

Introduction to Experimental Neuroscience: Neurocomputation | July 2021

Single neuron and network dynamics

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Computational Neuroscience

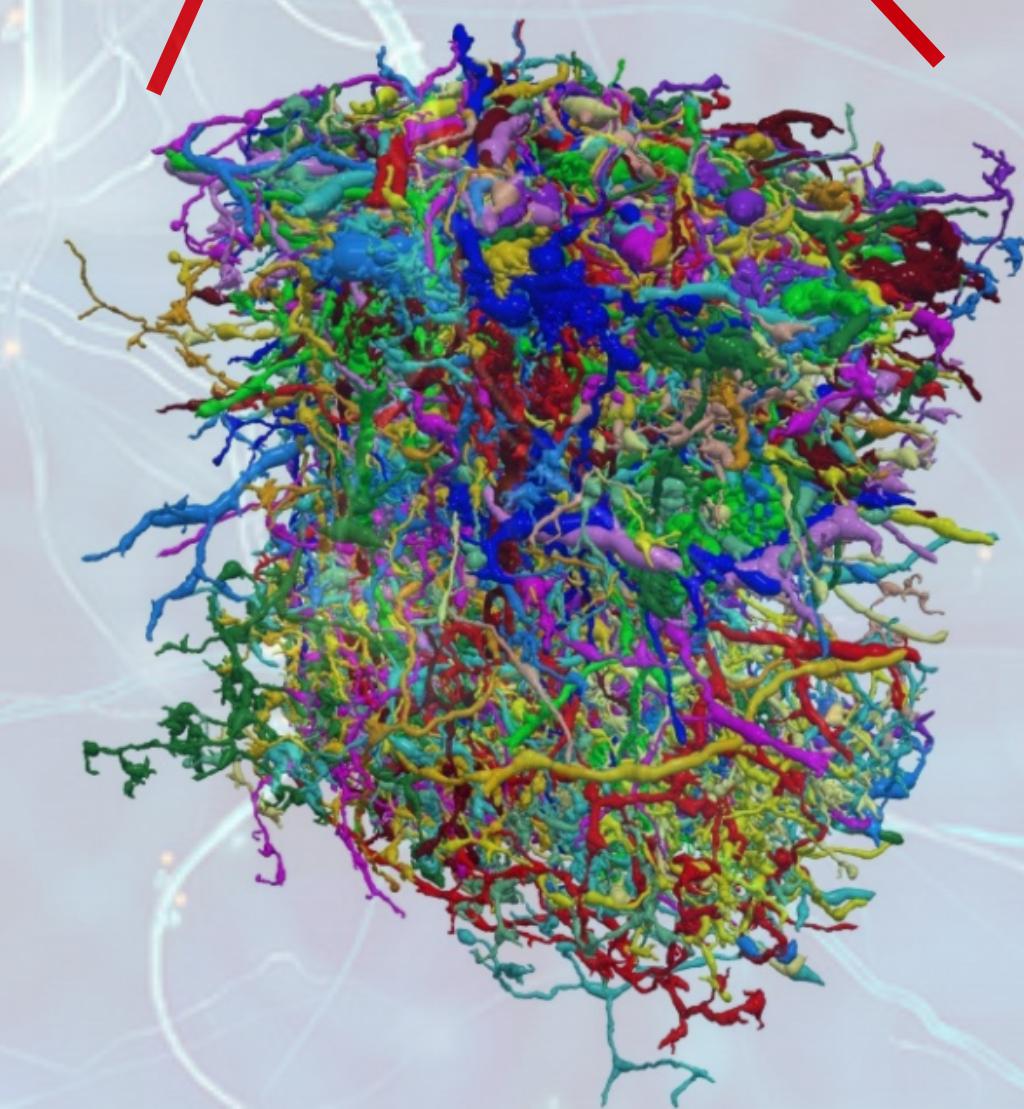
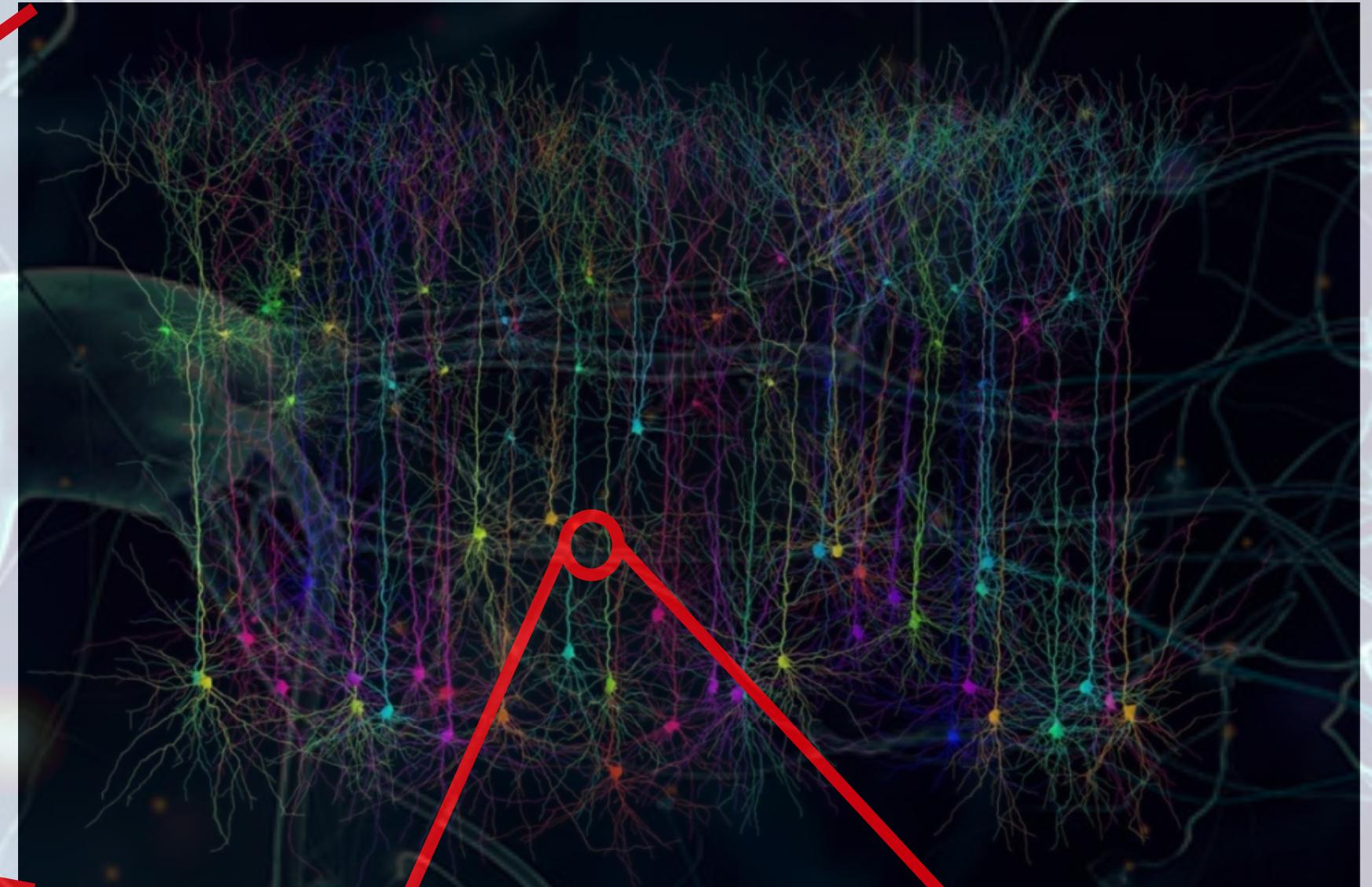
Wikipedia: Computational neuroscience is the study of the computational principles underlying brain function and neuronal activity, i.e. the generic algorithms allowing the implementation of cognitive functions in our central nervous system.

