# Lecture 8 –Rotation with translation in CM and Lab Frame


..\video MOW\Push Me Pull You - YouTube (360p).mp4

# Collisions in C.M and Lab Frame for objects that is rotating and translating

## Properties of Lab Frame:

1. Momentum is conserved in elastic and inelastic collision. In some special cases, net momentum before and after collision will be zero.

2. Speed of the particles before and after collision will be different. In some special cases, it will be same.

3. In, in-elastic collision, the loss in kinetic energy is dissipated as heat.

## Properties of C.M Frame:

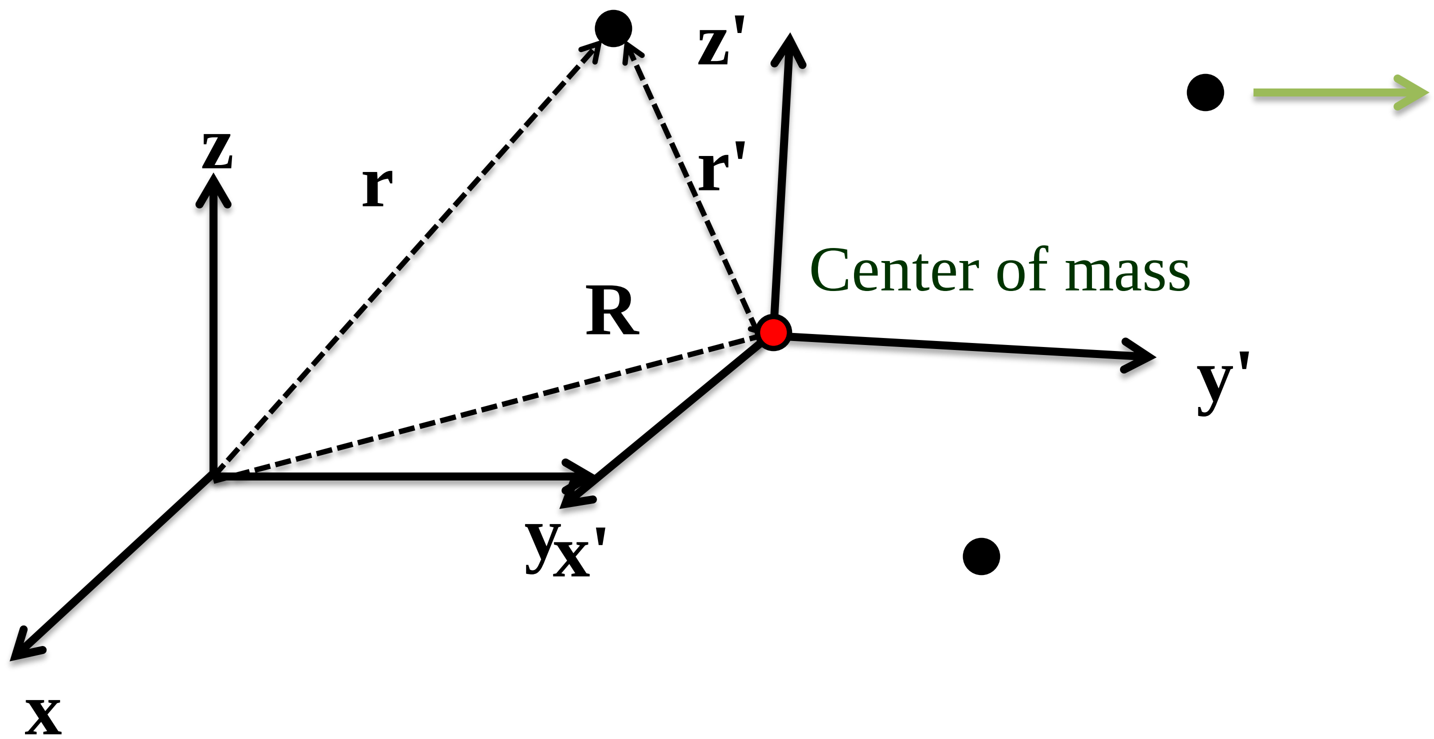
**Momentum is always zero in C.M Frame**

**Speed of particles before and after collisions in C.M Frame is always the same.**

**In, in-elastic collision, the kinetic energy is completely lost in C. M. Frame**

# Angular Momentum of Body Both Rotating and Translating

# Rotation and Translation




$$r = R + r^{'}$$

$$L = r \times m\dot{r}$$ Angular momentum in Lab frame

# Rotation and Translation

$$L = r \times m\dot{r} \qquad\qquad r = R + r'$$

$$L = (R + r') \times m(\dot{r}' + \dot{R})$$

$$L = R \times m\dot{r}' + R \times m\dot{R} + r' \times m\dot{r}' + r' \times m\dot{R}$$

$$L_{total} = R \times \sum m_j \dot{r}_j' + R \times \sum m_j \dot{R} + r_j' \times \sum m_j \dot{r}_j' + r_j' \times \sum m_j \dot{R}$$

$$\sum m_j r_j' = \sum m_j (r_j - R) = \sum m_j r_j - MR = 0$$

# Rotation and Translation

$$L_{total} = \cancel{R \times \sum m_j \dot{r}_j'} + R \times \sum m_j \dot{R} + r_j' \times \sum m_j \dot{r}_j' + \cancel{r_j' \times \sum m_j \dot{R}}$$

$$L_{total} = R \times \sum m_j \dot{R} + r_j' \times \sum m_j \dot{r}_j'$$

$$\boxed{1^{st}\, term = R \times \sum m_j \dot{R} = R \times MV}$$

**Angular momentum due to the translation of center of mass with respect to lab frame**

# Rotation and Translation

$$2^{nd}\,term = r_j' \times \sum m_j \dot{r}_j'$$

Origin of $r_j$' is the center of mass

The particle rotate about center of mass without changing its magnitude.

$$\boxed{r_j' \times \sum m_j \dot{r}_j' = \sum m_j r_j' \times r_j' \omega = I\omega}$$

**Angular momentum due to rotation about the center of mass**

# Rotation and Translation

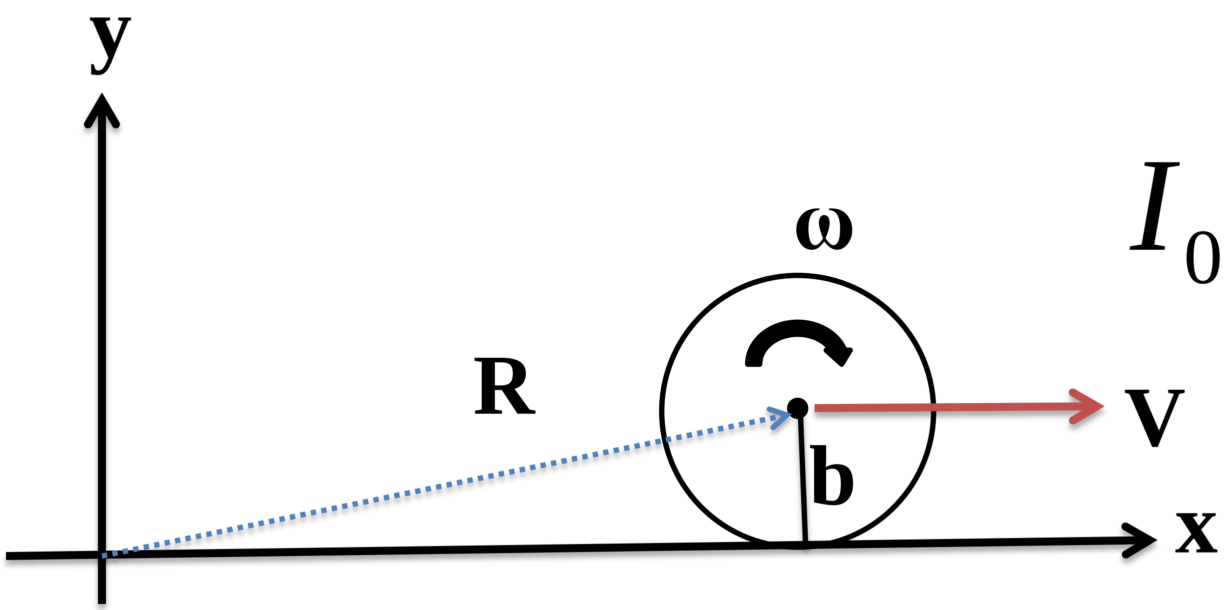
$$L_{total} = R \times \sum m_j \dot{R} + r_j' \times \sum m_j \dot{r}_j'$$

$$L_{total} = R \times MV + I\omega$$

**Statement:**

**Angular momentum of a rigid body is the sum of angular momentum of the center of mass about the origin and angular momentum about its center of mass**

# Example – Angular Momentum of a Rolling Wheel

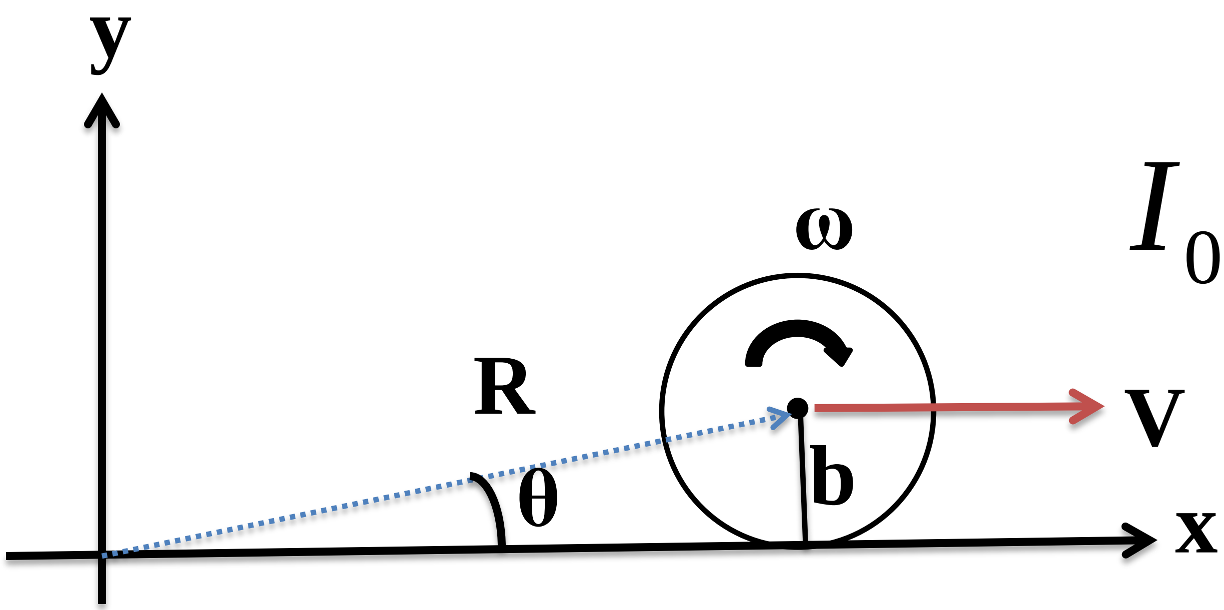



$$I_0 = m\frac{b^2}{2}$$

**Angular Momentum about the Center of Mass**

$$L = -I_0\omega = -m\frac{b^2}{2}\omega$$

# Example – Angular Momentum of a Rolling Wheel

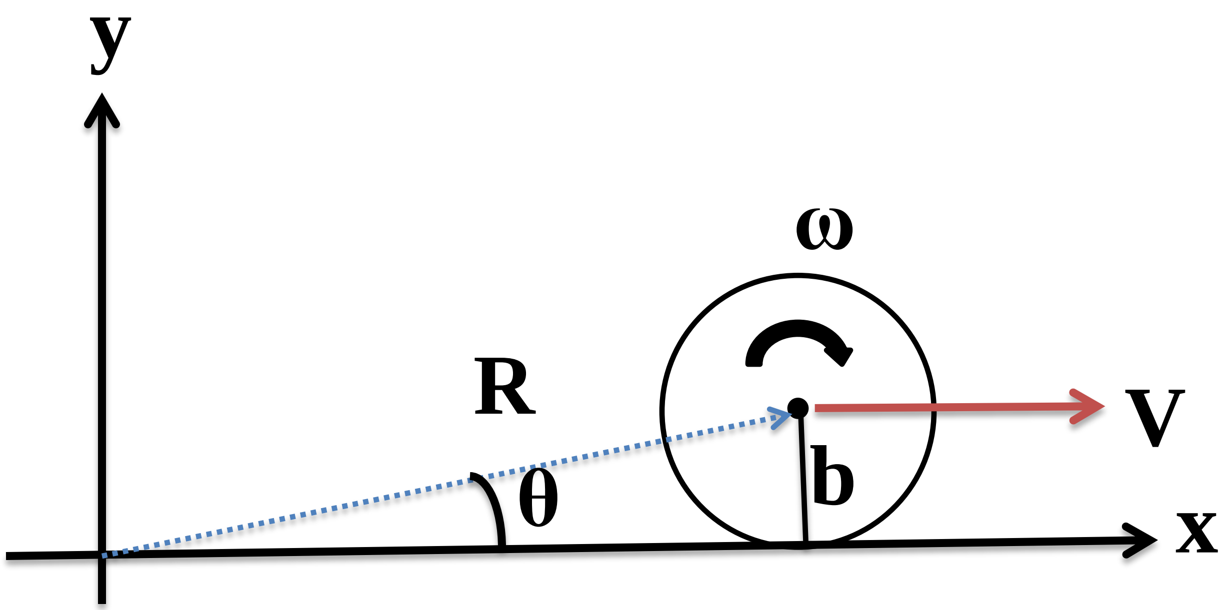



$$I_0 = m\frac{b^2}{2}$$

**Angular Momentum of the Center of Mass**

$$L = -R_\perp \times mv = -mb^2\omega$$

# Example – Angular Momentum of a Rolling Wheel




**Total Angular Momentum of the Wheel about origin**

$$L = -m\frac{b^2}{2}\omega - mb^2\omega = -\frac{3}{2}mb^2\omega$$

# Summary

**Angular momentum of a rigid body is the sum of angular momentum of the center of mass about the origin and angular momentum about its center of mass**

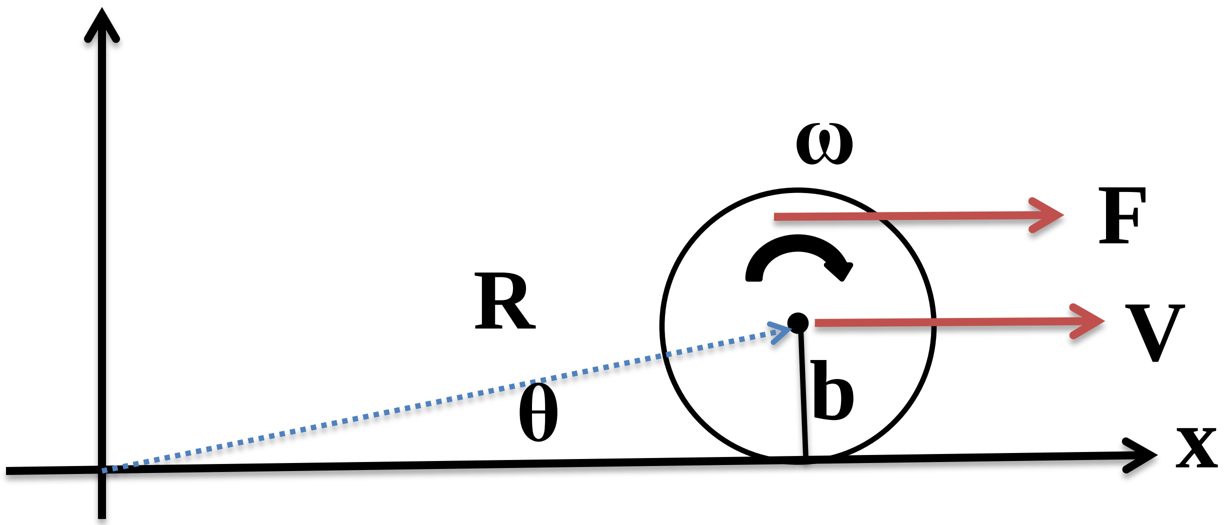
**What about TORQUE? Does it follow the same rule?**

# Example – Angular Momentum of a Rolling Wheel

**Angular momentum of a rigid body is the sum of angular momentum of the center of mass about the origin and angular momentum about its center of mass**

**What about TORQUE? Does it follow the same rule?**

# Example – Angular Momentum of a Rolling Wheel




$$\tau = \sum r_j \times f_j = \sum (r_j' + R) \times f_j$$

# Example – Angular Momentum of a Rolling Wheel

**Total Angular Momentum of the Wheel about origin**

$$L_{total} = R \times MV + I\omega$$

$$\frac{dL_{total}}{dt} = \frac{d}{dt}(R \times MV) + \frac{d}{dt}(I\omega)$$

-

# Example – Torque of a Rolling Wheel

**Comparing the two results, we get**

$$\tau = \sum (r_j^{'}) \times f_j + R \times F$$

$$\tau = I\alpha + R \times F$$

**Torque acting on the rigid body is the sum of torque about its center of mass and the torque acting on the center of mass**

$$\tau_0 = \sum (r_j^{'}) \times f_j = I\alpha$$

# Example – Torque of a Rolling Wheel

$$\tau_0 = \sum (r_j^{'}) \times f_j = I\alpha$$

**Rotational Motion about the center of Mass depends only on the torque about the center of mass, independent of the motion of the center of mass.**

# Example – Kinetic Energy of a Rolling Wheel

**So torque follows angular momentum, how about kinetic energy?**

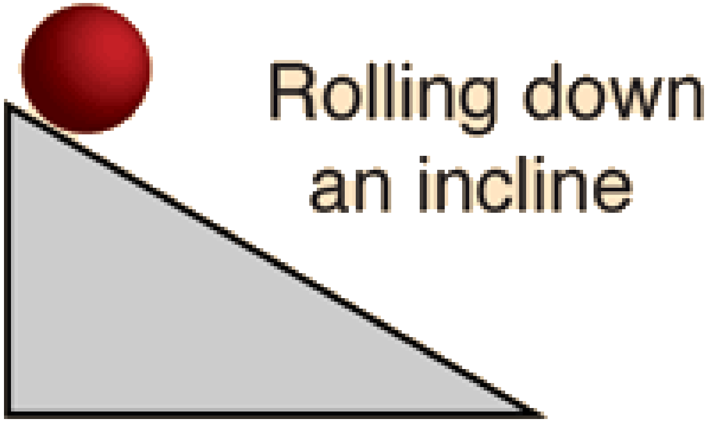
$$K.E = \frac{1}{2}\sum(m_j v_j^2) = \frac{1}{2}\sum m_j(\dot{r}_j + V)^2$$

$$K.E = \frac{1}{2}I_0\omega^2 + \frac{1}{2}MV^2$$

# Sum – up of Physical Quantities

| Pure Rotation | Translation + Rotation |
| --- | --- |
| $L = I\omega$ | $L = I\omega + (R \times MV)$ |
| $\tau = I\alpha$ | $\tau = I\alpha + (R \times F)$ |
| $K = \frac{1}{2} I\omega^2$ | $K = \frac{1}{2} I\omega^2 + \frac{1}{2} MV^2$ |

