,; T_5 = 357.9611\,; T_6 = 357.7195\,; T_7 = 122.6628\,; T_8 = 293.4843\,; T_9 = 358.5903\,; T_{10} = 358.515\,; T_{11} = 293.1343$

Lista 7 — Exercício 4.b

Compare os resultados obtidos.

A diferença entre $\Delta t = 225{,}51$ e $225{,}54$ s é de $0{,}01330\,\%$. Repare que o quinto algarismo é duvidoso.

Explícito: $T_5 = 357{,}9608$; $T_6 = 357{,}7171$; $T_7 = 122{,}5382$; $T_8 = 293{,}4656$; $T_9 = 358{,}5942$; $T_{10} = 358{,}5184$; $T_{11} = 293{,}1317$ K.

Implícito: $T_5 = 357{,}9608$; $T_6 = 357{,}7170$; $T_7 = 122{,}5390$; $T_8 = 293{,}4658$; $T_9 = 358{,}5943$; $T_{10} = 358{,}5184$; $T_{11} = 293{,}1317$ K.

A maior diferença foi em T_7 de $0{,}0006310\,\%$. As temperaturas são quase idênticas.

Lista 7 — Exercício 4.c

Determine o tempo necessário para a placa entrar em equilíbrio.

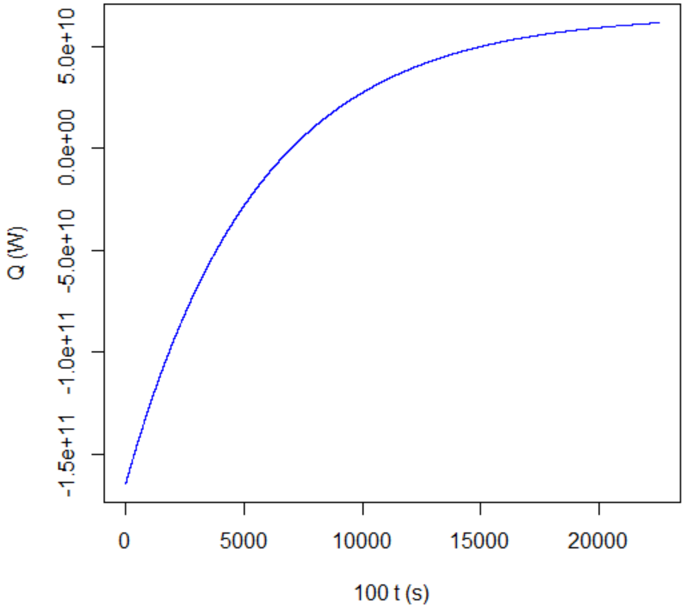
$$\Delta t(\text{equilíbrio}) = \frac{225{,}51 + 225{,}54}{2} = 225{,}53 \text{ s.}$$

Lista 7 — Exercício 4.d

Determine o calor dissipado pela fronteira norte e pelo canal.

$$
\begin{aligned}
Q_1 &= \frac{k \cdot 0.5\Delta x}{\Delta y}(T_1 - T_5) \\
Q_2 &= \frac{k\Delta x}{\Delta y}(T_2 - T_6) \\
Q_3 &= \frac{k\Delta x}{\Delta y}(T_3 - T_7) \\
Q_4 &= \frac{k \cdot 0.5\Delta x}{\Delta y}(T_4 - T_8) \\
Q_5 &= q'' \cdot 2\Delta x\Delta y \\
Q_6 &= h \cdot 0.5\Delta y(T_\infty - T_7) \\
Q_7 &= h \cdot 0.5\Delta x(T_\infty - T_7) \\
Q_8 &= h \cdot 0.5\Delta x(T_\infty - T_8) \\
Q_9 &= h \cdot 0.5\Delta y(T_\infty - T_{11}) \\
\sum_{i=1}^{9}\sum_{n=1}^{22551} Q_i(n) &= \frac{2{,}194501 + 2{,}194254}{2} \times 10^{14} = 2{,}1944 \times 10^{14}\, W.
\end{aligned}
$$

Para cada instante, a soma parcial encontrada foi:



Anexos os códigos-fonte em R.

```
# explícito
a <- matrix(0, 7, 7)
b <- matrix(0, 7)
qq <- matrix(0, 22555)
xx <- matrix(0, 22555)
t <- 85 + 273
ti <- 20 + 273
tinf <- 20 + 273
c <- c(t,t,t,t,t,t,t)
alfa <- 1.5e-6
rho <- 2.4
cc <- 790 # 0.79 J/g /deg C
k <- rho * cc / alfa
q <- 5
h <- 15
x <- 0.5
y <- 0.75
```

```
t <- 0.01 # ???
c <- t(c)
c <- t(c)
soma <- 0
for (N in 1:1000000) {
if (N %% 800 == 1) {
print(paste("t &=", (N - 1)/100
, "\\,;\\,T_5 = ", round(c[1], 4)
, "\\,;\\,T_6 =", round(c[2], 4)
, "\\,;\\,T_7 =", round(c[3], 4)
, "\\,;\\,T_8 =", round(c[4], 4)
, "\\,;\\,T_9 =", round(c[5], 4)
, "\\,;\\,T_{10} =", round(c[6], 4)
, "\\,;\\,T_{11} =", round(c[7], 4)
, "\\\\"))
}

xx[N] <- N
qq[N] <- qq[N] + k * 0.5 * x / y * (ti - c[1])
qq[N] <- qq[N] + k * x / y * (ti - c[2])
qq[N] <- qq[N] + k * x / y * (ti - c[3])
qq[N] <- qq[N] + k * 0.5 * x / y * (ti - c[4])
qq[N] <- qq[N] + q * 2 * x * y
qq[N] <- qq[N] + h * 0.5 * y * (tinf - c[3])
qq[N] <- qq[N] + h * 0.5 * x * (tinf - c[3])
qq[N] <- qq[N] + h * 0.5 * x * (tinf - c[4])
qq[N] <- qq[N] + h * 0.5 * y * (tinf - c[7])
soma <- soma + qq[N]

a[1,1] <- (- alfa * y / x - alfa * x / y) * t/0.5/x/y + 1 # T5
# t <= 57692.32
a[1,2] <- alfa * y / x * t/0.5/x/y # T6
a[1,3] <- 0 # T7
a[1,4] <- 0 # T8
a[1,5] <- alfa * 0.5 * x / y * t/0.5/x/y # T9
a[1,6] <- 0 # T10
a[1,7] <- 0 # T11
b[1] <- alfa * 0.5 * x / y * ti * t/0.5/x/y

a[2,1] <- alfa * y / x * t/x/y # T5
a[2,2] <- (- 2 * alfa * y / x - 2 * alfa * x / y) * t/x/y + 1 # T6
a[2,3] <- alfa * y / x * t/x/y # T7
a[2,4] <- 0 # T8
```

```
a[2,5] <- 0 # T9
a[2,6] <- alfa * x / y * t/x/y # T10
a[2,7] <- 0 # T11
b[2] <- alfa * x / y * ti * t/x/y

a[3,1] <- 0 # T5
a[3,2] <- alfa * y / x * t/0.75/x/y # T6
a[3,3] <- (- 1.5 * alfa * y / x - h/alfa/k * 0.5 * y - alfa * 1.5 * x / y - h/alfa/k * 0.5 * x) * t/0.75/x/y + 1 # T7
a[3,4] <- alfa * 0.5 * y / x * t/0.75/x/y # T8
a[3,5] <- 0 # T9
a[3,6] <- 0 # T10
a[3,7] <- alfa * 0.5 * x / y * t/0.75/x/y # T11
b[3] <- (h/alfa/k * 0.5 * y + h/alfa/k * 0.5 * x * tinf + alfa * x / y * ti) * t/0.75/x/y

a[4,1] <- 0 # T5
a[4,2] <- 0 # T6
a[4,3] <- alfa * 0.5 * y / x * t/0.25/x/y # T7
a[4,4] <- (- alfa * 0.5 * y / x - h/alfa/k * 0.5 * x - alfa * 0.5 * x / y) * t/0.25/x/y + 1 # T8
a[4,5] <- 0 # T9
a[4,6] <- 0 # T10
a[4,7] <- 0 # T11
b[4] <- (h/alfa/k * 0.5 * x * tinf + alfa * 0.5 * x / y * ti) * t/0.25/x/y

a[5,1] <- alfa * 0.5 * x / y * t/0.25/x/y # T5
