Superposition of SHMs

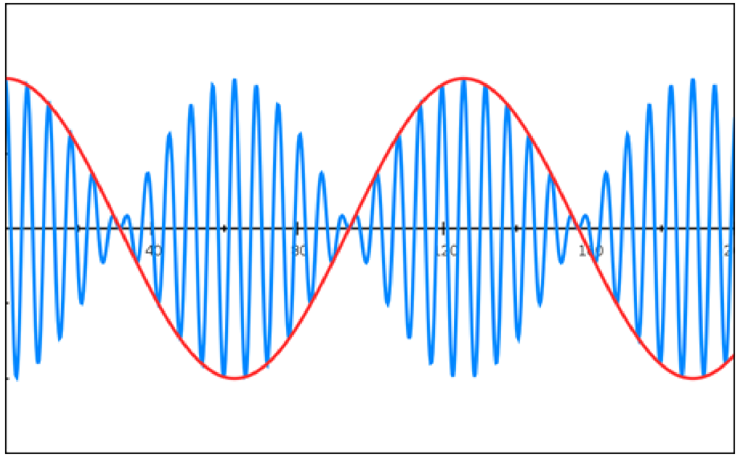
COURSE NAME: Mechanics, Oscillations and Waves (MOW)

PHY F111

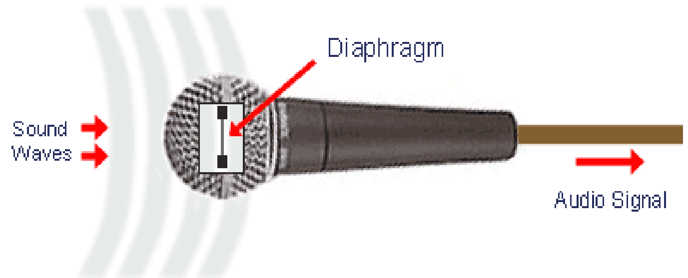
Instructor: Dr. Indrani Chakraborty

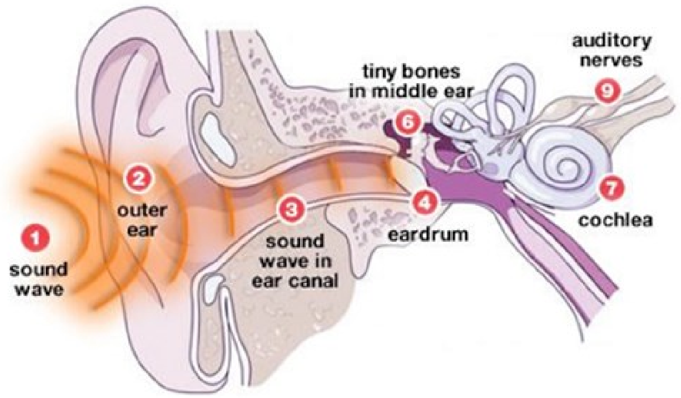
Semester II 2021

e-mail: indranic@goa.bits-pilani.ac.in



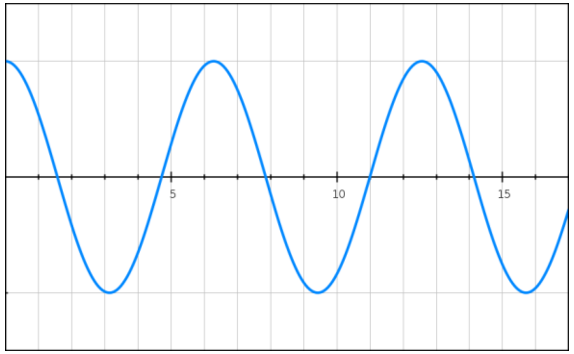
Superposing vibrations



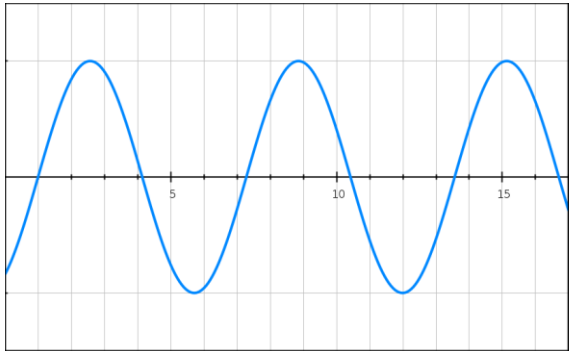


- Most physical vibrations are superpositions of vibrations
- Think about the diaphragm of a microphone or the ear drum of our ears!
- As long as the system is linear, that is Displacement $\propto$ force,
  SHMs can simply be added mathematically!

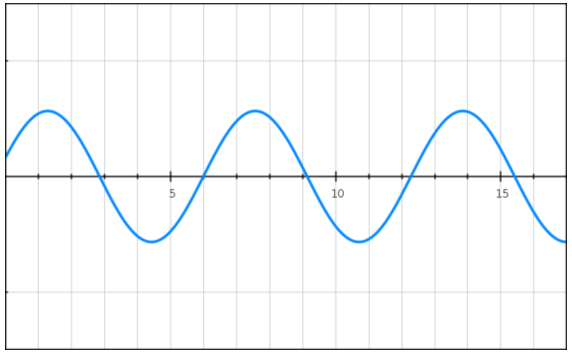
Two SHMs: same frequency, different phase



+



=



$$x_1 = A_1 \cos(\omega t + \varphi_1)$$

$$x_2 = A_2 \cos(\omega t + \varphi_2)$$

$$x = x_1 + x_2 = A_1 \cos(\omega t + \varphi_1) + A_2 \cos(\omega t + \varphi_2)$$

