$$\frac{76.2}{p(x)=0}$$

$$y''(x) + q(x)y(x) = 0$$

*B* arbitrário

$$z = \frac{1}{\sqrt{B}} \int \sqrt{q} \, dx$$

$$A = \frac{q' + 2pq}{2q^{\frac{3}{2}}} \sqrt{B}$$
$$y''(z) + Ay'(z) + By = 0$$

$$y''(z) + Ay'(z) + By = 0$$

$$\Delta = A^2 - 4B$$

$$y(z) = C_1 \exp\left(\frac{-A + \sqrt{\Delta}}{2}z\right) + C_2 \exp\left(\frac{-A - \sqrt{\Delta}}{2}z\right)$$

**Prontos** 

43.1

$$\beta$$
:  $x = \ln y, y \in (0, \infty)$ 

 $\alpha: y = e^x, x \in \Re \Rightarrow \ln y = x$ : traços iguais

70.2A

$$\alpha = \begin{pmatrix} t \\ t^2 \\ t^3 \end{pmatrix} \Rightarrow \alpha' = \begin{pmatrix} 1 \\ 2t \\ 3t^2 \end{pmatrix} \Rightarrow \alpha'' = \begin{pmatrix} 0 \\ 2 \\ 6t \end{pmatrix} \Rightarrow \alpha' \times \alpha'' = w = 2 \begin{vmatrix} i & j & k \\ 1 & 2t & 3t^2 \\ 0 & 1 & 3t \end{vmatrix} = 2 \begin{pmatrix} 3t^2 \\ -3t \\ 1 \end{pmatrix}$$

$$k_1 = \frac{|w|}{v^3} = \frac{2\sqrt{9t^4 + 9t^2 + 1}}{(9t^4 + 4t^2 + 1)^{\frac{3}{2}}}$$

$$\alpha''' = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} \Rightarrow -\frac{w \cdot \alpha'''}{|w|^2} = \mathbf{V} = -\frac{12}{9t^4 + 9t^2 + 1}$$

função comprimento ñ deu

$$\tau = \frac{\alpha'}{v} \Rightarrow \tau' = \frac{\alpha''v - \alpha'v'}{v^2} = \frac{2}{v} \begin{bmatrix} 0\\1\\3t \end{bmatrix} - \frac{9t^3 + 2t}{9t^4 + 4t^2 + 1} \begin{bmatrix} 1\\2t\\3t^2 \end{bmatrix}$$

$$k_2 = \frac{|\tau'|}{v} = \frac{2}{v^2} \sqrt{\left[ -\frac{9t^3 + 2t}{9t^4 + 4t^2 + 1} \right]^2 + \left[ 1 - \frac{9t^3 + 2t}{9t^4 + 4t^2 + 1} 2t \right]^2 + \left[ 3t - \frac{9t^3 + 2t}{9t^4 + 4t^2 + 1} 3t^2 \right]^2}$$

$$k_2 = \frac{2}{v^4} \sqrt{81t^8 + 117t^6 + 54t^4 + 13t^2 + 1} = k_1 \text{ conferido!}$$

70.2B

$$\alpha = \begin{pmatrix} \cos t \\ \sin t \\ \exp t \end{pmatrix} \Rightarrow \alpha' = \begin{pmatrix} -\sin t \\ \cos t \\ \exp t \end{pmatrix} \Rightarrow \alpha'' = \begin{pmatrix} -\cos t \\ -\sin t \\ \exp t \end{pmatrix} \Rightarrow \alpha' \times \alpha'' = w = \begin{vmatrix} i & j & k \\ -\sin t & \cos t & \exp t \\ -\cos t & -\sin t & \exp t \end{vmatrix} = \begin{pmatrix} e^t \cos t + e^t \sin t \\ -e^t \cos t + e^t \sin t \\ 1 \end{vmatrix}$$

$$k = \frac{|w|}{v^3} = \frac{\sqrt{2e^{2t} + 1}}{(1 + e^{2t})^{\frac{3}{2}}}; \quad \alpha''' = \begin{pmatrix} \sin t \\ -\cos t \\ \exp t \end{pmatrix} \Rightarrow -\frac{w \cdot \alpha'''}{|w|^2} = \mathbf{V} = -\frac{2e^t}{2e^{2t} + 1}$$

