$$\sqrt{A_{mn}} = X_{pq} : X_{pq}X_{pq} = A_{mn} \Rightarrow q = p = m = n$$
$$\sqrt{A_{nn}} = X_{nn} \Leftrightarrow A = XX$$
$$\sqrt{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} = \begin{bmatrix} w & y \\ x & z \end{bmatrix} \Leftrightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} w & y \\ x & z \end{bmatrix}\begin{bmatrix} w & y \\ x & z \end{bmatrix} = \begin{bmatrix} w^2 + xy & wy + yz \\ wx + xz & xy + z^2 \end{bmatrix}$$
$$\left.\begin{array}{l} y(w+z) = b \\ x(w+z) = c \\ b \neq 0, c \neq 0 \end{array}\right\} \Rightarrow w + z = \frac{b}{y} = \frac{c}{x} \Rightarrow \begin{cases} y = \dfrac{b}{c}x \\ z = \dfrac{c}{x} - w \end{cases}$$
$$\left.\begin{array}{l} w^2 + xy = a \\ xy + z^2 = d \end{array}\right\} \Rightarrow xy = a - w^2 = d - z^2$$
$$\frac{b}{c}x^2 = a - w^2 = d - \frac{c^2}{x^2} + 2\frac{c}{x}w - w^2$$
$$w^2 = a - \frac{b}{c}x^2$$
$$0 = d - \frac{c^2}{x^2} + 2\frac{c}{x}w - a \Rightarrow c^2 + (a-d)x^2 = 2cx \cdot w \Rightarrow c^4 + (a-d)^2 x^4 + 2c^2(a-d)x^2 = 4ac^2x^2 - 4bcx^4$$
$$\left[(a-d)^2 + 4bc\right]x^4 - 2c^2(a+d)x^2 + c^4 = 0$$
$$b \neq -\frac{(a-d)^2}{4c} \Rightarrow \Delta = 4c^4\left[(a+d)^2 - (a-d)^2 - 4bc\right] = 16c^4(ad - bc)$$
$$x = \pm c\sqrt{\frac{a + d \pm 2\sqrt{ad - bc}}{(a-d)^2 + 4bc}}; y = \pm b\sqrt{\frac{a + d \pm 2\sqrt{ad - bc}}{(a-d)^2 + 4bc}}; w = \pm\sqrt{a - bc\frac{a + d \pm 2\sqrt{ad - bc}}{(a-d)^2 + 4bc}}$$
$$z = \frac{c}{x} - w = \pm\left[\sqrt{\frac{(a-d)^2 + 4bc}{a + d \pm 2\sqrt{ad - bc}}} - \sqrt{a - bc\frac{a + d \pm 2\sqrt{ad - bc}}{(a-d)^2 + 4bc}}\right]$$
$$b = -\frac{(a-d)^2}{4c}, d \neq -a \Rightarrow x = \pm\frac{c}{\sqrt{2(a+d)}}; y = \pm\frac{b}{\sqrt{2(a+d)}}; w = \pm\sqrt{a - \frac{bc}{2(a+d)}}$$
$$z = \frac{c}{x} - w = \pm\left[\sqrt{2(a+d)} - \sqrt{a - \frac{bc}{2(a+d)}}\right]$$
$$b = -\frac{a^2}{c}, d = -a \Rightarrow x, \sqrt{\frac{a}{c}\begin{bmatrix} c & -a \\ c^2 & -c \end{bmatrix}} \in \varnothing$$
$$\sqrt{\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}} \in \left\{\pm\frac{\sqrt{6}}{6}\begin{bmatrix} 2 & 2 \\ 7 & 4 \end{bmatrix}, \pm\frac{\sqrt{10}}{10}\begin{bmatrix} 4 & 2 \\ 7 & 6 \end{bmatrix}\right\}$$
$$\frac{1}{6}\begin{bmatrix} 2 & 2 \\ 7 & 4 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 7 & 4 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 18 & 12 \\ 42 & 30 \end{bmatrix} satisfaz$$
$$\frac{1}{10}\begin{bmatrix} 4 & 2 \\ 7 & 6 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 7 & 6 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 30 & 20 \\ 70 & 50 \end{bmatrix} satisfaz$$

