EEE 473 Medical Imaging
Homework #1
Boran Kılıç
22103444

Q1.

X-ray:

Fig. 1.1 X-ray image of hand of a person
[https://www.setonimaging.com/facts-may-not-known-x-rays/]

CT:

Fig. 1.2 CT image of a brain of a person
[https://www.radiologymasterclass.co.uk/gallery/ct_brain/ct_brain_stacks/ventricles_ct_brain]

PET:

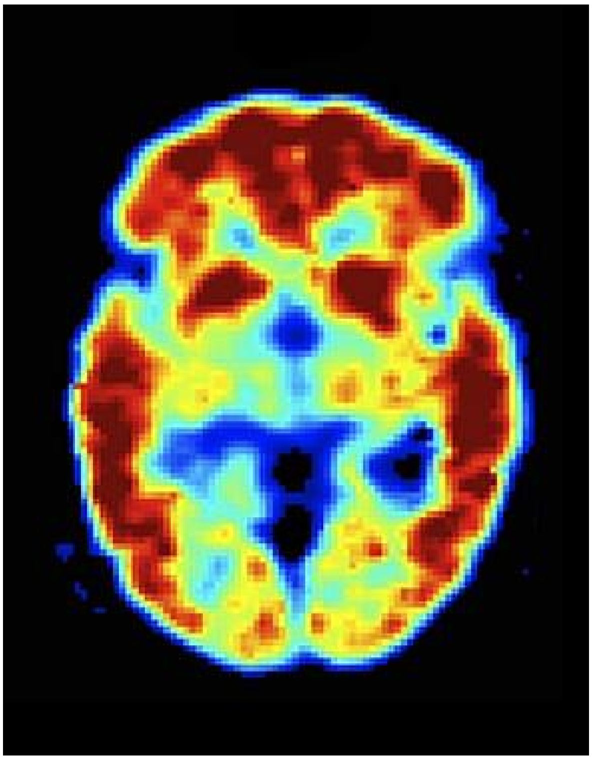

Fig. 1.3 PET scan of a brain of a person
[https://www.health.harvard.edu/blog/pet-scans-peer-into-the-heart-of-dementia-201310166761]

Ultrasound:

Fig. 1.4 Ultrasound image of a fetus in the mothers womb
[https://mtauburnobgyn.com/2018/08/what-to-expect-during-your-ultrasound/]

MRI:

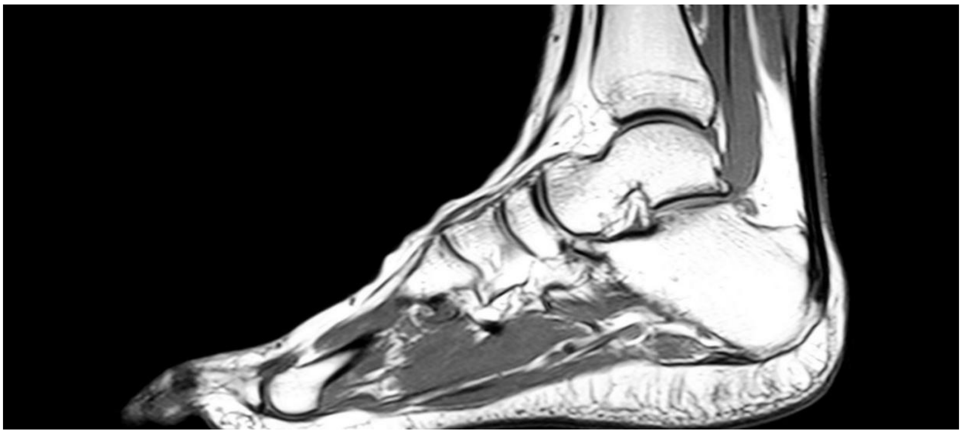

Fig. 1.5 MRI image of a foot of a person
[https://i-med.com.au/procedures/foot-ankle-mri#gsc.tab=0]

Q2)

a) new input: $f'(x,y) = \sum_{k=1}^{K} w_k f_k(x,y)$

$g'(x,y) = 3f'(x-1,y+1) + f'(0,y) = 3\sum_{k=1}^{K} w_k f_k(x-1,y+1) + \sum_{k=1}^{K} w_k f_k(0,y) = \sum_{k=1}^{K} w_k \underbrace{(3 f_k(x-1,y-1) + f_k(0,y))}_{g_k(x,y)}$