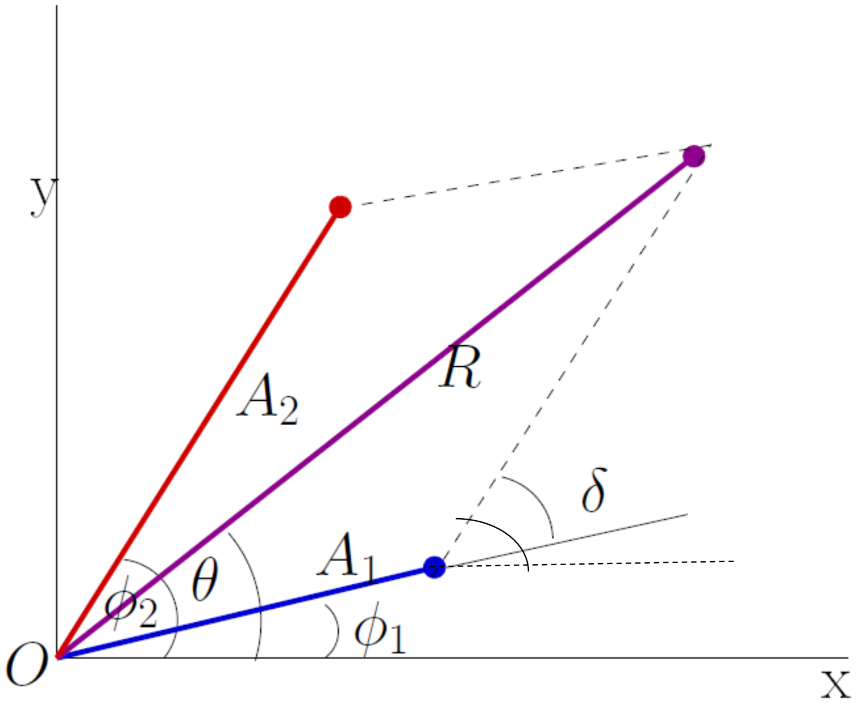
$$= R \cos(\omega t + \theta)$$

How do we find R and θ?

There are two methods…

Note: Here, the SHMs are in the same direction:
Parallel superposition!

The rotating vector method



By vector addition,

$$R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\delta$$

$$\tan\theta = \frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2}$$

$$\delta = \varphi_2 - \varphi_1$$

When $A_1 = A_2 = A$,

$$\theta = \frac{(\phi_1+\phi_2)}{2}$$

$$R = 2A\cos\frac{\delta}{2}$$

The complex exponential method

$$z_1 = A_1 e^{i(\omega t+\varphi_1)}$$

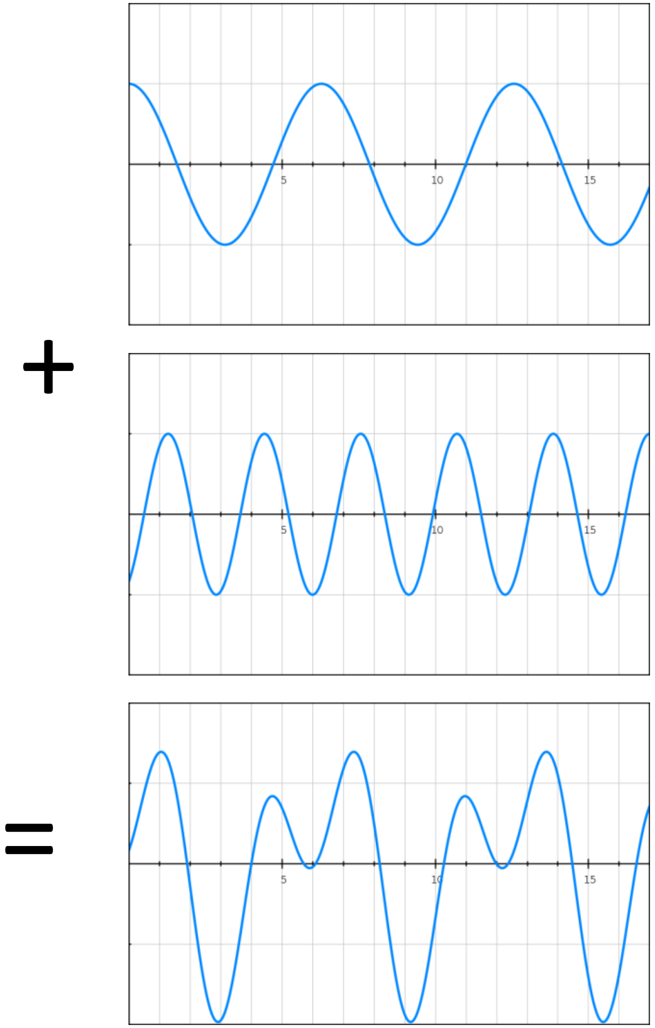
$$z_2 = A_2 e^{i(\omega t+\varphi_2)}$$

$$z = z_1 + z_2 = A_1 e^{i(\omega t+\varphi_1)} + A_2 e^{i(\omega t+\varphi_2)}$$

$z = Re^{i(\omega t+\theta)} = e^{i(\omega t+\varphi_1)}\left(A_1 + A_2 e^{i\delta}\right)$ where $\delta = \varphi_2 - \varphi_1$

See that R and θ have the same values as that in the rotating vector method for $A_1 = A_2 = A$.

Two SHMs: same amplitude, different frequency and phase



$$x_1 = A\cos(\omega_1 t + \varphi_1)$$

$$x_2 = A\cos(\omega_2 t + \varphi_2)$$

Now the result of addition is a complicated function of time!

