Universal Journal of Applied Mathematics 4(2): 39-41, 2016 DOI: 10.13189/ujam.2016.040202

A Standard Generator/Parity Check Matrix for Codes from the Cayley Tables Due to the Non-associative (123)-Avoiding Patterns of AUNU Numbers

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Abstract In this paper, we aim at utilizing the Cayley tables demonstrated by the Authors[1] in the construction of a Generator/Parity check Matrix in standard form for a Code say C Our goal is achieved first by converting the Cayley tables in [1] using Mod2 arithmetic into a Matrix with entries from the binary field. Echelon Row operations are then performed (carried out) on the matrix in line with existing algorithms and propositions to obtain a matrix say G whose rows spans C and a matrix say H whose rows spans \mathbb{C}^{\perp} , the dual code of C, where G and H are given as, $G = (I_k | X)$ and $H=(-X^{T}|I_{n-k})$. The report by Williem (2011) that the adjacency Matrix of a graph can be interpreted as the generator matrix of a Code [3] is in this context extended to the Cayley table which generates matrices from the permutations of points of the AUNU numbers of prime cardinality.

Keywords Carley Tables, AUNU Permutation Patterns, Generator Matrix, Parity Check Matrix, Standard Form, Pattern Avoidance, Echelon Row Operation, Non-associative, Non-commutative

1. Introduction

In Coding theory, the generator matrix of a Code plays an important role. Once the generator matrix of a code is known, such a code can easily be encoded and decoded, since procedures for obtaining the parity check matrix say of the code from the generator matrix is obvious through existing algorithms and theorems. As such, we shall not be out of place to mention here that once we have the generator matrix for a particular code, then a message can be encoded, decoded and analyzed by the matrix.

The generation and analysis of some small classes of linear and cyclic codes from the adjacency matrices of Eulerian graphs due to AUNU patterns had been reported in [2] and [3]. This special class of the (132) and (123)-avoiding class of permutation patterns which were first reported [4], where some group and graph theoretic properties were identified, had enjoyed a wide range of applications in various areas of applied Mathematics. In[1], we described how the non-associative and non-commutative properties of the patterns can be established using their Cayley tables where a binary operation was defined to act on the (132) and (123)-avoiding patterns of the AUNU numbers using a pairing scheme.

In this paper, we utilize the Cayley tables generated, first as matrices, then as matrices over the binary field and lastly transform such matrices into generator/parity check matrices for some codes in standard form $G = (I_k | I_{n-k \times k}) / H = (-X^T | I_{n-k})$.

We review some basic concepts and propositions for the easy understanding of this paper.

1.1. Permutation Patterns:

An arrangement of the objects $1, 2, \ldots, n$ is a sequence consisting of these objects arranged in any order. When in addition, a particular order is desired, such an arrangement becomes an ordered permutation governed by a pattern σ and such a permutation pattern $\sigma \in S_n$ naturally results into a certain arrangement of $1, 2, \ldots, n$ given by

$$\sigma(1) \ \sigma(2)$$
. . $\sigma(n)$ (1)

which is called the arrangement associated with a permutation pattern σ of points of a non-empty set,

$$\Omega = \{1, 2, \dots, n\} \tag{2}$$

Given a sequence π consisting of n elements arranged in a given pattern and another sequence σ having m elements such that m < n, then σ is said to be

contained as a pattern in $^{\pi}$ provided $^{\pi}$ has a subsequence which its order is isomorphic to $^{\sigma}$. If $^{\pi}$ does not contain $^{\sigma}$, it is said to avoid it. The set of all $^{\sigma}$ -avoiding permutation is denoted by $S_n(\sigma)$.

It is useful to differentiate between a subsequence and a subword. if $\sigma = 4132 \in S_4$, $\pi = 78364521 \in S_8$ contains σ as a subword since $\rho(8364) = 4132$. $\pi = 54321 \in S_5$ does not contain σ as a subword although it does contain it as a subsequence. Occurrences of subwords can be overlapped. As an example, the sequence $5716243 \in S_{\pi}$ contains two occurrences of $\sigma = 7162$ and 6234. Determination of $\left|S_n(\sigma)\right|$ has remained a hard and intractable problem for a given σ containing more than three elements. This is among the reasons that motivate the Author to construct some class of pattern-avoiding permutations using some special subword (instead of subsequences) governed some succession scheme (Ibrahim, 2005). Thus $A_n(132)$ in this context, represents the subwords of length $n \ge 3$ that are (132)-avoiding in A_n being the set of strictly consecutive succession scheme containing pairs i, j such that

$$j = i + 1$$
 $i, j \in \Omega$.