The problem

Brain: 10^{11} neurons (100 billions)

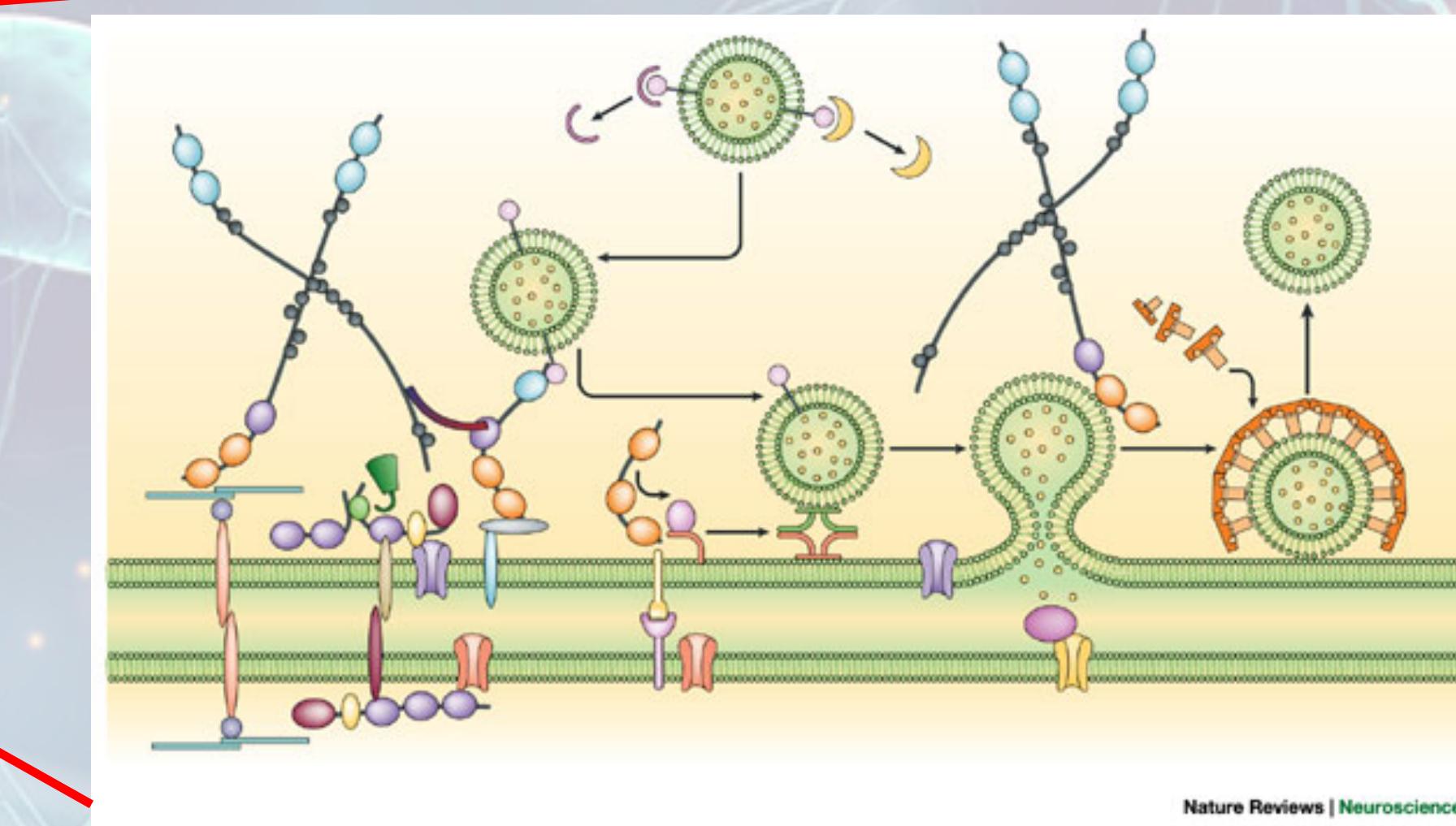
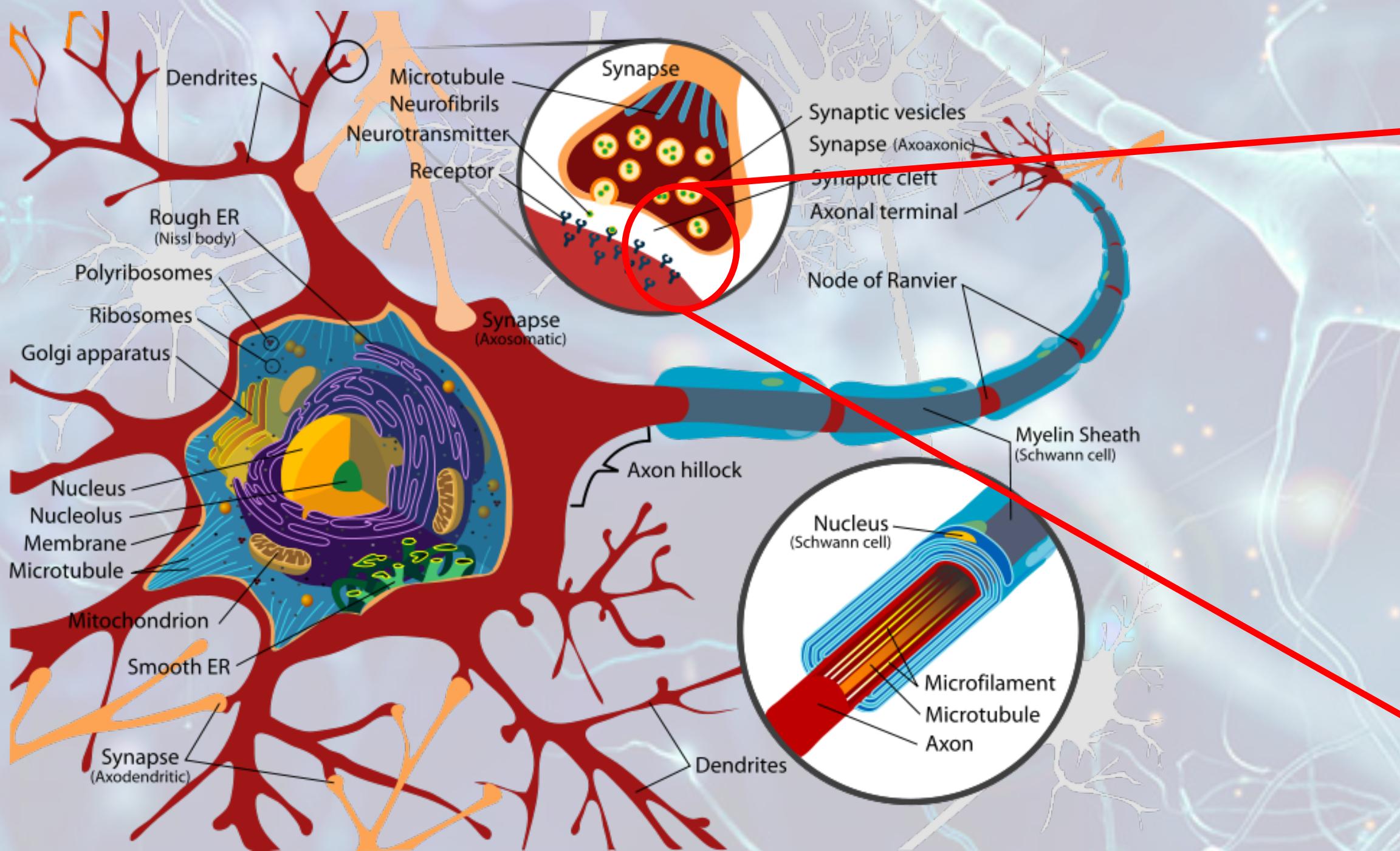


