EEE 473/573 Medical Imaging – Fall 2024-2025
Homework #1
Due 13 October 2024, Sunday at 23:59

GUIDELINES FOR HOMEWORK SUBMISSION

1. Submit your solution via Moodle. No submission via e-mail (all email submissions will be discarded).
2. Submit a single PDF file. Other file types will not be accepted. If there are any handwritten parts, you can scan them (make sure they are legible) and insert into the PDF file. No partial credits to unjustified answers.
3. This is a Turnitin submission. The Turnitin system requires the submitted file to contain at least 20 words in it. If you are submitting a Word file with scanned pages only, the file may be rejected by the system. You can type your name multiple times at the beginning of the file to overcome this problem.
4. For the part labeled as "MATLAB Question", you can choose to use MATLAB or other software (e.g., Python). Make sure to include the relevant codes at the end of the PDF file to be submitted. If your codes are missing, that question will NOT be graded.
5. Submission system will remain open for 1 day after the deadline. No points will be lost if you submit your assignment within 12 hours of the deadline. There will be a 50% penalty if you submit after 12 hours but within 24 hours past the deadline. No submissions beyond 24 hours past the deadline.

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1) Find one image from each of the following medical imaging modalities: X-ray, CT, PET, ultrasound, and MRI. Clearly label the image type and indicate which body part is shown in the image (e.g., head, torso, heart, kidneys, etc.). For reference, include the URLs of the source webpages under each image or DOI number if images are taken from an article.

2) For each system with the following input-output equation, determine whether the system is (1) linear and (2) shift-invariant.

a) $g(x,y) = 3f(x-1, y+1) + f(0,y)$

b) $g(x,y) = |f(x,y)| + f(x-1,y)$

c) $g(x,y) = \begin{cases} 2f(x,3y) + f(x-1,y), & x \geq 0 \; and \; y \geq 0 \\ 0, & otherwise \end{cases}$

3) Answer the following questions for $f(x,y) = e^{-j2\pi(x+y)}$:

a) Simplify the following expression: $f(x,y)\delta(ax+b, cy-d)$

b) Evaluate the following expression: $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(ax+b, cy-d)\, dx\, dy$

c) Calculate the following 2D convolution (**Hint**: Use 2D FT): $sin(2\pi x + 3\pi y) * sinc(3x, 2y)$

4) In (a)-(b), find the 2D Fourier transforms of the continuous 2D signals. In (c), find the inverse 2D Fourier transform of the 2D spectrum. Simplify your answers as much as possible. You can use known Fourier transform pairs and properties.

a) $f(x,y) = \delta(y+1)$

b) $f(x,y) = rect(\frac{x}{4}, \frac{y}{3})e^{j\pi(2x+y)}$

c) $F(u,v) = rect(5u-2, \frac{v}{3})$

<u>MATLAB PREPARATION 1)</u> Centered FFT: In medical imaging, the preferred way to display the image and the Fourier domain data is such that the origin is at the center of the image or data array. The usual convention for the FFT in MATLAB, however, is that the origin is at the beginning of the array, or the upper left corner of a 2D array. To do a centered FFT, we need to perform fftshift/ifftshift before/after the FFT. To do this, define the following two functions in MATLAB and save them as two separate .m files:

```
function d = fft2c(im)
% d = fft2c(im)
%
% fft2c performs a centered fft2
d = fftshift(fft2(ifftshift(im)));
end

function im = ifft2c(d)
% im = fft2c(d)
%
% ifft2c performs a centered ifft2
im = fftshift(ifft2(ifftshift(d)));
end
```

When you type 'help fft2c' in MATLAB, you will now see the commented text that gives the function usage. Pay attention to including "help" sections when you create your own functions/scripts.

You may also need the corresponding one dimensional versions fftc and ifftc. Note that fftshift and ifftshift give exactly the same result when the array size is even-valued, but are different otherwise.

<u>MATLAB PREPARATION 2)</u> Displaying the magnitude spectrum: To display the magnitude spectrum as an image, we typically do the following:

```
>> imshow(log(abs(F)+1),[])
```

where F is the 2D Fourier transform of an image. In Fourier domain, the value at DC (i.e., at the origin of Fourier domain) is much larger than the values elsewhere. The "log" operation brings these values closer together, so that we can display and inspect the entire magnitude spectrum more easily. The addition of 1 is to avoid the log(0) problem.

5) MATLAB QUESTION: Generate a "phantom" image in MATLAB using the following command:

```
P = phantom('Modified Shepp-Logan',500);
```

This digital phantom presents an axial cross-section of a human body, showing the lungs, the heart, and a few blood vessels. Assume that this is our object of interest, with its physical x-axis and y-axis ranging from -25 cm $< x \leq 25$ cm and -25 cm $< y \leq 25$ cm (i.e., 500 pixels corresponding to a physical extent of 50 cm). "P" is our "ideal" image.

a) Display the "ideal" image P and its magnitude spectrum.

b) We image this phantom using a medical imaging systems with the following point spread function (PSF): $h(x,y) = sinc\left(\frac{x}{\sigma_x}, \frac{y}{\sigma_y}\right)$, where x and y are given in units of cm.

Assume that $\sigma_x = 0.3$ and $\sigma_y = 0.3$. Display the PSF $h(x, y)$ and the magnitude of the corresponding transfer function $H(u, v)$. Compute and display the image that we would get if we image P with this medical imaging system. Also, display the magnitude spectrum of this image.

c) Repeat part (b) for $\sigma_x = 0.3$ and $\sigma_y = 3$.

d) Repeat part (b) for $\sigma_x = 1.5$ and $\sigma_y = 1.5$.

e) Explain what you see in the resulting images in parts (b)-(c)-(d), and comment on how the image quality changes with different σ_x and σ_y values.