April/2016. Cardinality Transformations. By Vinicius Claudino Ferraz

Def. 1: $D = \{0, 1\} = \mathbb{Z}_2$
Def. 2: $\mathbb{N} \in \mathbb{M}(0,0) \Leftrightarrow \exists f : \mathbb{N} \times \mathbb{N} \to D$
Def. 3: $P(P(\mathbb{N})) = P^2(\mathbb{N})$
$P(P^k(\mathbb{N})) = P^{k+1}(\mathbb{N})$
$P^k(\mathbb{N}) \times P^k(\mathbb{N}) = (P^k)^2(\mathbb{N})$
$P^k(\mathbb{N}) \times (P^k)^{n-1}(\mathbb{N}) = (P^k)^n(\mathbb{N}) = \mathbb{P}_k^n = \mathbb{P}_k^{n\times 1}$
$\mathbb{P}_k^{p\times q} = [a_{ij}], 1 \le i \le p, 1 \le j \le q, a_{ij} \in \mathbb{P}_k$
Def. 4: $\mathbb{R} = P(\mathbb{N}) = \mathbb{P}_1 \in \mathbb{M}(1,0) \Leftrightarrow \exists f : \mathbb{R} \times \mathbb{N} \to D$
We take $\mathbb{R}$ as y-axis and $\mathbb{N}$ as x-axis.
Def. 5: $\#\mathbb{N}_0 = \aleph_0$; $\#\mathbb{R}_1 = \aleph_1$; $\#X_k = \aleph_k$
Example 1: $\pi = \pi_{1\times\aleph_0}(\mathbb{Z}_{10}) = [\cdots, 5, 1, 4, 1, 3, 0, 0, 0, \cdots] = [\cdots, f(-4), f(-3), f(-2), f(-1), f(0), f(1), f(2), f(3), \cdots]$
Example 2: $-\pi = (-\pi)_{1\times\aleph_0}(\mathbb{Z}_{10}) = [\cdots, 5, 1, 4, 1, 3, 0, 1, 1, 1, \cdots]$
Notation: $x_i \in Y_j \Leftrightarrow$ Exists decomposition $d : Y_j \times X_i \to D$, such that $d(x, \lambda)$ is the λ-th digit of x.
Def. 6: $A \in \mathbb{M}(i,j) \Leftrightarrow \exists f : B_j \times C_i \to D$
Def. 7: $A^T \in \mathbb{M}(j,i) \Leftrightarrow A \in \mathbb{M}(i,j)$
$\mathbb{N}^T \in \mathbb{M}(0,0)$
$\mathbb{R}^T \in \mathbb{M}(0,1)$
$\mathbb{B} = P(\mathbb{R}) = \mathbb{R}^{\mathbb{R}} = P^2(\mathbb{N}) = \mathbb{P}_2 \in \mathbb{M}(2,j)$

Theorem 1: $j = 1$
Demo: $\mathbb{B} = \{f : \mathbb{R} \to \mathbb{R}\}$
$\Gamma f \subset \mathbb{R}^2 \Rightarrow \aleph_0 < \#f \le \#\mathbb{R}^2 = \aleph_1$

$\mathbb{B}^T \in \mathbb{M}(1,2)$
$\mathbb{N}^n \in \mathbb{M}(0,0)$
$\mathbb{C}, \mathbb{H}, \mathbb{R}^n \in \mathbb{M}(1,0)$
$\mathbb{N}^\omega = \mathbb{N}^{\mathbb{N}} = \mathbb{R} \in \mathbb{M}(1,0)$
$\mathbb{Q}^\omega \in \mathbb{M}(1,0)$

Def. 8: $M \in \mathbb{M}(i,j) \Rightarrow T : M^{(B_k)} \to D^{B_k \times C_j}$; $T(d) = d'$; $T^{-1}(d') = d'' = d$
$d : B_k \to M$; $d(x_k) = y_{1\times\aleph_j} \in M$
$d'(x_k, \lambda_j) = y_\lambda \in D$
$d''(x) = \sum\limits_{\lambda \in C_j} d'(x, \lambda)$; This sum provides a matrix $1 \times \aleph_j$. $y_{1,\lambda} = d'(x,\lambda)$. We name Σ a **transfinite concatenation**.

