$d^{a+bi} = d^a \exp(bi\ln d)$

$\exp i\theta = \cos\theta + i\sin\theta$

$d^{a+bi} = d^a[\cos(b\ln d) + i\sin(b\ln d)]$

$$f(a,b) = \frac{1}{1-2^{1-s}} = \frac{2^s}{2^s-2} = \frac{1}{1+2^{1-a}[-\cos(b\ln 2)+i\sin(b\ln 2)]} = \frac{1}{1-p+qi} = \frac{1-p-qi}{1-2p+p^2+q^2} = \frac{1-2^{1-a}[\cos(b\ln 2)+i\sin(b\ln 2)]}{1-2^{2-a}\cos(b\ln 2)+2^{2-2a}}$$

$$= 2^a\frac{2^a-2\cos(b\ln 2)-2i\sin(b\ln 2)}{2^{2a}-2^{a+2}\cos(b\ln 2)+4} = P+Qi = \begin{bmatrix}P\\Q\end{bmatrix} = \frac{2^a}{2^{2a}-2^{a+2}\cos(b\ln 2)+4}\begin{bmatrix}2^a-2\cos(b\ln 2)\\-2\sin(b\ln 2)\end{bmatrix}$$

$$a>0 \Rightarrow \zeta(a+bi) = f\sum_{d=1}^{\infty}\frac{(-1)^{d+1}}{d^{a+bi}} = f\sum_{d=1}^{\infty}(-1)^{d+1}\frac{\cos(b\ln d)-i\sin(b\ln d)}{d^a} \underset{?}{=} \begin{bmatrix}P\\Q\end{bmatrix}\begin{bmatrix}S_1\\S_2\end{bmatrix} = \begin{bmatrix}PS_1-QS_2\\PS_2+QS_1\end{bmatrix} = 0 \underset{\text{Hipótese de Riemman}}{\Rightarrow} a = \frac{1}{2}$$

$$S_1 = \sum_{d=1}^{\infty}(-1)^{d+1}\frac{\cos(b\ln d)}{d^a} = 1 - \frac{\cos(b\ln 2)}{2^a} + \frac{\cos(b\ln 3)}{3^a} - \ldots$$

$$S_2 = -\sum_{d=1}^{\infty}(-1)^{d+1}\frac{\sin(b\ln d)}{d^a} = \frac{\sin(b\ln 2)}{2^a} - \frac{\sin(b\ln 3)}{3^a} + \ldots$$

$$a \neq \frac{1}{2} \underset{?}{\Rightarrow} \zeta \neq 0 \Rightarrow PS_1 \neq QS_2 \vee QS_1 \neq -PS_2$$

Recíproca

$$f\left(\frac{1}{2},b\right) = \frac{\sqrt{2}}{2-4\sqrt{2}\cos(b\ln 2)+4}\begin{bmatrix}\sqrt{2}-2\cos(b\ln 2)\\-2\sin(b\ln 2)\end{bmatrix}$$

$$S_1\left(\frac{1}{2},b\right) = \sum_{d=1}^{\infty}(-1)^{d+1}\frac{\cos(b\ln d)}{\sqrt{d}} = 1 - \frac{\cos(b\ln 2)}{\sqrt{2}} + \frac{\cos(b\ln 3)}{\sqrt{3}} - \ldots$$

$$S_2\left(\frac{1}{2},b\right) = \sum_{d=1}^{\infty}(-1)^{d}\frac{\sin(b\ln d)}{\sqrt{d}} = \frac{\sin(b\ln 2)}{\sqrt{2}} - \frac{\sin(b\ln 3)}{\sqrt{3}} + \ldots$$

$$PS_1 - QS_2 = \frac{1}{3\sqrt{2}-4\cos(b\ln 2)}\sum_{d=1}^{\infty}(-1)^{d+1}\left\{\left[\sqrt{2}-2\cos(b\ln 2)\right]\frac{\cos(b\ln d)}{\sqrt{d}} - 2\sin(b\ln 2)\frac{\sin(b\ln d)}{\sqrt{d}}\right\}$$

$$PS_2 + QS_1 = \frac{1}{3\sqrt{2}-4\cos(b\ln 2)}\sum_{d=1}^{\infty}(-1)^{d}\left\{\left[\sqrt{2}-2\cos(b\ln 2)\right]\frac{\sin(b\ln d)}{\sqrt{d}} + 2\sin(b\ln 2)\frac{\cos(b\ln d)}{\sqrt{d}}\right\}$$

$$\sum_{d=1}^{\infty}(-1)^{d+1}\left[\sqrt{\frac{2}{d}}\cos(b\ln d) - \frac{2}{\sqrt{d}}\cos\left(b\ln\frac{2}{d}\right)\right] \underset{?}{=} 0$$

$$\sum_{d=1}^{\infty}(-1)^{d+1}\left[\sqrt{\frac{2}{d}}\sin(b\ln d) + \frac{2}{\sqrt{d}}\sin\left(b\ln\frac{2}{d}\right)\right] \underset{?}{=} 0,\ \text{para alguns } b.\ \text{Seja } b=0.\ \text{Não satisfaz a 1ª igualdade.}$$

$f = 0 \Rightarrow \ldots$

$$\zeta\left(1+\frac{2k\pi}{\ln 2}i\right) = f\sum_{d=1}^{\infty}\frac{(-1)^{d+1}}{d^{a+bi}} = \frac{2}{4-8+4}\begin{bmatrix}2-2\\0\end{bmatrix}\sum_{d=1}^{\infty}\frac{(-1)^{d+1}}{d^{a+bi}} = 0\ !!!$$

$$S = 0 \Rightarrow \begin{cases}S_1 = 0 \Rightarrow \ldots\\ S_2 = 0 \Rightarrow \ldots\end{cases}$$

A ideia agora é comparar

$$f(x) = \left|\zeta\left(\frac{1}{2}+xi\right)\right| \underset{?}{\Rightarrow} \inf f = 0 \underset{?}{\Rightarrow} \#\{x : f(x)=0\} = \#N$$

$$g(x) = \left|\zeta\left(\frac{1}{4}+xi\right)\right| \underset{?}{\Rightarrow} \inf f = \delta > 0$$

Vamos dar só uma derivadinha

$$f(x)=\sqrt{(PS_1-QS_2)^2+(PS_2-QS_1)^2}$$

$$\frac{df}{dx}=\frac{1}{2f}\left[2(PS_1-QS_2)\left(\frac{\partial P}{\partial x}S_1+P\frac{\partial S_1}{\partial x}-\frac{\partial Q}{\partial x}S_2-Q\frac{\partial S_2}{\partial x}\right)+2(PS_2-QS_1)\left(\frac{\partial P}{\partial x}S_2+P\frac{\partial S_2}{\partial x}-\frac{\partial Q}{\partial x}S_1-Q\frac{\partial S_1}{\partial x}\right)\right]$$

$$\frac{\partial P}{\partial x}=\left(2^a-2\cos(x\ln 2)\right)\frac{\partial h}{\partial x}+2\ln 2\cdot h\sin(x\ln 2)$$

$$h(x)=\frac{2^a}{2^{2a}-2^{a+2}\cos(x\ln 2)+4}\Rightarrow\frac{\partial h}{\partial x}=-4\ln 2\cdot h^2\sin(x\ln 2)$$

$$\frac{\partial Q}{\partial x}=-2\sin(x\ln 2)\frac{\partial h}{\partial x}-2\ln 2\cdot h\cos(x\ln 2)$$

$$\frac{\partial S_1}{\partial x}=-\sum_{d=1}^{\infty}(-1)^{d+1}\frac{\ln d\sin(x\ln d)}{d^a}$$

$$\frac{\partial S_2}{\partial x}=-\sum_{d=1}^{\infty}(-1)^{d+1}\frac{\ln d\cos(x\ln d)}{d^a}$$