$= \sum_{k=1}^{K} w_k g_k(x,y)$ ✓ Linear

new input: $f'(x,y) = f(x-x_0, y-y_0)$

$g'(x,y) = 3f'(x-1,y+1) + f'(0,y) = 3f(x-1-x_0, y+1-y_0) + f(-x_0, y-y_0)$

$g(x-x_0, y-y_0) = 3f(x-x_0-1, y-y_0+1) + f(0, y-y_0)$

$g'(x,y) \neq g(x-x_0, y-y_0)$ ✗ not shift-invariant

b) new input: $f'(x,y) = \sum_{k=1}^{K} w_k f_k(x,y)$

$g'(x,y) = |f'(x,y)| + f'(x-1,y) = \left|\sum_{k=1}^{K} w_k f_k(x,y)\right| + \sum_{k=1}^{K} w_k f_k(x-1,y)$

since $|a| + |b| \geqslant |a+b|$,

$g'(x,y) \neq \sum_{k=1}^{K} w_k g_k(x,y)$ ✗ not-linear

↳ not necessarily equal

new input: $f'(x,y) = f(x-x_0, y-y_0)$

$g'(x,y) = |f'(x,y)| + f'(x-1,y) = |f(x-x_0, y-y_0)| + f(x-1-x_0, y-y_0)$

$g(x-x_0, y-y_0) = |f(x-x_0, y-y_0)| + f(x-x_0-1, y-y_0) = g'(x,y)$ ✓ Shift invariant

c) new input: $f'(x,y) = \sum_{k=1}^{K} w_k f_k(x,y)$

$x \geqslant 0$ and $y \geqslant 0$

$g'(x,y) = 2f'(x,3y) + f'(x-1,y) = 2\sum_{k=1}^{K} w_k f_k(x,3y) + \sum_{k=1}^{K} w_k f_k(x-1,y) = \sum_{k=1}^{K} w_k \underbrace{(2f_k(x,3y) + f_k(x-1,y))}_{g_k(x,y)}$ ✓

otherwise

$g(x,y) = 0 = g'(x,y) = \sum_{k=1}^{K} w_k g_k(x,y)$ ✓ Linear

new input: $f'(x,y) = f(x-x_0, y-y_0)$

$x \geqslant 0$ and $y \geqslant 0$

$g'(x,y) = 2f'(x,3y) + f'(x-1,y) = 2f(x-x_0, 3y-y_0) + f(x-1-x_0, y-y_0)$

$g(x-x_0, y-y_0) = 2f(x-x_0, 3y-3y_0) + f(x-x_0-1, y-y_0) \neq g'(x,y)$ ✗ not shift invariant

Q3)

a)

$$f(x,y)\,\delta(ax+b,\,cy-d) = f(x,y)\,\delta\left(a\left(x+\frac{b}{a}\right),\,c\left(y-\frac{d}{c}\right)\right) = f(x,y)\,\frac{1}{|a.c|}\,\delta\left(x+\frac{b}{a},\,y-\frac{d}{c}\right)$$

$$= \frac{1}{|a.c|}\,f\left(-\frac{b}{a},\frac{d}{c}\right)\delta\left(x+\frac{b}{a},\,y-\frac{d}{c}\right) = \boxed{\frac{1}{|a.c|}\,e^{-j2\pi\left(-\frac{b}{a}+\frac{d}{c}\right)}\,\delta\left(x+\frac{b}{a},\,y-\frac{d}{c}\right)}$$

b) Using the result of part 3-a.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(ax+b,\,cy-d)\,dx\,dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{|ac|}\,e^{-j2\pi\left(-\frac{b}{a}+\frac{d}{c}\right)}\,\delta\left(x+\frac{b}{a},\,y-\frac{d}{c}\right)dx\,dy$$

$$= \frac{1}{|a.c|}\,e^{-j2\pi\left(-\frac{b}{a}+\frac{d}{c}\right)}\underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\delta\left(x+\frac{b}{a},\,y-\frac{d}{c}\right)dx\,dy}_{1} = \boxed{\frac{1}{|a.c|}\,e^{-j2\pi\left(-\frac{b}{a}+\frac{d}{c}\right)}}$$

c) $$\mathcal{F}_{2D}\{\sin(2\pi x+3\pi y) * \operatorname{sinc}(3x,2y)\} = \mathcal{F}_{2D}\{\sin(2\pi x+3\pi y)\}\cdot\mathcal{F}_{2D}\{\operatorname{sinc}(3x,2y)\}$$

$$\mathcal{F}_{2D}\left\{\sin\left(2\pi\left(x+\frac{3}{2}y\right)\right)\right\} = \frac{1}{2j}\left[\delta\left(u-1,\,v-\frac{3}{2}\right)-\delta\left(u+1,\,v+\frac{3}{2}\right)\right]$$

$$\mathcal{F}_{2D}\{\operatorname{sinc}(3x,2y)\} = \frac{1}{|3.2|}\operatorname{rect}\left(\frac{u}{3},\frac{v}{2}\right)$$

$$\frac{1}{2j}\left[\delta\left(u-1,\,v-\frac{3}{2}\right)-\delta\left(u+1,\,v+\frac{3}{2}\right)\right]\frac{1}{6}\operatorname{rect}\left(\frac{u}{3},\frac{v}{2}\right)$$

$$\frac{1}{12j}\left[\underbrace{\operatorname{rect}\left(\frac{1}{3},\frac{3}{4}\right)}_{1}-\underbrace{\operatorname{rect}\left(-\frac{1}{3},-\frac{3}{4}\right)}_{1}\right] = 0 \xrightarrow{\mathcal{F}_{2D}^{-1}} 0$$