Neuron: >1000 synapses



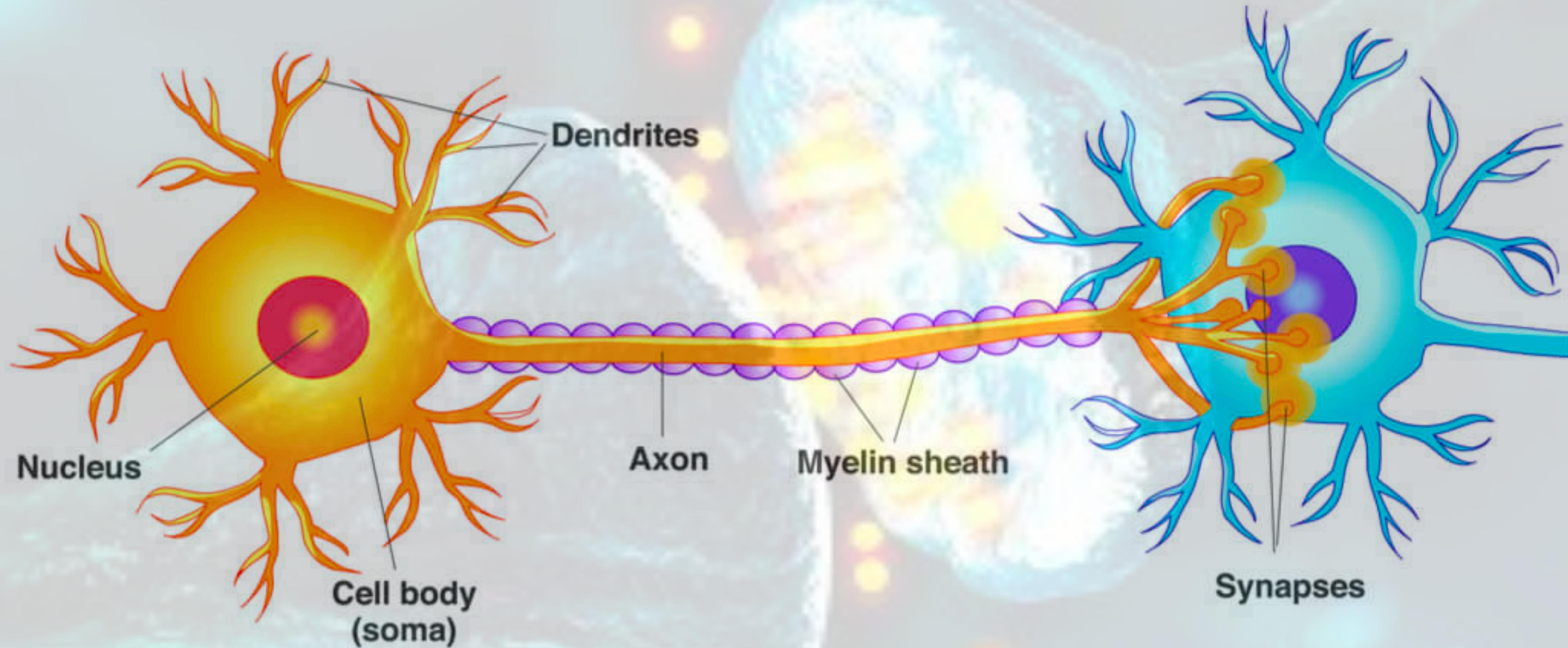
The problem

Neuron: $\sim 10^6$ ion channels, complex interactions between various intracellular actors....

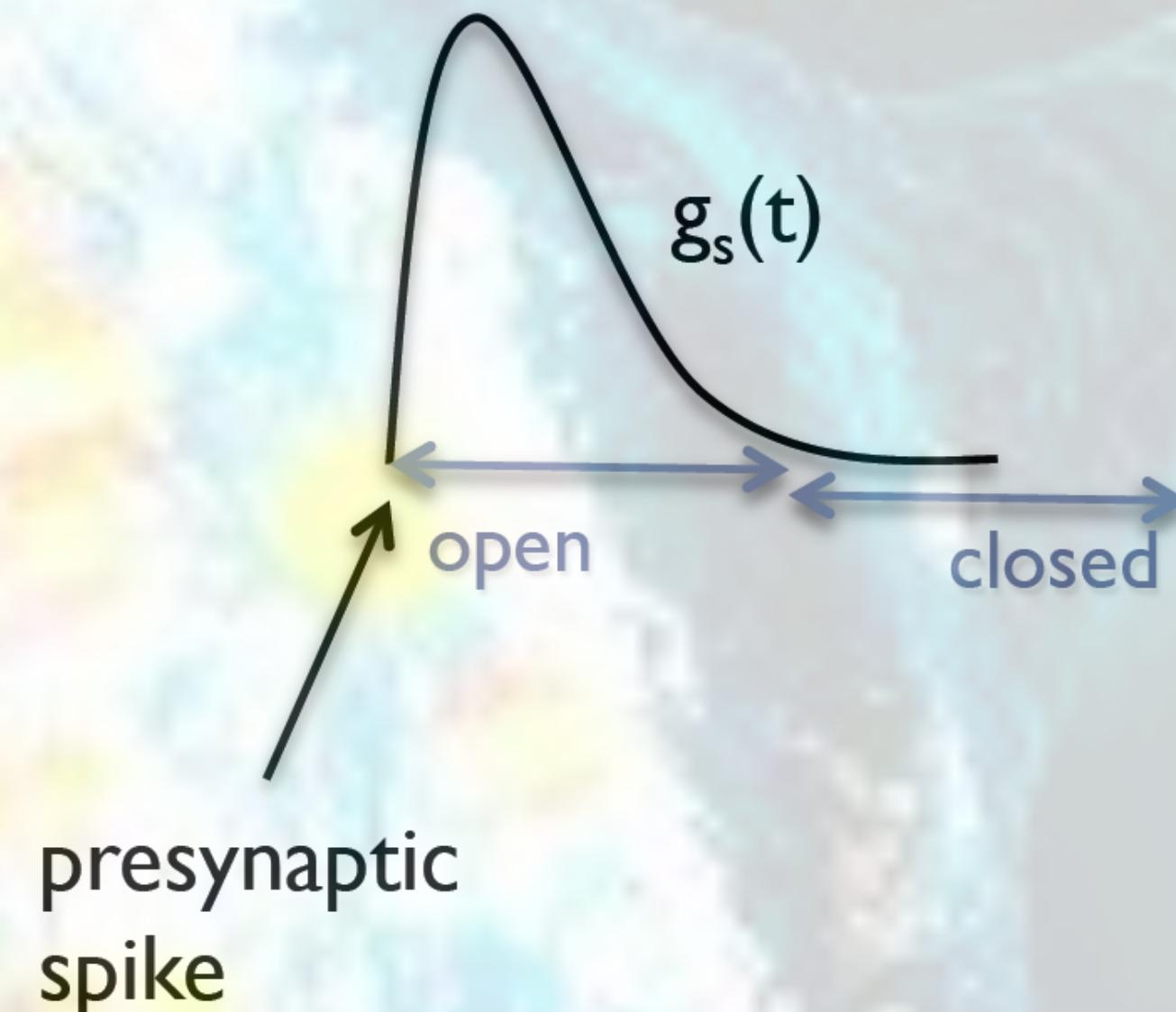
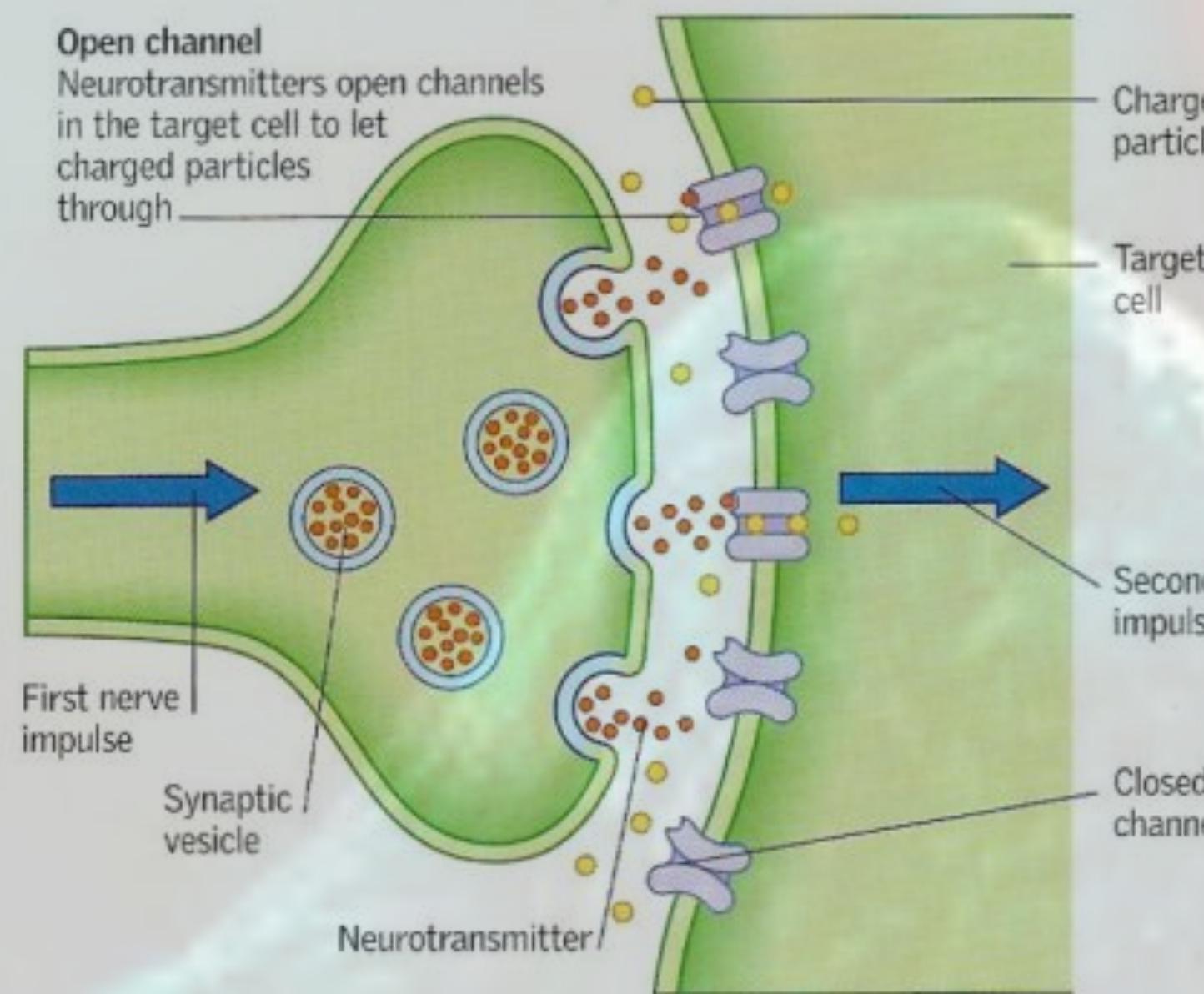