$$(a-d)^2 + 4bc = 0 \Leftrightarrow b = -\frac{(a-d)^2}{4c}, c \neq 0 \vee \begin{cases} b = 0 \vee c = 0 \\ a = d \end{cases}$$
$$a + d \pm 2\sqrt{ad - bc} = 0 \Rightarrow 4(ad - bc) = a^2 + 2ad + d^2 \Rightarrow -4bc = (a-d)^2$$

$$b=0$$

$$y(w+z)=0 \Rightarrow y=0 \vee z=-w$$

$$y=0 \Rightarrow \begin{cases} w=\pm\sqrt{a}; z=\pm\sqrt{d} \\ a=d=0 \Rightarrow 0x=c \Rightarrow \begin{cases} c=0 \Rightarrow \forall x, \sqrt{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}} = x\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \\ c \neq 0 \Rightarrow x \in \varnothing \end{cases} \\ (a,d) \neq (0,0) \Rightarrow x=\pm\dfrac{c}{\sqrt{a}+\sqrt{d}} \therefore \sqrt{\begin{bmatrix} a & 0 \\ c & d \end{bmatrix}} = \pm\begin{bmatrix} \sqrt{a} & 0 \\ \dfrac{c}{\sqrt{a}+\sqrt{d}} & \sqrt{d} \end{bmatrix} \end{cases}$$

$$z=-w \Rightarrow \begin{cases} 0x=c \\ xy=a-w^2=d-w^2 \\ 0w=d-a \end{cases}$$

$$c \neq 0 \Rightarrow x \in \varnothing$$

$$d \neq a \Rightarrow w \in \varnothing$$

$$c=0, d=a \Rightarrow y=\frac{a-w^2}{x}, \forall x \neq 0$$

$$c=0, d=a, x=0 \Rightarrow 0y=a-w^2 \Rightarrow \begin{cases} w \neq \pm\sqrt{a} \Rightarrow y \in \varnothing \\ w=\pm\sqrt{a} \Rightarrow \forall y, \sqrt{\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}} = \pm\begin{bmatrix} \sqrt{a} & y \\ 0 & -\sqrt{a} \end{bmatrix} \end{cases}$$

$$c=0$$

$$x(w+z)=0 \Rightarrow x=0 \vee z=-w$$

$$x=0 \Rightarrow \begin{cases} w=\pm\sqrt{a}; z=\pm\sqrt{d} \\ a=d=0 \Rightarrow 0y=b \Rightarrow \begin{cases} b=0 \Rightarrow \forall y, \sqrt{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}} = y\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \\ b \neq 0 \Rightarrow y \in \varnothing \end{cases} \\ (a,d) \neq (0,0) \Rightarrow y=\pm\dfrac{b}{\sqrt{a}+\sqrt{d}} \end{cases}$$

$$z=-w \Rightarrow \begin{cases} 0y=b \\ xy=a-w^2=d-w^2 \\ 0w=d-a \end{cases}$$

$$b \neq 0 \Rightarrow y \in \varnothing$$

$$d \neq a \Rightarrow w \in \varnothing$$

$$b=0, d=a \Rightarrow x=\frac{a-w^2}{y}, \forall y \neq 0$$

$$b=0, d=a, y=0 \Rightarrow 0x=a-w^2 \Rightarrow \begin{cases} w \neq \pm\sqrt{a} \Rightarrow x \in \varnothing \\ w=\pm\sqrt{a} \Rightarrow \forall x, \sqrt{\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}} = \pm\begin{bmatrix} \sqrt{a} & 0 \\ x & -\sqrt{a} \end{bmatrix} \end{cases}$$