Q4)

a) $f(x,y) = f_1(x).f_2(y)$ than $\mathcal{F}_{2D}(f)(u,v) = F_1(u).F_2(v)$

$f_1(x) = 1 \longrightarrow F_1(u) = \delta(u)$

$f_2(y) = \delta(y+1) \longrightarrow F_2(v) = e^{j2\pi v}$

$$\boxed{\mathcal{F}_{2D}(f)(u,v) = \delta(u)\,e^{j2\pi v}}$$

b) $\mathcal{F}_{2D}\left\{ rect\left(\frac{x}{4}, \frac{y}{3}\right)\right\} = |4 \cdot 3| \; sinc(4u, 3v)$

$\mathcal{F}_{2D}\left\{ e^{j2\pi(x+\frac{1}{2}y)} \right\} = \delta(u-1, v-\frac{1}{2})$

sifting property

$$\mathcal{F}_{2D}\left\{ f(x,y) \right\} = 12\, sinc(4u, 3v) * \delta(u-1, v-\tfrac{1}{2}) \xlongequal{\downarrow} \boxed{12\, sinc\left(4(u-1), 3(v-\tfrac{1}{2})\right)}$$

c) $rect\left(5u-2, \frac{v}{3}\right) = rect\left(5(u-\frac{2}{5}), \frac{v}{3}\right) = rect\left(5u, \frac{v}{3}\right) * \delta(u-\frac{2}{5}, v)$

$\mathcal{F}_{2D}^{-1}\left\{ rect\left(5u, \frac{v}{3}\right) * \delta\left(u-\frac{2}{5}\right)\right\} = \mathcal{F}_{2D}^{-1}\left\{ rect\left(5u, \frac{v}{3}\right)\right\} \mathcal{F}^{-1}\left\{\delta\left(u-\frac{2}{5}\right)\right\}$

$$= \boxed{\frac{3}{5}\, sinc\left(\frac{x}{5}, 3y\right) e^{j2\pi\left(\frac{2}{5}x\right)}}$$

Q5.

A)

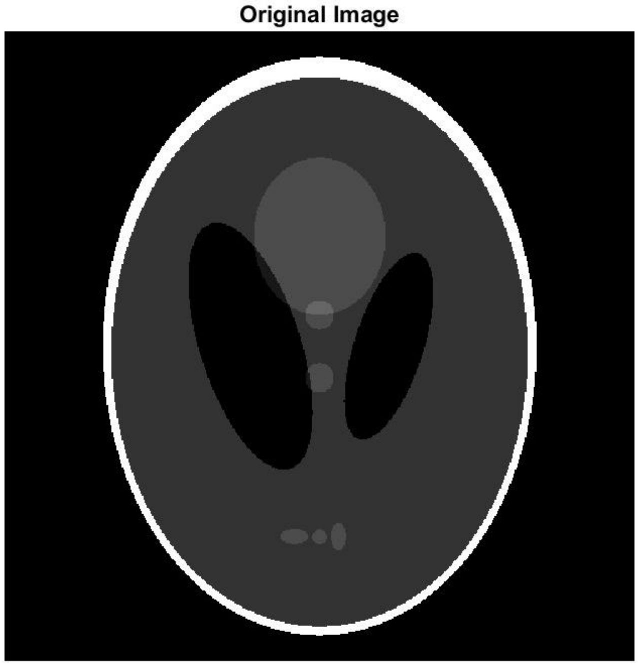

Fig. 5.1

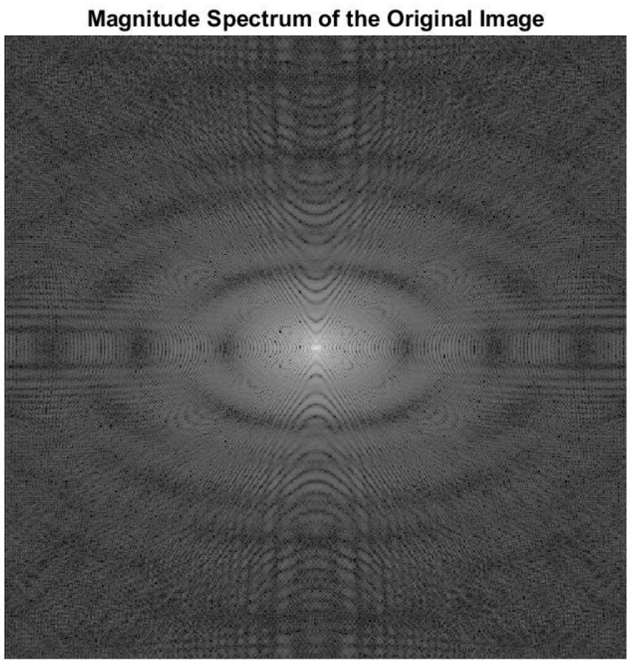

Fig. 5.2

B)

