EEE 473 Medical Imaging
Homework #4
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Q1)

a) $\alpha = \gamma \int B_1^e(t)\, e^{-j2\pi(\nu - \frac{\gamma}{\delta})t}\, e^{j2\pi\nu t}\, dt$

$= \gamma A\, \mathcal{F}_{1D}\Big\{ \underbrace{e^{-\frac{t^2}{\sigma^2}} \mathrm{rect}\left(\frac{t}{10\sigma}\right)}_{\approx e^{-\frac{t^2}{\sigma^2}}} \Big\}(\nu) \approx \gamma A \sqrt{\pi}\,\sigma\, e^{-\sigma^2\pi^2\nu^2}$

because at $t = 10\sigma$ $e^{-100} = 3.72 \times 10^{-44} \approx 0$

$\Rightarrow e^{-\sigma^2\pi^2\nu^2} = \frac{1}{2} \Rightarrow -\pi^2\sigma^2\nu^2 = -\ln(2) \Rightarrow \nu^2 = \frac{\ln(2)}{\pi^2\sigma^2} \Rightarrow \nu = \frac{\sqrt{\ln(2)}}{\pi\sigma}$

$\mathrm{FWHM} = \Delta\nu = \frac{2\sqrt{\ln(2)}}{\pi\sigma} = \frac{2\sqrt{\ln(2)}}{\pi\,(3\,ms)} = \boxed{1.77\ kHz}$

b) $\Delta\nu = \gamma G_z \Delta z \Rightarrow G_z = \frac{\Delta\nu}{\gamma\Delta z}$

$G_z = \frac{1.77\ kHz}{42.58\ \frac{MHz}{T} \cdot 4mm} = 0.1\ \frac{mT}{cm} = 10\ \frac{G}{cm}$

c) $\alpha(z=0) = \gamma\int_{-\infty}^{\infty} B_1^e(t)\,dt = \gamma\int_{-\infty}^{\infty} A\, \underbrace{e^{-\frac{t^2}{\sigma^2}} \mathrm{rect}\left(\frac{t}{10\sigma}\right)}_{\approx e^{-\frac{t^2}{\sigma^2}}} dt = \frac{\pi}{3}$

$\alpha(z=0) \approx \gamma A \int_{-\infty}^{\infty} e^{-\frac{t^2}{0.3}}\, dt = \gamma A \sqrt{2\pi}\,\frac{\sigma}{\sqrt{2}} = \gamma A\sqrt{\pi}\,\sigma$

$\alpha(z=0) = \gamma A\sqrt{\pi}\,\sigma = \frac{\pi}{3} \Rightarrow A = \frac{\pi/3}{2\pi\gamma\sqrt{\pi}\,\sigma} = \frac{\pi/3}{2\pi\,(42.58\ MHz/T)\,(\sqrt{\pi}\ 0.3\,ms)}$

$\Rightarrow A = \frac{1}{6\sqrt{\pi}\cdot 42.58\times 10^6 \cdot 0.3\times 10^{-3}} = \boxed{0.0736\ G}$

Q2) Let us find the pattern:

$M_z^0(t) = M_0\left(1 - e^{-\frac{t}{T_1}}\right) + M_0\cos(\alpha)\, e^{-\frac{t}{T_1}}$ for $t \in [0^+, T_R^-]$

$M_z^1(t) = M_0\left(1 - e^{-\frac{t - T_R}{T_1}}\right) + M_z^0(T_R^-)\cos(\alpha)\, e^{-\frac{(t - T_R)}{T_1}}$ for $t \in [T_R^+, 2T_R^-]$

$\vdots$

$M_z^n(t) = M_0\left(1 - e^{-\frac{t - nT_R}{T_1}}\right) + M_z^{n+1}(nT_R^-)\cos(\alpha)\, e^{-\frac{t - nT_R}{T_1}}$ for $t \in [nT_R^+, (n+1)T_R^-]$

