Introduction to Experimental Neuroscience: Neurocomputation | July 2021

Single neuron and network dynamics

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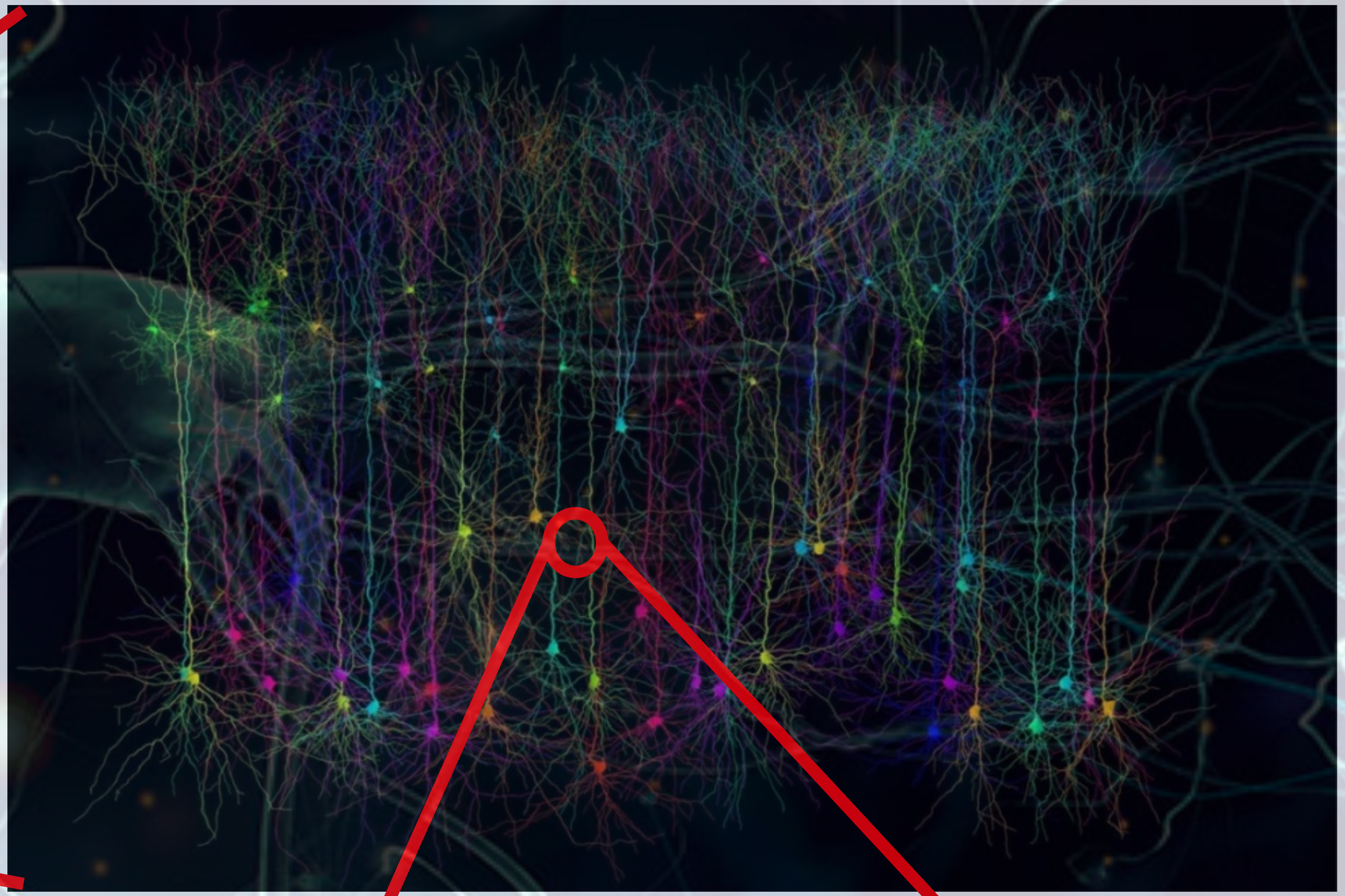
Computational Neuroscience

