TETRAEDRO E ESFERA

E Circunscrita

$A=(0,0,0); B=(c,0,0), c>0; C=(p,q,0), p,q>0; D=(r,s,t), t>0$

$$X=C_{circunscrita}=(x,y,z):\begin{cases} x=\dfrac{c}{2}; y=\dfrac{p^2+q^2-cp}{2q} \\ z=\dfrac{q(r^2+s^2+t^2)+s(cp-p^2-q^2)-cqr}{2qt} \end{cases}$$

$$AX^2=\frac{c^2}{4}+\frac{(p^2+q^2-cp)^2}{4q^2}+\left[\frac{q^2((c-p)^2+q^2)+u(cp-p^2-q^2)-c(c-p)q^2}{2q\sqrt{q^4-u^2}}\right]^2=R^2$$

$BX^2=R^2=CX^2=DX^2$

E Inscrita

$Y=C_{inscrita}=(x,y,z)$

$\alpha=ABC: z=0$

$\beta=ABD: ty-sz=0$

$\gamma=ACD: qtx-pty+(ps-qr)z=0$

$\delta=BCD: qtx+(c-p)ty+[cq-qr+(p-c)s]z-cqt=0$

$$dist(Y,\pi:ax+by+cz+d=0)=\frac{|ax+by+cz+d|}{\sqrt{a^2+b^2+c^2}}\Rightarrow \alpha Y=z>0$$

$$\beta Y=\frac{|ty-sz|}{f_1}=z\Rightarrow y=\frac{s\pm f_1}{t}z; f_1=\sqrt{s^2+t^2}>s$$

dois planos perpendiculares, entao escolho inclinaçao positiva em $x=0$

$$\gamma Y=\frac{|qtx-pty+(ps-qr)z|}{f_2}=z\Rightarrow qtx-pty=z(qr-ps\pm f_2)\Rightarrow x=\frac{qr+pf_1\pm f_2}{qt}z$$

$$f_2=\sqrt{(p^2+q^2)t^2+(ps-qr)^2}>qr-ps$$

$$\delta Y=\frac{|qtx+(c-p)ty+gz-cqt|}{f_3}=z\Rightarrow qtx+(c-p)ty-cqt=z(-g\pm f_3); g=cq-qr+(p-c)s$$

aqui escolher o negativo, pois $-cqt<0$

$$f_3=\sqrt{[q^2+(c-p)^2]t^2+g^2}>g$$

$$z=\frac{cqt}{cq+cf_1+f_2+f_3}; y=cq\frac{s+f_1}{cq+cf_1+f_2+f_3}; x=c\frac{qr+pf_1+f_2}{cq+cf_1+f_2+f_3}$$

$$Y=\frac{c}{cq+cf_1+f_2+f_3}(qr+pf_1+f_2, q(s+f_1), qt)$$

Resultado para faces opostas congruentes

$$D=\left(c-p,\frac{u}{q},\frac{\sqrt{q^4-u^2}}{q}\right),u=2p^2+q^2-2cp$$

$$AB^2=c^2=CD^2;AC^2=p^2+q^2=BD^2;AD^2=(c-p)^2+q^2=BC^2$$

$$X=\left(\frac{c}{2},\frac{p^2+q^2-cp}{2q},\frac{\sqrt{q^2-u^2}}{4q}\right)=(x_X,y_X,z_X)$$

$$Y=\frac{c}{cq\pm cf_1\pm f_2\mp f_3}(qr\pm pf_1\pm f_2,q(s\pm f_1),qt)$$

$$Y=\frac{c}{cq+\Sigma_1 cf_1+\Sigma_2 f_2-\Sigma_3 f_3}(qr+\Sigma_1 pf_1+\Sigma_2 f_2,q(s+\Sigma_1 f_1),qt),\Sigma_i\in\{1,-1\}$$

$$f_1=q;f_2=cq=f_3;Y=\frac{1}{q(1+\Sigma_1+\Sigma_2-\Sigma_3)}\left((\Sigma_1-1)pq+(\Sigma_2+1)cq,u+\Sigma_1 q^2,\sqrt{q^4-u^2}\right)$$

