Znani integrali

$$\int e^{ax}\sin(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\big(a\sin(bx)-b\cos(bx)\big)+C$$

$$\int e^{ax}\cos(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\big(a\cos(bx)+b\sin(bx)\big)+C$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\Big(\frac{x}{a}\Big)+C$$

Parcialni ulomki

Ulomek	Parcialni razcep
$\frac{px+q}{(x-a)(x-b)}, a\neq b$	$\frac{A}{x-a}+\frac{B}{x-b}$
$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}$
$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$

kjer se x^2+bx+c se ne da razstaviti naprej.

Trigonometrične formule

$$\sin^2 x = \frac{1-\cos 2x}{2},\quad \cos^2 x = \frac{1+\cos 2x}{2}$$

Univerzalna substitucija

$$t=\tan\tfrac{x}{2},\ \sin x = \frac{2t}{1+t^2},\ \cos x = \frac{1-t^2}{1+t^2},\ dx = \frac{2}{1+t^2}dt$$

Inverzi matrik

$$\begin{bmatrix} a & b \\ c & d\end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a\end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A}\,adj(A),\qquad adj(A) = \big[(-1)^{i+j}D_{ij}\big]^T$$

Cramerjevo pravilo

$x_i = \frac{D_i}{D}$, $\quad D=\det(A)$, $\quad D_i = \det(A$ z i-tim stolpcem $\vec{b})$

Lastne vrednosti

$\det(A-\lambda I)=0$, lastni vektorji rešijo $(A-\lambda I)\vec{v}=0$.

Linearne preslikave, podobnost, prehod med bazama

...

Skalarni, vektorski in mešani produkt

$$\vec{a}\cdot\vec{b} = |a||b|\cos\varphi = a_1b_1+a_2b_2+a_3b_3,\quad |\vec{a}| = \sqrt{\vec{a}\cdot\vec{a}}$$

$$\vec{a}\times\vec{b} = \det\begin{bmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3\end{bmatrix},\quad |\vec{a}\times\vec{b}| = |a||b|\sin\varphi$$

$$(\vec{a}\times\vec{b})\cdot\vec{c} = \det[a,b,c]$$

Dvojni vektorski produkt, Lagrangeva identiteta

$$(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}$$

$$|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$$

Razdalje

$$d(T,p) = \frac{|\overrightarrow{T_0T}\times\vec{s}|}{|\vec{s}|}$$

$$d(T,\Sigma) = \frac{|at_x+bt_y+ct_z-d|}{|\vec{n}|}$$

$$d(p_1,p_2) = \frac{|(\overrightarrow{T_1T_2},\vec{s_1},\vec{s_2})|}{|\vec{s_1}\times\vec{s_2}|}$$

Vektorski prostor in podprostor

dodamo? (Vaje 6)

Dvojni integral

$$V = \int_D f(x,y)\,dx\,dy,\quad P = \int_D dx\,dy$$

Polarne koordinate: $x=r\cos\varphi,\ y=r\sin\varphi,\ J=r$

Trojni integral

$$V = \int_D dV,\quad M = \int_D \rho\,dV,\quad I = \int_D r^2\rho\,dV$$

Cilindrične in sferične koordinate:

$$x=r\cos\varphi,\ y=r\sin\varphi,\ z=z,\quad J=r$$

$$x=r\sin\vartheta\cos\varphi,\ y=r\sin\vartheta\sin\varphi,\ z=r\cos\vartheta,\quad J=r^2\sin\vartheta$$

Krivulje v prostoru

$$\vec{r}(t) = (x(t),y(t),z(t)),\quad |\dot{\vec{r}}(t)| = \sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}$$

$$l = \int_a^b |\dot{\vec{r}}(t)|\,dt$$

$$\int_C f(\vec{r})\,ds = \int_a^b f(\vec{r}(t))|\dot{\vec{r}}(t)|\,dt$$

$$\int_C \vec{F}\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\dot{\vec{r}}(t)\,dt$$

Ločni parameter, ukrivljenost, zvitost

$$s(t) = \int_{t_0}^t |\dot{\vec{r}}(\tau)|\,d\tau$$

$$\vec{u}(s) = \vec{r}'(s),\quad \vec{p}(s) = \frac{\vec{u}'(s)}{\kappa(s)},\quad \vec{b}(s) = \vec{u}(s)\times\vec{p}(s)$$

$$\kappa(s) = |\vec{u}'(s)|,\quad \tau(s) = -\vec{p}(s)\cdot\vec{b}'(s)$$

$$\vec{u}'(s) = \kappa(s)\vec{p}(s),\ \vec{p}'(s) = -\kappa(s)\vec{u}(s)+\tau(s)\vec{b}(s),\ \vec{b}'(s) = -\tau(s)\vec{p}(s)$$

Krivuljni integrali 2. reda

$$\int_C \vec{F}\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\dot{\vec{r}}(t)\,dt$$

Če je $\vec{F} = \nabla u$, potem

$$\int_C \vec{F}\cdot d\vec{r} = u(\vec{r}(b)) - u(\vec{r}(a))$$

Ploskve

$z = f(x,y), \quad \vec{n} = \pm(-f_x, -f_y, -1)$
$\vec{r}(u,v) = (x(u,v), y(u,v), z(u,v)), \quad \vec{n} = \pm\vec{r}_u \times \vec{r}_v$

$$P = \int_d |\vec{n}|\,dS$$

Ploskovni integrali

$$\int_S g\,dS = \int_D g(x,y,f(x,y))|\vec{n}|\,dx\,dy$$

$$\iint_S g\,dS = \iint_D g(\vec{r}(u,v))\,|\vec{n}|\,du\,dv$$

$$\Psi_{\vec{F}} = \int_S \vec{F}\cdot d\vec{S} = \int_S \vec{F}\cdot\vec{n}\,dS$$

Izreki

Gaussov izrek:

$$\int_S \vec{F} d\vec{S} = \int_V \mathrm{div}\vec{F}\,dV$$

Stokesov izrek:

$$\int_C \vec{F}\,d\vec{r} = \int_S \mathrm{rot}\vec{F}\,dS$$

Greenova formula:

$$\int_C \vec{F}\,d\vec{r} = \int_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right)\,dx\,dy$$

Sistemi DE

...

Faktorizacija in defaktorizacija

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad \sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad \cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta))$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \qquad \sin\alpha\sin\beta = \frac{1}{2}(\cos(\alpha-\beta) - \cos(\alpha+\beta))$$