70.2C

$$\alpha = \begin{pmatrix} t \\ \cosh t \\ \sinh t \end{pmatrix} \Rightarrow \alpha' = \begin{pmatrix} 1 \\ \sinh t \\ \cosh t \end{pmatrix} \Rightarrow \alpha'' = \begin{pmatrix} 0 \\ \cosh t \\ \sinh t \end{pmatrix} \Rightarrow \alpha' \times \alpha'' = w = \begin{vmatrix} i & j & k \\ 1 & \sinh t & \cosh t \\ 0 & \cosh t & \sinh t \end{vmatrix} = \begin{pmatrix} 1 \\ 0 \\ \cosh t \end{pmatrix}$$

$$k = \frac{|w|}{v^3} = \frac{1}{2\sqrt{2}\sinh^2 t}; \quad \alpha''' = \begin{pmatrix} 0 \\ \sinh t \\ \cosh t \end{pmatrix} \Rightarrow -\frac{w \cdot \alpha'''}{|w|^2} = \mathbf{V} = -\frac{1}{\tanh^2 t}$$

$$\alpha = \begin{pmatrix} 4\cos t \\ 5 - 5\sin t \\ -3\cos t \end{pmatrix} \Rightarrow \alpha' = \begin{pmatrix} -4\sin t \\ -5\cos t \\ 3\sin t \end{pmatrix} \Rightarrow v = 5; \alpha'' = \begin{pmatrix} -4\cos t \\ 5\sin t \\ 3\cos t \end{pmatrix} \Rightarrow \alpha''' = \begin{pmatrix} 4\sin t \\ 5\cos t \\ -3\sin t \end{pmatrix}$$

$$L(t) = 5\int_0^t du = 5t \Rightarrow L^{-1}(s) = \frac{s}{5} \Rightarrow \beta(s) = \alpha\left(\frac{s}{5}\right) = \left(4\cos\frac{s}{5}; 5 - 5\sin\frac{s}{5}; -3\cos\frac{s}{5}\right)$$

$$\tau = \frac{\alpha'}{5} \Rightarrow \tau' = \frac{1}{5}\alpha'' \Rightarrow \mathbf{n} = \frac{\tau'}{|\tau'|} = \frac{\alpha''}{|\alpha''|} = \frac{\alpha''}{5}$$

$$\vec{w} = \alpha' \times \alpha'' = \begin{vmatrix} i & j & k \\ -4\sin t & -5\cos t & 3\sin t \\ -4\cos t & 5\sin t & 3\cos t \end{vmatrix} = \begin{pmatrix} -15 \\ 0 \\ -20 \end{pmatrix}; \mathbf{b} = \frac{w}{|w|} = -\frac{1}{5}\begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}; k = \frac{|w|}{v^3} = \frac{25}{5^3} = \frac{1}{5}; \mathbf{V} = -\frac{w \cdot \alpha'''}{|w|^2} = 0$$

## 70.1B

$$\alpha = \begin{pmatrix} 1 - \cos t \\ \sin t \\ t \end{pmatrix} \Rightarrow \alpha' = \begin{pmatrix} \sin t \\ \cos t \\ 1 \end{pmatrix} \Rightarrow v = \sqrt{2}; \alpha'' = \begin{pmatrix} \cos t \\ -\sin t \\ 0 \end{pmatrix} \Rightarrow \alpha''' = \begin{pmatrix} -\sin t \\ -\cos t \\ 0 \end{pmatrix}$$

$$L(t) = \sqrt{2} \int_0^t du = t\sqrt{2} \Rightarrow L^{-1}(s) = \frac{s}{\sqrt{2}} \Rightarrow \beta(s) = \alpha \left( \frac{s}{\sqrt{2}} \right) = \left( 1 - \cos \frac{s}{\sqrt{2}}; \sin \frac{s}{\sqrt{2}}; \frac{s}{\sqrt{2}} \right)$$

$$\tau = \frac{\alpha'}{\sqrt{2}} \Rightarrow \tau' = \frac{\alpha''}{\sqrt{2}} \Rightarrow \mathbf{n} = \frac{\tau'}{|\tau'|} = \frac{\alpha''}{|\alpha''|} = \alpha''$$

$$\vec{w} = \alpha' \times \alpha'' = \begin{vmatrix} i & j & k \\ \sin t & \cos t & 1 \\ \cos t & -\sin t & 0 \end{vmatrix} = \begin{pmatrix} \sin t \\ \cos t \\ -1 \end{pmatrix}; \mathbf{b} = \frac{w}{|w|} = \frac{w}{\sqrt{2}}; k = \frac{|w|}{v^3} = \frac{1}{2}; \mathbf{V} = -\frac{w \cdot \alpha'''}{|w|^2} = \frac{1}{2}$$

