$$\mathbf{X} = x_0 + x_1 i + x_2 j + x_3 k + x_4 A + x_5 B + x_6 C + x_7 D$$

$$ij = k$$

$$ik = D \qquad jk = B$$

$$iA = -j \qquad jA = i \qquad kA = C$$

$$iB = C \qquad jB = -k \qquad kB = j \qquad AB = D$$

$$iC = -B \qquad jC = D \qquad kC = -A \qquad AC = k \qquad BC = i$$

$$iD = -k \qquad jD = -C \qquad kD = i \qquad AD = -B \qquad BD = A \qquad CD = j$$

$$\mathbf{XY} = \begin{pmatrix} x_0y_0 & -x_1y_1 & -x_2y_2 & -x_3y_3 & -x_4y_4 & -x_5y_5 & -x_6y_6 & -x_7y_7 \\ x_1y_0 & +x_0y_1 & -x_4y_2 & -x_7y_3 & +x_2y_4 & -x_6y_5 & +x_5y_6 & +x_3y_7 \\ x_2y_0 & +x_4y_1 & +x_0y_2 & -x_5y_3 & -x_1y_4 & +x_3y_5 & -x_7y_6 & +x_6y_7 \\ x_3y_0 & +x_7y_1 & +x_5y_2 & +x_0y_3 & -x_6y_4 & -x_2y_5 & +x_4y_6 & -x_1y_7 \\ x_4y_0 & -x_2y_1 & +x_1y_2 & +x_6y_3 & +x_0y_4 & -x_7y_5 & -x_3y_6 & +x_5y_7 \\ x_5y_0 & +x_6y_1 & -x_3y_2 & +x_2y_3 & +x_7y_4 & +x_0y_5 & -x_1y_6 & -x_4y_7 \\ x_6y_0 & -x_5y_1 & +x_7y_2 & -x_4y_3 & +x_3y_4 & +x_1y_5 & +x_0y_6 & -x_2y_7 \\ x_7y_0 & -x_3y_1 & -x_6y_2 & +x_1y_3 & -x_5y_4 & +x_4y_5 & +x_2y_6 & +x_0y_7 \end{pmatrix} (1, i, j, k, A, B, C, D)$$

$$\mathbf{XY} = \mathbf{YX}$$

$$\begin{pmatrix} & -x_4y_2 & -x_7y_3 & +x_2y_4 & -x_6y_5 & +x_5y_6 & +x_3y_7 \\ +x_4y_1 & & -x_5y_3 & -x_1y_4 & +x_3y_5 & -x_7y_6 & +x_6y_7 \\ +x_7y_1 & +x_5y_2 & & -x_6y_4 & -x_2y_5 & +x_4y_6 & -x_1y_7 \\ -x_2y_1 & +x_1y_2 & +x_6y_3 & & -x_7y_5 & -x_3y_6 & +x_5y_7 \\ +x_6y_1 & -x_3y_2 & +x_2y_3 & +x_7y_4 & & -x_1y_6 & -x_4y_7 \\ -x_5y_1 & +x_7y_2 & -x_4y_3 & +x_3y_4 & +x_1y_5 & & -x_2y_7 \\ -x_3y_1 & -x_6y_2 & +x_1y_3 & -x_5y_4 & +x_4y_5 & +x_2y_6 & \end{pmatrix} (i, j, k, A, B, C, D) = 0$$

$$\begin{pmatrix} 0 & -x_4 & -x_7 & +x_2 & -x_6 & +x_5 & +x_3 \\ +x_4 & 0 & -x_5 & -x_1 & +x_3 & -x_7 & +x_6 \\ +x_7 & +x_5 & 0 & -x_6 & -x_2 & +x_4 & -x_1 \\ -x_2 & +x_1 & +x_6 & 0 & -x_7 & -x_3 & +x_5 \\ +x_6 & -x_3 & +x_2 & +x_7 & 0 & -x_1 & -x_4 \\ -x_5 & +x_7 & -x_4 & +x_3 & +x_1 & 0 & -x_2 \\ -x_3 & -x_6 & +x_1 & -x_5 & +x_4 & +x_2 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{pmatrix} = 0$$