EEE 431 - Digital Communications
Computational Assignment 2

Due: April 26, 2025, 23:59

1 Filtering of Random Processes

In this part, you will generate a random process, estimate its PSD and filter it.

(a) Consider the AR random process given by

$$W_n = 0.9W_{n-1} + V_n$$

where V_n is a zero mean IID Gaussian noise. Show that W_n is a WSS process.

(b) Generate a realization of W_n for $n = 1, \ldots, N$ with $N = 10^6$. Plot a small section of W_n.

(c) Input W_n to a system described by

$$Y_n = (W_n + W_{n-1} + \cdots + W_{n-19})/20 \tag{1}$$

assume $W_n = 0$ for $n < 0$. Zoom to a smaller range of samples and plot the W_n on top of Y_n. Comment on the differences.

(d) Estimate the PSDs of W_n and Y_n. Plot these estimations. Also estimate magnitude response of the system in (1). Analytically find the impulse response and magnitude response of this system. Plot and compare the estimated and analytically found magnitude responses. (**HINT:** Check out *pwelch* function in MATLAB. To estimate the system's magnitude response, use the PSDs you have estimated.)

(e) Generate a white Gaussian noise of length N and variance 1. Repeat parts (c) and (d) for that noise. Plot the PSD of Gaussian noise and W_n together.

2 DSB-SC and Frequency Offset

In this part, we will investigate a message signal $m(t)$ being modulated using DSB-SC and demodulated with a frequency offset. The message $m(t)$ is given as

$$m(t) = \sin(2\pi 100t)$$

to represent the analog signal $m(t)$ for $t \in [0, T]$ in MATLAB, choose a sampling frequency F_s and use the following snippet

```
Fs = 100000;
t = [0:T*Fs]'/Fs;
mt = sin(200*pi*t)
```

where sampling frequency $F_s = 100$kHz is given. The same snippet will be used to represent signals through this part. See here for further details and implementation ideas. Note that the guide in the link is for SSB-AM modulation and demodulation. You should implement the coherent detector yourself.

To modulate the signal, choose the carrier frequency $f_c = 5$kHz. In real life, every oscillator will have a small frequency offset. In this part, we will investigate the phenomena called Carrier Frequency Offset (CFO) and its effect on demodulation. Consider coherent detection, but now local oscillator generates the signal

$$x(t) = \cos(2\pi(f_c + \Delta)t)\ . \tag{2}$$

(a) In MATLAB, modulate $m(t)$ with DSB-SC using the given frequencies.

(b) Implement coherent detection for the signal in (2) with $\Delta \in \{0, 50, 100, 500, 1000\}$. Plot a small section of the demodulated signals on top of each other. Comment on your results.

(c) CFO is an advanced topic in telecommunications. Provide a short discussion on what can be done to eliminate this problem.

(d) Another sub optimality is phase difference between the modulated and generated signal. For this question, assume $\Delta = 0$, but the local oscillator has phase difference ϕ. Namely, we have

$$x(t) = \cos(2\pi f_c t + \phi) \ ,$$

Simulate coherent detection with $\phi \in \{0, \pi/6, \pi/3, \pi/2\}$. Plot your results and comment.