EEE-321: Signals and Systems

Section-1

LAB Assignment 6

Boran KILIÇ

22103444

16/12/2023

Part 1

$$y[0] = \underbrace{\sum_{l=1}^{N} a[l]\, y[-l]}_{0} + \sum_{k=0}^{M} b[k]\, x[-k] = b[0]\, x[0]$$

$$y[1] = \sum_{l=1}^{N} a[l]\, y[1-l] + \sum_{k=0}^{M} b[k]\, x[1-k] = a[1]\, y[0] + b[0]\, x[1] + b[1]\, x[0]$$

since $y[0] = b[0]\, x[0]$

$$y[1] = a[1]\, b[0]\, x[0] + b[0]\, x[1] + b[1]\, x[0]$$

Taking the z-transform:

$$Y(z) = \sum_{n=0}^{\infty} \sum_{l=1}^{N} a[l]\, y[n-l]\, z^{-n} + \sum_{n=0}^{\infty} \sum_{k=0}^{M} b[k]\, x[n-k]\, z^{-n}$$

$$= \sum_{l=1}^{N} a[l] \sum_{n=0}^{\infty} y[n-l]\, z^{-n} + \sum_{k=0}^{M} b[k] \sum_{n=0}^{\infty} x[n-k]\, z^{-n}$$

$$= \sum_{l=1}^{N} a[l]\, z^{-l}\, Y(z) + \sum_{k=0}^{M} b[k]\, z^{-k}\, X(z)$$

$$Y(z)\left(1 - \sum_{l=1}^{N} a[l]\, z^{-l}\right) = \sum_{k=0}^{M} b[k]\, z^{-k}\, X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b[k]\, z^{-k}}{1 - \sum_{l=1}^{N} a[l]\, z^{-l}} = \frac{\sum_{k=0}^{M} b[k]\, z^{-k}}{\sum_{l=0}^{N} - a[l]\, z^{-l}}$$

where $a[0] = 1$

$P = M$, $Q = N$

$c_n[p] = b[k]$

$c_d[q] = -a[l]$

Matlab code of the function DTLTI:

```
function y = DTLTI(a, b, x, Ny)
y = zeros(1, Ny);
for n = 1:Ny
for k = 1:length(b)
if n- k+1 > 0
y(n) = y(n) + b(k) * x(n-k+1);
end
end
for l = 2:length(a)
if n-l+1 > 0
y(n) = y(n) + a(l) * y(n-l+1);
end
end
end
end
```

Part 2

A)

Matlab code of the impulse response of the filter with a[l] = 0 for $1 \leq l \leq N$ and b[k] = exp(−k):

```
clear all
close all
D = 22103444; %my ID number
D4 = rem(D, 4); %D4 = 0
M = 5 + D4; % M = 5
Ny = 11;
a = zeros(1, M);
b = exp(-(0:M-1));
n = 0:Ny-1;
x = [1 zeros(1, Ny-1)];
h = DTLTI(a, b, x, Ny);
figure;
stem(n, h);
xlabel('n');ylabel('h[n]');
```

Impulse response plot:

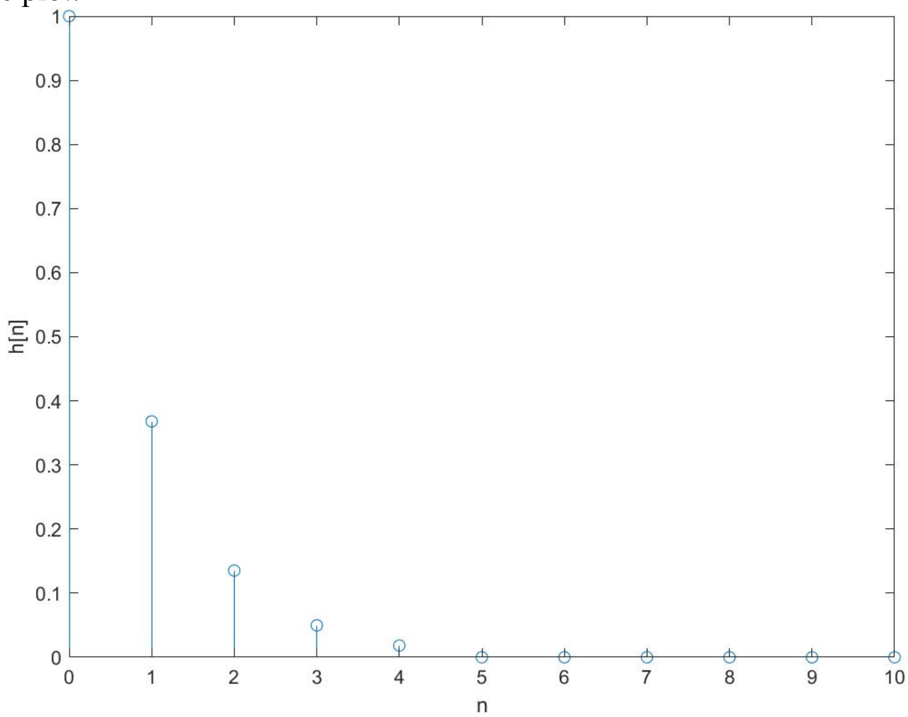


Figure 1: Impulse response of the filter in part 2-a

B) Nonzero values of the impulse response are same as the nonzero values of b[k]. The derivation of the impulse response is given below:

$$b[k] = e^{-k}, \quad M = 5+0 = 5, \quad \text{nonzero values } 0 \leq k \leq M-1 = 4$$

$$h[n] = \sum_{k=0}^{M-1} b[k]\,\delta[n-k] = \sum_{k=0}^{4} b[k] = \sum_{k=0}^{4} e^{-k}$$

C) The impulse response has finite length as it is zero after n = M-1 = 4 and before n = 0. Therefore the system is Finite Impulse Response (FIR). Length of the impulse response is 5 which is equal to M.

D) Z-transform and discrete-time Fourier transform (DTFT) are computed below:

Z-transform.

$$H(z) = \sum_{k=0}^{M-1} \sum_{n=-\infty}^{\infty} e^{-k}\,\delta[n-k]\,z^{-n} = \sum_{k=0}^{M-1} e^{-k} \sum_{n=-\infty}^{\infty} \delta[n-k]\,z^{-n}$$

$$H(z) = \sum_{k=0}^{M-1} e^{-k} z^{-k} = \sum_{k=0}^{M-1} (ez)^{-k} = \frac{1-(ez)^{-M}}{1-(ez)^{-1}} = \frac{1-(ez)^{-M}}{1-(ez)^{-1}} = \frac{1-(ez)^{-5}}{1-(ez)^{-1}}$$

In order to find DTFT of the impulse response we should substitute $z = e^{j\Omega}$

$$H(e^{j\Omega}) = \frac{1-(e\,e^{j\Omega})^{-M}}{1-(ez)^{-1}} = \frac{1-e^{-j5\Omega-5}}{1-e^{-j\Omega-1}}$$

E)

Matlab code for the magnitude response of the impulse response for the frequency interval [-π,π):

```
clear all
close all
D = 22103444; %my ID number
D4 = rem(D, 4); %D4 = 0
M = 5 + D4; % M = 5
omega = -pi:1/1000:pi-1/1000;
FTh = (1-exp(-M*(1i*omega+1)))./(1-exp(-(1i*omega+1)));
figure;
plot(omega,abs(FTh));
title({'$|\mathcal{H}(e^{j\omega})|$ vs Frequency (Hz)'},'Interpreter','latex');
xlim([-pi,pi]); xlabel('Frequency (Hz)','Interpreter','latex');
ylabel('$|\mathcal{H}(e^{j\omega})|$','Interpreter','latex');
```

Magnitude response plot:

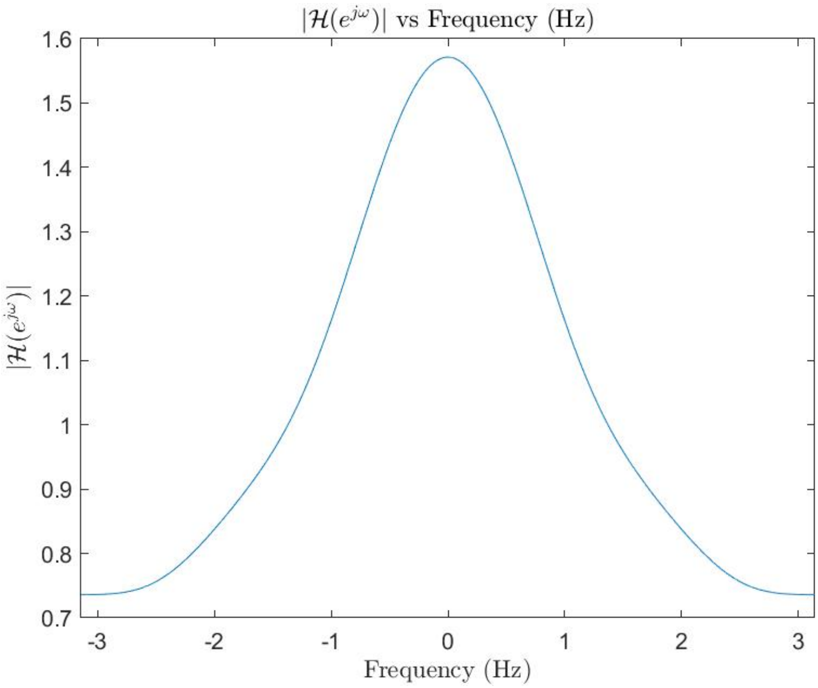


Figure 2: Magnitude response of the filter in part 2-a

This filter is a lowpass filter as it can be seen from Figure 2. Its magnitude response is high in the low frequencies but it decays to zero for high frequencies. 3dB bandwidth is a measure to specify the filters passband. It is determined by the frequencies where the outputs magnitude is decreased by 3dB from its maximum value which is equal to maximum value times $1/\sqrt{2}$. Cut off frequencies represent the filters passband, those are the frequencies where the amplitude is decreased by 3dB.

For this filter,

-3db amplitude is approximately $1.57/\sqrt{2} = 1.11$,

Cut off frequencies are approximately wc1 = -1.1, wc2 = +1.1,

3dB bandwidth is approximately 2.2.

Those values are found from the graph.

F) In this step we will examine the behavior of our filter using a chirp filter.

Matlab code for computing the output for 3 different chirp signals:

```
clear all
close all
D = 22103444; %my ID number
D4 = rem(D, 4); %D4 = 0
M = 5 + D4; % M = 5
a = zeros(1, M);
b = exp(-(0:M-1));
t = 0:1/1400:1-1/1400;
x = cos(2*pi*700*(t.^2)/2);
omega = linspace(0,pi,length(x));
y = DTLTI(a,b,x,length(x));
figure;
subplot(1,3,1);
```

```
plot(omega,abs(y));
title({'$0 < t < 1$'}, 'Interpreter', 'latex');
xlabel('$f$', 'Interpreter', 'latex');xlim([0,pi]);
ylabel('$y$', 'Interpreter', 'latex');
t = 0:1/1400:10-1/1400;
x = cos(2*pi*70*(t.^2)/2);
omega = linspace(0,pi,length(x));
y = DTLTI(a,b,x,length(x));
subplot(1,3,2);
plot(omega,abs(y));
title({'$0 < t < 10$'}, 'Interpreter', 'latex');
xlabel('$f$', 'Interpreter', 'latex');xlim([0,pi]);
ylabel('$y$', 'Interpreter', 'latex');
t = 0:1/1400:1000-1/1400;
x = cos(2*pi*0.7*(t.^2)/2);
omega = linspace(0,pi,length(x));
y = DTLTI(a,b,x,length(x));
subplot(1,3,3);
plot(omega,abs(y));
title({'$0 < t < 1000$'}, 'Interpreter', 'latex');
xlabel('$f$', 'Interpreter', 'latex');xlim([0,pi]);
ylabel('$y$', 'Interpreter', 'latex');
```

The output plots of the chirp signal:

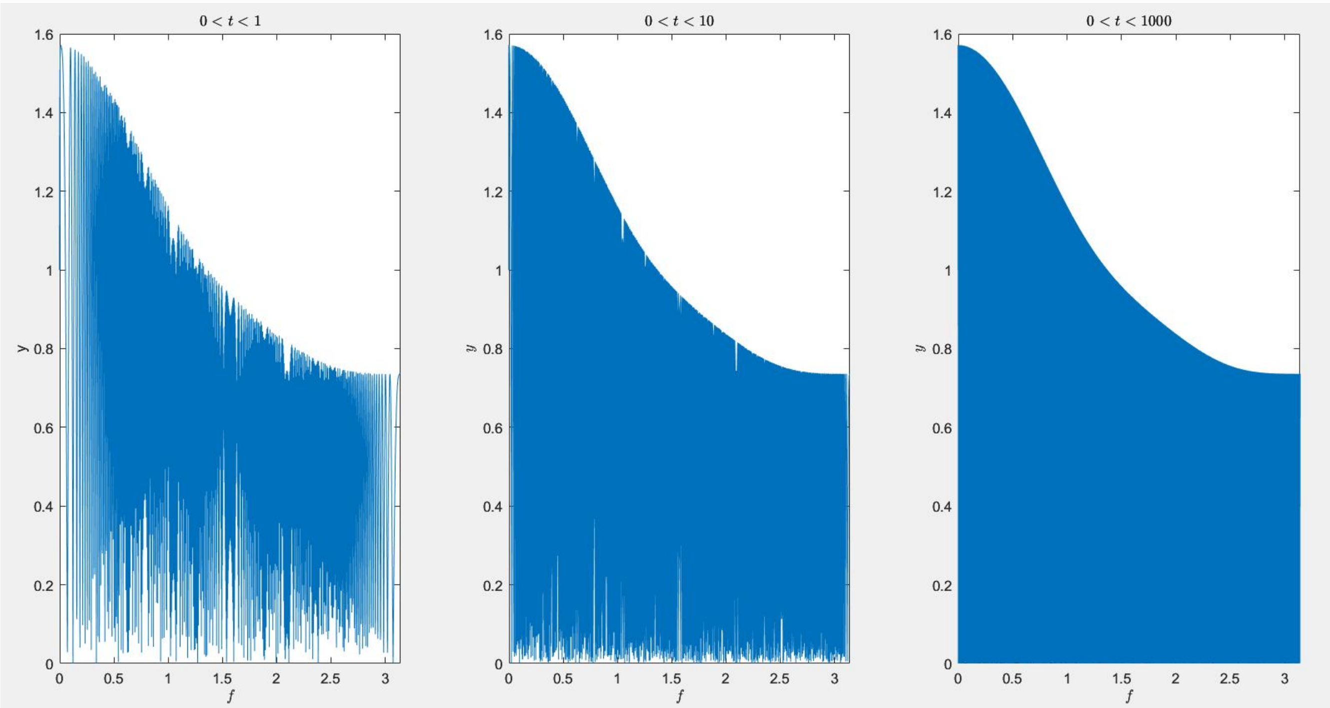


Figure 3: Chirp signal responses

As it can be seen from Figure 3 the output signal is processed chirp signal, it represents the bode plot for a filter as frequency increases as time passes. It can seen from the response of the filter, general trend of this decaying in magnitude, therefore, this filter is a low pass filter because amplitude decreased as frequency increased. The plots look like the right side of the plot in part e, Figure 2. Since, they both represent the magnitude response of the filter. The function gets smoother as interval of t increases and there are no sudden jumps in the output signals.

Part 3

A)

Z-transform

ID = 22103444

$n_2 = 2+2 = 4$, $n_3 = 2+1 = 3$, $n_4 = 2+2 = 4$, $n_5 = 2+3 = 5$, $n_8 = 2+4 = 6$, $n_6 = 2+4 = 6$

$z_1 = \frac{4+3j}{5}$, $p_1 = \frac{4+5j}{\sqrt{42}}$, $p_2 = \frac{6+6j}{\sqrt{73}}$

$z_1 = 0.8 + 0.6j$, $p_1 = 0.617 + 0.772j$, $p_2 = 0.702 + 0.702j$

$|p_2| > |p_1| \therefore$ ROC $|z| > |p_2|$

Since the signal is right-hand sided.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z_1 z^{-1}}{(1-p_1 z^{-1})(1-p_2 z^{-1})}$$

B)

Converting the form to eq.1

$$Y(z) - Y(z)p_1 z^{-1} - Y(z)p_2 z^{-1} + p_1 p_2 z^{-2} Y(z) = X(z) - z_1 z^{-1} X(z)$$

$$y[n] - p_1 y[n-1] - p_2 y[n-1] + p_1 p_2 y[n-2] = x[n] - z_1 x[n-1]$$

$$y[n] = p_1 y[n-1] + p_2 y[n-1] - p_1 p_2 y[n-2] + x[n] - z_1 x[n-1]$$

$$y[n] = \sum_{\ell=1}^{2} a[\ell]\, y[n-\ell] + \sum_{k=0}^{1} b[k]\, x[n-k]$$

$a[1] = p_1 + p_2$ $\quad$ $b[0] = 1$

$a[2] = -p_1 p_2$ $\quad$ $b[1] = -z_1$

C)

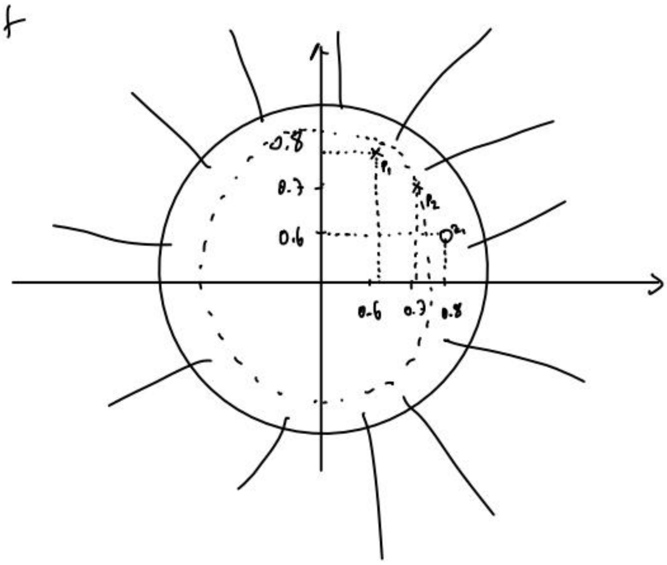
Impulse response

$$H(z) = (1 - z_1 z^{-1}) \frac{1}{(1-p_1 z^{-1})(1-p_2 z^{-1})} = (1 - z_1 z^{-1}) \left(\frac{\frac{p_2}{p_2 - p_1}}{1 - p_1 z^{-1}} + \frac{\frac{p_1}{p_1 - p_2}}{1 - p_2 z^{-1}} \right)$$

$$h[n] = \frac{p_2}{p_2 - p_1}(p_1)^n u[n] + \frac{p_1}{p_1 - p_2}(p_2)^n u[n] - z_1 \frac{p_2}{p_2 - p_1}(p_1)^{n-1} u[n-1] - z_1 \frac{p_1}{p_1 - p_2}(p_2)^{n-1} u[n-1]$$

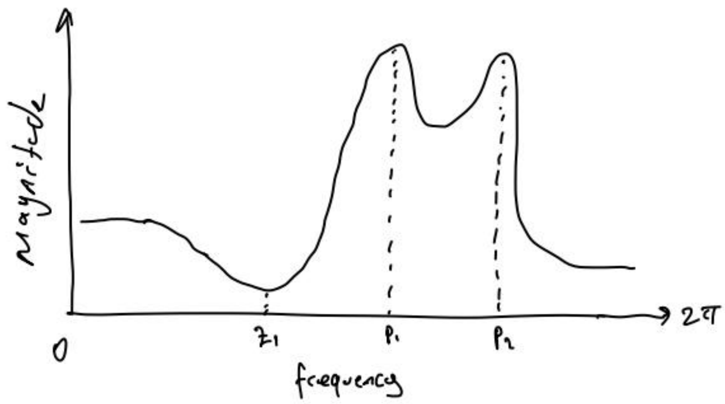
D)

pole-zero plot



ROC: $|z| > |p_2|$

Magnitude plot



E)The magnitude of both poles are less than 1 as they are both inside the unit circle which can be seen from pole-zero plot in part d, therefore the system is stable.

F)Since the system has poles the impulse response has feedback terms. Since, there are feedback terms, the system is not FIR, hence the system is IIR.

G)

In order to find DTFT of the filter $z = e^{j\Omega}$ is substituted

$$H(e^{j\Omega}) = \frac{1 - z_1 e^{-j\Omega}}{(1 - p_1 e^{-j\Omega})(1 - p_2 e^{j\Omega})}$$

Using the result magnitude response of the filter is plotted with the following Matlab code:

```
clear all
close all
omega = -pi:1/1000:pi-1/1000;
```

```
Fth = (exp(1i*omega) - (0.8 + 1i*0.6))./((exp(1i*omega) - (0.617 + 1i*0.77)) ...
.*(exp(1i*omega) - (0.70 + 1i*0.70)));
figure;
plot(omega,abs(Fth));
title({'$|\mathcal{H}(e^{j\omega})|$ vs Frequency (Hz)'},'Interpreter','latex');
xlim([-pi,pi]); xlabel('Frequency (Hz)','Interpreter','latex');
ylabel('$|\mathcal{H}(e^{j\omega})|$','Interpreter','latex');
```

Magnitude response plot:

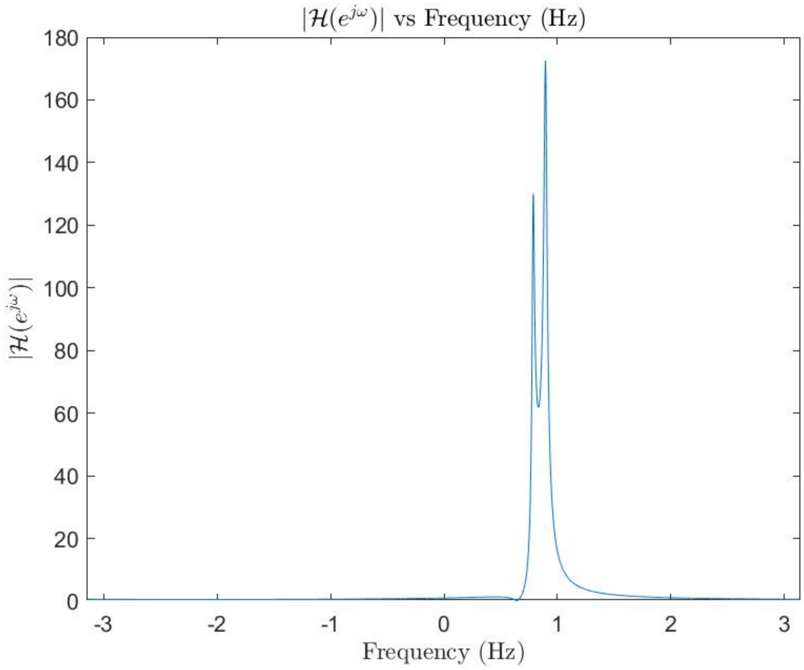


Figure 4: Magnitude response of the filter in part 3

This magnitude response plot is very similar to the one in part d. From Figure 4 it can be seen that this filter is a band-pass filter.

H) In this step we will examine the behavior of our filter using a chirp filter.

Matlab code for computing the 3 different chirp signals and their outputs:

```
clear all
close all
a = [(0.70 + 1i*0.70) + (0.617 + 1i*0.772), - (0.70 + 1i*0.70)*(0.617 + 1i*0.772)];
b = [1, - (0.80 + 1i*0.60)];

t = 0:1/1400:1-1/1400;
x1 = exp(2i*pi*(1400*(t.^2)/2-700*t));
y = DTLTI(a,b,x1,length(x1));
figure;
subplot(1,3,1);
omega = linspace(-pi,pi,length(x1));
plot(omega,abs(y));
title({'$0 < t < 1$'}, 'Interpreter', 'latex');
xlabel('$f$', 'Interpreter', 'latex');xlim([-pi,pi]);
ylabel('$y$', 'Interpreter', 'latex');

t = 0:1/1400:10-1/1400;
x2 = exp(2i*pi*(140*(t.^2)/2-700*t));
omega = linspace(-pi,pi,length(x2));
y = DTLTI(a,b,x2,length(x2));
```

```
subplot(1,3,2);
plot(omega,abs(y));
title({'$0 < t < 10$'}, 'Interpreter', 'latex');
xlabel('$f$', 'Interpreter', 'latex');xlim([-pi,pi]);
ylabel('$y$', 'Interpreter', 'latex');

t = 0:1/1400:1000-1/1400;
x3 = exp(2i*pi*(1.4*(t.^2)/2-700*t));
omega = linspace(-pi,pi,length(x3));
y = DTLTI(a,b,x3,length(x3));
subplot(1,3,3);
plot(omega,abs(y));
title({'$0 < t < 1000$'}, 'Interpreter', 'latex');
xlabel('$f$', 'Interpreter', 'latex');xlim([-pi,pi]);
ylabel('$y$', 'Interpreter', 'latex');

figure;
subplot(1,3,1);
omega = linspace(-pi,pi,length(x1));
plot(omega,x1);
title({'$0 < t < 1$'}, 'Interpreter', 'latex');
xlabel('$f$', 'Interpreter', 'latex');xlim([-pi,pi]);
ylabel('$x$', 'Interpreter', 'latex');

subplot(1,3,2);
omega = linspace(-pi,pi,length(x2));
plot(omega,x2);
title({'$0 < t < 10$'}, 'Interpreter', 'latex');
xlabel('$f$', 'Interpreter', 'latex');xlim([-pi,pi]);
ylabel('$x$', 'Interpreter', 'latex');

subplot(1,3,3);
omega = linspace(-pi,pi,length(x3));
plot(omega,x3);
title({'$0 < t < 1000$'}, 'Interpreter', 'latex');
xlabel('$f$', 'Interpreter', 'latex');xlim([-pi,pi]);
ylabel('$x$', 'Interpreter', 'latex');
```

The output plots of the chirp signal:

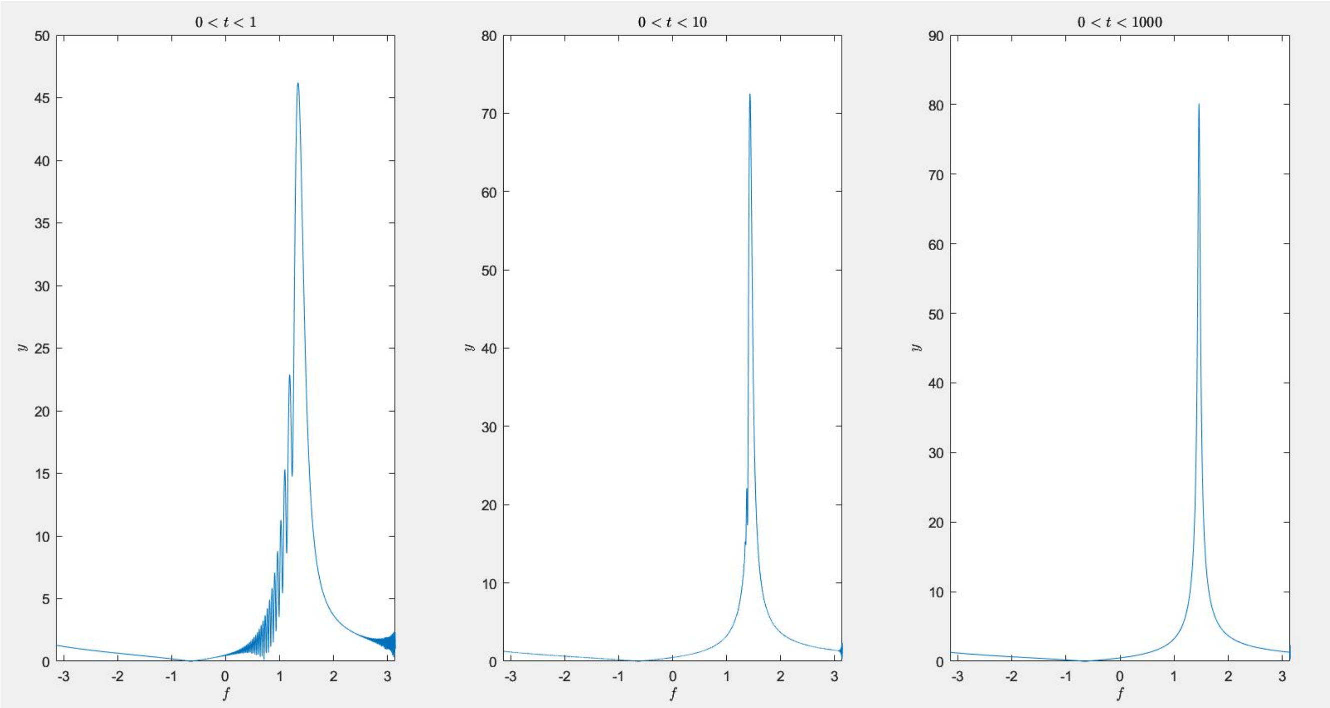


Figure 5: Chirp signal responses

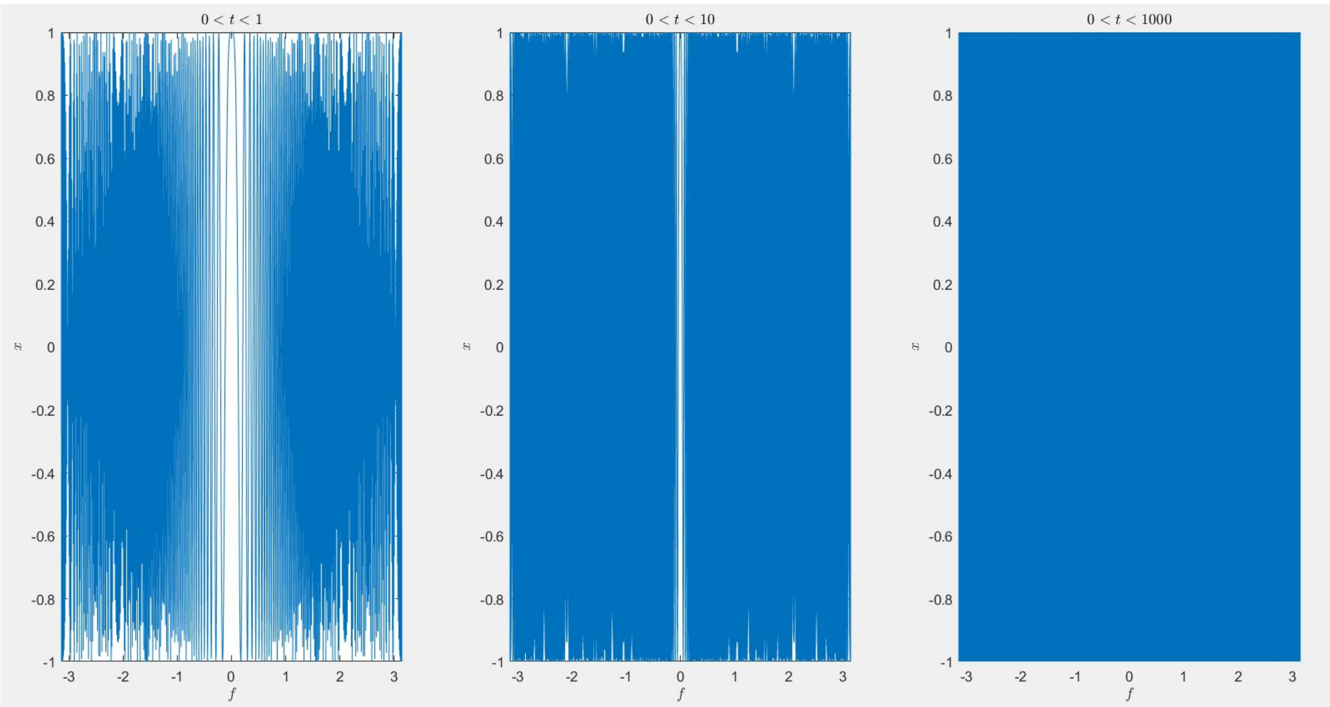


Figure 6: Chirp signals

Comparing the plots in Figure 5 with the previous plot the magnitude of the plot in Figure 5 is smaller. Since the filters coefficients are not symmetric, the magnitude plot is not symmetric with respect to origin.

According to Nyquist criteria the sampling frequency should be greater or equal to the highest frequency component of the signal. Therefore if the chirp was sweeping from -600 to 800, fs should be at least 2·800 = 1600Hz.