Σ_1	+	−	+	−	+	−	+	−
Σ_2	+	+	−	−	+	+	−	−
Σ_3	+	+	+	+	−	−	−	−
$1+\Sigma_1+\Sigma_2-\Sigma_3$	2	0	0	−2	4	2	2	0

$$Y_{++-}=X$$

$$Y_{+++}=2X=A'$$

$$Y_{--+}=\left(p,p\frac{c-p}{q},-2z_X\right)=D'$$

$$Y_{-+-}=\left(c-p,p\frac{p-c}{q},2z_X\right)=C'$$

$$Y_{+--}=2(0,y_X,z_X)=B'$$

$$X=\frac{A+A'}{2}=\frac{B+B'}{2}=\frac{C+C'}{2}=\frac{D+D'}{2}$$

$$Y_0\in\varnothing\because 0z=cqt$$

Tetraedro regular : 5 pontos equidistantes das faces

$$A=(0,0,0);B=(1,0,0);C=\left(\frac{1}{2},\frac{\sqrt{3}}{2},0\right);D=\left(\frac{1}{2},\frac{\sqrt{3}}{6},\frac{\sqrt{6}}{3}\right);X=\left(\frac{1}{2},\frac{\sqrt{3}}{6},\frac{\sqrt{6}}{12}\right);2X=\left(1,\frac{\sqrt{3}}{3},\frac{\sqrt{6}}{6}\right)=A'$$

$$D'=\left(\frac{1}{2},\frac{\sqrt{3}}{6},-\frac{\sqrt{6}}{6}\right);C'=\left(\frac{1}{2},-\frac{\sqrt{3}}{6},\frac{\sqrt{6}}{6}\right);B'=\left(0,\frac{\sqrt{3}}{3},\frac{\sqrt{6}}{6}\right)$$

$$\alpha=ABC:z=0$$

$$\beta=ABD:2y\sqrt{2}-z=0$$

$$\gamma=ACD:6x-2y\sqrt{3}-z\sqrt{6}=0$$

$$\delta=BCD:6x+2y\sqrt{3}+z\sqrt{6}-6=0$$

<u>Bissetriz de ângulo</u>

$$A,O,B,u=\frac{A-O}{|OA|},v=\frac{B-O}{|OB|},X:\begin{cases}u\in pl(A,O,B)\\ \angle u,X=\angle v,X\end{cases}\Rightarrow\begin{cases}X=O+\lambda u+\mu v\\ \dfrac{u\cdot(X-O)}{\|u\|\cdot\|X\|}=\dfrac{v\cdot(X-O)}{\|v\|\cdot\|X\|}\end{cases}$$

$$(u-v)\cdot(\lambda u+\mu v)=0\Rightarrow\lambda u\cdot(u-v)+\mu v\cdot(u-v)=0\Rightarrow\mu=\lambda\frac{1-u\cdot v}{1-u\cdot v}$$

$$X=O+\lambda(u+v)$$

Tangente às arestas: regular

$$A=(1,0,0);B=\left(-\frac{1}{2},\frac{\sqrt{3}}{2},0\right);C=\left(-\frac{1}{2},-\frac{\sqrt{3}}{2},0\right)$$

$$D=\left(0,0,\sqrt{2}\right)\Rightarrow|AD|=|AB|\Rightarrow\sqrt{1+2}=\sqrt{\frac{9}{4}+\frac{3}{4}}$$

Determinando $s=\text{bissetriz}\left(B\hat{A}C\right)$

$$u=\frac{B-A}{\sqrt{3}}=\left(-\frac{\sqrt{3}}{2},\frac{1}{2},0\right);v=\frac{C-A}{\sqrt{3}}=\left(-\frac{\sqrt{3}}{2},-\frac{1}{2},0\right);s:X=A+\lambda\left(-\sqrt{3},0,0\right)\Rightarrow s:X=\lambda A$$

