Model of F-16 Fighter Aircraft

- Equation of Motions -

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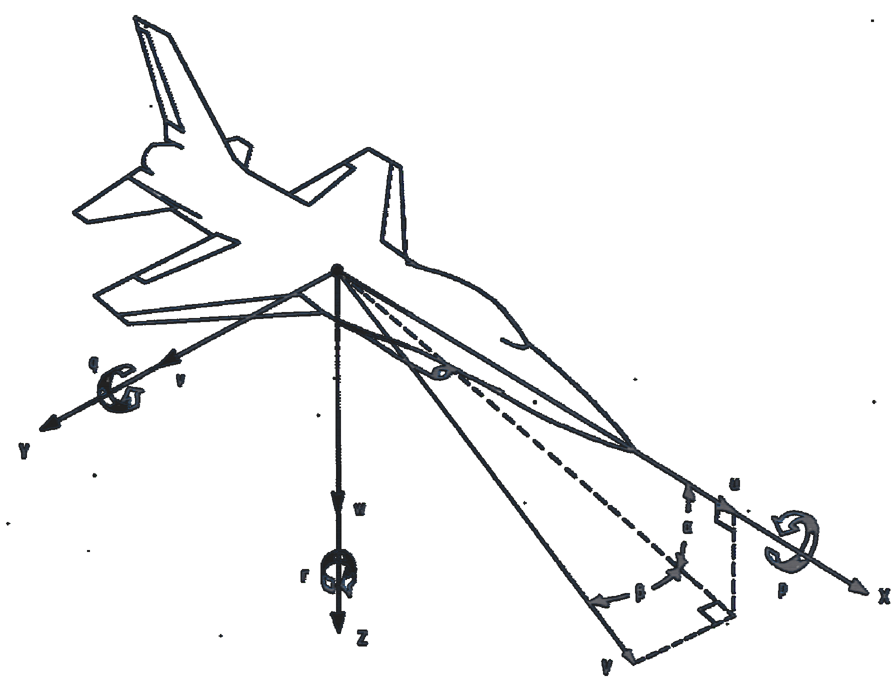

Ref : **[1].** Brian L. Stevens, Frank L. Lewis, Aircraft Control and Simulation, John Wiley & Sons, Inc. 1992
[2]. Nguyen, L.T., et al., Simulator study of stall/post-stall characteristics of a fighter airplane with relaxed longitudinal static stability, NASA Tech. Pap. 1538, NASA, Washington, D.C., Dec. 1979

" The mathematical model given here uses the wind-tunnel data from NASA-Langley wind-tunnel tests on a scale model of an F-16 airplane. The data apply to the speed range up to about Mach=0.6, and were used in a MASA-piloted simulation to study the maneuvering and stall/post-stall characteristics of a relaxed static-stability airplane."

Nomenclature

State Variables:

V	- true velocity, ft/sec
α	- angle of attack, radian (range –10° ~ 45°)
β	- sideslip angle, radian (range –30° ~ 30°)
ϕ	- Euler (roll) angle, rad
θ	- Euler (pitch) angle, rad
φ	- Euler (yaw) angle, rad
p	- roll rate, rad/sec
q	- pitch rate, rad/sec
r	- yaw rate, rad/sec
N_{dis}	- north displacement, ft
E_{dis}	- east displacement, ft
h	- altitude, ft
P_{pow}	- power

Control Variables:

δ_T	- throttle setting, (0.0 – 1.0)
δ_E	- elevator setting, degree
δ_A	- aileron setting, degree
δ_R	- rudder setting, degree

Parameters:

ρ	- air density, slugs/ft^3
M	- Mach number
T	- total instantaneous engine thrust, N (lb)
m	- total airplane mass, slugs
$C_{X,t}$	- total x-axis force coefficient
$C_{Y,t}$	- total y-axis force coefficient
$C_{Z,t}$	- total z-axis force coefficient
$\overline{q}$	- dynamic pressure, psf
p_s	- static pressure, psf
$C_{L,t}$	- total rolling-moment coefficient

$C_{M,t}$	- total pitching-moment coefficient
$C_{N,t}$	- total yawing-moment coefficient
t	- temperature, R
u	- velocity in x-axis direction, ft/sec
v	- velocity in y-axis direction, ft/sec
w	- velocity in z-axis direction, ft/sec
W	- vehicle weight (lbs)
b	- wing span (ft)
S	- wing platform area (ft^2)
$\bar{c}$	- mean aerodynamic chord (ft)
I_x	- roll moment of inertia (slug-ft^2)
I_y	- pitch moment of inertia (slug-ft^2)
I_z	- yaw moment of inertia (slug-ft^2)
I_{xz}	- product moment of inertia (slug-ft^2)
I_{xy}	- product moment of inertia (slug-ft^2)
I_{yz}	- product moment of inertia (slug-ft^2)
X_{cgR}	- reference cg location (ft)
X_{cg}	- center of gravity location (ft)
g	- gravitational constant (ft/sec^2)
h_E	- engine angular momentum (slug-ft^2/s)
d_r	- radian-to-degree constant, $d_r = 57.29578$