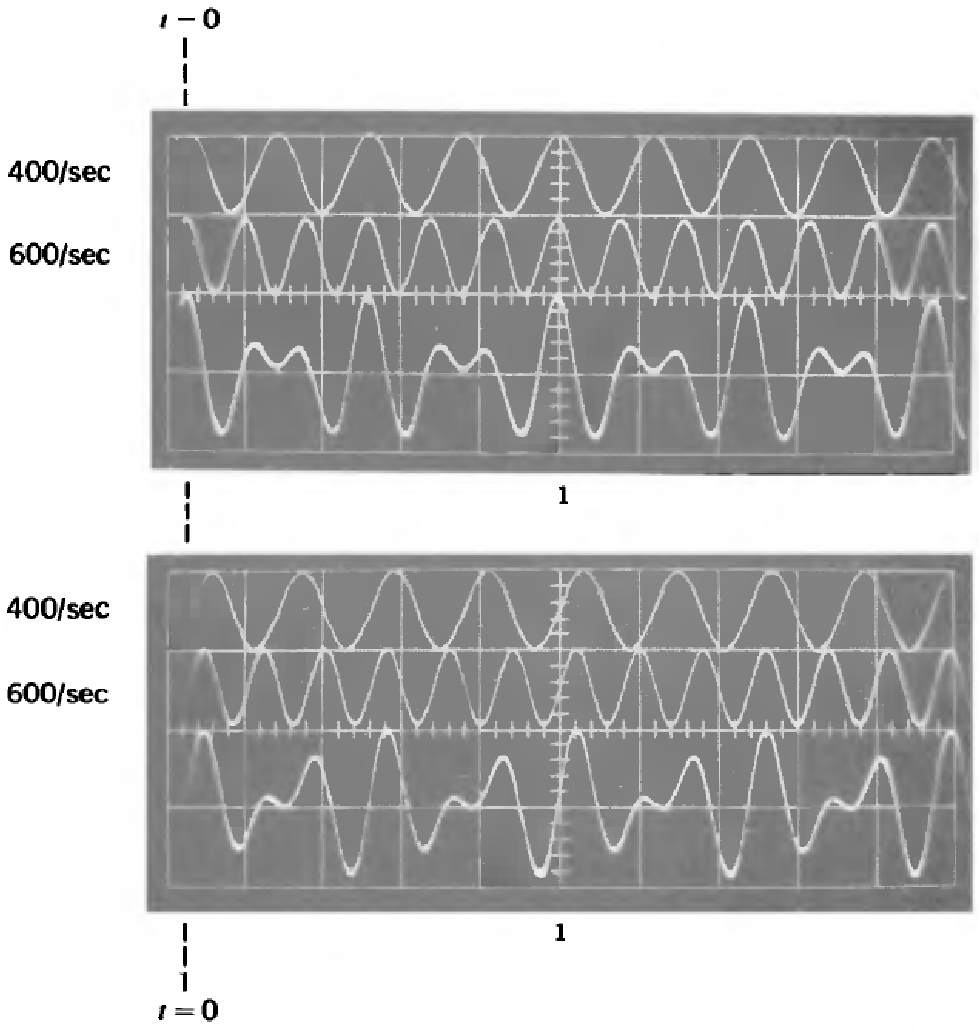
It would be periodic with period T if T_1 and T_2 are **commensurable**, that is:

$$n_1 T_1 = n_2 T_2 = T$$

where n_1 and n_2 are integers.

The resultant motion has a periodicity T in this case.

Two SHMs: same amplitude, different frequency and phase

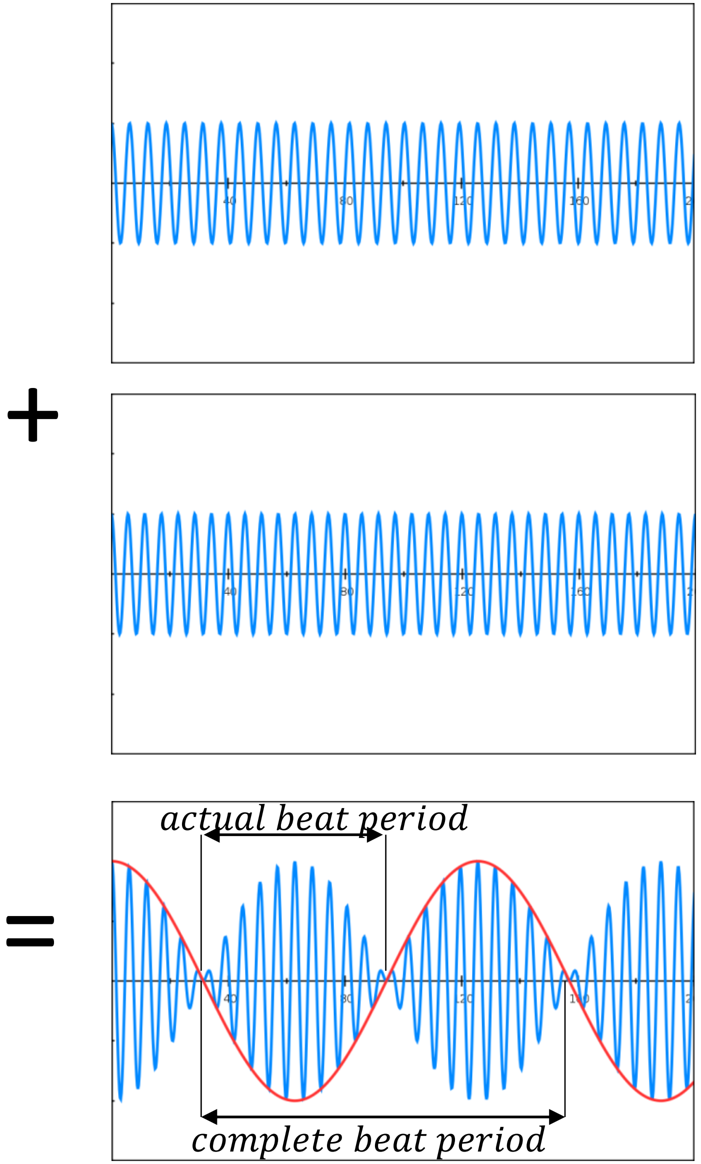


What about the phases?

- Resultant wave nature depends markedly on the initial phases.
- Human ears are quite insensitive to phase in a mixture of SHMs, so we won't distinguish the two cases here by hearing.
- However, superposition of large number of SHMs with different phase relationships might drastically modify the resultant wave, and produce new aural effects!

.

When the frequencies are nearly equal: ‘Beats’



$$x_1 = A\cos(\omega_1 t)$$

$$x_2 = A\cos(\omega_2 t)$$

$$x = x_1 + x_2 = A\cos(\omega_1 t) + A\cos(\omega_2 t)$$

$$= 2A\cos\frac{(\omega_1+\omega_2)t}{2}\cos\frac{(\omega_1-\omega_2)t}{2}$$

- Physically meaningful when $|\omega_1 - \omega_2| \ll \omega_1 + \omega_2$, as then it represents a cosine wave, with an average frequency $\frac{(\omega_1+\omega_2)}{2}$ with its amplitude modulated by an envelope:

$$\pm\, 2A\cos\frac{(\omega_1-\omega_2)t}{2}$$

However since time between 2 successive zeros = ½ of one complete beat period, and our ears can only perceive this ‘stops’,
The perceived **Beat frequency $\boldsymbol{\omega_b = \omega_1 - \omega_2}$**