Need to Simplify & Reduce!

The problem



The synapse



$$I_s = g_s (E_s - V_m)$$

If:

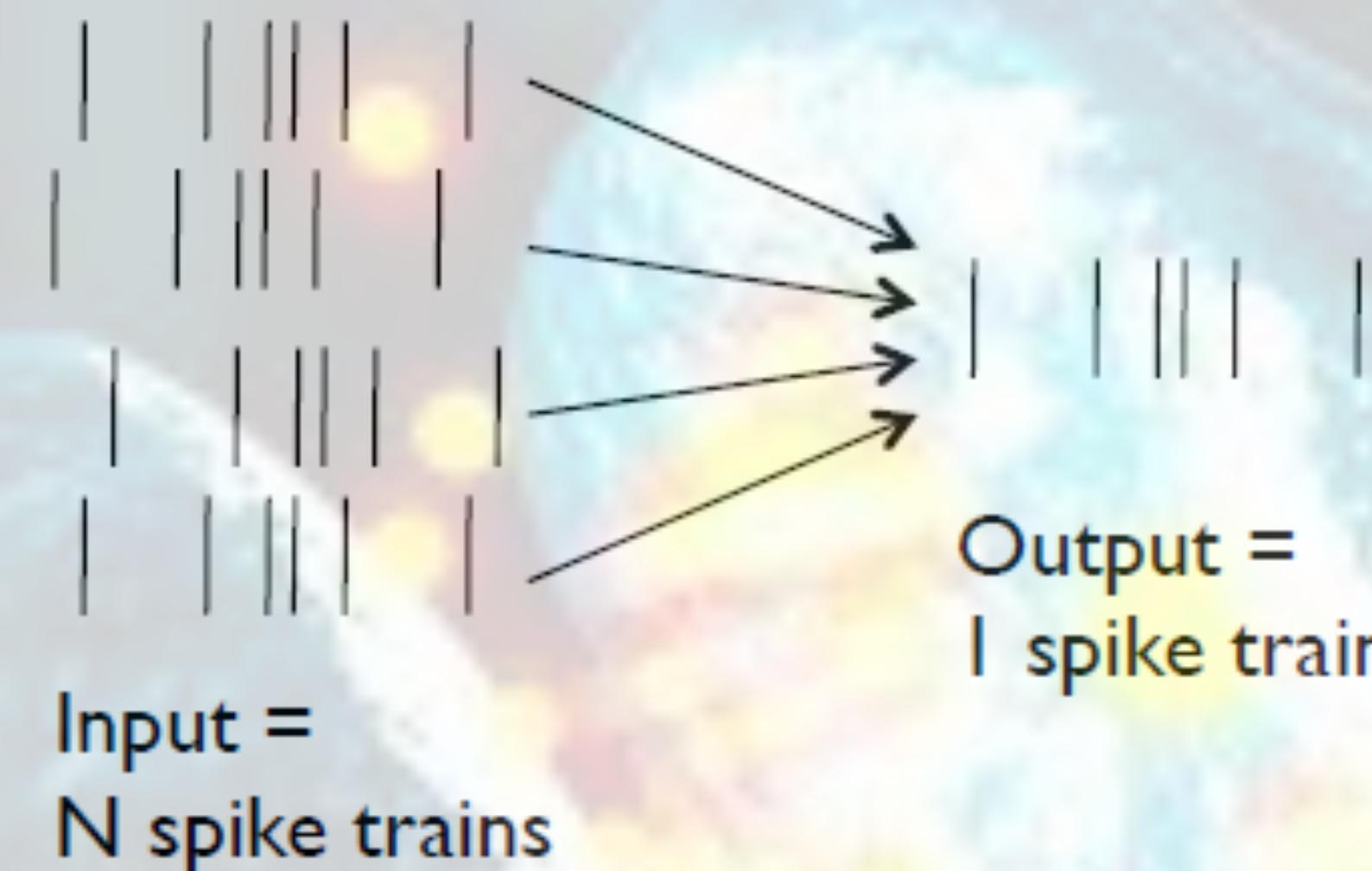
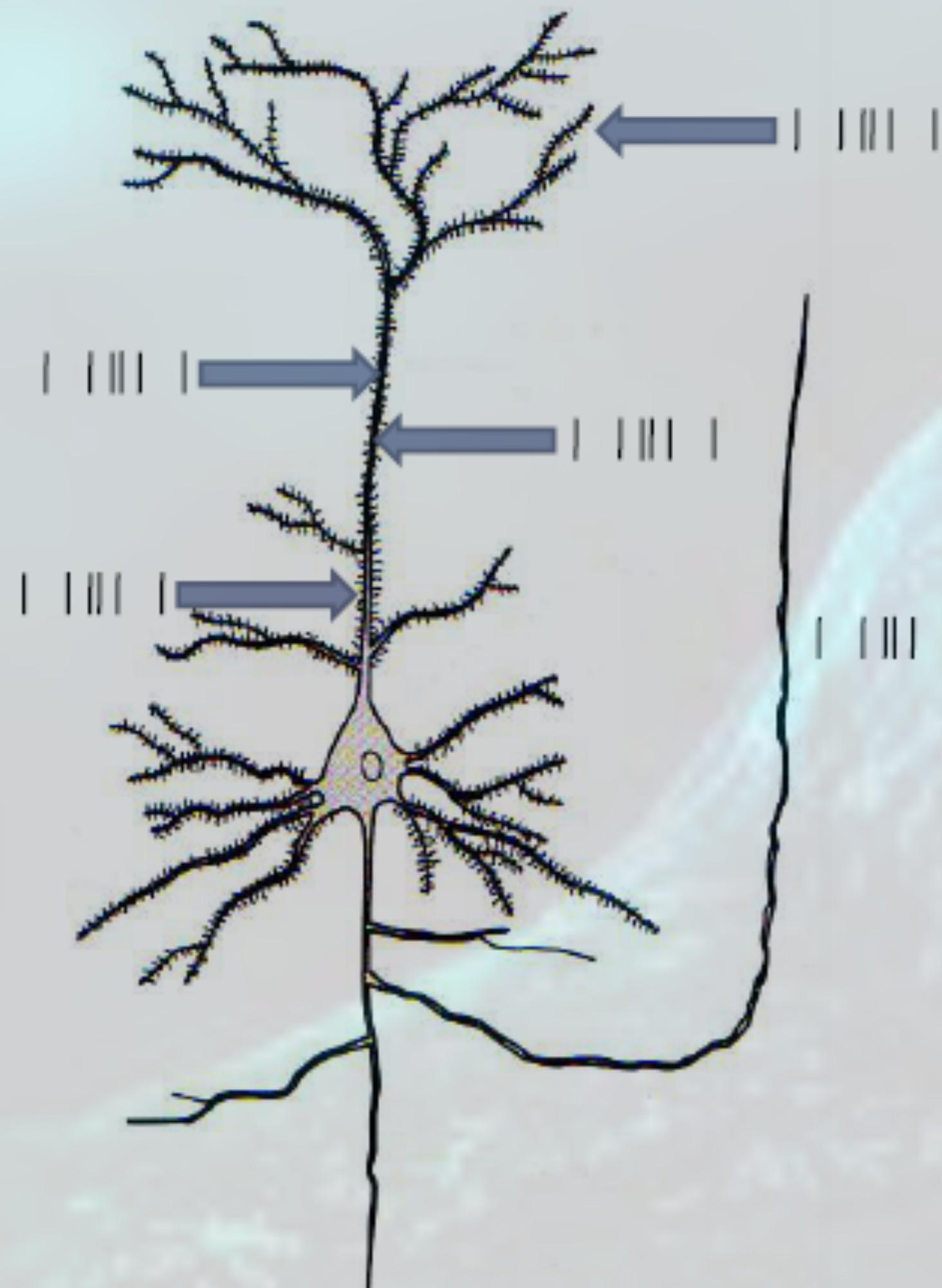
- E_s is « far » from resting and threshold pot ($V_{th}-V_{rest} \ll E_s-V = \Delta V$)
- The maximum conductance is the same for each spike ($G_{max}=cst$)

Then:

$$I_s = G_{max} \Delta V \alpha(t)$$

$$\alpha(t) = \frac{1}{\tau_1 - \tau_2} (e^{-\frac{(t-t_{spike})}{\tau_1}} - e^{-\frac{(t-t_{spike})}{\tau_2}})$$