Table 1. Mass and Geometry Properties

Symbol	Parameter	Value
W	Vehicle weight (lbs)	20500
b	Wing span (ft)	30
S	Wing area (ft^2)	300
$\overline{c}$	Mean aerodynamic chord (ft)	11.32
I_x	Roll moment of inertia (slug-ft^2)	9496
I_y	Pitch moment of inertia (slug-ft^2)	55814
I_z	Yaw moment of inertia (slug-ft^2)	63100
I_{xz}	Product moment of inertia (slug-ft^2)	982
I_{xy}	Product moment of inertia (slug-ft^2)	0
I_{yz}	Product moment of inertia (slug-ft^2)	0

Table 2. Control Surface Actuator Models

Symbol	Command name	Deflection limit	Rate limit	Time constant	Positive sign convention	Effect
δ_E	Elevator	$\pm 25.0^\circ$	60°/s	0.0495sec lag	Trailing edge down	Negative pitching moment
δ_A	Ailerons	$\pm 21.5^\circ$	80°/s	0.0495sec lag	Right-wing trailing edge down	Negative rolling moment
δ_R	Rudder	$\pm 30.0^\circ$	120°/s	0.0495sec lag	Trailing edge left	Negative yawing moment, positive rolling moment

Table 3. Other parameters used in the model

Symbol	Parameter	Value
X_{cgR}	Reference CG Location (ft)	$0.35\overline{c}$
g	Gravitational constant (ft/sec^2)	32.174
h_E	Engine Angular Momentum (slug-ft^2/s) (assume fixed !)	160.0
d_r	Radian-to-degree constant	57.29578

Six-degree-of-freedom Motion Equations

The equations used to describe the motions of the airplanes were nonlinear, six-degree-of-freedom, rigid-body equations referenced to a body-fixed axis coordinate system.

Force Equations

$$u = V\cos\alpha\cos\beta$$
$$v = V\sin\beta$$
$$w = V\sin\alpha\cos\beta$$
$$V = \sqrt{u^2 + v^2 + w^2}$$

$$\dot{u} = rv - qw - g\sin\theta + \frac{1}{m}(\bar{q}SC_{X,t} + T)$$
$$\dot{v} = pw - ru + g\cos\theta\sin\phi + \frac{\bar{q}S}{m}C_{Y,t}$$
$$\dot{w} = qu - pv + g\cos\theta\cos\phi + \frac{\bar{q}S}{m}C_{Z,t}$$

$$\dot{V} = \frac{u\dot{u} + v\dot{v} + w\dot{w}}{V}$$

$$\alpha = \tan^{-1}(\frac{w}{u})$$
$$\beta = \sin^{-1}(\frac{v}{V})$$

$$\dot{\alpha} = \frac{u\dot{w} - w\dot{u}}{(V\cos\beta)^2}$$
$$\dot{\beta} = \frac{V\cos\beta\dot{v} - v\cos\beta\dot{V}}{(V\cos\beta)^2}$$

Kinetics

$$\dot{\phi} = p + \tan\theta(q\sin\phi + r\cos\phi)$$

$$\dot{\theta} = q\cos\phi - r\sin\phi$$

$$\dot{\varphi} = \frac{q\sin\phi + r\cos\phi}{\cos\theta}$$

Moments

$$\dot{p} = \frac{I_y - I_z}{I_x} qr + \frac{I_{xz}}{I_x}(\dot{r} + pq) + \frac{\overline{q}Sb}{I_x} C_{L,t}$$

$$\dot{q} = \frac{I_z - I_x}{I_y} pr + \frac{I_{xz}}{I_y}(r^2 - p^2) + \frac{\overline{q}S\overline{c}}{I_y} C_{M,t} - h_E r$$

$$\dot{r} = \frac{I_x - I_y}{I_z} pq + \frac{I_{xz}}{I_z}(\dot{p} - qr) + \frac{\overline{q}Sb}{I_z} C_{N,t} + h_E q$$

or

$$\dot{p} = \frac{1}{I_x I_z - I_{xz}^2}\{I_{xz}(I_x - I_y + I_z)pq + [I_z(I_y - I_z) - I_{xz}^2]qr + I_{xz}N + I_z\overline{L} + I_{xz}I_z h_E q\}$$

$$\dot{q} = \frac{I_z - I_x}{I_y} pr + \frac{I_{xz}}{I_y}(r^2 - p^2) + \frac{M}{I_y} - h_E r$$

$$\dot{r} = \frac{1}{I_x I_z - I_{xz}^2}\{(I_x^2 - I_x I_y + I_{xz}^2)pq + I_{xz}(I_y - I_z - I_x)qr + I_x N + I_{zz}\overline{L} + I_x I_z h_E q\}$$

where $\overline{L} = \overline{q}sbC_{L,t}$, $M = \overline{q}s\overline{c}C_{M,t}$, $N = \overline{q}sbC_{N,t}$

Navigation

$$\dot{N}_{dis} = V\cos\alpha\cos\beta\cos\theta\cos\varphi + V\sin\beta(\sin\phi\cos\varphi\sin\theta - \cos\phi\sin\varphi)$$
$$+ V\sin\alpha\cos\beta(\cos\phi\sin\theta\cos\varphi + \sin\phi\sin\varphi)$$

$$\dot{E}_{dis} = V\cos\alpha\cos\beta\cos\theta\sin\varphi + V\sin\beta(\sin\phi\sin\varphi\sin\theta + \cos\phi\cos\varphi)$$
$$+ V\sin\alpha\cos\beta(\cos\phi\sin\theta\sin\varphi - \sin\phi\cos\varphi)$$

$$\dot{h} = V\cos\alpha\cos\beta\sin\theta - V\sin\beta\sin\phi\cos\theta - V\sin\alpha\cos\phi\cos\theta$$

Coefficients

$$\rho = 2.377\times 10^{-3}(1.0 - 0.703\times 10^{-5}h)^{4.14}$$

$$t = \begin{cases} 519(1.0 - 0.703\times 10^{-5}h) & h < 35000.0 \\ 390.0 & h \geq 35000.0 \end{cases}$$

$$\begin{cases} \bar{q} = \frac{1}{2}\rho V^2 & \text{dynamic pressure} \\ p_s = 1715\rho t & \text{static pressure} \end{cases}$$

$$M = \frac{V}{\sqrt{1.4\times 1716.3\times t}}$$

$$C_{X,t} = \frac{\bar{c}}{2V}C_{Xq}(\alpha_d)q + C_x(\alpha_d,\delta_E)$$

$$\begin{aligned} C_{Y,t} &= C_y(\beta_d,\delta_A,\delta_R) + \frac{b}{2V}[C_{Yr}(\alpha_d)r + C_{Yp}(\alpha_d)p] \\ &= -0.02\beta_d + \frac{b}{2V}[rC_{Yr}(\alpha_d) + C_{Yp}(\alpha_d)p] + 0.021\frac{\delta_A}{20.0} + 0.086\frac{\delta_R}{30.0} \end{aligned}$$

$$\begin{aligned} C_{Z,t} &= C_z(\alpha_d,\beta_d,\delta_E) + \frac{\bar{c}}{2V}C_{Zq}(\alpha_d)q \\ &= C_{z,1}(\alpha_d,\beta_d) + \frac{\bar{c}}{2V}C_{Zq}(\alpha_d)q - 0.19\frac{\delta_E}{25.0} \end{aligned}$$

$$\begin{aligned} C_{L,t} &= C_l(\alpha_d,\beta_d,\delta_A,\delta_R) + \frac{b}{2V}[rC_{Lr}(\alpha_d) + C_{Lp}(\alpha_d)p] \\ &= C_{l,1}(\alpha_d,\beta_d) + \frac{b}{2V}[C_{Lr}(\alpha_d)r + C_{Lp}(\alpha_d)p] + C_{l,2}(\alpha_d,\beta)\frac{\delta_A}{20.0} + C_{l,3}(\alpha_d,\beta_d)\frac{\delta_R}{30} \end{aligned}$$

$$C_{M,t} = \frac{\bar{c}}{2V}C_{Mq}(\alpha_d)q + C_{Z,t}(X_{cgR} - X_{cg}) + C_m(\alpha_d,\delta_E)$$

$$\begin{aligned} C_{N,t} &= C_n(\alpha_d,\beta_d,\delta_A,\delta_R) + \frac{b}{2V}[C_{Nr}(\alpha_d)r + C_{Np}(\alpha_d)p] - \frac{\bar{c}}{b}C_{Y,t}(X_{cgr} - X_{cg}) \\ &= C_{n,1}(\alpha_d,\beta_d) + \frac{b}{2V}[C_{Nr}(\alpha_d)r + C_{Np}(\alpha_d)p] - \frac{\bar{c}}{b}C_{Y,t}(X_{cgr} - X_{cg}) \\ &\quad + C_{n,2}(\alpha_d,\beta_d)\frac{\delta_A}{20.0} + C_{n,3}(\alpha_d,\beta_d)\frac{\delta_R}{30} \end{aligned}$$

Table 4. Source of aerodynamic coefficients

Coefficients	**Source**	**Independent variables** ($\alpha_d = d_r \alpha$, $\beta_d = d_r \beta$)
C_{Xq}	Table	α_d
C_x	Table	α_d, δ_E
C_{Yr}	Table	α_d
C_{Yp}	Table	α_d
$C_{Z,1}$	Table	α_d
C_{Zq}	Table	α_d
$C_{l,1}$	Table	α_d, β_d
C_{Lr}	Table	α_d
C_{Lp}	Table	α_d
$C_{l,2}$	Table	α_d, β_d
$C_{l,3}$	Table	α_d, β_d
C_{Mq}	Table	α_d
C_m	Table	α_d, δ_E
$C_{n,1}$	Table	α_d, β_d
C_{Nr}	Table	α_d
C_{Np}	Table	α_d
$C_{n,2}$	Table	α_d, β_d
$C_{n,3}$	Table	α_d, β_d