Average frequency $\boldsymbol{\omega_a = \frac{(\omega_1+\omega_2)}{2}}$

A short demo: Beats

https://academo.org/demos/wave-interference-beat-frequency/

Moving on to 2D: perpendicular vibrations

A General treatment:

$$x = A_1 \cos \omega t$$

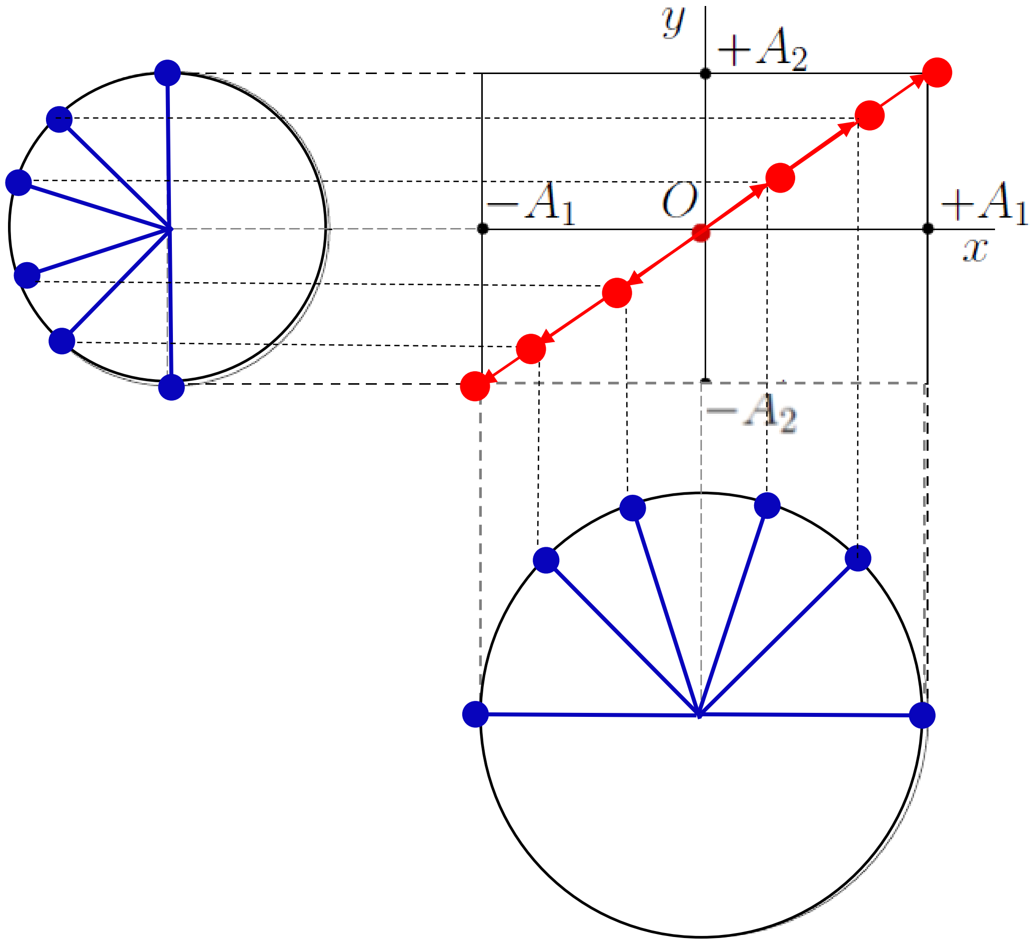
$$y = A_2 \cos(\omega t + \delta)$$

Let, $\frac{x}{A_1} = \cos \omega t = \tilde{x}$ and $\frac{y}{A_2} = \cos(\omega t + \delta) = \tilde{y}$

Then, $\boxed{\tilde{x}^2 + \tilde{y}^2 - 2\tilde{x}\tilde{y}cos\delta = sin^2\delta}$

when $\delta = 0$, a straight line; when $\delta = \frac{\pi}{2}$, an ellipse

Moving on to 2D: perpendicular vibrations



Phase difference ($\delta = 0$):