$t=b\text{issetriz}\left(A\hat{B}C\right)$

$$u=\frac{A-B}{\sqrt{3}}=\left(\frac{\sqrt{3}}{2},-\frac{1}{2},0\right);v=\frac{C-B}{\sqrt{3}}=(0,-1,0);t:X=B+\lambda\left(\frac{\sqrt{3}}{2},-\frac{3}{2},0\right)\Rightarrow t:X=\lambda\left(1,-\sqrt{3},0\right)$$

$O=s\cap t=\mathbf{0}$

Determinando interseções da esfera com ΔABC : M, N, P

$$\begin{cases}M\in AB\\ OM\perp AB\end{cases}\Rightarrow\begin{cases}M=A+\lambda(B-A)\\ \left[A+\lambda(B-A)\right]\cdot(B-A)=0\Rightarrow A\cdot(B-A)+\lambda|AB|^2=0\Rightarrow\lambda=\dfrac{3/2}{3}=\dfrac{1}{2}\Rightarrow M=\left(1-\dfrac{3}{4},\dfrac{\sqrt{3}}{4},0\right)\end{cases}$$

$$\begin{cases}N\in AC\\ ON\perp AC\end{cases}\Rightarrow\begin{cases}N=A+\lambda(C-A)\\ \lambda=-\dfrac{A\cdot(C-A)}{|AC|^2}=\dfrac{3/2}{3}\Rightarrow N=\left(\dfrac{1}{4},-\dfrac{\sqrt{3}}{4},0\right)\end{cases}$$

$$\begin{cases}P\in BC\\ OP\perp BC\end{cases}\Rightarrow\begin{cases}P=B+\lambda(C-B)\\ \lambda=-\dfrac{B\cdot(C-B)}{|BC|^2}=\dfrac{3/2}{3}\Rightarrow P=\left(-\dfrac{1}{2},0,0\right)\end{cases}$$

Determinando $s_2=\text{bissetriz}\left(D\hat{B}C\right)$

$$u=\frac{D-B}{\sqrt{3}}=\left(\frac{\sqrt{3}}{6},-\frac{1}{2},\frac{\sqrt{6}}{3}\right);v=\frac{C-B}{\sqrt{3}}=(0,-1,0);s_2:X=B+\lambda(u+v)=\left(-\frac{1}{2},\frac{\sqrt{3}}{2},0\right)+\lambda\left(\frac{\sqrt{3}}{6},-\frac{3}{2},\frac{\sqrt{6}}{3}\right)$$

$t_2=\text{bissetriz}\left(B\hat{C}D\right)$

$$u=\frac{D-C}{\sqrt{3}}=\left(\frac{\sqrt{3}}{6},\frac{1}{2},\frac{\sqrt{6}}{3}\right);v=\frac{B-C}{\sqrt{3}}=(0,1,0);t_2:X=C+\lambda(u+v)=\left(-\frac{1}{2},-\frac{\sqrt{3}}{2},0\right)+\lambda\left(\frac{\sqrt{3}}{6},\frac{3}{2},\frac{\sqrt{6}}{3}\right)$$

$$I=s_2\cap t_2\Rightarrow I=\left(-\frac{1}{2},\frac{\sqrt{3}}{2},0\right)+\lambda\left(\frac{\sqrt{3}}{6},-\frac{3}{2},\frac{\sqrt{6}}{3}\right)=\left(-\frac{1}{2},-\frac{\sqrt{3}}{2},0\right)+\mu\left(\frac{\sqrt{3}}{6},\frac{3}{2},\frac{\sqrt{6}}{3}\right)$$

$$\mu=\lambda=\frac{\sqrt{3}}{3}\Rightarrow I=\left(-\frac{1}{3},0,\frac{\sqrt{2}}{3}\right)$$

Interseção da esfera com BD : Q

$$\begin{cases}Q\in BD\\ IQ\perp BD\end{cases}\Rightarrow\begin{cases}Q=B+\lambda(D-B)\\ \left[B+\lambda(D-B)-I\right]\cdot(D-B)=0\Rightarrow(B-I)\cdot(D-B)+\lambda|BD|^2=0\end{cases}$$

$$3\lambda=\left(-\frac{1}{6},\frac{\sqrt{3}}{2},-\frac{\sqrt{2}}{3}\right)\left(-\frac{1}{2},\frac{\sqrt{3}}{2},-\sqrt{2}\right)=\frac{1}{12}+\frac{3}{4}+\frac{2}{3}=\frac{1+9+8}{12}\Rightarrow\lambda=\frac{1}{2}\Rightarrow Q=\left(-\frac{1}{4},\frac{\sqrt{3}}{4},\frac{\sqrt{2}}{2}\right)$$