Neuronal integration

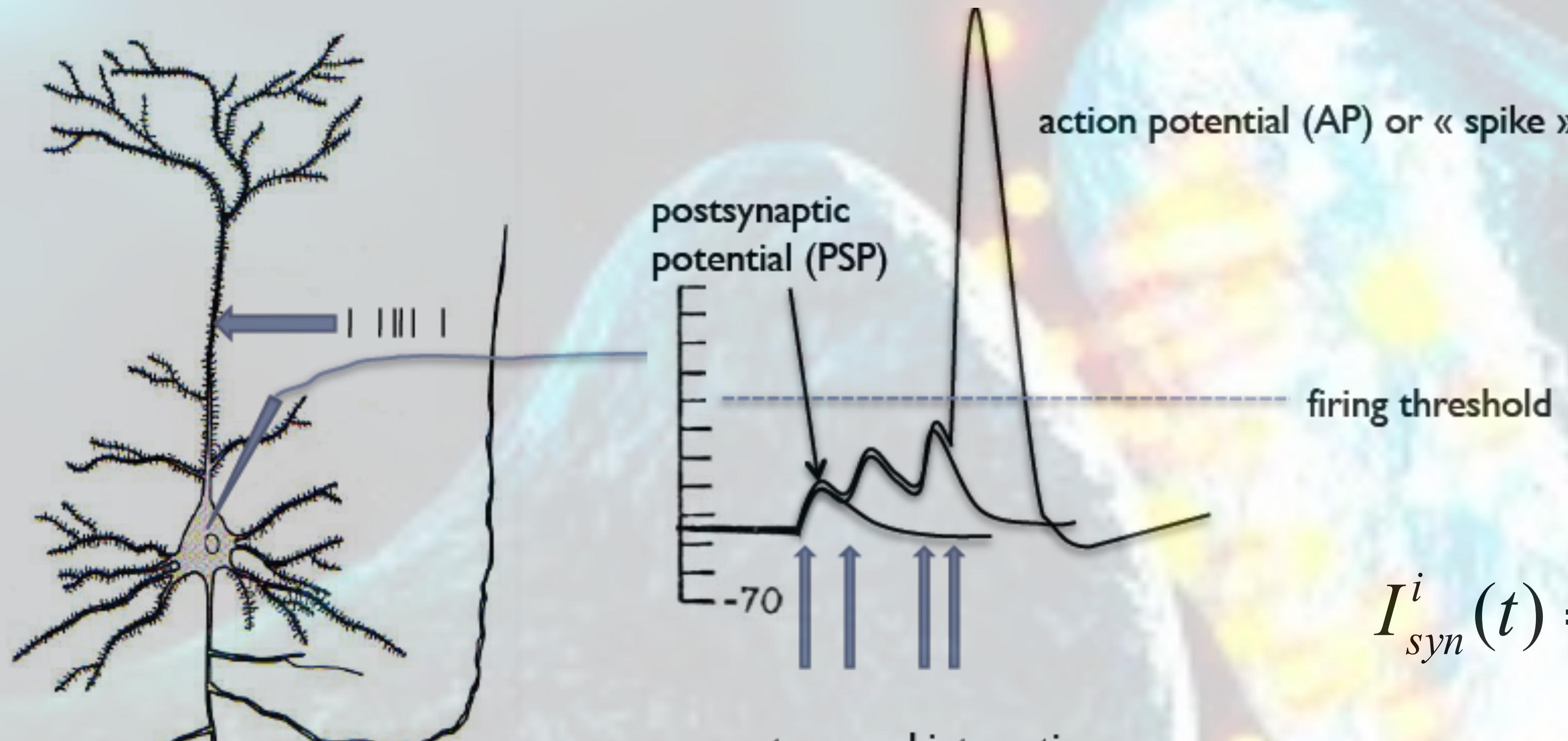


$$I_s = G_{max} \Delta V \alpha(t) = w_{ij} \alpha(t)$$

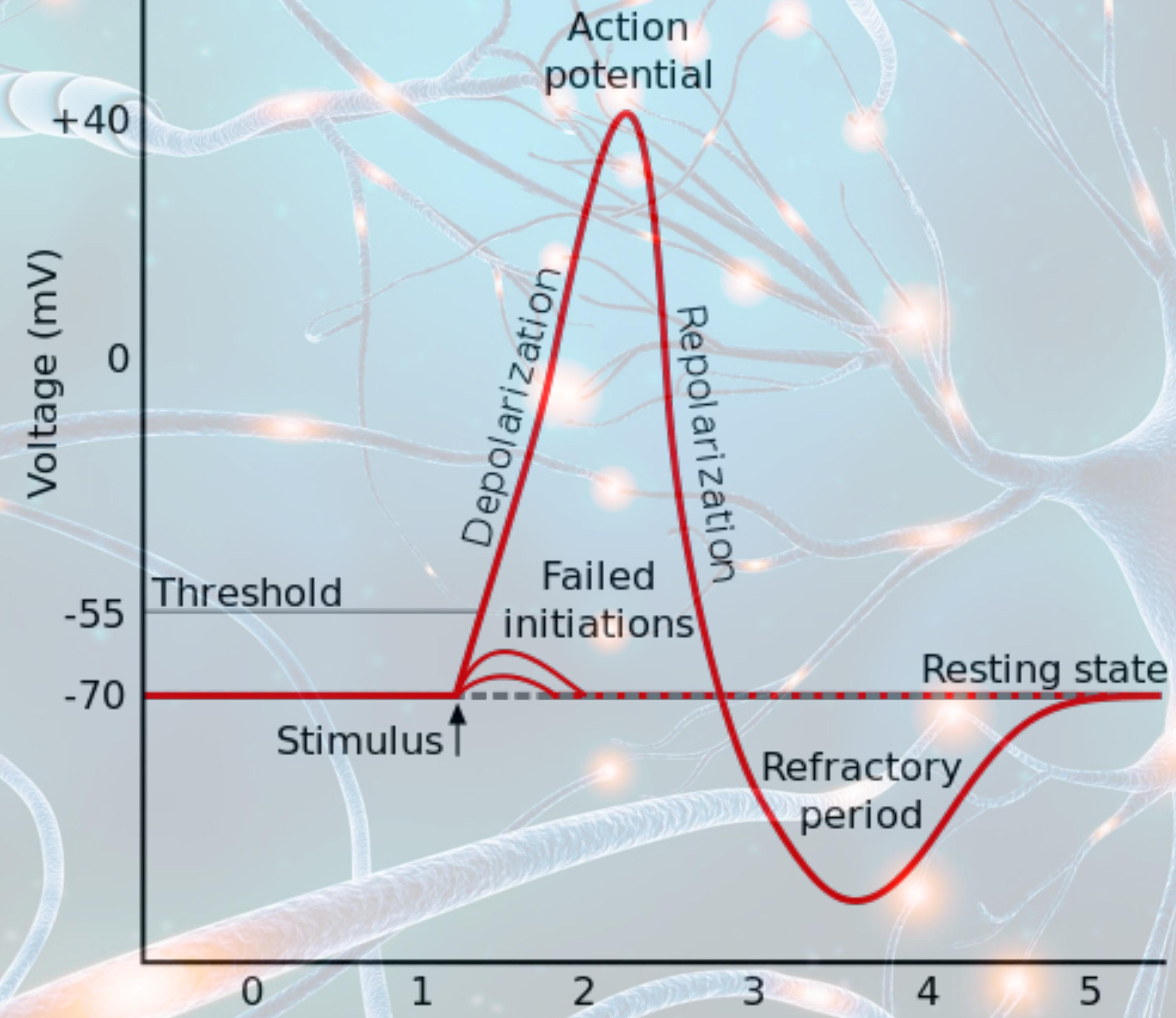
Total synaptic current in neuron i at time t :