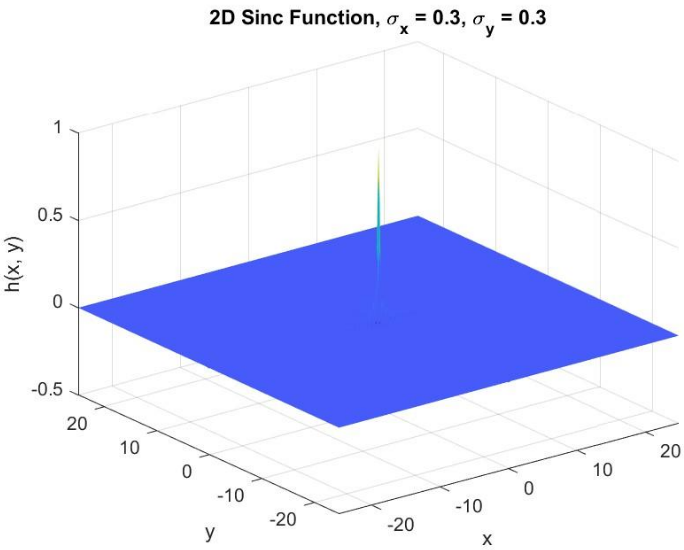

Fig. 5.3

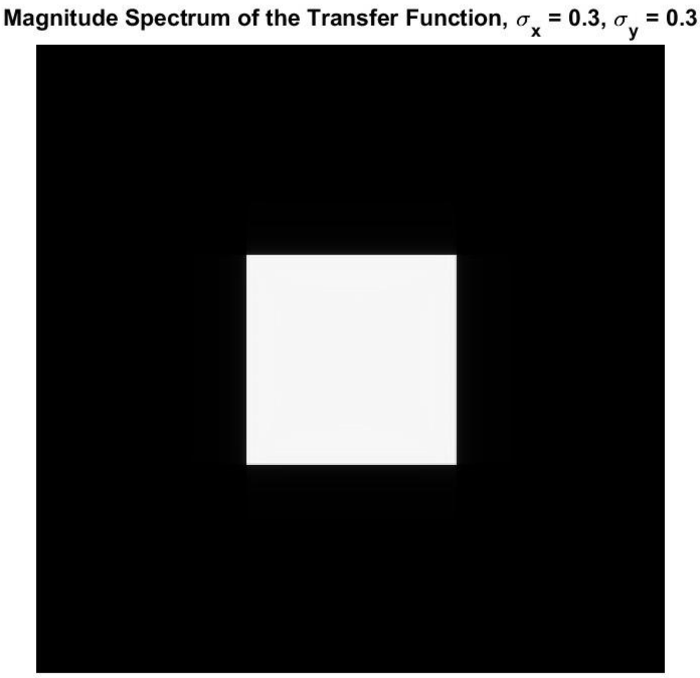

Fig. 5.4

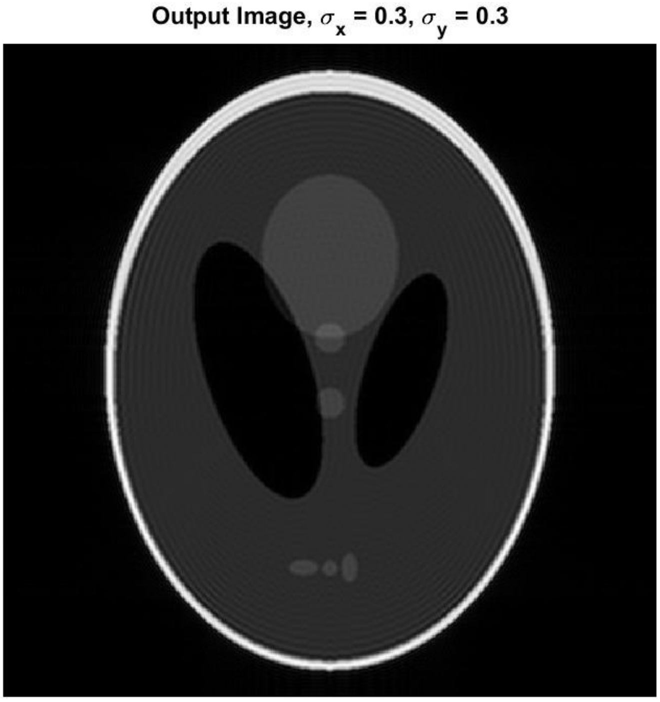
