Exercícios

(1) $$d(n)=\#\{1\le x\le n; x\mid n\}\Rightarrow\sum_{n=1}^{\infty}\frac{d(n)}{n^s}\to\zeta(s)^2$$

(2) $$\phi(n)=\#\{1\le x\le n;(x,n)=1\}\Rightarrow\sum_{n=1}^{\infty}\frac{\phi(n)}{n^s}\to\frac{\zeta(s)}{\zeta(s+1)}$$

(3) $$\ln\Gamma(s)=\left(s-\frac{1}{2}\right)\ln s-s+\frac{1}{2}\ln(2\pi)-\int_0^\infty\frac{B_1(t)}{t+s}dt$$

$$\Gamma(s)=\frac{s^{s-\frac{1}{2}}(2\pi)^{\frac{1}{2}}}{e^s\exp\int_0^\infty\frac{B_1(t)}{t+s}dt}$$

(4) $$\gamma=\frac{1}{2}+\sum_{n=2}^{\infty}\frac{B_n}{n}$$

(5) $$\zeta'(0)=\zeta(0)\ln(2\pi)$$

(6) $$\frac{\Gamma'(t)}{\Gamma(t)}=\frac{1}{t}+\gamma+\sum_{k=1}^{\infty}(-1)^k\zeta(k+1)$$

(7) $$\prod_{k=0}^{n-1}\Gamma\left(s+\frac{k}{n}\right)=n^{\frac{1}{2}-ns}(2\pi)^{\frac{n-1}{2}}\Gamma(ns)$$

Sabendo que

$s=\sigma+i\tau$; $x\in R\Rightarrow x-\lfloor x\rfloor=\langle x\rangle\in[0,1)$; Integrais pela esquerda

$\sigma>1\Rightarrow$ $$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}=\prod_{p\in P}\left(1-\frac{1}{p^s}\right)^{-1}=\prod_{p\in P}\sum_{n=1}^{\infty}\frac{1}{p^{ns}}=\frac{1}{\Gamma(s)}\int_0^\infty\frac{x^{s-1}\,\mathrm{d}x}{e^x-1}$$

$\sigma>0\Rightarrow$ $$\zeta(s)=\frac{1}{1-2^{1-s}}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^s}$$

Pólo único em $s=1$

$$\zeta(s)=\frac{s}{s-1}-s\int_1^\infty\langle t\rangle t^{-s-1}\mathrm{d}t$$

$\sigma>-k\Rightarrow$ $$\zeta(s)=\frac{s}{s-1}+\sum_{n=0}^{k}\frac{B_{n+1}}{n+1}\binom{s+n-1}{n}-\binom{s+k}{k+1}\int_1^\infty B_{k+1}(t)t^{-s-k-1}\mathrm{d}t$$

$\sigma<0\Rightarrow$ $$\zeta(1-s)=(2\pi i)^{-s}\frac{I(s)}{e^{\pi is}-1}$$

$$\zeta(s)=2^s\pi^{s-1}\sin\frac{\pi s}{2}\Gamma(1-s)\zeta(1-s)$$

$s\neq1\Rightarrow$ $$I(s)=\int_{C_\delta}\frac{z^{s-1}}{e^z-1}\mathrm{d}z\Rightarrow\zeta(s)=\frac{1}{e^{2\pi is}-1}\frac{I(s)}{\Gamma(s)}=\frac{e^{-\pi is}}{2\pi i}\Gamma(1-s)I(s)$$

$$\zeta(-n)=\frac{(-1)^n}{n+1}B_{n+1}$$

$$\sum_{n=1}^{k}\frac{1}{n}-\ln k\xrightarrow{k\to\infty}\gamma$$

$$\Gamma(s)=\int_0^\infty e^{-x}x^{s-1}\,\mathrm{d}x$$

$$\Gamma(k)=(k-1)\Gamma(k-1)$$

$$\Gamma(x)=\lim_{n\to\infty}\frac{n^x n!}{\prod_{k=0}^{n}(x+k)}=\frac{e^{-\gamma x}}{x}\prod_{n=1}^{\infty}\frac{e^{\frac{x}{n}}}{1+\frac{x}{n}}$$

$$\Gamma(x)\Gamma(1-x)\sin(\pi x)=\pi$$

$$\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}=\int_0^1 t^{x-1}(1-t)^{y-1}\mathrm{d}t$$

$$2^x\Gamma\left(\frac{x}{2}\right)\Gamma\left(\frac{x+1}{2}\right)=2\sqrt{\pi}\,\Gamma(x)$$

$$b_k:[0,1]\to R$$

$$b_0(t)\equiv 1$$

$$b_k'(t)=kb_{k-1}(t)$$

$$\int_0^1 b_k(t)\,\mathrm{d}t=0$$

$$\sum_{n=0}^{\infty}b_n(t)\frac{y^n}{n!}=\frac{ye^{ty}}{e^y-1}$$

$$\beta_n:R\to R$$

Período $T=1$

$$t\in[0,1]\Rightarrow\beta_n(t)=b_n(t)$$

$$B_n=b_n(0)$$

$$\therefore\frac{y}{e^y-1}=\sum_{0}^{\infty}B_n\frac{y^n}{n!}$$

$$\sum_{n=a\in Z}^{b\in Z}f(n)=\int_a^b f(t)\,\mathrm{d}t+\sum_{k=1}^{N}\frac{(-1)^k}{k!}B_k\left[f^{(k-1)}(b)-f^{(k-1)}(a)\right]+\frac{(-1)^{N+1}}{N!}\int_a^b f^{(N)}(t)\,\mathrm{d}B_{N+1}(t)$$