Esfera que passa por 4 pontos

$$M, N, P, Q \in E : (x-a)^2 + (y-b)^2 + (z-c)^2 = r^2 \Leftrightarrow a^2 + b^2 + c^2 - r^2 = 2ax + 2by + 2cz - x^2 - y^2 - z^2$$

$$\left(\frac{1}{4} - a\right)^2 + \left(\frac{\sqrt{3}}{4} - b\right)^2 + c^2 = r^2 = a^2 - \frac{1}{2}a + \frac{1}{16} + b^2 \pm \frac{\sqrt{3}}{2}b + \frac{3}{16} + c^2$$

$$\left(\frac{1}{4} - a\right)^2 + \left(-\frac{\sqrt{3}}{4} - b\right)^2 + c^2 = r^2 = a^2 - \frac{1}{2}a + \frac{1}{16}$$

$$\left(-\frac{1}{2} - a\right)^2 + b^2 + c^2 = r^2 = a^2 + a + \frac{1}{4} + b^2 + c^2$$

$$\left(-\frac{1}{4} - a\right)^2 + \left(\frac{\sqrt{3}}{4} - b\right)^2 + \left(\frac{\sqrt{2}}{2} - c\right)^2 = r^2$$

$$\underbrace{a^2 + b^2 + c^2 - r^2}_{i} = \underbrace{\frac{1}{2}a + \frac{\sqrt{3}}{2}b - \frac{1}{16} - \frac{3}{16}}_{ii} = \underbrace{\frac{1}{2}a - \frac{\sqrt{3}}{2}b - \frac{1}{16} - \frac{3}{16}}_{iii} = \underbrace{-a - \frac{1}{4}}_{iv} = \underbrace{-\frac{1}{2}a + \frac{\sqrt{3}}{2}b + c\sqrt{2} - \frac{1}{16} - \frac{3}{16} - \frac{1}{2}}_{v}$$

$$\begin{cases} iv = ii \Rightarrow -4a - 1 = 2a + 2b\sqrt{3} - 1 \\ iv = iii \Rightarrow -4a - 1 = 2a - 2b\sqrt{3} - 1 \end{cases} \Rightarrow \begin{cases} 0 = 6a + 2b\sqrt{3} \\ 0 = 6a - 2b\sqrt{3} \end{cases} \Rightarrow 0 = 12a \Rightarrow a = b = 0$$

$$iv = v \Rightarrow 0 = c\sqrt{2} - \frac{1}{2} \Rightarrow c = \frac{\sqrt{2}}{4}$$

$$i = iv \Rightarrow \frac{1}{8} - r^2 = -\frac{1}{4} \Rightarrow r^2 = \frac{3}{8} \Rightarrow r = \frac{\sqrt{6}}{4}$$

Tangente às arestas: genérico

$A=(0,0,0); B=(c,0,0), c>0; C=(p,q,0), p,q>0; D=(r,s,t), t>0$

$U=(p-c)^2+q^2; V=(r-c)^2+s^2+t^2; W=(r-p)^2+(s-q)^2+t^2$

Determinando $s=\text{bissetriz}(B\hat{A}C)$

$$u=\frac{1}{c}B; v=\frac{C}{\sqrt{p^2+q^2}}; s: X=\lambda\left(\frac{p}{\sqrt{p^2+q^2}}+1, \frac{q}{\sqrt{p^2+q^2}}, 0\right)$$