Output Image, σ_x = 0.3, σ_y = 0.3

Fig. 5.5

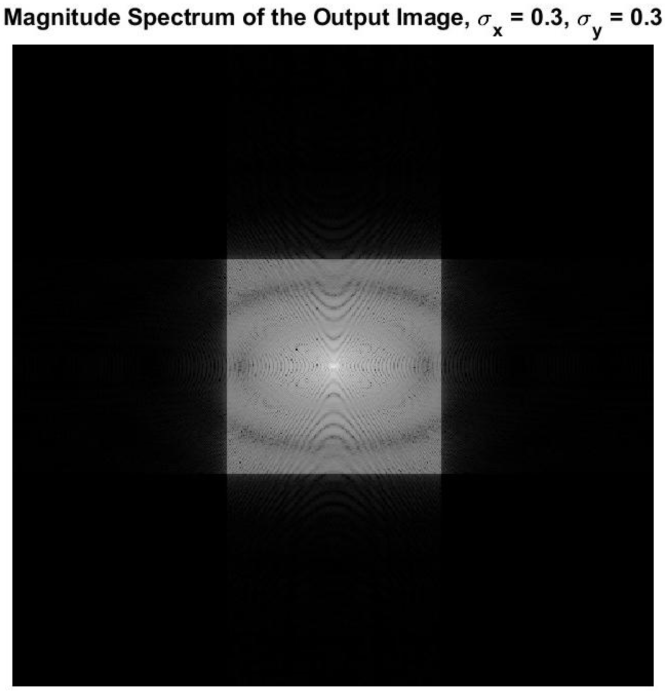
Magnitude Spectrum of the Output Image, σ_x = 0.3, σ_y = 0.3

Fig. 5.6

C)

