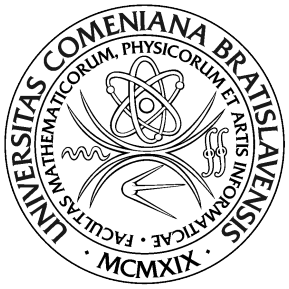
UNIVERZITA KOMENSKÉHO V BRATISLAVE
FAKULTA MATEMATIKY, FYZIKY A INFORMATIKY



GRUPY AUTOMORFIZMOV LINEÁRNYCH KÓDOV

Diplomová práca

2022 Bc. Branislav Boráň

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Diplomová práca

Študijný program:	Aplikovaná informatika
Študijný odbor:	2511 Aplikovaná informatika
Školiace pracovisko:	Katedra algebry a geometrie
Školiteľ:	doc. RNDr. Róbert Jajcay, DrSc.

Bratislava, 2022 Bc. Branislav Boráň

Univerzita Komenského v Bratislave
Fakulta matematiky, fyziky a informatiky

ZADANIE ZÁVEREČNEJ PRÁCE

Meno a priezvisko študenta: Branislav Boráň
Študijný program: aplikovaná informatika (Jednoodborové štúdium, magisterský II. st., denná forma)
Študijný odbor: informatika
Typ záverečnej práce: diplomová
Jazyk záverečnej práce: anglický
Sekundárny jazyk: slovenský

Názov: Automorphism groups of linear codes and linear codes with prescribed automorphism groups
Grupy automorfizmov lineárnych kódov a lineárne kódy s predpísanou grupou automorfizmov

Anotácia: Lineárne kódy sú podpriestory konečnorozmerných vektorových priestorov nad konečnými poľami. Majú preto bohaté grupy automorfizmov, ktoré zároveň obsahujú množstvo informácií o uvažovanom kóde. Určenie úplnej grupy automorfizmov kódu je výpočtovo náročná úloha. Namiesto určenia grupy automorfizmov pre daný kód sa preto uvažuje obrátená úloha zostrojenia kódu s predpísanou grupou automorfizmov. Cieľom práce je preskúmať oba smery tejto interakcie.

Cieľ: Cieľom navrhovanej problematiky je poskytnúť študentovi výpočtovo zložitý problém vyžadujúci dôkladné porozumenie štruktúry uvažovaných objektov ako aj programátorské a organizačné schopnosti.

Literatúra:
R. Hill, A first course in coding theory, Oxford University Press, 1993
S. Roman, Coding and information theory, Springer, 1992
R. Jajcay, P. Potocnik and Stephen E. Wilson, Half-cyclic, dihedral and half-dihedral codes,
J. of Applied Mathematics and Computing 64 (2020), 691-708.

Kľúčové slová: lineárny kód, grupa automorfizmov, konečné pole

Vedúci: doc. RNDr. Róbert Jajcay, DrSc.
Katedra: FMFI.KAG - Katedra algebry a geometrie
Vedúci katedry: doc. RNDr. Pavel Chalmovianský, PhD.

Dátum zadania: 09.12.2020

Dátum schválenia: 10.12.2020

prof. RNDr. Roman Ďurikovič, PhD.
garant študijného programu

.....
študent

.....
vedúci práce

Čestne prehlasujem, že túto diplomovú prácu som vypracoval samostatne len s použitím uvedenej literatúry a za pomoci konzultácií u môjho školiteľa.

. . . . . .

Bratislava, 2022 Bc. Branislav Boráň

Poďakovanie

Chcel by som sa v prvom rade poďakovať môjmu školiteľovi doc. RNDr. Róbertovi Jajcayovi, DrSc. za odbornú pomoc a usmernenia pri písaní tejto práce, za materiály, cenné rady, ktoré mi veľmi pomohli pri riešení tejto diplomovej práce. V neposlednom rade chcem tiež poďakovať všetkým mojím kamarátom a celej mojej rodine za podporu počas môjho štúdia.

Abstrakt

Táto práca sa venuje problematike xxxxxxxxx. Súčasťou tejto práce je prehľad existujúcich riešení a ich krátke zhodnotenie. Ďalej je tu xxxxx. Čo sa tu rieši.

Kľúčové slová: automorfizmus grúp, ...

Abstract

english abstract

Keywords: Automorphism groups, ...

Obsah

Kapitola 1

Úvod

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Kapitola 2

Motivácia

xxxxxx

Kapitola 3

Analýza problému

3.1 Lineárny kód

Lineárny kód (n, k) je k-rozmerný lineárny podpriestor priestoru F_n^2. F_n^2 je priestor n–rozmerných vektorov, kde koordináty berieme z poľa F^2. k-rozmerný lineárny podpriestor obsahuje práve k lineárne nezávislých vektorov. Ak by sme zobrali k takých vektorov, potom tieto vektory generujú daný k-rozmerný podpriestor a hovoríme, že tvoria bázu podpriestoru. Ak je splnená vlastnosť modulo 2 (q) súčtu 2 kódových slov je kódové slovo, tak vieme nájsť Generačnú maticu lineárneho kódu.

3.1.1 Generujúca matica lineárneho kódu

Generujúca matica lineárneho kódu (G) je zostrojená z bázy lineárneho kódu tak, že riadky matice predstavujú prvky bázy. Riadky generujúcej matice sú lineárne nezávislé vektory dĺžky n. Nech $\vec{m}$ je vstup (nekódované slovo), $\vec{v}$ je výstup (kódované slovo), C je označenie lineárneho kódu, potom platí:

$$C = \{\vec{m} \times G : \vec{m} \in F_2^k\}, \quad \vec{v} = \vec{m} \times G \tag{3.1}$$

3.1.2 Kontrolná matica lineárneho kódu

V k-rozmernom linearnom kóde (C) v F_n^2 potom existuje $n-k$ lineárne nezávislých vektorov $\vec{v}$ takých, že každé kódové slovo je kolmé na všetky tieto vektory. Keď týchto $n-k$ vektorov zoberieme ako riadky matice, dostaneme kontrolnú maticu lineárneho kódu H. Ľubovoľný vektor $\vec{v}$ je kódovým slovom práve vtedy, ak platí:

$$C = \{\vec{v} \in F_2^n : H \times \vec{v}^T = 0\} \tag{3.2}$$

3.1.3 LDPC kódy

LDPC kódy (z angl. low density parity check code) sú lineárne samoopravné kódy, ktoré jednak umožňujú prenos dát rýchlosťou blízkou kapacite kanálu a zároveň pre ne existujú vysoko účinné dekódovacie algoritmy. Kódy majú veľmi riedku kontrolnú maticu, pomocou ktorej sa dajú opraviť chyby v kódových slovách. Ich kontrolná matica obsahuje menej ako 1% jednotiek. Hlavnou nevýhodou väčšiny LDPC kódov je vysoká časová náročnosť ich kódovacieho algoritmu. Výhodou je paralelizmus pri dekódovaní a jednoduché výpočtové operácie. Dekódovacie výpočty sú rozdelené do 2 množín uzlov a to do kontrolných uzlov a premenných uzlov. Uzol na jednej strane je spojený s uzlom na druhej strane, čo umožňuje paralelné výpočty na každej strane.

3.1.4 Kvázicyklický kód

Lineárny kód (C) je kvázicyklický kód, ak existuje kontrolná matica H, ktorá má tvar:

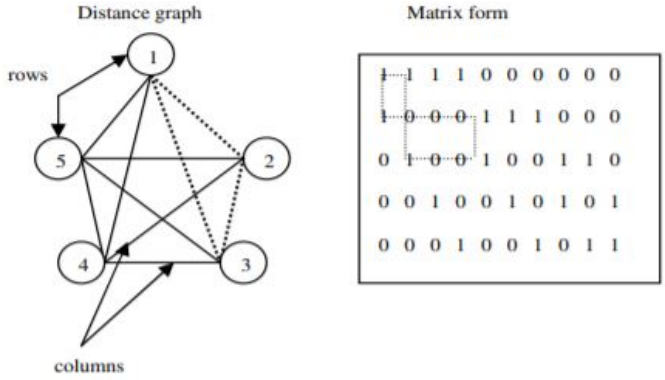
$$H = (H_0|H_1|...|H_{n_0}^{-1}) \tag{3.3}$$

H_i sú cyklické matice. Ak sú LDPC kvázicyklické, nazývame ich QC-

LDPC kódy.