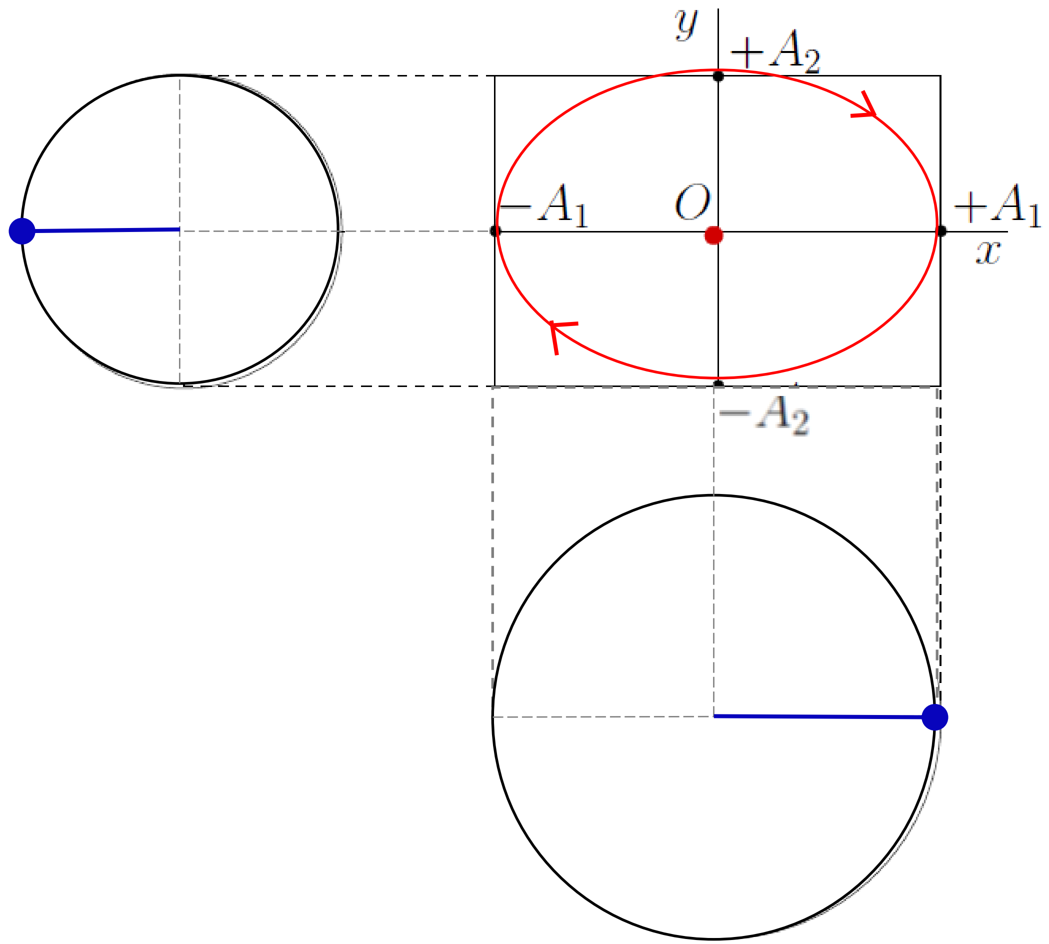
$$x = A_1 \cos \omega t$$

$$y = A_2 \cos \omega t$$

The resultant motion is a straight line with slope

$$\frac{y}{x} = \frac{A_2}{A_1}$$

Moving on to 2D: perpendicular vibrations



Phase difference ($\delta = \frac{\pi}{2}$):

$$x = A_1 \cos \omega t$$

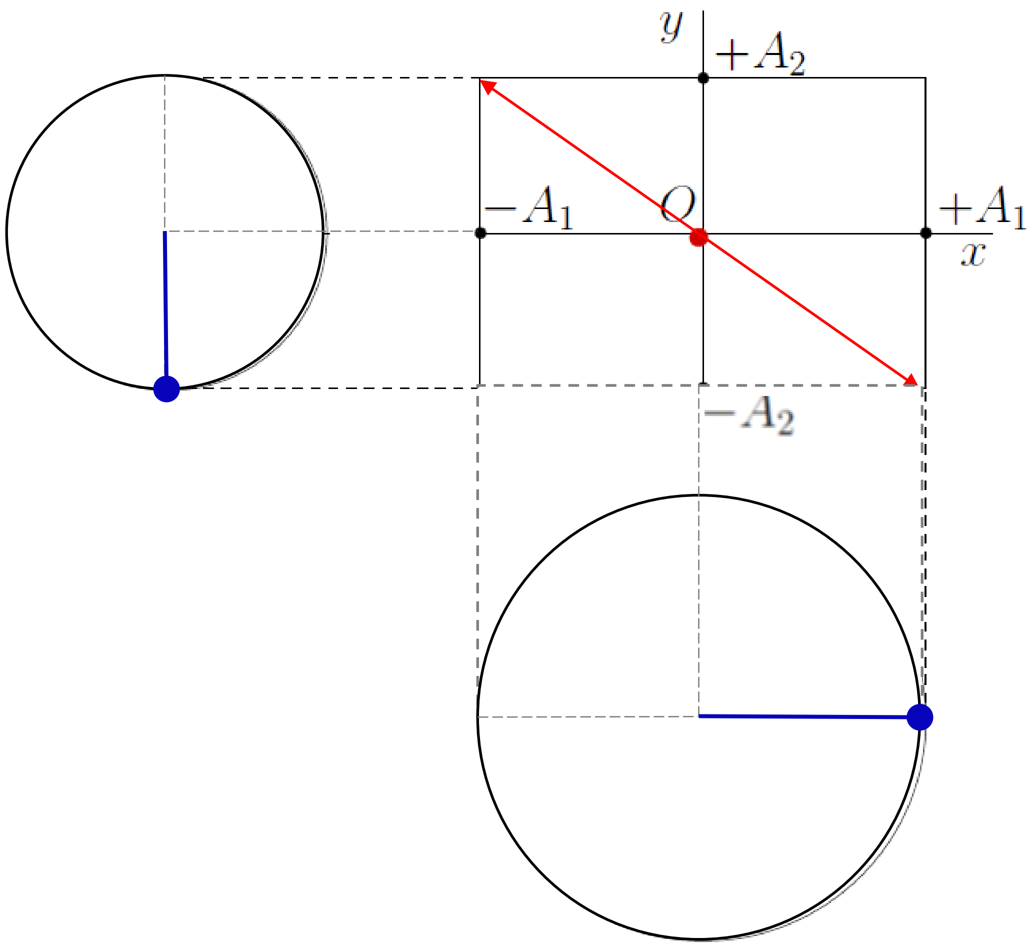
$$y = A_2 \cos(\omega t + \frac{\pi}{2}) = -A_2 \sin \omega t$$

The resultant motion is an ellipse with:

$$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = 1$$

Directionality: clockwise.

Moving on to 2D: perpendicular vibrations



Phase difference ($\delta = \pi$):

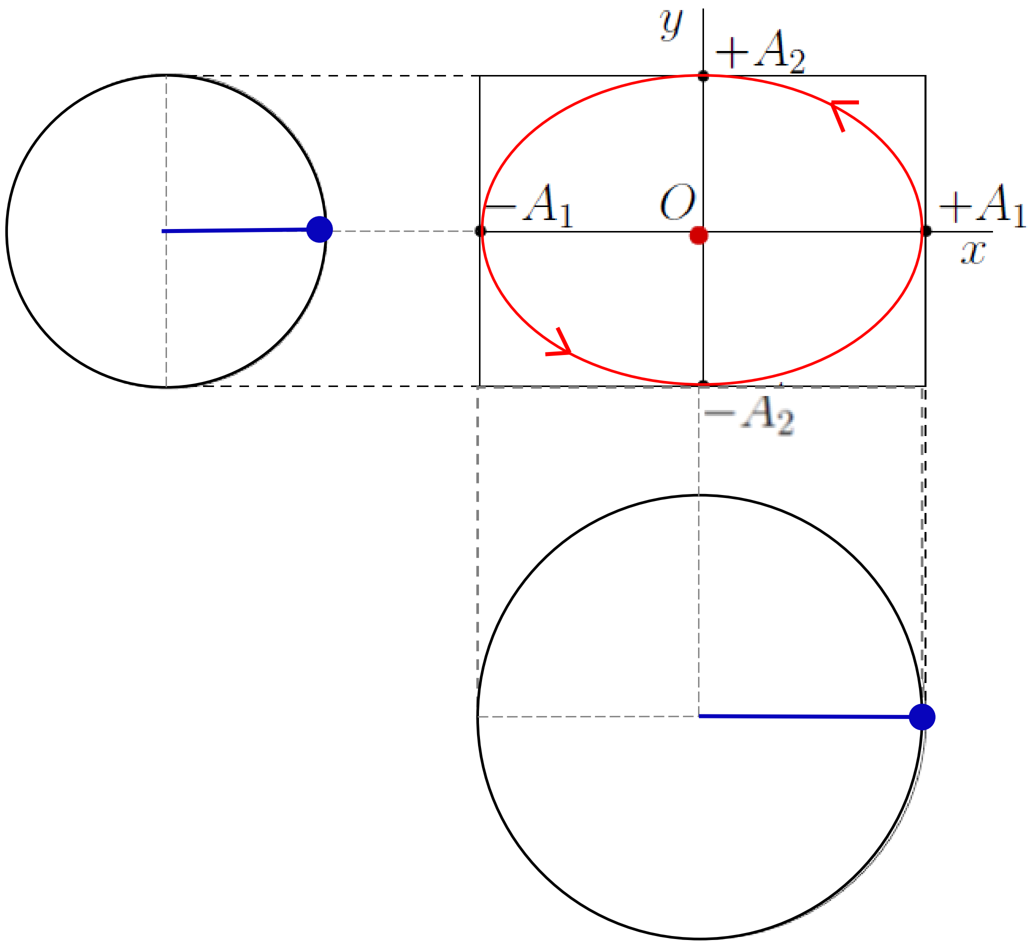
$$x = A_1 \cos \omega t$$

$$y = A_2 \cos(\omega t + \pi) = -A_2 \cos \omega t$$

The resultant motion is a straight line with slope

$$\frac{y}{x} = -\frac{A_2}{A_1}$$

Moving on to 2D: perpendicular vibrations



Phase difference ($\delta = \frac{3\pi}{2}$):

$$x = A_1 \cos \omega t$$

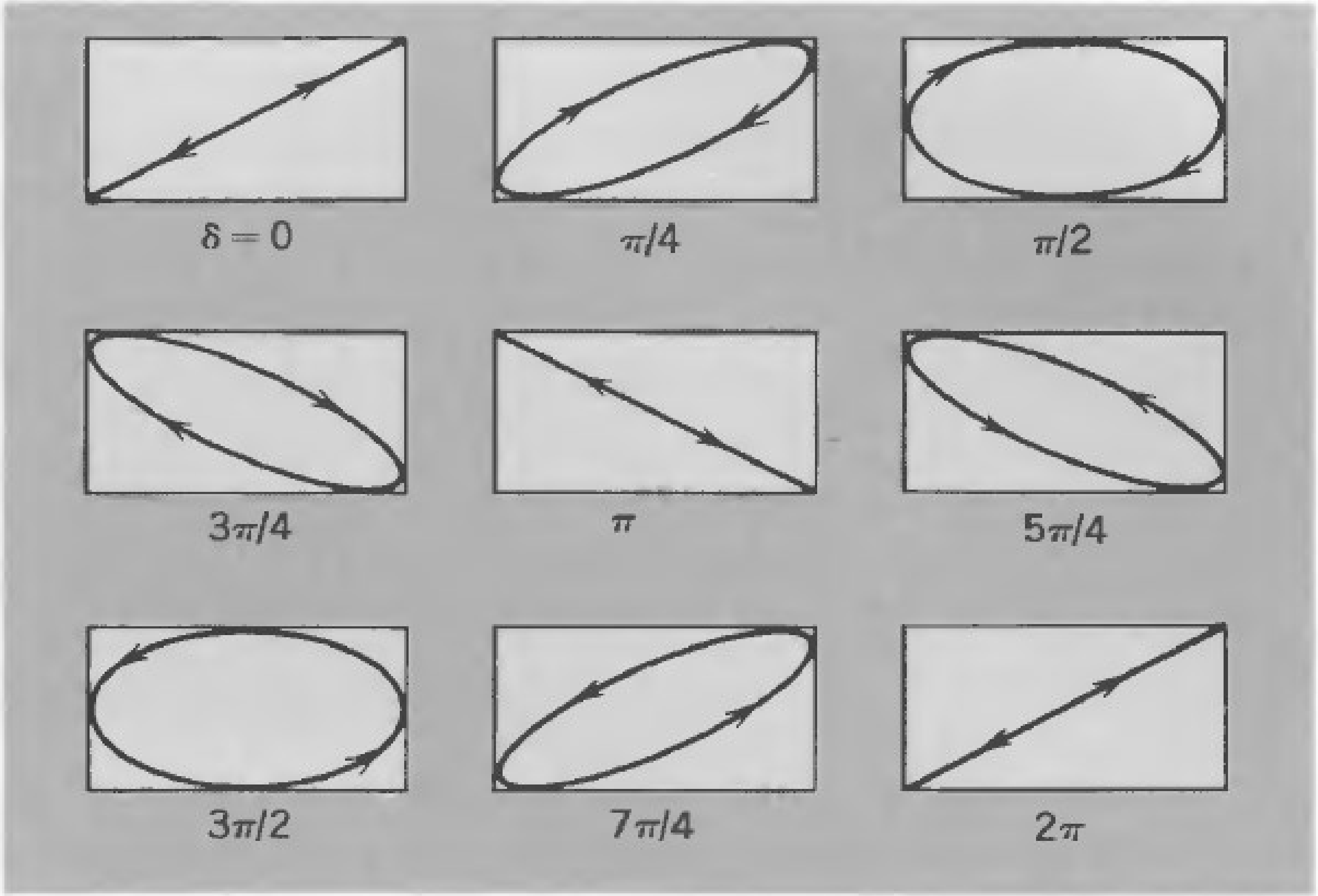
$$y = A_2 \cos(\omega t + \frac{3\pi}{2}) = A_2 \sin \omega t$$

The resultant motion is an ellipse with:

$$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = 1$$

Directionality: anti-clockwise.