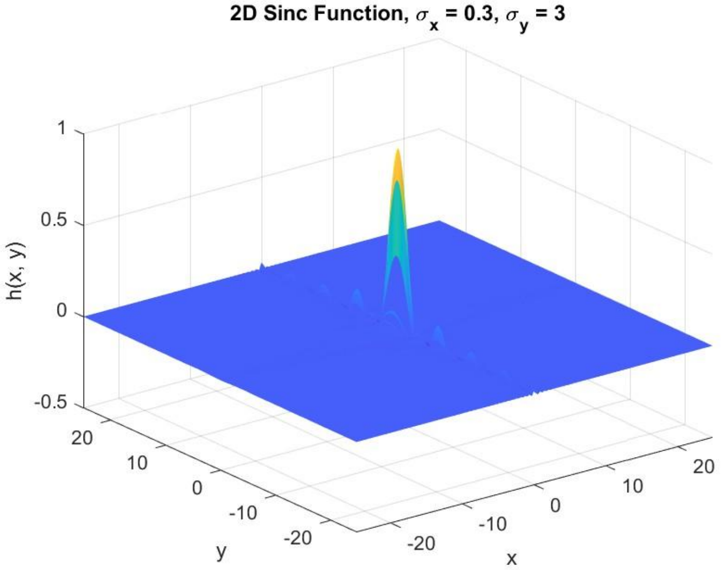

Fig. 5.7

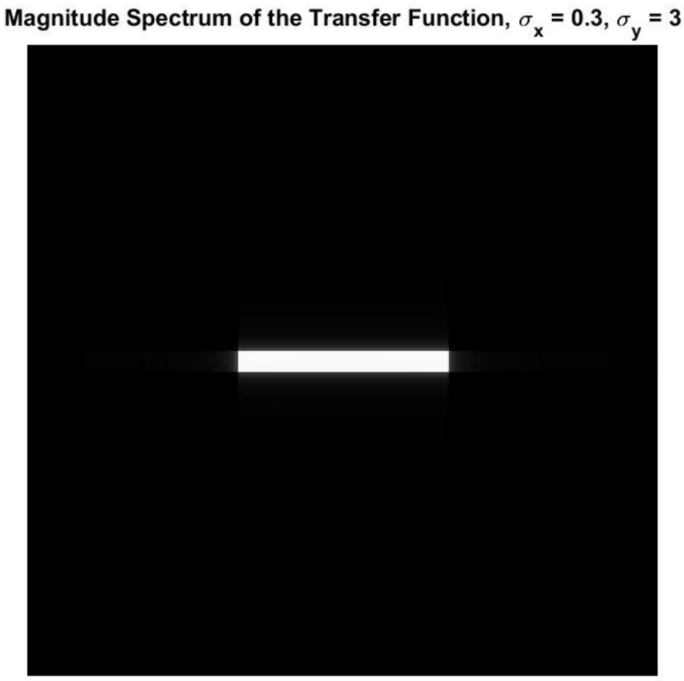

Fig. 5.8

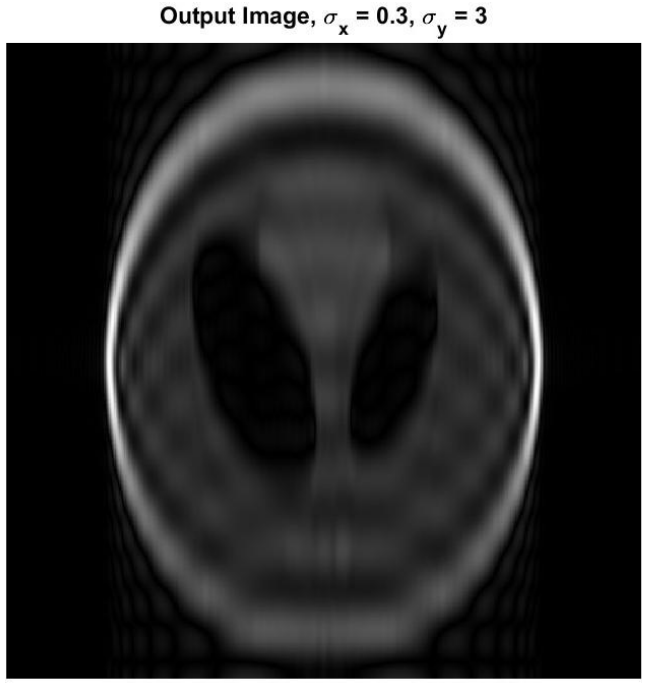

Fig. 5.9

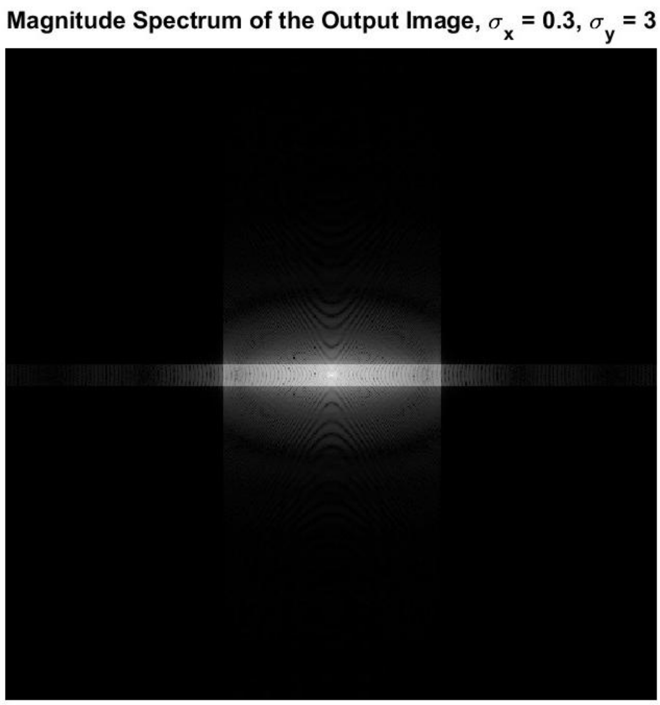

Fig. 5.10

D)