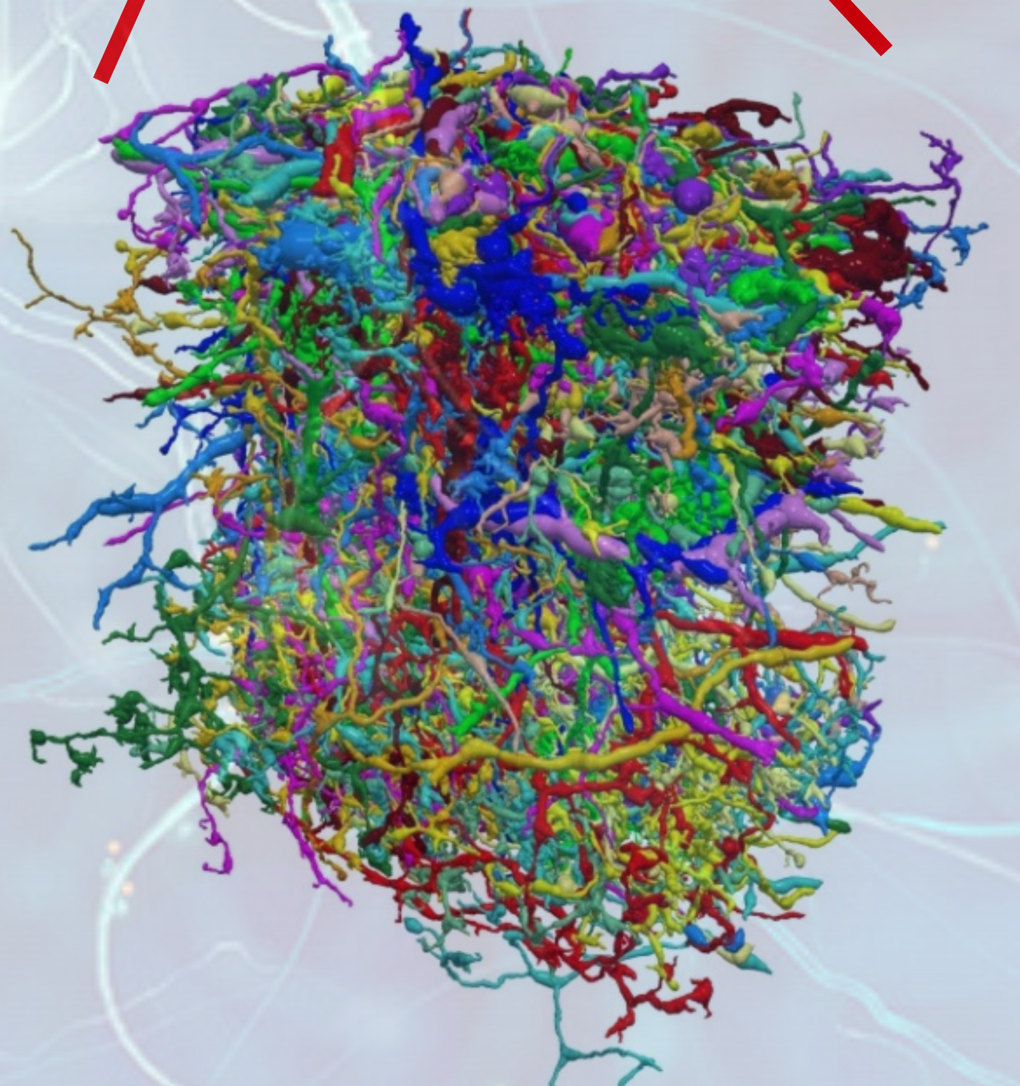
***Wikipedia*:** Computational neuroscience is the study of the computational principles underlying brain function and neuronal activity, i.e. the generic algorithms allowing the implementation of cognitive functions in our central nervous system.