Lissajous Figures

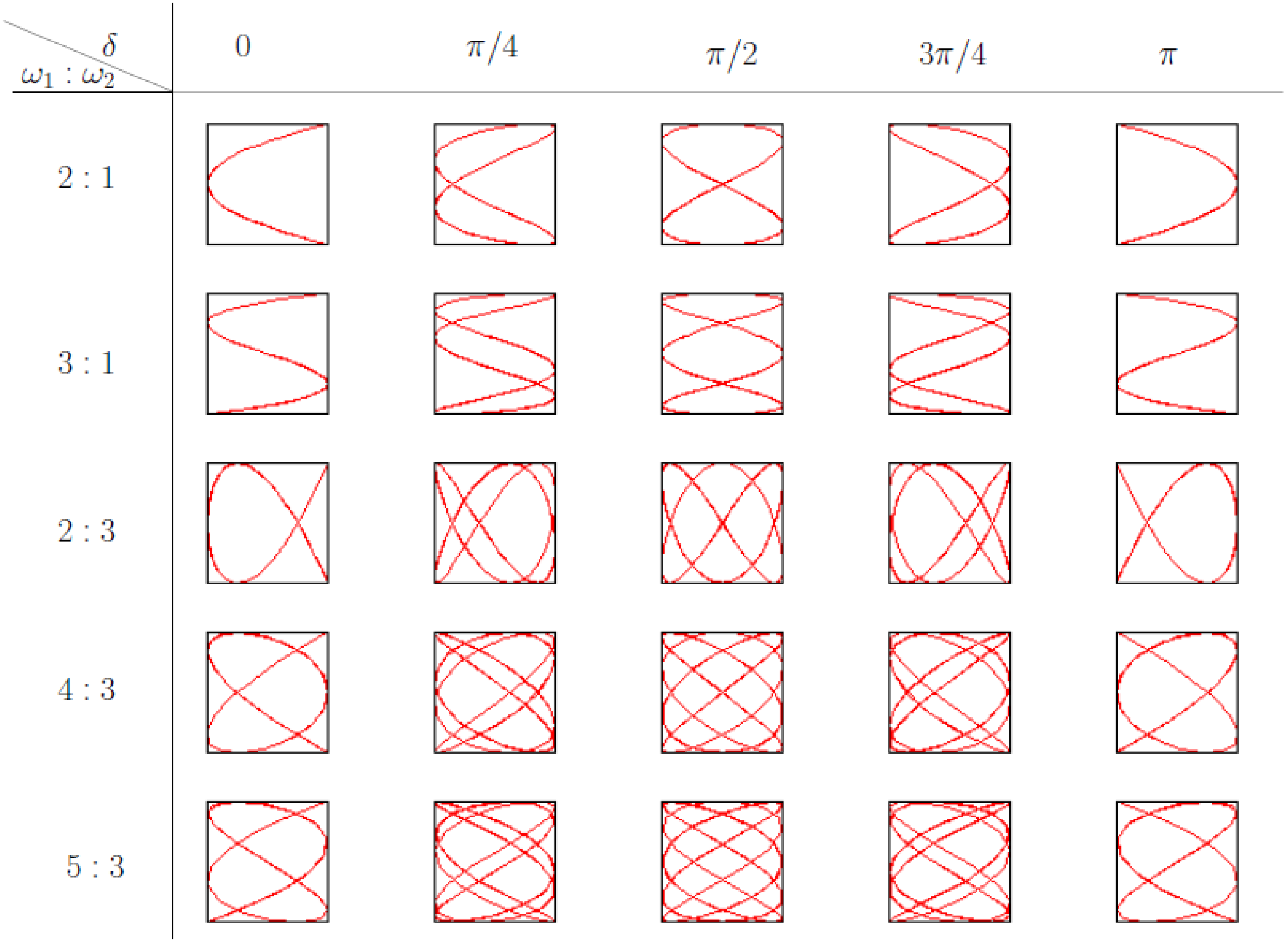


Lissajous Figures: demo

https://academo.org/demos/lissajous-curves/

Lissajous Figures

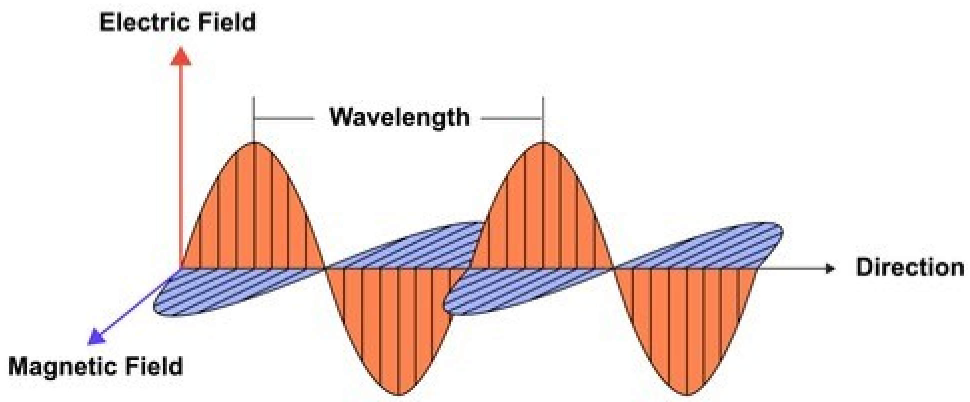
For different frequency ratios



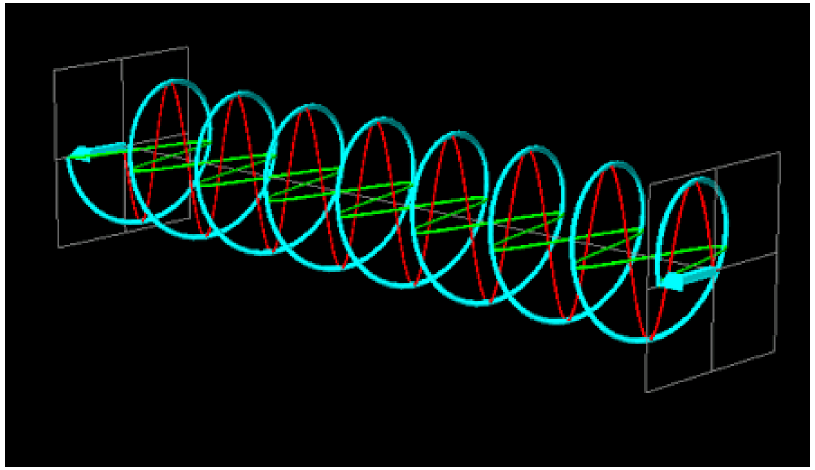
J. A. Lissajous
(1822-1880)

