# EEE 473 Medical Imaging
# Homework #3

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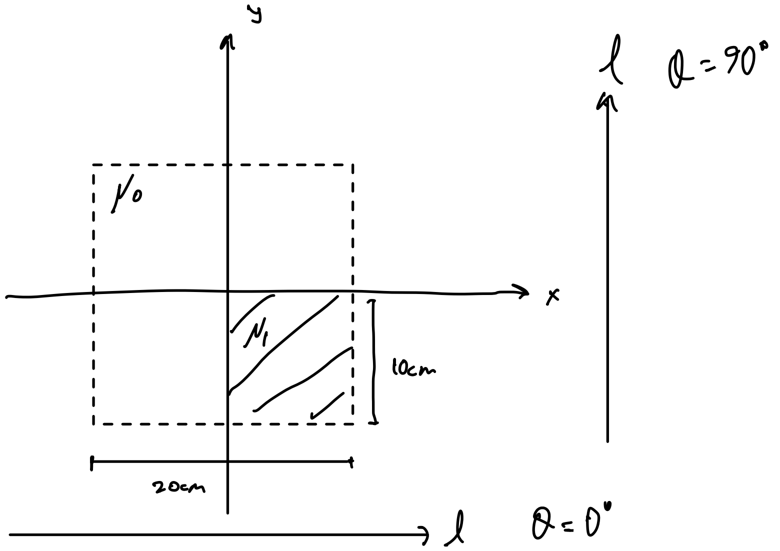
Q1)




$$g(l,\theta) = \int_{-\infty}^{\infty} f(l\cos\theta - s\sin\theta,\ l\sin\theta + s\cos\theta)\, ds$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, \delta(x\cos\theta + y\sin\theta - l)\, dx\, dy$$

$$f(x,y) = \mu(x,y;\bar{E})$$

$$g(l,\theta) = -\ln\left(\frac{I_d}{I_0}\right) = \int_0^d \mu(s;E)\, ds$$

a)

$$g(l,0) = \int_{-\infty}^{\infty} f(l, s)\, ds = \int_{-\infty}^{\infty} f(l,y)\, dy$$

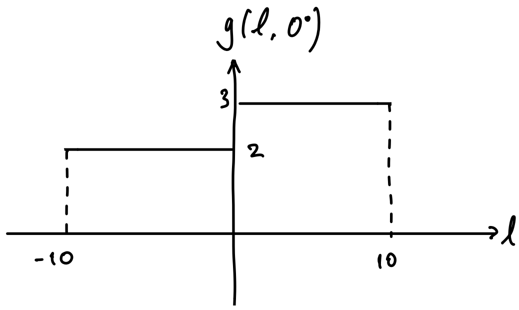
for $-10 < l < 0$

$$\int_{-10}^{10} 0.1\, dy = 2$$

for $0 < l < 10$

$$\int_{-10}^{0} 0.2\, dy + \int_{0}^{10} 0.1\, dy = 2 + 1 = 3$$

$$g(l,0°) = \begin{cases} 2 & \text{if } -10 < l < 0 \\ 3 & \text{if } 0 < l < 10 \\ 0 & \text{ow} \end{cases}$$



b)

$$g(l, 90^\circ) = \int_{-\infty}^{\infty} f(l, s)\, ds = \int_{-\infty}^{\infty} f(l, x)\, dx$$

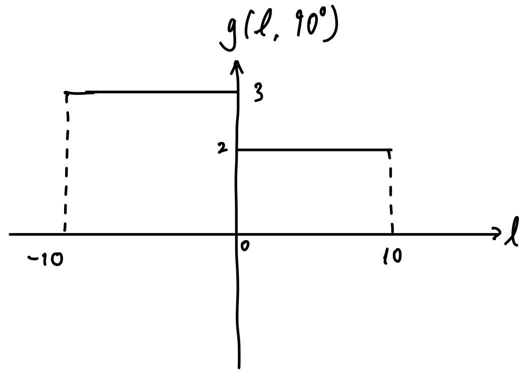
$$g(l, 90^\circ) = \begin{cases} 3 & \text{if } -10 < l < 0 \\ 2 & \text{if } 0 < l < 10 \\ 0 & \text{ow} \end{cases}$$

for $-10 < l < 0$

$$\int_{-10}^{0} 0.2\, dx + \int_{0}^{10} 0.1\, dx = 3$$

for $0 < l < 10$

$$\int_{-10}^{10} 0.1\, dx = 2$$




Q2) $G(\rho, \theta) = \mathcal{F}_{1D}\{g(l, \theta)\} = F(\rho\cos\theta, \rho\sin\theta) \Rightarrow \mathcal{F}_{1D}^{-1}\{F(\rho\cos\theta, \rho\sin\theta)\} = g(l, \theta)$

$$f(x, y) = f(x) f(y) = e^{-x^2} \Rightarrow F(u, v) = F(u) F(v)$$

- $f(x) = e^{-x^2}$

$$F(u) = \mathcal{F}_{1D}\{e^{-x^2}\} = \mathcal{F}_{1D}\left\{e^{-\pi\left(\frac{x^2}{\pi}\right)}\right\} = \sqrt{\pi}\, e^{-\pi^2 u^2}$$