The problem

Brain: 10^{11} neurons (100 billions)

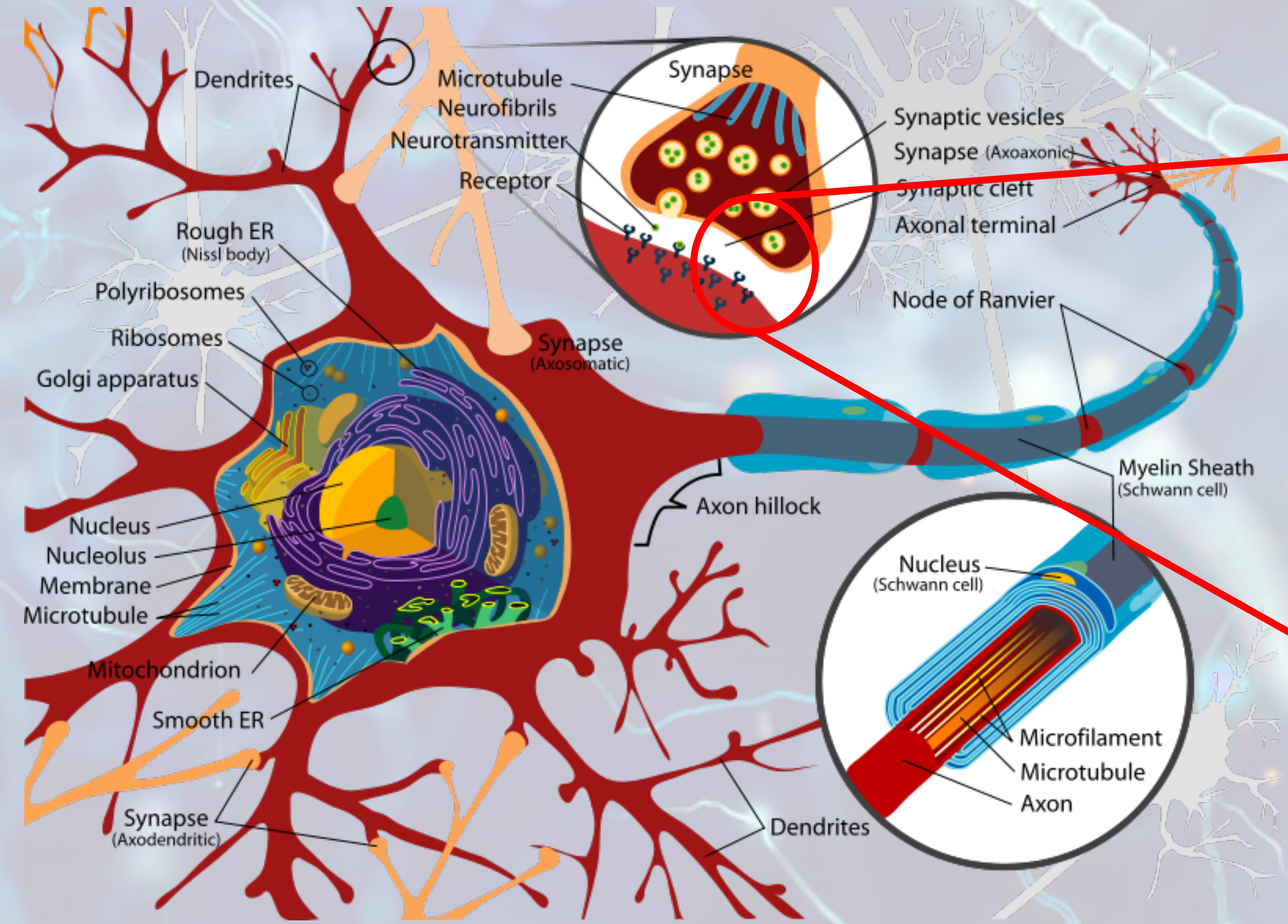