$$\alpha = \begin{pmatrix} e^{t} \\ e^{-t} \\ t\sqrt{2} \end{pmatrix} \Rightarrow \alpha' = \begin{pmatrix} e^{t} \\ -e^{-t} \\ \sqrt{2} \end{pmatrix} \Rightarrow v = \sqrt{e^{2t} + e^{-2t} + 2} = e^{t} + e^{-t}; \alpha'' = \begin{pmatrix} e^{t} \\ e^{-t} \\ 0 \end{pmatrix} \Rightarrow \alpha''' = \begin{pmatrix} e^{t} \\ -e^{-t} \\ 0 \end{pmatrix}$$

$$L(t) = 2 \int_0^t \cosh u \ du = 2 \sinh t \Rightarrow L^{-1}(s) = \arg \sinh \frac{s}{2} = \ln(s + \sqrt{4 + s^2}) - \ln 2$$

$$\beta(s) = \alpha \left( \operatorname{arg sinh} \frac{s}{2} \right) = \left( \frac{s + \sqrt{4 + s^2}}{2}; \frac{2}{s + \sqrt{4 + s^2}}; \sqrt{2} \operatorname{arg sinh} \frac{s}{2} \right)$$

$$\vec{w} = \alpha' \times \alpha'' = \begin{vmatrix} i & j & k \\ e^t & -e^{-t} & \sqrt{2} \\ e^t & e^{-t} & 0 \end{vmatrix} = \begin{pmatrix} -e^{-t}\sqrt{2} \\ e^t\sqrt{2} \\ 2 \end{pmatrix} = \sqrt{2} \begin{pmatrix} -e^{-t} \\ e^t \\ \sqrt{2} \end{pmatrix} \Rightarrow |w| = v\sqrt{2}; \mathbf{b} = \frac{w}{|w|} = \frac{w}{\sqrt{2}(e^t + e^{-t})}$$

$$k = \frac{|w|}{v^3} = \frac{\sqrt{2}}{e^{2t} + e^{-2t} + 2}; \quad \mathbf{V} = -\frac{w \cdot \alpha'''}{|w|^2} = \frac{2\sqrt{2}}{2v^2} = \frac{\sqrt{2}}{e^{2t} + e^{-2t} + 2}; \quad \mathbf{\tau} = \frac{\alpha'}{v} = \frac{\alpha'}{e^t + e^{-t}}; \quad \mathbf{n} = \mathbf{b} \times \mathbf{\tau} = \frac{w}{v\sqrt{2}} \times \frac{\alpha'}{v}$$

$$\mathbf{n} = \frac{1}{v^{2}\sqrt{2}} w \times \alpha' = \frac{1}{e^{2t} + e^{-2t} + 2} \begin{vmatrix} i & j & k \\ -e^{-t} & e^{t} & \sqrt{2} \\ e^{t} & -e^{-t} & \sqrt{2} \end{vmatrix} = \frac{1}{e^{2t} + e^{-2t} + 2} \begin{pmatrix} \sqrt{2}(e^{t} + e^{-t}) \\ \sqrt{2}(e^{t} + e^{-t}) \\ e^{-2t} - e^{2t} \end{pmatrix} = \frac{1}{1 + e^{2t}} \begin{pmatrix} \sqrt{2}e^{t} \\ \sqrt{2}e^{t} \\ \frac{1 - e^{2t}}{1 + e^{2t}} \end{pmatrix}$$

### 36.7 - Ciclóide - 43.3

Centro 
$$O = (x_c, y_c) = (vt, R)$$

$$\alpha(t) = O(t) + R\hat{u}(t), \begin{cases} \hat{u}(0) = (0, -1) \\ \hat{u}(90^{\circ}) = (-1, 0) \\ \hat{u}(180^{\circ}) = (0, 1) \end{cases} \Rightarrow \alpha(t) = \begin{pmatrix} vt - R\sin t \\ R - R\cos t \end{pmatrix}$$
$$\hat{u}(270^{\circ}) = (1, 0)$$

