Part 5

$x_1(t) = A_1 \cos(2\pi f_0 t + \phi_1)$, $x_2(t) = A_2 \cos(2\pi f_0 t + \phi_2)$

whee $A_1 \geq A_2 \geq 0$, $x_3(t) = x_1(t) + x_2(t)$, $x_3(t) = A_3 \cos(2\pi f_3 t + \phi_3)$

$A_3 \geq 0$

$x_1(t) = A_1 e^{i(2\pi f_0 t + \phi_1)}$, $x_2(t) = A_2 e^{i(2\pi f_0 t + \phi_2)}$

$x_3(t) = A_3 e^{i(2\pi f_3 t + \phi_3)}$

$$A_3 e^{i(2\pi f_3 t + \phi_3)} = A_1 e^{i(2\pi f_0 t + \phi_1)} + A_2 e^{i(2\pi f_0 t + \phi_2)}$$

$$= (A_1 e^{i\phi_1} + A_2 e^{i\phi_2}) e^{i2\pi f_0 t} = A_3 e^{i\phi_3} \cdot e^{i2\pi f_3 t}$$

$e^{x+iy} = e^x(\cos y + i\sin y)$

$$e^{i2\pi f_0 t} = e^{i2\pi f_3 t} \Rightarrow \boxed{f_0 = f_3}$$

$$A_1 e^{i\phi_1} + A_2 e^{i\phi_2} = A_3 e^{i\phi_3}$$

$$A_1 \cos(\phi_1) + A_1 i \sin(\phi_1) + A_2 \cos(\phi_2) + A_2 i \sin(\phi_2) = A_3 \cos(\phi_3) + A_3 i \sin(\phi_3)$$

$$A_1 \cos(\phi_1) + A_2 \cos(\phi_2) = A_3 \cos(\phi_3) \qquad (1)$$

$$A_1 \sin(\phi_1) + A_2 \sin(\phi_2) = A_3 \sin(\phi_3) \qquad (2)$$

$$A_3^2 (\underbrace{\cos^2(\phi_3) + \sin^2(\phi_3)}_{1})$$

$$= A_1^2 (\underbrace{\cos^2(\phi_1) + \sin^2(\phi_1)}_{1}) + A_2^2 (\underbrace{\cos^2(\phi_2) + \sin^2(\phi_2)}_{1})$$

$$+2A_1A_2\left(\cos(\phi_1)\cos(\phi_2)+\sin(\phi_1)\sin(\phi_2)\right)$$

$$A_3^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1-\phi_2)$$

since $A_3 \geq 0$

$$\boxed{A_3 = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1-\phi_2)}}$$

in order to find ϕ_3 I will divide equation (2) by (1)

$$\tan(\phi_3) = \frac{A_1\cos(\phi_1)+A_2\cos(\phi_2)}{A_1\sin(\phi_1)+A_2\sin(\phi_2)}$$

$$\boxed{\phi_3 = \arctan\left(\frac{A_1\cos(\phi_1)+A_2\cos(\phi_2)}{A_1\sin(\phi_1)+A_2\sin(\phi_2)}\right)}$$

$$A_3 = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1-\phi_2)}$$

a) for A_3 to be minimum $\cos(\phi_1-\phi_2) = -1$

therefore $\phi_1-\phi_2 = (2k-1)\pi \quad , k \in \mathbb{Z}$

b) for A_3 to be maximum $\cos(\phi_1+\phi_2) = 1$

therefore $\phi_1-\phi_2 = 2k\pi \quad , k \in \mathbb{Z}$