Odvodi in inverse funkcije

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$
$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$
$$(\arctan x)' = \frac{1}{1+x^2}$$

Kot med premicama

$$\cos\varphi = |\frac{k_1-k_2}{1+k_1k_2}|$$

Stacionarne točke

$f'(x_0)=0$. Če $f''(x_0)>0 \to$ minimum, če $f''(x_0)<0 \to$ maksimum, če $f''(x_0)=0 \to$ test višjih odvodov ali infleksijska točka.

Konveksnost / konkavnost

$f''(x)>0 \to$ konveksna, $f''(x)<0 \to$ konkavna.

Taylorjeva vrsta okrog a

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$$

Per partes

$$\int u\,dv = uv - \int v\,du$$

Znani integrali

$$\int e^{ax}\sin(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\big(a\sin(bx) - b\cos(bx)\big) + C$$
$$\int e^{ax}\cos(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\big(a\cos(bx) + b\sin(bx)\big) + C$$
$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\Big(\frac{x}{a}\Big) + C$$

Parcialni ulomki

Ulomek	Parcialni razcep
$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a} + \frac{B}{x-b}$
$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$

kjer se x^2+bx+c se ne da razstaviti naprej.

Trigonometrične formule

$$\sin^2 x = \frac{1-\cos 2x}{2}, \quad \cos^2 x = \frac{1+\cos 2x}{2}$$

Univerzalna substitucija

$t = \tan\frac{x}{2}$, $\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$, $dx = \frac{2}{1+t^2}dt$

Prostornina in površina vrtenine, dolžina loka

$$V = \pi\int_a^b f(x)^2 dx$$
$$pl = 2\pi\int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx$$
$$L = \int_a^b \sqrt{1+(f'(x))^2}\,dx$$

Normala in tangentna ravnina

Normala $\vec{n} = (f_x, f_y, -1)$
Tangentna ravnina $z = f_x(x_0,y_0)x + f_y(x_0,y_0)y + d$.

Ekstremi 2D

$\det H_f > 0, f_{xx} > 0$: minimum
$\det H_f > 0, f_{xx} < 0$: maksimum
$\det H_f < 0$: sedlo
$\det H_f = 0$: ne vemo

Bernoullijeva enačba

$$y' + p(x)y = q(x)y^\alpha, \quad \alpha \neq 0,1$$

Substitucija $z = y^{1-\alpha}$

LDE 2. reda in Wronski

$$y'' + p(x)y' + q(x)y = f(x)$$

Neodvisnost rešitev y_1, y_2 testiramo z Wronskim:

$$W(y_1,y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

Če $W \neq 0$ na intervalu $\to$ rešitvi linearno neodvisni.

$$y_p = C_1(x)y_1 + C_2(x)y_2, \text{ kjer}$$

$$C_1'(x)y_1 + C_2'(x)y_2 = 0$$
$$C_1'(x)y_1' + C_2'(x)y_2' = f(x)$$

$$C_1(x) = -\int\frac{f(x)y_2(x)}{W(x)}dx \quad C_2(x) = \int\frac{f(x)y_1(x)}{W(x)}dx$$

Homogena LDE s konstantnimi koef.

$$y'' + py' + qy = 0$$

Nastavek $y = e^{\lambda x}$, $\lambda^2 + p\lambda + q = 0$

- $\lambda_1 \neq \lambda_2 \to y_1 = e^{\lambda_1 x}$, $y_2 = e^{\lambda_2 x}$
- $\lambda_1 = \lambda_2 \to y_1 = e^{\lambda_1 x}$, $y_2 = xe^{\lambda_1 x}$
- $\lambda_1 = a+bi$, $\lambda_2 = a-bi \to y_1 = e^{ax}\sin(bx)$, $y_2 = e^{ax}\cos(bx)$

Metoda nedoločenih koeficientov (nastavki y_p)

$$y'' + py' + qy = f(x)$$

1. $f(x) = e^{ax}P(x)$: (kjer je P polinom stopnje n)

$$y_P(x) = x^r e^{ax} Q(x),$$

kjer je Q polinom stopnje n z neznanimi koeficienti in je $\lambda = a$ r-kratni koren karakteristične enačbe.

2. $f(x) = e^{ax}(P_1(x)\cos(bx) + P_2(x)\sin(bx))$:

$$y_P(x) = x^r e^{ax}(Q_1(x)\cos(bx) + Q_2(x)\sin(bx)),$$

kjer sta Q_1 in Q_2 polinoma stopnje $\max\{n_1, n_2\}$ z neznanimi koeficienti in je $\lambda = a + ib$ r-kratni koren karakteristične enačbe.

3. $f = f_1 + f_2 + \cdots + f_m$: Partikularno rešitev poiščemo kot vsoto zgoraj omenjenih nastavkov $y_P = y_{P1} + y_{P2} + \cdots + y_{Pm}$.

Vezani ekstremi

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Faktorizacija in defaktorizacija

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad \sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad \cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta))$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \qquad \sin\alpha\sin\beta = \frac{1}{2}(\cos(\alpha-\beta) - \cos(\alpha+\beta))$$