- $f(y) = 1$

$$F(v) = \mathcal{F}_{1D}\{1\} = \delta(v)$$

- $F(u, v) = \sqrt{\pi}\, e^{-\pi^2 u^2}\, \delta(v)$

$$F(\rho\cos\theta, \rho\sin\theta) = \sqrt{\pi}\, e^{-\pi^2(\rho\cos\theta)^2}\, \delta(\rho\sin\theta)$$

$$g(l, \theta) = \mathcal{F}_{1D}^{-1}\{F(\rho\cos\theta, \rho\sin\theta)\} = \mathcal{F}_{1D}^{-1}\left\{\sqrt{\pi}\, e^{-\pi^2(\cos\theta\, \rho)^2}\, \delta(\rho\sin\theta)\right\}$$

sifting

$$= \mathcal{F}_{1D}^{-1}\left\{\sqrt{\pi}\, e^{-\pi^2\cos^2\theta\,(0)^2}\, \delta(\rho\sin\theta)\right\}$$

$$= \sqrt{\pi}\, \mathcal{F}_{1D}^{-1}\{\delta(\rho\sin\theta)\}$$

$$\Rightarrow g(l, \theta) = \frac{\sqrt{\pi}}{|\sin\theta|} \quad //$$

Q3) $g(l,\theta) = \text{sinc}\left(l - a \cdot \sin\theta\right)$

$$\mathcal{F}_{1D}\{g(l.\theta)\} = G(\rho,\theta) = \text{rect}(\rho)\, e^{-j2\pi a \sin\theta \rho} = F(\rho\cos\theta, \rho\sin\theta) = F(u, v)$$

$$F(u,v) = \text{rect}(\rho)\, e^{-j2\pi a v}$$

$$\mathcal{F}_{2D}^{-1}\{F(u,v)\} = \mathcal{F}_{2D}^{-1}\left\{\text{rect}(\rho)\, e^{-j2\pi a v}\right\}(x,y) = \text{jinc}(r) * \delta(y-a)$$

