Q1)

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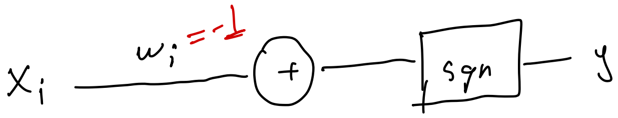
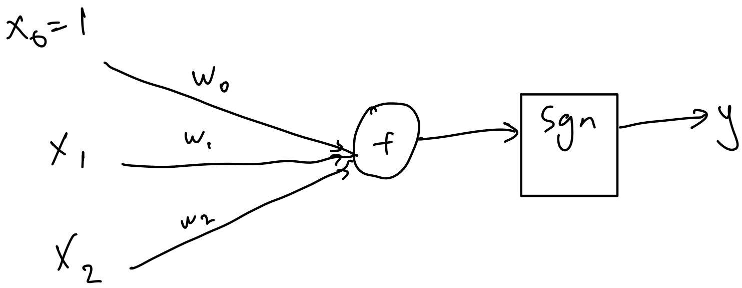
NOT Gate

NOT gate can simply be achieved by reversing signs by weights.



$w_i = -1$, $y = \overline{x_i}$

AND Gate



Truth table:

x_1	x_2	y
-1	-1	-1
-1	1	-1
1	-1	-1
1	1	1

$w_0 - w_1 - w_2 < 0$

$w_0 - w_1 + w_2 < 0$

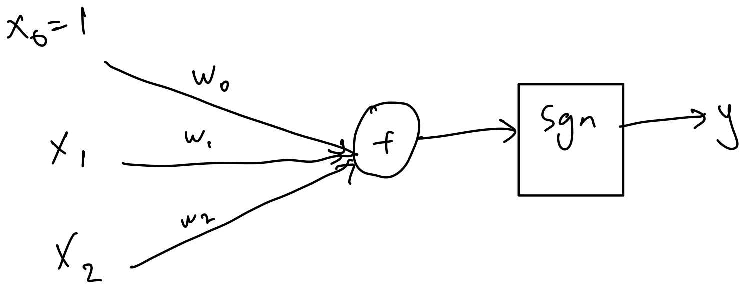
$w_0 + w_1 - w_2 < 0$

$w_0 + w_1 + w_2 > 0$

if we let $w_1 = w_2 = 1$ and $w_0 = -\frac{3}{2}$ $y = x_1 x_2$

In general, $w_0 = -m + \frac{1}{2}$ where $w_i = 1$ $(i \neq 0)$ and m is the number of inputs to the AND gate.

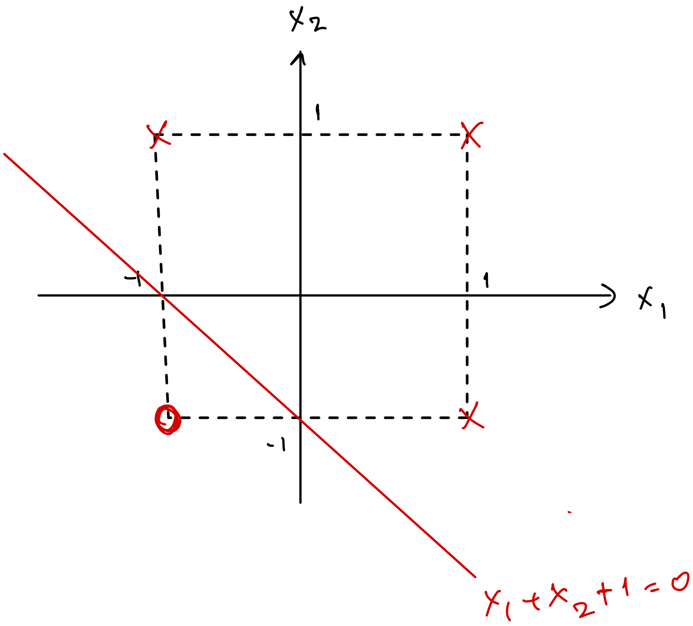
OR Gate



Truth table:

x_1	x_2	y
-1	-1	-1
-1	1	1
1	-1	1
1	1	1

$$w_0 - w_1 - w_2 < 0$$
$$w_0 - w_1 + w_2 > 0$$
$$w_0 + w_1 - w_2 > 0$$
$$w_0 + w_1 + w_2 > 0$$