Theorem 2: $P^k(\mathbb{N}) = \mathbb{P}_k \in \mathbb{M}(k, k-1), \forall k \ge 1$
Demo: Induction on k. $\#\mathbb{P}_{k+1} = \aleph_{k+1}$
$f \in \mathbb{P}_{k+1} \Rightarrow f : \mathbb{P}_k \to \mathbb{P}_k \Rightarrow \Gamma f \subset \mathbb{P}_k^2$
$\#f \le \aleph_k \Rightarrow \mathbb{P}_{k+1} \in \mathbb{M}(k+1, k)$

Theorem 3: $\mathbb{R}^{J_k} \in \mathbb{M}(k+1, k), \forall k \ge 1$
Demo: $\#\mathbb{R}^J = \#\{g : J \to \mathbb{R}\} = \#\{g' : J \times \mathbb{N} \to D\} = 2^{\#J} = \aleph_{k+1}$
$\Gamma g \subset J \times \mathbb{R} \Rightarrow \#g \le \#J = \aleph_k$

$x, y \in \mathbb{M}(i,j) \Rightarrow x \simeq y$
Corollary 3.1: $\mathbb{R}^{J_k} \simeq \mathbb{P}_{k+1}$

In particular: $\mathbb{R}^{\mathbb{B}} \in \mathbb{M}(3,2) \Rightarrow \mathbb{R}^{\mathbb{B}} \simeq \mathbb{P}_3$

Theorem 4: $M \in \mathbb{M}(i,j) \Rightarrow M^\omega \in \mathbb{M}(j+1, i)$
Demo: $\#M^\omega = \#\{f : \mathbb{N} \to M\} = \#\{f' : \mathbb{N} \times C_j \to D\} = 2^{\#C} = \aleph_{j+1}$
$\Gamma f \subset \mathbb{N} \times M \Rightarrow \#f \le \#M = \aleph_i$

In particular: $(\mathbb{P}_k)^\omega \in \mathbb{M}(k,k)$
In particular: $\mathbb{R}^\omega = \mathbb{R}^{\mathbb{N}} \in \mathbb{M}(1,1) \Rightarrow (\mathbb{R}^\omega)^T \simeq \mathbb{R}^\omega$
Corollary 4.1: $M_0 \in \mathbb{M}(i,j) \Rightarrow (M_0^\omega)^T = M_1 \in \mathbb{M}(i, j+1)$
$(M_1^\omega)^T = M_2 \in \mathbb{M}(i, j+2)$
Corollary 4.2: $(M_{\ell-1}^\omega)^T = M_\ell \in \mathbb{M}(i, j+\ell) \Rightarrow (M_\ell^\omega)^T = M_{\ell+1} \in \mathbb{M}(i, j+\ell+1)$
Demo: Reader's work.
In particular: $M_0 = \mathbb{P}_k \in \mathbb{M}(k, k-1) \Rightarrow M_1 \in \mathbb{M}(k,k) \Rightarrow M_\ell \in \mathbb{M}(k, k-1+\ell)$

Theorem 5: $\mathbb{M}(i,j) = \{g : Y_j \times X_i \to D\} \in \mathbb{M}(\ell, \ell+1), \ell = \max\{i,j\}$
Demo: $\#g \le \#(Y \times X \times D) = \aleph_\ell$
$\#\mathbb{M}(i,j) = 2^{\#(Y\times X)} = \aleph_{\ell+1}$

In particular: $i < j \Rightarrow \mathbb{M}(i,j) \in \mathbb{M}(j+1, j)$
In particular: $i > j \Rightarrow \mathbb{M}(i,j) \in \mathbb{M}(i+1, i)$

In particular: $\mathbb{M}(k,k) \in \mathbb{M}(k+1,k)$
In particular: $\mathbb{M}(0,0) \in \mathbb{M}(1,0) \in \mathbb{M}(2,1) \in \mathbb{M}(3,2) \in \mathbb{M}(4,3) \in \mathbb{M}(5,4) \in \mathbb{M}(6,5) \in \mathbb{M}(7,6) \in \cdots \in \mathbb{M}(k+1,k) \in \cdots$