$$= \text{jinc}\left(\sqrt{x^2+y^2}\right) * \delta(y-a)$$

$$= \text{jinc}\left(\sqrt{x^2+(y-a)^2}\right) \quad //$$

**Q4)**
**a)**

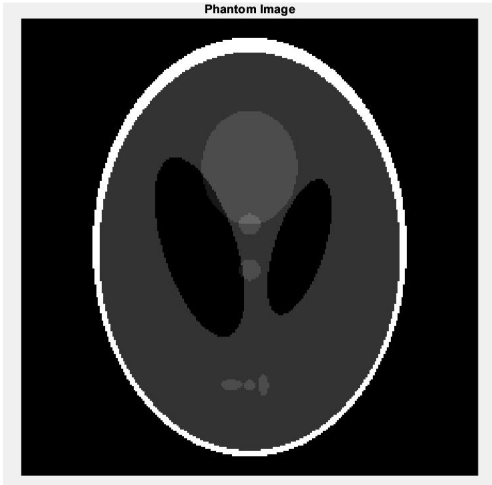



*Fig.1 Phantom Image*

**b)**

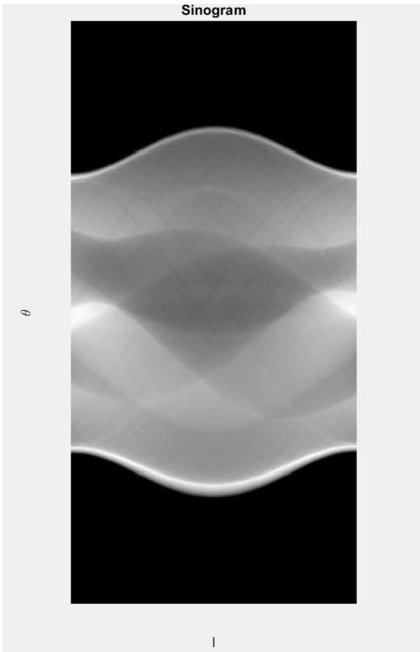



*Fig.2 Sinogram*

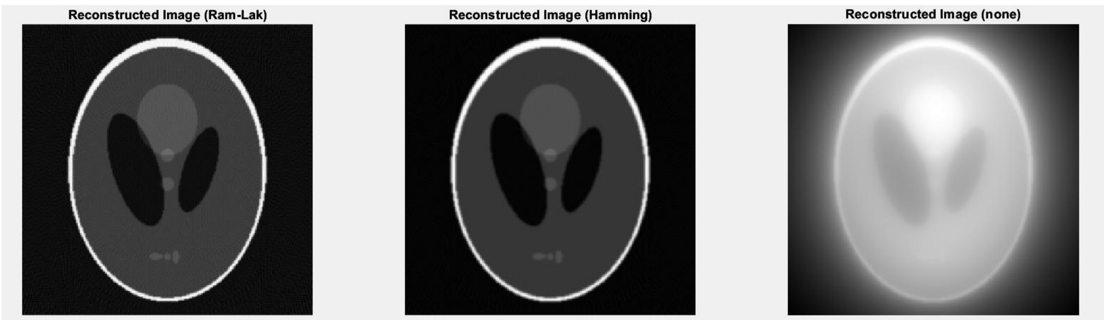



*Fig.3 Reconstructed Images using 3 different filters*

When reconstructing the image with 3 different filters, distinct variations in image quality are observed. The Ram-Lak filter produces sharper edges due to its ramp nature, ideal for highlighting structures but susceptible to noise. The Hamming filter, being smoother, reduces noise artifacts but also slightly blurs details. The “none” option, which uses direct backprojection, results in significant blurring and lacks definition, leading to a less accurate reconstruction. Each filter affects the clarity and noise handling of the reconstructed image.

**c)**

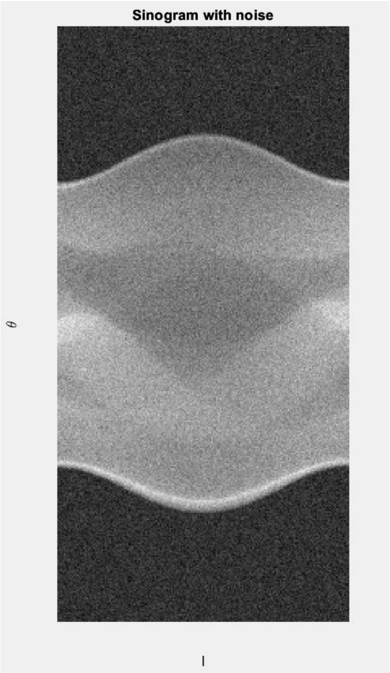



*Fig.4 Sinogram with noise*

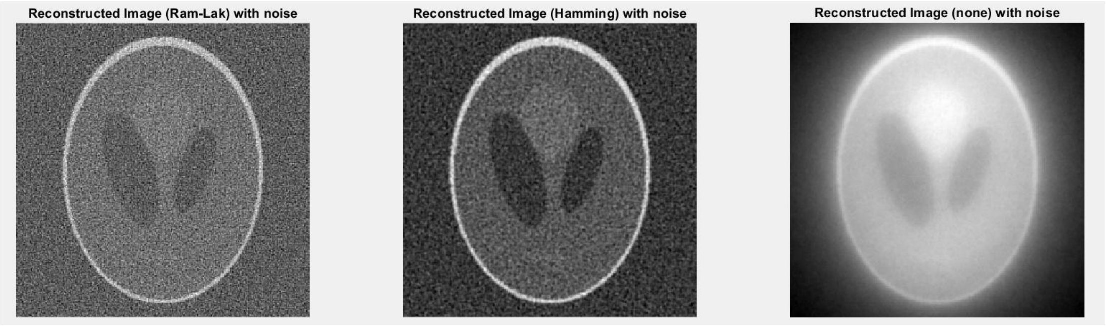



*Fig.5 Reconstructed Images using 3 different filters with noise*

With added zero mean Gaussian noise with standard deviation 5, the reconstructed images differ further. The Ram-Lak filter amplifies the noise alongside details, making the result grainier, while the Hamming filter reduces noise better due to its smoother characteristics but sacrifices sharpness. The “none” option performs poorly, as direct backprojection without filtering intensifies noise, leading to a highly blurred and noisy image. This demonstrates the importance of filtering in managing noise during reconstruction.