# Problem Solving – Rotation and Translation

# Sum – up of Physical Quantities

**Pure Rotation**

$$L = I\omega$$

$$\tau = I\alpha$$

$$K = \frac{1}{2}I\omega^2$$

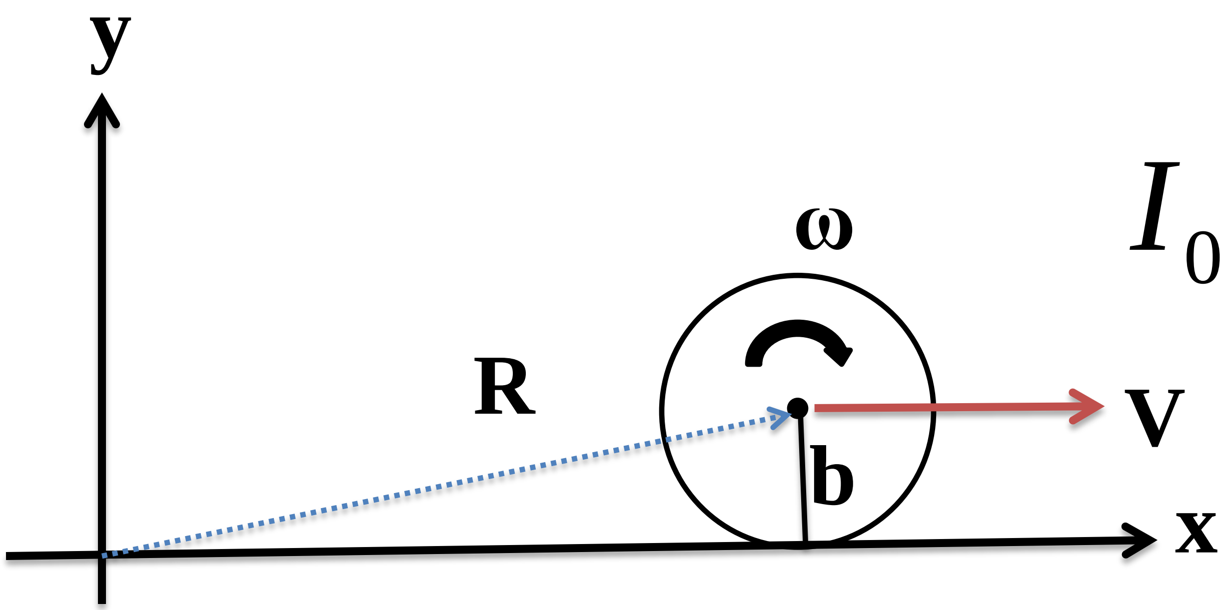
**Translation + Rotation**

$$L = I\omega + (R \times MV)$$

$$\tau = I\alpha + (R \times F)$$

$$K = \frac{1}{2}I\omega^2 + \frac{1}{2}MV^2$$

# Example – Angular Momentum of a Rolling Wheel

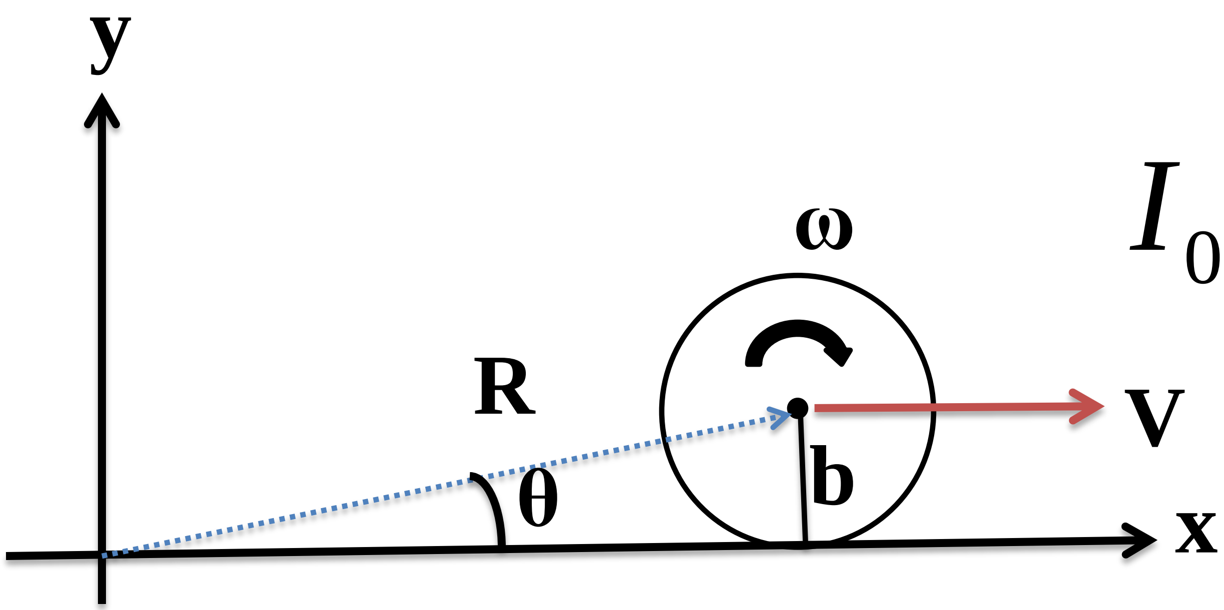



$$I_0 = m\frac{b^2}{2}$$

**Angular Momentum about the Center of Mass**

$$L = -I_0\omega = -m\frac{b^2}{2}\omega$$

# Example – Angular Momentum of a Rolling Wheel

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**Angular Momentum of the Center of Mass**

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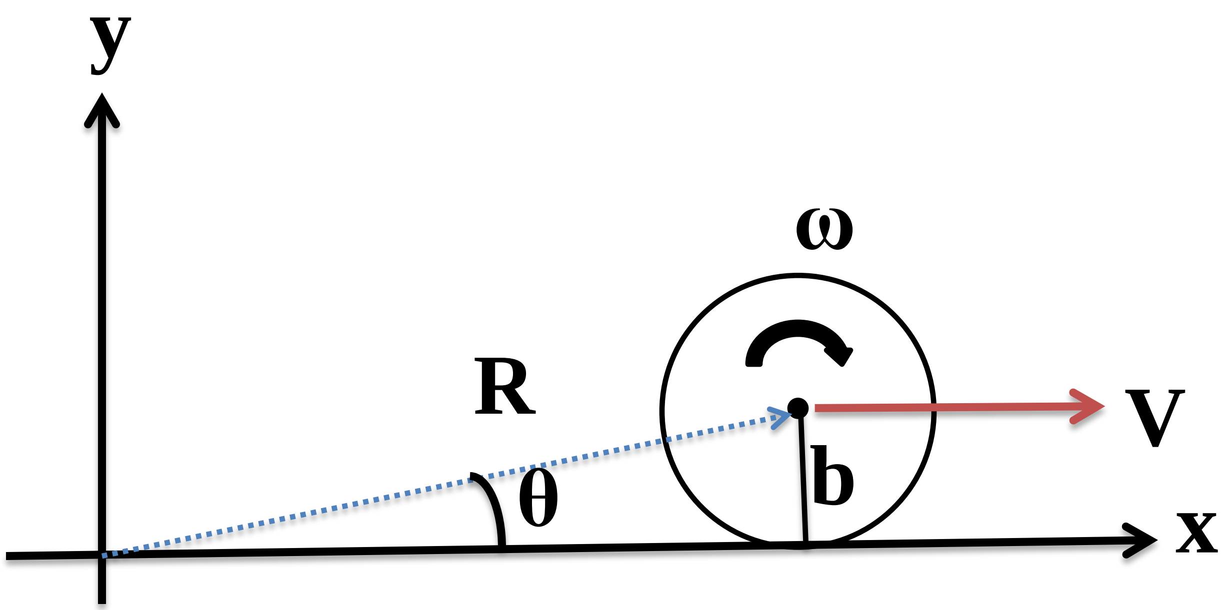
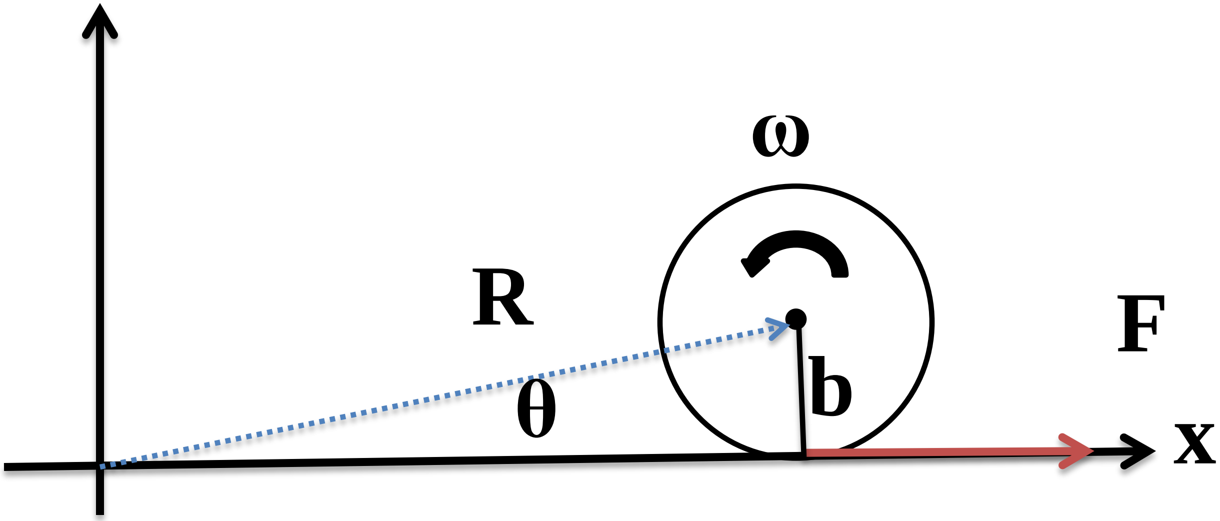
# Example – Angular Momentum of a Rolling Wheel




**Total Angular Momentum of the Wheel about origin**

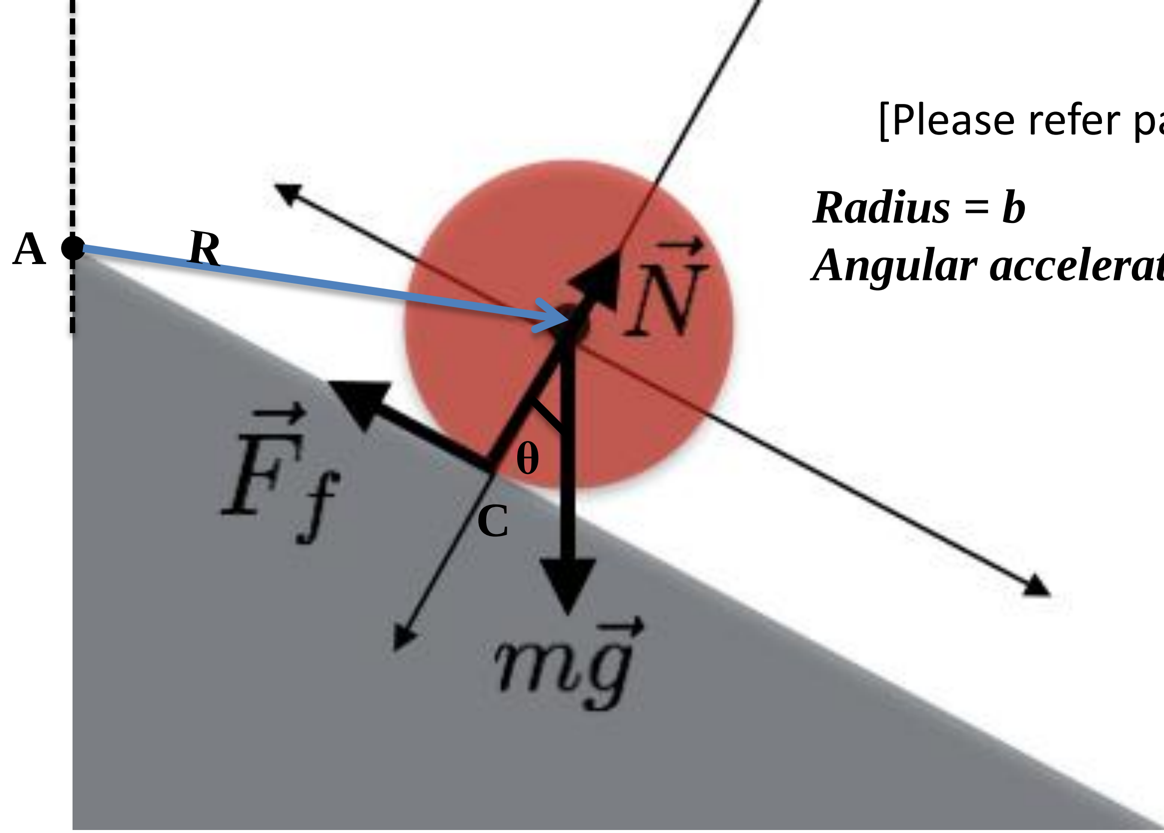
$$L = -m\frac{b^2}{2}\omega - mb^2\omega = -\frac{3}{2}mb^2\omega$$

# Example – Torque of a Rolling Wheel on a frictionless surface




$$\tau = \sum r_j \times f_j = \sum (r_j^{'} + R) \times f_j$$

[Please refer page 265 for solution]

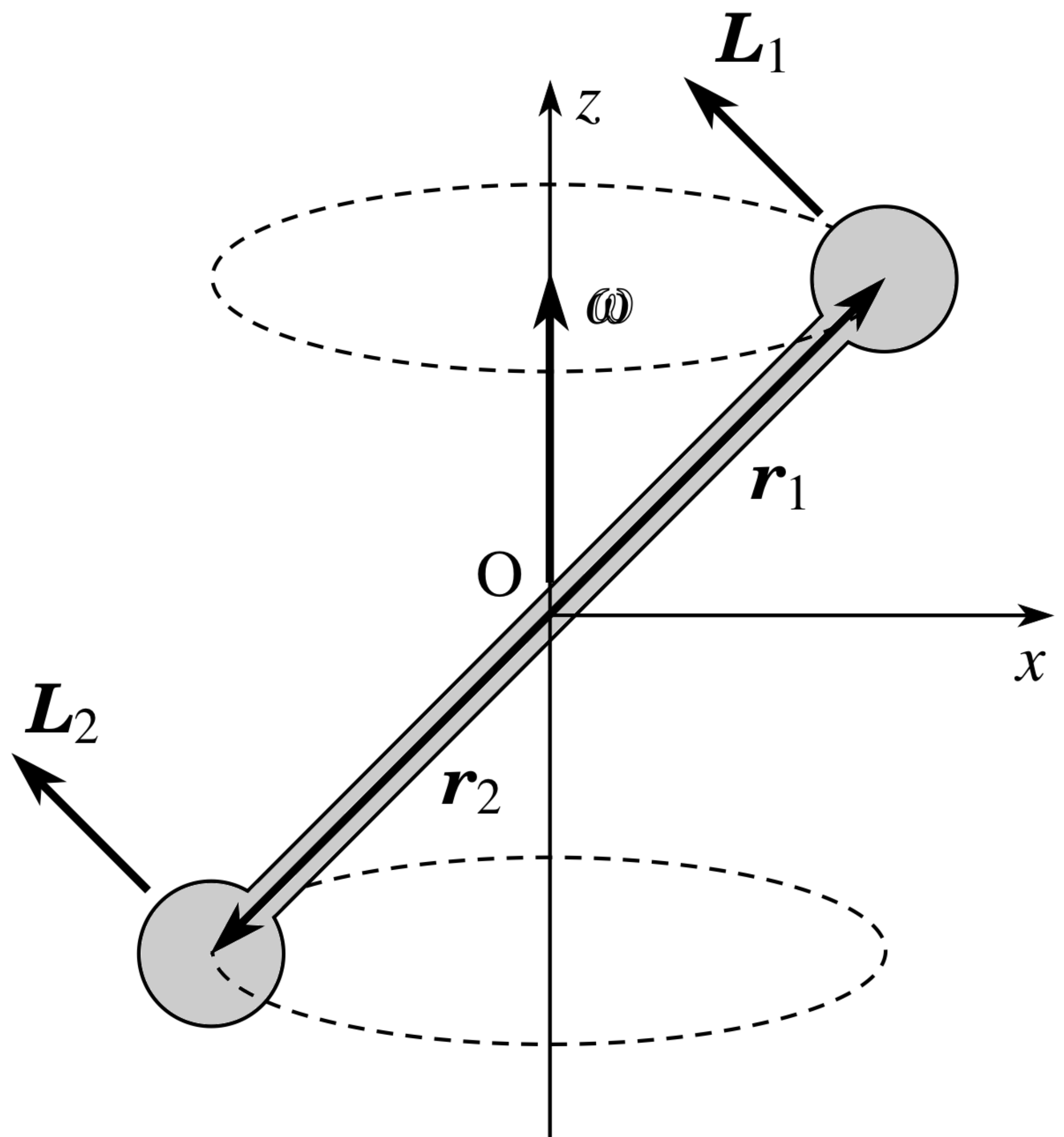



***Radius = b***
***Angular acceleration = α***

(a)Find torque about center of mass, point A and point of contact C.

(b)Prove that linear and angular acceleration determined about center of mass, point A and point of contact C are all identical.

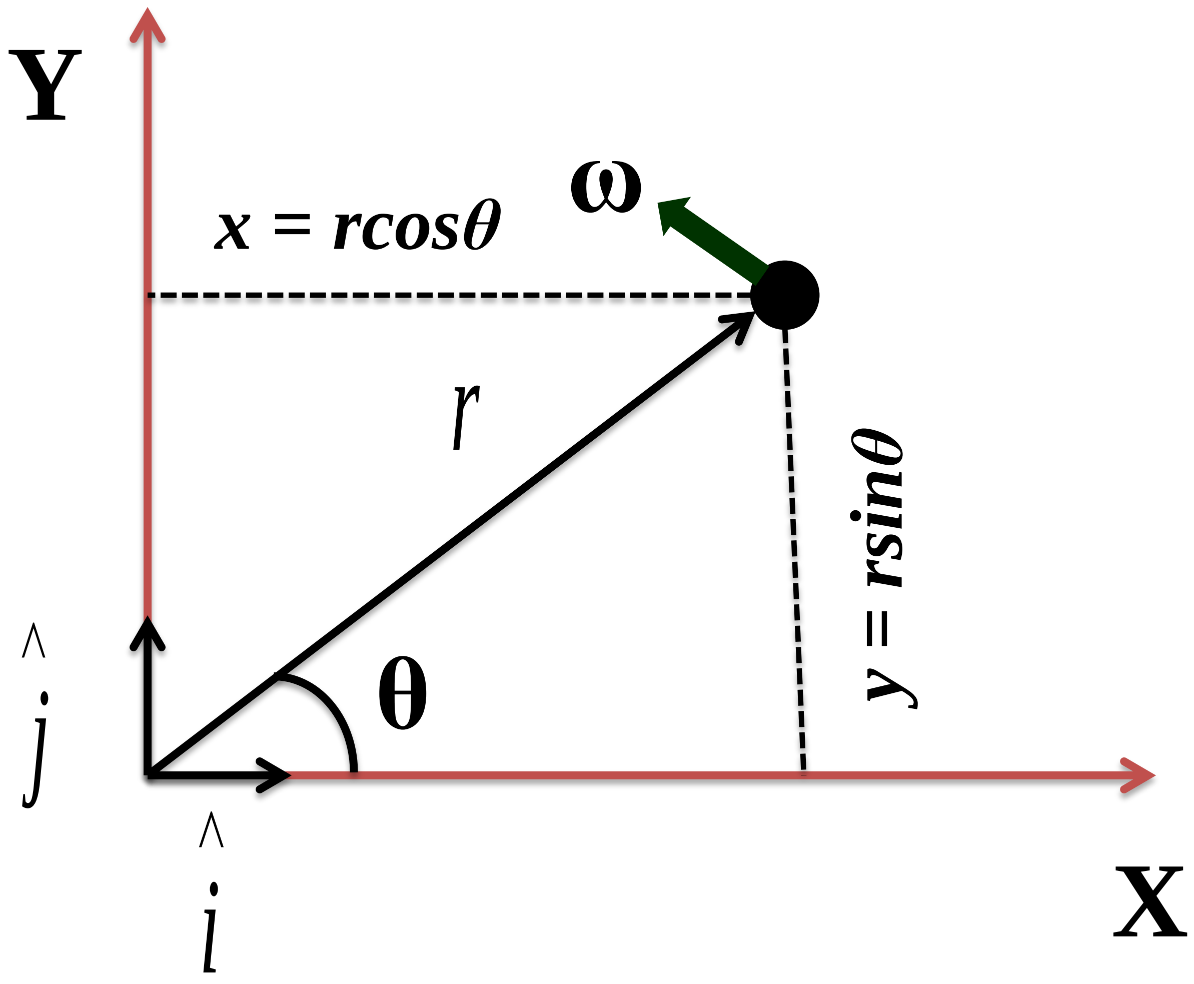
# Vector Nature of Angular Velocity and Momentum



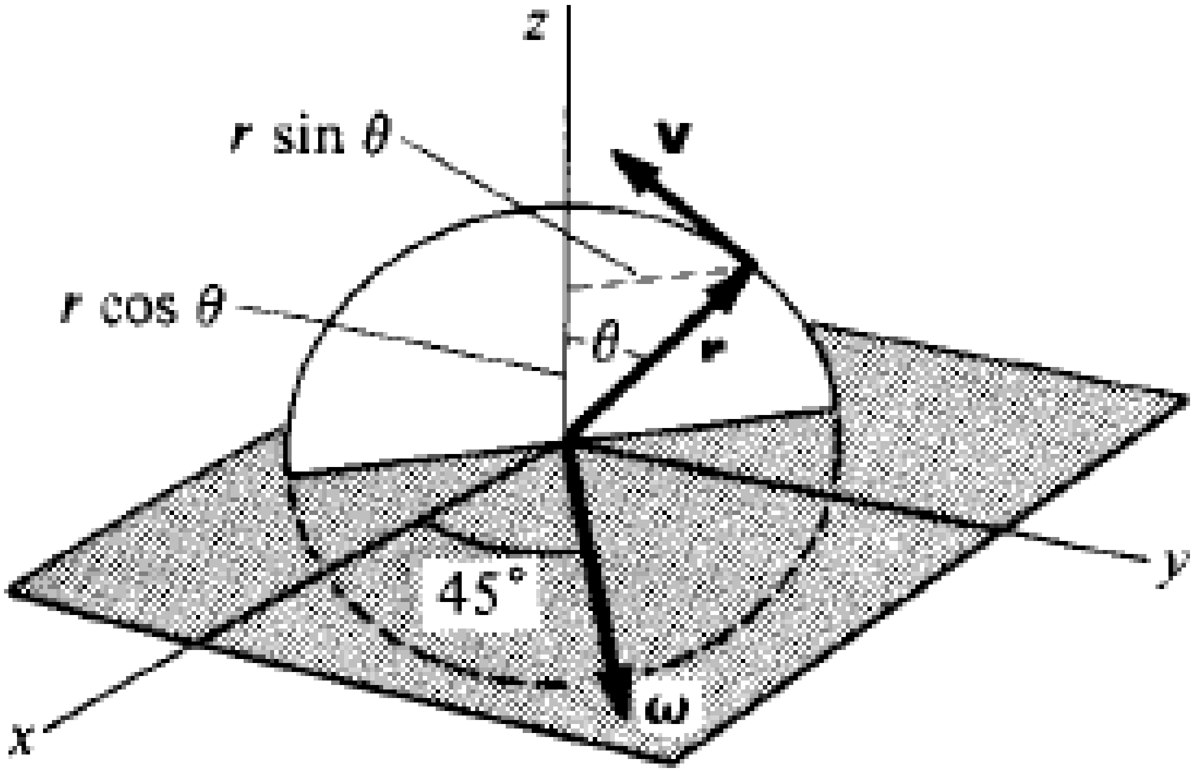
# Angular Position not a Vector?

Despite having direction and magnitude, angular displacement ($\theta$) is not a vector, Why?