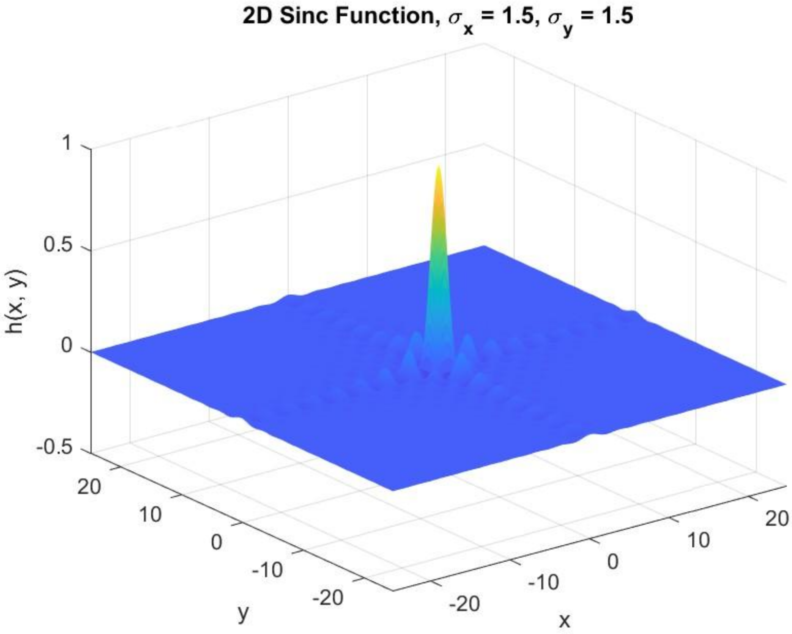

Fig. 5.11

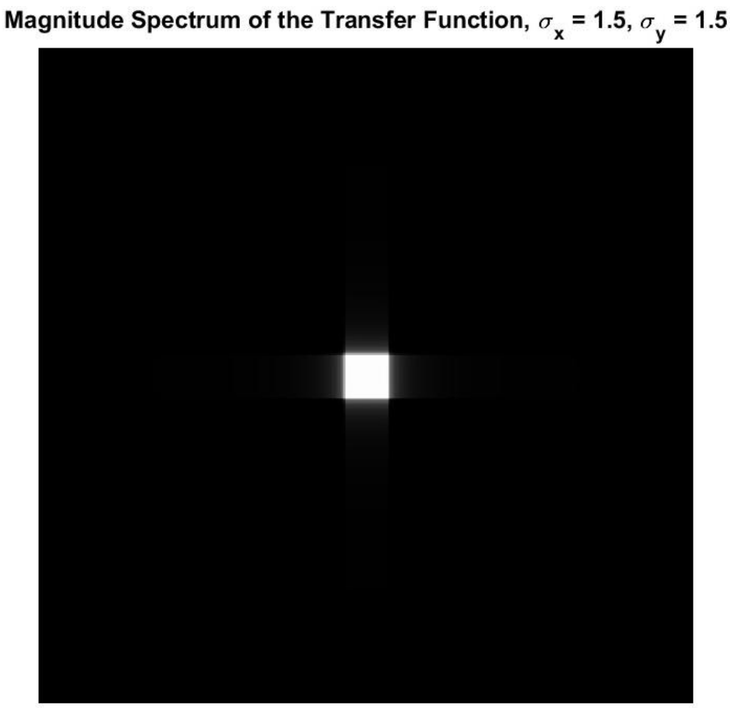

Fig. 5.12

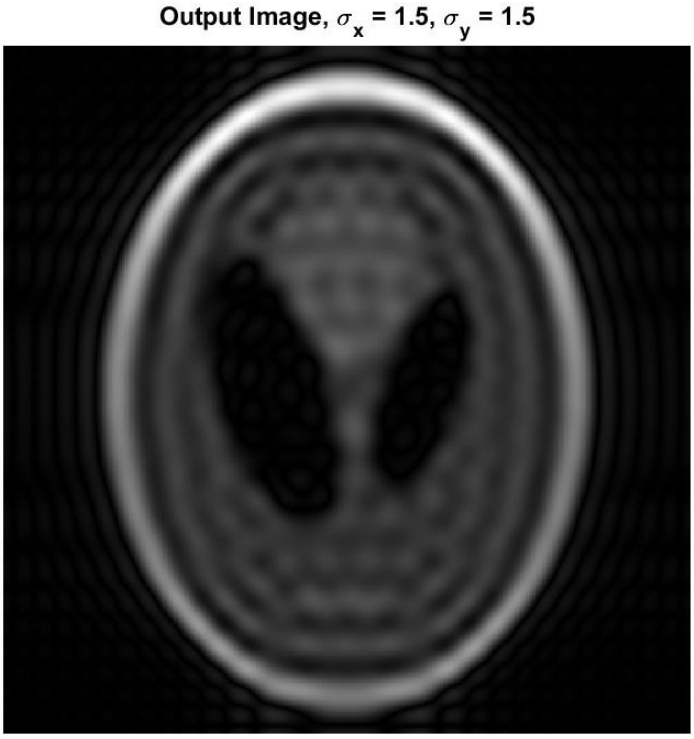

Fig. 5.13

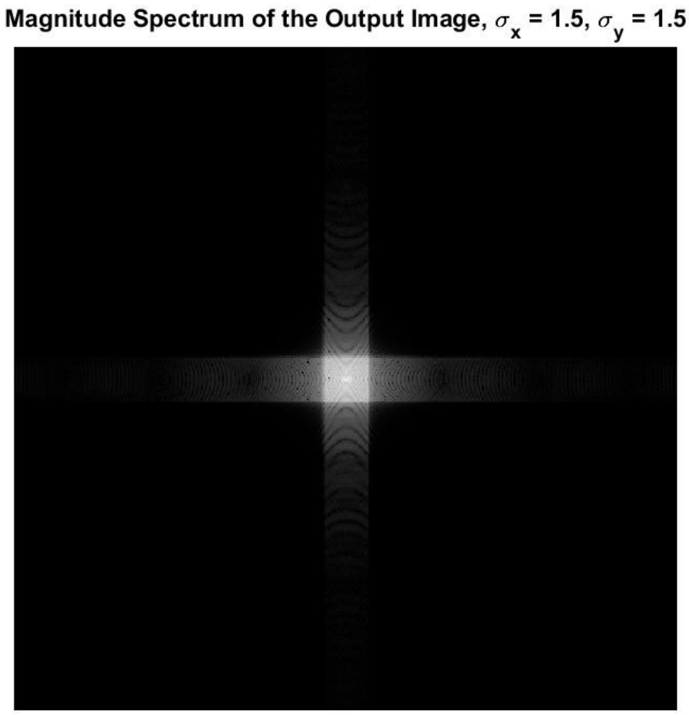

Fig. 5.14

E)

$$h(x,y) = sinc\,(\frac{x}{\sigma_x}, \frac{y}{\sigma_y})$$
$$H(u,v) = \sigma_x \sigma_y rect(\sigma_x x, \sigma_y y)$$

The given PSF is a sinc function therefore the corresponding transfer function is a rectangular function. In frequency domain rectangular function acts as a low-pass filter. σ_x, σ_y are the variances of the sinc function, they determine the boundaries of the rectangular function. If σ increases for a direction in the spatial domain threshold of the low-pass filter in the frequency domain decreases in that direction. Which means higher frequency components are cut off that results a less detailed (blurry) image.
Comparing part (b) with part (d) higher σ values resulted a expansion in the spatial domain which then results a shrinkage in the frequency domain.
Comparing the Fig 5.4 and 5.12 it can be seen that the higher σ values resulted a narrower transfer function. That resulted a cut off on the magnitude spectrum of the output image.
Comparing part (b) with part (c) it is visible that expansion in y axis in spatial domain resulted a shrinkage on the y axis of the frequency domain.
If the original image in Fig 5.1 is compared with the result of the case where σ_x = 1.5, σ_y = 1.5 in Fig 5.13 it is clearly visible that the image lost the details therefore blurry.

Appendix

MATLAB code:

```
clear
close all
P = phantom('Modified Shepp-Logan',500);
figure();
imshow(P);
title('Original Image');
