EEE 431 - Digital Communications

Computational Assignment 1

Due: March 28, 2025, 23:59

In this assignment, you will implement various topics you learned in the class using MATLAB. Whenever a question requires plotting, numerical computation, or simulation, use MATLAB. Certain parts will also require analytical work. Please use a fixed random seed in your code so that your results are reproducible.

You can discuss the problems with your colleagues, but you need to write your code on your own without any collaboration with others/other tools (colleagues, Chegg, AI Chatbots, etc.) and you need to write your comments in your report with your own words and your own understanding.

The March 28 deadline is sharp, i.e., no deadline extensions will be granted.

1 Quantization

In this part you will perform uniform and non-uniform quantization. Let X and Y be independent, identically distributed random variables, both following an exponential distribution with parameter $\lambda = 1$. Their joint PDF is:

$$f_{X,Y}(x, y) = e^{-(x+y)}, \quad x > 0, y > 0 \ .$$

(a) Let $Z = X - Y$. Derive $f_Z(z)$ and plot the resulting theoretical PDF. Provide a clear, step-by-step derivation in your report.

(b) Generate 100 realizations of (X, Y), and calculate the corresponding realizations of Z. Plot the histogram for the realizations of Z and compare it to $f_Z(z)$. Repeat the process for 100000 realizations. Compare the two histograms and comment on how they differ.

(c) Suppose that a random source emits i.i.d. samples with distribution of Z. What is the average source power $\mathbb{E}[Z^2]$? Then, using 100000 realizations of Z, calculate the average value of squares of these realizations, and compare it with the theoretical result you found.

(d) Calculate the quantized versions of Z (denoted by $\tilde{Z}$) by using a uniform quantizer with $N = 8$ levels. Calculate the quantization errors corresponding to the source samples. Then, calculate the average power of the quantization error and compute the resulting SQNR in dB ($10 \log(\text{SQNR})$).

(e) Determine the probability of each possible quantization output based on their frequency of occurrence, and plot the resulting PMF.

(f) Now, you will use a compander (compressor, uniform quantizer, expander) to study non-uniform quantization. If you haven't derived the CDF of Z, $F_Z(z)$, in part (a), derive it. Obtain the inverse of CDF of Z, $F_Z^{-1}(z)$. Implement both $F(z)$ and $F_Z^{-1}(z)$ in MATLAB.

(g) Implement a non-uniform PCM quantizer with compressor $g(z) = F_Z(z)$ and expander $g^{-1}(z) = F_Z^{-1}(z)$, and a uniform quantizer with $N = 8$ levels. Plot the quantization regions and corresponding reconstruction points using MATLAB, which is $Q(z)$ versus z.

(h) Calculate the quantized versions of Z (denoted by $\tilde{Z}$) by this quantizer. Calculate the quantization errors corresponding to the source samples. Then, calculate the average power of the quantization error and compute the resulting SQNR in dB ($10\log(\text{SQNR})$). Compare the results with part (d).

(i) Determine the probability of each possible quantization output based on their frequency of occurrence, and plot the resulting PMF. Compare the results with part (e).

(j) Now, you will use the Lloyd-Max algorithm. Determine and report the boundaries of the quantization regions and the reconstruction levels. You should implement the algorithm yourself, and explain your implementation.

(k) Calculate the quantized versions of Z (denoted by $\tilde{Z}$) by this quantizer. Calculate the quantization errors corresponding to the source samples. Then, calculate the average power of the quantization error and compute the resulting SQNR in dB ($10\log(\text{SQNR})$). Compare the results with part (d) and (h).

(l) Determine the probability of each possible quantization output based on their frequency of occurrence, and plot the resulting PMF. Compare the results with part (e) and (i).

(m) Discuss whether the Lloyd-Max algorithm is guaranteed to outperform other techniques in all scenarios. Explain any reasons why it might not.