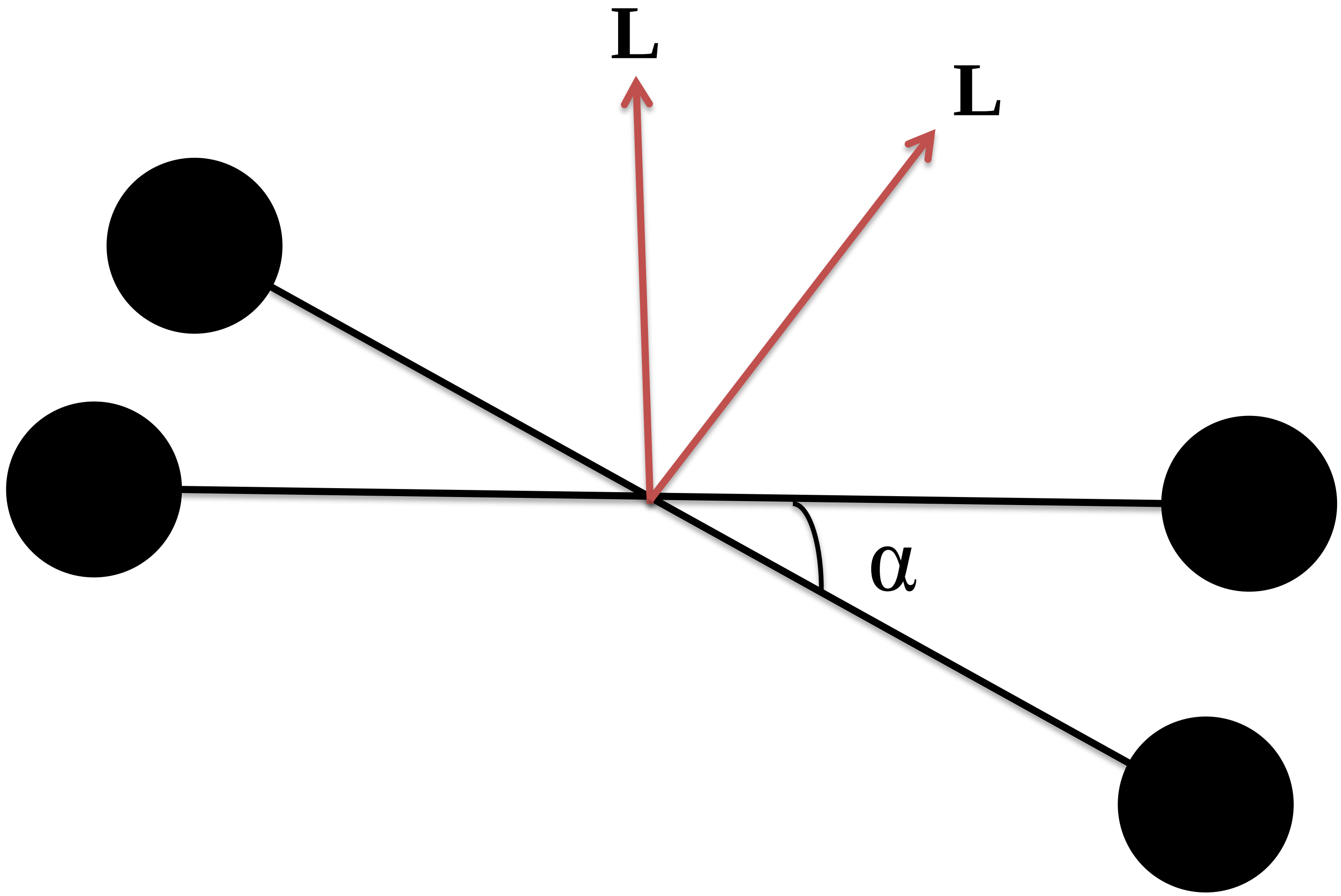
# Angular Velocity as vector



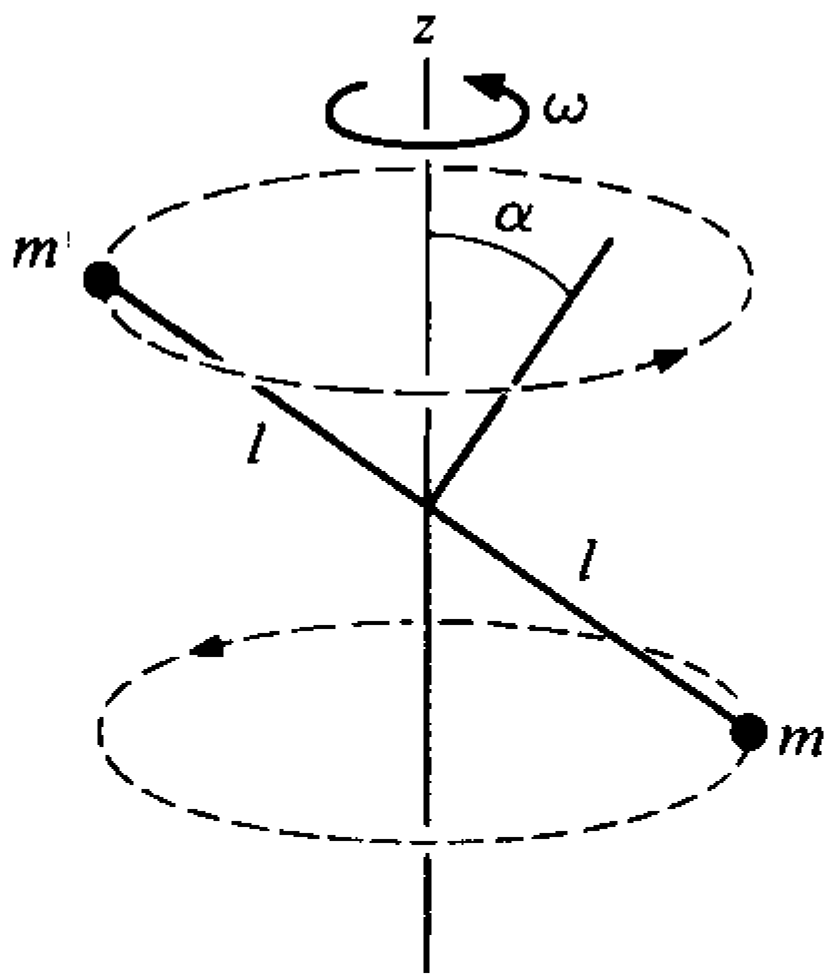
# Angular Velocity as Vector!



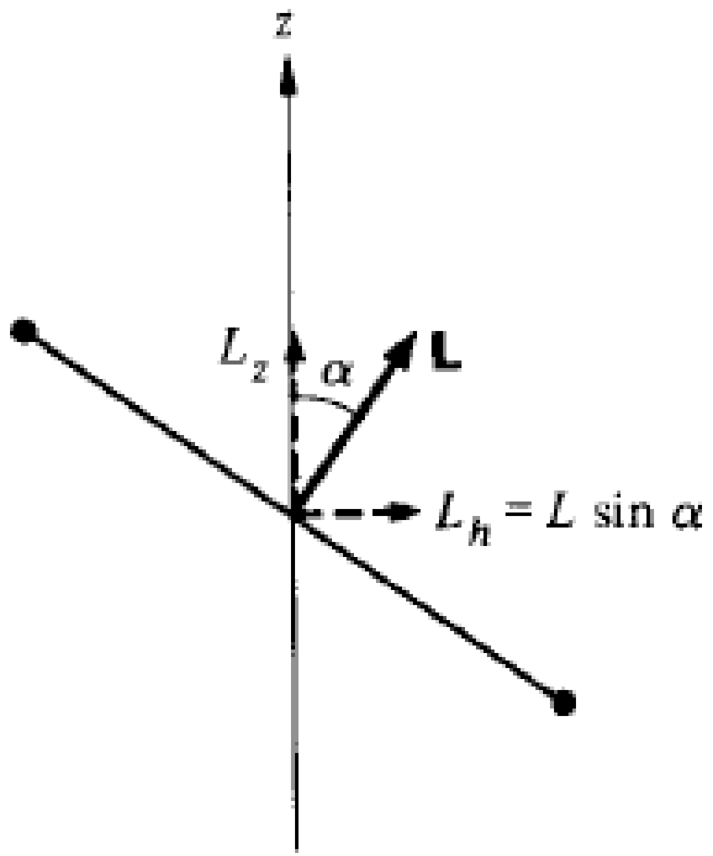
# Angular Momentum of a Rotating Skew Rod

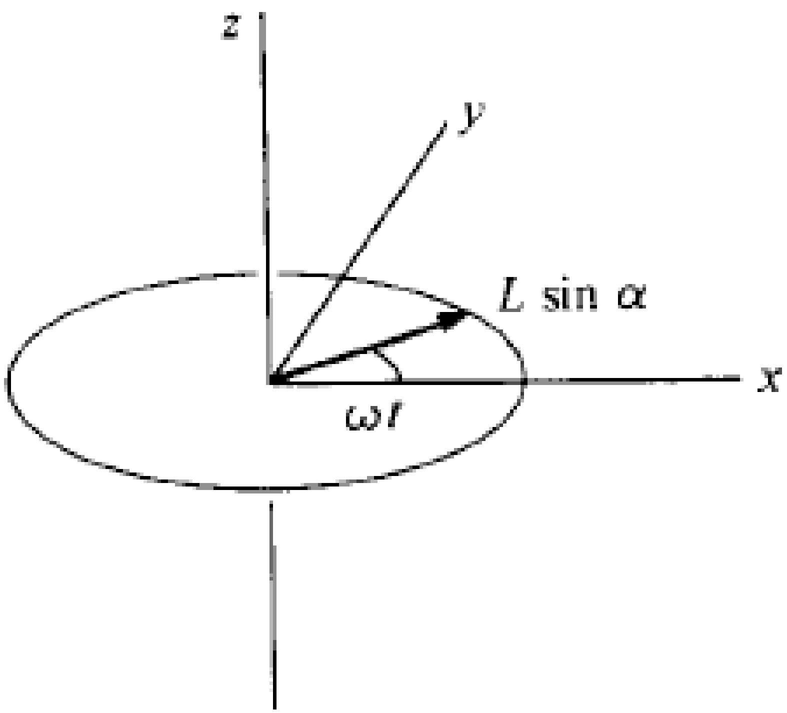


# Angular Momentum of a Rotating Skew Rod



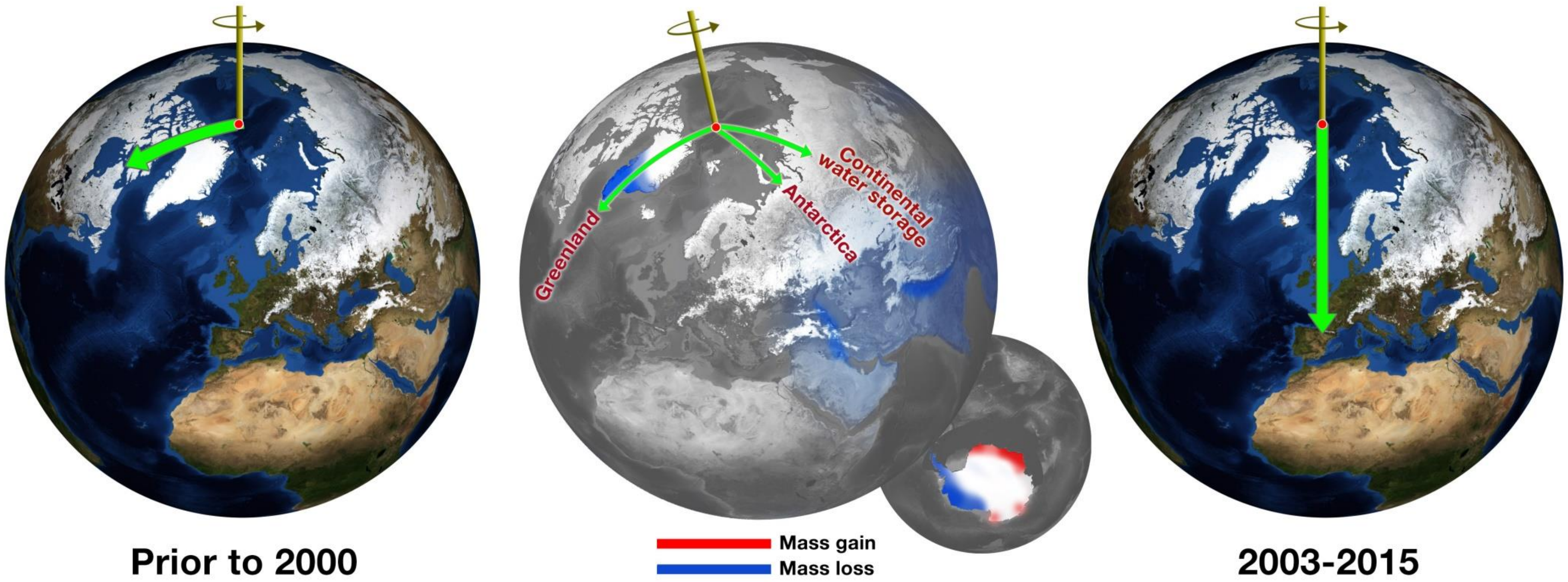
# Torque of a Rotating Skew Rod




**Torque is zero when $\alpha$ = 0 or $90^0$ why?**

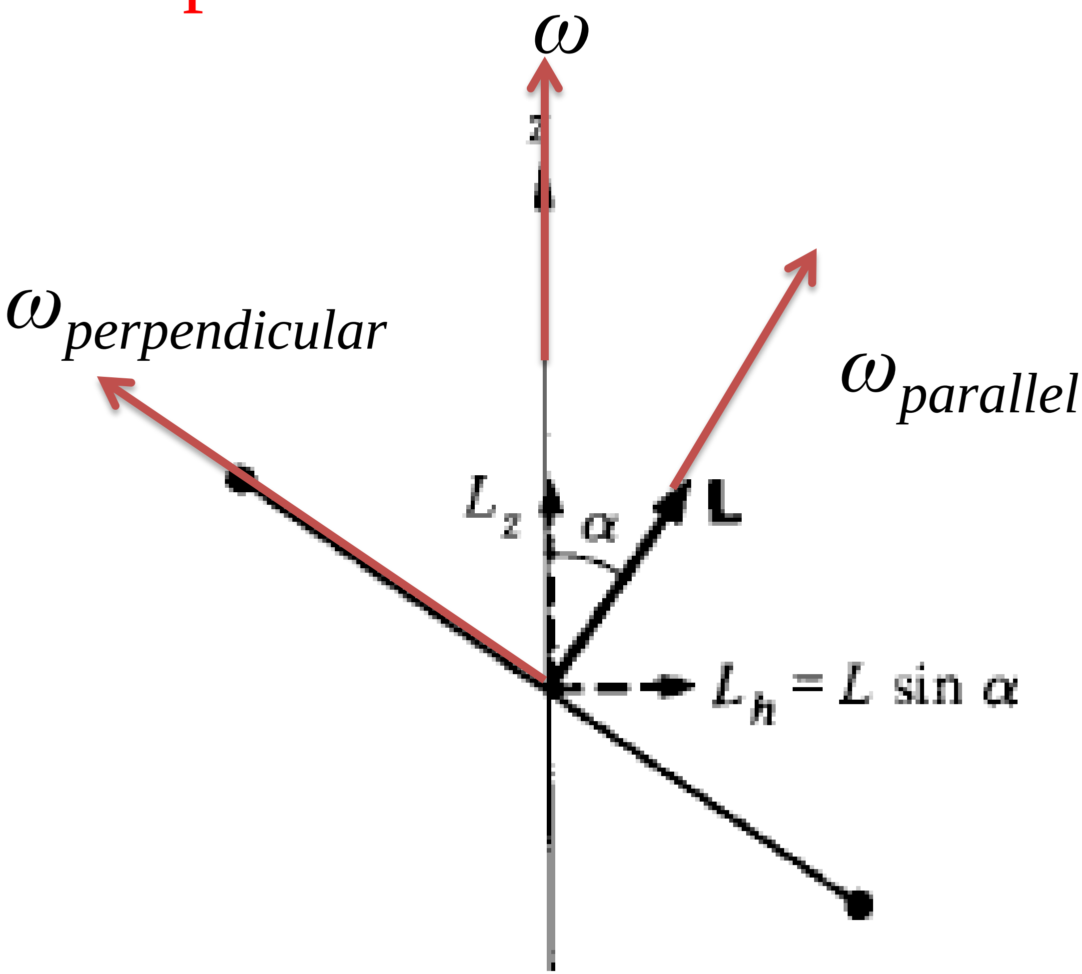
# Lecture 11 – Precession and Gryoscope



# How do racers make sharp turn without reducing speed?

# Concepts to remember

Angular Velocity and Angular Momentum may not point in the same direction always!



# Concepts to remember

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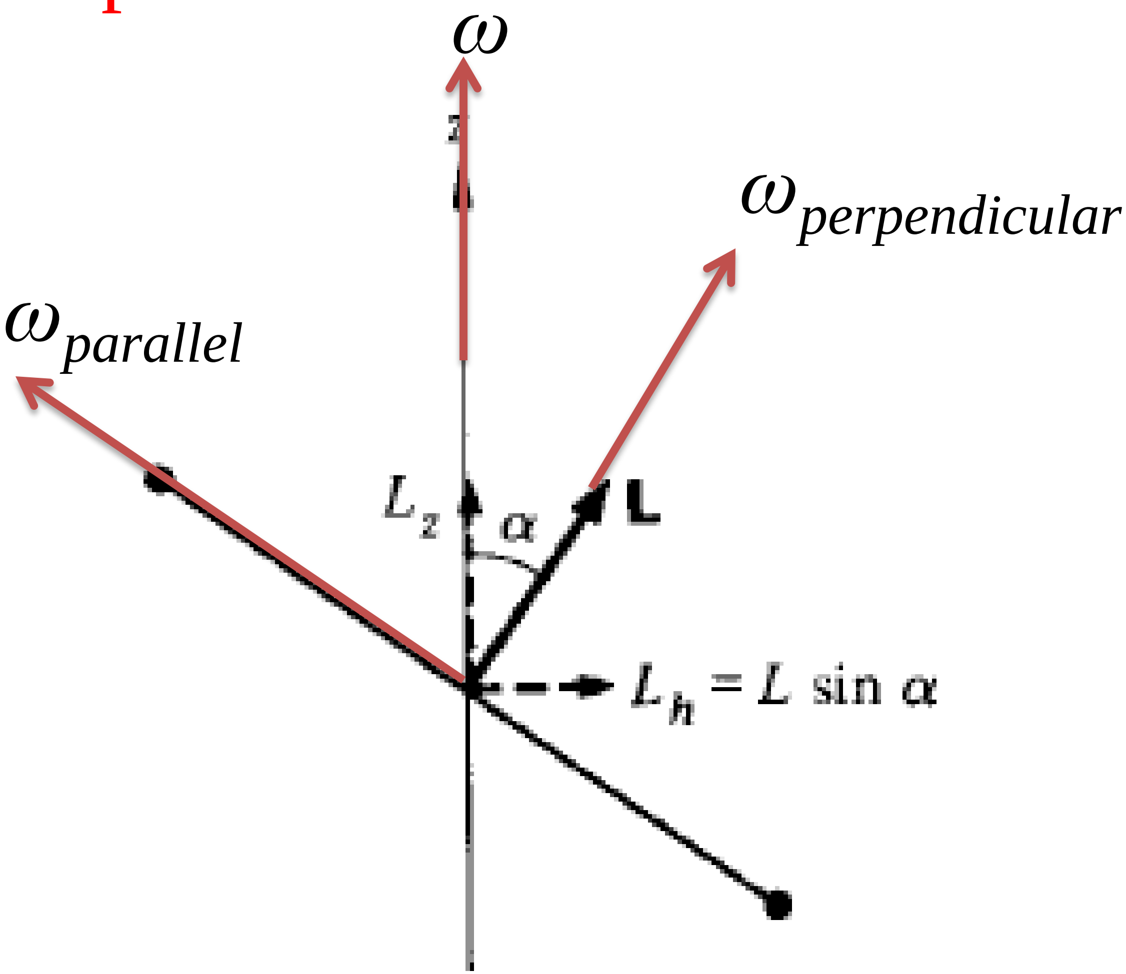
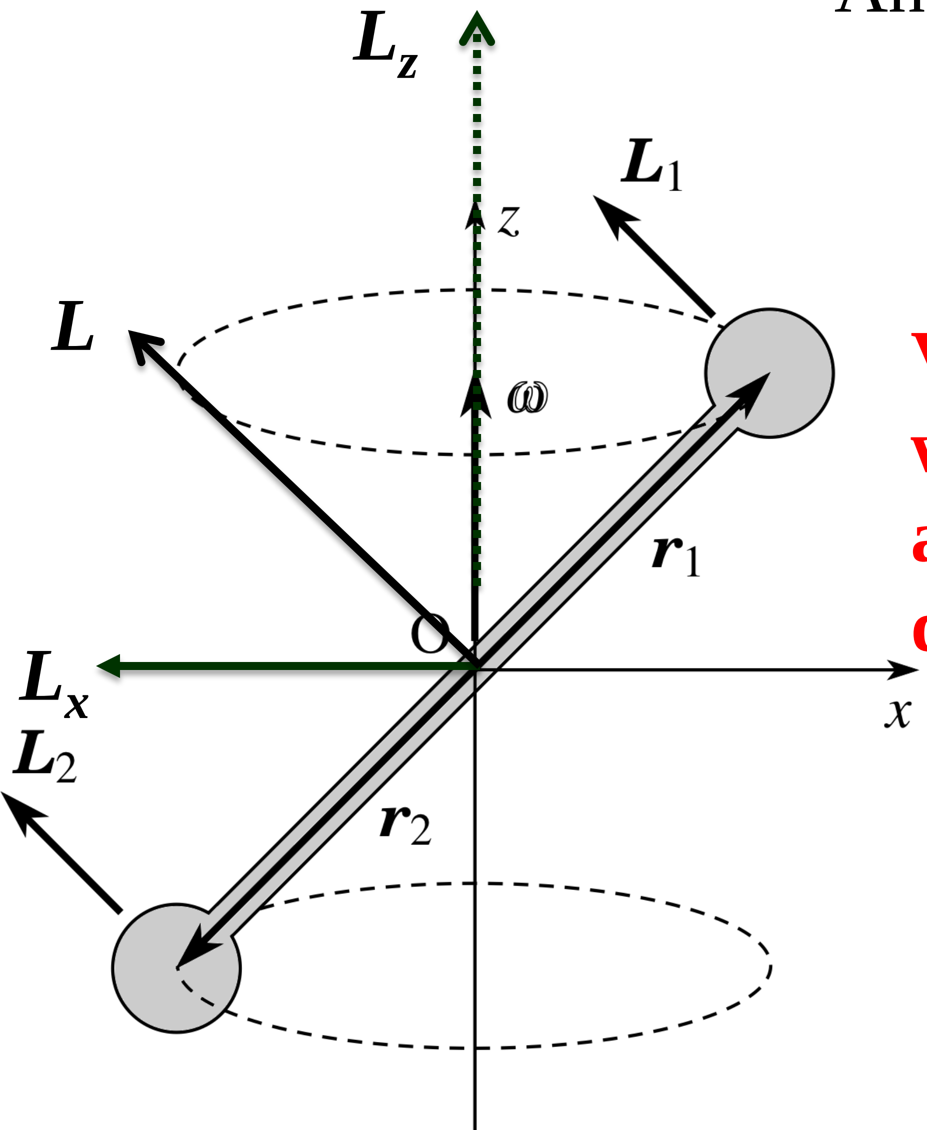