$$\alpha(2\pi) = (2\pi v, 0)$$

Sem deslize  $\Rightarrow$  Comprimento =  $2\pi R = 2\pi v \Rightarrow v = R$ 

$$\therefore \alpha(t) = R \binom{t - \sin t}{1 - \cos t}$$

Singularidades: 
$$\alpha' = R \binom{1 - \cos t}{\sin t} = 0 \Rightarrow \begin{cases} \cos t = 1 \\ \sin t = 0 \end{cases} \Rightarrow t = 2k\pi$$

$$0 < t < 2\pi$$

$$v = R\sqrt{(1-\cos t)^2 + \sin^2 t} = R\sqrt{2-2\cos t}$$

$$L(t) = R\sqrt{2} \int_0^t \sqrt{1 - \cos u} \ du$$

$$u = 2x \Rightarrow du = 2dx$$

$$L(t) = 2R\sqrt{2} \int_0^{\frac{t}{2}} \sqrt{1 - \cos^2 x + \sin^2 x} \, dx = 2R\sqrt{2} \sqrt{2} \int_0^{\frac{t}{2}} |\sin x| \, dx$$

$$t < 2\pi \Rightarrow \frac{t}{2} < \pi \Rightarrow \sin t > 0$$

$$L(t) = 4R \int_0^{\frac{t}{2}} \sin x \, dx = 4R \left( 1 - \cos \frac{t}{2} \right) \Rightarrow L^{-1}(s) = 2\arccos \left( 1 - \frac{s}{4R} \right) = 2\theta(s) = h(s)$$

$$\beta(s) = \alpha \circ h(s) = R \binom{2\theta - \sin(2\theta)}{1 - \cos(2\theta)} = 2R \binom{\theta - \sin\theta\cos\theta}{\sin^2\theta}$$

$$\cos\theta = \frac{4R - s}{4R} \Rightarrow \sin^2\theta + \frac{16R^2 - 16Rs + s^2}{16R^2} = 1 \Rightarrow \sin^2\theta = \frac{16Rs - s^2}{16R^2} \Rightarrow \sin\theta = \frac{\sqrt{16Rs - s^2}}{4R}$$

$$\beta(s) = 32R^{3} \begin{pmatrix} 16R^{2}\theta + (s - 4R)\sqrt{16Rs - s^{2}} \\ 16Rs - s^{2} \end{pmatrix}$$

### 71.9

$$\beta(t) = \alpha(-t) \Rightarrow \begin{cases} k_{\alpha} = k_{\beta} \\ \mathbf{V}_{\alpha} = -\mathbf{V}_{\beta} \end{cases}$$

$$w_1 = \alpha' \times \alpha''; v_1 = \sqrt{(x')^2 + (y')^2 + (z')^2}; k_{\alpha} = \frac{|w_1|}{v_1^3}; \mathbf{V}_{\alpha} = \frac{w \cdot \alpha'''}{|w|^2}$$

$$\boldsymbol{\beta}' = -\boldsymbol{\alpha}'; \boldsymbol{\beta}'' = -\boldsymbol{\alpha}'' \Rightarrow w_2 = \boldsymbol{\beta} \times \boldsymbol{\beta}'' = \boldsymbol{\alpha}' \times \boldsymbol{\alpha}'' = w_1 : \boldsymbol{V}_{\boldsymbol{\beta}} = \frac{w_1 \cdot \left(-\boldsymbol{\alpha}'''\right)}{\left|w_1\right|^2} = -\boldsymbol{V}_{\boldsymbol{\alpha}}$$

$$v_2 = \sqrt{(-x')^2 + (-y')^2 + (-z')^2} = v_1 : k_\beta = k_\alpha$$

$$\alpha = \begin{pmatrix} r\cos t \\ r\sin t \\ f \end{pmatrix} \Rightarrow \alpha' = \begin{pmatrix} -r\sin t \\ r\cos t \\ f' \end{pmatrix} \Rightarrow \alpha'' = \begin{pmatrix} -r\cos t \\ -r\sin t \\ f'' \end{pmatrix}$$

$$v = \sqrt{r^2 + (f')^2} \Rightarrow v' = \frac{f'f''}{v}$$

$$\tau = \frac{\alpha'}{v} \Rightarrow \tau' = \frac{\alpha''v - \alpha'\frac{f'f''}{v}}{v^2} = \frac{\vec{u}}{v^3}; \vec{u} = \begin{pmatrix} -rv^2\cos t + f'f''r\sin t \\ -rv^2\sin t - f'f''r\cos t \\ f''v^2 - (f')^2f'' \end{pmatrix}$$