3.2 Grafová reprezentácia LDPC kódov

Matica LDPC je reprezentovaná Tannerovým grafom. Matica môže byť tiež reprezentovaná nebipartitným grafom alebo grafom vzdialenosti, v ktorom riadky matice predstavujú vrcholy a stĺpce matice reprezentujú hrany grafu. Stĺpec je potom množina hrán formujúca kompletný graf medzi vrcholmi spojenými v stĺpci. Nasledujúci obrázok ilustruje grafovú reprezentáciu matice LDPC kódu odvodenú z grafu vzdialenosti:



Obr. 3.1: Vztah medzi grafom a maticou [Mal07]

Graf vzdialenosti je formovaný cestami hrán alebo vrcholov Cyklus dĺžky g v grafe korešponduje s cyklom dĺžky $2g$ v maticovej forme.

3.2.1 Základné pojmy

- Dĺžka kódu - špecifikuje dimenzie $(M \times N)$ kontrolnej matice H. M predstavuje počet riadkov matice a N je počet stĺpcov.

- Kódová váha a rate (R) - predstavuje počet bitov (informácií) nad celkovým počtom prenesených bitov. Rate možno vyjadriť vzťahom:

$$R = (N - M)/N \tag{3.4}$$

- Minimálna Hammingová (kódová) vzdialenosť $minHW(\vec{u}, \vec{v})$ - Nech sú vektory $\vec{u}$ a $\vec{v}$ kódové slová. Minimálna Hammingová vzdialenosť 2 vektorov $\vec{u} \in F_n^2$ a $\vec{v} \in F_n^2$ je počet koordinátov, na ktorých sa vektory $\vec{u}$ a $\vec{v}$ líšia.

- Obvod (g) - ovplyvňuje dekódovanie LDPC kódu. V grafovej reprezentácií LDPC kódu sa jedná o najmenší cyklus v grafe. Jeho dĺžku zrátavame iba pomocou vrcholov alebo hrán. V matici LDPC kódu je dĺžka obvodu $2g$, pretože cyklus alternuje medzi riadkami a stĺpcami z čoho vyplýva, že cyklus grafu reprezentuje iba polovicu maticového kódu.

- Moorov graf - Pravidelný graf stupňa d a parametra k vo forme stromu vyhľadávania do šírky začínajúceho z ľubovoľného vrcholu V, ktorého počet vrcholov vieme dostať ako:

$$1 + d\sum_{i=0}^{k-1}(d-1)^i \tag{3.5}$$

- Rád grafu - Predstavuje počet vrcholov daného grafu

3.2.2 Automorfizmus grafu

Automorfizmus grafu je permutácia ϕ všetkých vrcholov grafu, ktorá zachováva jeho štruktúru takým spôsobom, že akékoľvek 2 vrcholy U a V susedia

iba vtedy a len vtedy ak platí, že $\phi(U)$ susedí s $\phi(V)$. Zjednodušene môžme povedať, že sa jedná o bijektívne zobrazenie, pri ktorom sa každý vrchol grafu a každá hrana zobrazí na iný vrchol a hranu, hovoríme tiež, že ide o jeho obraz. Množina všetkých automorfizmov grafu G tvorí grupu automorfizmov $Aut(G)$. Moorové grafy vlastnia grupu automorfizmov, ktorá prechodne pôsobí na vrcholy daného grafu.

3.2.3 Klietky

Na konštrukciu LDPC kódov môžme využiť grafy vzdialenosti. Tieto grafy delíme na pravidelné s vrcholmi rovnakého stupňa (Moorové grafy) a nepravidelné s vrcholmi rôznych stupňov. Klietka $cage(k, g)$ je k-pravidelný graf obvodu g s najmenším možným počtom vrcholov m. Výpočet minimálneho počtu vrcholov pre klietku sa líši podľa toho, či je jej obvod párny alebo nepárny:

- g - nepárne:

$$m = 1 + \sum_{i=0}^{(g-3)/2} k(k-1)^i = \frac{k(k-1)^{(g-1)/2} - 2}{k-2} \tag{3.6}$$

- g - párne:

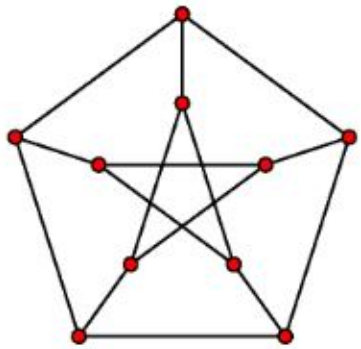
$$m = 2 \sum_{i=0}^{(g-2)/2} k(k-1)^i = \frac{2(k-1)^{g/2} - 2}{k-2} \tag{3.7}$$

Takéto dolné ohraničenie počtu vrcholov m je tiež nazývané ako Moorové ohraničenie a označuje sa tiež $M(k, g)$. Pre klietku ako Moorov graf platí:

$$d = k \tag{3.8}$$

Aj keď neexistuje jednotná konštrukcia klietok, existuje niekoľko známych klietok pre stupeň vrchola k a obvod g. Ukážeme si niektoré z nich:

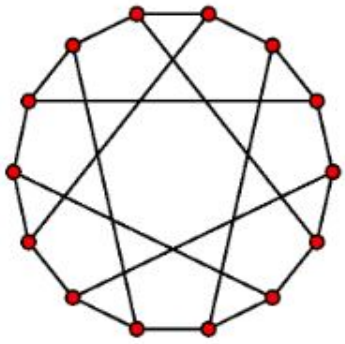
- Petersenov graf - $cage(3, 5)$:



Obr. 3.2: Petersenov graf [EJ11]

Petersenov graf má rád 10. Automorfizmus grúp je izomorfný k Sym(5). Graf je vrcholovo tranzitívny.

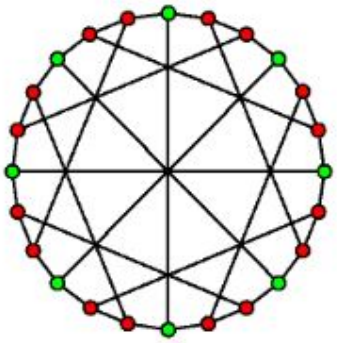
- Heawoodov graf - $cage(3, 6)$:



Obr. 3.3: Heawoodov graf [EJ11]

Heawoodov graf má rád 14 a počet grúp automorfizmov je 336. Graf je vrcholovo tranzitívny.

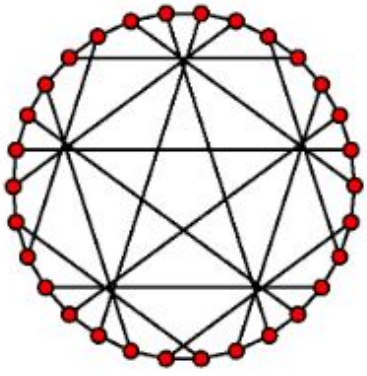
- McGeeho graf - $cage(3, 7)$:



Obr. 3.4: McGeeho graf [EJ11]

McGeeho graf má rád 24 a počet grúp automorfizmov je 32. Graf nie je vrcholovo tranzitívny.

- Tutte-Coxeterov graf - $cage(3, 8)$:



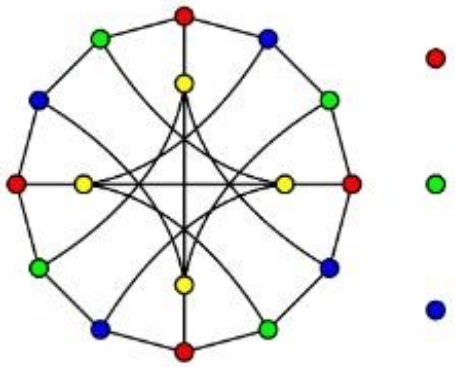
Obr. 3.5: Tutte-Coxeterov graf [EJ11]

Tutte-Coxterov graf má rád 30 a počet grúp automorfizmov je 1440. Graf je vrcholovo tranzitívny.