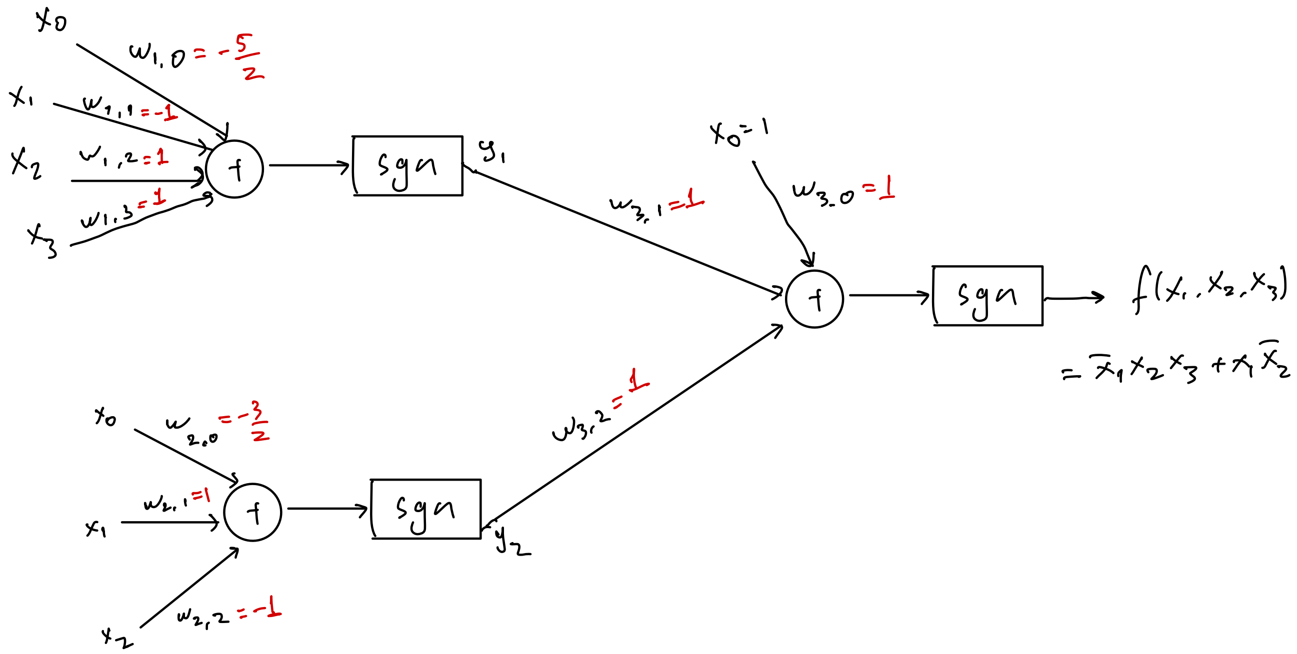
From the left graph it is evident that the OR gate can be achieved by the following weights.

$w_0 = 1$, $w_1 = 1$, $w_2 = 1$

Final Network

$$f(x_1, x_2, x_3) = \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2$$

We need 2 AND and 1 OR gates.



$$w_{1,0} = -3 + \frac{1}{2} = -\frac{5}{2}$$

$$w_{2,0} = -2 + \frac{1}{2} = -\frac{3}{2}$$

$w_{1,1} = w_{2,2} = -1$ because of the NOT gate

All the other weights are 1 according to AND and OR gate rules that are determined before.

Q2) In the first layer there are three perceptrons resulting three different decision boundaries.