$t=b\text{issetriz}(A\hat{B}C)$

$$u=-\frac{1}{c}B; v=\frac{C-B}{|BC|}=\left(\frac{p-c}{\sqrt{U}}, \frac{q}{\sqrt{U}}, 0\right); t: X=B+\lambda\left(\frac{p-c}{\sqrt{U}}-1, \frac{q}{\sqrt{U}}, 0\right)$$

$$I_1=s\cap t\Rightarrow I_1=\lambda\left(\frac{p}{\sqrt{p^2+q^2}}+1, \frac{q}{\sqrt{p^2+q^2}}, 0\right)=B+\mu\left(\frac{p-c}{\sqrt{U}}-1, \frac{q}{\sqrt{U}}, 0\right)$$

$$\begin{cases}\lambda\sqrt{\dfrac{U}{p^2+q^2}}=\mu \\ \lambda=\dfrac{c\sqrt{p^2+q^2}}{c+\sqrt{p^2+q^2}+\sqrt{U}}\end{cases}\Rightarrow I_1=\frac{c}{c+\sqrt{p^2+q^2}+\sqrt{U}}\left(p+\sqrt{p^2+q^2}, q, 0\right)$$

Determinando interseções da esfera com $\Delta\mathrm{ABC}:\mathrm{M}, \mathrm{N}, \mathrm{P}$

$$\begin{cases}M\in AB \\ I_1M\perp AB\end{cases}\Rightarrow\begin{cases}M=\lambda B \\ (\lambda B-I_1)\cdot B=0\Rightarrow\lambda\|B\|^2=I_1\cdot B\end{cases}\Rightarrow M=\frac{p+\sqrt{p^2+q^2}}{c+\sqrt{p^2+q^2}+\sqrt{U}}B$$

$$\begin{cases}N\in AC \\ I_1N\perp AC\end{cases}\Rightarrow\begin{cases}N=\lambda C \\ (\lambda C-I_1)\cdot B=0\Rightarrow\lambda\|C\|^2=I_1\cdot C\end{cases}\Rightarrow N=\frac{p^2+q^2+p\sqrt{p^2+q^2}}{c+\sqrt{p^2+q^2}+\sqrt{U}}\frac{c}{p^2+q^2}C$$

$$\begin{cases}P\in BC \\ I_1P\perp BC\end{cases}\Rightarrow\begin{cases}P=B+\lambda(C-B) \\ [B+\lambda(C-B)-I_1]\cdot(C-B)=0\Rightarrow\lambda|BC|^2=(I_1-B)\cdot(C-B)\end{cases}$$

$$U\lambda=\frac{c}{c+\sqrt{p^2+q^2}+\sqrt{U}}\left(p-c-\sqrt{U}, q, 0\right)(p-c, q, 0)\Rightarrow\lambda=\frac{c}{U}\frac{U+(c-p)\sqrt{U}}{c+\sqrt{p^2+q^2}+\sqrt{U}}$$

$$P=\frac{c}{U\left(c+\sqrt{p^2+q^2}+\sqrt{U}\right)}\left(pU+\left(2cp+q^2-2c^2p\right)\sqrt{U}+U\sqrt{p^2+q^2}, q\left[U+(c-p)\sqrt{U}\right], 0\right)$$

Determinando $s_2=\text{bissetriz}(D\hat{B}C)$

$$u=\frac{D-B}{\sqrt{V}}; v=\frac{C-B}{\sqrt{U}}; s_2: X=B+\lambda\left(\frac{r-c}{\sqrt{V}}+\frac{p-c}{\sqrt{U}}, \frac{s}{\sqrt{V}}+\frac{q}{\sqrt{U}}, \frac{t}{\sqrt{V}}\right)$$

$t_2=\text{bissetriz}(B\hat{C}D)$

$$u=\frac{D-C}{\sqrt{W}}; v=\frac{B-C}{\sqrt{U}}; t_2: X=C+\lambda\left(\frac{r-p}{\sqrt{W}}+\frac{c-p}{\sqrt{U}}, \frac{s-q}{\sqrt{W}}-\frac{q}{\sqrt{U}}, \frac{t}{\sqrt{W}}\right)$$