Rings
$\mathbb{Z}$ is a ring.
$f', g' \in \mathbb{M}(0,0) = M \Rightarrow f'', g'' : \mathbb{N} \to \mathbb{N} \Rightarrow f'' +_{\mathbb{R}} g'' \in \mathbb{N}^{\mathbb{N}} \Rightarrow f' +_M g' \in \mathbb{M}(0,0)$, which is a ring, after a bijection $b : \mathbb{N} \to \mathbb{Z}$. **Example 3:** $b(0,1,2,3,4,5,6,\cdots) = (0,1,-1,2,-2,3,-3,\cdots)$.
$\mathbb{N}^n + \mathbb{N}^T + (\mathbb{Z}^T)^n + (\mathbb{Q}^n)^T = (?)$
$\mathbb{N}^n \cdot \mathbb{N}^T \cdot (\mathbb{Z}^T)^n \cdot (\mathbb{Q}^n)^T = (??)$
$xx = x^2; x^n x = x^{n+1}$
$p(x) \in M[x], \deg p(x) = g \Leftrightarrow p(x) = \sum_{i=0}^{g} a_i x^i = [a_0, a_1, \cdots, a_g, 0, 0, 0, \cdots] = [a_i]_{i\in\mathbb{N}}$
$p(x,y) \in M[x,y], \deg p(x,y) = g \Leftrightarrow p(x,y) = \sum_{i=0}^{m_1}\sum_{j=0}^{m_2} a_{ij}x^i y^j = [a_{ij}]_{i,j\in\mathbb{N}}; i+j > g \Rightarrow a_{ij} = 0$
$X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}, p(X) \in M[X], \deg p(X) = g \Leftrightarrow p(X) = \sum_{i_1=0}^{m_1} \cdots \sum_{i_n=0}^{m_n} a_{i_1\cdots i_n} x_1^{i_1} \cdots x_n^{i_n} = [a_{i_1\cdots i_n}]_{i\in\mathbb{N}^n}; \sum_{k=1}^{n} i_k > g \Rightarrow a_{i_1\cdots i_n} = 0$
$a : \mathbb{N}^n \to M; x_i \in (G, \cdot)$, which is a group. $x_i^0 = \mathrm{Id}_G; \cdot : M \times G \to U; y_M \cdot \mathrm{Id}_G = y_U; + : U \times U \to U; p(X_0) \in U, \forall X_0 \in G^n$
$0 \notin M \Rightarrow \deg p(X) = \infty, \forall p(X) \in M[X]$; but we want that $0 \in M \Rightarrow 0 \in M[X], \deg 0 \notin \mathbb{N}$.

Probability
Let $X_{2,1} : P(\mathbb{R}) \to \mathbb{R}$ be a random variable.
$X'_{2,0} : P(\mathbb{R}) \times \mathbb{N} \to D$

Basis and Representations
$[\mathbb{R}]_2 = \{f : \mathbb{R} \times \mathbb{N} \to \mathbb{Z}_2\} \sim [\mathbb{R}]_{10} = \{f : \mathbb{R} \times \mathbb{N} \to \mathbb{Z}_{10}\} \sim [\mathbb{R}]_b \in \mathbb{M}(\underline{1,0}, \mathbb{Z}_b)$
$\exists!\, B(\mathbb{R}) = \{2,3,4,\cdots\}$. Define $B(\mathbb{P}_2)$. $\exists!\, \overline{[\mathbb{R}]} = \{[\mathbb{R}]_b; b \in B(\mathbb{R})\}; \exists!\, \overline{[\mathbb{P}_k]} = \{[\mathbb{P}_k]_b; b \in B(\mathbb{P}_k)\}$

Def. 9: $F_{j,i} : A_i \to B_j; G_{k,j} : B_j \to C_k \Rightarrow G_{k,j} \circ F_{j,i} = H_{k,i}$
$F' : A \times B \to D$
$G' : B \times C \to D$
$H' : A \times C \to D; H' = G' \otimes F'$

Norms and ordinations in $\mathbb{P}_k$
$f \in \mathbb{B}; f : \mathbb{R} \to \mathbb{R}; |f_{11}| = \sup\{|f(x)|; x \in \mathbb{R}\} = |s| \in \mathbb{R}$
$\sup\{|f_1|, |f_2|, \cdots\} = |g| : \mathbb{R} \to \mathbb{R}; \forall x \in \mathbb{R}, \exists!\, \sup_{n\in\mathbb{N}} |f_n(x)| = |g(x)| \in \mathbb{R}$
$f \in \mathbb{P}_3; f : \mathbb{B} \to \mathbb{B}; |f_{22}| = \sup\{|f_{11}(x)| \in \mathbb{R}^{\mathbb{R}}; x \in \mathbb{B}\} = |s| \in \mathbb{B}$
$\sup\{|f_1|, |f_2|, \cdots\} = |g| : \mathbb{B} \to \mathbb{B}; \forall x \in \mathbb{B}, \exists!\, \sup_{n\in\mathbb{N}} |f_n(x)| = |g(x)| \in \mathbb{B}$
$f \in \mathbb{P}_{k+1}; f : \mathbb{P}_k \to \mathbb{P}_k; |f_{k,k}| = \sup\{|f_{k-1,k-1}(x)| : \mathbb{P}_{k-1} \to \mathbb{P}_{k-1}; x \in \mathbb{P}_k\} = |s| \in \mathbb{P}_k$
$\sup\{|f_1|, |f_2|, \cdots\} = |g| : \mathbb{P}_k \to \mathbb{P}_k; \forall x \in \mathbb{P}_k, \exists!\, \sup_{n\in\mathbb{N}} |f_n(x)| = |g(x)| \in \mathbb{P}_k$
Sups may be infinite. Prove that **the norm** is a norm.
$g : \Lambda \to \mathbb{P}_k; \|g\|_M = \sup\{|g(x)| \in \mathbb{P}_k; x \in \Lambda\}; \|g\|_S = \sum_{x\in\Lambda} |g(x)|$; Define Lebesgue sum over uncountable.