$$I_{syn}^i(t) = \sum_j w_{ij} \sum_k \alpha(t - t_j^{(k)})$$

Neuronal integration

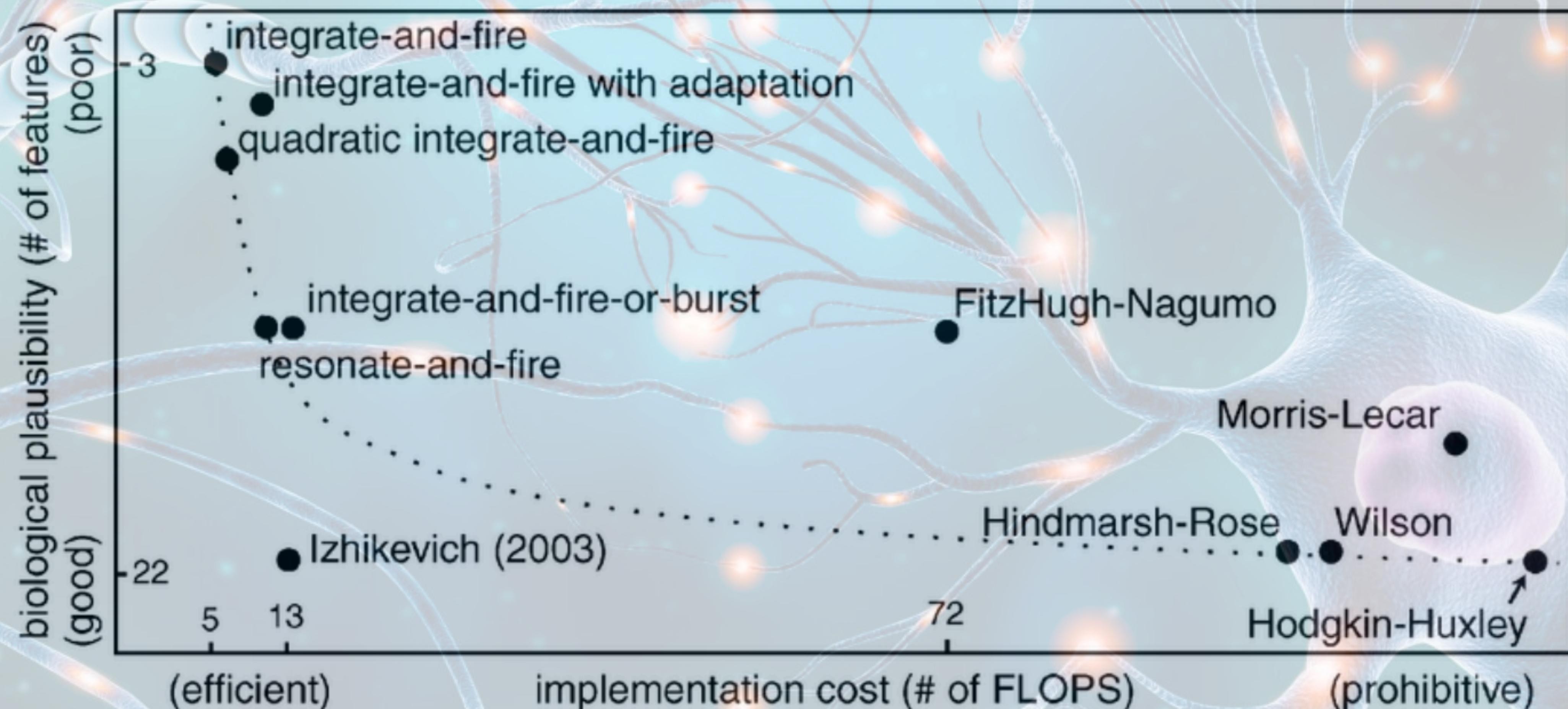


$$I_{syn}^i(t) = \sum_j w_{ij} \sum_k \alpha(t - t_j^{(k)})$$



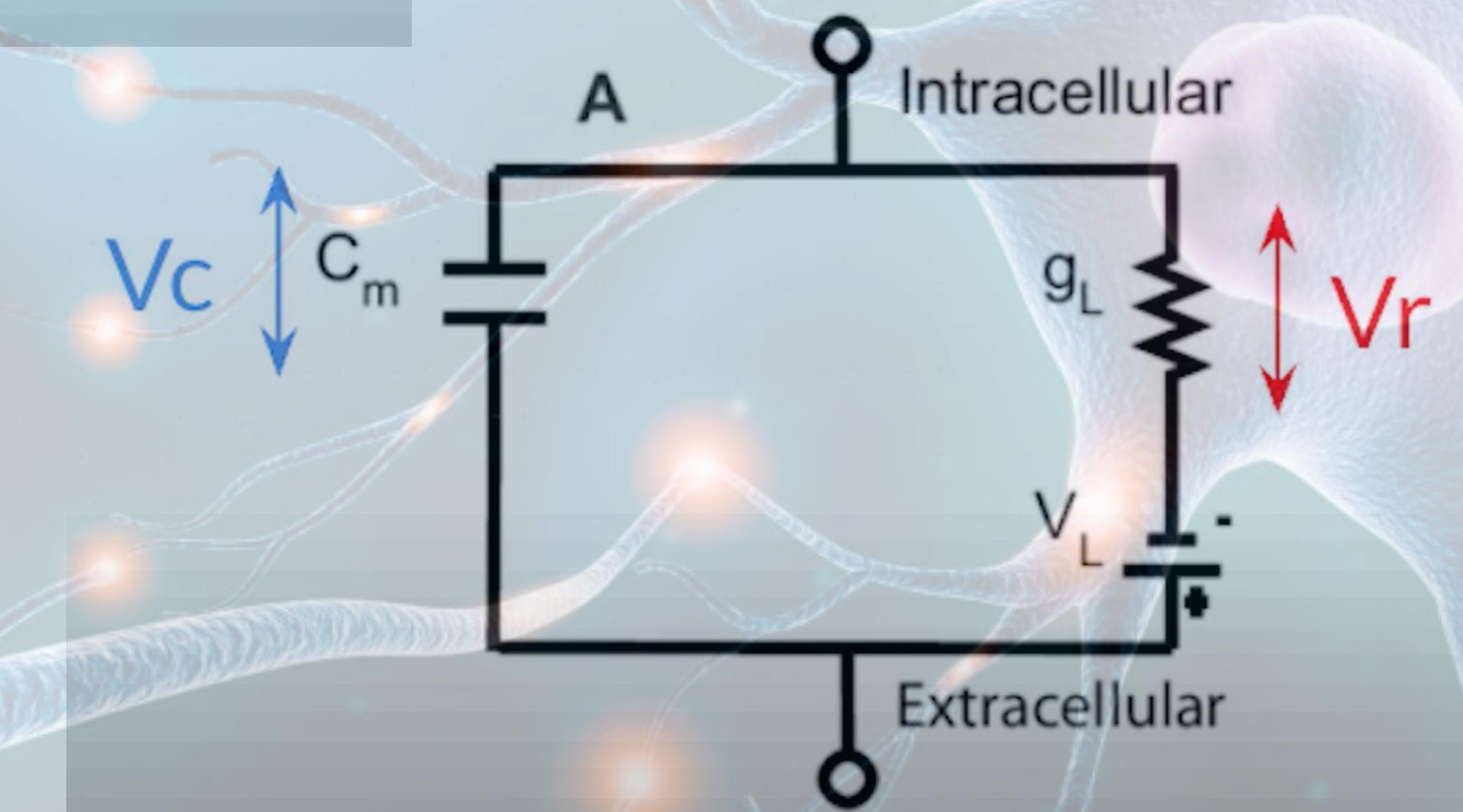
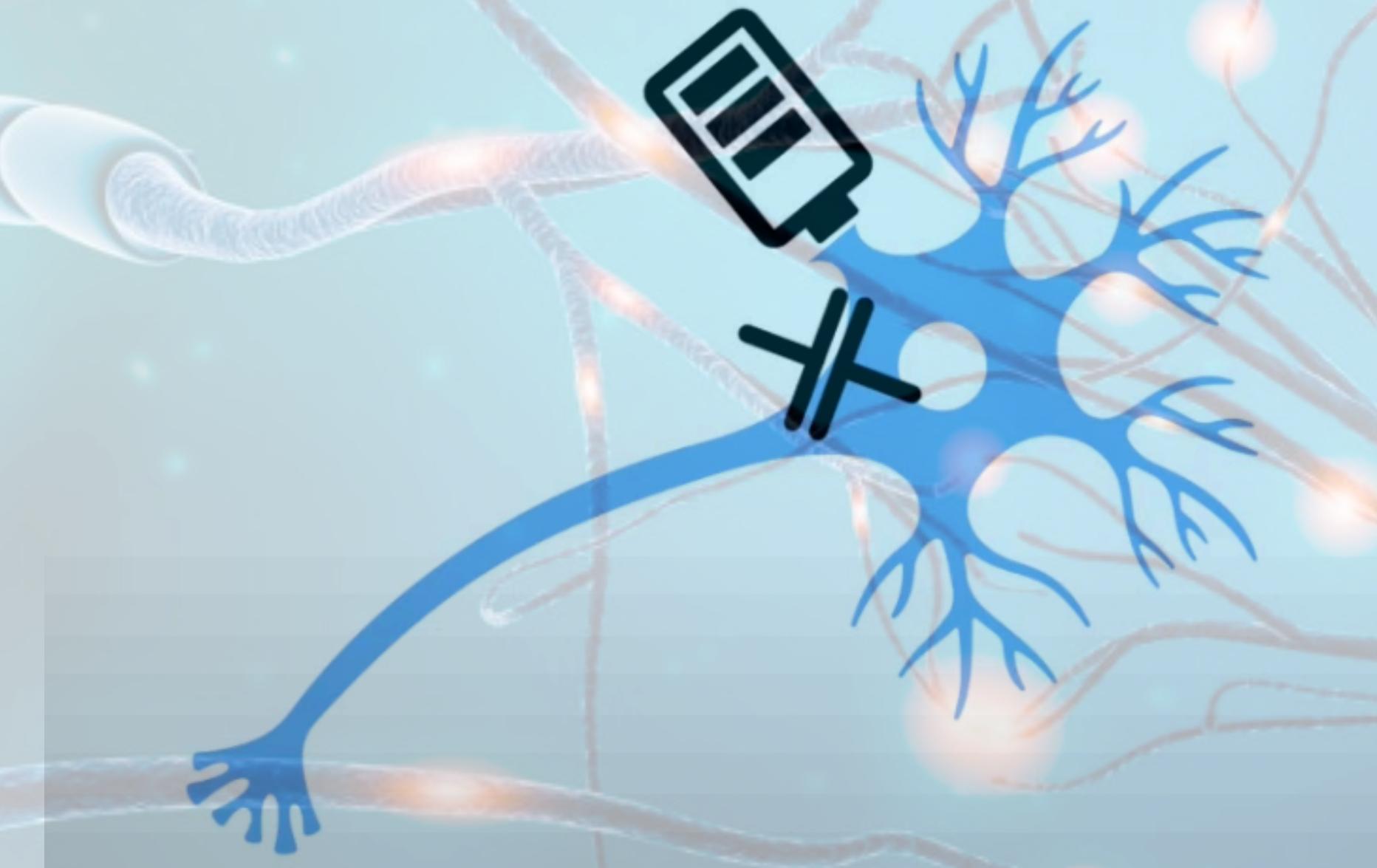
Single neuron models:

- Hodgkin&Huxley: description of ion channel dynamics
- Integrate-and-fire: description of membrane potential
- Rate models: description of the average firing rate



Trade-off between biological plausibility (with respect to a number of features known to exhibited by neurons) and implementation cost in FLOPS per 1ms simulation of the model (from: Izhikevich 2004).