**d)**

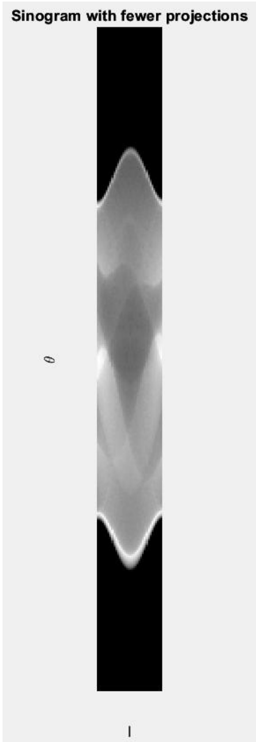



*Fig.6 Sinogram with fewer projections*

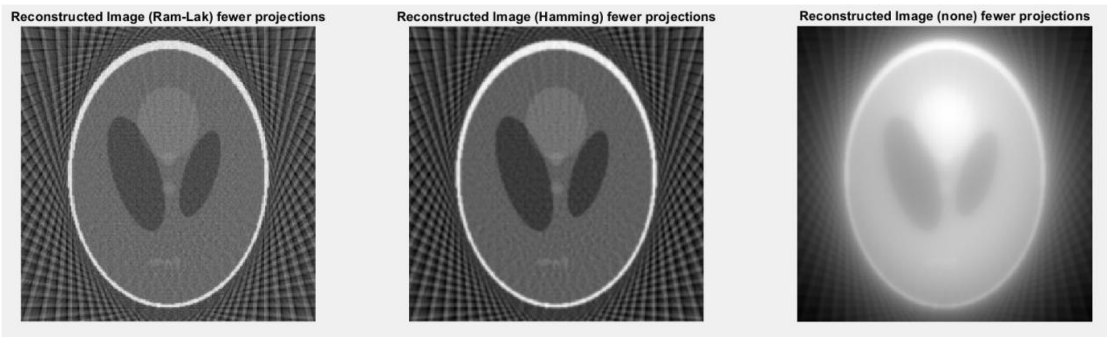



*Fig.7 Reconstructed Images using 3 different filters with fewer projections*

Using fewer projections introduces "streaking artifacts". This is due to insufficient data for accurate reconstruction. The Ram-Lak filter produces visible streaks and noise. The Hamming filter softens these artifacts but at the cost of additional blurring. The “none” filter performs worst, with pronounced streaking and a very low-quality image, highlighting how the number of projections impacts reconstruction quality.

## Appendix - MATLAB code

### hw3q4.m

```
close all;clear;

%a
P = phantom('Modified Shepp-Logan',256);
figure;
imshow(P,[]);
title("Phantom Image");

%b
theta= 0:179;
R = radon(P,theta);
figure;imshow(R,[]);
title("Sinogram");
xlabel("l", 'Interpreter', 'tex');
ylabel('\theta', 'Interpreter', 'tex')

figure;
I1 = iradon(R,theta,"linear","Ram-Lak");
subplot(1,3,1);
imshow(I1,[]);
title("Reconstructed Image (Ram-Lak)");

I2 = iradon(R,theta,"linear","Hamming");
subplot(1,3,2);
imshow(I2,[]);
title("Reconstructed Image (Hamming)");

I3 = iradon(R,theta,"linear","none");
subplot(1,3,3);
imshow(I3,[]);
title("Reconstructed Image (none)");

%c
noise_std = 5;
noise = noise_std * randn(size(R));
R = R + noise;

theta= 0:179;
figure;imshow(R,[]);
title("Sinogram with noise");
xlabel("l", 'Interpreter', 'tex');
ylabel('\theta', 'Interpreter', 'tex')

figure;
I1 = iradon(R,theta,"linear","Ram-Lak");
subplot(1,3,1);
imshow(I1,[]);
title("Reconstructed Image (Ram-Lak) with noise");

I2 = iradon(R,theta,"linear","Hamming");
subplot(1,3,2);
imshow(I2,[]);
title("Reconstructed Image (Hamming) with noise");

I3 = iradon(R,theta,"linear","none");
```

```
subplot(1,3,3);
imshow(I3,[]);
title("Reconstructed Image (none) with noise");

%d
theta= 0:5:179;
R = radon(P,theta);
figure;imshow(R,[]);
title("Sinogram with fewer projections");
xlabel("l", 'Interpreter', 'tex');
ylabel('\theta', 'Interpreter', 'tex')

figure;
I1 = iradon(R,theta,"linear","Ram-Lak");
subplot(1,3,1);
imshow(I1,[]);
title("Reconstructed Image (Ram-Lak) fewer projections");

I2 = iradon(R,theta,"linear","Hamming");
subplot(1,3,2);
imshow(I2,[]);
title("Reconstructed Image (Hamming) fewer projections");

I3 = iradon(R,theta,"linear","none");
subplot(1,3,3);
imshow(I3,[]);
title("Reconstructed Image (none) fewer projections");

clear;clc;
```