a[5,2] <- 0 # T6
a[5,3] <- 0 # T7
a[5,4] <- 0 # T8
a[5,5] <- (- alfa * 0.5 * y / x - alfa * 0.5 * x / y) * t/0.25/x/y + 1 # T9
a[5,6] <- alfa * 0.5 * y / x * t/0.25/x/y # T10
a[5,7] <- 0 # T11
b[5] <- q/alfa/k * 0.25 * x * y * t/0.25/x/y

a[6,1] <- 0 # T5
a[6,2] <- alfa * x / y * t/0.5/x/y # T6
a[6,3] <- 0 # T7
a[6,4] <- 0 # T8
a[6,5] <- alfa * 0.5 * y / x * t/0.5/x/y # T9
a[6,6] <- (- alfa * y / x - alfa * x / y) * t/0.5/x/y + 1 # T10
a[6,7] <- alfa * 0.5 * y / x * t/0.5/x/y # T11
b[6] <- q/alfa/k * 0.5 * x * y * t/0.5/x/y

a[7,1] <- 0 # T5
a[7,2] <- 0 # T6
```

```
a[7,3] <- alfa * 0.5 * x / y * t/0.25/x/y # T7
a[7,4] <- 0 # T8
a[7,5] <- 0 # T9
a[7,6] <- alfa * 0.5 * y / x * t/0.25/x/y # T10
a[7,7] <- (- alfa * 0.5 * y / x - h/alfa/k * 0.5 * y - alfa * 0.5 * x / y) * t/0.25/x/y + 1 # T11
b[7] <- (h/alfa/k * 0.5 * y * tinf + q/alfa/k * 0.25 * x * y) * t/0.25/x/y

c1 <- a %*% c + b

if (norm(c - c1) < 1e-3)
  break
c <- c1
}
t(c - 273)
N
qq1 <- qq

#implícito
a <- matrix(0, 7, 7)
b <- matrix(0, 7)
qq <- matrix(0, 22555)
xx <- matrix(0, 22555)
t <- 85 + 273
ti <- 20 + 273
tinf <- 20 + 273
c <- c(t,t,t,t,t,t,t)
alfa <- 1.5e-6
rho <- 2.4
cc <- 790 # 0.79 J/g /deg C
k <- rho * cc / alfa
q <- 5
h <- 15
x <- 0.5
y <- 0.75
t <- 0.01 # ???
c <- t(c)
c <- t(c)
soma2 <- 0
for (N in 1:10000000) {
if (N %% 800 == 1) {
print(paste("t &=", (N - 1)/100
, "\\,;\\,T_5 = ", round(c[1], 4)
, "\\,;\\,T_6 =", round(c[2], 4)
```

```
, "\\,;\\,T_7 =", round(c[3], 4)
, "\\,;\\,T_8 =", round(c[4], 4)
, "\\,;\\,T_9 =", round(c[5], 4)
, "\\,;\\,T_{10} =", round(c[6], 4)
, "\\,;\\,T_{11} =", round(c[7], 4)
, "\\\\"))
}

xx[N] <- N
qq[N] <- qq[N] + k * 0.5 * x / y * (ti - c[1])
qq[N] <- qq[N] + k * x / y * (ti - c[2])
qq[N] <- qq[N] + k * x / y * (ti - c[3])
qq[N] <- qq[N] + k * 0.5 * x / y * (ti - c[4])
qq[N] <- qq[N] + q * 2 * x * y
qq[N] <- qq[N] + h * 0.5 * y * (tinf - c[3])
qq[N] <- qq[N] + h * 0.5 * x * (tinf - c[3])
qq[N] <- qq[N] + h * 0.5 * x * (tinf - c[4])
qq[N] <- qq[N] + h * 0.5 * y * (tinf - c[7])
soma2 <- soma2 + qq[N]

a[1,1] <- - alfa * y / x - alfa * x / y - 0.5 * x * y / t # T5
a[1,2] <- alfa * y / x # T6
a[1,3] <- 0 # T7
a[1,4] <- 0 # T8
a[1,5] <- alfa * 0.5 * x / y # T9
a[1,6] <- 0 # T10
a[1,7] <- 0 # T11
b[1] <- - alfa * 0.5 * x / y * ti - 0.5 * x * y / t * c[1]

a[2,1] <- alfa * y / x # T5
a[2,2] <- - 2 * alfa * y / x - 2 * alfa * x / y - x * y / t # T6
