EEE 473/573 Medical Imaging – Fall 2024-2025
Homework 4
Due 23 December 2024, Monday at 11:59AM (No grace period)

GUIDELINES FOR HOMEWORK SUBMISSION

1. Submit your solution via Moodle. No submission via e-mail (all email submissions will be discarded).
2. Submit a single PDF file. Other file types will not be accepted. If there are any handwritten parts, you can scan them (make sure they are legible) and insert into the PDF file. No partial credits to unjustified answers.
3. This is a Turnitin submission. The Turnitin system requires the submitted file to contain at least 20 words in it. If you are submitting a Word file with scanned pages only, the file may be rejected by the system. You can type your name multiple times at the beginning of the file to overcome this problem.
4. For the part labeled as "MATLAB Question", you can choose to use MATLAB or other software (e.g., Python). Make sure to include the relevant codes at the end of the PDF file to be submitted. If your codes are missing, that question will NOT be graded.

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1) The RF pulse in the transverse plane can be expressed as $B_1(t) = B_1^e(t)e^{-j2\pi\nu_0 t}$ given in the units of Gauss (G) and the envelope of the RF pulse is given as:

$$B_1^e(t) = Ae^{-t^2/\sigma^2} rect\left(\frac{t}{10\sigma}\right).$$

Note that ν_0 is the Larmor frequency at z=0 of the MRI scanner with magnetic field strength of 3 T, and $\sigma = 0.3\ ms$. Hence, total duration of the RF pulse is $8\sigma = 3\ ms$.

a) Using FWHM metric, calculate the bandwidth $\Delta\nu$ of the RF pulse.
b) Calculate the value of the slice selection gradient G_z to select a slice with a thickness of 4 mm with this RF pulse.
c) Calculate the value of A to make $\alpha(z=0) = \pi/3$, where $\alpha(z)$ is the flip angle as a function of z-position, also called the "slice profile".

2) If $\boldsymbol{M}$ is initially in equilibrium, the components of $\boldsymbol{M}$ after an RF pulse with tip angle α applied at t=0 are as follows:

$$M_z(t) = M_0\left(1 - e^{-\frac{t}{T_1}}\right) + M_0\cos(\alpha)\, e^{-\frac{t}{T_1}}$$

$$M_{xy}(t) = M_0 \sin(\alpha)\, e^{j\theta} e^{-\frac{t}{T_2}}$$

Suppose that the sample is excited with repeated RF pulses of tip angle α, separated by a repetition time T_R. Show that after a sufficient number of repetitions, a steady-state will be reached as follows:

$$M_{xy}(t) = M_z^{ss} \sin(\alpha)\, e^{j\theta} e^{-\frac{t}{T_2}}$$

where,

$$M_z^{ss} = M_0 \frac{1 - e^{-\frac{T_R}{T_1}}}{1 - \cos(\alpha)\, e^{-\frac{T_R}{T_1}}}$$

Here, M_z^{SS} is the steady-state value for $M_z(t)$ right before each RF pulse. Assume that the transverse magnetization has completely decayed before each RF pulse, i.e., $T_R \gg T_2$ and $M_{xy}(T_R) = 0$.

Hint: Relate M_z^n and M_z^{n+1}, i.e., the M_z values after n^{th} and $(n+1)^{th}$ pulses respectively.

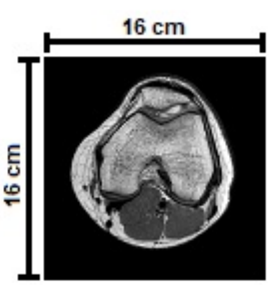
3) We would like to image an axial cross-section of the knee using a 3 T MRI scanner. We want field-of-view in both the x- and y- directions to be 16 cm. We want 1 mm x 1 mm resolution. The samples are acquired 8 μs apart during data acquisition. We want to design a typical gradient-echo sequence (i.e. line-by-line k-space acquisition).