# Angular Momentum of a rotating skew rod

Angular Velocity point along $\hat{k}$



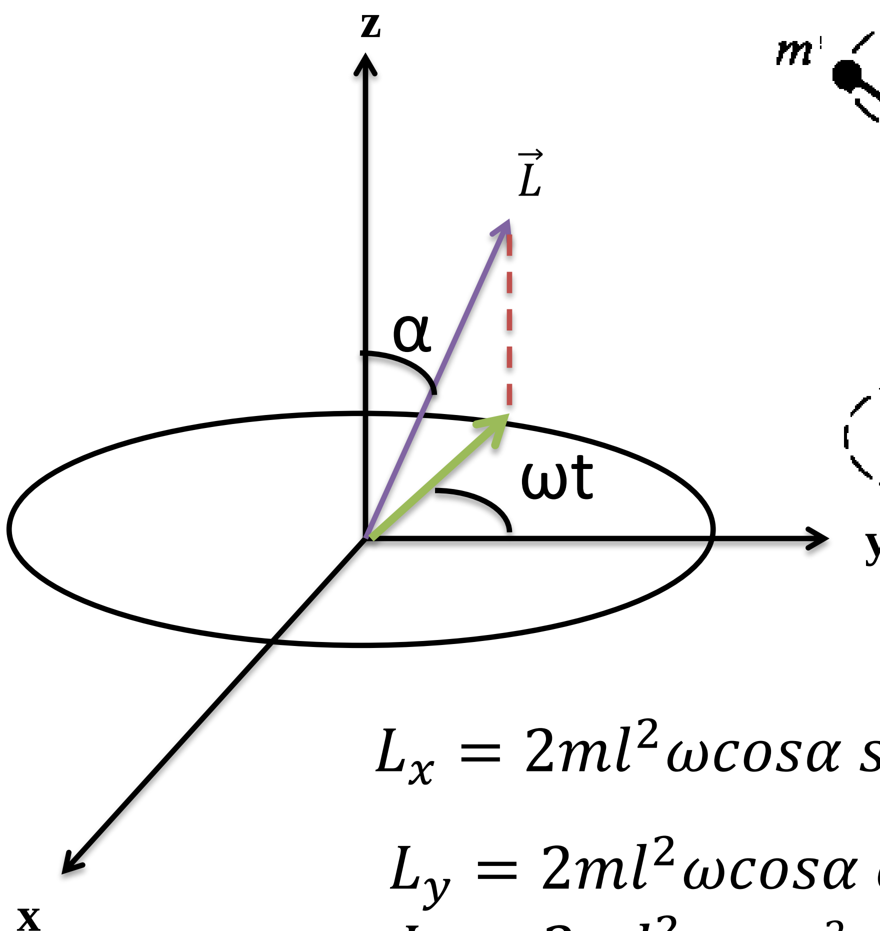

**Varying ω along one direction will impact the magnitude of angular momentum in other directions**

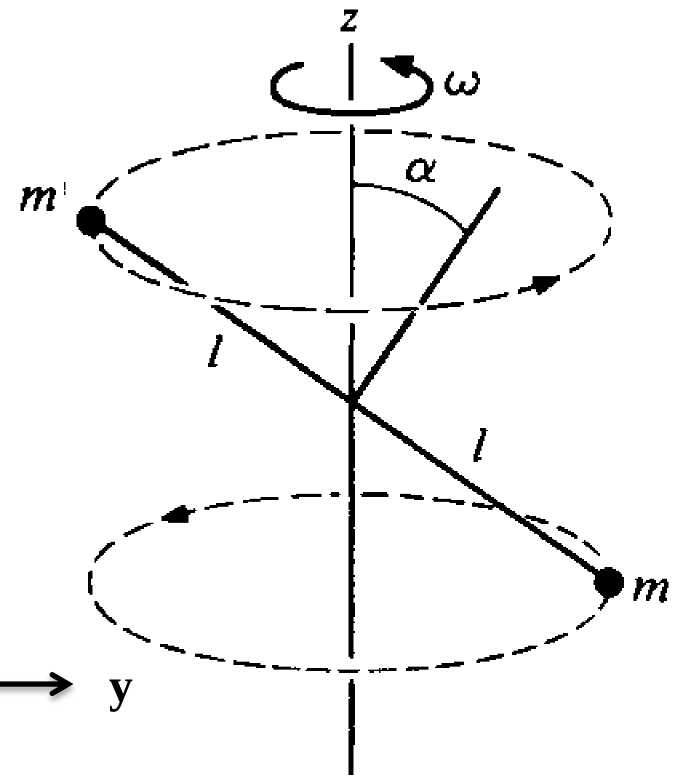
# Torque due on a rotating skew rod

$$\vec{L} = 2ml^2\omega cos\alpha(\hat{\imath} + \hat{\jmath} + \hat{k})$$

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

$$\tau = L\omega sin\alpha$$



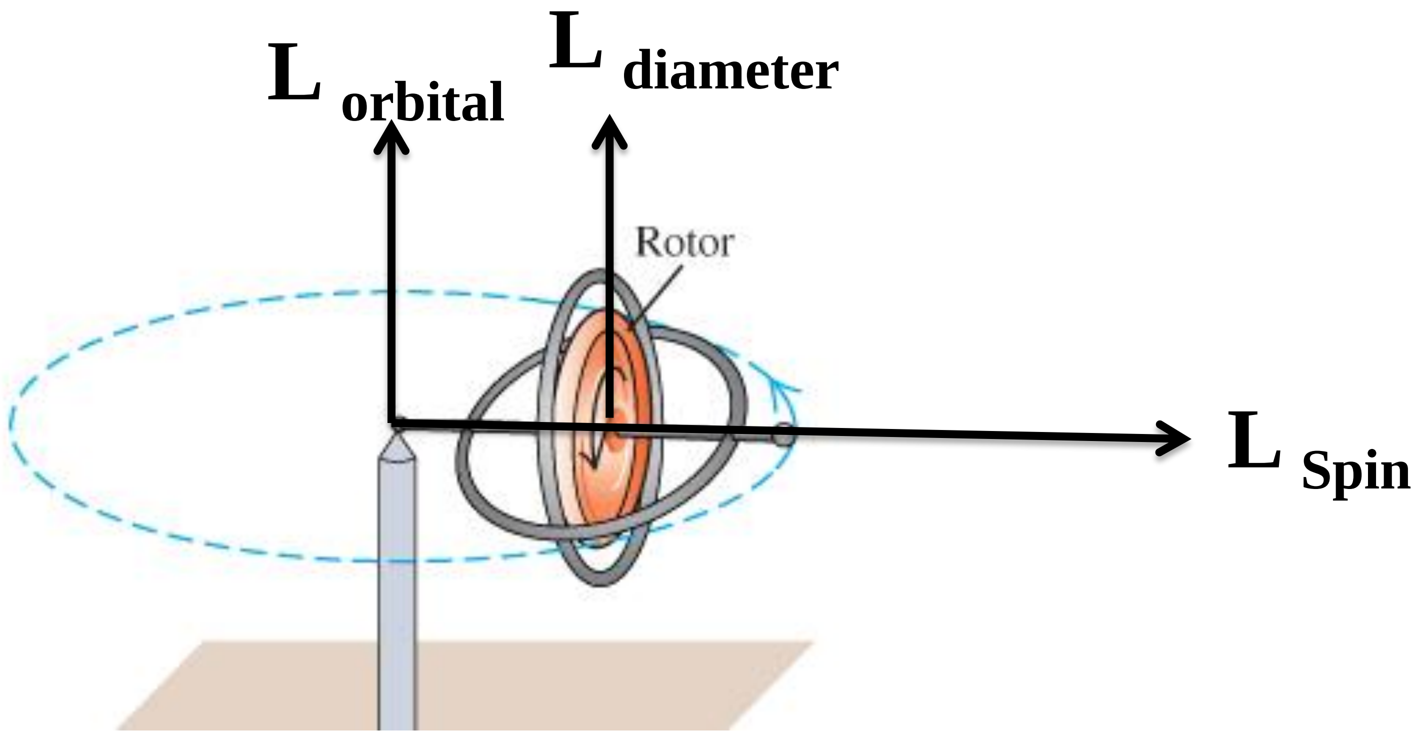

$$L_x = 2ml^2\omega cos\alpha \; sin\omega t$$

$$L_y = 2ml^2\omega cos\alpha \; cos\omega t$$

$$L_z = 2ml^2\omega cos^2\alpha$$

# Concepts to remember

An object moving around in a circle has an angular momentum component along its diameter



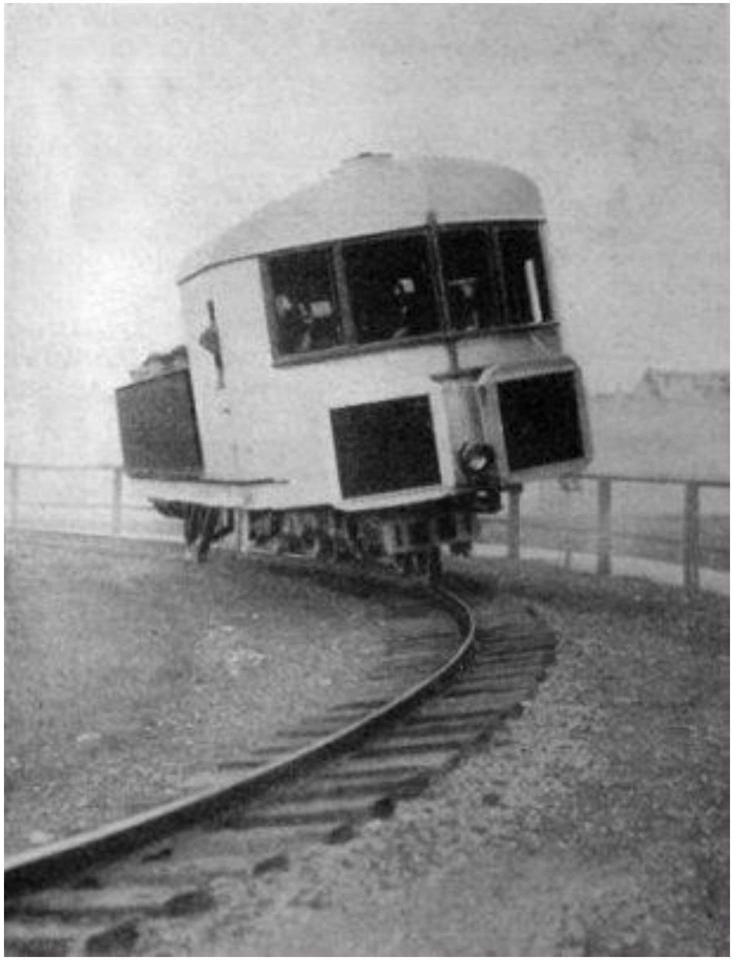

$$L_{total} = L_{spin} + L_{orbital} + L_{diameter}$$

# High Speed Trains

In high speed trains such as TGV (France), KTX (South Korea), Shinkansen (Japan), the turning radius is 7 Km which limits the deployment of these trains in all terrains.

Engineers claim that if these high speed trains are turned into mono-rail trains, this difficulty can be eliminated. Is it possible to really make a mono-rail train?

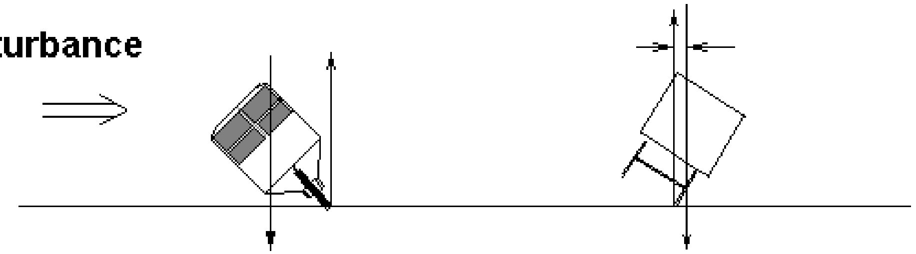
# World's First Mono-Rail Train



Monorail - Resisting Moment Increases

Two Track- Resisting Moment Decreases

Disturbance

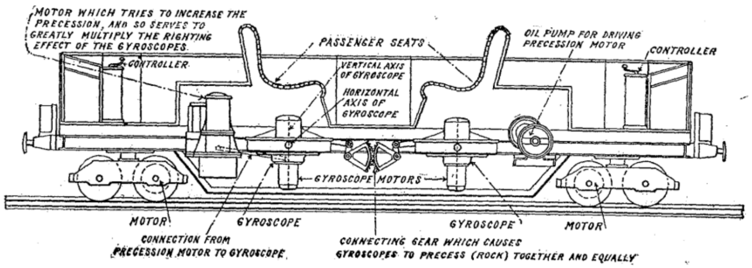


Leans Progressively into disturbance

Once Toppling starts - cannot recover

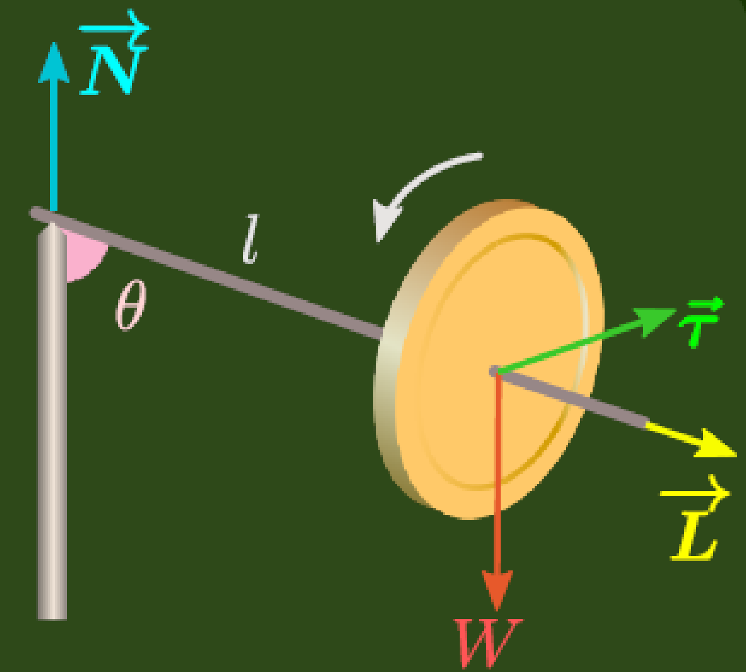
Early warning of danger

No Warning



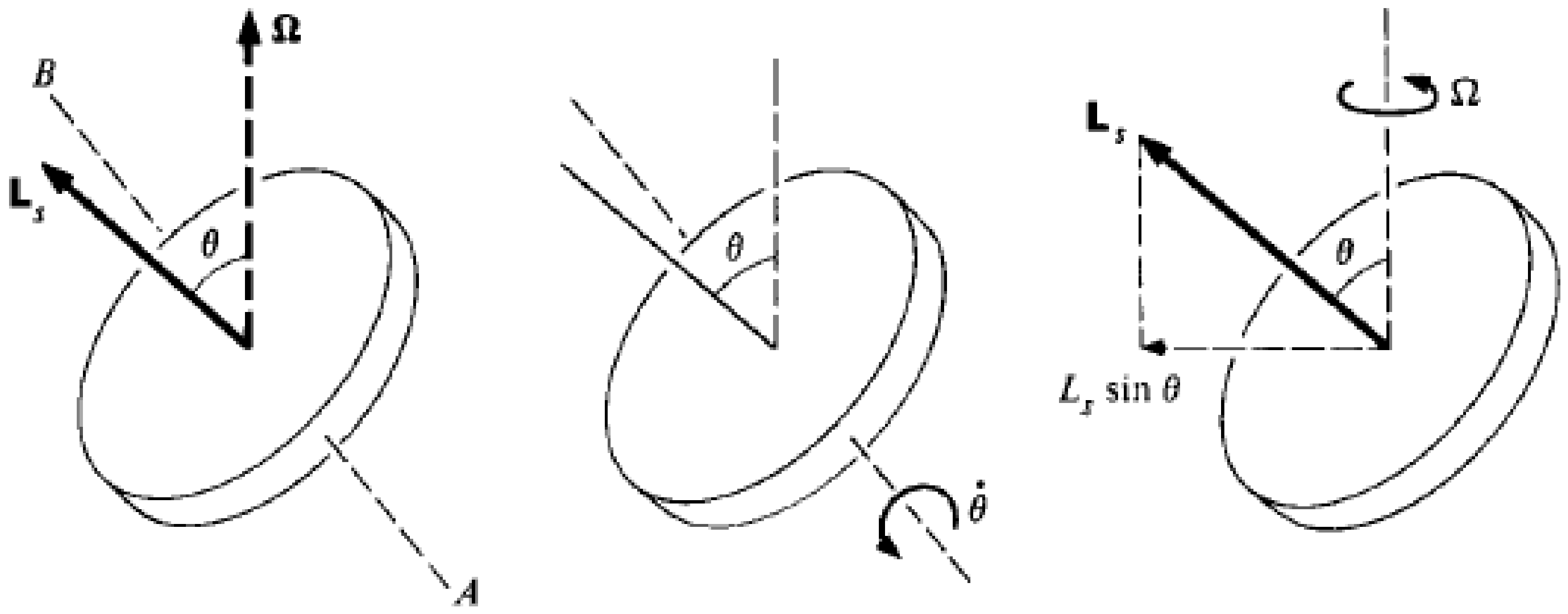
# Gyroscope pivoted at one end

Spinning Gyroscope does not fall but precesses about the $z$-axis at a constant rate



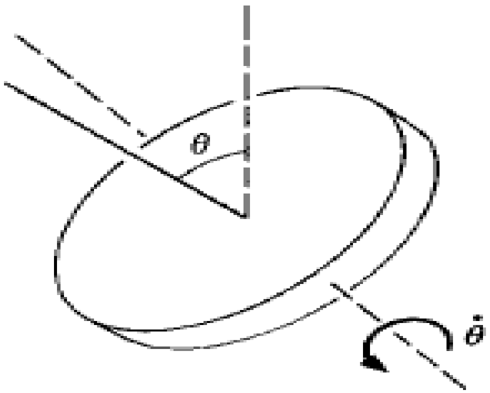

$$
\begin{aligned}
\vec{\tau} &= \vec{r} \times \vec{W} \\
|\vec{\tau}| &= W l \sin\theta \\
\vec{\tau} &= \frac{d\vec{L}}{dt} \perp \vec{L} \\
\frac{d\vec{L}}{dt} &= L \sin\theta \frac{d\phi}{dt} = L\sin\theta(\Omega) \\
& (\perp \vec{L})
\end{aligned}
$$