3ª ordem

$$\begin{bmatrix} r & s & t \\ u & v & w \\ x & y & z \end{bmatrix}\begin{bmatrix} r & s & t \\ u & v & w \\ x & y & z \end{bmatrix}=\begin{bmatrix} r^2+su+tx\,(\mathbf{VII}) & rs+sv+ty\,(\mathbf{II}) & rt+sw+tz\,(\mathbf{IV}) \\ ru+uv+wx\,(\mathbf{I}) & su+v^2+wy\,(\mathbf{VIII}) & tu+vw+wz\,(\mathbf{VI}) \\ rx+uy+xz\,(\mathbf{III}) & sx+vy+yz\,(\mathbf{V}) & tx+wy+z^2\,(\mathbf{IX}) \end{bmatrix}=\begin{bmatrix} a & b & c \\ f & g & h \\ m & p & q \end{bmatrix}$$

$$(III),(IV)\Rightarrow r+z=\frac{m-uy}{x}=\frac{c-sw}{t}\Rightarrow t=x\frac{c-sw}{m-uy}\,(\mathbf{T_0})$$

$$(I),(II)\Rightarrow r+v=\frac{f-wx}{u}=\frac{b-ty}{s}\Rightarrow s\frac{f-wx}{u}=b-xy\frac{c-sw}{m-uy}\Rightarrow s=u\frac{bm-buy-cxy}{fm-fuy-mwx}\,(\mathbf{S_1})$$

$$(T_0)\Rightarrow t=x\frac{cf-cwx-buw}{fm-fuy-mwx}\,(\mathbf{T_1})$$

$$(V),(VI)\Rightarrow v+z=\frac{p-sx}{y}=\frac{h-tu}{w}\Rightarrow w\left(p-ux\frac{bm-buy-cxy}{fm-fuy-mwx}\right)=y\left(h-ux\frac{cf-cwx-buw}{fm-fuy-mwx}\right)$$

$$pw(fm-fuy-mwx)-uwx(bm-\underline{\underline{buy}}-\underline{\underline{cxy}})=hy(fm-fuy-mwx)-uxy(cf-\underline{\underline{cwx}}-\underline{\underline{buw}})$$

$$(pw-hy)(fm-mwx)=u(fhy^2-cfxy+fpwy+bmwx)\Rightarrow u=m\frac{(pw-hy)(f-wx)}{fhy^2+fpwy+bmwx-cfxy}\,(\mathbf{U_2})$$

$$(S_1)\Rightarrow s=\frac{pw-hy}{fhy^2+fpwy+bmwx-cfxy}\cdot\frac{A}{x(bmw-cfy)}\,(\mathbf{S_2})$$

$$A=(bm-cxy)(fhy^2+fpwy+bmwx-cfxy)-bmy(pw-hy)(f-wx)$$

$$(T_1)\Rightarrow t=\frac{x}{m}\cdot\frac{c(fhy^2+fpwy+bmwx-cfxy)-bmw(pw-hy)}{2fhy^2+bmwx-cfxy}\,(\mathbf{T_2})$$

$$(III)+(I)-(V)\Rightarrow 2r=\frac{m-uy}{x}+\frac{f-wx}{u}-\frac{p-sx}{y}\Rightarrow r=\frac{1}{2}(B+C-D)$$

$$B=\frac{m}{x}\cdot\frac{fhy^2+fpwy+bmwx-cfxy-y(pw-hy)(f-wx)}{fhy^2+fpwy+bmwx-cfxy};C=\frac{fhy^2+fpwy+bmwx-cfxy}{m(pw-hy)}$$

$$D=\frac{p(bmw-cfy)(fhy^2+fpwy+bmwx-cfxy)-(pw-hy)A}{y(bmw-cfy)(fhy^2+fpwy+bmwx-cfxy)}$$

$$-(III)+(I)+(V)\Rightarrow v=\frac{1}{2}(-B+C+D);(III)-(I)+(V)\Rightarrow z=\frac{1}{2}(B-C+D)$$

$$su=\frac{mA}{x}\cdot\frac{f-wx}{bmw-cfy}\cdot\frac{(pw-hy)^2}{(fhy^2+fpwy+bmwx-cfxy)^2}$$