- Balabanov graf - $cage(3, 11)$: Balabanov graf má 112 vrcholov a počet grúp automorfizmov je 64. Graf nie je vrcholovo tranzitívny.

- Bensonov graf - $cage(3, 12)$: Bensonov graf má 126 vrcholov a počet grúp automorfizmov je 12096. Graf je vrcholovo tranzitívny.

- Robertsonov graf - $cage(4, 5)$:



Obr. 3.6: Robertsonov graf [EJ11]

- ďalšie známe klitky: $cage(4, 7)$ - Exoo, McKay,a Nadonov graf, $cages(5, 5)$: počet grúp automorfizmov je 20,30 a 120, $cage(7, 5)$ - Hoffman - Singletonov graf, $cage(7, 6)$ - O'Keefe a Wongov graf

Kapitola 4

Návrh riešenia

Problematiku riešime v programe Sage [sag], ktorý je založený na programovacom jazyku Python. Zvolili sme ho, pretože ponúka veľké množstvo vopred naimplementovaných funkcií, ktoré nám podstatne uľahčia prácu s grafmi, maticami a grupami automorfizmov. Využili sme online aplikáciu CoCalc [coc], ktorá nám umožňuje vytvárať Sage projekty priamo na internete. CoCalc prevádzkuje prostredie Ubuntu Linux, s ktorým je možné komunikovať cez terminál a taktiež poskytuje prístup k ďalším možnostiam Linuxu.

4.1 Generovanie a skúmanie incidenčných matíc, grúp automorfizmov z klietok

4.1.1 Klietka je zadaná

Uvažujeme známe klietky a na základe nich vieme vygenerovať incidenčnú maticu a zistiť grupu automorfizmov. Tieto známe klietky vieme rozdeliť do 2 kategórií. V prvej kategórií využijeme tie grafy, ktoré má už Program Sage naimplementované. V druhej kategórií uvažujeme generovanie na základe zo-

znamu susedností jednotlivých vrcholov. Tieto dáta sme získali zo stránky p. Exooa a bolo ich potrebné spracovať do vhodnej grafovej štruktúry.

4.1.2 Generovanie klietky

Uvažujeme existujúce klietky, ktoré je potrebné zostrojiť spoločne s grafom, na základe nich potom vygenerujeme incidenčnú maticu a zistíme grupu automorfizmov. Pre vygenerovanú klietku vieme zistiť všetky potrebné informácie ako v predošlom prípade. Generovanie ja zatiaľ experimentálne ja zatiaľ experimentálne na základe dostupných metód jazyka Sage.

4.2 Generovanie a skúmanie klietok, grúp automorfizmov z incidenčných matíc

4.2.1 Incidenčná matica je zadaná

Uvažujeme známe incidenčné matice, ktoré je potrebné zostrojiť, na základe nich potom vygenerujeme klietku a zistíme grupu automorfizmov.

4.2.2 Generovanie incidenčnej matice

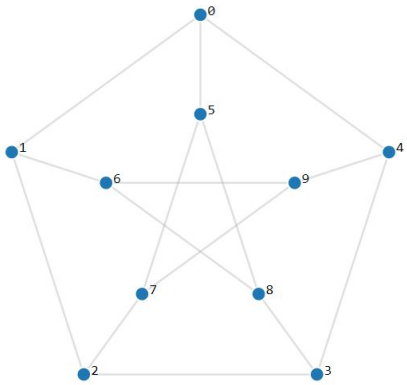
Uvažujeme existujúce incidenčné matice, ktoré je potrebné zostrojiť, na základe nich potom vygenerujeme klietku a zistíme grupu automorfizmov. Generovanie ja zatiaľ experimentálne na základe dostupných metód jazyka Sage.

Kapitola 5

Výsledky

5.1 Generovanie a skúmanie incidenčných matíc, grúp automorfizmov zo zadaných klietok

5.1.1 Petersenov graf - $cage(3, 5)$



Obr. 5.1: Petersenov graf 2D [coc]

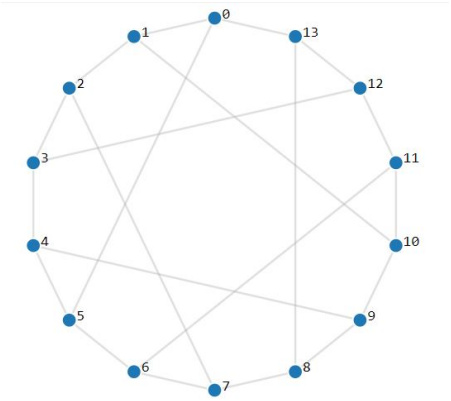
Obr. 5.2: Petersenov graf 3D [coc]

- minimálny počet vrcholov: 10
- rozmer minimálnej matice kódu: 10×15
- obvod cyklu v matici: 10
- minimálny počet hrán: 15
- cykly v grafe: 6 cyklov
  $[[1, 6, 8, 5, 0], [4, 9, 6, 8, 5, 0], [7, 9, 6, 8, 5], [4, 3, 8, 5, 0], [1, 2, 3, 8, 5, 0], [7, 2, 3, 8, 5]]$
- hrany v grafe: 15 hrán
  $[(0, 1, None), (0, 4, None), (0, 5, None), (1, 2, None), (1, 6, None), (2, 3, None),$
  $(2, 7, None), (3, 4, None), (3, 8, None), (4, 9, None), (5, 7, None), (5, 8, None),$
  $(6, 8, None), (6, 9, None), (7, 9, None)]$
- počet grúp automorfizmov: 120
- matica lineárneho kódu:

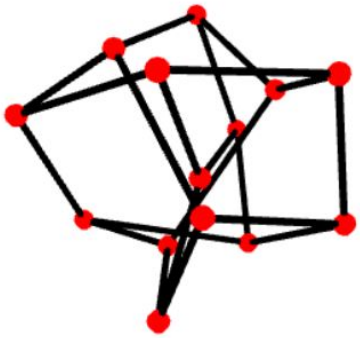
```
[1 1 1 0 0 0 0 0 0 0 0 0 0 0 0]
[1 0 0 1 1 0 0 0 0 0 0 0 0 0 0]
[0 0 0 1 0 1 1 0 0 0 0 0 0 0 0]
[0 0 0 0 0 1 0 1 1 0 0 0 0 0 0]
[0 1 0 0 0 0 0 1 0 1 0 0 0 0 0]
[0 0 1 0 0 0 0 0 0 0 1 1 0 0 0]
[0 0 0 0 1 0 0 0 0 0 0 0 1 1 0]
[0 0 0 0 0 0 1 0 0 0 1 0 0 0 1]
[0 0 0 0 0 0 0 0 1 0 0 1 1 0 0]
[0 0 0 0 0 0 0 0 0 1 0 0 0 1 1]
```

Obr. 5.3: Matica Petersenovho grafu [coc]

5.1.2 Heawoodov graf - $cage(3, 6)$



Obr. 5.4: Heawoodov graf 2D [coc]



Obr. 5.5: Heawoodov graf 3D [coc]