In order to reach steady-state:

$$M_z^{ss} = M_z^{n-1}(n T_R^-) = M_z^{n}((n+1) T_R^-)$$

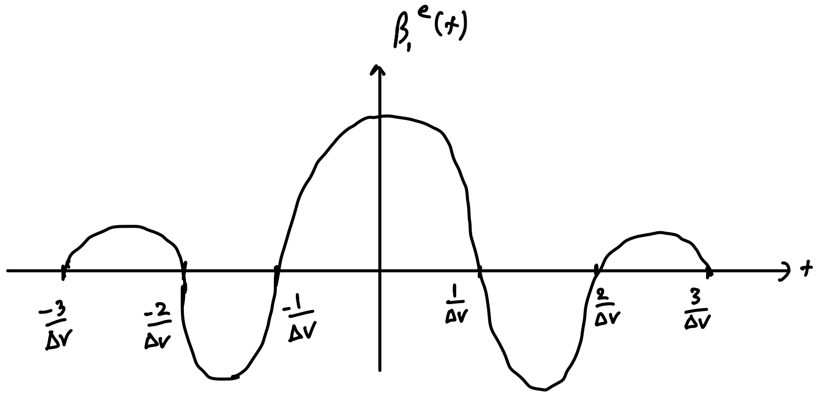
$$M_z^{ss} = M_0 \left(1 - e^{-\frac{T_R}{T_1}}\right) + M_z^{ss} \cos(\alpha)\, e^{-\frac{T_R}{T_1}}$$

$$\Rightarrow M_z^{ss}\left(1 - \cos(\alpha)\, e^{-\frac{T_R}{T_1}}\right) = M_0 \left(1 - e^{-\frac{T_R}{T_1}}\right)$$

$$\Rightarrow \boxed{M_z^{ss} = M_0 \frac{1 - e^{-\frac{T_R}{T_1}}}{1 - \cos(\alpha)\, e^{-\frac{T_R}{T_1}}}}$$

Q3)

a) $B_1^e(t) = A \operatorname{sinc}(\Delta v\, t)$



$$\frac{6}{\Delta v} = T_p = 4\,ms$$

$$\Rightarrow \Delta v = 1.5\ kHz$$

$$\Delta v = \gamma G_z \underbrace{\Delta z}_{\text{slice thickness} = 3mm} \Rightarrow G_z = \frac{\Delta v}{\gamma \Delta z} = \frac{1.5k\ Hz}{42.58\ \frac{MHz}{T} \cdot 3mm} \Rightarrow \boxed{G_z = 11.74\ \frac{mT}{m}}$$

$$\alpha = \gamma \int_{-2}^{2} B_1^e(t)\, dt = \gamma \int_{-2}^{2} A \operatorname{sinc}(1.5 t)\, dt = \gamma A\, 0.71 = \frac{\pi}{2}$$

$$A = \frac{\pi/2}{2\pi \cdot 42.58\ \frac{MHz}{T} \cdot 0.71ms} = \frac{1}{4 \cdot 42.58 \times 10^6 \cdot 0.71 \times 10^{-3}} = \boxed{8.27\ \mu T}$$

b) $N_y = N = \frac{16cm}{1mm} = \boxed{160}$

c) $T_s = N_x T = 160 \cdot 8\mu s = \boxed{1.28ms}$

$$FOV_x = \frac{1}{\Delta k_x} = \frac{1}{\gamma G_x T}$$

$$\Rightarrow G_x = \frac{1}{\gamma\, FOV_x\, T} = \frac{1}{\left(42.58\ \frac{MHz}{T}\right)(16\,cm)(8\mu s)} = \boxed{0.18\ \frac{mT}{cm}}$$

d) $B_1(t) = B_1^e(t)\, e^{-j2\pi \nu_1 t}$ where $\nu_1 = \nu_0 + \gamma G_z z \Big|_{z=1cm}$

$$\nu_1 = \gamma B_0 + \gamma G_z(1cm) = 42.58\,\frac{MHz}{T} \cdot 3T + 42.58\,\frac{MHz}{T}\; 11.74\,\frac{mT}{m}\; 0.01\,m$$

$$\boxed{\nu_1 = 127.745\ MHz}$$

Q4) a)

$$\frac{IMG_1}{IMG_2} = \frac{AM_0(x,y)\,\sin(\alpha)\,\exp\left(-\frac{TE_1}{T_2(x,y)}\right)}{AM_0(x,y)\,\sin(\alpha)\,\exp\left(\frac{-TE_2}{T_2(x,y)}\right)} = \frac{\exp\left(\frac{-TE_1}{T_2(x,y)}\right)}{\exp\left(-\frac{TE_2}{T_2(x,y)}\right)} = \exp\left(\frac{TE_2 - TE_1}{T_2(x,y)}\right)$$

$$\ln\left(\frac{IMG_1}{IMG_2}\right) = \frac{TE_2 - TE_1}{T_2(x,y)}$$

$$\boxed{T_2(x,y) = \frac{TE_2 - TE_1}{\ln\left(\frac{IMG_1}{IMG_2}\right)}}$$

b)

