4)

$i = 2$ (I didn't consider the $i = i - 1$ case because it is going to be a coefficient of the n.)

$i = 4$ (2×2)

$i = 16$ (4×4)

$i = 256$ (16×16) the growth rate of i is 2^{2^k}.

Since i is the index and when this index reach n, the program will terminate, the time complexity of the program is $O(\log(\log(n)))$.

5)

We can seperate the inputs by inputs when $n \leq 100$ and $n > 100$. when $n \leq 100$ the maximum comparision we do is at most 100 and we can say that the time complexity of inputs when $n \leq 100$ is bounded by $O(100)$ which is constant. (for worst and average)

For inputs $n > 100$, the possibility of that we couldn't find any even number in first 100 element is $(0.8)^{100}$ which is 2.037^{-10}, since this possibility is almost zero, this case almost never occures on average. And we can say that this program's average time complexity is $O(1)$.

2)

$$\underset{(a)}{\frac{1}{2n}},\ \underset{(b)}{\log(n)},\ \underset{(c)}{\sqrt{n+5}},\ \underset{(d)}{n+1},\ \underset{(e)}{n^2\log(n)},\ \underset{(f)}{2^n},\ \underset{(g)}{10^n},\ \underset{(e)}{n!},\ n^{2^n}$$

(a) since $1/2n$ gets smaller when n increases, growth rate of $1/2n$ is less than $\log(n)$.

(b) $\lim\limits_{n\to\infty} \dfrac{\log(n)}{\sqrt{n+5}} = \dfrac{\frac{1}{n\ln 2}}{\frac{1}{2\sqrt{n}}} = \dfrac{2\sqrt{n}}{n\ln 2} = \lim\limits_{n\to\infty} \dfrac{2}{\sqrt{n}\ln 2} = 0$ which means time comp. of $\log(n)$ is less than $\sqrt{n+5}$.

(c) Since $\sqrt{n+5}$ is a sub-linear function and $n+1$ is a linear function, the growth rate of $\sqrt{n+5}$ is less than $n+1$.

d) Since $n^2\log(n)$ is an exponential and $n+1$ is linear, the growth rate of $n+1$ is less than $n^2\log(n)$.

e) Since 2^n grows faster than $n^2\log(n)$, the growth rate of $n^2\log(n)$ is less than 2^n.

f) Since 10^n grows faster than 2^n, the growth rate of 2^n is less than 10^n.

g) 10^n = n factors of 10, $n!$ is n factors of numbers until n. so the growth rate of $n!$ is more than 10^n.

e) $n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$ and this function grows slower than n^{2^n}. Because of this, growth rate of $n!$ is less than n^{2^n}.

I used l'hopital rule after ","s.

a) $f(n) \in O(g(n))$
because 2^{2^n} grows much faster than 2^n.

b)
$\lim_{n\to\infty} \frac{n^2}{n^3}$, $\lim_{n\to\infty} \frac{2n}{3n^2}$, $\lim_{n\to\infty} \frac{2}{6n} = 0$.

since the $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ is 0, $f(n) \in O(g(n))$

c) $\lim_{n\to\infty} \frac{3n+1}{2n-5}$, $\lim_{n\to\infty} \frac{3}{2} = \frac{3}{2}$.

since the $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ is a constant, $f(n) \in \Theta(g(n))$.

d) $\lim_{n\to\infty} \frac{4n^2}{n^2}$, $\lim_{n\to\infty} \frac{8n}{2n}$, $\lim_{n\to\infty} \frac{8}{2} = 4$.

since the $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ is constant, $f(n) \in \Theta(g(n))$

e) since $\log_{10} n = \frac{\log_2 n}{\log_2 10}$, and $\log_2 10$ is a constant,
$f(n) \in \Theta(g(n))$

f) $\lim_{n\to\infty} \frac{2^n}{3^n} = \lim_{n\to\infty} \left(\frac{2}{3}\right)^n = 0$

since it is 0, $f(n) \in O(g(n))$

g) $\lim_{n\to\infty} \frac{n^3}{1000n^2}$, $\lim_{n\to\infty} \frac{3n^2}{2000n}$,

$\lim_{n\to\infty} \frac{6n}{2000} = \infty$, since
it is ∞, $f(n) \in \Omega(g(n))$

h) $\lim_{n\to\infty} \frac{5n+4}{2n+2}$, $\lim_{n\to\infty} \frac{5}{2}$,
since it is constant $f(n) \in \Theta(g(n))$

i) $\lim_{n\to\infty} \frac{\sqrt{n}}{\log_2(n)}$, $\lim_{n\to\infty} \frac{\frac{1}{2} n^{-\frac{1}{2}} (n \ln 2)}{\cancel{\frac{1}{n(\ln 2)}}}$

$= \lim_{n\to\infty} \frac{1}{2} \sqrt{n} \ln 2 = \infty$
since it is ∞, $\sqrt{n} \in \Omega(\log_2(n))$

j) $\frac{2^n}{2^{n+1}} = \frac{2^n}{2 \times 2^n} = \frac{1}{2}(1)^n = \frac{1}{2}$, $\lim_{n\to\infty} \frac{1}{2} = \frac{1}{2}$

since it is constant $f(n) \in \Theta(g(n))$