$$I_2=s_2\cap t_2\Rightarrow I_2=B+\lambda\left(\frac{r-c}{\sqrt{V}}+\frac{p-c}{\sqrt{U}}, \frac{s}{\sqrt{V}}+\frac{q}{\sqrt{U}}, \frac{t}{\sqrt{V}}\right)=C+\mu\left(\frac{r-p}{\sqrt{W}}+\frac{c-p}{\sqrt{U}}, \frac{s-q}{\sqrt{W}}-\frac{q}{\sqrt{U}}, \frac{t}{\sqrt{W}}\right)$$

$$\begin{cases}\dfrac{t}{\sqrt{V}}\lambda=\dfrac{t}{\sqrt{W}}\mu\Rightarrow\mu=\sqrt{\dfrac{W}{V}}\lambda \\ \left(\dfrac{s}{\sqrt{V}}+\dfrac{q}{\sqrt{U}}\right)\lambda=q+\left(s-q-q\sqrt{\dfrac{W}{U}}\right)\dfrac{\lambda}{\sqrt{V}}\Rightarrow\lambda=\dfrac{\sqrt{UV}}{\sqrt{U}+\sqrt{V}+\sqrt{W}} \\ c+\left(\dfrac{r-c}{\sqrt{V}}+\dfrac{p-c}{\sqrt{U}}\right)\lambda=p+\left(r-p+\dfrac{c-p}{\sqrt{U}}\sqrt{W}\right)\dfrac{\lambda}{\sqrt{V}}\Rightarrow 0(p-c)=0\end{cases}\Rightarrow I_2=\frac{\left(r\sqrt{U}+p\sqrt{V}+c\sqrt{W}, s\sqrt{U}+q\sqrt{V}, t\sqrt{U}\right)}{\sqrt{U}+\sqrt{V}+\sqrt{W}}$$

Interseção da esfera com BD : Q

$$\begin{cases} Q \in BD \\ I_2Q \perp BD \end{cases} \Rightarrow \begin{cases} Q = B + \lambda(D-B) \\ [B+\lambda(D-B)-I_2]\cdot(D-B) = 0 \Rightarrow \lambda|BD|^2 = (I_2-B)\cdot(D-B) \end{cases}$$

$$Q = \frac{1}{V}\frac{V\sqrt{U}+[(r-c)(p-c)+qs]\sqrt{V}}{\sqrt{U}+\sqrt{V}+\sqrt{W}}\left(r-c+cV\left(\sqrt{U}+\sqrt{V}+\sqrt{W}\right), s, t\right)$$

$$M = \frac{p+\sqrt{p^2+q^2}}{c+\sqrt{p^2+q^2}+\sqrt{U}}(c,0,0)$$

$$N = \frac{p^2+q^2+p\sqrt{p^2+q^2}}{c+\sqrt{p^2+q^2}+\sqrt{U}}\frac{c}{p^2+q^2}(p,q,0)$$

$$P = \frac{c}{U\left(c+\sqrt{p^2+q^2}+\sqrt{U}\right)}\left(pU+\left(2cp+q^2-2c^2p\right)\sqrt{U}+U\sqrt{p^2+q^2}, q\left[U+(c-p)\sqrt{U}\right], 0\right)$$

$$M,N,P,Q \in E: (x-a)^2+(y-b)^2+(z-c)^2 = r^2 \Leftrightarrow a^2+b^2+c^2-r^2 = 2ax+2by+2cz-x^2-y^2-z^2$$

$$\underbrace{a^2+b^2+c^2-r^2}_{i} = \underbrace{2ax_M-x_M{}^2}_{ii} = \underbrace{2ax_N+2by_N-x_N{}^2-y_N{}^2}_{iii} = \underbrace{2ax_P+2by_P-x_P{}^2-y_P{}^2}_{iv}$$

$$= \underbrace{2ax_Q+2by_Q+2cz_Q-x_Q{}^2-y_Q{}^2-z_Q{}^2}_{v}$$

$$ii = iii \Rightarrow a = \frac{x_M{}^2-x_N{}^2-y_N{}^2+2by_N}{2(x_M-x_N)}$$

$$ii = iv \Rightarrow b = \frac{(x_M-x_P)\left(x_M{}^2-x_N{}^2-y_N{}^2+2by_N\right)+\left(x_P{}^2+y_P{}^2-x_M{}^2\right)(x_M-x_N)}{2y_P(x_M-x_N)}$$