$$\mathbf{n} = \frac{\tau'}{|\tau|} = \frac{\vec{u}}{|u|} \perp \mathbf{k} \Rightarrow f''v^2 - (f')^2f'' = 0$$

$$f'' = 0 \lor r^2 + (f')^2 = (f')^2 \Rightarrow r = 0, \forall f(t)$$

$$f''' = 0 \Rightarrow f' = A \Rightarrow f(t) = At + B; A, B \in \Re$$

$$\mathbf{f}'' = \frac{1}{|\tau'|} = \frac{1}{|u|} + \mathbf{K} \Rightarrow f \quad \forall \quad -(f) \quad f = 0$$
$$f'' = 0 \lor r^2 + (f')^2 = (f')^2 \Rightarrow r = 0, \forall f(t)$$

 $\forall t \in I, \alpha' \times \alpha' = \mathbf{0} \Rightarrow \alpha = \vec{P} + t\vec{u}$ 

$$\begin{vmatrix} i & j & k \\ x' & y' & z' \\ x'' & y'' & z'' \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} y' & z' \\ y'' & z'' \end{vmatrix} = \begin{vmatrix} x' & z' \\ x'' & z'' \end{vmatrix} = \begin{vmatrix} x' & y' \\ x'' & y'' \end{vmatrix} = 0 \Rightarrow \text{"Alguém" \'e zero ou}$$

$$\begin{cases} y'' = x'' \frac{y'}{x'} \\ z'' = y'' \frac{z'}{y'} = x'' \frac{z'}{x'} \Rightarrow x'' \frac{y'}{x'} \frac{z'}{y'} = x'' \frac{z'}{x'} \end{cases} (identidade) \Rightarrow \begin{cases} f(t) = x''/x' \\ y'' = f(t)y' \\ z'' = f(t)z' \end{cases} \Rightarrow \begin{cases} y'' = z'' \\ \frac{y''}{z''} = \frac{y'}{z'} = 1 \\ y' = z' \end{cases}$$

$$x' = g, y' = h \Rightarrow \frac{dh}{dt} = fh \Rightarrow \int \frac{dh}{h} = \int f \ dt \Rightarrow \ln h = \int \frac{dg}{dt} \frac{dt}{dx} dt = \int \frac{dg}{dx} dt = \int \frac{d}{dx} \left(\frac{dx}{dt}\right) dt = \int \frac{d}{dt} \left(\frac{dx}{dx}\right) dt = 0 + c$$

$$h = e^c \Rightarrow \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} A \\ C \end{pmatrix} t + \begin{pmatrix} B \\ D \end{pmatrix}, A, B > 0 \Rightarrow x'' = 0 \Rightarrow x = Et + F, \underline{eqd}$$

$$\alpha: I \to \mathfrak{R}^3$$

$$\forall t \in I, \exists r_t : X_{\lambda} = \alpha(t) + \lambda \alpha'(t)$$

$$C = \bigcap_{t \in I} r_t \neq \emptyset \Rightarrow \alpha(t) = \vec{P} + t\vec{u}$$

Consideremos, sem perda de generalidade,  $\vec{P} = 0 \in C$ . Vamos mostrar que  $\alpha(t) = t\vec{u}$ 

Sejam 
$$t_0 = a, t_1 = b$$

$$\mathbf{0} = \alpha_a + \lambda \alpha'_a = \alpha_b + \mu \alpha'_b$$

$$\lambda = -\frac{x_a}{x'_a} = -\frac{y_a}{y'_a} = -\frac{z_a}{z'_a}; \mu = -\frac{x_b}{x'_b} = -\frac{y_b}{y'_b} = -\frac{z_b}{z'_b}$$

$$x' = -\frac{1}{\lambda}x \Rightarrow \frac{dx}{dt} = f(t) \cdot x \Rightarrow \int \frac{dx}{x} = \int f(t) dt \Rightarrow \ln x = F(t) + C \Rightarrow x(t) = e^{c}g(t)$$