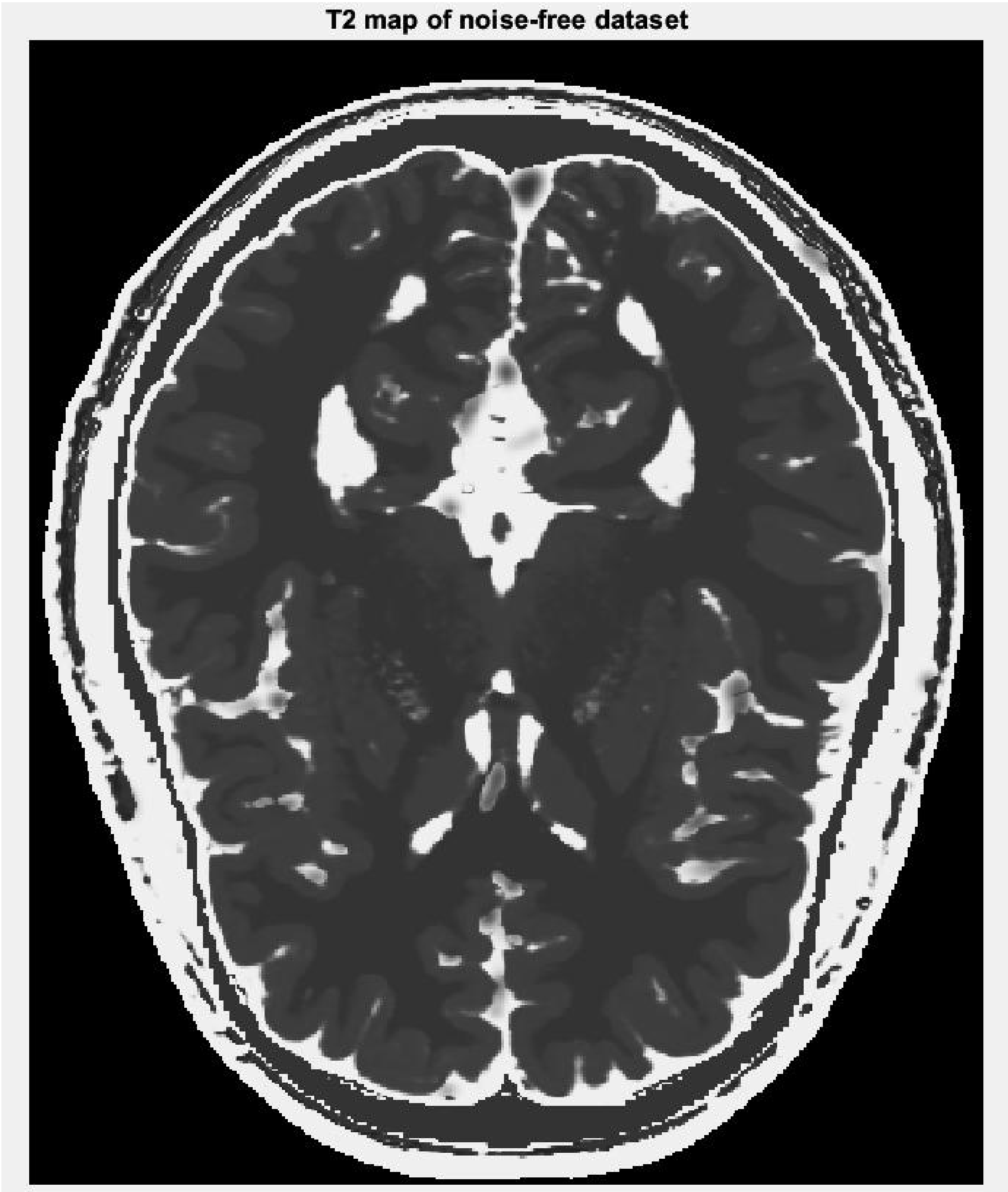


Fig.1 T_2 map of noise-free dataset