2. Methodology

We consider the Cayley table below, which is constructed using $A_n(132)$ for n = 5 as in [2]

Table 1. Cayley Table for n = 5 showing generated points of Ω as permutations of (132) and (123)-avoiding patterns of AUNU scheme under the action of Θ .

Θ	1	2	3	4	5
1	1	3	5	2	4
2	1	4	2	5	3
3	1	5	4	3	2

We now convert the entries of the Cayley table above to the binary system using *Modulus 2 arithmetic*. The table thus becomes;

Θ	1	2	3	4	5
1	1	1	1	0	0
2	1	0	0	1	1
3	1	1	0	1	0

The above Cayley table is the matrix A below;

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Proposition:

The matrix
$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$
 from the Cayley

table 1 above is equivalent to a matrix $G^I = \begin{bmatrix} I_k \mid X_{k \times n - k} \end{bmatrix}$, the generator matrix in standard form of a code C spanned by the rows of A.

Proof: suppose A above is a generator matrix for some linear code C. Then C has dimension k=3 and length n=5. According to an established result, the rows of a generator matrix G are independent, which is obvious for our G=A above. Now, since every generator matrix can either be put in standard form or is equivalent to a generator

matrix in standard form i.e $\begin{bmatrix} I_k \mid X_{k \times n-k} \end{bmatrix}$. Apply the following Reduced Row Echelon form (RREF) operations on A; i. $R_2 = R_3$ and ii. $R_3 = R_2 + R_3$ clearly gives $us G^* = \begin{bmatrix} X_{k \times n-k} \mid I_k \end{bmatrix} = X_{3 \times 5-3} \mid I_3 = X_{3 \times 2} \mid I_3$

where
$$X_{k \times n-k} = X_{3 \times 2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$
 and $I_k = I_3$ the identity

matrix of order 3. On juxtaposition of G^* , we obtain $G^I = \begin{bmatrix} I_k | X_{k \times n-k} \end{bmatrix}$ which is our required result. \Box

Next, we transform the A to a matrix G in standard form using the above proposition. Now, applying the following row operations;

i.
$$R_2 = R_3$$
 on $A \rightarrow A = A^I = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$ and

ii.
$$R_3 = R_2 + R_3$$
 on $A^I \rightarrow A^I = A^{II} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$

Clearly,
$$A^{II}$$
 can be written as $A^{II} = G^I = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} X_{k \times n-k} & I_k \end{bmatrix}$ and on

juxtaposition of G^I , we have

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} I_k & X_{k \times n - k} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

which is the required generator matrix in standard form for a Code of length n = 5 and dimension

$$k = 3$$
 Efficiency = $Rate = \frac{3}{5}$ and

and 3 message digits with 2 parity check digits.

For the dual code C^{\perp} of the code C whose generator matrix in standard form is H (also the parity check matrix of C). We define a matrix $H^{I} = \left[-X^{T} \left| I_{n-k} \right. \right]$ from

$$G^{I}$$
 above. *i.e* $H^{I} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$ and on

juxtaposing
$$H^I$$
, we have $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$. And

we conclude that H is the parity check matrix of C and the generator matrix of the orthogonal complement of C, (C^{\perp}).

3. Findings

The Cayley table 1, when interpreted as a binary Matrix using $Modulus\ 2$ arithmetic, has been transformed into a generator matrix in standard form for a code of length n=5, dimension k=3, $Efficiency=Rate=\frac{3}{5}$.

Moreover, using existing algorithms and results, the parity check matrix H of the code which is also the generator matrix of the dual code C^{\perp} has also been obtained.

4. Conclusions

The Cayley tables due to the non-associative property of the (123)- avoiding class of AUNU permutation pattern has found suitable application in determining the generator /parity check matrices in standard form for some classes of codes.

Acknowledgements

We acknowledge the support enjoyed in this research endeavour by the Institution Based Research (IBR) TETFund Grand 2015, of the Sokoto State University, Sokoto, Nigeria.

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