EXAMPLE 1: Sand patterns

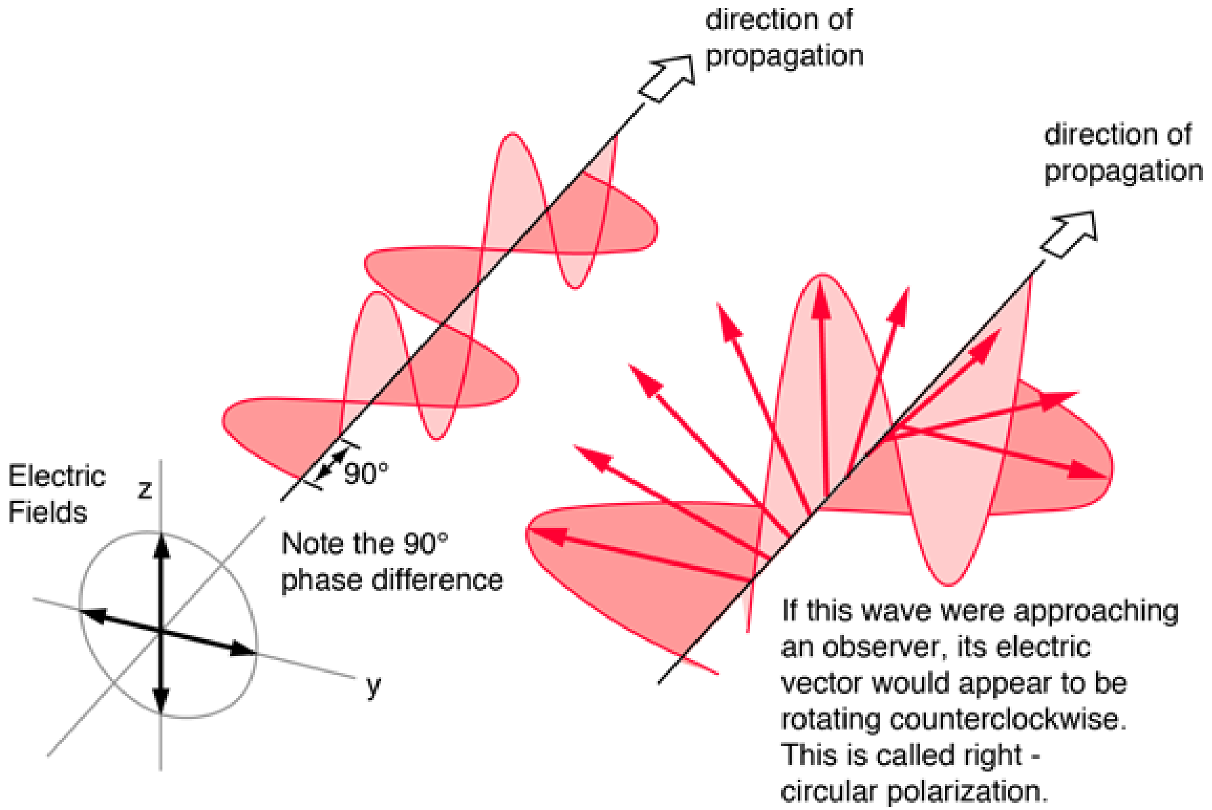
Courtesy: xmdemo



Plane polarized light



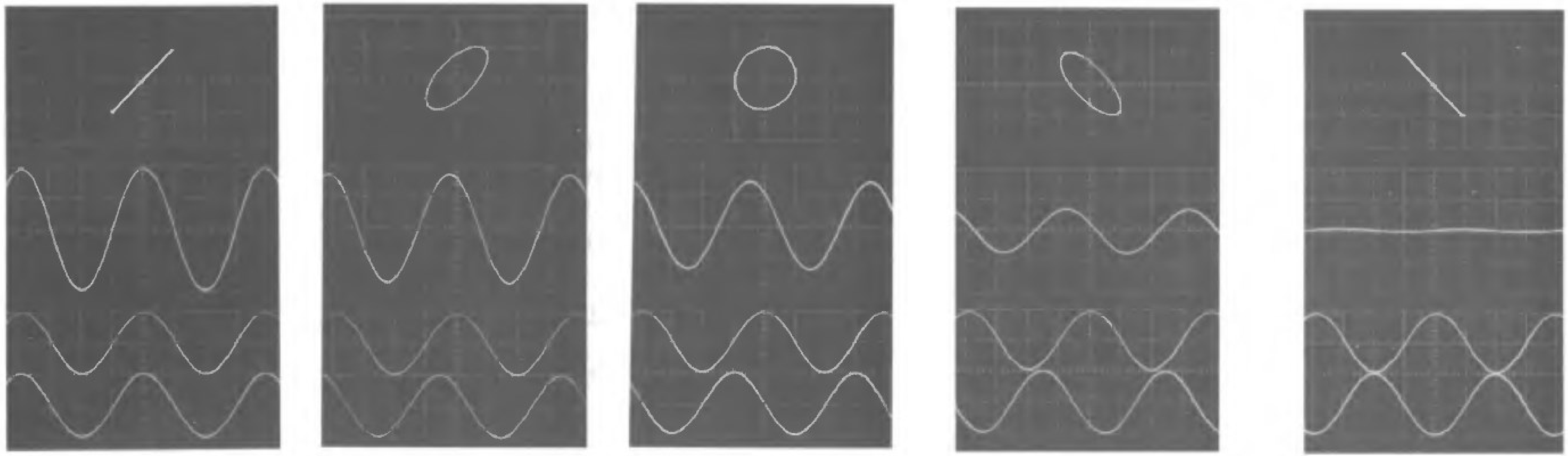
Circularly polarized light



Courtesy: hyperphysics.phy-astr.gsu.edu

EXAMPLE 2: Circularly polarized light

Comparison of parallel and perpendicular



$\delta = 0$ $\delta = \pi/4$ $\delta = \pi/2$ $\delta = 3\pi/4$ $\delta = \pi$