# Gyro-Oscillation

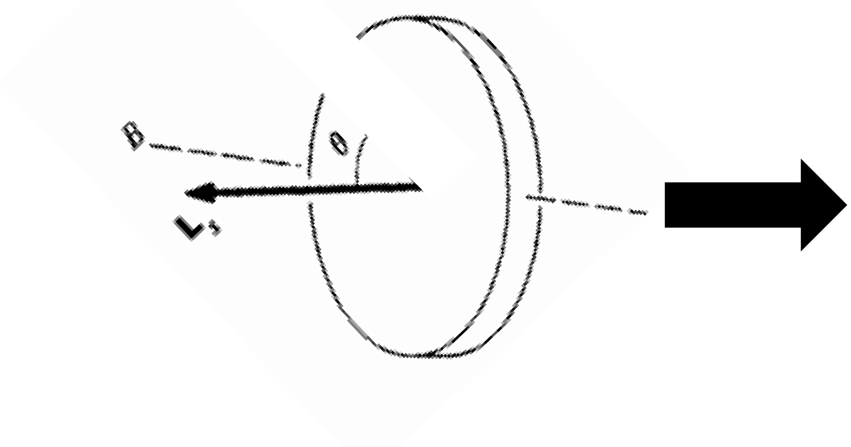


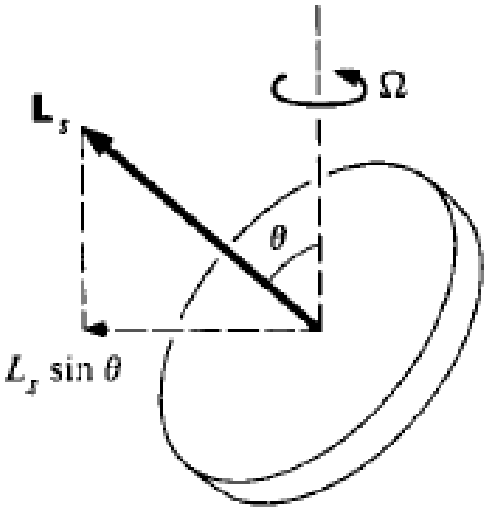
# Gyro-Oscillation

Angular momentum change about the axis arises due to two factors




1. Due to angular rotation of the gyro
2. Due to change in the magnitude of $L_s$



# Gyro-Oscillation

Torque due to rotation $= I\ddot{\theta}$

Torque due to change in $L_s$ $= L_s \sin\theta\Omega$

$$\tau = I\ddot{\theta} + L_s \sin\theta\Omega = 0$$

# Gyro-Oscillation

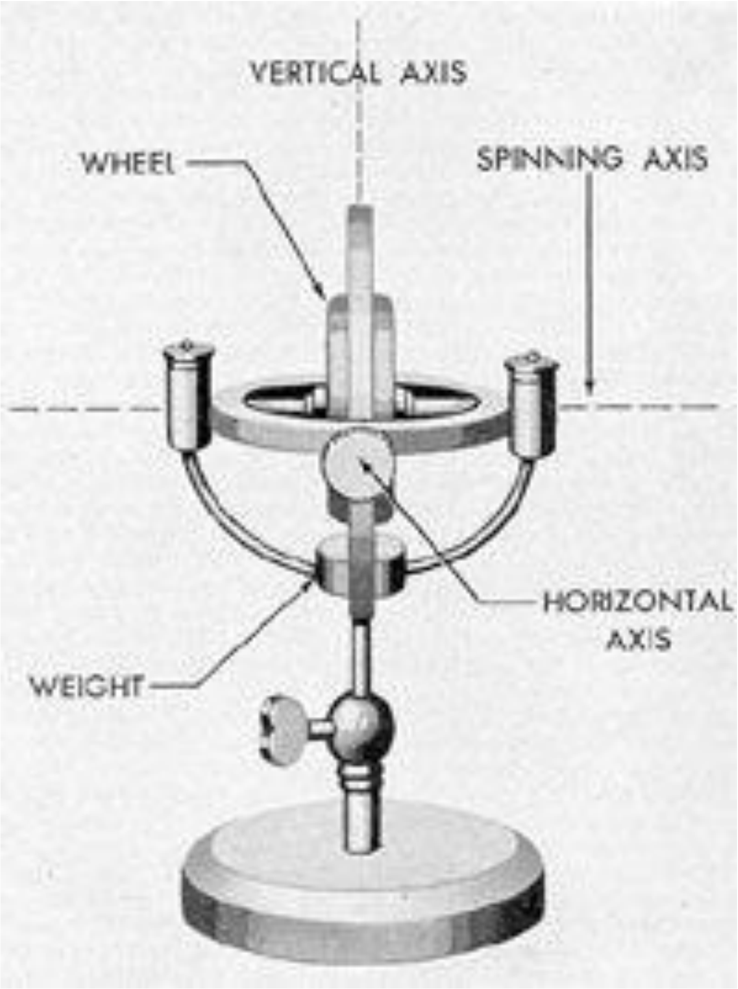
$$\tau = I\ddot{\theta} + L_s \sin\theta\Omega = 0$$

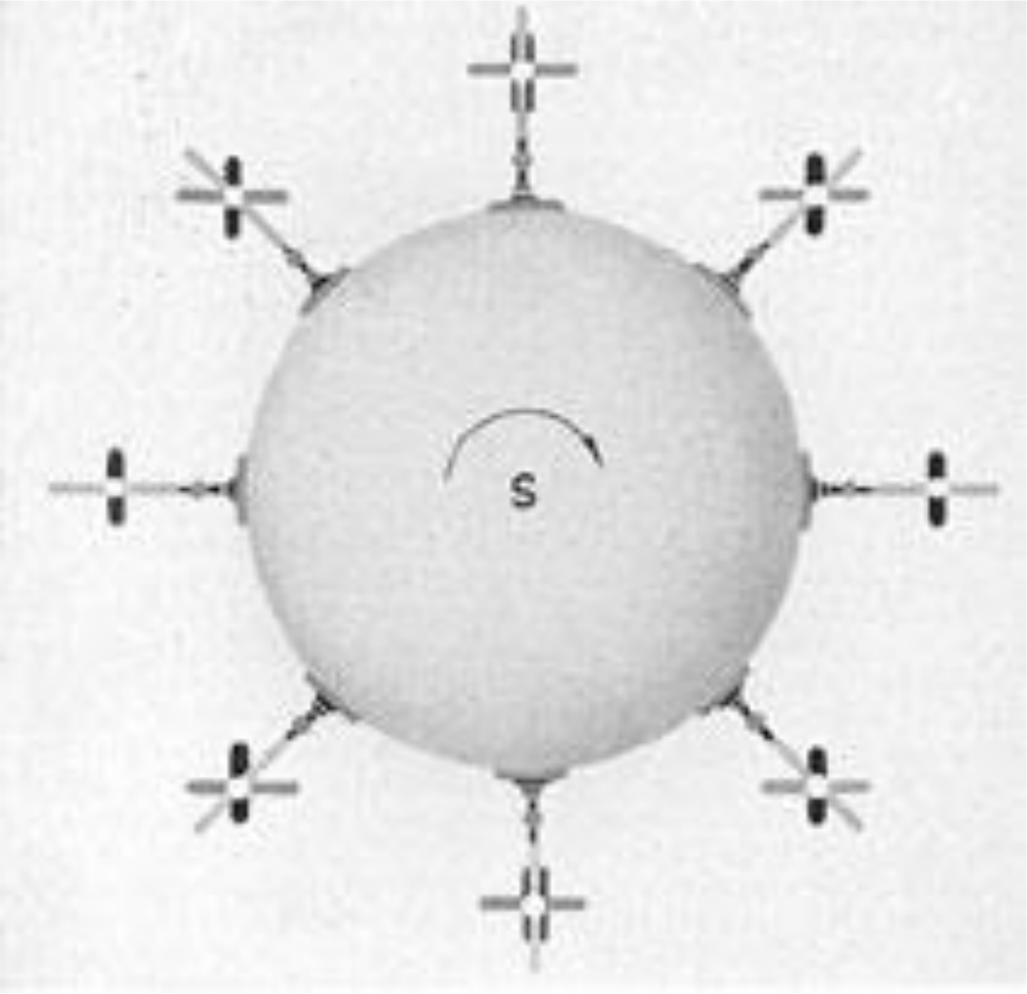
$$I\ddot{\theta} + L_s\theta\Omega = 0$$

$$\ddot{\theta} + \frac{L_s\Omega}{I}\theta = 0$$

$$\omega = \sqrt{\frac{L_s\Omega}{I}}$$

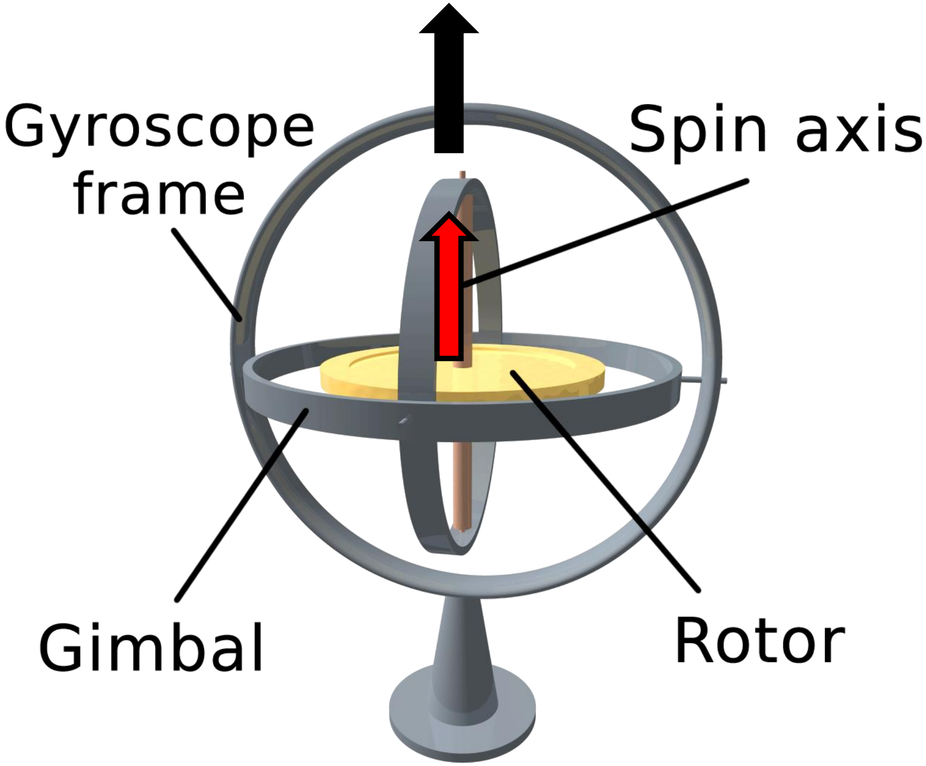
# Gyro-Compass



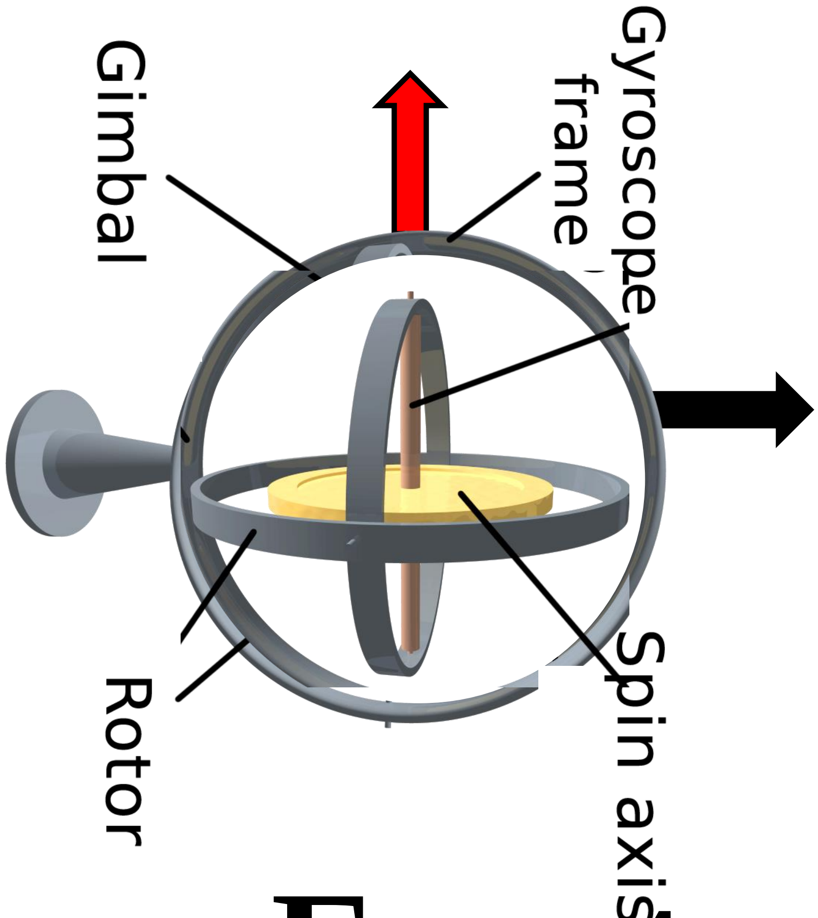



# Gyro-Compass

Spin of gyro maintains its orientation

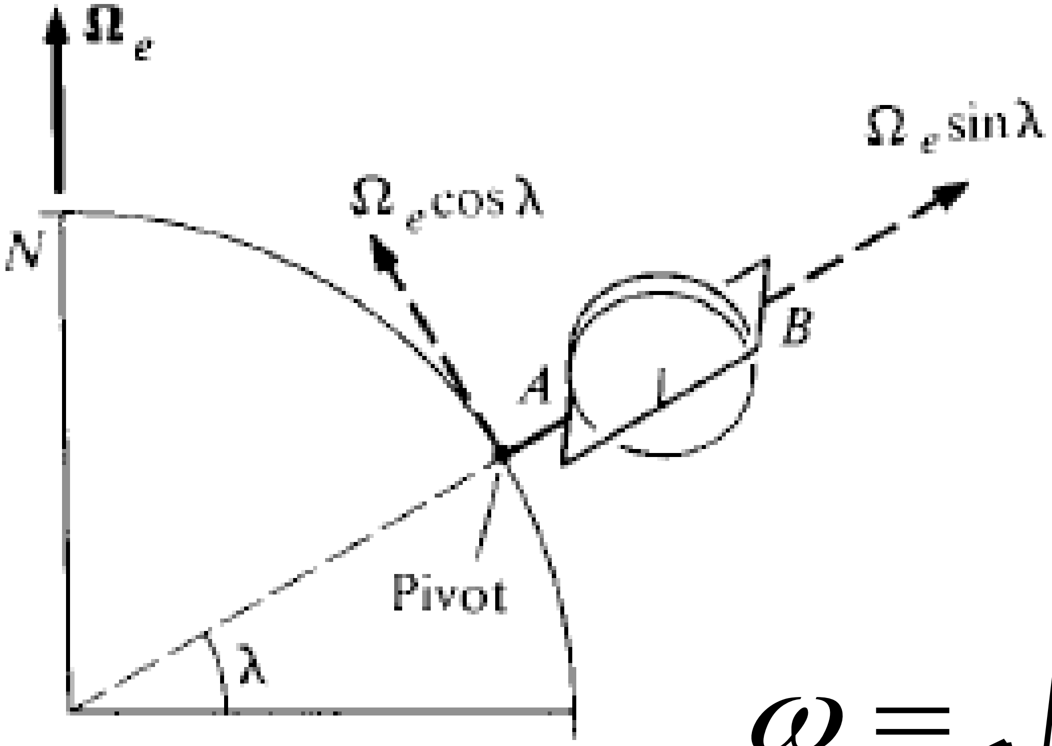



**Pole**



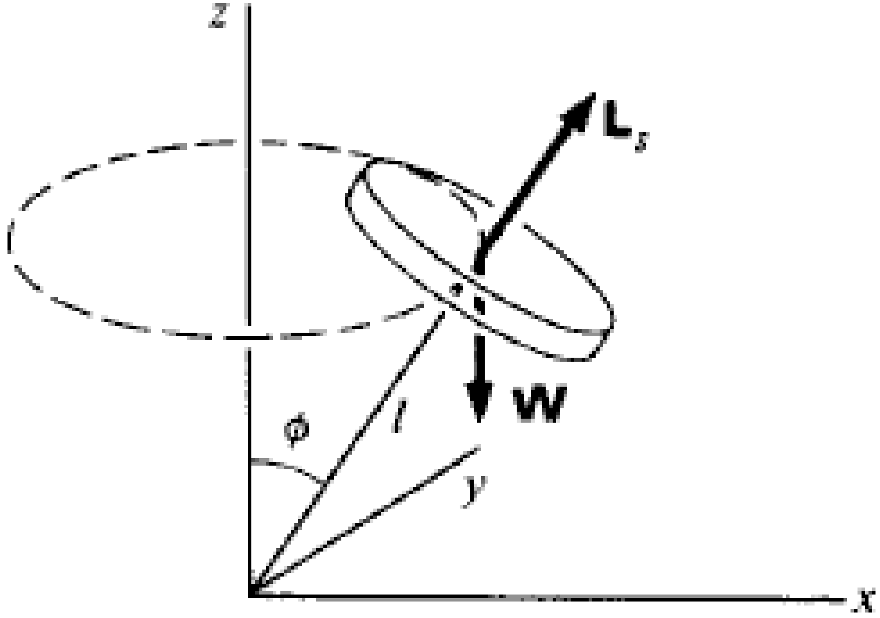

**Equator**

# Gyro-Compass




$$\omega = \sqrt{\frac{L_s \Omega_e \cos\lambda}{I}}$$

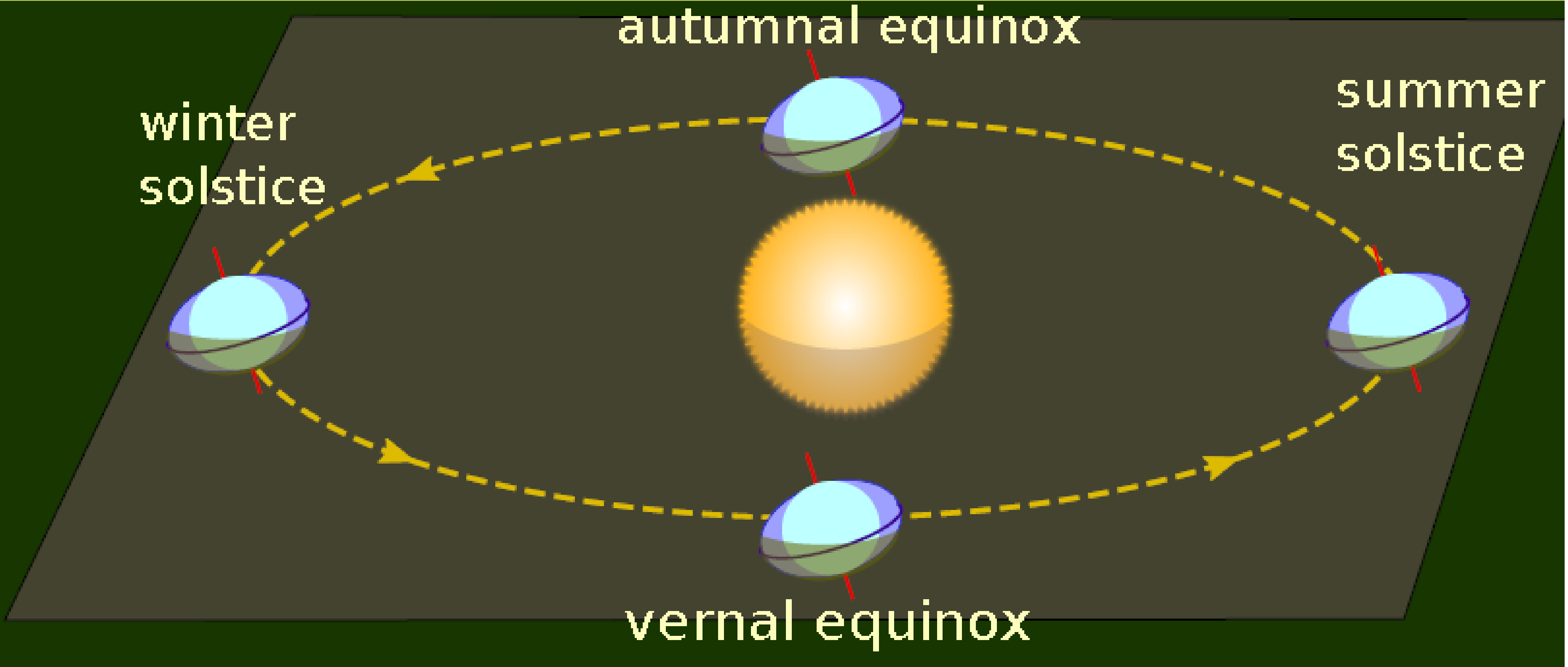
# Gyro-Precession




[Please see page 298 for solution]

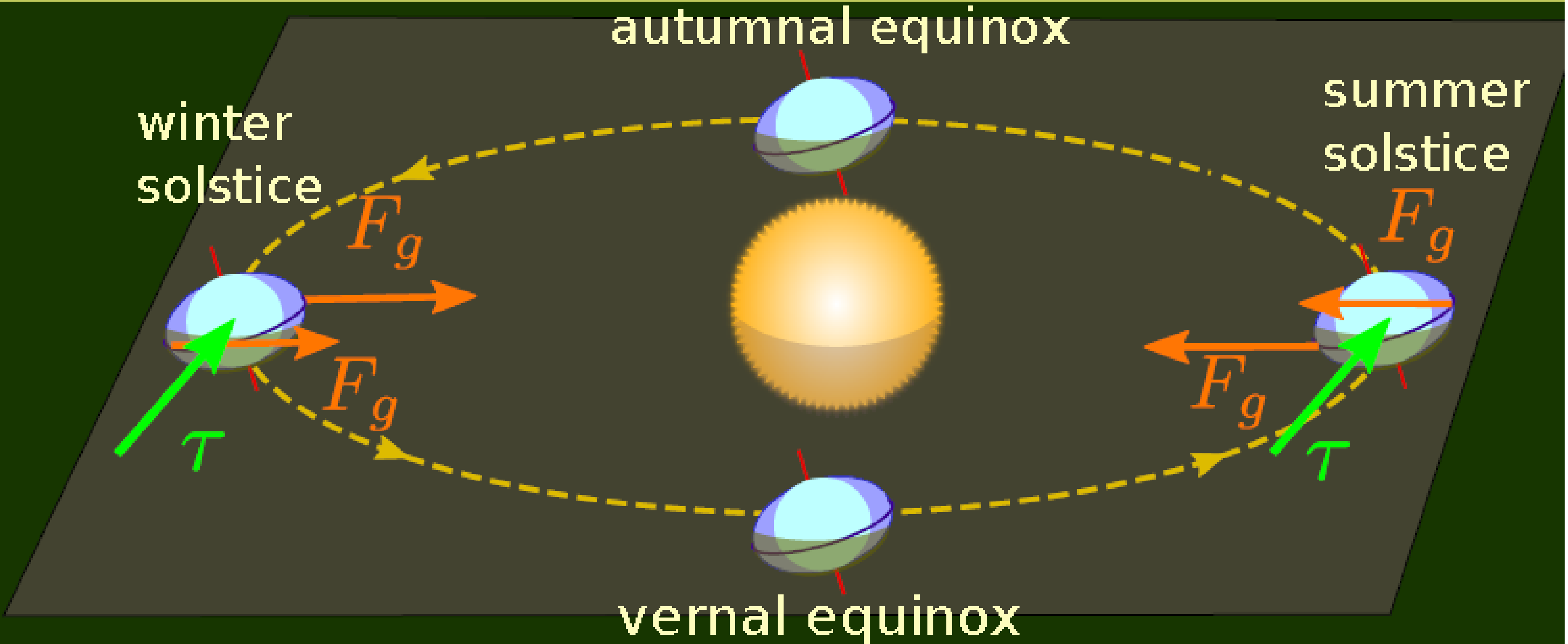
What happens to precession frequency on increasing φ?