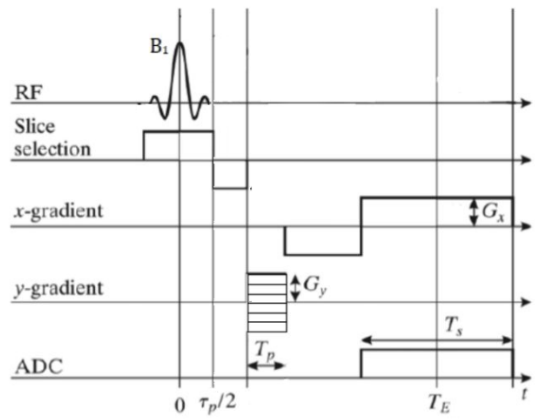
a) We want a 90° excitation and 3 mm slice thickness. We are using a sinc-shaped RF pulse with main lobe and two side lobes on each side, as shown in the figure below. Note that $B_1(t) = B_1^e(t)e^{-j2\pi\nu_0 t}$, where $B_1^e(t)$ is the envelope of the RF pulse. If we want the duration of the RF pulse to be 4 ms (i.e., τ_p = 4 ms), what is the required gradient strength G_z for this slice selection? What is the amplitude B_1 for this τ_p?

b) Find number of phase encoding lines N_y and readout samples N_x?

c) What is T_s? What is G_x?

d) If we want to select a slice at z = 1 cm with the same slice thickness of 3 mm, how should we change the RF pulse? Assume slice selection gradient G_z stays the same.





4) MATLAB Question: Include your MATLAB codes in your solution.

T2 mapping: If you estimate the T_2 for every pixel in an MRI image and display this as an image, it is called a "T_2 map". So, every pixel in this "T_2 map" image corresponds to the estimated T_2 of the corresponding pixel in the MRI image.

Download the files *brainT2_mri.mat* and roiellipse.m posted on Moodle. The file *brainT2_mri.dat* contains two sets of simulated brain MRI images:

• *image1* and *image2*: Simulated T_2-weighted MRI images with TE = 60 ms and TE = 120 ms, respectively. For both images, = 90° and TR = 4000 ms. No noise added.
• *image1_noisy* and *image2_noisy*: Same as above, with a very small amount of Gaussian noise added to both images (noise is added in k-space, which is where the measurements are taken).

a) Derive an equation for how the T_2 map, $T_2(x, y)$ can be calculated from images at two different echo times: IMG_1 at TE_1 and IMG_2 at TE_2. You can assume the following for each image:

$$IMG_i(x,y) = A\, M_0(x,y) sin\alpha\; e^{-TE_i/T_2(x,y)}$$, where A is some constant.

b) Calculate and display the T_2 map for the noise-free dataset (i.e., image1 and image2), and the noisy dataset (i.e., image1_noisy and image2_noisy).
We recommend doing the following to restrict the range of T_2 values displayed to between 0 and

350 ms, and to make sure the estimated T_2 map is real-valued:

```
figure, imshow(abs(T2map),[0 350])
```

c) For the noise-free data set, estimate T_2 for white matter using roiellipse.m. Here is the MATLAB snippet that you can use to do that:

```
figure, imshow(T2map,[])
mask = roiellipse; % type "help roiellipse" to see how to use it
T2_est = mean(T2map(mask))
```

This code will create a figure displaying your T_2 map. It will then place a draggable and scalable ellipse on it. Drag this ellipse on a part of the image that contains ***white matter only*.** Select a reasonably large ellipse, so that the mean T_2 estimation is more accurate. We recommend maximizing the window containing the figure first, so that you can move/scale the ellipse more carefully. When you are ready, click the "Continue" button on the bottom left of the window. This will create a "mask" of the selected elliptical region, and then calculate the mean T_2 value in that region using the mask.
In your solution, include the image with ellipse showing the selected region, together with the estimated T_2 value.

d) Use the same "mask" in part (c) to estimate the T_2 for white matter for the noisy dataset. What is the estimated value? How much did it deviate (percentage-wise) from the noise-free dataset estimation?

e) Repeat parts (c-d) for gray matter. Include the image with ellipse in your solution, together with the estimated T_2 values.

f) Repeat parts (c-d) for CSF (cerebrospinal fluid). Include the image with ellipse in your solution, together with the estimated T_2 values.

g) Which noisy T_2 estimate showed the biggest deviation from its noise-free version? Why?

Hint: If you are not sure which part of the brain is gray matter, white matter, or CSF, here is a segmented version of the simulated MRI image: