EEE-321: Signals and Systems

Section-1

LAB Assignment 5

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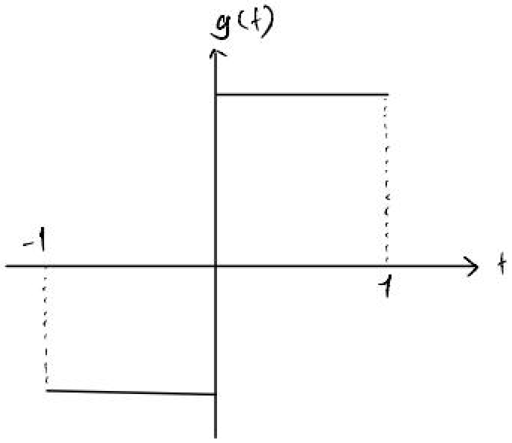
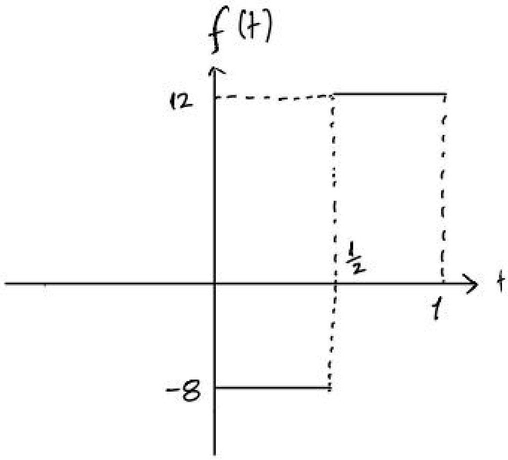
Part 1

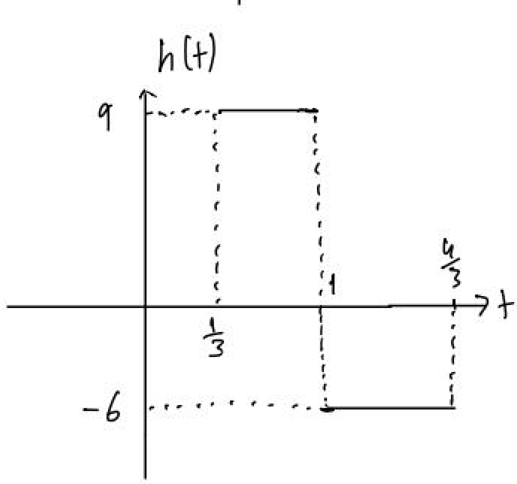


$$g(t) = \begin{cases} -2 & , \text{ if } -1 \leq t < 0 \\ 3 & , \text{ if } 0 < t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$



$$f(t) = 4g(2t-1) = 4g(2(t-\tfrac{1}{2}))$$

- t axis of $g(t)$ is scaled by $\frac{1}{2}$ $\Rightarrow$ $g(2t)$
- $g(2t)$ is shifted by $\frac{1}{2}$ to the right $\Rightarrow$ $g(2t-1)$
- $g(2t-1)$ is scaled by 4 $\Rightarrow$ $4g(2t-1)$



$$h(t) = 3g(-3(t-1))$$

- $g(t)$ is reflected with respect to $g(t)$ axis $\Rightarrow$ $g(-t)$
- t axis is scaled by $\frac{1}{3}$ $\Rightarrow$ $g(-3t)$
- $g(-3t)$ is shifted 1 to the right $\Rightarrow$ $g(-3(t-1))$
- $g(-3(t-1))$ is scaled by 3 $\Rightarrow$ $3g(-3(t-1))$

Part 2

$$\tilde{x}(t) = x(t)\sum_{n=-\infty}^{\infty} \delta(t-nT_s) = \sum_{n=-\infty}^{\infty} x(t)\,\delta(t-nT_s) = \sum_{n=-\infty}^{\infty} x(nT_s)\cdot\delta(t-nT_s)$$

$$x(nT_s) = \bar{x}[n]$$

It is desired that $x_R(t) = x(t)$

Interpolation:

$$x_R(t) = \int_{-\infty}^{\infty} \tilde{x}(t)\cdot p(t-t')\,dt' = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t'-nT_s)\,p(t-t')\,dt'$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\int_{-\infty}^{\infty} \delta(t'-nT_s)\,p(t-t')\,dt' = \sum_{n=-\infty}^{\infty} x(nT_s)\,p(t-nT_s) = x_R(t)$$

Since $p(0)=1$ and $p(kT_s)=0$ for non-zero integers k we have:

$$\boxed{x_R(nT_s) = x(nT_s) = \bar{x}[n]}$$

a) $p_Z(0) = rect(0) = 1$, $p_L(0) = tri(0) = 1 - \frac{0}{0.5} = 1$, $p_I(t) = sinc(0) = 1$

b) $p_Z(kT_s) = rect(kT_s) = 0$ since $kT_s \neq 0$.

$p_L(kT_s) = tri\left(\frac{kT_s}{T_s}\right) = tri(k) = 0$ since, $k = \mp1, \mp2, \mp3 \ldots$ (non-zero integer)

$p_I(kT_s) = sinc\left(\frac{kT_s}{T_s}\right) = sinc(k) = \frac{\sin(k\pi)}{k\pi} = \frac{0}{k\pi} = 0$ since, $k = \mp1, \mp2 \ldots$ (non-zero integer)

c) As shown above if $p(0)=1$ and $p(kT_s)$ for non-zero integer k, the interpolation is consistent. $P_Z(t)$, $P_L(t)$, $P_I(t)$ are all satisfies this conditions. Therefore, all of them are consistent.

Since g(t) is not a band-limited signal, it is not possible to fully recover g(t) from its samples. This contradicts the first criteria of the sampling theorem. G(jw) is a sinc function which is not bandlimited.

$$G(j\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt = \int_{0}^{1} e^{-j\omega t}\, dt + \int_{-1}^{0} (-e^{-j\omega t})\, dt = \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{0}^{1} + \left.\frac{e^{-j\omega t}}{j\omega}\right|_{-1}^{0}$$

$$= \frac{1}{j\omega}\left(1 - e^{-j\omega} + 1 - e^{j\omega}\right) = \frac{1}{j\omega}\left(2 - 2\cos(j\omega)\right)$$

$$G(j\omega) \neq 0 \quad \text{for } |\omega| > \omega_m$$

Part 3

MATLAB code of the generateInterp function:

```
function p = generateInterp(type,Ts,dur)
t=-dur/2:Ts/500:dur/2-Ts/500;
p=zeros(1,length(t));
if type == 0
p(-Ts/2 <= t & t < Ts/2-Ts/500)=1;
elseif type == 1
p(-Ts/2 <= t & t <= Ts/2) = 1-2*abs(t(-Ts/2 <= t & t <= Ts/2))/Ts;
elseif type == 2
p=sin(pi*t/Ts)./(pi*t/Ts);
p(t==0)=1;
end
end
```

Plots:

```
dur = rem(22103444,7); %dur=6
```

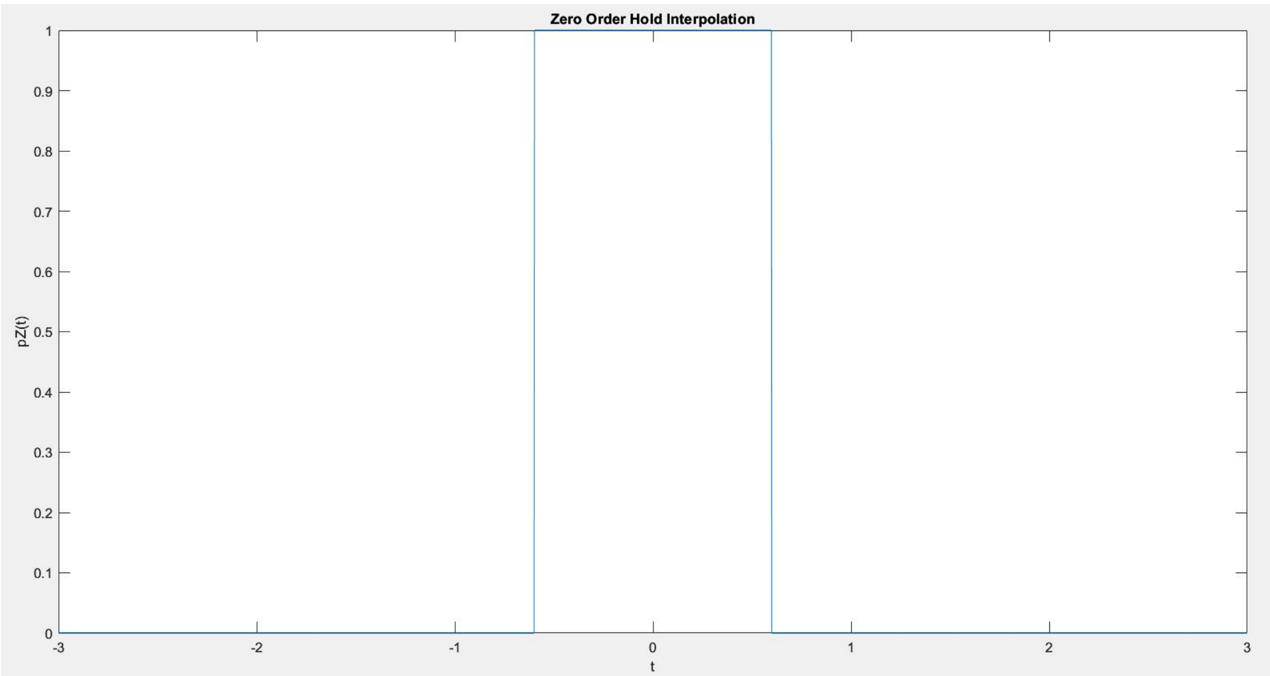


Figure 1: Zero Order Hold Interpolation

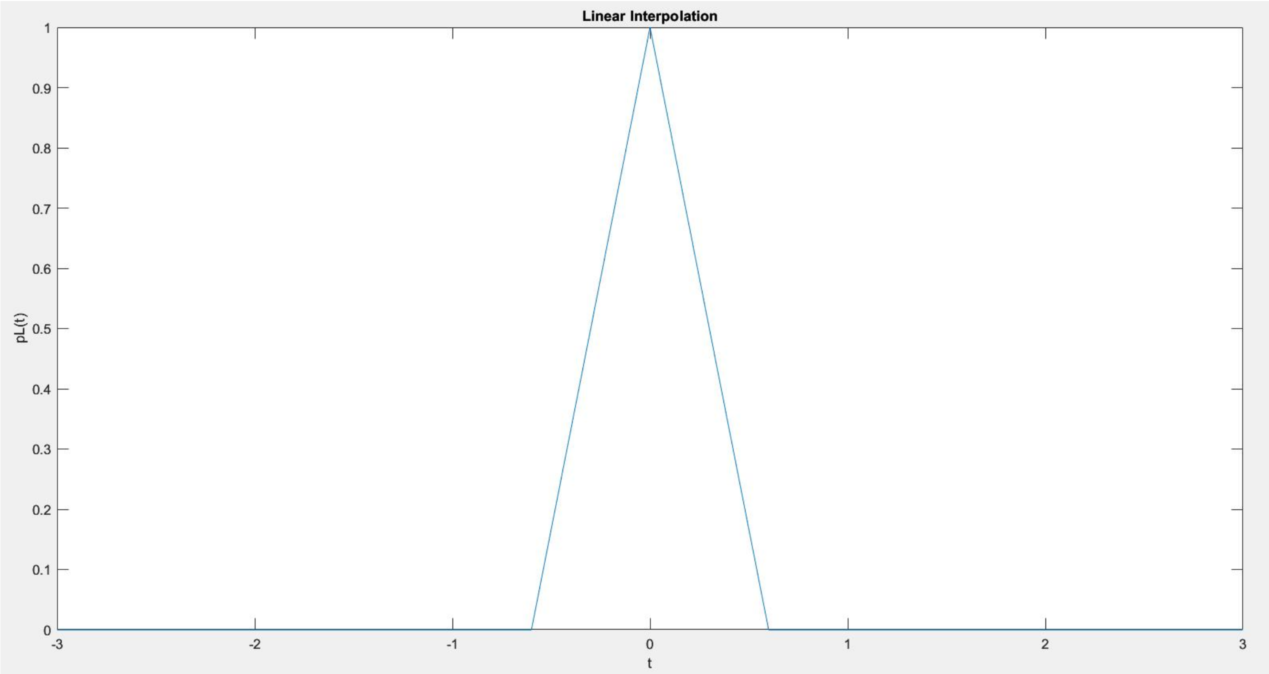


Figure 2: Linear Interpolation

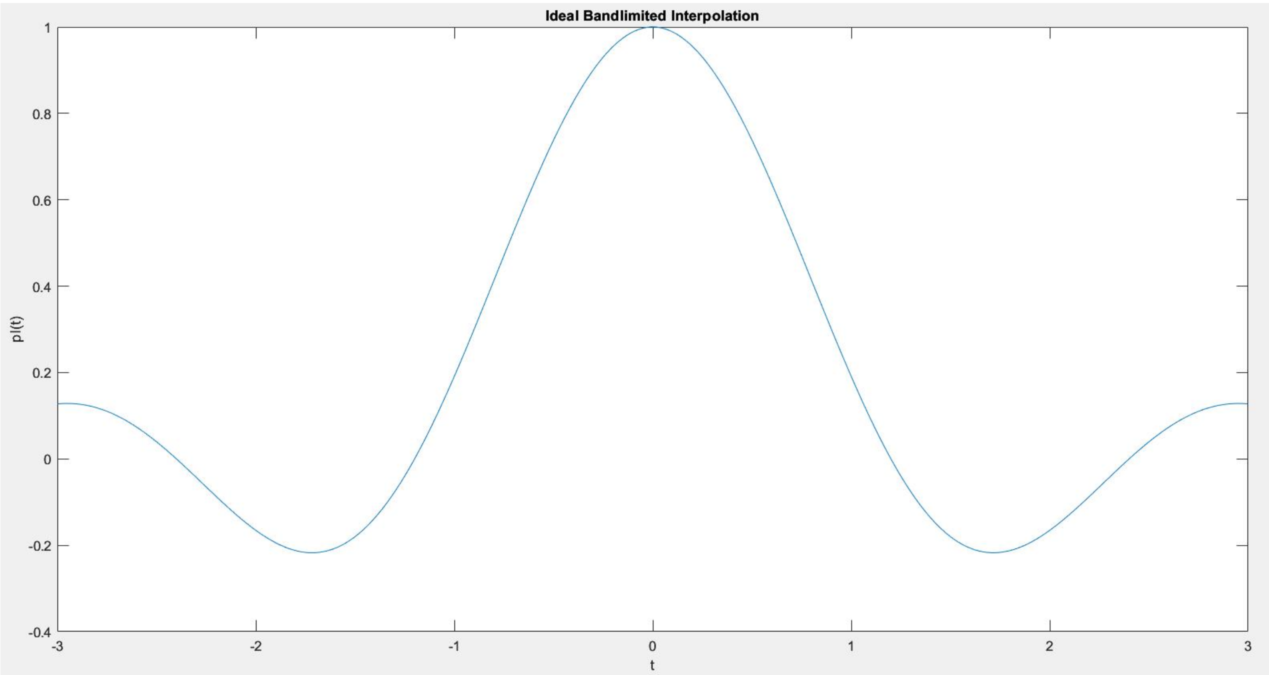


Figure 3: Ideal Bandlimited Interpolation

Part 4

MATLAB code of the DtoA function:

```
function xR=DtoA(type,Ts,dur,Xn)
Tf=Ts/500;
N=length(Xn);
p=generateInterp(type,Ts,dur*Ts);
xR=zeros(1,round((dur+N-1)*Ts/Tf));
for n=0:N-1
```

```
xR(1+round(n*Ts/Tf):round((dur+n)*Ts/Tf))=xR(1+round(n*Ts/Tf):round((dur+n)*Ts/Tf))+Xn
(n+1)*p;
end
end
```

Part 5

MATLAB code for generating g(t):

```
clear
a = randi([2 6],1);
Ts = 1/(25*a);
dur=6;
t = -dur/2:Ts:dur/2-Ts;

g=zeros(1,length(t));
g(-1<=t & t<0) = -2;
g(0<t & t<=1) = 3;

figure;
stem(t,g);
title("Stem of g(nTs)");
xlabel("n");
ylabel("g(nTs)");

figure;
gR1=DtoA(0,Ts,dur,g);
plot(linspace(-3,3,length(gR1)),gR1);
title("Zero Order Hold Reconstruction");
xlabel("n");
ylabel("gR1");

figure;
gR2=DtoA(1,Ts,dur,g);
plot(linspace(-3,3,length(gR2)),gR2);
title("Linear Reconstruction");
xlabel("t");
ylabel("gR2");

figure;
gR3=DtoA(2,Ts,dur,g);
plot(linspace(-3,3,length(gR3)),gR3);
title("Ideal Bandlimited Reconstruction");
xlabel("t");
ylabel("gR3");
```

Plots of the reconstructed signals:

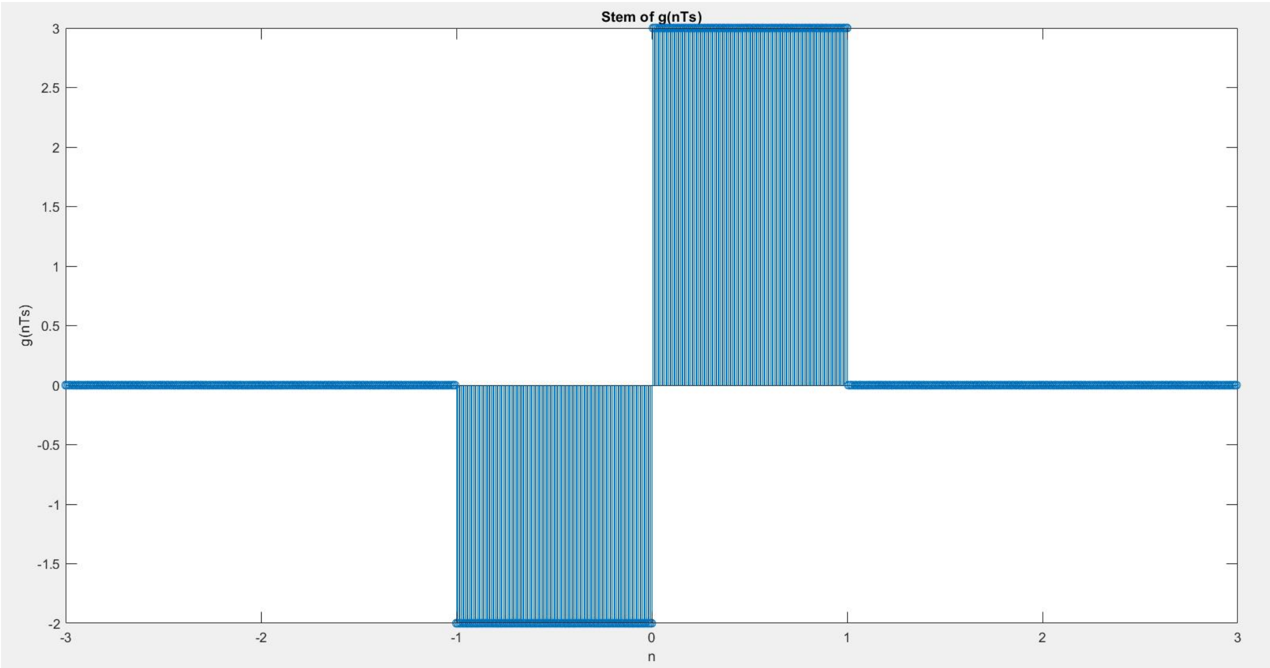


Figure 4: Stem plot of g(nTs)

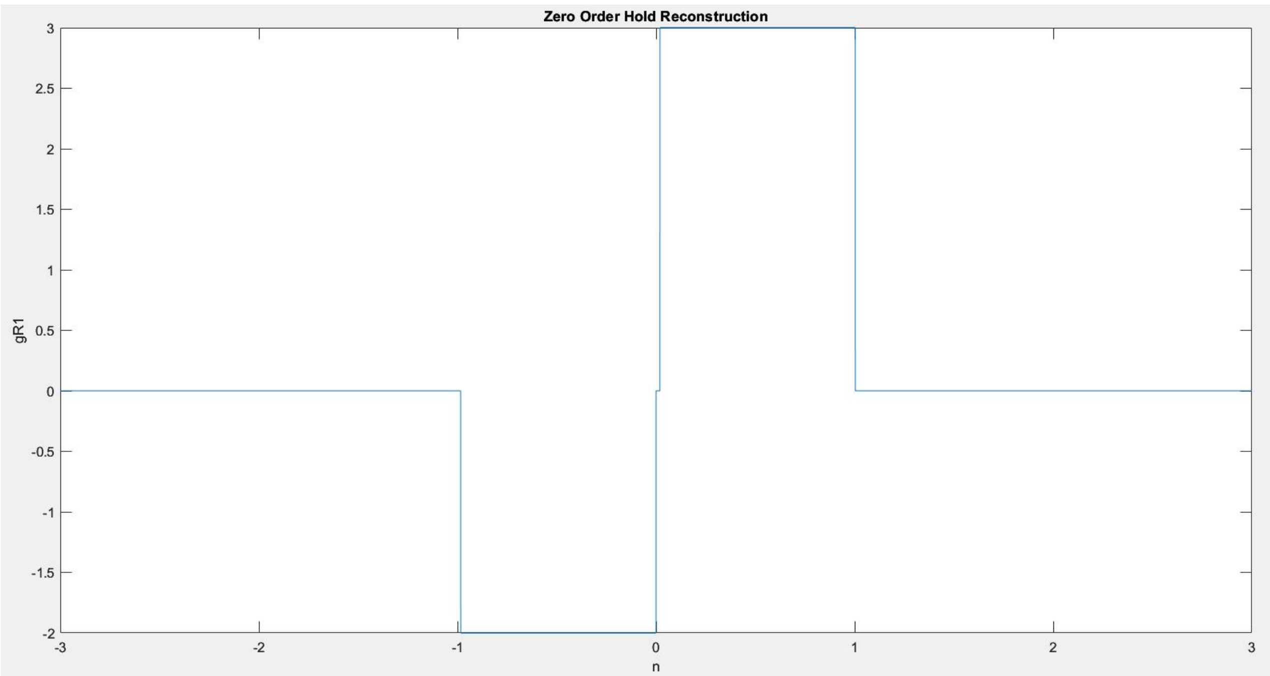


Figure 5: Zero Order Hold Reconstruction of g(t)

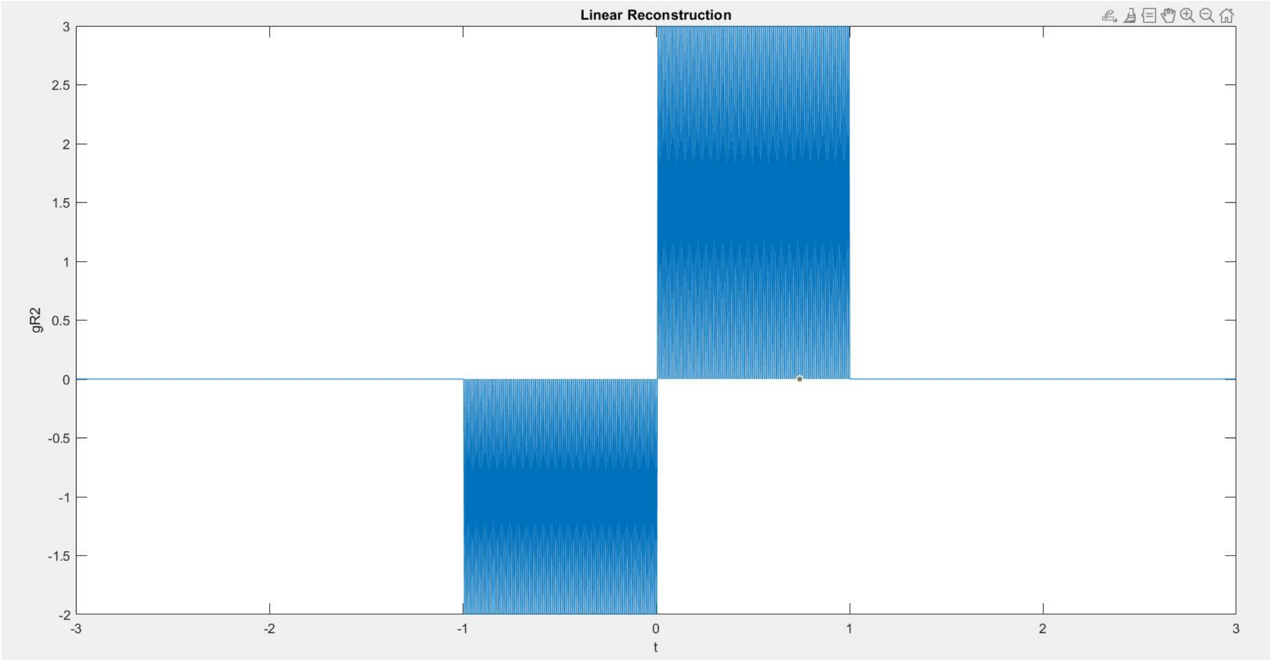


Figure 6: Linear Reconstruction of g(t)

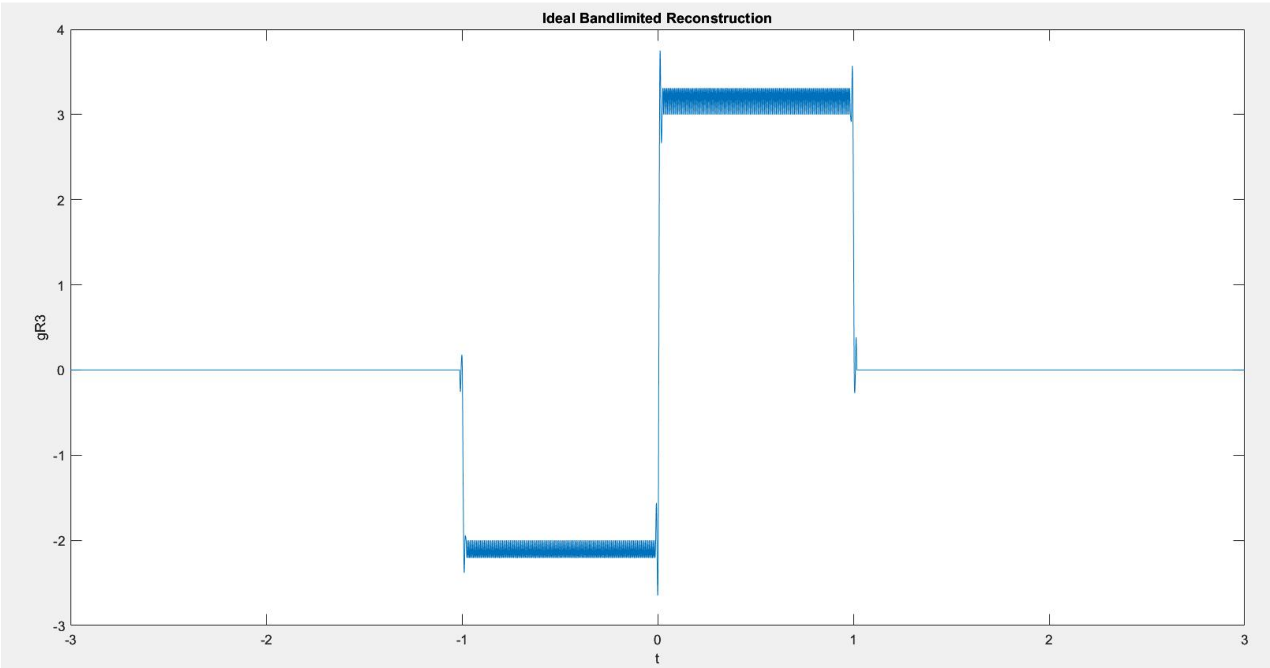


Figure 7: Ideal Bandlimited Reconstruction of g(t)

If we compare the three interpolation methods it can be seen form Figure 5 that the most successful method can be considered as Zero Order Hold Interpolation method as it generates a closer approximation to the original signal g(t). The reason for that may be the shapes of the signals g(t) and zero order pulse are similar to each other as they both are rectangular functions.

Linear interpolation method has the same envelope with the g(t) function but its frequency is much more higher this is because the duration of the pulse was smaller then the optimal value. In fact, if we double the duration of the pulse we get a better approximation. However, I kept it as it was in the lab manual for the sake of consistency.
Ideal bandlimited interpolation method was not ideal for the reconstruction of this signal. Even though, it offers a good approximation for the original signal there are oscillations on the discontinuities which referred as Gibbs phenomenon.
As the sampling period Ts decreases, the sampling frequency increases which means we get more samples from the unit interval. The result of decreasing Ts is we get more accurate approximations. Naturally, as Ts increases, the approximations get less accurate. The trade of for decreasing Ts it makes computations longer and makes the arrays larger which means it requires more time and storage are which is usually not desired. One should find the optimal value for Ts in order to get accurate and efficient approximations.

Part 6

```
D7 = rem(22103444,7); %D7=6
```

A) `Ts=0.005*(D7+1);`

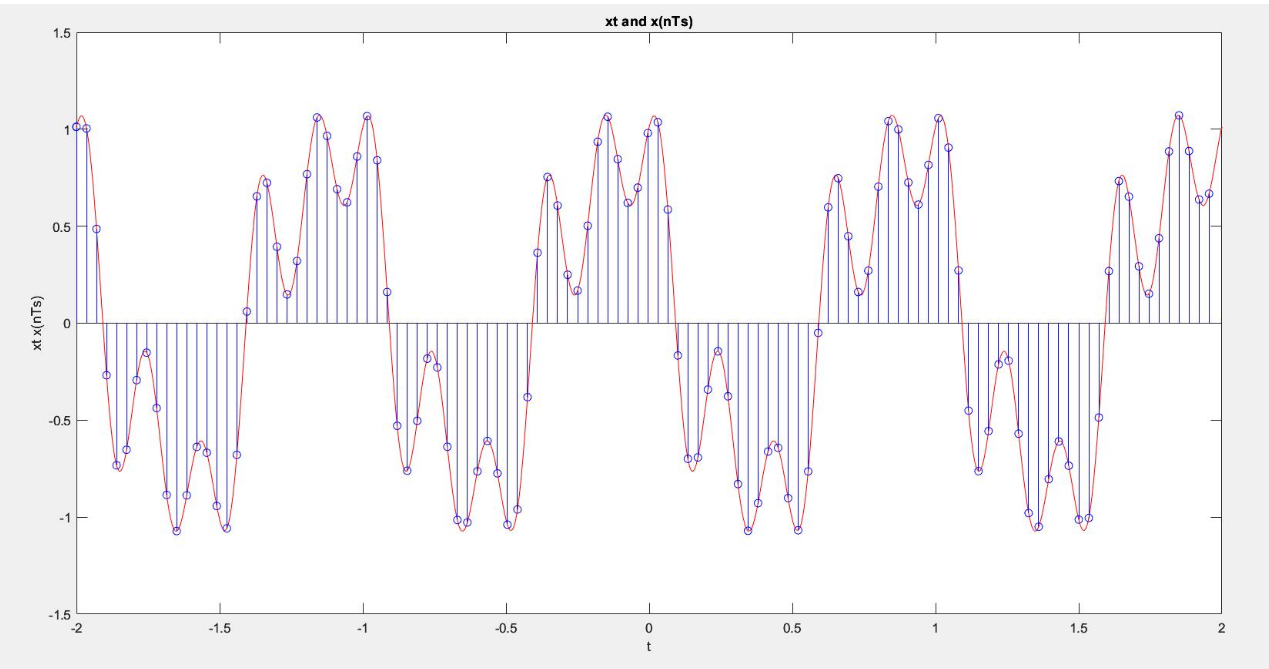


Figure 8: x(nTs) and x(t) when Ts=0.005(D7+1)*

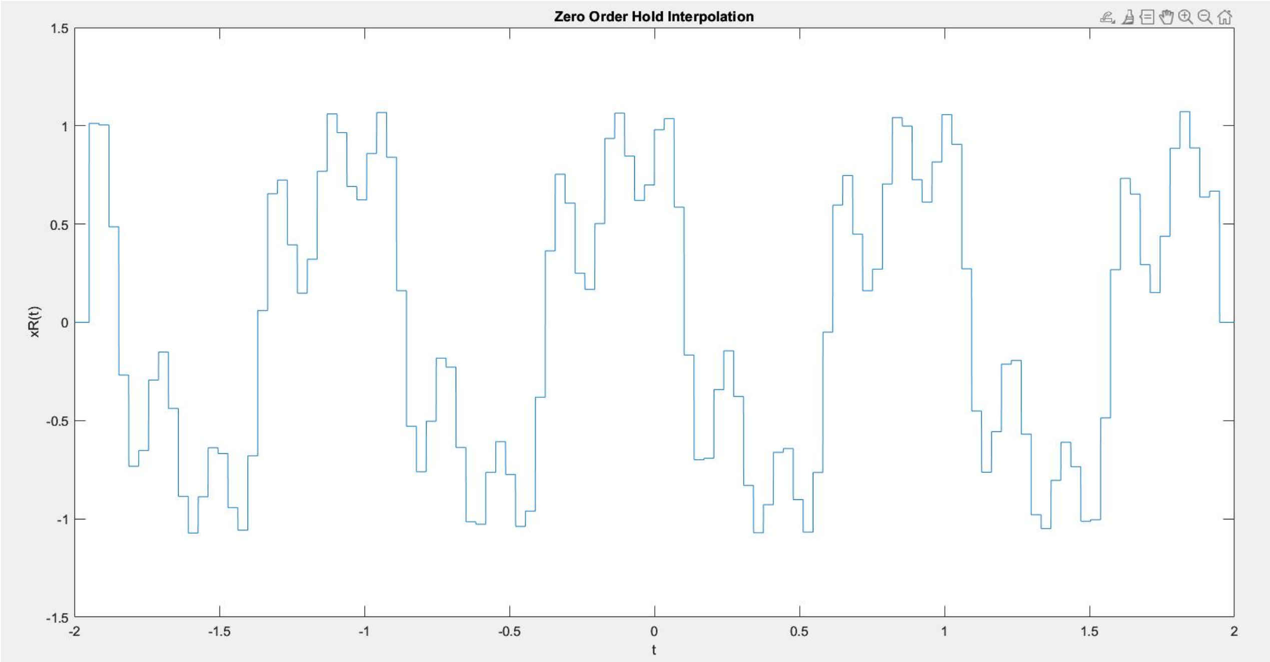


Figure 9: Zero Order Hold Reconstruction of x(t) when Ts=0.005(D7+1)*

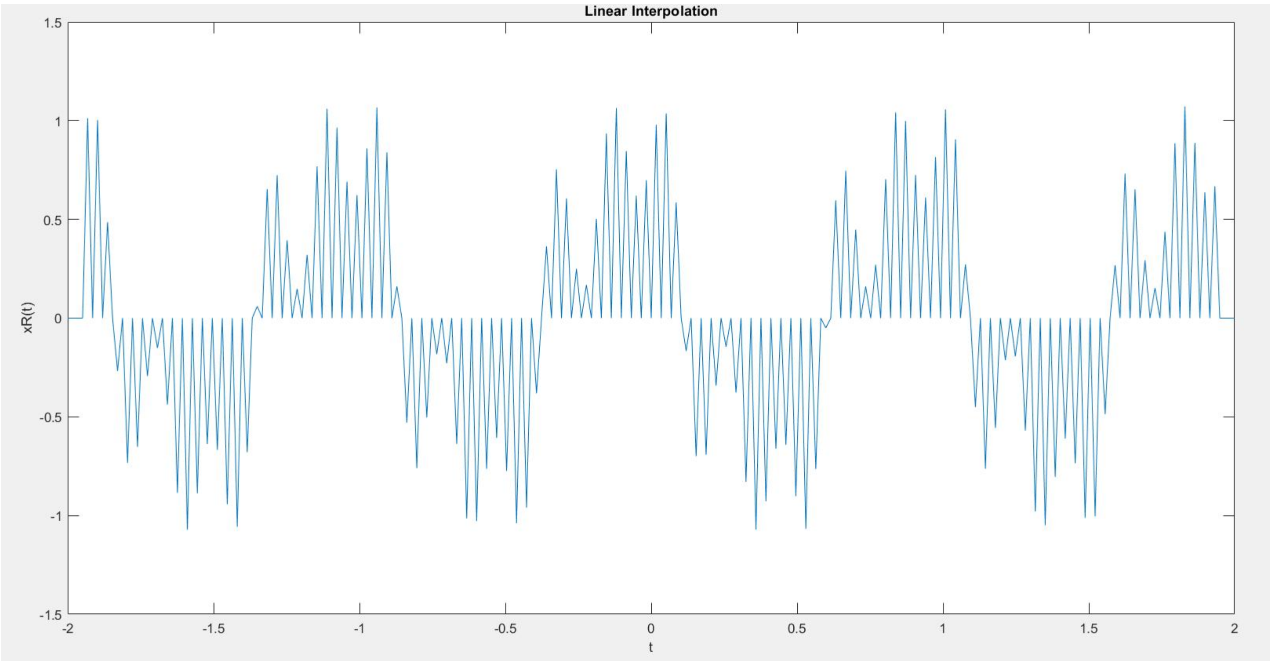


Figure 10: Linear Reconstruction of x(t) when Ts=0.005(D7+1)*

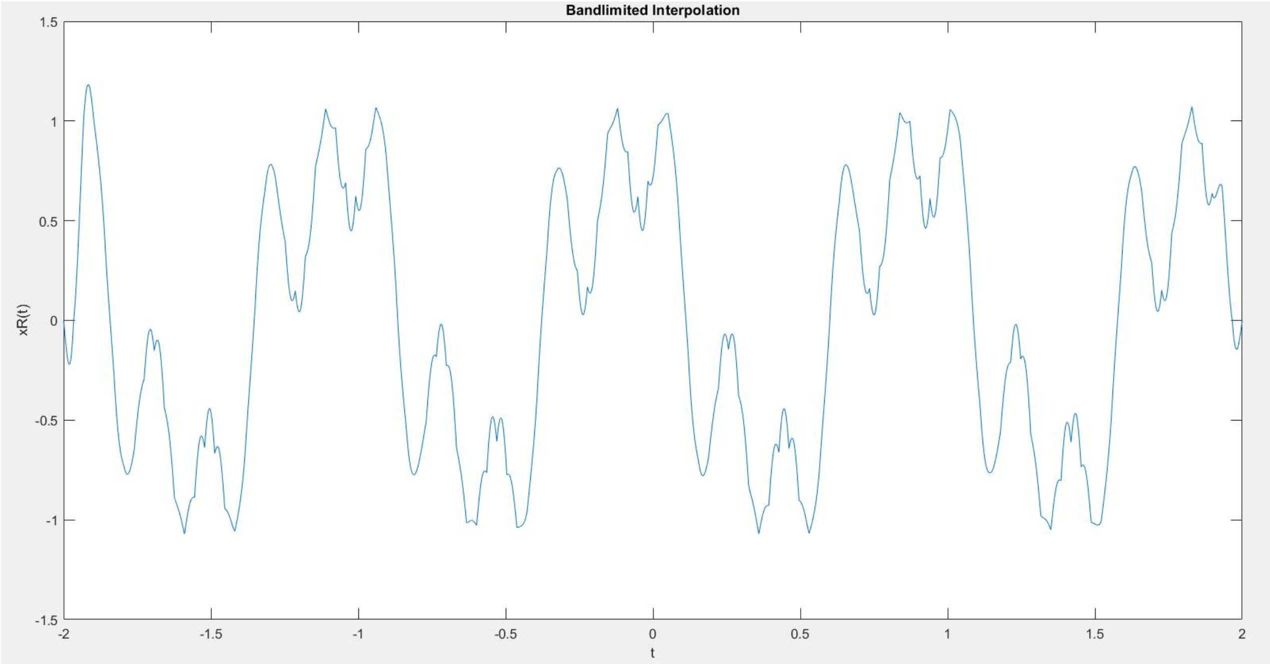


Figure 11: Ideal Bandlimited Reconstruction of x(t) when Ts=0.005(D7+1)*

B) `Ts=0.25+0.01*D7;`

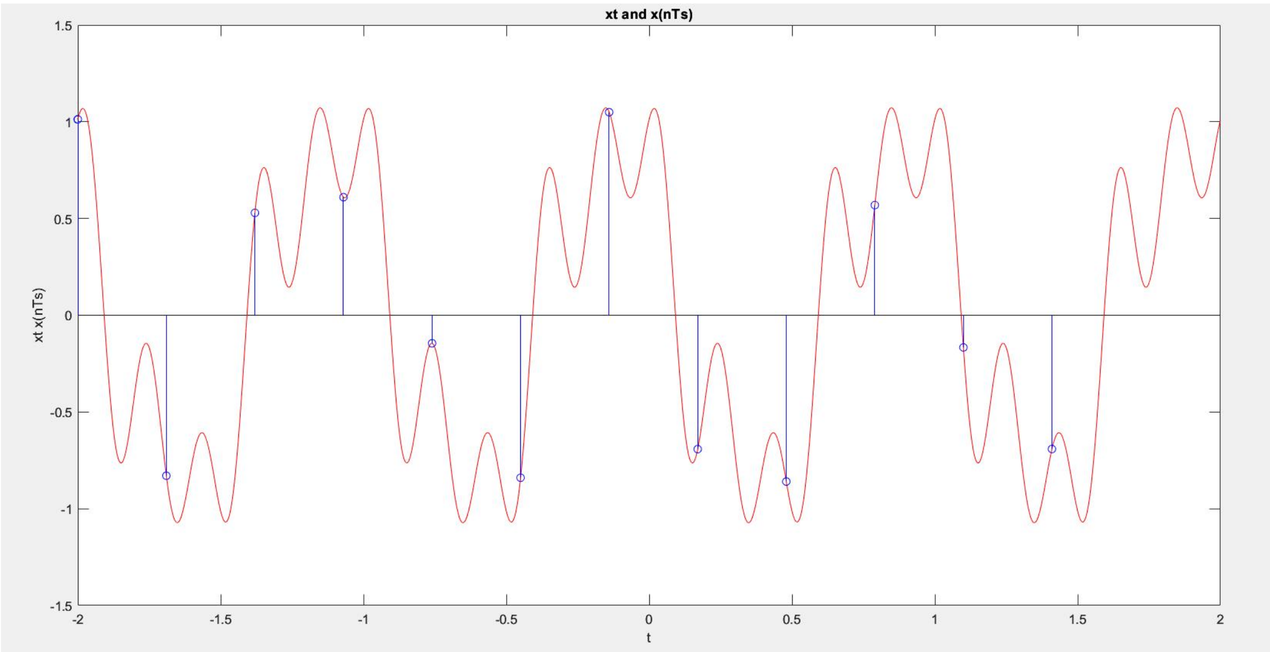


*Figure 12: x(nTs) and x(t) when Ts=0.25+0.01*D7*

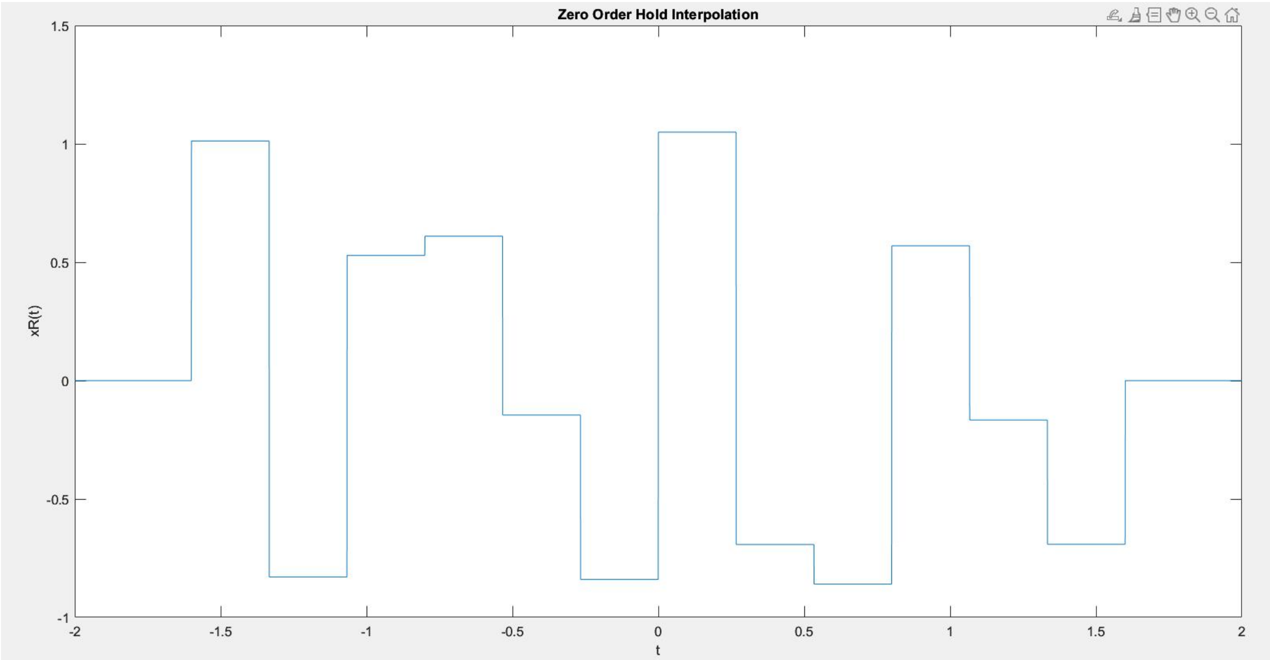


*Figure 13: Zero Order Hold Reconstruction of x(t) when Ts=0.25+0.01*D7*

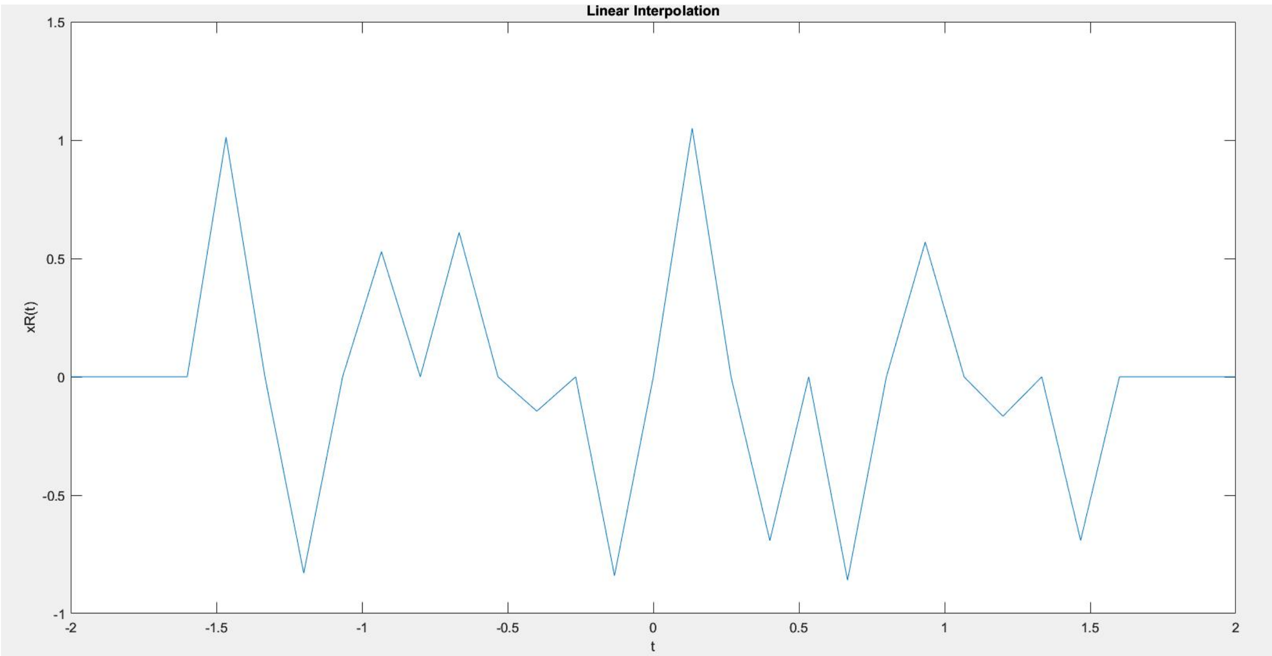


*Figure 14: Linear Reconstruction of x(t) when Ts=0.25+0.01*D7*

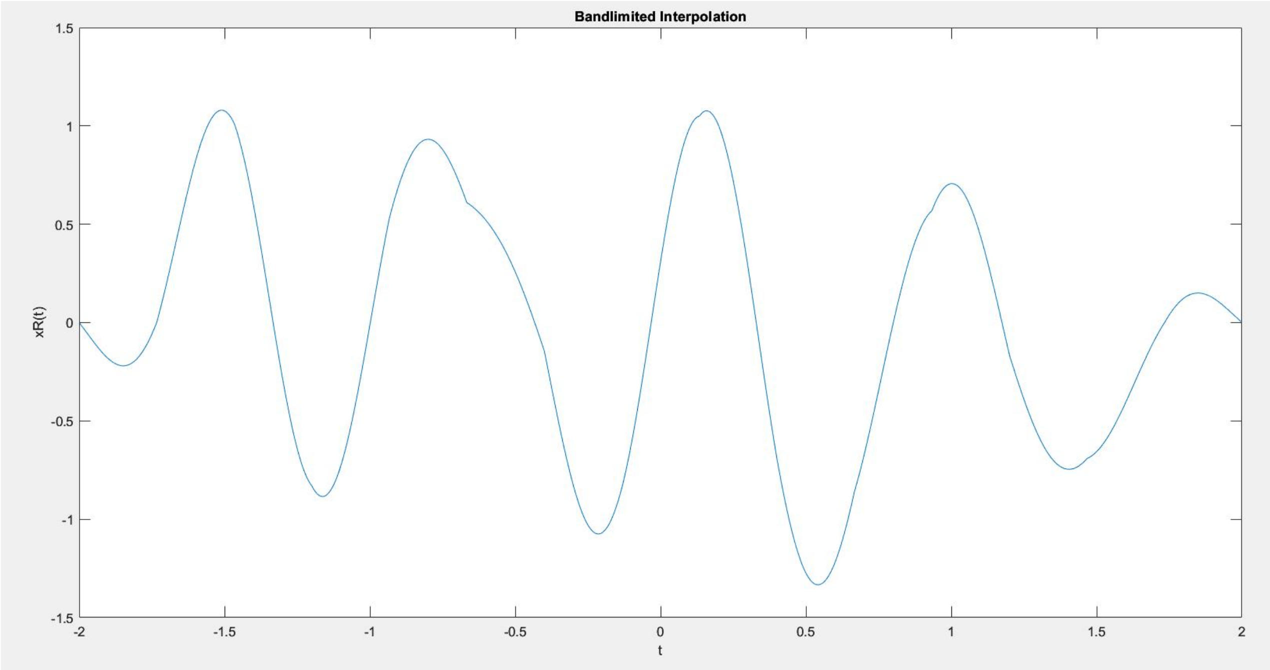


*Figure 15: Ideal Bandlimited Reconstruction of x(t) when Ts=0.25+0.01*D7*

C) `Ts=0.18+0.005*(D7+1);`

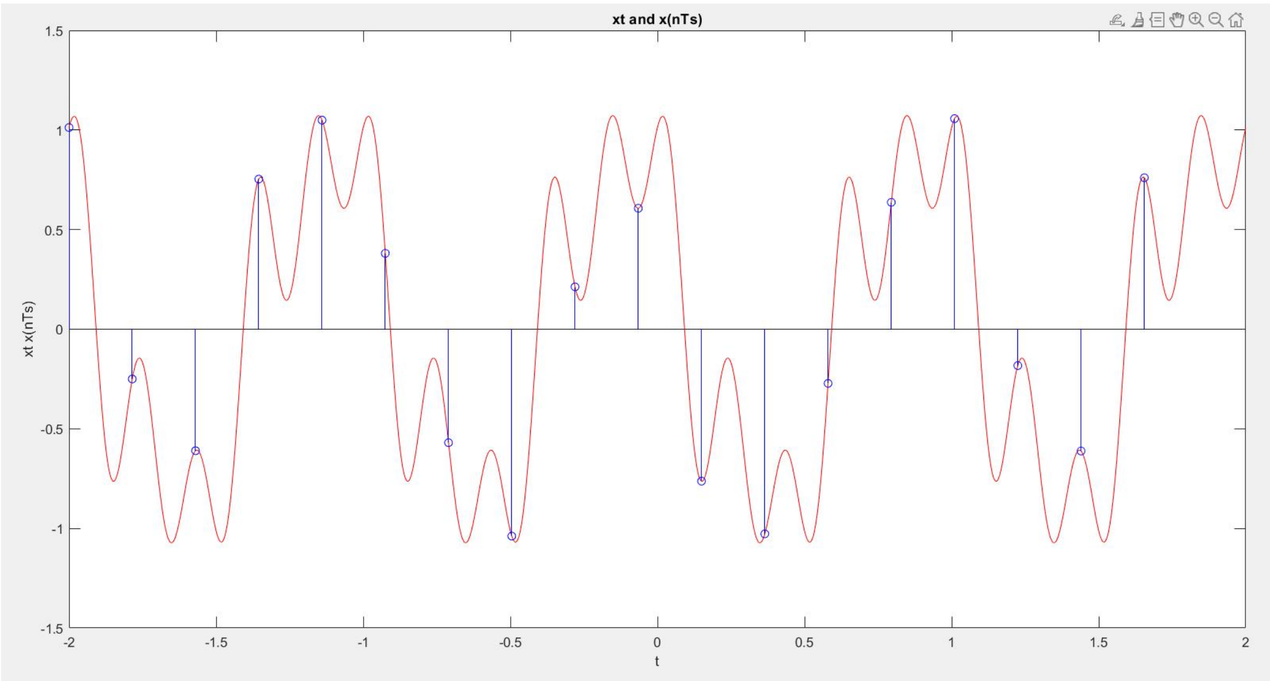


Figure 16: x(nTs) and x(t) when Ts=0.18+0.005(D7+1)*

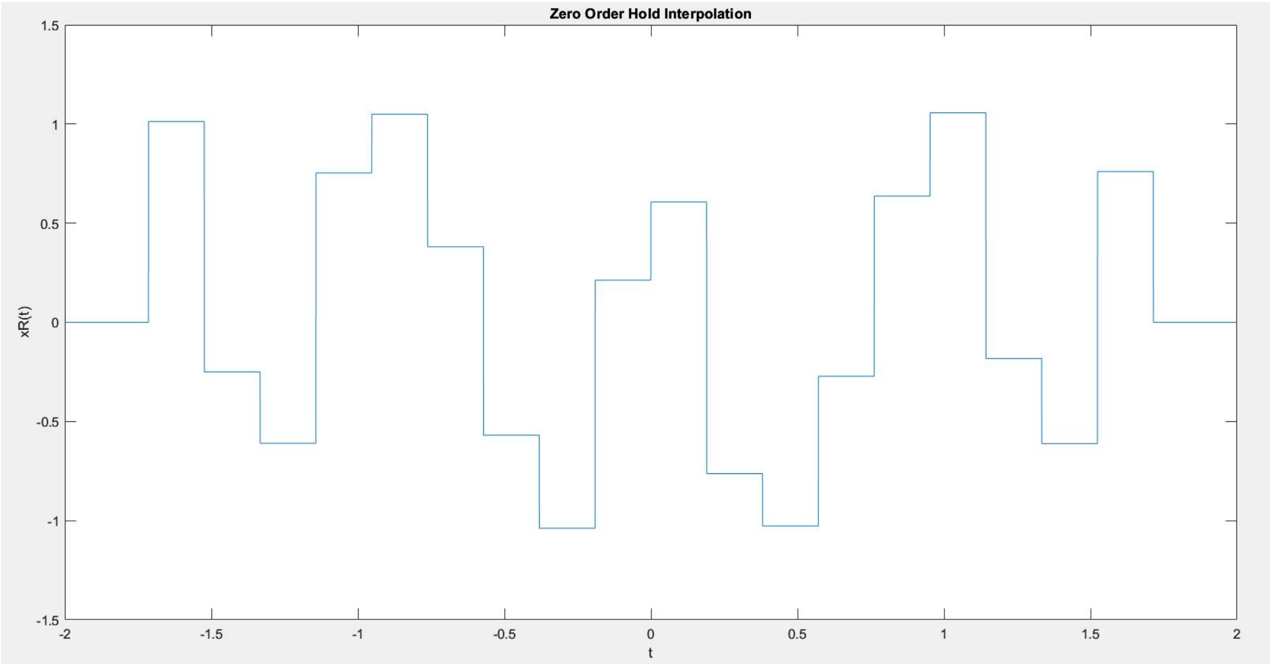


Figure 17: Zero Order Hold Reconstruction of x(t) when Ts=0.18+0.005(D7+1)*

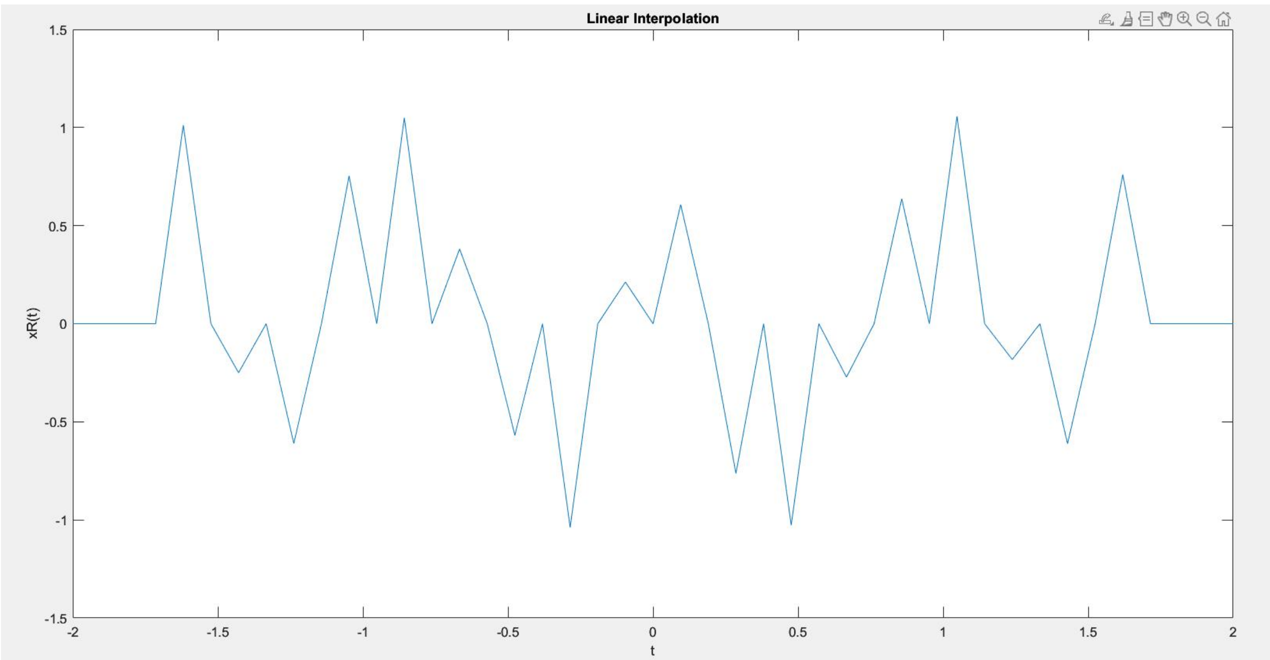


Figure 18: Linear Reconstruction of x(t) when Ts=0.18+0.005(D7+1)*

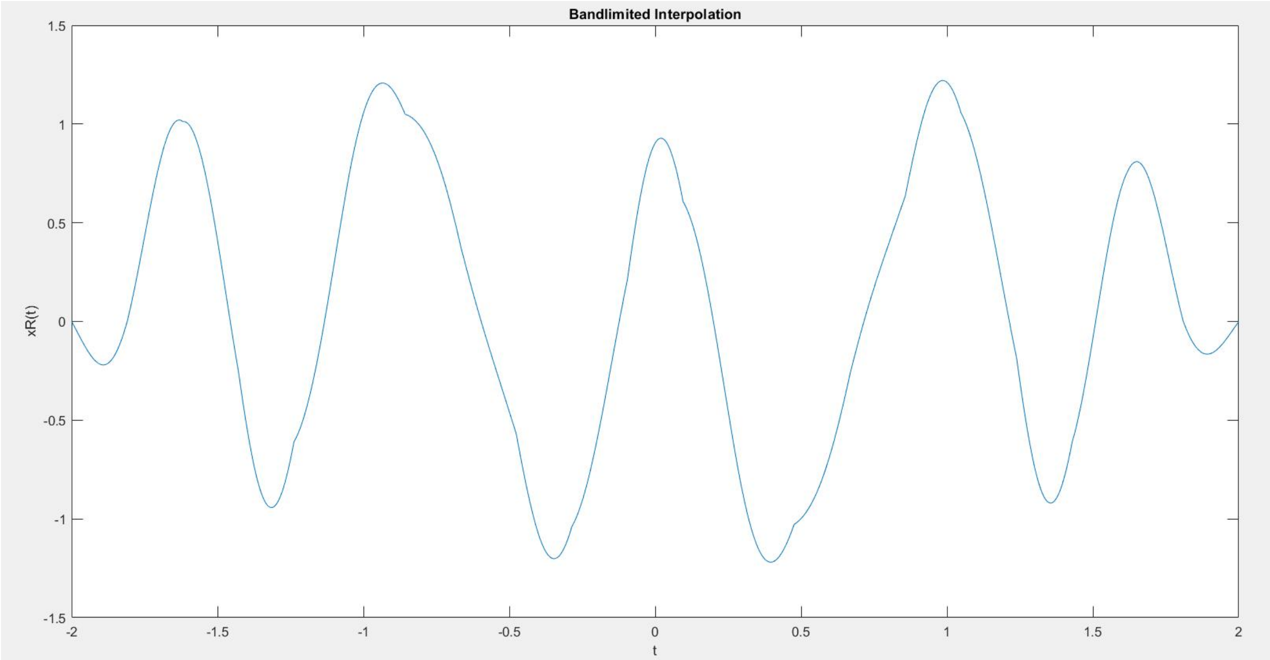


Figure 19: Ideal Bandlimited Reconstruction of x(t) when Ts=0.18+0.005(D7+1)*

D) `Ts=0.099;`

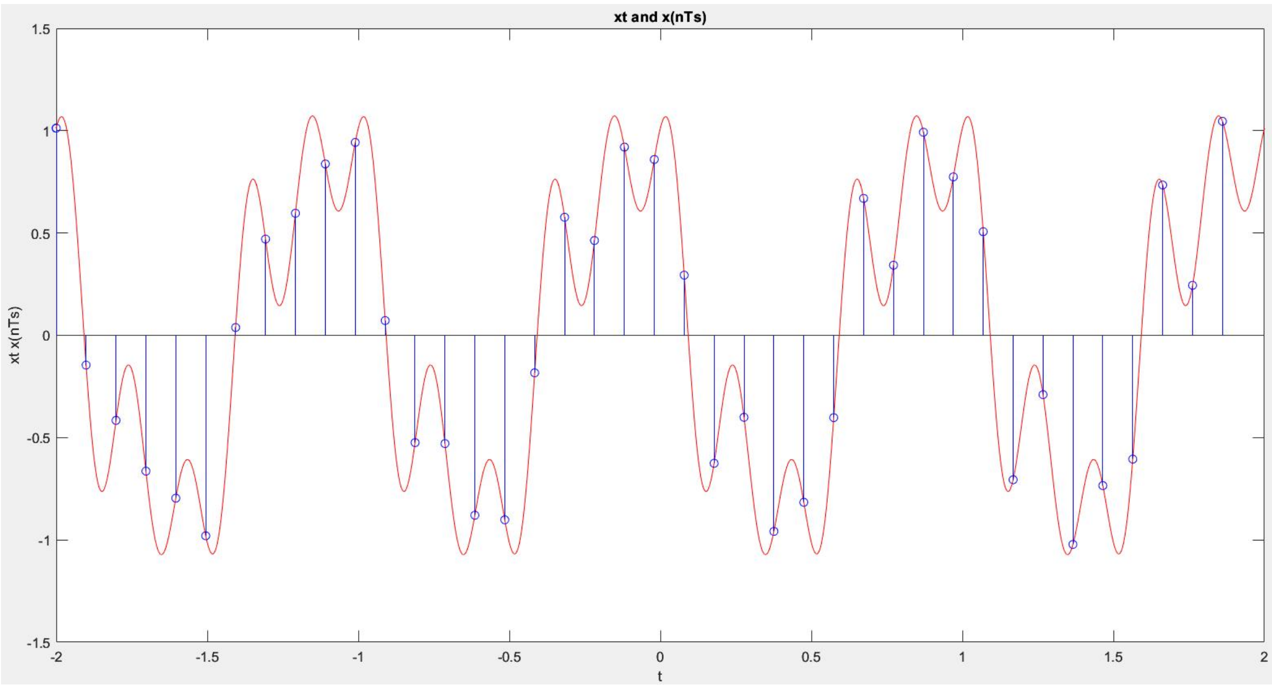


Figure 20: x(nTs) and x(t) when Ts=0.099

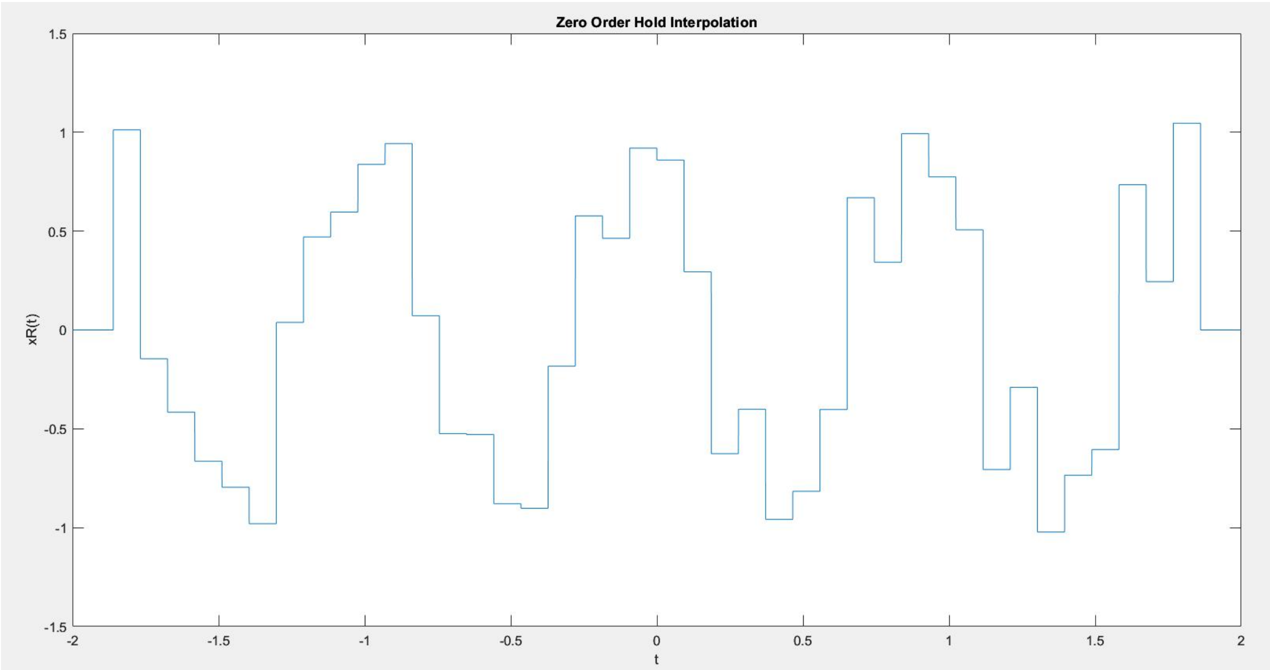


Figure 21: Zero Order Hold Reconstruction of x(t) when Ts=0.099

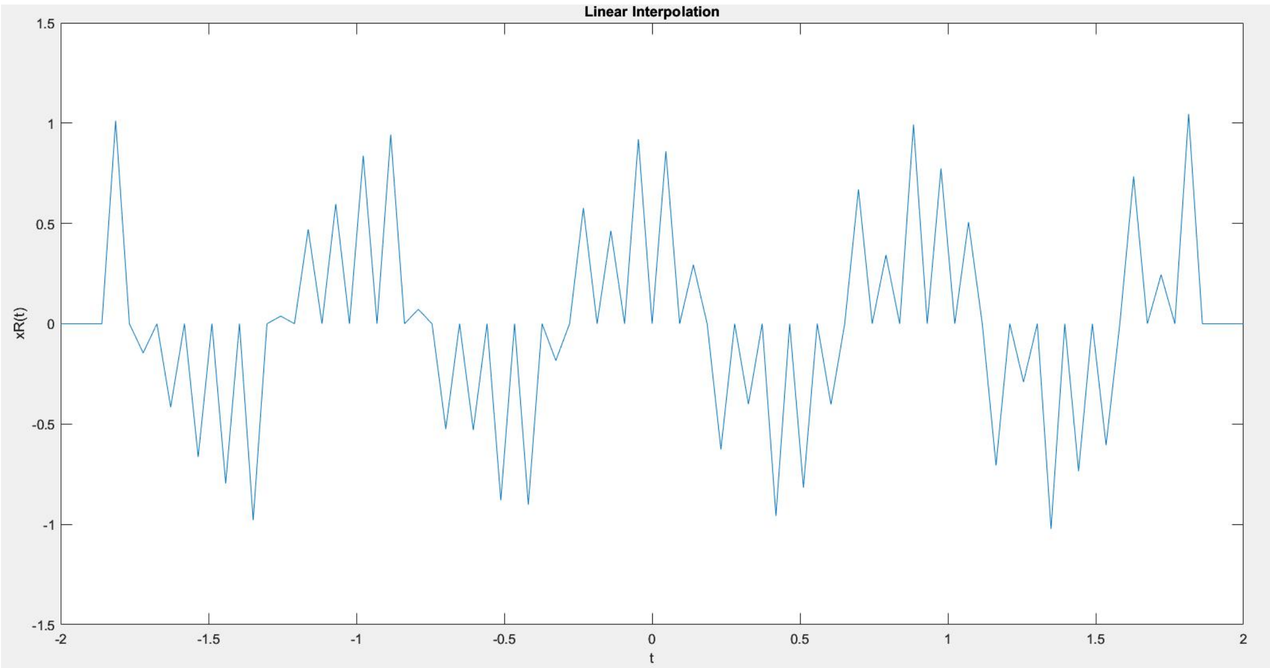


Figure 22: Linear Reconstruction of x(t) when Ts=0.099

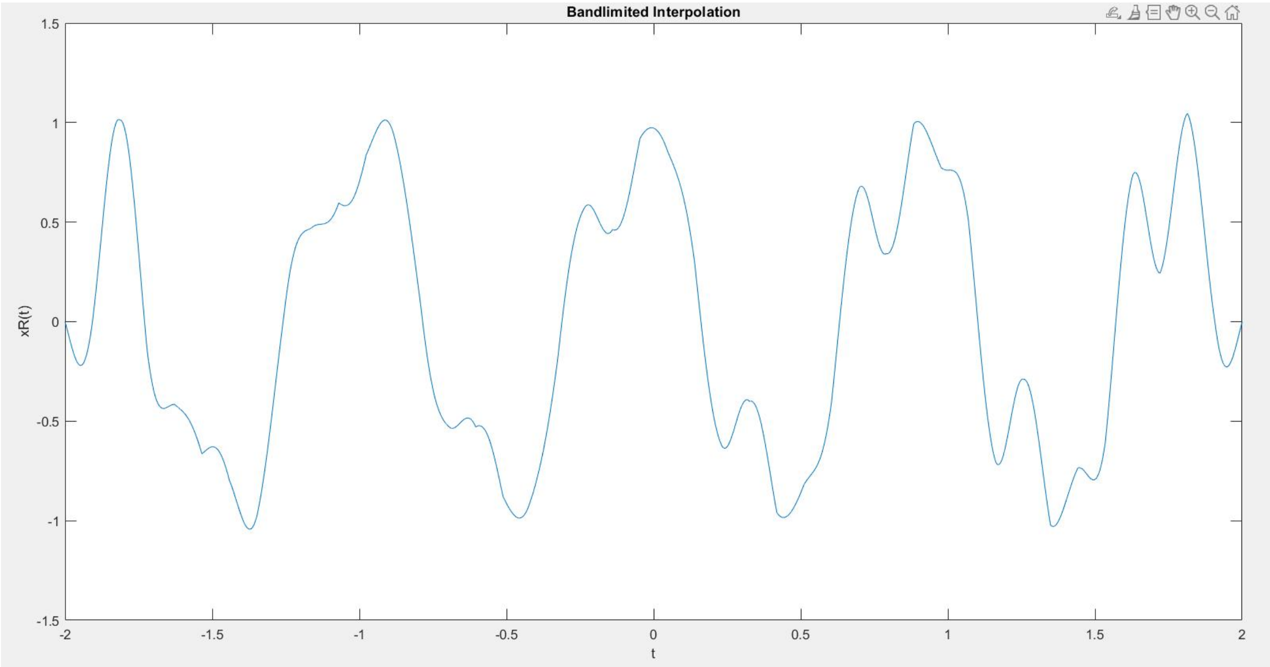


Figure 23: Ideal Bandlimited Reconstruction of x(t) when Ts=0.099

If we compare the interpolation methods, Ideal Bandlimited reconstuction method was the most accurate one for this case. The sampled signal x(t) is a sinusoidal function which means it has a curvy nature. However, zero order interpolation method does not shows to the sinusoidal nature of the signal for this sampling period. If the sampling period was smaller the result would be better. However, this offers a trade-off which explained above, it would require more time and storage space. The same issue with the duration of the linear interpolation method also occurred in this part but even when this issue is fixed sampling rate issue would be problem for this method as it was for the zero order method, therefore this method is not sufficient as well. On the other hand, ideal bandlimited reconstruction method offers a better approximation as it requires less storage space (less sampling rate) and reflects the nature of the sampled signal more accurately.

When Ts is in the interval [0.01, 0.1] the results were relatively accurate, especially for ideal bandlimited interpolation method. However, when Ts is larger than 0.1, i.e. when Ts is in the range [0.1, 0.2], the results become less accurate for each interpolation method. The is due to Nyquist Criteria. Nyquist criteria states that the sampling frequency must be at least 2 times the highest frequency component. Highest frequency component for the sampled signal x(t) is 5Hz. Double of the highest frequency is 10Hz therefore sampling rate fs must be bigger than 10 Hz which means Ts must be less than 0.1. This derivation is written more explicitly below:

$$fs > 2fmax \quad , \quad fmax = 5Hz$$

$$fs > 2 \cdot 5Hz \quad \Rightarrow \quad Ts = 1/fs < 1/10 = 0.1 \quad \Rightarrow \quad Ts < 0.1$$