$$ii = v \Rightarrow c = \frac{\begin{gathered} y_P\left(x_M{}^2-x_N{}^2-y_N{}^2+2by_N\right)(x_M-x_Q)-y_Q(x_M-x_P)\left(x_M{}^2-x_N{}^2-y_N{}^2+2by_N\right) \\ -y_Q\left(x_P{}^2+y_P{}^2-x_M{}^2\right)(x_M-x_N)+y_P(x_M-x_N)\left(x_Q{}^2+y_Q{}^2+z_Q{}^2-x_M{}^2\right) \end{gathered}}{2y_Pz_Q(x_M-x_N)}$$

$$i = ii \Rightarrow r^2 = a^2+b^2+c^2-2ax_M+x_M{}^2$$

Geografia

$A=[0,0]$

$B=[60,60]$

$C=[\lambda,\mu]$

$AB=c;\cos c=\frac{1}{4}$

$AC=b;\cos b=\cos\lambda\cos\mu$

$BC=a;\cos a=\frac{1}{2}\cos\lambda\cos(\mu-60)+\frac{\sqrt{3}}{2}\sin\lambda$

$\cos(\mu-60)=\frac{1}{2}\cos\mu+\frac{\sqrt{3}}{2}\sin\mu$

$AB=AC=BC\Rightarrow\begin{cases}4\cos\lambda\cos\mu=k_1\\ \cos\lambda(\cos\mu+\sqrt{3}\sin\mu)+2\sqrt{3}\sin\lambda=k_2\end{cases}$

$\begin{cases}\cos\mu=\frac{k_1}{4\cos\lambda}\Rightarrow\sin^2\mu=\frac{16\cos^2\lambda-1}{16\cos^2\lambda}\\ \sin\lambda=x\\ 60x^2+4(k_1-4k_2)\sqrt{3}x-7-2k_1k_2=0\end{cases}$

$\Delta=16*6(26+k_1k_2)$

$\sin\lambda=\sqrt{3}\,\frac{4k_2-k_1+k_3\sqrt{2(26+k_1k_2)}}{30}$

$\cos\lambda=\pm\sqrt{\frac{231+6k_1k_2+2(k_1-4k_2)k_3\sqrt{2(26+k_1k_2)}}{300}}$

$\cos\mu=\pm\frac{5\sqrt{3}k_1}{2\sqrt{231+6k_1k_2+2(k_1-4k_2)k_3\sqrt{2(26+k_1k_2)}}}$

k_1	+	+	+	+	−	−	−	−
k_2	+	+	−	−	+	+	−	−
k_3	+	−	+	−	+	−	+	−
$\sin\lambda$	A	B	C	$-D$	D	$-C$	$-B$	$-A$
$\cos\mu$	$\pm E$	$\pm F$	$\pm G$	$\pm H$	$\mp H$	$\mp G$	$\mp F$	$\mp E$

$A=\frac{\sqrt{3}+3\sqrt{2}}{10}\rightarrow 36{,}68885;B=\frac{\sqrt{3}-3\sqrt{2}}{10}\rightarrow\text{-}14{,}54;C=\frac{\sqrt{6}-\sqrt{3}}{6}\rightarrow 6{,}8674688;D=\frac{\sqrt{6}+\sqrt{3}}{6}\rightarrow 44{,}18$

$(-A,-E),(B),(-C),(-D)$

$E=\frac{5}{2\sqrt{79-6\sqrt{6}}}\rightarrow 71{,}8345;F=\frac{5}{2\sqrt{79+6\sqrt{6}}};G=\frac{\sqrt{3}}{2\sqrt{9+2\sqrt{2}}};H=\frac{\sqrt{3}}{2\sqrt{9-2\sqrt{2}}}$

$\cos b=1/4$

$\cos a=0{,}40*\cos(\mu-60)-0{,}517$

- 0,12499993651388152595875715261479

- 0,78484685682830825643429815223337