# Precession of the Equinoxes



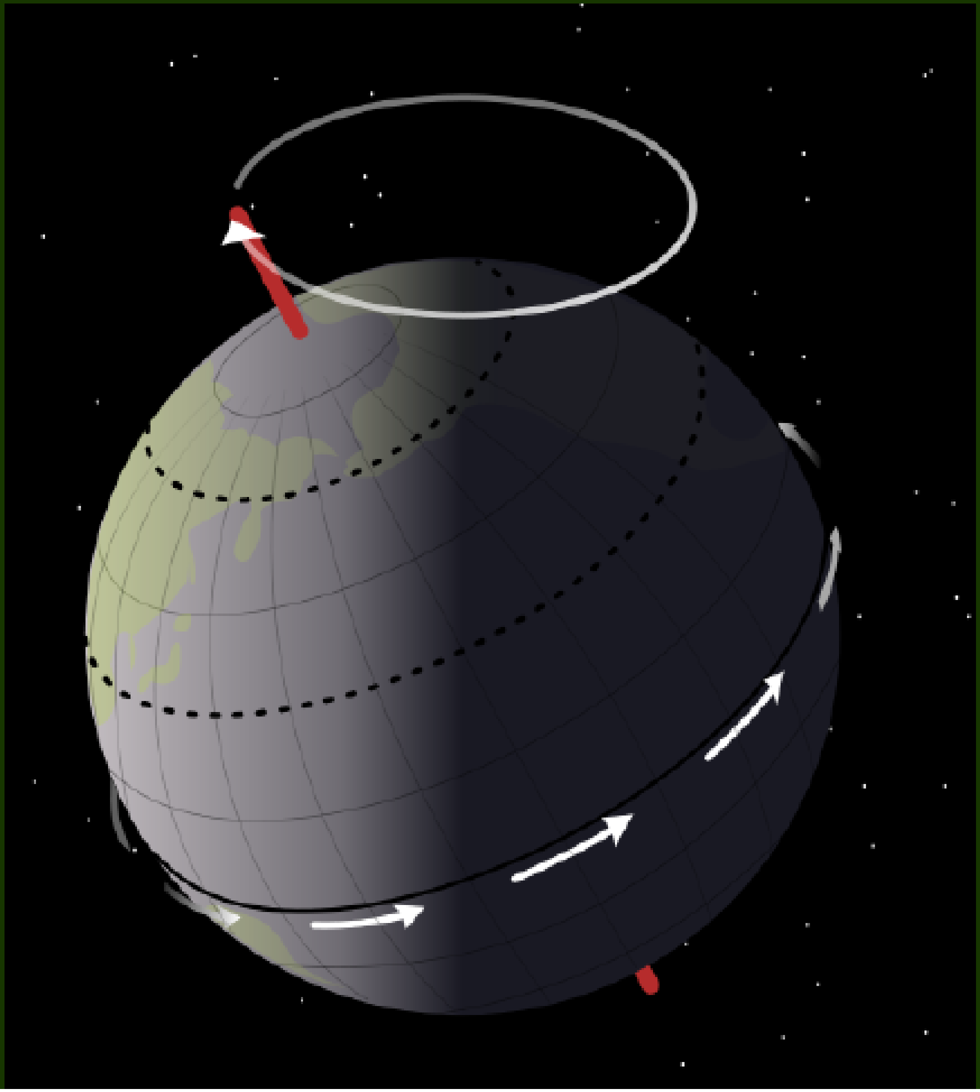

- Earth nonspherical: 21km bulge at equator
- Earth's spin axis tilted at 23.5° from ecliptic pole
- Net torque (about centre of earth) due to sun

# Precession of the Equinoxes




- Earth nonspherical: 21km bulge at equator
- Earth's spin axis tilted at 23.5° from ecliptic pole
- Net torque (about centre of earth) due to sun
- Torque maximum in winter & summer

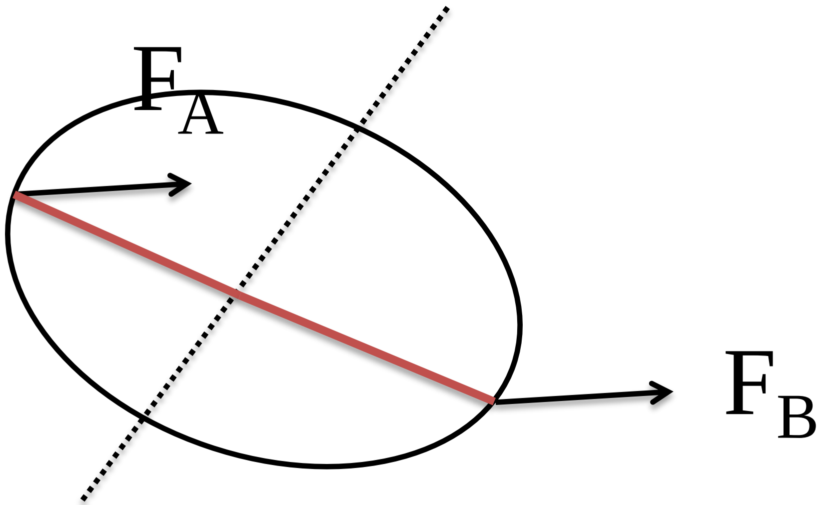
# Precession of the Equinoxes

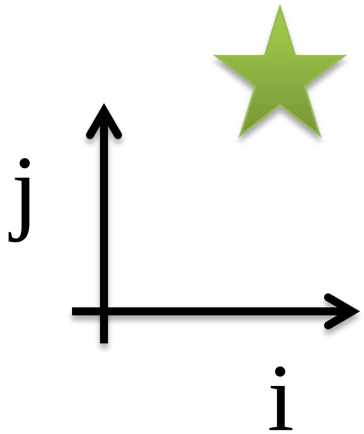


- Earth's axis precesses with period of 26,000 yrs.

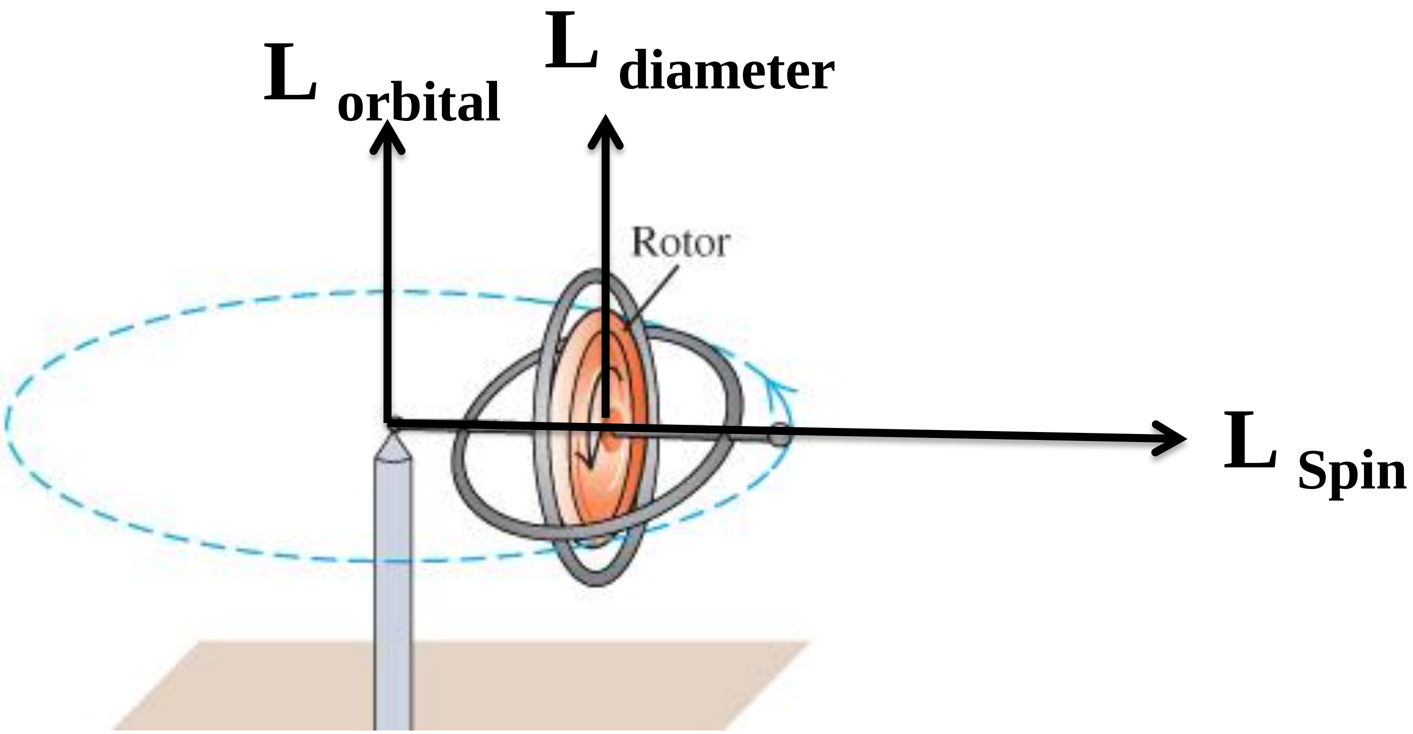
# Precession of Equinoxes

Please refer page 300 for detailss



# Lecture 12 – Physics of Gyroscope




$$L_{total} = L_{spin} + L_{orbital} + L_{diameter}$$

# Gyroscope pivoted at one end

Spinning Gyroscope does not fall but precesses about the $z$-axis at a constant rate

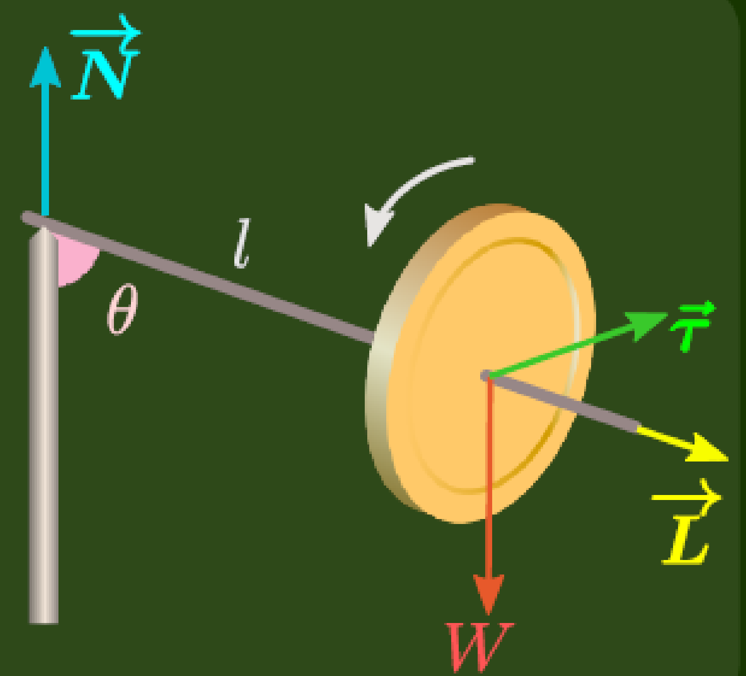
$$\vec{\tau} = \vec{r} \times \vec{W}$$

$$|\vec{\tau}| = W l \sin\theta$$

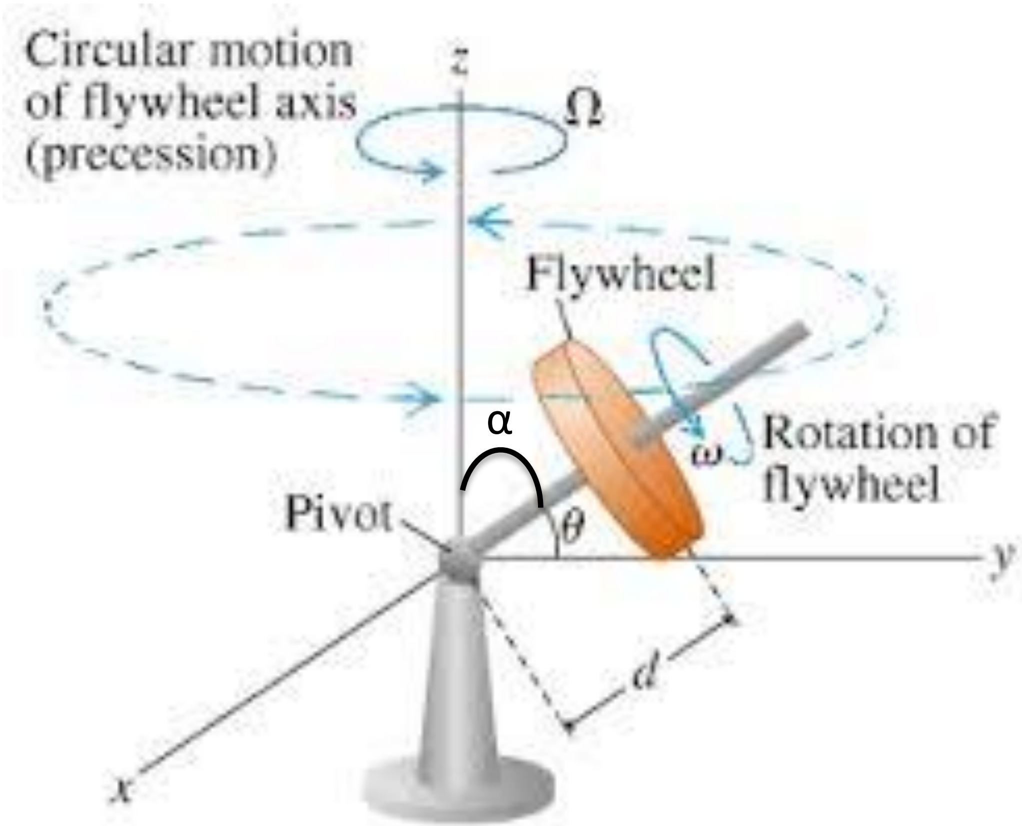
$$\vec{\tau} = \frac{d\vec{L}}{dt} \perp \vec{L}$$

$$\frac{d\vec{L}}{dt} = L\sin\theta\frac{d\phi}{dt} = L\sin\theta(\Omega)$$

$$(\perp \vec{L})$$

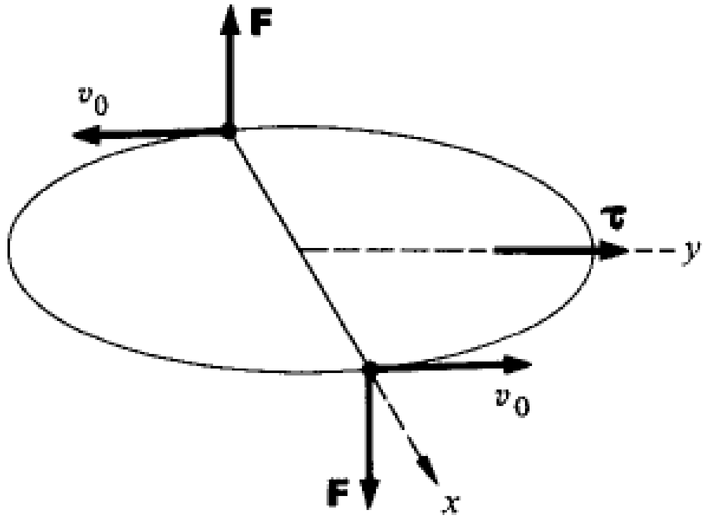


# Gyro-Precession Frequency

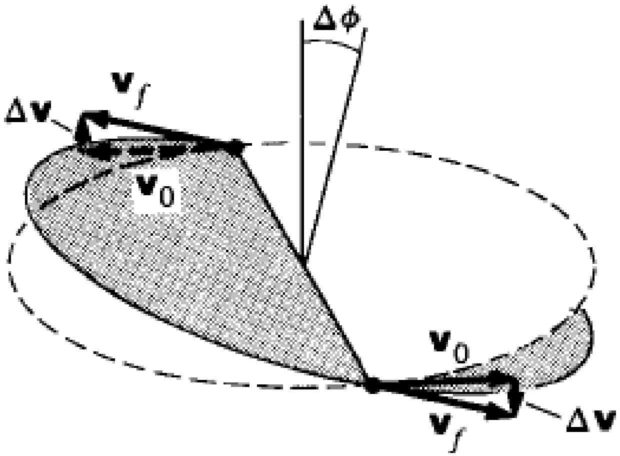



**The precession frequency of the gyroscope is independent of the angle α it makes with the precession axis.**

# An alternate mathematical treatment about Gyroscope Precessesiom




$$\Delta\phi = \frac{F\,\Delta t}{mv_0} = \frac{2lF\,\Delta t}{2lmv_0} = \frac{\tau\,\Delta t}{L_s}.$$




The rate of precession while the torque is acting is therefore

$$\Omega = \frac{\Delta\phi}{\Delta t} = \frac{\tau}{L_s},$$

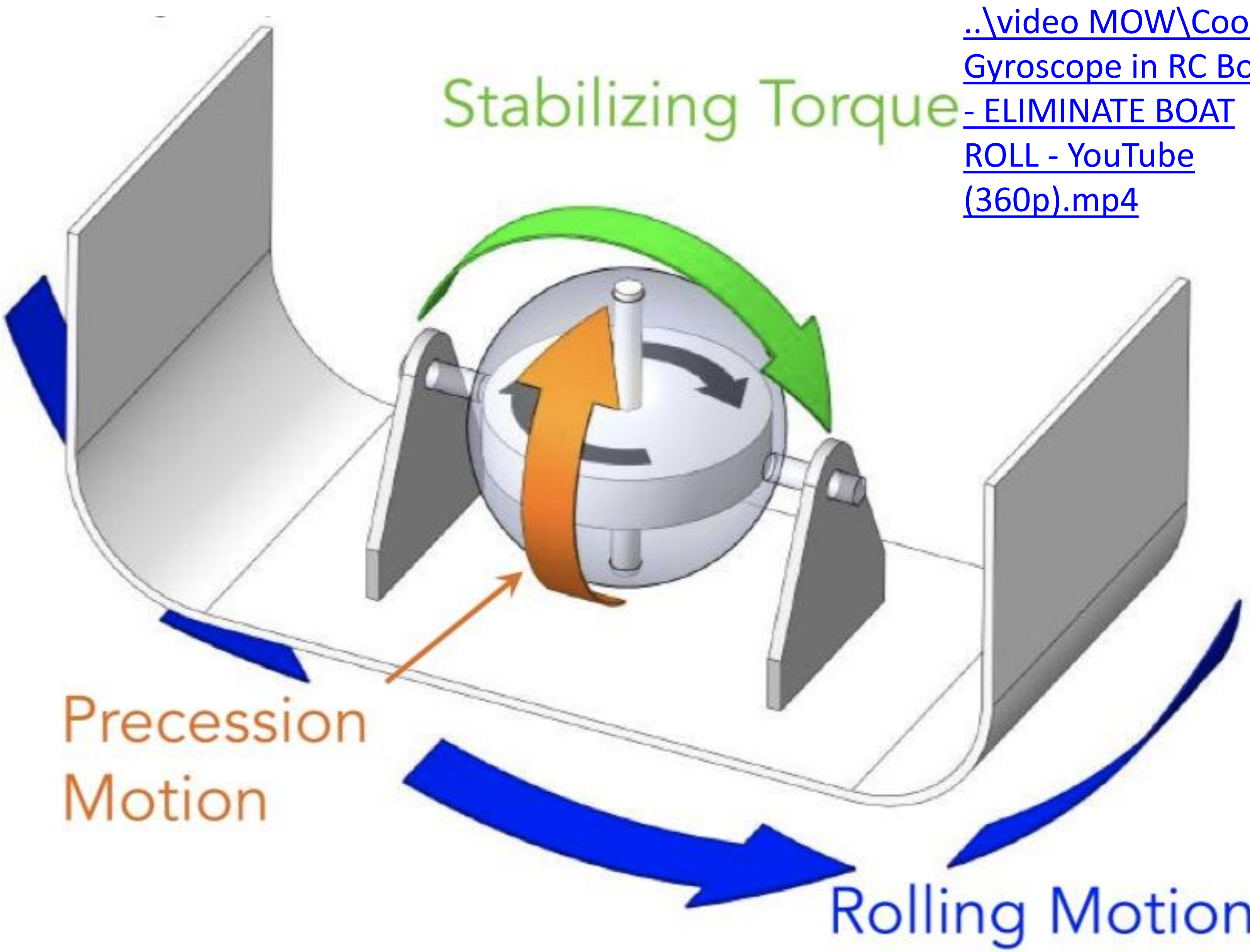
# LECTURE 14 – APPLICATION OF GYROSCOPIC CONCEPT


Are we prisoners in a ship? I thought we were on land. How come I not know this?