Single neuron models: Integrate-and-fire



Single neuron models: Integrate-and-fire

Hypothesis: no active currents ($g = \text{cste}$)

$$C \frac{dV}{dt} = g_{Na}(V_{Na} - V) + g_K(V_K - V) + g_L(V_L - V) + I_{ext}$$

$$C \frac{dV}{dt} = g_{Na}V_{Na} + g_KV_K + g_LV_L - (g_{Na} + g_K + g_L)V + I_{ext}$$

$$C \frac{dV}{dt} = G_{tot}(V_0 - V) + \tilde{I}_{ext}$$

$$\boxed{\tau \frac{dV}{dt} = (V_0 - V) + \frac{\tilde{I}_{ext}}{G_{tot}}}$$

$$\tau = \frac{C}{G_{tot}}$$

Single neuron models: Integrate-and-fire

$$\tau \frac{dV}{dt} = (V_0 - V) + \frac{\tilde{I}_{ext}}{G_{tot}}$$

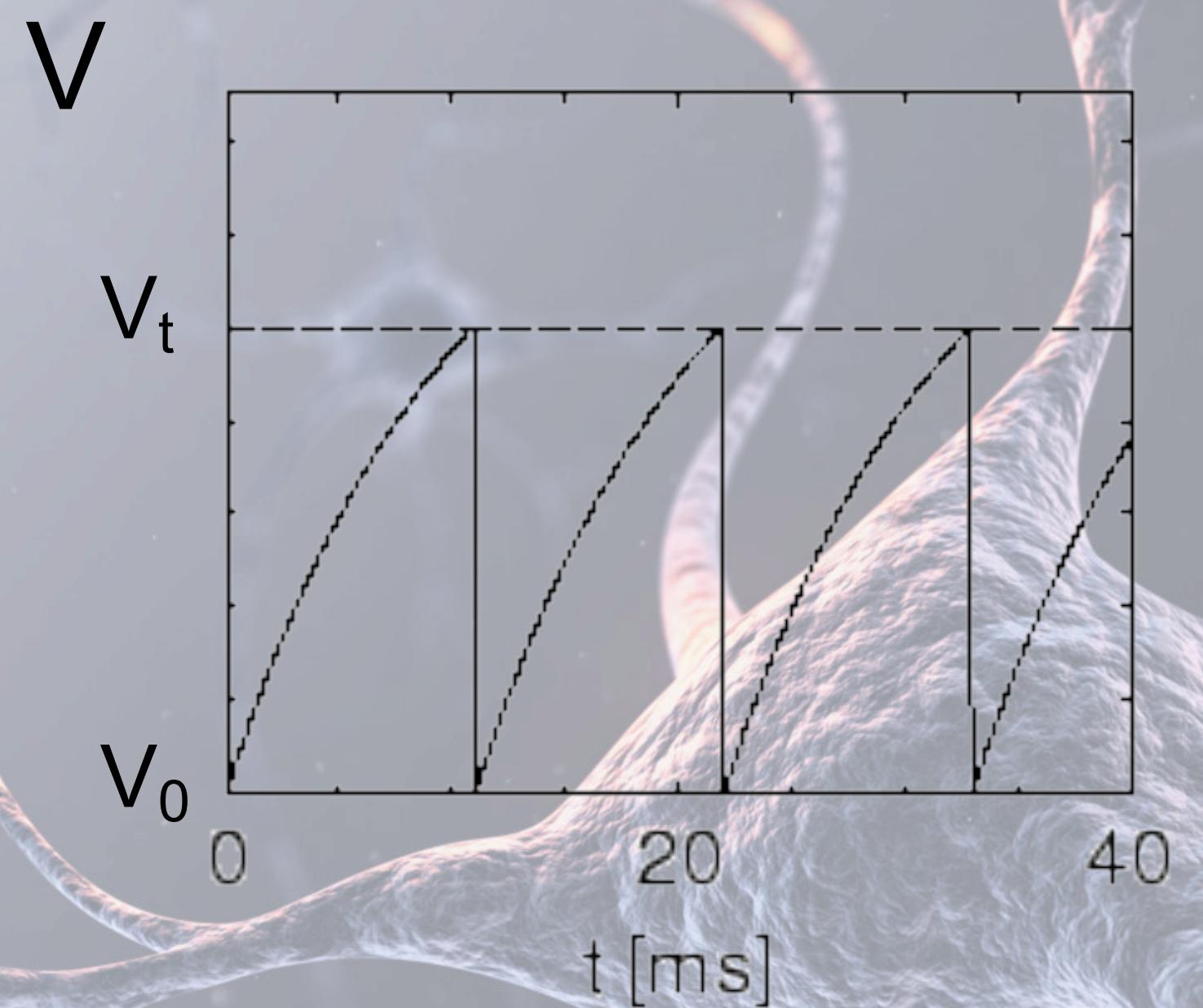
- V_0 : resting potential
- τ : membrane time constant
- I : external current (synaptic)
- G_{tot} : total conductance

$$V = V_0 + \frac{I_0}{G_{tot}} \left(1 - e^{-\frac{t-t_0}{\tau}}\right)$$

- If $V=V_t$ (threshold), neuron spikes and $V \rightarrow V_0$

Single neuron models: Integrate-and-fire

Voltage as a function of time



$$V_t - V_0 = \frac{I_0}{G_{tot}} \left(1 - e^{-\frac{T}{\tau}}\right)$$

$$T = \tau \ln\left(\frac{I_0}{I_0 - G_{tot}(V_t - V_0)}\right)$$

With a refractory period, the firing rate is:

$$\nu = 1 / (D_{ref} + \tau \ln\left(\frac{I_0}{I_0 - G_{tot}(V_t - V_0)}\right))$$

How to build the network model

Important anatomical and physiological data:

- Time constant of connections and integration: synaptic delays, synaptic time constants, neuronal time constants
- Pattern of connectivity: between neuronal populations (circuit), within neuronal populations (recurrent), within each interconnection (connectivity pattern)
- External inputs: sensory information, inputs from other brain areas.
- Sources of variability: synaptic noise, random connectivity, heterogeneities.
- Neuromodulation?

Step 1: a simplified network for mathematical analysis

- Simple neuron model (linear rate model or Integrate-and-Fire)
- All-to-all connectivity or simple connectivity pattern (Gaussian?)
- No noise, no heterogeneities

Step 2: numerical simulations in a realistic model

- Realistic neuron model (non-linear input-output, H&H, conductance based...)
- Realistic connectivity pattern (with some randomness)
- Synaptic noise
- Heterogeneities in single neuron parameters (threshold, gain)

