EEE 473/573 Medical Imaging – Fall 2024-2025
Homework 2
Due 3 November 2024, Sunday at 23:59

GUIDELINES FOR HOMEWORK SUBMISSION

1. Submit your solution via Moodle. No submission via e-mail (all email submissions will be discarded).
2. Submit a single PDF file. Other file types will not be accepted. If there are any handwritten parts, you can scan them (make sure they are legible) and insert into the PDF file. No partial credits to unjustified answers.
3. This is a Turnitin submission. The Turnitin system requires the submitted file to contain at least 20 words in it. If you are submitting a Word file with scanned pages only, the file may be rejected by the system. You can type your name multiple times at the beginning of the file to overcome this problem.
4. For the part labeled as "MATLAB Question", you can choose to use MATLAB or other software (e.g., Python). Make sure to include the relevant codes at the end of the PDF file to be submitted. If your codes are missing, that question will NOT be graded.
5. Submission system will remain open for 1 day after the deadline. No points will be lost if you submit your assignment within 12 hours of the deadline. There will be a 50% penalty if you submit after 12 hours but within 24 hours past the deadline. No submissions beyond 24 hours past the deadline.

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1) For the following questions, you can use known Hankel transform pairs and Fourier Transform pairs, and their properties.

- **a)** Find the Hankel transforms of $f(r) = \dfrac{\delta(ar)}{r}$.
- **b)** Find the Hankel transforms of $f(r) = rect(r - a)$, where $a > 1$.
  Hint: Try to visualize what this function looks like.
- **c)** Assume that $f(r)$ is a delta ring of radius 3, with its center shifted to position $(x, y) = (1,2)$. Express $f(r)$ in closed form. Is this function circularly symmetric? Find $F(u, v)$, the 2D Fourier transform of $f(r)$.

2) Consider the LSI system given below,

$$h(x,y) = e^{-\frac{x^2+8y^2}{2\sigma^2}} \quad with \quad \sigma = 8$$

- **a)** Calculate the Modulation Transfer Function, $MTF(u, v)$, associated with this system.
- **b)** Sketch $MTF(u, 0)$, $MTF(0, v)$. For plotting purposes, clearly mark the points where the MTF is equal to 1/2.
- **c)** An object $f(x, y) = 4 + 3\sin\left(\frac{2\pi x}{20}\right)$ is imaged using this system. What is the modulation of this object? What is the modulation of the image generated by the system?
- **d)** An object $f(x, y) = 4 + 3\sin\left(\frac{2\pi(x+y)}{20}\right)$ is imaged using this system. What is the modulation of this object? What is the modulation of the image generated by the system?

3) Consider a 1D medical imaging system that is a cascade of two subsystems with the following PSFs:

$$h_1(x) = rect\left(\frac{x}{4}\right) \qquad\qquad h_2(x) = tri\left(\frac{x}{3}\right)$$

Here, $tri(x)$ is the triangle function extending from -1 to 1 with a peak value of 1.

a) Calculate the full-width half-maximum (FWHM) resolutions of each system. Which of these two systems have better spatial resolution?

b) Using the approximate relation for FWHM, calculate the FWHM of the overall system.

4) *"If a test to detect a disease whose prevalence is 1/1000 has a false positive rate of 5%, what is the chance that a person found to have a positive result actually has the disease, assuming you know nothing about the person's symptoms or signs?"*

This is the question asked by Harvard Medical School researchers in a scientific study published in 2014, where approximately three-quarters of the surveyed doctors answered incorrectly. The most common incorrect answer was "95%". Here is the link to this study:

https://pmc.ncbi.nlm.nih.gov/articles/PMC4955674/

a) What is the name of the test parameter that the question is asking for?

b) What is the correct answer to the question?

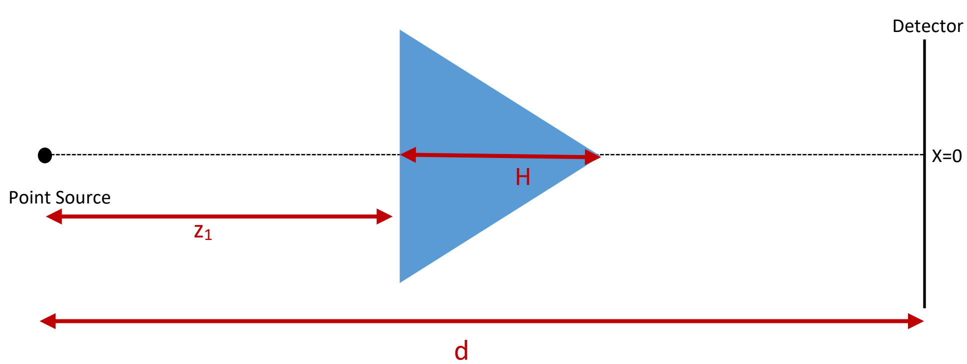
Note: The article already states the correct answer. You are expected to draw a contingency table and show your steps in finding this answer. You will need to make a simplifying assumption at one point due to missing information (this assumption is also stated in the article).

c) Does the simplifying assumption place an upper bound or a lower bound on the performance of the test? Explain briefly.

5) An equilateral triangle object with height $H = 15\,cm$ and constant linear attenuation coefficient $\mu_0 = 0.2\,cm^{-1}$ is imaged with a point source x-ray imaging system, $d = 1\,m$ away from the detector. The object is placed between the source and the detector.

a) Formulate the intensity on the detector along the x-axis $I_d(x)$, for $z_1 = 0.3\,m$. Do not ignore obliquities.

b) Plot $I_d(x)$ using a tool such as MATLAB.



6) **MATLAB Question:** Let's say we developed a new type of iron oxide nanoparticle tracer for magnetic particle imaging (MPI). We name this tracer TraceX and we now want to characterize its response with an imaging experiment. Since there is no "gold standard" tracer in MPI, we decide to compare our new tracer with two different commercially available tracers for MPI, namely Tracer A and Tracer B. We prepare a phantom with point sources of these three tracers (i.e., new tracer and two commercial tracers) that are separated 5-mm apart from each other. The tracers are placed in the following order from left to right: Tracer A, TraceX, and Tracer B. To have a fair comparison, all of the tracers have the same iron concentration level.

a) Load the experiment data ("MPI_data.mat") into your workspace and display the MPI image ("MPI_image") in grayscale. The image corresponds to a physical extent of 30 mm × 90 mm, with a matrix size of 500×1500 pixels. Properly place ticks and labels to the x-axis and y-axis of the displayed image.
Hint: You can place ticks for "imshow" function via "set(gca, 'XTick', xTicks, 'YTick', yTicks);".

b) Measure the approximate FWHM of these three tracers. Note that the tracer responses are circularly symmetric.

c) Calculate the SNR for each tracer in the image, using the amplitude SNR metric (i.e., the peak pixel intensity level for a tracer divided by the standard deviation (STD) of noise).

Note: For measuring standard deviation (STD) for noise, first choose a rectangular region in the background where there is negligible tracer signal. Then compute the STD in that region using the built-in *std* function of MATLAB.

d) Load the vessel phantom ("vessel_phantom.mat") into your workspace and display the vessel phantom ("vessel_phantom"), where 500×500 pixels correspond to a physical extent of 30 mm × 30 mm. Assume that we 3D print this vessel phantom, fill the vessel channels with a tracer, and image it on our MPI system. We repeat this procedure for all three tracers, separately. Display the resulting image that we would get for each tracer. Explain how each tracer's FWHM affects the resulting MPI image.
Hint: Take 500×500 regions from "MPI_image" in part (a) as the PSF for each tracer.