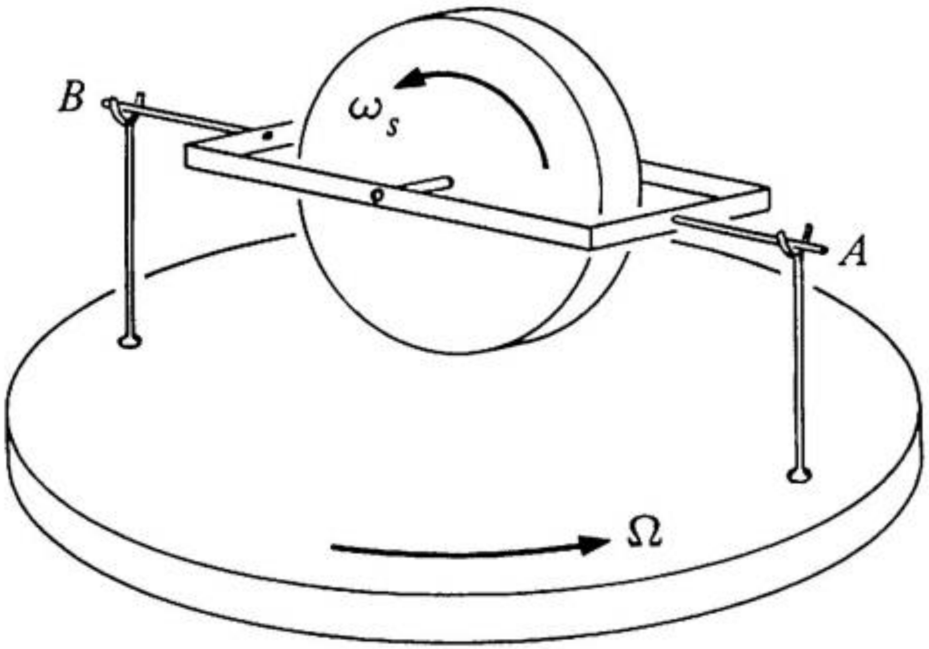
Because you are criminal and not a physicist.

# Principle of Gyrostabilizers

..\video MOW\Cool Gyroscope in RC Boat - ELIMINATE BOAT ROLL - YouTube (360p).mp4

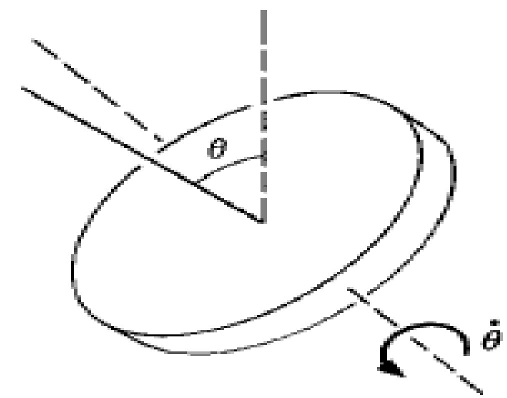


# Gyrocompass

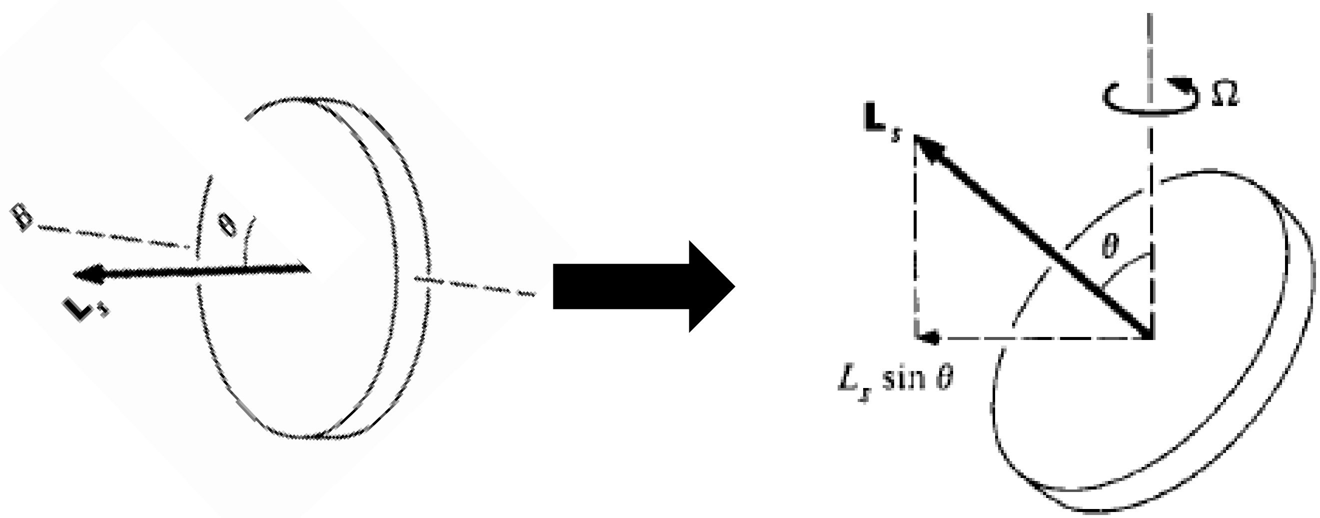


# Gyro-Oscillation

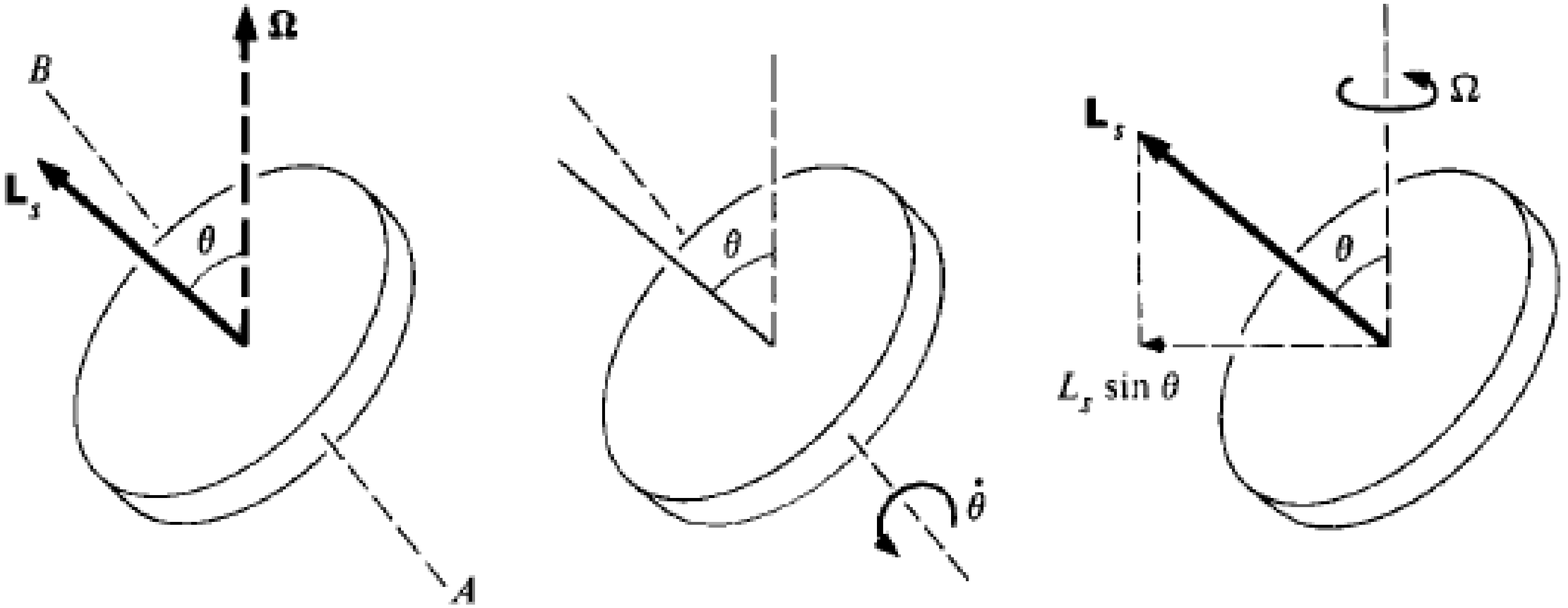
Angular momentum change about the axis arises due to two factors




1. Due to angular rotation of the gyro
2. Due to change in the magnitude of $L_s$



# Gyro-Oscillation



# Gyro-Oscillation

Torque due to rotation $= I\ddot{\theta}$

Torque due to change in $L_s$ $= L_s \sin\theta\Omega$

$$\tau = I\ddot{\theta} + L_s \sin\theta\Omega = 0$$

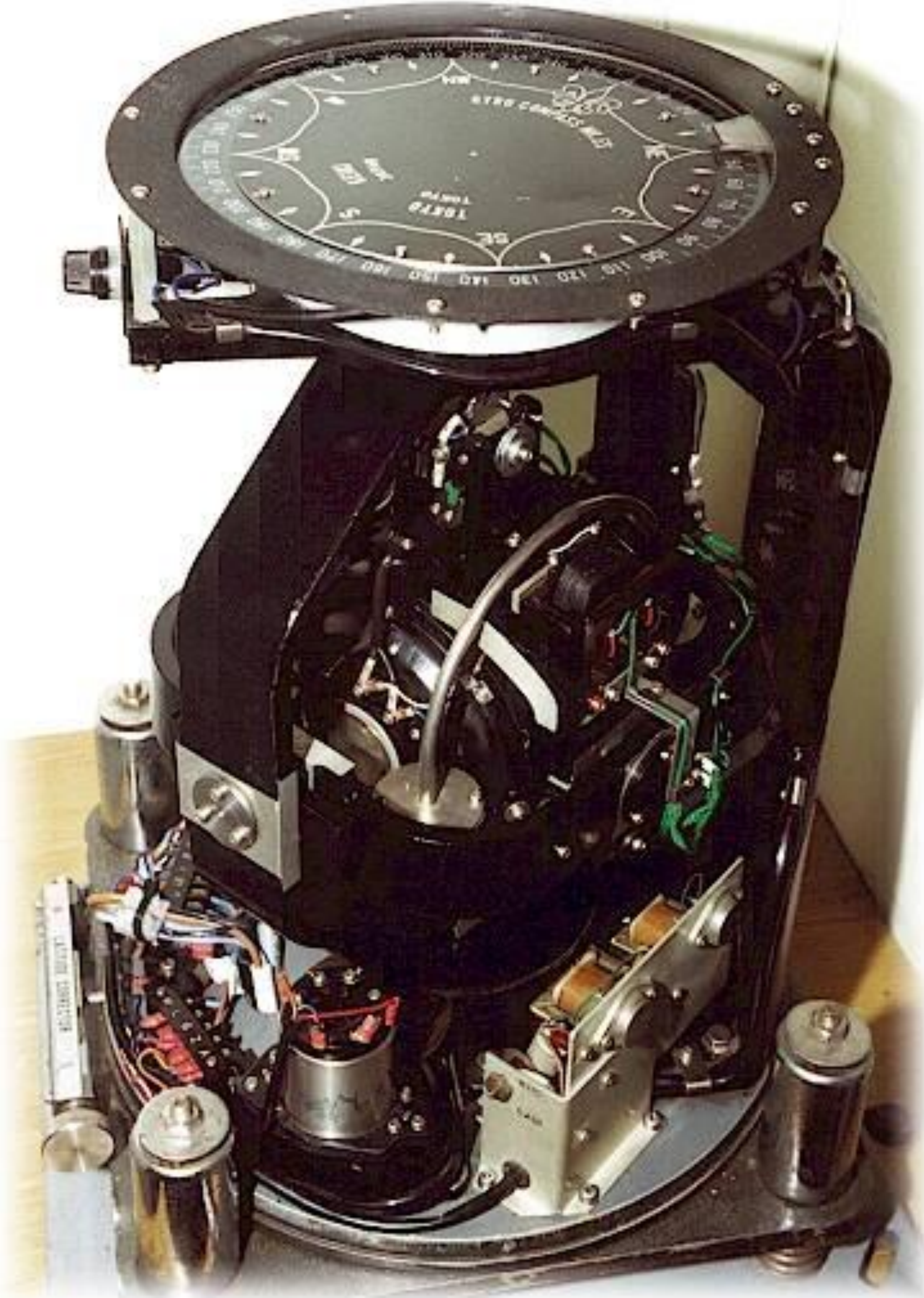
# Gyro-Oscillation

$$\tau = I\ddot{\theta} + L_s \sin\theta\,\Omega = 0$$

$$I\ddot{\theta} + L_s\theta\Omega = 0$$

$$\ddot{\theta} + \frac{L_s\Omega}{I}\theta = 0$$

$$\omega = \sqrt{\frac{L_s\Omega}{I}}$$



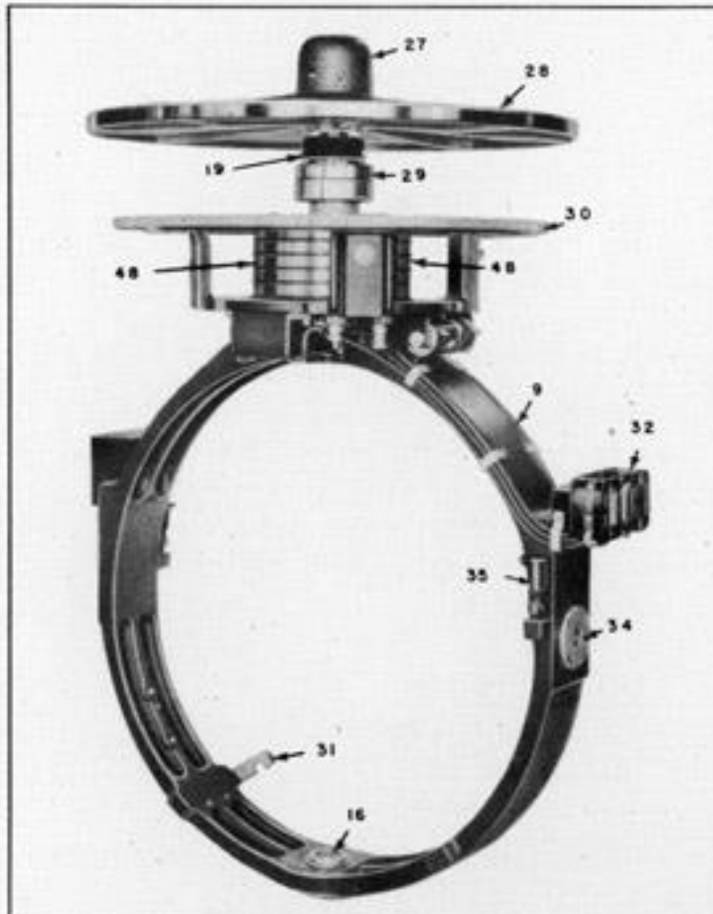
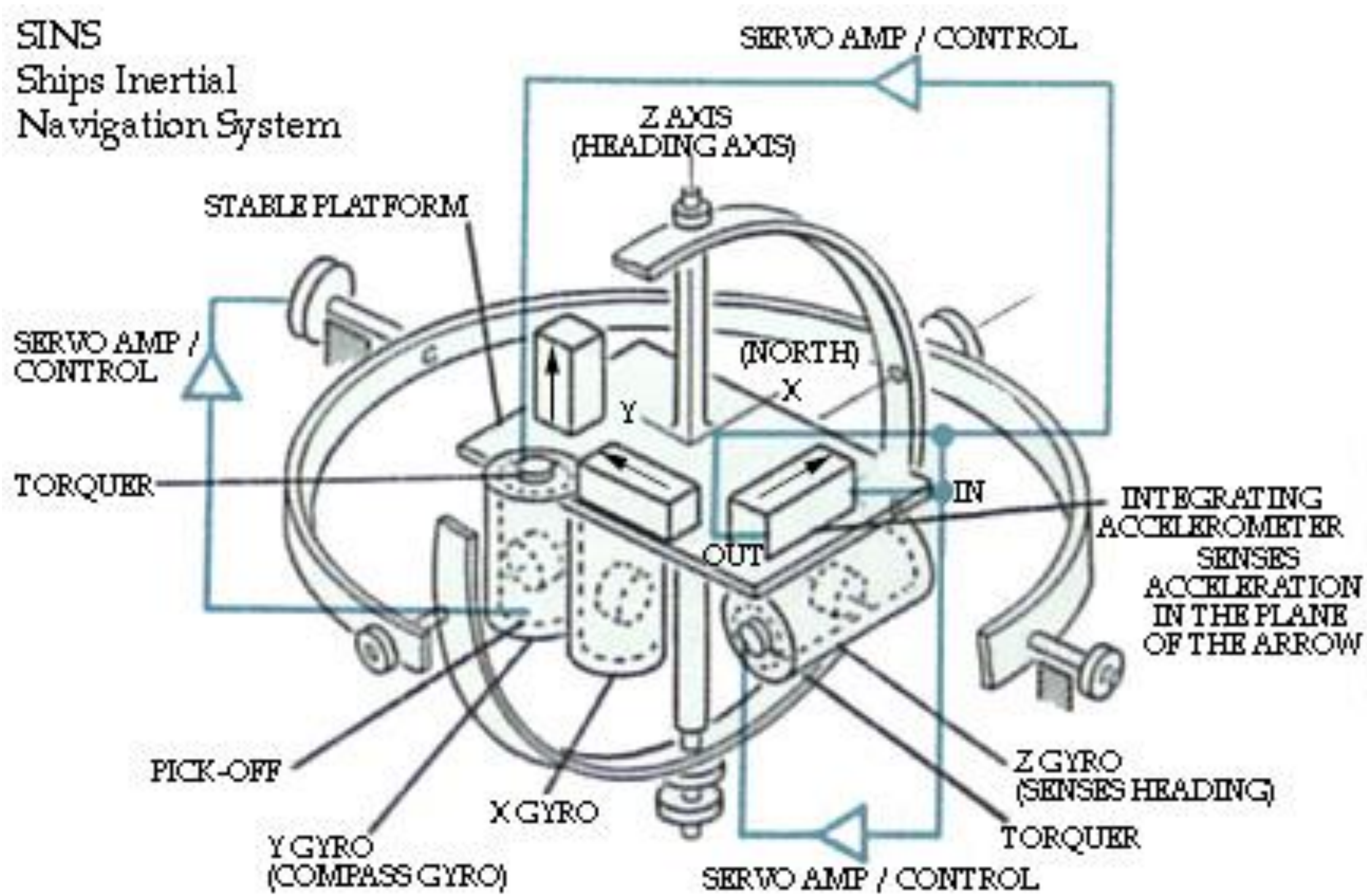



FIGURE 3
## PHANTOM ELEMENT

9. PHANTOM RING
16. LOWER VERTICAL RING GUIDE BEARING
19. STEM THRUST BEARING
27. SUSPENSION CAP
28. COMPASS CARD
29. UPPER STEM BEARING
30. AZIMUTH GEAR
31. VERTICAL RING LOCK
32. FOLLOW-UP TRANSF'M'R
34. MERCURY BALLISTIC BEARING
35. MERCURY BALLISTIC BEARING OIL CUP
48. STEM SLIP RINGS

# Inertial Navigation System (INS)



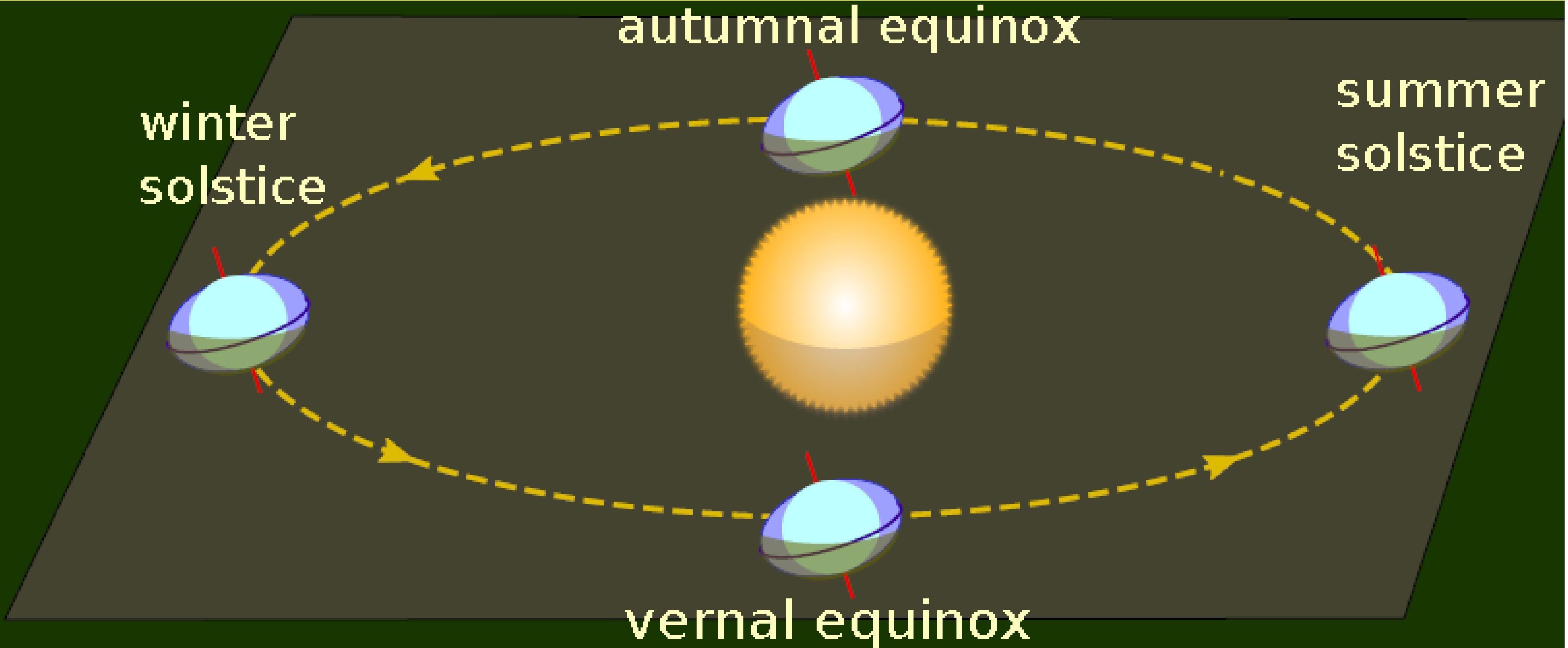
# What determines the stability of rotating objects?