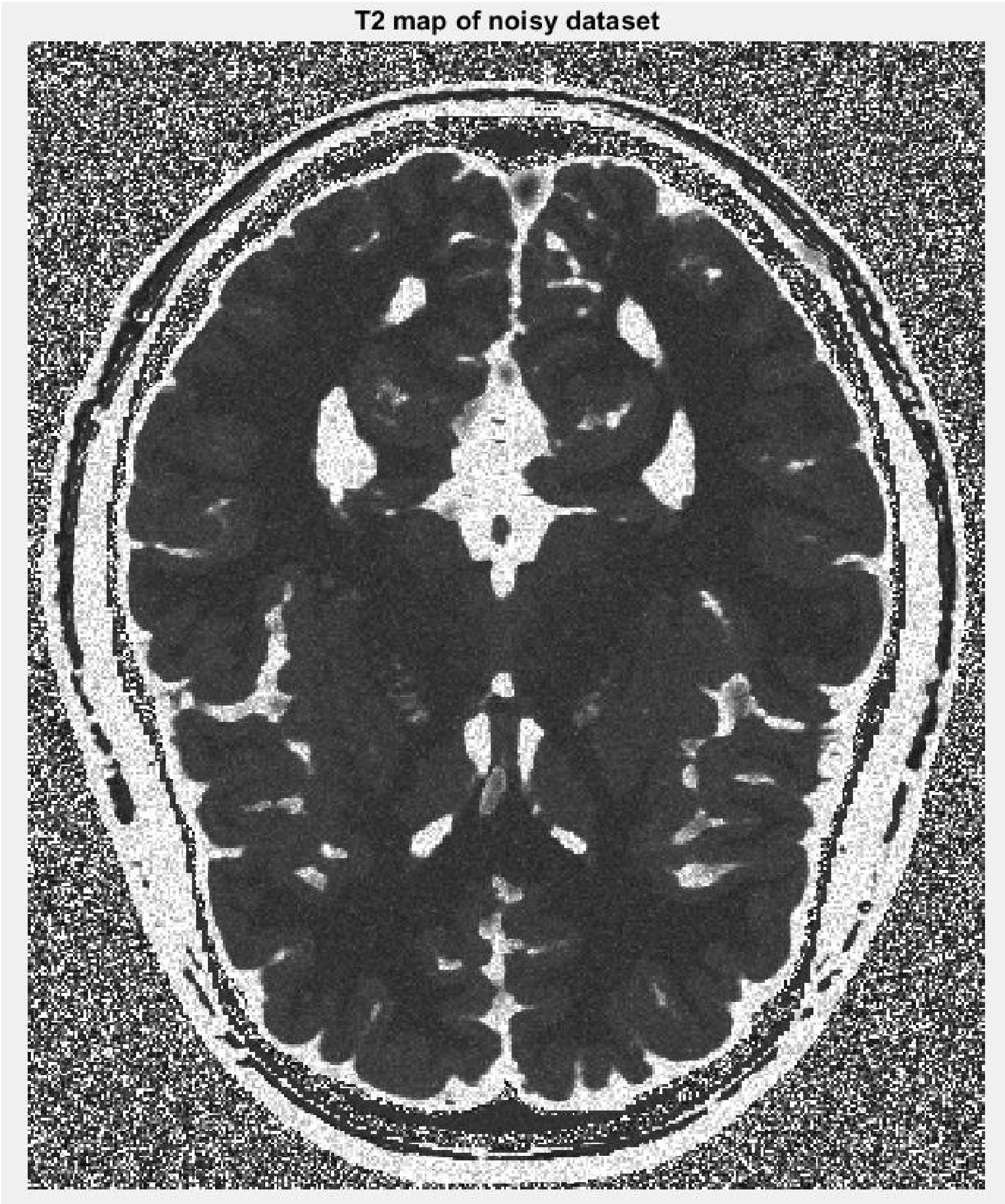


Fig.2 T_2 map of noisy dataset

c) d)

