Odvodi in inverse funkcije

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

Kot med premicama

$$\cos\varphi = \left|\frac{k_1-k_2}{1+k_1k_2}\right|$$

Stacionarne točke

$f'(x_0)=0$. Če $f''(x_0)>0 \to$ minimum, če $f''(x_0)<0 \to$ maksimum, če $f''(x_0)=0 \to$ test višjih odvodov ali infleksijska točka.

Konveksnost / konkavnost

$f''(x)>0 \to$ konveksna, $f''(x)<0 \to$ konkavna.

Taylorjeva vrsta okrog a

$$f(x) = f(a) + f'(a)(x-a) + \tfrac{f''(a)}{2!}(x-a)^2 + \cdots$$

Per partes

$$\int u\,dv = uv - \int v\,du$$

Znani integrali

$$\int e^{ax}\sin(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\big(a\sin(bx) - b\cos(bx)\big) + C$$

$$\int e^{ax}\cos(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\big(a\cos(bx) + b\sin(bx)\big) + C$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\Big(\frac{x}{a}\Big) + C$$

Parcialni ulomki

Ulomek	Parcialni razcep
$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a} + \frac{B}{x-b}$
$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$

kjer se x^2+bx+c se ne da razstaviti naprej.

Trigonometrične formule

$$\sin^2 x = \frac{1-\cos 2x}{2}, \quad \cos^2 x = \frac{1+\cos 2x}{2}$$

Univerzalna substitucija

$$t = \tan\tfrac{x}{2},\ \sin x = \frac{2t}{1+t^2},\ \cos x = \frac{1-t^2}{1+t^2},\ dx = \frac{2}{1+t^2}dt$$

Prostornina in površina vrtenine, dolžina loka

$$V = \pi\int_a^b f(x)^2 dx$$

$$pl = 2\pi\int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx$$

$$L = \int_a^b \sqrt{1+(f'(x))^2}\,dx$$

Normala in tangentna ravnina

Normala $\vec{n} = (f_x, f_y, -1)$
Tangentna ravnina $z = f_x(x_0,y_0)x + f_y(x_0,y_0)y + d$.

Ekstremi 2D

det $H_f > 0, f_{xx} > 0$: minimum
det $H_f > 0, f_{xx} < 0$: maksimum
det $H_f < 0$: sedlo
det $H_f = 0$: ne vemo

Inverzi matrik

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A}\,adj(A), \qquad adj(A) = \big[(-1)^{i+j}D_{ij}\big]^T$$

Cramerjevo pravilo

$$x_i = \frac{D_i}{D}, \quad D = \det(A), \quad D_i = \det(A \text{ z } i\text{-tim stolpcem } \vec{b})$$

Lastne vrednosti

$\det(A-\lambda I)=0$, lastni vektorji rešijo $(A-\lambda I)\vec{v}=0$.

Skalarni, vektorski in mešani produkt

$$\vec{a}\cdot\vec{b} = |a||b|\cos\varphi = a_1b_1 + a_2b_2 + a_3b_3, \quad |\vec{a}| = \sqrt{\vec{a}\cdot\vec{a}}$$

$$\vec{a}\times\vec{b} = \det\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}, \quad |\vec{a}\times\vec{b}| = |a||b|\sin\varphi$$

$$(\vec{a}\times\vec{b})\cdot\vec{c} = \det[a,b,c]$$

Dvojni vektorski produkt, Lagrangeva identiteta

$$(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}$$

$$|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$$

Razdalje

$$d(T,p) = \frac{|\overrightarrow{T_0T}\times\vec{s}|}{|\vec{s}|}$$

$$d(T,\Sigma) = \frac{|at_x + bt_y + ct_z - d|}{|\vec{n}|}$$

$$d(p_1,p_2) = \frac{|(\overrightarrow{T_1T_2}, \vec{s_1}, \vec{s_2})|}{|\vec{s_1}\times\vec{s_2}|}$$

Faktorizacija in defaktorizacija

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$$

$$\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta))$$

$$\sin\alpha\sin\beta = \frac{1}{2}(\cos(\alpha-\beta) - \cos(\alpha+\beta))$$