Analogamente,  $y(t) = e^k g(t)$ ;  $z(t) = e^q g(t)$ 

# p. 36, 6

F:  $\mathbb{R}^2 \to \mathbb{R}$ ,  $(x_0, y_0) \in \mathbb{R}^2$ (x, y): F(x, y) = 0 forma curva C no plano xy

$$F(x_0, y_0) = 0$$

$$F_x(x_0, y_0) \neq 0$$

$$F_{y}(x_0, y_0) \neq 0$$

Na vizinhança de  $(x_0, y_0)$ , podemos aproximar C como  $\alpha = (x, f(x))$  ou  $\beta = (f(y), y)$ 

$$\alpha' = \begin{pmatrix} x' \\ \frac{df}{dx} x' \end{pmatrix}; \beta' = y' \begin{pmatrix} \frac{df}{dy} \\ 1 \end{pmatrix}$$

$$\alpha' = \mathbf{0} \Rightarrow x' = 0 \Rightarrow x(t) = k$$
, mas nesse caso  $\beta' \neq \mathbf{0}$ 

$$v = 1; k \neq 0;$$

$$r_{t} : X_{\lambda} = \alpha_{t} + \lambda \mathbf{n}_{t}$$

$$\mathbf{0} = P \in \bigcap_{t \in I} r_{t}$$

$$0 = \alpha_{a} + \lambda n_{a} = \alpha_{b} + \lambda n_{b}$$

$$\lambda = -\frac{x_{a}}{n_{x,a}} = -\frac{y_{a}}{n_{y,a}} = -\frac{z_{a}}{n_{z,a}}; \mu = -\frac{z$$

$$n = \frac{\tau'}{|\tau'|} = \frac{\alpha''}{|\alpha''|} = \frac{\alpha''}{k} = \frac{1}{\sqrt{(x'')^2 + (y'')^2 + (z'')^2}} \begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix}$$

$$k = \frac{|w|}{v^3} \neq 0 \Rightarrow \alpha'' \neq C\alpha'$$

$$k = |\alpha''| \neq 0 \Rightarrow x'', y'' \neq 0$$

$$V = \frac{w \cdot \alpha'''}{\left|w\right|^2}$$

$$\frac{1}{v}b' = vn$$

$$qmq\begin{vmatrix} x' & y' & z' \\ x'' & y'' & z'' \\ z''' & y''' & z''' \end{vmatrix} = 0$$

$$x'y^2z^3 + x^3y'z^2 + x^2y^3z' = x'y^3z^2 + x^2y'z^3 + x^3y^2z'$$

$$\lambda n_y = -y$$

$$\lambda \frac{y''}{|\alpha''|} = -y$$

$$\lambda y'' = -y\sqrt{(x'')^2 + (y'')^2 + (z'')^2}$$

$$\lambda = -\frac{x_a}{n_{x,a}} = -\frac{y_a}{n_{y,a}} = -\frac{z_a}{n_{z,a}}; \mu = -\frac{x_b}{n_{x,b}} = -\frac{y_b}{n_{y,b}} = -\frac{z_b}{n_{z,b}}$$

$$qmq \alpha(t) \subset S: x^2 + y^2 + z^2 = R^2$$

$$qmq \mathbf{V} = 0$$
, ou seja,  $\alpha(t) \in \gamma : ax + by + cz + d = 0$ 

$$w = \alpha' \times \alpha'' = \begin{vmatrix} i & j & k \\ x' & y' & z' \\ x'' & y'' & z'' \end{vmatrix} = i \begin{vmatrix} y' & z' \\ y'' & z'' \end{vmatrix} - j \begin{vmatrix} x' & z' \\ x'' & z'' \end{vmatrix} + k \begin{vmatrix} x' & y' \\ x'' & y'' \end{vmatrix}$$

$$|w| = \sqrt{\begin{vmatrix} y' & z' \\ y'' & z'' \end{vmatrix}^2 + \begin{vmatrix} x' & z' \\ x'' & z'' \end{vmatrix}^2 + \begin{vmatrix} x' & y' \\ x'' & y'' \end{vmatrix}^2}$$

$$|w|' = \frac{\begin{vmatrix} y' & z' & y' & z' \\ y'' & z'' & y'' & z'' \end{vmatrix} + \begin{vmatrix} x' & z' & x' & z' \\ x'' & z'' & x'' & z'' \end{vmatrix} + \begin{vmatrix} x' & y' & x' & y' \\ x'' & y'' & x'' & y'' \end{vmatrix}}{|w|} = \frac{u}{|w|} = \frac{w' \cdot w}{|w|}$$

 $n = b \times \tau = \frac{1}{|w|} (\alpha' \times \alpha'' \times \alpha')$ 

$$w' = i \begin{vmatrix} y' & z' \\ y'' & z'' \end{vmatrix} - j \begin{vmatrix} x' & z' \\ x'' & z'' \end{vmatrix} + k \begin{vmatrix} x' & y' \\ x'' & y'' \end{vmatrix}$$