[..\video MOW\The Bizarre Behavior of Rotating Bodies, Explained - YouTube (360p).mp4]

1. Spin Angular Momentum of the object.

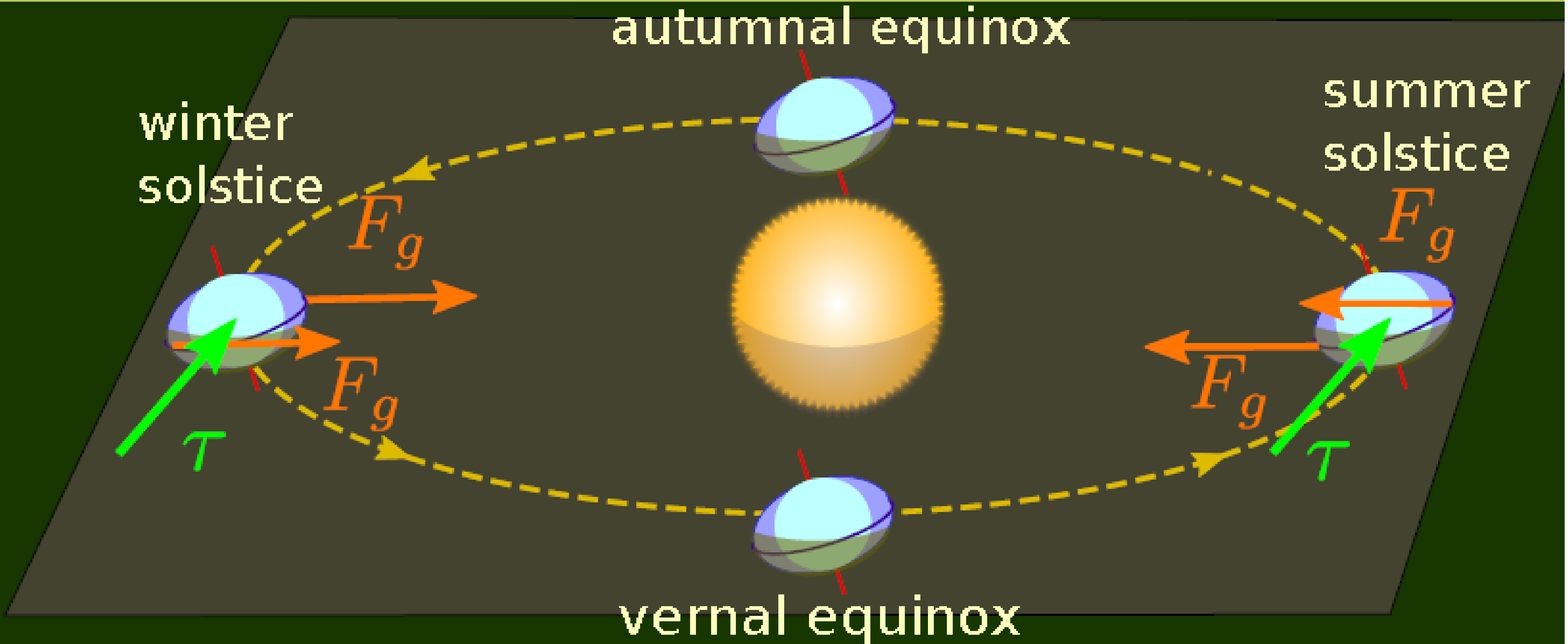
2. System should be spun about an axis passing through the center of symmetry for which moment of inertia is the MAXIMUM.

# Precession of the Equinoxes



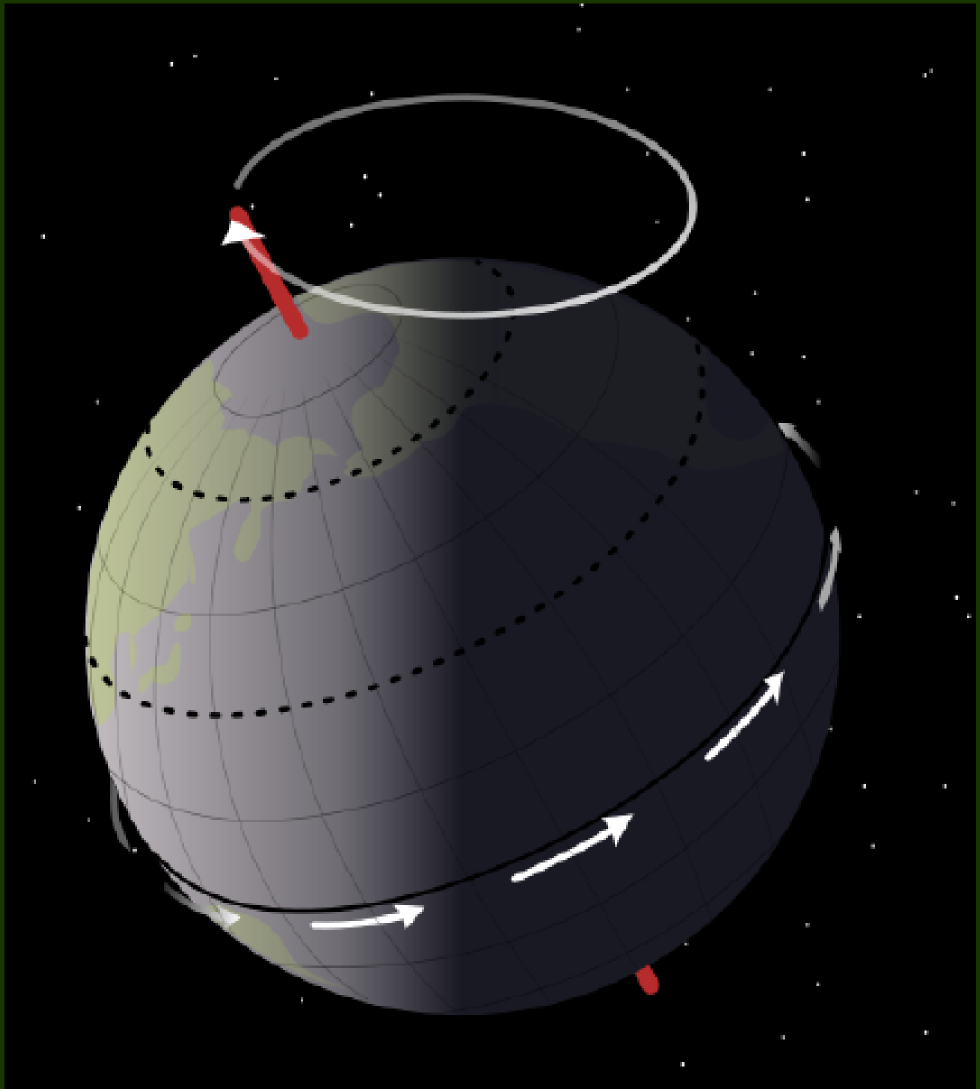

- Earth nonspherical: 21km bulge at equator
- Earth's spin axis tilted at 23.5° from ecliptic pole
- Net torque (about centre of earth) due to sun

# Precession of the Equinoxes




- Earth nonspherical: 21km bulge at equator
- Earth's spin axis tilted at 23.5° from ecliptic pole
- Net torque (about centre of earth) due to sun
- Torque maximum in winter & summer

# Precession of the Equinoxes



- Earth's axis precesses with period of 26,000 yrs.

# Lecture 14 - Moment of Inertia Tensor

PAGE 2 THE CHICAGO SUN, THURSDAY, JUNE 26, 1947

In These United States

## Supersonic Flying Saucers Sighted by Idaho Pilot

*Speed Estimated at 1,200 Miles an Hour When Seen 10,000 Feet Up Near Mt. Rainier*

PENDLETON, Ore., June 25.—(AP)

NINE bright, saucer-like objects flying at "incredible" speed at 10,000 feet altitude were reported here today by Kenneth Arnold, Boise (Idaho), pilot, who said he could not hazard a guess as to what they were.

Arnold, a U.S. Forest Service employee searching for a missing plane, said he sighted the mystery craft yesterday at 3 p.m. They were flying between Mount Rainier and Mount Adams, in Washington state, he said, and appeared to weave in and out of formation. Arnold said he clocked them and estimated their speed at 1,200 miles an hour.

Inquiries at Yakima last night brought only blank stares, he said, but he added he talked today with an unidentified man from Ukiah, south of here, who said he had seen similar objects over the mountains near Ukiah yesterday.

"It seems impossible," Arnold said, "but there it is."

Roswell Daily Record

Claims Army Is Stacking Courts Martial

## RAAF Captures Flying Saucer On Ranch in Roswell Region

House Passes Tax Slash by Large Margin

Security Council Paves Way to Talks On Arms Reductions

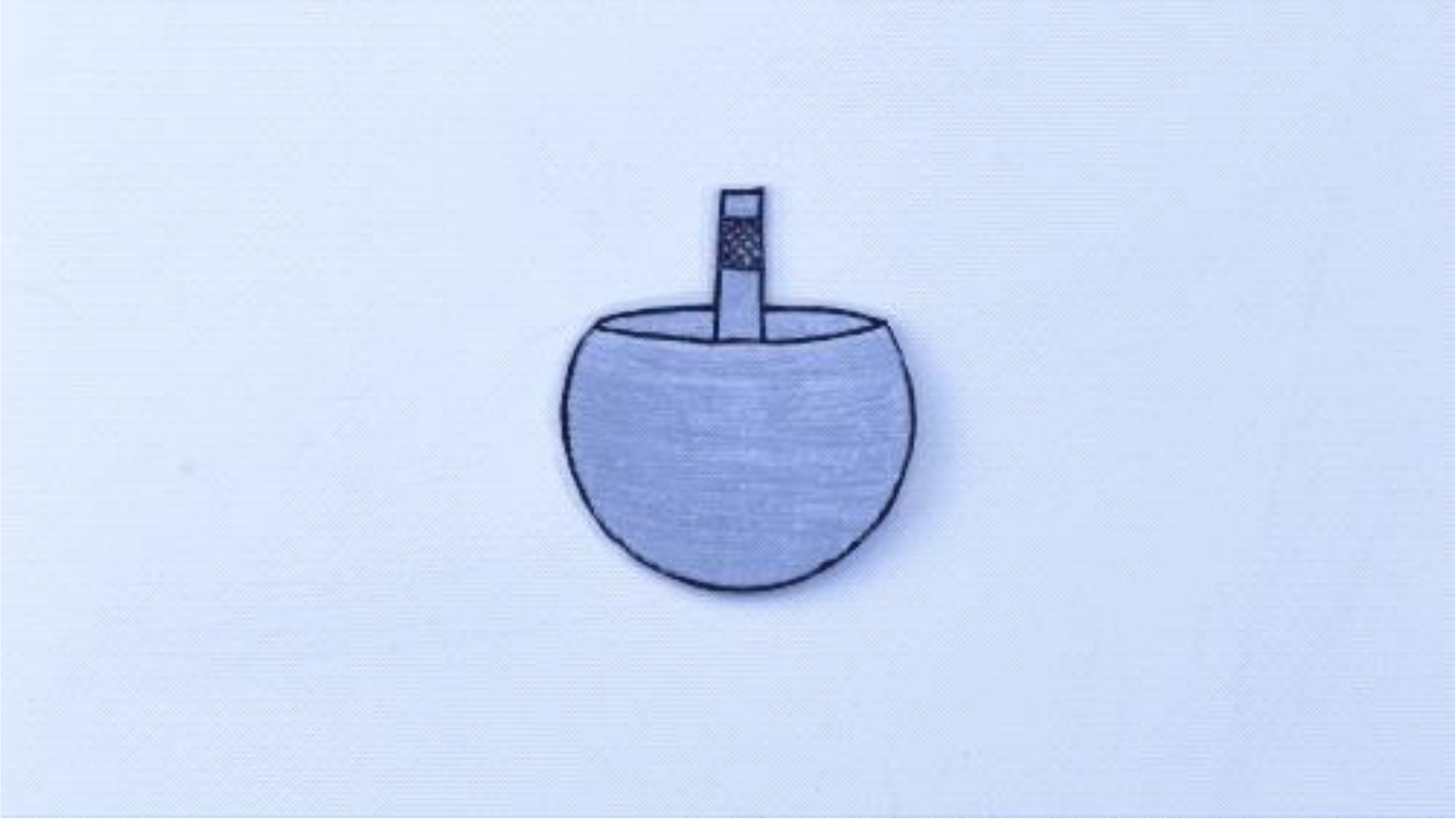
No Details of Flying Disk Are Revealed

Some of Soviet Satellites May Attend Paris Meeting

Miners and Operators Sign



[..\video MOW\Tippe Top.mp4]



..\video MOW\The Bizarre Behavior of Rotating Bodies.mp4

# What determines the stability of rotating objects?

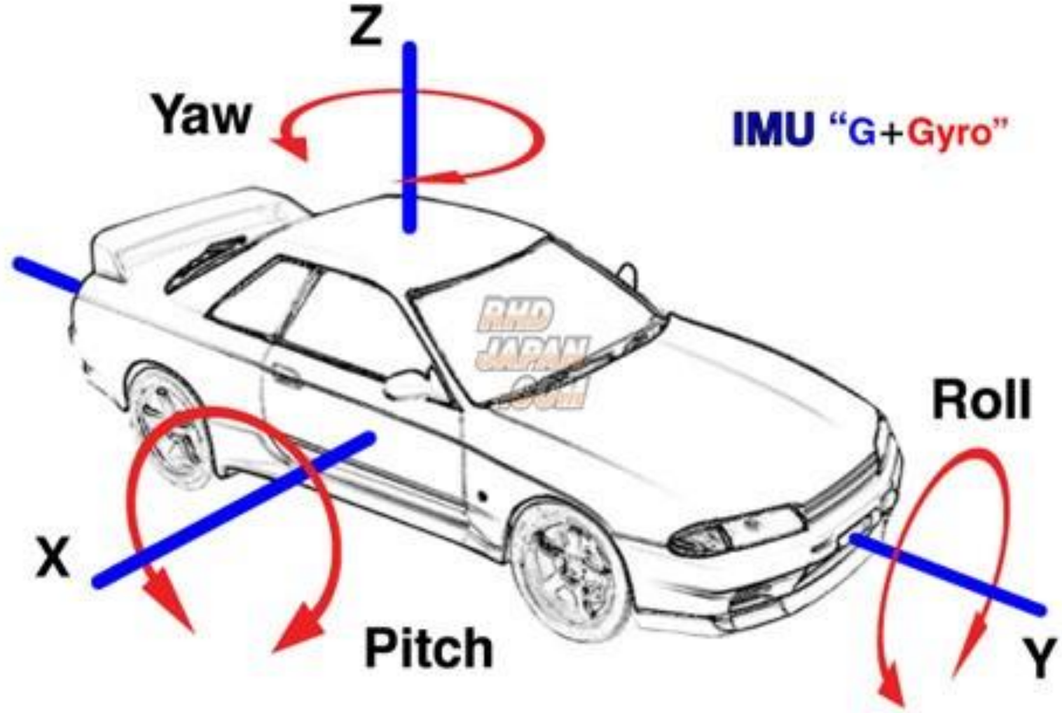
..\video MOW\Intermediate Axis Theorem.mp4

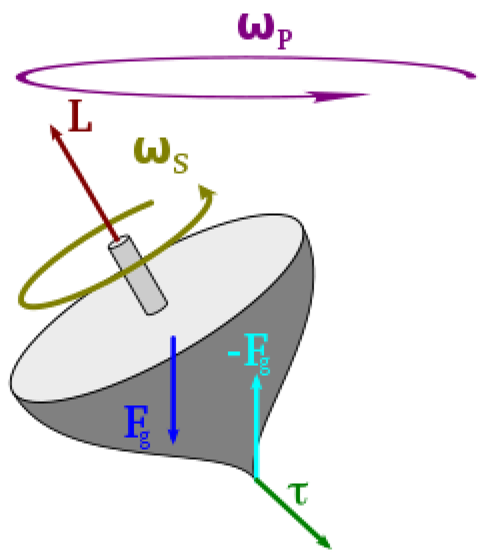
1. Spin Angular Momentum of the object.

2. System should be spun about an axis passing through the center of symmetry for which moment of inertia is the greatest.

# Moment of Inertia Tensor

..\video MOW\VIMM.wmv



# Simple Facts about Moment of Inertia Tensor

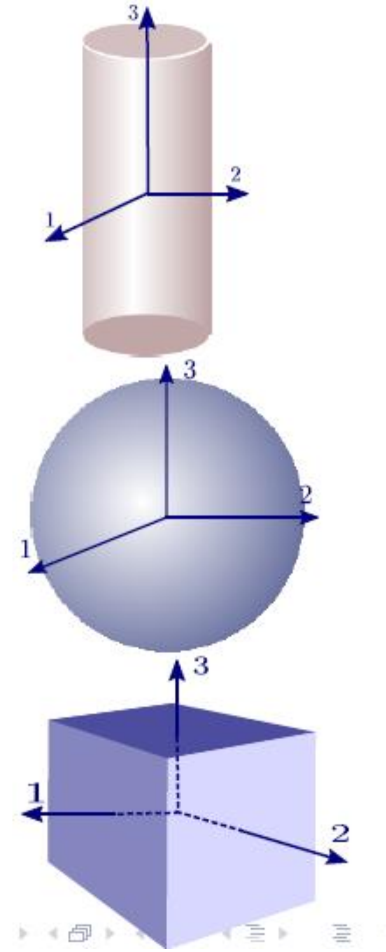
The inertia tensor ($I_{ij}$) quantifies an object's resistance to rotation in different directions. It is represented mathematically by a symmetric 3×3 matrix

$$I_p = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & I_{-yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}$$

# Simple Facts about Moment of Inertia Tensor

The diagonal terms (moments of inertia) capture the object's rotational inertia (resistance to rotational acceleration) with respect to three orthogonal axes passing through the center of symmetry.

$$I_p = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & I_{-yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}$$



# Simple Facts about Moment of Inertia Tensor

The off-diagonal terms (products of inertia) reflect asymmetries of the object's mass distribution about axes defined at O.

Example : Moment of inertia of a fan without wobbling

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$I_p = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & I_{-yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}$$

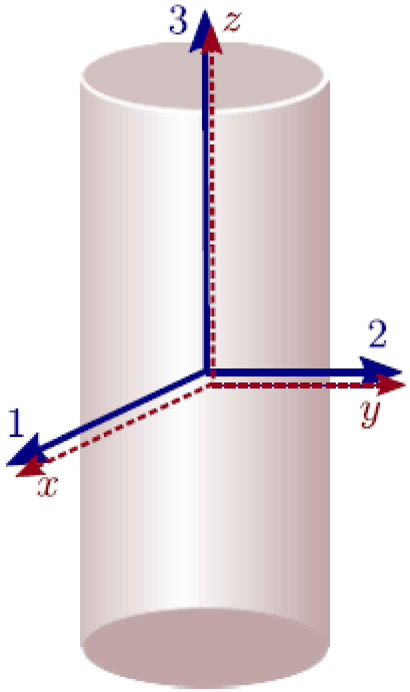
**<u>Example</u>** : Moment of inertia of a plastic bottle half filled with water. For symmetric systems, products of inertia vanish and for non-symmetric system it doesn’t.

# Simple Facts about Moment of Inertia Tensor

**A particular set of orthogonal axes can be chosen such that the products of inertia disappear, thereby rendering the inertia tensor in its diagonal form. Such axes are called principal axes.**

Principal Axes $(1,2,3)$: Symmetry axes of rigid body
When Body Axes chosen to coincide with the Principal axes:

$$\mathcal{I} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}$$




In Principal Axes:

$$\begin{aligned} L_1 &= I_1\omega_1 \\ L_2 &= I_2\omega_2 \\ L_3 &= I_3\omega_3 \end{aligned}$$

# Moment of Inertia tensor

$$\begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

$$\begin{aligned} \bar{I} &= [I] = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \\ &= \begin{bmatrix} \int dm(y^2+z^2) & -\int dmxy & -\int dmxz \\ -\int dmyx & \int dm(x^2+z^2) & -\int dmyz \\ -\int dmzx & -\int dmzy & \int dm(x^2+y^2) \end{bmatrix} \end{aligned}$$

$\bar{I} = [I]_{3\times3}$: 3X3 Moment of Inertia Matrix

# Moment of Inertia tensor

- $\bar{I}$ is a 3×3 symmetric matrix

  6 independent components

- $$\left.\begin{array}{l} I_{xx} = \int dm(y^2 + z^2) \\ I_{yy} = \int dm(x^2 + z^2) \\ I_{zz} = \int dm(x^2 + y^2) \end{array}\right\}$$

  Moments of inertia abt $x$, $y$ & $z$

- $$\left.\begin{array}{l} I_{xy}(= I_{yx}) = -\int dmxy \\ I_{yz}(= I_{zy}) = -\int dmzy \\ I_{xz}(= I_{zx}) = -\int dmxz \end{array}\right\}$$

  Products of Inertia

# Lets find Moment of Inertia Tensor for rotating dumbell and skewed rod