$$\begin{cases}(VII)\Rightarrow r^2+su+tx=a\\(VIII)\Rightarrow su+v^2+wy=g\\(IX)\Rightarrow tx+wy+z^2=q\end{cases}\Rightarrow\begin{cases}(B+C-D)^2+4\frac{mA}{x}\cdot\frac{f-wx}{bmw-cfy}\cdot\frac{(pw-hy)^2}{(fhy^2+fpwy+bmwx-cfxy)^2}\\+4\frac{x^2}{m}\cdot\frac{c(fhy^2+fpwy+bmwx-cfxy)-bmw(pw-hy)}{2fhy^2+bmwx-cfxy}=4a\\4\frac{mA}{x}\cdot\frac{f-wx}{bmw-cfy}\cdot\frac{(pw-hy)^2}{(fhy^2+fpwy+bmwx-cfxy)^2}+(-B+C+D)^2+4wy=4g\\4\frac{x^2}{m}\cdot\frac{c(fhy^2+fpwy+bmwx-cfxy)-bmw(pw-hy)}{2fhy^2+bmwx-cfxy}+4wy+(B-C+D)^2=4q\end{cases}$$

$$E = pw - hy$$
$$D = bmw - cfy$$
$$C = 2fhy^2 + bmwx - cfxy$$
$$B = fhy^2 + fpwy + bmwx - cfxy$$
$$A = (bm - cxy)B - bmy(pw - hy)(f - wx)$$

$$CD^2E^2m^4y^2[B - Ey(f - wx)]^2$$
$$+ B^4CD^2x^2y^2$$
$$+ CE^2m^2x^2(BDp - AE)^2$$
$$+ 2B^2CD^2Em^2xy^2[B - Ey(f - wx)]$$
$$- 2CDE^2m^2xy(BDp - AE)[B - Ey(f - wx)]$$
$$- 2B^2CDEmx^2y(BDp - AE)$$
$$+ 4ACDE^4m^3xy^2(f - wx)$$
$$+ 4B^2D^2E^2mx^4y^2(Bc - bEmw)$$
$$- 4aB^2CD^2E^2m^2x^2y^2$$
$$= 0$$

(segunda)

$$4ADE^4m^3xy^2(f - wx)$$
$$+ D^2E^2m^4y^2[B - yE(f - wx)]^2$$
$$+ B^4D^2x^2y^2$$
$$+ E^2m^2x^2(pDB - EA)^2$$
$$- 2B^2D^2Em^2xy^2[B - yE(f - wx)]$$
$$- 2DE^2m^3xy(pDB - EA)[B - yE(f - wx)]$$
$$+ 2B^2DEmx^2y(pDB - EA)$$
$$+ 4B^2D^2E^2m^2wx^2y^3$$
$$- 4B^2D^2E^2m^2x^2y^2g$$
$$= 0$$

(terceira)

$$4B^2D^2E^2mx^4y^2(cB - bmwE)$$
$$+ 4B^2CD^2E^2m^2wx^2y^3$$
$$+ CD^2E^2m^4y^2[B - yE(f - wx)]^2$$
$$+ B^4CD^2x^2y^2$$
$$+ CE^2m^2x^2(pDB - EA)^2$$
$$- 2B^2CD^2Em^2xy^2[B - yE(f - wx)]$$
$$+ 2CDE^2m^3xy(pDB - EA)[B - yE(f - wx)]$$
$$- 2B^2CDEmx^2y(pDB - EA)$$
$$- 4B^2CD^2E^2m^2qx^2y^2$$
$$= 0$$

$$\frac{\textbf{(primeira)} - \textbf{(terceira)}}{4CDEm^2xy}$$

$$B^2D\ldots[B - E\ldots(f - \ldots)] - E(BD\ldots - AE)[B - E\ldots(f - \ldots)] + AE^3\ldots(f - \ldots) - B^2DE\ldots - B^2DE\ldots{}^2 + B^2DE\ldots = 0$$