a[2,3] <- alfa * y / x # T7
a[2,4] <- 0 # T8
a[2,5] <- 0 # T9
a[2,6] <- alfa * x / y # T10
a[2,7] <- 0 # T11
b[2] <- - alfa * x / y * ti - x * y / t * c[2]

a[3,1] <- 0 # T5
a[3,2] <- alfa * y / x # T6
a[3,3] <- - 1.5 * alfa * y / x - h/alfa/k * 0.5 * y - alfa * 1.5 * x / y - h/alfa/k * 0.5 * x - 0.75 * x * y / t # T7
a[3,4] <- alfa * 0.5 * y / x # T8
a[3,5] <- 0 # T9
```

```
a[3,6] <- 0 # T10
a[3,7] <- alfa * 0.5 * x / y # T11
b[3] <- - h/alfa/k * 0.5 * y - h/alfa/k * 0.5 * x * tinf - alfa * x / y * ti - 0.75 * x * y / t * c[3]

a[4,1] <- 0 # T5
a[4,2] <- 0 # T6
a[4,3] <- alfa * 0.5 * y / x # T7
a[4,4] <- - alfa * 0.5 * y / x - h/alfa/k * 0.5 * x - alfa * 0.5 * x / y - 0.25 * x * y / t # T8
a[4,5] <- 0 # T9
a[4,6] <- 0 # T10
a[4,7] <- 0 # T11
b[4] <- - h/alfa/k * 0.5 * x * tinf - alfa * 0.5 * x / y * ti - 0.25 * x * y / t * c[4]

a[5,1] <- alfa * 0.5 * x / y # T5
a[5,2] <- 0 # T6
a[5,3] <- 0 # T7
a[5,4] <- 0 # T8
a[5,5] <- - alfa * 0.5 * y / x - alfa * 0.5 * x / y - 0.25 * x * y / t # T9
a[5,6] <- alfa * 0.5 * y / x # T10
a[5,7] <- 0 # T11
b[5] <- - q/alfa/k * 0.25 * x * y - 0.25 * x * y / t * c[5]

a[6,1] <- 0 # T5
a[6,2] <- alfa * x / y # T6
a[6,3] <- 0 # T7
a[6,4] <- 0 # T8
a[6,5] <- alfa * 0.5 * y / x # T9
a[6,6] <- - alfa * y / x - alfa * x / y - 0.5 * x * y / t # T10
a[6,7] <- alfa * 0.5 * y / x # T11
b[6] <- - q/alfa/k * 0.5 * x * y - 0.5 * x * y / t * c[6]

a[7,1] <- 0 # T5
a[7,2] <- 0 # T6
a[7,3] <- alfa * 0.5 * x / y # T7
a[7,4] <- 0 # T8
a[7,5] <- 0 # T9
a[7,6] <- alfa * 0.5 * y / x # T10
a[7,7] <- - alfa * 0.5 * y / x - h/alfa/k * 0.5 * y - alfa * 0.5 * x / y - 0.25 * x * y / t # T11
b[7] <- - h/alfa/k * 0.5 * y * tinf - q/alfa/k * 0.25 * x * y - 0.25 * x * y / t * c[7]

c2 <- solve(a) %*% b

if (norm(c - c2) < 1e-3)
```

```
  break
c <- c2
}
t(c - 273)
N
t(c1)
t(c2)
soma
qq2 <- qq
Mx1 <- 0
Mx2 <- 22555
My1 <- min(qq[1:N])
My2 <- max(qq[1:N])
dev.off()
plot(xx[1:N],qq[1:N],type = 'l',col='blue',xlim=c(Mx1,Mx2),ylim = c(My1,My2),xlab='100 t (s)',ylab='Q (W)')

max(abs(c1 - c2))/c1[3] * 100
max(abs(c1 - c2))/c2[3] * 100

# explícito
soma
soma2
Mx1 <- 0
Mx2 <- 22555
My1 <- min(qq[1:N])
My2 <- max(qq[1:N])
par(new=T)
plot(xx[1:N],qq[1:N],type = 'l',col='red',xlim=c(Mx1,Mx2),ylim = c(My1,My2),xlab='100 t (s)',ylab='Q (W)')
```

Versão de 16/dezembro/2021[*] por Vinicius Claudino Ferraz.
Matrícula = 2019435823.

[*]Fora da caridade não há salvação.