

$$\begin{array}{ccccccc} 3 & \longrightarrow & 2 & \longrightarrow & -2 & \longrightarrow & 4 \\ \mathbb{N} & & 6\mathbb{N} & & \mathbb{Z} & & \mathbb{N} \end{array}$$

$$\beta(0, 1, 2, 3, 4, \dots) = \begin{pmatrix} 0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 & 0 \end{pmatrix}$$

$$5 \xrightarrow{\beta} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \xrightarrow{\downarrow_1} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \xrightarrow{\beta^{-1}} 6$$

$$\searrow_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} \rightarrow 8$$

$$7 \xrightarrow{\beta} \begin{pmatrix} -1 \\ -1 \end{pmatrix} \xrightarrow{\downarrow_1} \begin{pmatrix} +1 \\ -1 \end{pmatrix} \rightarrow 8$$

$$\searrow_2 \begin{pmatrix} -1 \\ +1 \end{pmatrix} \rightarrow 6$$

$$f: (\mathbb{Q}_+, \mathbb{R}) \longrightarrow (M^0, G) \subset \mathbb{N}$$

$$p/q \longmapsto 2^p 3^q$$

$$\tilde{f}: (\mathbb{Q}, \mathbb{R}) \longrightarrow (\tilde{M}, G) \subset \mathbb{Q}_+$$

$$\textcircled{1} \begin{array}{ccc} 2^a 3^b < 2^p 3^q & \Rightarrow & a/b < p/q \\ \begin{array}{c} M \\ \tilde{M} \end{array} & & \begin{array}{c} \mathbb{Q}_+ \\ \mathbb{Q} \end{array} \end{array}$$

$$\textcircled{2} \frac{a}{b} < \frac{p}{q} \Rightarrow 2^a 3^b < 2^p 3^q$$

$$\tilde{f}: \xrightarrow{\beta^{-1}} \mathbb{N}$$