$$x_u = \frac{d}{du}\frac{4R^2u}{u^2+v^2+4R^2} = \frac{4R^2}{\left(u^2+v^2+4R^2\right)^2}\left(v^2-u^2+4R^2\right)$$

$$y_u = \frac{d}{du}\frac{4R^2v}{u^2+v^2+4R^2} = -2uv\frac{4R^2}{\left(u^2+v^2+4R^2\right)^2}$$

$$z_u = \frac{d}{du}R\frac{u^2+v^2-4R^2}{u^2+v^2+4R^2} = 4Ru\frac{4R^2}{\left(u^2+v^2+4R^2\right)^2}$$

$$x_v = \frac{d}{dv}\frac{4R^2u}{u^2+v^2+4R^2} = -2uv\frac{4R^2}{\left(u^2+v^2+4R^2\right)^2}$$

$$y_v = \frac{d}{dv}\frac{4R^2v}{u^2+v^2+4R^2} = \frac{4R^2}{\left(u^2+v^2+4R^2\right)^2}\left(u^2-v^2+4R^2\right)$$

$$z_v = \frac{d}{dv}R\frac{u^2+v^2-4R^2}{u^2+v^2+4R^2} = 4Rv\frac{4R^2}{\left(u^2+v^2+4R^2\right)^2}$$

$$\phi_u = \frac{4R^2}{\left(u^2+v^2+4R^2\right)^2}\begin{pmatrix}-u^2+v^2+4R^2\\-2uv\\4Ru\end{pmatrix}$$

$$\phi_v = \frac{4R^2}{\left(u^2+v^2+4R^2\right)^2}\begin{pmatrix}-2uv\\u^2-v^2+4R^2\\4Rv\end{pmatrix}$$

$$E = \frac{16R^4}{\left(u^2+v^2+4R^2\right)^4}\left[\left(-u^2+v^2+4R^2\right)^2+4u^2v^2+16R^2u^2\right] = \frac{16R^4}{\left(u^2+v^2+4R^2\right)^2}$$

$$F = 0$$

$$G = \frac{4R^2}{\left(u^2+v^2+4R^2\right)^2}\left[4u^2v^2+\left(-v^2+u^2+4R^2\right)^2+16R^2v^2\right] = E$$

$$\left(-a+b+c\right)^2+4ab+4ac = a^2+b^2+c^2+2ab+2ac+2bc = \left(a+b+c\right)^2$$