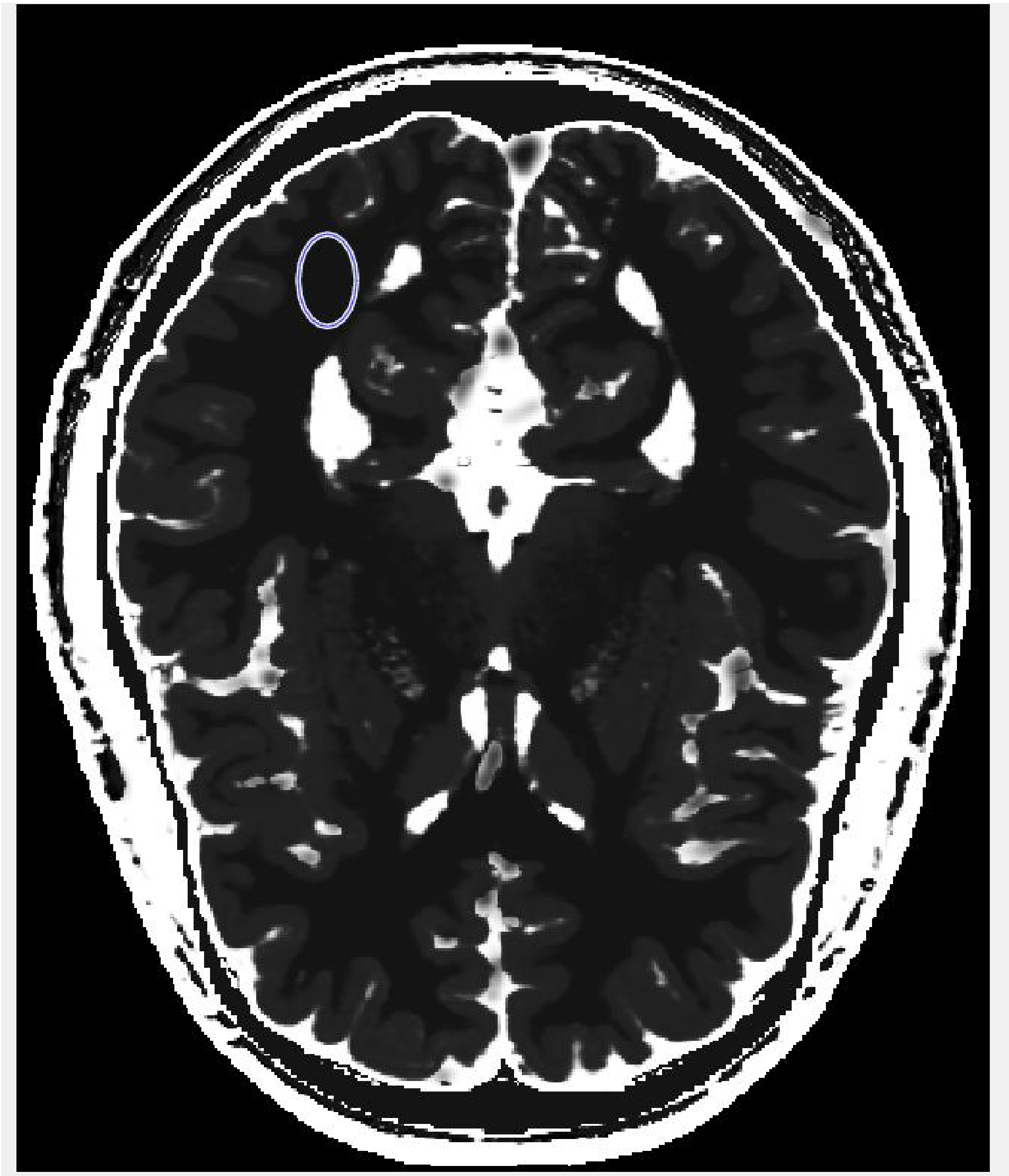


Fig.3 T_2 map with an ellipse on a region that contains white matter only

e)

Fig.4 T_2 map with an ellipse on a region that contains gray matter only

f)

