Comutação de matrizes, do + específico ao + geral

geral: encaixa se $a, d \neq 0$

(iA) $$x\begin{pmatrix} a & b^* \\ c^* & d \end{pmatrix}\begin{pmatrix} a-d & b \\ c & 0 \end{pmatrix} = x\begin{pmatrix} a-d & b \\ c & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = x\begin{pmatrix} a(a-d)+bc & ab \\ ac & bc \end{pmatrix}$$

Ex.: $$A = \begin{pmatrix} 5 & 7 \\ 11 & 13 \end{pmatrix}; B = x\begin{pmatrix} -8 & 7 \\ 11 & 0 \end{pmatrix} \Rightarrow AB = BA = x\begin{pmatrix} 37 & 35 \\ 55 & 77 \end{pmatrix}$$

1 *zero*: encaixa se $a = 0$ xor $d = 0$

a (iA_0) $$x\begin{pmatrix} 0 & b^* \\ c^* & d \end{pmatrix}\begin{pmatrix} -d & b \\ c & 0 \end{pmatrix} = x\begin{pmatrix} -d & b \\ c & 0 \end{pmatrix}\begin{pmatrix} 0 & b \\ c & d \end{pmatrix} = bcx\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Ex.: $$A = \begin{pmatrix} 0 & 3 \\ 5 & 7 \end{pmatrix}; B = x\begin{pmatrix} -7 & 3 \\ 5 & 0 \end{pmatrix} \Rightarrow AB = BA = 15x\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

d (iB) $$y\begin{pmatrix} a & b^* \\ c^* & 0 \end{pmatrix}\begin{pmatrix} x & b \\ c & x-a \end{pmatrix} = y\begin{pmatrix} x & b \\ c & x-a \end{pmatrix}\begin{pmatrix} a & b \\ c & 0 \end{pmatrix} = y\begin{pmatrix} ax+bc & bx \\ cx & bc \end{pmatrix}$$

Ex.: $$A = \begin{pmatrix} 3 & 5 \\ 7 & 0 \end{pmatrix}; B = y\begin{pmatrix} x & 5 \\ 7 & x-3 \end{pmatrix} \Rightarrow AB = BA = y\begin{pmatrix} 3x+35 & 5x \\ 7x & 35 \end{pmatrix}$$

1 *a* 3 *zeros*: $\forall a, d$

c (ii) $$\begin{pmatrix} a & b^* \\ 0 & d \end{pmatrix}\begin{pmatrix} x & by \\ 0 & x+(d-a)y \end{pmatrix} = \begin{pmatrix} x & by \\ 0 & x+(d-a)y \end{pmatrix}\begin{pmatrix} a & b \\ 0 & d \end{pmatrix} = \begin{pmatrix} ax & b(x+dy) \\ 0 & d[x+(d-a)y] \end{pmatrix}$$

Ex.: $$A = \begin{pmatrix} 3 & 5 \\ 0 & 7 \end{pmatrix}; B = \begin{pmatrix} x & 5y \\ 0 & x+4y \end{pmatrix} \Rightarrow AB = BA = \begin{pmatrix} 3x & 5(x+7y) \\ 0 & 7(x+4y) \end{pmatrix}$$

b (iii) $$\begin{pmatrix} a & 0 \\ c^* & d \end{pmatrix}\begin{pmatrix} (d-a)x & 0 \\ -c(x+y) & (d-a)y \end{pmatrix} = \begin{pmatrix} (d-a)x & 0 \\ -c(x+y) & (d-a)y \end{pmatrix}\begin{pmatrix} a & 0 \\ c & d \end{pmatrix} = \begin{pmatrix} (d-a)ax & 0 \\ c[(d-2a)y-ax] & (d-a)dy \end{pmatrix}$$

Ex.: $$A = \begin{pmatrix} 3 & 0 \\ 5 & 7 \end{pmatrix}; B = \begin{pmatrix} 4x & 0 \\ -5(x+y) & 4y \end{pmatrix} \Rightarrow AB = BA = \begin{pmatrix} 12x & 0 \\ 5(y-3x) & 28y \end{pmatrix}$$

2 *ou* 3 *zeros*: encaixa se $a, b \neq 0$

(iv) $$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} = \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} ax & 0 \\ 0 & by \end{pmatrix}$$

Ex.: $$A = \begin{pmatrix} 3 & 0 \\ 0 & 5 \end{pmatrix}; B = \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} \Rightarrow AB = BA = \begin{pmatrix} 3x & 0 \\ 0 & 5y \end{pmatrix}$$

2 *ou* 4 *zeros*: $\forall a$

(v) $$a\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x & y \\ z & w \end{pmatrix} = a\begin{pmatrix} x & y \\ z & w \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = a\begin{pmatrix} x & y \\ z & w \end{pmatrix}$$

Ex.: $$A = 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; B = \begin{pmatrix} x & y \\ z & w \end{pmatrix} \Rightarrow AB = BA = 3\begin{pmatrix} x & y \\ z & w \end{pmatrix}$$

Cópia da solução do sistema:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} x & y \\ z & w \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}; S = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

$$\underbrace{b = c = 0, d = a}_{\text{múltipla de identidade}} \Rightarrow \langle S \rangle = R^4; \underbrace{b = c = 0}_{\text{diagonal}} \Rightarrow \underbrace{S = xe_1 + we_4}_{\text{diagonal}}$$

$$b = 0 \neq c \Rightarrow S = x\begin{pmatrix} a-d \\ 0 \\ c \\ 0 \end{pmatrix} + w\begin{pmatrix} 0 \\ 0 \\ c \\ a-d \end{pmatrix}; c = 0 \neq b \Rightarrow S = x\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} + y\begin{pmatrix} 0 \\ b \\ 0 \\ d-a \end{pmatrix}; b, c \neq 0 \Rightarrow S = z\begin{pmatrix} a-d \\ b \\ c \\ 0 \end{pmatrix}$$