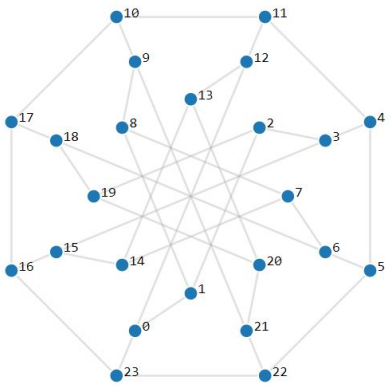
- minimálny počet vrcholov: 14
- rozmer minimálnej matice kódu: 14×21
- obvod cyklu v matici: 12
- minimálny počet hrán: 21
- cykly v grafe: 8 cyklov
  $[[1, 10, 11, 12, 13, 0], [8, 9, 10, 11, 12, 13], [3, 4, 9, 10, 11, 12],$
  $[5, 4, 9, 10, 11, 12, 13, 0], [5, 6, 11, 12, 13, 0], [8, 7, 6, 11, 12, 13],$
  $[1, 2, 7, 6, 11, 12, 13, 0], [3, 2, 7, 6, 11, 12]]$
- hrany v grafe: 21 hrán
  $[(0, 1, None), (0, 5, None), (0, 13, None), (1, 2, None), (1, 10, None), (2, 3, None),$
  $(2, 7, None), (3, 4, None), (3, 12, None), (4, 5, None), (4, 9, None), (5, 6, None),$
  $(6, 7, None), (6, 11, None), (7, 8, None), (8, 9, None), (8, 13, None),$
  $(9, 10, None), (10, 11, None), (11, 12, None), (12, 13, None)]$
- počet grúp automorfizmov: 336

- matica lineárneho kódu:

```
[1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 1 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 1 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 1 0 1 1 0 0 0 0 0 0 0 0 0 0]
[0 1 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0]
[0 0 0 0 0 0 1 0 0 0 0 0 1 0 1 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 1 0 0 0]
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 1 0]
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1 1]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1]
```

Obr. 5.6: Matica Heawoodovho grafu [coc]

5.1.3 McGeeho graf - $cage(3, 7)$



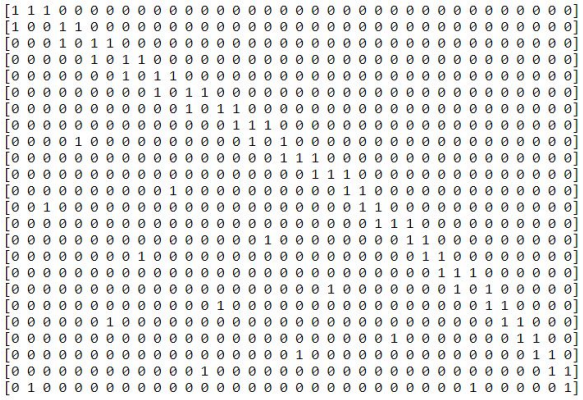
Obr. 5.7: McGeeho graf 2D [coc]

Obr. 5.8: McGeeho graf 3D [coc]

- minimálny počet vrcholov: 22
- rozmer minimálnej matice kódu: 22×33
- obvod cyklu v matici: 14
- minimálny počet hrán: 33
- cykly v grafe: 13 cyklov
  $[[23, 16, 15, 14, 13, 12, 0], [20, 19, 18, 17, 16, 15, 14, 13], [1, 2, 19, 18, 17, 16,$
  $15, 14, 13, 12, 0], [3, 2, 19, 18, 17, 16, 15], [7, 6, 18, 17, 16, 15, 14], [23, 22, 5, 6, 18, 17, 16],$
  $[20, 21, 22, 5, 6, 18, 17, 16, 15, 14, 13], [10, 9, 21, 22, 5, 6, 18, 17],$
  $[1, 8, 9, 21, 22, 5, 6, 18, 17, 16, 15, 14, 13, 12, 0], [7, 8, 9, 21, 22, 5, 6], [3, 4, 5, 6,$
  $18, 17, 16, 15], [11, 4, 5, 6, 18, 17, 16, 15, 14, 13, 12], [11, 10, 17, 16, 15, 14, 13, 12]]$
- hrany v grafe: 36 hrán
  $[(0, 1, None), (0, 12, None), (0, 23, None), (1, 2, None), (1, 8, None), (2, 3, None),$
  $(2, 19, None), (3, 4, None), (3, 15, None), (4, 5, None), (4, 11, None), (5, 6, None),$
  $(5, 22, None), (6, 7, None), (6, 18, None), (7, 8, None), (7, 14, None), (8, 9, None),$
  $(9, 10, None), (9, 21, None), (10, 11, None), (10, 17, None), (11, 12, None),$

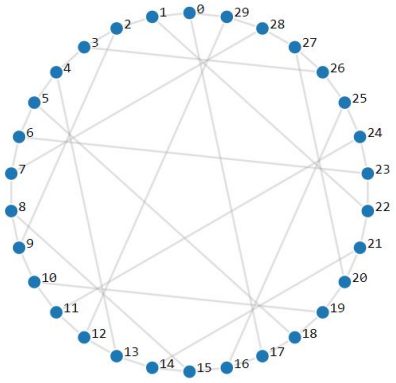
$(12, 13, None), (13, 14, None), (13, 20, None), (14, 15, None), (15, 16, None),$
$(16, 17, None), (16, 23, None), (17, 18, None), (18, 19, None), (19, 20, None),$
$(20, 21, None), (21, 22, None), (22, 23, None)]$

- počet grúp automorfizmov: 32
- matica lineárneho kódu:

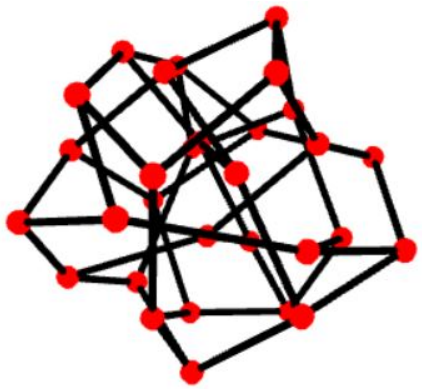


Obr. 5.9: Matica McGeeho grafu [coc]

5.1.4 Tutte - Coxeterov graf - $cage(3, 8)$



Obr. 5.10: Tutte-Coxeterov graf 2D [coc]



Obr. 5.11: Tutte-Coxeterov graf 3D [coc]

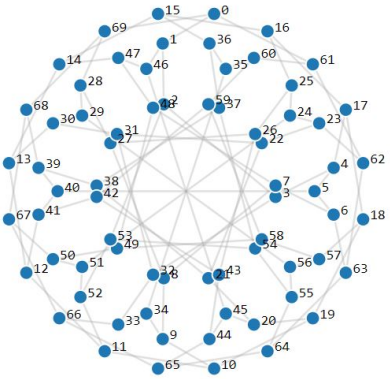
- minimálny počet vrcholov: 30
- rozmer minimálnej matice kódu: 30×45
- obvod cyklu v matici: 16
- minimálny počet hrán: 45
- cykly v grafe: 16 cyklov
  $[[1, 22, 23, 24, 25, 26, 27, 28, 29, 0], [20, 21, 22, 23, 24, 25, 26, 27], [16, 15, 14, 21,$
  $22, 23, 24, 25], [7, 8, 15, 14, 21, 22, 23, 24, 25, 26, 27, 28], [11, 10, 9, 8, 15, 14, 21, 22, 23, 24],$
  $[20, 19, 10, 9, 8, 15, 14, 21], [17, 18, 19, 10, 9, 8, 15, 14, 21, 22, 23, 24, 25, 26, 27, 28, 29, 0],$
  $[6, 5, 18, 19, 10, 9, 8, 15, 14, 21, 22, 23], [3, 4, 5, 18, 19, 10, 9, 8, 15, 14, 21, 22, 23, 24, 25, 26],$
  $[13, 4, 5, 18, 19, 10, 9, 8, 15, 14], [1, 2, 9, 8, 15, 14, 21, 22],$
  $[3, 2, 9, 8, 15, 14, 21, 22, 23, 24, 25, 26], [12, 13, 14, 21, 22, 23, 24, 25, 26, 27, 28, 29],$
  $[7, 6, 23, 24, 25, 26, 27, 28], [12, 11, 24, 25, 26, 27, 28, 29], [17, 16, 25, 26, 27, 28, 29, 0]]$
- hrany v grafe: 45 hrán
  $[(0, 1, None), (0, 17, None), (0, 29, None), (1, 2, None), (1, 22, None), (2, 3, None),$
  $(2, 9, None), (3, 4, None), (3, 26, None), (4, 5, None), (4, 13, None), (5, 6, None),$

