Discrete Time Signals

1 Introduction

Welcome to the CNCC 2021 workshop! This document serves as an introduction to signal processing. After reading through this document you should have a decent understanding of how discrete-time signals are represented using mathematical notation and what are their properties.

2 Basic Signals in D.T.

A discrete-time (D.T.) signal is any signal in the form:

$$x[n] : n \in \mathbb{Z} \to \mathbb{R} \; or \; \mathbb{C}$$

2.1 The unit impulse function $\delta[n]$

This signal is denoted by $\delta[n]$ and it's formula is given below:

$$\delta[n] = \begin{cases} 1, & \text{if } n = 0 \\ 0, & \text{otherwise} \end{cases}$$

As you can see, this signal is zero everywhere except at the point $n = 0$.

2.2 The unit step function $u[n]$

An extension of the unit impulse function is the unit step function, given by the formula:

$$u[n] = \begin{cases} 1, & \text{if } n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

The signal takes the value 1 for every $n \geq 0$. It can also be represented as an infinite sum of delayed unit impulse signals, which will be the topic of a later exercise.

2.3 The ramp function $r[n]$

The ramp function is given by the formula:

$$u[n] = \begin{cases} n, & \text{if } n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Its value keeps increasing linearly with n, if $n \geq 0$.

2.4 Sinusoidal signals

All continuous-time sinusoidal signals are periodic. However, in discrete-time, a sinusoidal sequence may or may not be periodic. Periodicity depends on the value of the signal's angular frequency Ω. For a discrete time signal to be periodic, its angular frequency must be a rational multiple of 2π. The general formula for a discrete-time sinusoidal signal is given as:

$$x[n] = sin(\Omega n + \phi)$$

Where the discrete-time angular frequency Ω is equal to $2\pi f_0$ and f_0 is the signal's frequency in Hz.

3 Even and Odd Signals

3.1 Even Signals in D.T.

A signal is said to be *even* or *symmetric* if it satisfies the following condition:

$$x[-n] = x[n]$$

3.2 Odd Signals in D.T.

A signal is said to be *odd* if it satisfies the following condition:

$$x[-n] = -x[n]$$

3.3 Periodic Signals in D.T.

A signal is said to be periodic **iff** it satisfies the following condition:

$$x[n + N] = x[n]$$

The condition above specifies that the signal is periodic if **and only if** the signal repeats itself after a period of N samples. For example, let's consider the following signal:

$$
\begin{aligned}
x[n] &= A\,cos(\Omega n + \phi) \\
x[n+N] &= A\,cos(\Omega(n+N) + \phi) \\
&\Rightarrow A\,cos(\Omega n + \Omega N + \phi)
\end{aligned}
$$

For the signal to be periodic, we know that the periodicity condition needs to be satisfied. From basic trigonometry, it follows that:

$$
\begin{aligned}
\Omega N &= 2\pi K \Rightarrow \\
N &= \frac{2\pi K}{\Omega} \Rightarrow \\
N &= \frac{K}{f_0}
\end{aligned}
$$

Where $K \in \mathbb{Z}$. The above states that the frequencies (in Hz) of discrete sinusoidal signals are separated by integral multiple of 2π.

Example #1

Let's consider the case where $x[n] = cos(\Omega\, n)$ with a frequency of $\Omega = \frac{\pi}{6}$;

$$
\begin{aligned}
x[n] &= cos(\frac{\pi\, n}{6}) \\
N &= \frac{2\pi K}{\Omega} = 12\,K
\end{aligned}
$$

If we choose $K = 1$, we get $N_0 = 12$; this is the *fundamental period* of our signal and it means that the signal $x[n]$ repeats itself every $N_0 = 12$ samples, and therefore is periodic.

Example #2

Now let's consider another case: $\Omega = \frac{1}{6}$;

$$
\begin{aligned}
x[n] &= cos(\frac{n}{6}) \\
N &= \frac{2\pi K}{\Omega} = 12\,\pi\,K
\end{aligned}
$$

As we can see, $\mathbb{N} \notin \mathbb{Z}^+$ for any $k \in \mathbb{Z}$ and thus the signal $x[n]$ is **not** periodic.

4 Signal Properties

Here we present a list of the basic properties of D.T. signals.

Time Shifting	$\{y[n]\} = \{x[n-k]\}$
Reflection	$\{y[n]\} = \{x[-n]\}$
Scaling	$\{y[n]\} = \alpha\{x[n]\}$
Addition	$\{y[n]\} = \{x_1[n]\} + \{x_1[n]\}$
Subtraction	$\{y[n]\} = \{x_1[n]\} + \{x_1[n]\}$
Linear Combination	$\{y[n]\} = \alpha_1\{x_1[n]\} + \alpha_2\{x_1[n]\}$

The properties listed above will be used in the exercises below. If you would like to read more, you can refer to this website:

TutorialsPoint: Digital Signal Processing - Operations on Signals; Shifting

5 Exercises

5.1 Draw the three basic signals $\delta[n]$, $u[n]$, $r[t]$ in the range $n \in [-5, 5]$

5.2 Draw the signal $y[n] = u[n] - u[n-3]$

5.3 Draw the signal $y[n] = 3(u[n] - u[n-3])$

5.4 Draw the signal $y[n] = \frac{2}{3}\, r[n-3]$

5.5 Draw the signal $y[n] = -\,\delta[n] + \delta[n-1] - \frac{1}{2}\, u[n-3] + \frac{1}{2} u[n-6]$

5.6 Draw the following signals in the specified range and classify them as even or odd

$\delta[n]$, $\forall n \in \mathbb{Z}$	even	odd
$u[n]$, $\forall n \in \{-5, 5\}$	even	odd
$r[n]$, $\forall n \in \{-5, 5\}$	even	odd
$u[n+3] - u[n-3]$, $\forall n \in \{-5, 5\}$	even	odd
$sin(\frac{\pi\, n}{5})$, $\forall n \in \{-20, 20\}$	even	odd

hint: use a computer to validate your results.

5.7 Show that the signal $A\,sin(2\pi fn + \phi)$ is periodic.

5.8 Is the signal $y[n] = cos(\frac{3\pi n}{12}) + cos(\frac{\pi n}{13})$ periodic?