Lines in $\mathbb{P}_k^2$
$+, \cdot : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$
$+, \cdot : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$
$+, \cdot : \mathbb{N}^{\mathbb{N}} \times \mathbb{N}^{\mathbb{N}} \to \mathbb{N}^{\mathbb{N}}$
$+, \cdot : \mathbb{B} \times \mathbb{B} \to \mathbb{B}$
$+, \cdot : \varphi(\mathbb{N},4) \times \varphi(\mathbb{N},4) \to \varphi(\mathbb{N},4)$
$+, \cdot : \mathbb{P}_k \times \mathbb{P}_k \to \mathbb{P}_k$
$+, \cdot : \varphi(\mathbb{N},2^k) \times \varphi(\mathbb{N},2^k) \to \varphi(\mathbb{N},2^k)$
$+ : \mathbb{P}_k^2 \times \mathbb{P}_k^2 \to \mathbb{P}_k^2; \cdot : \mathbb{P}_k \times \mathbb{P}_k^2 \to \mathbb{P}_k; \cdot : \mathbb{P}_k^2 \times \mathbb{P}_k \to \mathbb{P}_k; t(u,v) = (tu, tv); (u,v)t = (ut, vt)$
Define $+, \cdot : \mathbb{M}_{ij} \times \mathbb{M}_{ij} \to \mathbb{M}_{ij}$.
$t \in \mathbb{P}_k, X = (x,y) = (a,b) + t(u,v) \neq (a,b) + (u,v)t = Y$
Define $\mathrm{dist}(X,Y); \|X\|, \langle X, Y\rangle$; derivative ; C^∞ topology ; $C^\infty \xrightarrow{T} C^\infty$ transforms.
$\langle (a,b), (x,y) \rangle = ax + by \Rightarrow \langle X, X\rangle = xx + yy$. Prove that the inner product is an inner product. Define $\sqrt{\cdot} : \mathbb{P}_k \to \mathbb{P}_k$.
$\|(x,y)\|_M = \max\{|x|,|y|\}$, where $\max\{f_1, f_2\} = \sup\{f_1, f_2\} = g$. $\|(x,y)\|_S = |x| + |y|$
$\|X\| = \|Y\| = a \Rightarrow X, Y \in S^1(0,a)$, the obscure sphere with radius a.

Lines in $\mathbb{P}_k^n$
$$t \in \mathbb{P}_k, X = X_0 + tV \neq X_0 + Vt = Y \;;\; \left\langle \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}, \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \right\rangle = X^T Y = x_1 y_1 + \cdots + x_n y_n$$

$$\left\| \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \right\|_M = \max\{|x_1|, \cdots, |x_n|\}; \left\| \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \right\|_S = |x_1| + \cdots + |x_n|$$

Lines in $\mathbb{P}_k^{p\times q}$
$+ : \mathbb{P}_k^{p\times q} \times \mathbb{P}_k^{p\times q} \to \mathbb{P}_k^{p\times q}; \cdot : \mathbb{P}_k^{p\times n} \times \mathbb{P}_k^{n\times q} \to \mathbb{P}_k^{p\times q}$
$X_{p\times q} = X_0 + T_{p\times n}V_{n\times q} \neq X_0 + V_{p\times n}T_{n\times q} = Y_{p\times q}$
$\langle A_{p\times q}, B_{p\times r}\rangle = (A^T)_{q\times p}B_{p\times r} = C_{q\times r}$
$\|A_k^{p\times q}\| = \sup\{\|Av\| \in \mathbb{P}_k^p; v \in \mathbb{P}_k^q, \langle v, v\rangle = \mathrm{Id}_{\mathbb{P}_k}\}$

Immersion and submersion. Click here. I'm without PDF viewer.
$V = L$