$(5, 18, None), (6, 7, None), (6, 23, None), (7, 8, None), (7, 28, None), (8, 9, None),$
$(8, 15, None), (9, 10, None), (10, 11, None), (10, 19, None),$
$(11, 12, None), (11, 24, None), (12, 13, None), (12, 29, None),$
$(13, 14, None), (14, 15, None), (14, 21, None),$
$(15, 16, None), (16, 17, None), (16, 25, None), (17, 18, None), (18, 19, None),$
$(19, 20, None), (20, 21, None), (20, 27, None), (21, 22, None), (22, 23, None),$
$(23, 24, None), (24, 25, None), (25, 26, None), (26, 27, None),$
$(27, 28, None), (28, 29, None)]$

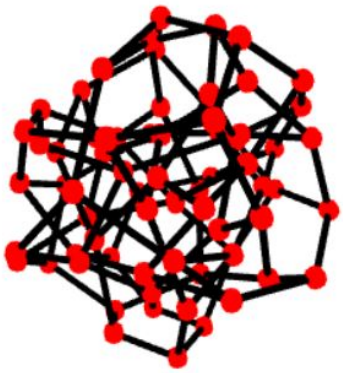
- počet grúp automorfizmov: 1440
- matica lineárneho kódu:

Obr. 5.12: Matica Tutte - Coxeterovho grafu [coc]

5.1.5 Balabanov graf - $cage(3, 10)$



Obr. 5.13: Balabanov(10) graf 2D [coc]



Obr. 5.14: Balabanov(10) graf 3D [coc]

- minimálny počet vrcholov: 62
- rozmer minimálnej matice kódu: 62×93
- obvod cyklu v matici: 20
- minimálny počet hrán: 93

- cykly v grafe: 36 cyklov

  [[69, 68, 67, 66, 65, 64, 63, 62, 61, 0], [60, 59, 42, 41, 40, 39, 68, 67, 66, 65, 64, 63, 62, 61],
  [55, 56, 57, 58, 59, 42, 41, 40, 39, 68, 67, 66, 65, 64],
  [33, 32, 31, 30, 29, 56, 57, 58, 59, 42, 41, 40, 39, 68, 67, 66],
  [5, 4, 3, 32, 31, 30, 29, 56, 57, 58, 59, 42, 41, 40],
  [18, 17, 4, 3, 32, 31, 30, 29, 56, 57],
  [43, 26, 25, 16, 17, 4, 3, 32, 31, 30, 29, 56, 57, 58, 59, 42],
  [28, 27, 26, 25, 16, 17, 4, 3, 32, 31, 30, 29],
  [12, 11, 10, 9, 8, 27, 26, 25, 16, 17, 4, 3, 32, 31, 30, 29, 56, 57, 58, 59, 42, 41],
  [2, 53, 52, 11, 10, 9, 8, 27, 26, 25, 16, 17, 4, 3],
  [55, 54, 53, 52, 11, 10, 9, 8, 27, 26, 25, 16, 17, 4, 3, 32, 31, 30, 29, 56],
  [38, 37, 54, 53, 52, 11, 10, 9, 8, 27, 26, 25, 16, 17, 4, 3, 32, 31,
  30, 29, 56, 57, 58, 59, 42, 41, 40, 39], [15, 36, 37, 54, 53, 52, 11, 10, 9, 8, 27, 26, 25, 16],
  [34, 35, 36, 37, 54, 53, 52, 11, 10, 9],
  [60, 35, 36, 37, 54, 53, 52, 11, 10, 9, 8, 27, 26, 25, 16, 17, 4, 3, 32, 31, 30, 29, 56, 57, 58, 59],
  [24, 51, 52, 11, 10, 9, 8, 27, 26, 25],
  [50, 51, 52, 11, 10, 9, 8, 27, 26, 25, 16, 17, 4, 3, 32,
  31, 30, 29, 56, 57, 58, 59, 42, 41, 40, 39, 68, 67], [18, 19, 10, 9, 8, 27, 26, 25, 16, 17],
  [44, 45, 20, 19, 10, 9, 8, 27, 26, 25, 16, 17, 4, 3, 32, 31, 30,
  29, 56, 57, 58, 59, 42, 41, 40, 39, 68, 67, 66, 65],
  [1, 46, 45, 20, 19, 10, 9, 8, 27, 26, 25, 16, 17, 4, 3, 32, 31, 30, 29, 56, 57, 58, 59, 42, 41, 40,
  39, 68, 67, 66, 65, 64, 63, 62, 61, 0], [7, 48, 47, 46, 45, 20, 19, 10, 9, 8],
  [49, 48, 47, 46, 45, 20, 19, 10, 9, 8, 27, 26, 25, 16, 17, 4, 3, 32, 31, 30, 29, 56, 57, 58],
  [13, 14, 47, 46, 45, 20, 19, 10, 9, 8, 27, 26, 25, 16, 17, 4, 3, 32, 31, 30],
  [15, 14, 47, 46, 45, 20, 19, 10, 9, 8, 27, 26, 25, 16],
  [22, 21, 20, 19, 10, 9, 8, 27, 26, 25, 16, 17, 4, 3, 32, 31],
  [38, 21, 20, 19, 10, 9, 8, 27, 26, 25, 16, 17, 4, 3, 32, 31, 30, 29, 56, 57, 58, 59, 42, 41, 40, 39],

$[33, 34, 9, 8, 27, 26, 25, 16, 17, 4, 3, 32]$,

$[6, 7, 8, 27, 26, 25, 16, 17, 4, 3, 32, 31,$

$30, 29, 56, 57, 58, 59, 42, 41, 40, 39, 68, 67, 66, 65, 64, 63]$,

$[23, 24, 25, 16, 17, 4, 3, 32, 31, 30, 29,$

$56, 57, 58, 59, 42, 41, 40, 39, 68, 67, 66, 65, 64, 63, 62]$,

$[1, 2, 3, 32, 31, 30, 29, 56, 57, 58, 59, 42, 41, 40, 39, 68, 67, 66, 65, 64, 63, 62, 61, 0]$,

$[23, 22, 31, 30, 29, 56, 57, 58, 59, 42, 41, 40, 39, 68, 67, 66, 65, 64, 63, 62]$,

$[12, 13, 30, 29, 56, 57, 58, 59, 42, 41]$,

$[69, 28, 29, 56, 57, 58, 59, 42, 41, 40, 39, 68]$,

$[50, 49, 58, 59, 42, 41, 40, 39, 68, 67]$,

$[44, 43, 42, 41, 40, 39, 68, 67, 66, 65]$,

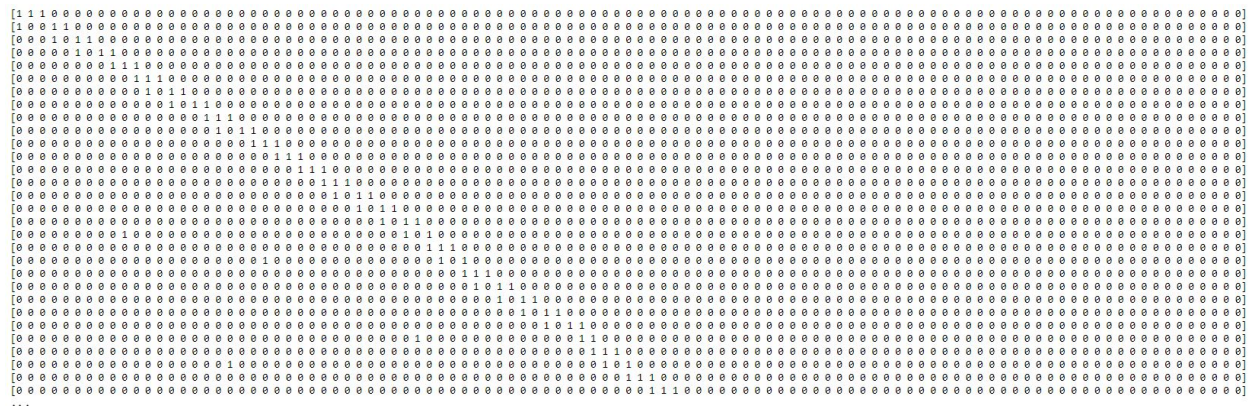
$[6, 5, 40, 39, 68, 67, 66, 65, 64, 63]]$

- hrany v grafe: 105 hrán

  $[(0, 1, None), (0, 61, None), (0, 69, None), (1, 2, None), (1, 46, None), (2, 3, None),$
  $(2, 53, None), (3, 4, None), (3, 32, None), (4, 5, None), (4, 17, None), (5, 6, None),$
  $(5, 40, None), (6, 7, None), (6, 63, None), (7, 8, None), (7, 48, None), (8, 9, None),$
  $(8, 27, None), (9, 10, None), (9, 34, None), (10, 11, None), (10, 19, None),$
  $(11, 12, None), (11, 52, None), (12, 13, None), (12, 41, None), (13, 14, None),$
  $(13, 30, None), (14, 15, None), (14, 47, None), (15, 16, None), (15, 36, None),$
  $(16, 17, None), (16, 25, None), (17, 18, None), (18, 19, None), (18, 57, None),$
  $(19, 20, None), (20, 21, None), (20, 45, None), (21, 22, None), (21, 38, None),$
  $(22, 23, None), (22, 31, None), (23, 24, None), (23, 62, None), (24, 25, None),$
  $(24, 51, None), (25, 26, None), (26, 27, None), (26, 43, None), (27, 28, None),$
  $(28, 29, None), (28, 69, None), (29, 30, None), (29, 56, None), (30, 31, None),$
  $(31, 32, None), (32, 33, None), (33, 34, None), (33, 66, None), (34, 35, None),$
  $(35, 36, None), (35, 60, None), (36, 37, None), (37, 38, None), (37, 54, None),$
  $(38, 39, None), (39, 40, None), (39, 68, None), (40, 41, None), (41, 42, None),$

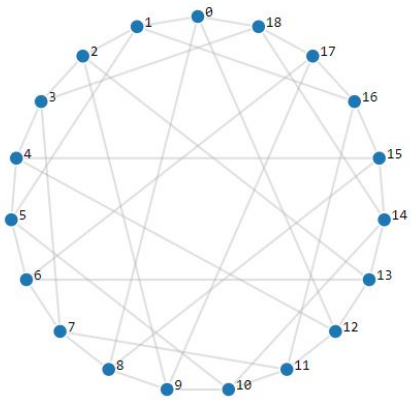
$(42, 43, None), (42, 59, None), (43, 44, None), (44, 45, None), (44, 65, None),$
$(45, 46, None), (46, 47, None), (47, 48, None), (48, 49, None), (49, 50, None),$
$(49, 58, None), (50, 51, None), (50, 67, None), (51, 52, None), (52, 53, None),$
$(53, 54, None), (54, 55, None), (55, 56, None), (55, 64, None), (56, 57, None),$
$(57, 58, None), (58, 59, None), (59, 60, None), (60, 61, None), (61, 62, None),$
$(62, 63, None), (63, 64, None), (64, 65, None), (65, 66, None), (66, 67, None),$
$(67, 68, None), (68, 69, None)]$

- počet grúp automorfizmov: 80
- matica lineárneho kódu:

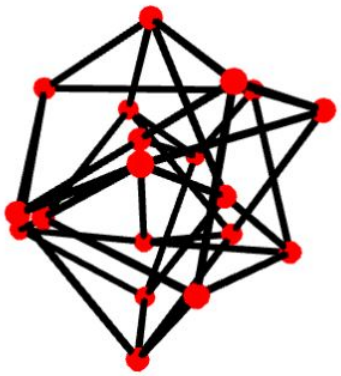


Obr. 5.15: Matica Balabanovho(10) grafu [coc]

5.1.6 Robertsonov graf - $cage(4, 5)$



Obr. 5.16: Robertsonov graf 2D [coc]



Obr. 5.17: Robertsonov graf 3D [coc]

- minimálny počet vrcholov: 17
- rozmer minimálnej matice kódu: 17×34
- obvod cyklu v matici: 10
- minimálny počet hrán: 34

- cykly v grafe: 20 cyklov

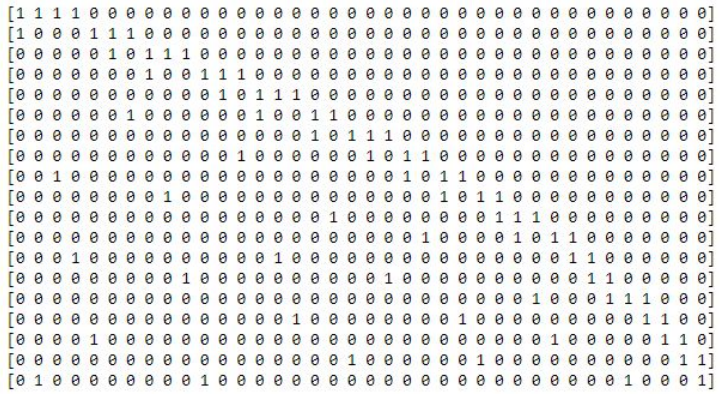
  $[[18, 14, 13, 12, 0], [4, 15, 14, 13, 12], [8, 15, 14, 13, 12, 0],$
  $[1, 16, 15, 14, 13, 12, 0], [11, 16, 15, 14, 13, 12], [6, 17, 16, 15, 14, 13],$
  $[18, 17, 16, 15, 14], [2, 9, 17, 16, 15, 14, 13], [8, 9, 17, 16, 15],$
  $[10, 9, 17, 16, 15, 14], [11, 10, 14, 13, 12], [1, 5, 10, 14, 13, 12, 0],$
  $[4, 5, 10, 14, 13, 12], [6, 5, 10, 14, 13], [8, 7, 6, 13, 12, 0],$
  $[11, 7, 6, 13, 12], [2, 3, 7, 6, 13], [18, 3, 7, 6, 13, 12, 0],$
  $[4, 3, 7, 6, 13, 12], [1, 2, 13, 12, 0]]$

- hrany v grafe: 38 hrán

  $[(0, 1, None), (0, 8, None), (0, 12, None), (0, 18, None), (1, 2, None), (1, 5, None),$
  $(1, 16, None), (2, 3, None), (2, 9, None), (2, 13, None), (3, 4, None), (3, 7, None),$
  $(3, 18, None), (4, 5, None), (4, 12, None), (4, 15, None), (5, 6, None), (5, 10, None),$
  $(6, 7, None), (6, 13, None), (6, 17, None), (7, 8, None), (7, 11, None), (8, 9, None),$
  $(8, 15, None), (9, 10, None), (9, 17, None), (10, 11, None), (10, 14, None), (11, 12, None),$
  $(11, 16, None), (12, 13, None), (13, 14, None), (14, 15, None), (14, 18, None), (15, 16, None),$
  $(16, 17, None), (17, 18, None)]$

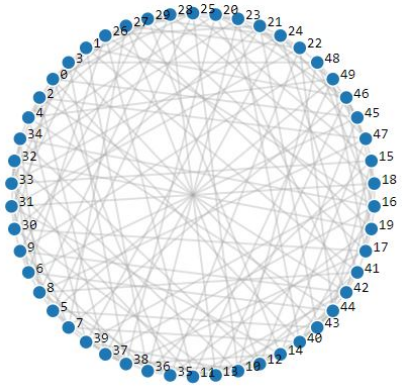
- počet grúp automorfizmov: 24

- matica lineárneho kódu:

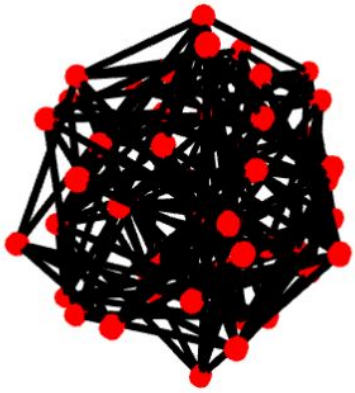


Obr. 5.18: Matica Robertsonovho grafu [coc]

5.1.7 Hoffman - Singletonov graf - $cage(7, 5)$



Obr. 5.19: Hoffman - Singletonov graf 2D [coc]



Obr. 5.20: Hoffman - Singletonov graf 3D [coc]

- minimálny počet vrcholov: 50
- rozmer minimálnej matice kódu: 50×175
- obvod cyklu v matici: 10
- minimálny počet hrán: 175
- cykly v grafe: 126 cyklov
  $[[45, 24, 22, 35, 0], [30, 21, 24, 22, 35, 0], [39, 21, 24, 22, 35], [2, 47, 21, 24, 22, 35, 0],$
  $[8, 47, 21, 24, 22, 35], [48, 47, 21, 24, 22], [44, 15, 47, 21, 24, 22], [25, 15, 47, 21, 24, 22, 35, 0],$
  $[36, 15, 47, 21, 24, 22, 35], [33, 15, 47, 21, 24], [31, 18, 15, 47, 21, 24, 22], [45, 18, 15, 47, 21, 24],$
  $[39, 18, 15, 47, 21], [3, 28, 18, 15, 47, 21, 24, 22, 35, 0], [23, 28, 18, 15, 47, 21],$
  $[27, 28, 18, 15, 47, 21, 24, 22], [8, 28, 18, 15, 47],$
  $[29, 28, 18, 15, 47, 21, 24], [46, 13, 28, 18, 15, 47],$
  $[44, 13, 28, 18, 15], [11, 13, 28, 18, 15, 47, 21, 24, 22, 35],$
  $[30, 13, 28, 18, 15, 47, 21], [37, 13, 28, 18, 15, 47, 21, 24],$
  $[25, 10, 13, 28, 18, 15], [41, 10, 13, 28, 18, 15, 47, 21, 24], [48, 10, 13, 28, 18, 15, 47],$
  $[39, 10, 13, 28, 18], [43, 12, 10, 13, 28, 18, 15, 47, 21],$
  $[45, 12, 10, 13, 28, 18], [14, 12, 10, 13, 28, 18, 15, 47],$

[27, 12, 10, 13, 28], [36, 12, 10, 13, 28, 18, 15], [16, 34, 12, 10, 13, 28, 18], [8, 34, 12, 10, 13, 28],
[33, 34, 12, 10, 13, 28, 18, 15], [30, 34, 12, 10, 13],
[20, 34, 12, 10, 13, 28, 18, 15, 47, 21, 24, 22],
[44, 4, 34, 12, 10, 13],
[2, 4, 34, 12, 10, 13, 28, 18, 15, 47], [29, 4, 34, 12, 10, 13, 28],
[39, 4, 34, 12, 10], [31, 1, 4, 34, 12, 10, 13, 28, 18],
[46, 1, 4, 34, 12, 10, 13], [3, 1, 4, 34, 12, 10, 13, 28], [26, 1, 4, 34, 12, 10, 13, 28, 18, 15, 47, 21],
[41, 1, 4, 34, 12, 10], [36, 1, 4, 34, 12], [17, 49, 4, 34, 12, 10, 13, 28, 18, 15],
[23, 49, 4, 34, 12, 10, 13, 28], [45, 49, 4, 34, 12], [11, 49, 4, 34, 12, 10, 13], [48, 49, 4, 34, 12, 10],
[31, 5, 49, 4, 34, 12, 10, 13, 28, 18], [43, 5, 49, 4, 34, 12], [25, 5, 49, 4, 34, 12, 10],
[8, 5, 49, 4, 34], [37, 5, 49, 4, 34, 12, 10, 13], [46, 7, 5, 49, 4, 34, 12, 10, 13], [33, 7, 5, 49, 4, 34],
[40, 7, 5, 49, 4, 34, 12, 10, 13, 28, 18, 15, 47, 21, 24, 22, 35, 0], [27, 7, 5, 49, 4, 34, 12],
[39, 7, 5, 49, 4], [30, 9, 7, 5, 49, 4, 34],
[42, 9, 7, 5, 49, 4, 34, 12, 10, 13, 28, 18],
[36, 9, 7, 5, 49, 4, 34, 12], [48, 9, 7, 5, 49],
[29, 9, 7, 5, 49, 4], [44, 6, 9, 7, 5, 49, 4], [45, 6, 9, 7, 5, 49],
[26, 6, 9, 7, 5, 49, 4, 34, 12, 10, 13, 28, 18, 15, 47, 21], [8, 6, 9, 7, 5],
[32, 6, 9, 7, 5, 49, 4, 34, 12, 10], [17, 38, 6, 9, 7, 5, 49], [14, 38, 6, 9, 7, 5, 49, 4, 34, 12],
[37, 38, 6, 9, 7, 5], [20, 38, 6, 9, 7, 5, 49, 4, 34], [3, 38, 6, 9, 7, 5, 49, 4, 34, 12, 10, 13, 28],
[39, 38, 6, 9, 7], [31, 32, 10, 13, 28, 18], [23, 32, 10, 13, 28], [2, 32, 10, 13, 28, 18, 15, 47],
[33, 32, 10, 13, 28, 18, 15], [19, 32, 10, 13, 28, 18, 15, 47, 21, 24, 22, 35], [43, 42, 18, 15, 47, 21],

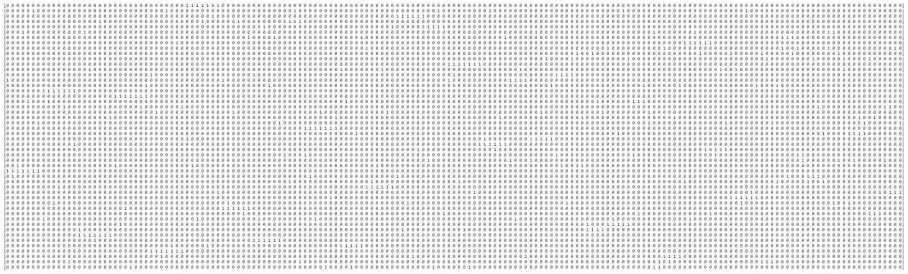
[2, 42, 18, 15, 47], [11, 42, 18, 15, 47, 21, 24, 22, 35], [41, 42, 18, 15, 47, 21, 24],
[20, 42, 18, 15, 47, 21, 24, 22], [40, 16, 18, 15, 47, 21, 24, 22, 35, 0], [37, 16, 18, 15, 47, 21, 24],
[19, 16, 18, 15, 47, 21, 24, 22, 35], [48, 16, 18, 15, 47], [26, 16, 18, 15, 47, 21],
[30, 17, 15, 47, 21], [41, 17, 15, 47, 21, 24],
[27, 17, 15, 47, 21, 24, 22], [19, 17, 15, 47, 21, 24, 22, 35],

$[31, 14, 47, 21, 24, 22], [11, 14, 47, 21, 24, 22, 35],$
$[29, 14, 47, 21, 24], [40, 14, 47, 21, 24, 22, 35, 0],$
$[20, 46, 47, 21, 24, 22], [45, 46, 47, 21, 24], [19, 46, 47, 21, 24, 22, 35],$
$[11, 26, 21, 24, 22, 35], [25, 26, 21, 24, 22, 35, 0], [27, 26, 21, 24, 22], [40, 23, 21, 24, 22, 35, 0],$
$[20, 23, 21, 24, 22], [36, 23, 21, 24, 22, 35], [44, 43, 21, 24, 22], [3, 43, 21, 24, 22, 35, 0],$
$[19, 43, 21, 24, 22, 35], [25, 29, 24, 22, 35, 0], [19, 29, 24, 22, 35], [2, 37, 24, 22, 35, 0],$
$[36, 37, 24, 22, 35], [3, 33, 24, 22, 35, 0], [11, 33, 24, 22, 35], [8, 41, 24, 22, 35],$
$[40, 41, 24, 22, 35, 0], [3, 48, 22, 35, 0], [2, 27, 22, 35, 0], [25, 20, 22, 35, 0],$
$[40, 44, 22, 35, 0], [30, 31, 22, 35, 0]]$

- hrany v grafe: 175 hrán
  $[(0, 2, None), (0, 3, None), (0, 25, None), (0, 30, None),$
  $(0, 35, None), (0, 40, None), (0, 45, None), (1, 3, None),$
  $(1, 4, None), (1, 26, None), (1, 31, None), (1, 36, None),$
  $(1, 41, None), (1, 46, None), (2, 4, None), (2, 27, None),$
  $(2, 32, None), (2, 37, None), (2, 42, None), (2, 47, None),$
  $(3, 28, None), (3, 33, None), (3, 38, None), (3, 43, None),$
  $(3, 48, None), (4, 29, None), (4, 34, None), (4, 39, None),$
  $(4, 44, None), (4, 49, None), (5, 7, None), (5, 8, None),$
  $(5, 25, None), (5, 31, None), (5, 37, None), (5, 43, None),$
  $(5, 49, None), (6, 8, None), (6, 9, None), (6, 26, None),$
  $(6, 32, None), (6, 38, None), (6, 44, None), (6, 45, None),$
  $(7, 9, None), (7, 27, None), (7, 33, None), (7, 39, None),$
  $(7, 40, None), (7, 46, None), (8, 28, None), (8, 34, None),$
  $(8, 35, None), (8, 41, None), (8, 47, None), (9, 29, None),$
  $(9, 30, None), (9, 36, None), (9, 42, None), (9, 48, None),$
  $(10, 12, None), (10, 13, None), (10, 25, None), (10, 32, None),$
  $(10, 39, None), (10, 41, None), (10, 48, None), (11, 13, None),$

$(11, 14, None), (11, 26, None), (11, 33, None), (11, 35, None),$
$(11, 42, None), (11, 49, None), (12, 14, None), (12, 27, None),$
$(12, 34, None), (12, 36, None), (12, 43, None), (12, 45, None),$
$(13, 28, None), (13, 30, None), (13, 37, None), (13, 44, None),$
$(13, 46, None), (14, 29, None), (14, 31, None), (14, 38, None),$
$(14, 40, None), (14, 47, None), (15, 17, None), (15, 18, None),$
$(15, 25, None), (15, 33, None), (15, 36, None), (15, 44, None),$
$(15, 47, None), (16, 18, None), (16, 19, None), (16, 26, None),$
$(16, 34, None), (16, 37, None), (16, 40, None), (16, 48, None),$
$(17, 19, None), (17, 27, None), (17, 30, None), (17, 38, None),$
$(17, 41, None), (17, 49, None), (18, 28, None), (18, 31, None),$
$(18, 39, None), (18, 42, None), (18, 45, None), (19, 29, None),$
$(19, 32, None), (19, 35, None), (19, 43, None), (19, 46, None),$
$(20, 22, None), (20, 23, None), (20, 25, None), (20, 34, None),$
$(20, 38, None), (20, 42, None), (20, 46, None), (21, 23, None),$
$(21, 24, None), (21, 26, None), (21, 30, None), (21, 39, None),$
$(21, 43, None), (21, 47, None), (22, 24, None), (22, 27, None),$
$(22, 31, None), (22, 35, None), (22, 44, None), (22, 48, None),$
$(23, 28, None), (23, 32, None), (23, 36, None), (23, 40, None),$
$(23, 49, None), (24, 29, None), (24, 33, None), (24, 37, None),$
$(24, 41, None), (24, 45, None), (25, 26, None), (25, 29, None),$
$(26, 27, None), (27, 28, None), (28, 29, None), (30, 31, None),$
$(30, 34, None), (31, 32, None), (32, 33, None), (33, 34, None),$
$(35, 36, None), (35, 39, None), (36, 37, None), (37, 38, None),$
$(38, 39, None), (40, 41, None), (40, 44, None), (41, 42, None),$
$(42, 43, None), (43, 44, None), (45, 46, None), (45, 49, None),$
$(46, 47, None), (47, 48, None), (48, 49, None)]$

- počet grúp automorfizmov: 252000
- matica lineárneho kódu:



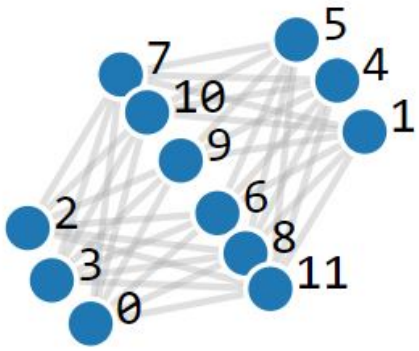
Obr. 5.21: Matica Hoffman - Singletonovho grafu [coc]

5.1.8 Generovanie klietky a následné skúmanie incidenčných matíc, grúp automorfizmov

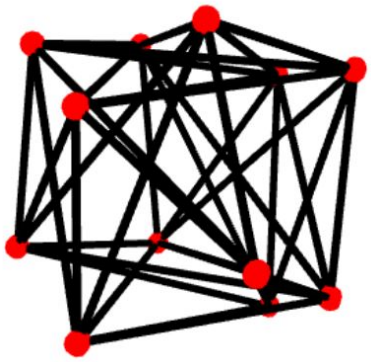
5.1.9 $cage(6, 4)$

Klietku vygenerujeme ako bipartitný graf. V Sage použijeme metódu $DegreeSequenceBipartite(s, s)$, ktorá bude mať 2 rovnaké parametre s, pričom každý predstavuje zoznam vrcholov. Najskôr je potrebné si vypočítať minimálny počet vrcholov m klietky $cage(6, 4)$ a na základe výpočtu viem určiť parameter. Každý zoznam obsahuje $m/2$ vrcholov stupňa k a teda platí:

$$s = [k, k, k, k, k, k] \quad len(s) = \frac{m}{2} \tag{5.1}$$



Obr. 5.22: Cage(6,4) graf 2D [coc]



Obr. 5.23: Cage(6,4) graf 3D [coc]

- minimálny počet vrcholov: 12
- rozmer minimálnej matice kódu: 12×36
- obvod cyklu v matici: 8
- minimálny počet hrán: 36
- cykly v grafe: 25 cyklov
  $[[6, 5, 11, 0], [7, 5, 11, 0], [8, 5, 11, 0], [9, 5, 11, 0], [10, 5, 11, 0], [6, 4, 11, 0],$

$[7,4,11,0],[8,4,11,0],[9,4,11,0],[10,4,11,0],[6,3,11,0],[7,3,11,0],$
$[8,3,11,0],[9,3,11,0],[10,3,11,0],[6,2,11,0],[7,2,11,0],[8,2,11,0],$
$[9,2,11,0],[10,2,11,0],[6,1,11,0],[7,1,11,0],[8,1,11,0],[9,1,11,0],[10,1,11,0]]$

- hrany v grafe: 36 hrán

  $[(0,6,None),(0,7,None),(0,8,None),(0,9,None),(0,10,None),(0,11,None),$
  $(1,6,None),(1,7,None),(1,8,None),(1,9,None),(1,10,None),(1,11,None),$
  $(2,6,None),(2,7,None),(2,8,None),(2,9,None),(2,10,None),(2,11,None),$
  $(3,6,None),(3,7,None),(3,8,None),(3,9,None),(3,10,None),(3,11,None),$
  $(4,6,None),(4,7,None),(4,8,None),(4,9,None),(4,10,None),(4,11,None),$
  $(5,6,None),(5,7,None),(5,8,None),(5,9,None),(5,10,None),(5,11,None)]$

- počet grúp automorfizmov: 1036800

- matica lineárneho kódu:

```
[1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1]
[1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0]
[0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0]
[0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0]
[0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1]
```

Obr. 5.24: Matica klietky Cage(6,4) [coc]

5.2 Generovanie a skúmanie klietok, grúp automorfizmov zo zadaných incidenčných matíc

5.3 Generovanie incidenčných matíc a následné skúmanie klietok, grúp automorfizmov

Kapitola 6

Záver

xxxxxx

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[thu]

[tnt]

[vie]

[wea]

[zhe]

Zoznam obrázkov