saveas(gcf, 'original.jpg');

F = fft2c(P);
figure();
imshow(log(1 + abs(F)), []);
title('Magnitude Spectrum of the Original Image');
saveas(gcf, 'magspec_original.jpg');

plot_sinc_and_spectrum(0.3, 0.3, F)
plot_sinc_and_spectrum(0.3, 3, F)
plot_sinc_and_spectrum(1.5, 1.5, F)

function plot_sinc_and_spectrum(sigma_x, sigma_y, F)
[x, y] = meshgrid(-25:0.1:25-0.1, -25:0.1:25-0.1);

h = sinc2D(x, y, sigma_x, sigma_y);
figure;
surf(x, y, h, 'EdgeColor', 'none');
title(['2D Sinc Function, \sigma_x = ', num2str(sigma_x), ', \sigma_y = ',
num2str(sigma_y)]);
xlabel('x'); ylabel('y'); zlabel('h(x, y)');
saveas(gcf, ['sinc_function_sigma_x_', num2str(sigma_x), '_sigma_y_',
num2str(sigma_y), '.jpg']);

H = fft2c(h);
figure();
imshow(log(1 + abs(H)), []);
title(['Magnitude Spectrum of the Transfer Function, \sigma_x = ',
num2str(sigma_x), ', \sigma_y = ', num2str(sigma_y)]);
saveas(gcf, ['spectrum_transfer_function_sigma_x_', num2str(sigma_x),
'_sigma_y_', num2str(sigma_y), '.jpg']);

Y = F .* H;

outimg = ifft2c(Y);
figure();
imshow(abs(outimg), []);
title(['Output Image, \sigma_x = ', num2str(sigma_x), ', \sigma_y = ',
num2str(sigma_y)]);
saveas(gcf, ['output_image_sigma_x_', num2str(sigma_x), '_sigma_y_',
num2str(sigma_y), '.jpg']);

figure();
imshow(log(1 + abs(Y)), []);
title(['Magnitude Spectrum of the Output Image, \sigma_x = ',
num2str(sigma_x), ', \sigma_y = ', num2str(sigma_y)]);
saveas(gcf, ['magnitude_spectrum_output_sigma_x_', num2str(sigma_x),
'_sigma_y_', num2str(sigma_y), '.jpg']);
end
```

```
function h = sinc2D(x, y, sigma_x, sigma_y)
x_scaled = x / sigma_x;
y_scaled = y / sigma_y;
h = (sinc(x_scaled) .* sinc(y_scaled));
end
```