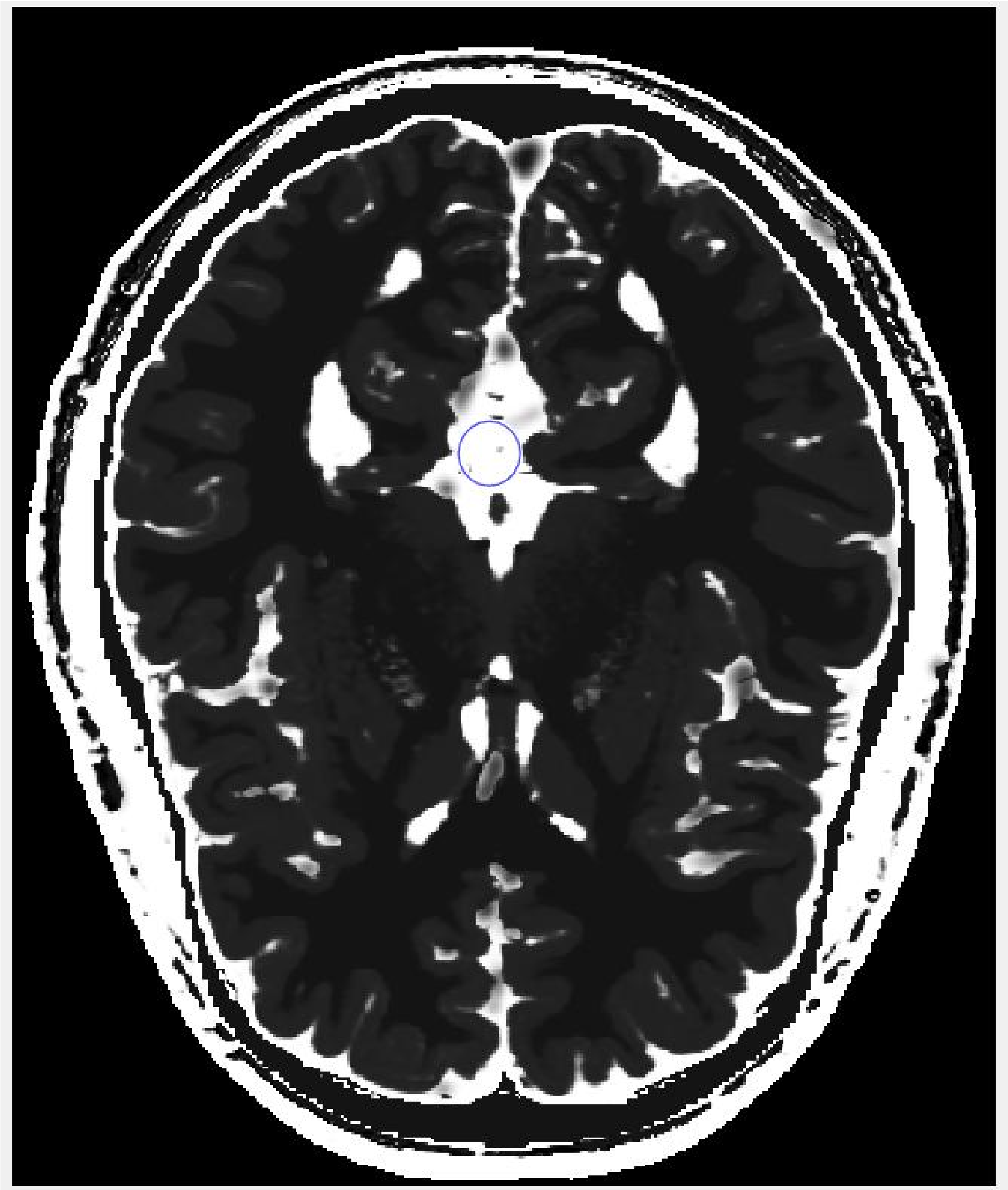


Fig.5 T_2 map with an ellipse on a region that contains CSF (cerebrospinal fluid) only

g)

Table.1 T_2 estimations and deviation percentages

T_2 estimations (ms)	Noise-Free Dataset	Noisy Dataset	Error (%)
White matter	70.0911	70.7212	0.8989
Gray matter	84.8766	86.1736	1.5280
CSF	324.5989	333.1363	2.6301

Table.1 depicts that the T_2 estimation of the CSF showed the biggest deviation in the noisy dataset compared to noise-free dataset with around 2.63% deviation. This higher deviation can be attributed to the inherent characteristics of CSF, which has a longer T2 relaxation time. As a result, the signal is more susceptible to noise interference, particularly in the presence of Gaussian noise. Additionally, the logarithmic calculation in the T2 mapping amplifies small differences in signal intensity, leading to a greater deviation for regions with high T2 values. In contrast, white matter, with its lower T2 value, exhibited the smallest deviation of 0.89%, as its signal is less affected by noise. These findings underscore the need for effective noise reduction techniques, particularly for tissues like CSF.

Appendix - MATLAB code

PART4.m

```
close all; clear; clc

load('brainT2_mri.mat');

TE1=60;
TE2=120;
T2map=(TE2-TE1)./log(image1./image2);
T2map_noisy=(TE2-TE1)./log(image1_noisy./image2_noisy);

figure;imshow(abs(T2map),[0,350]);
title("T2 map of noise-free dataset");

figure; imshow(abs(T2map_noisy),[0,350]);
title("T2 map of noisy dataset");

figure;imshow(T2map,[])
mask=roiellipse;

T2_est=mean(T2map(mask))
T2_est_noisy=mean(T2map_noisy(mask))
error = (abs(T2_est - T2_est_noisy)/T2_est)*100
```