Let's call the outputs of the first layer h_1, h_2, h_3 from top to bottom.
Then, we have:

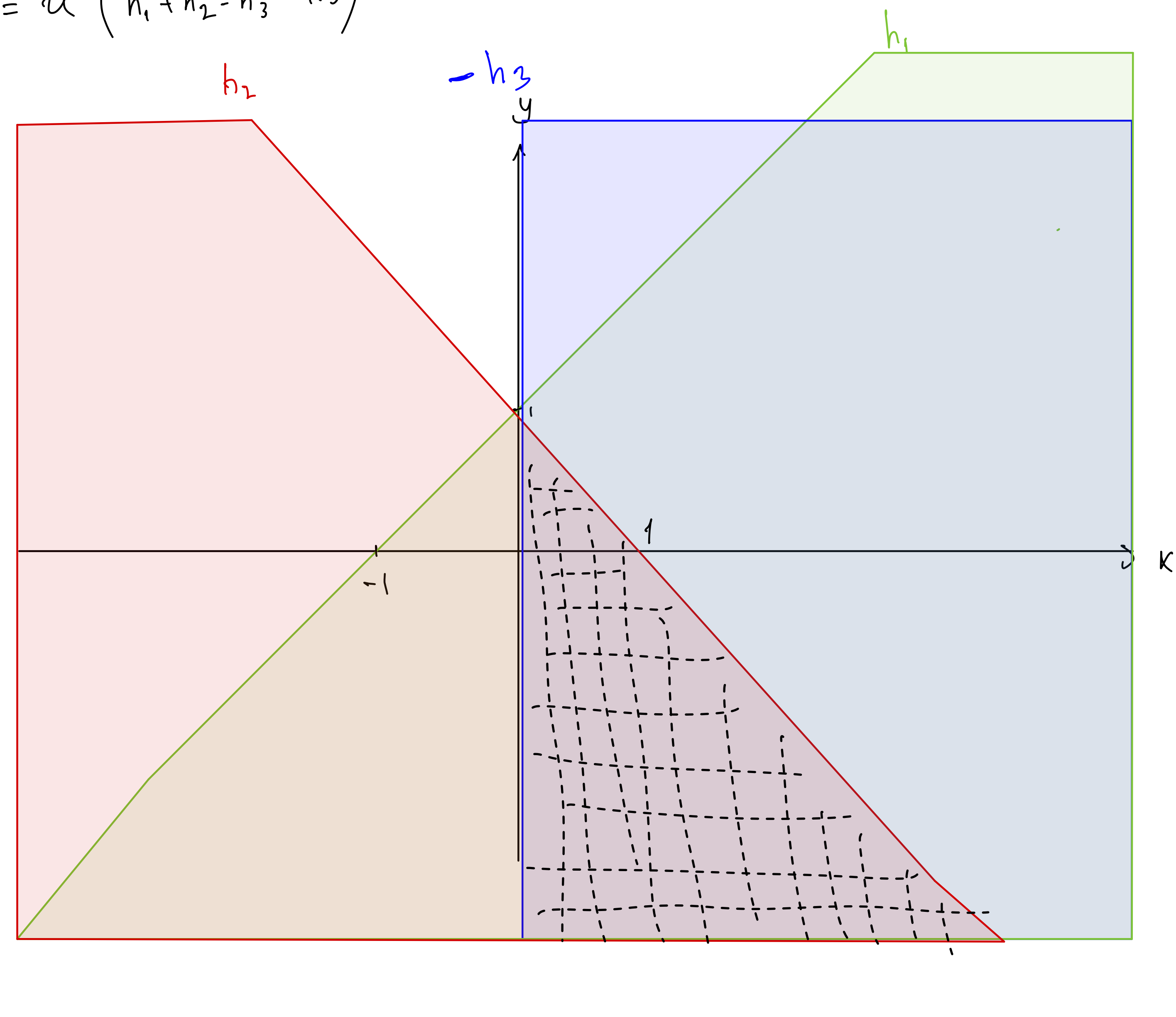
$h_1 = u(x - y + 1)$ $\qquad h_1 = \begin{cases} 1 & \text{if } x - y + 1 > 0 \\ 0 & \text{ow} \end{cases}$

$h_2 = u(-x - y + 1)$ $\qquad h_2 = \begin{cases} 1 & \text{if } -x - y + 1 > 0 \\ 0 & \text{ow} \end{cases}$

$h_3 = u(-x)$ $\qquad h_3 = \begin{cases} 1 & \text{if } -x > 0 \\ 0 & \text{ow} \end{cases}$

The output of the output layer z becomes:

$z = u(h_1 + h_2 - h_3 - 1.5)$



Comparing the above graph with the output plot of the Python code it can be seen that the output z corresponds to $h_1 h_2 \bar{h}_3$ that is the dotted region.

The decision region plot given by the python is as follows.