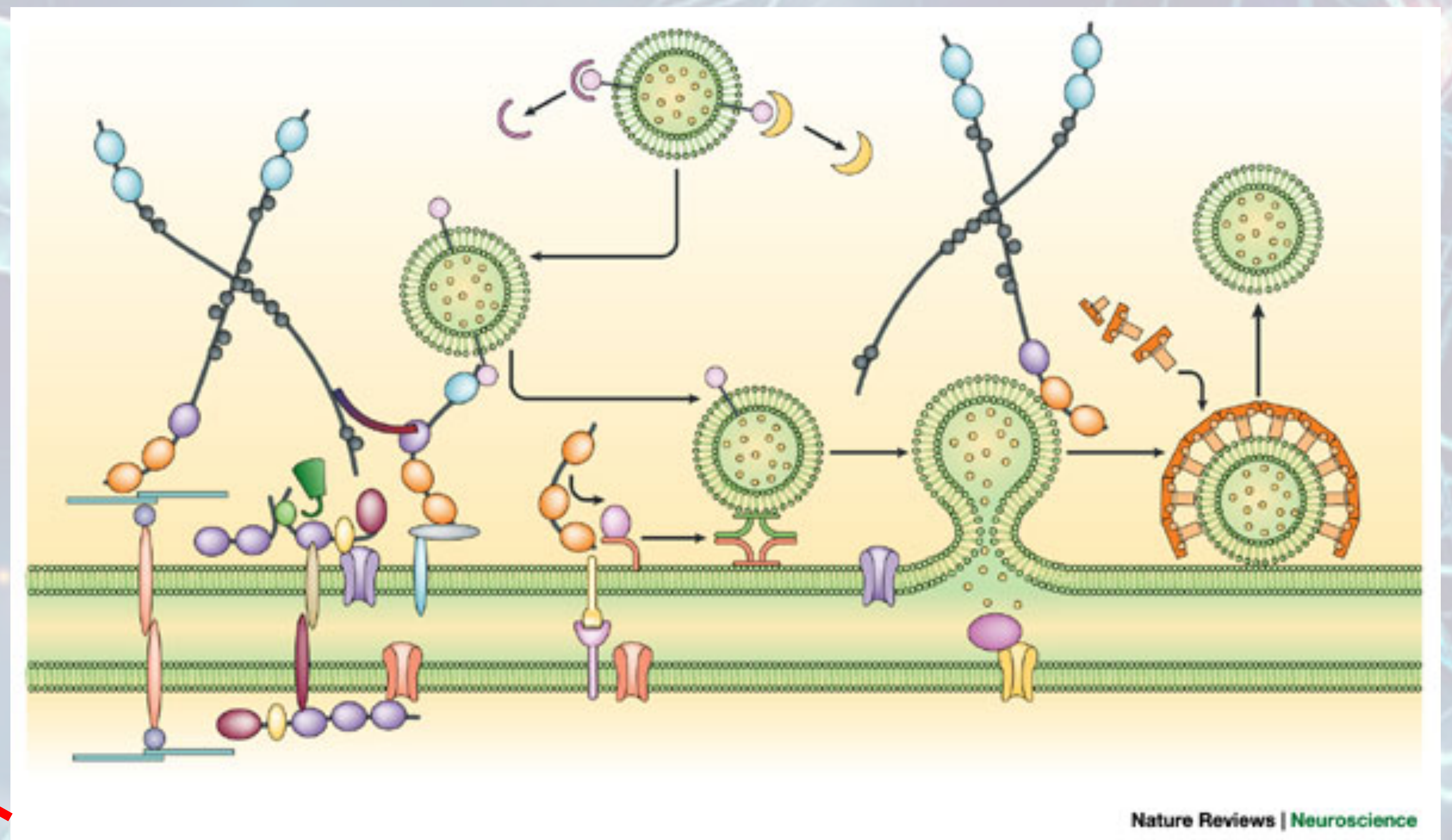
Neuron: >1000 synapses



The problem

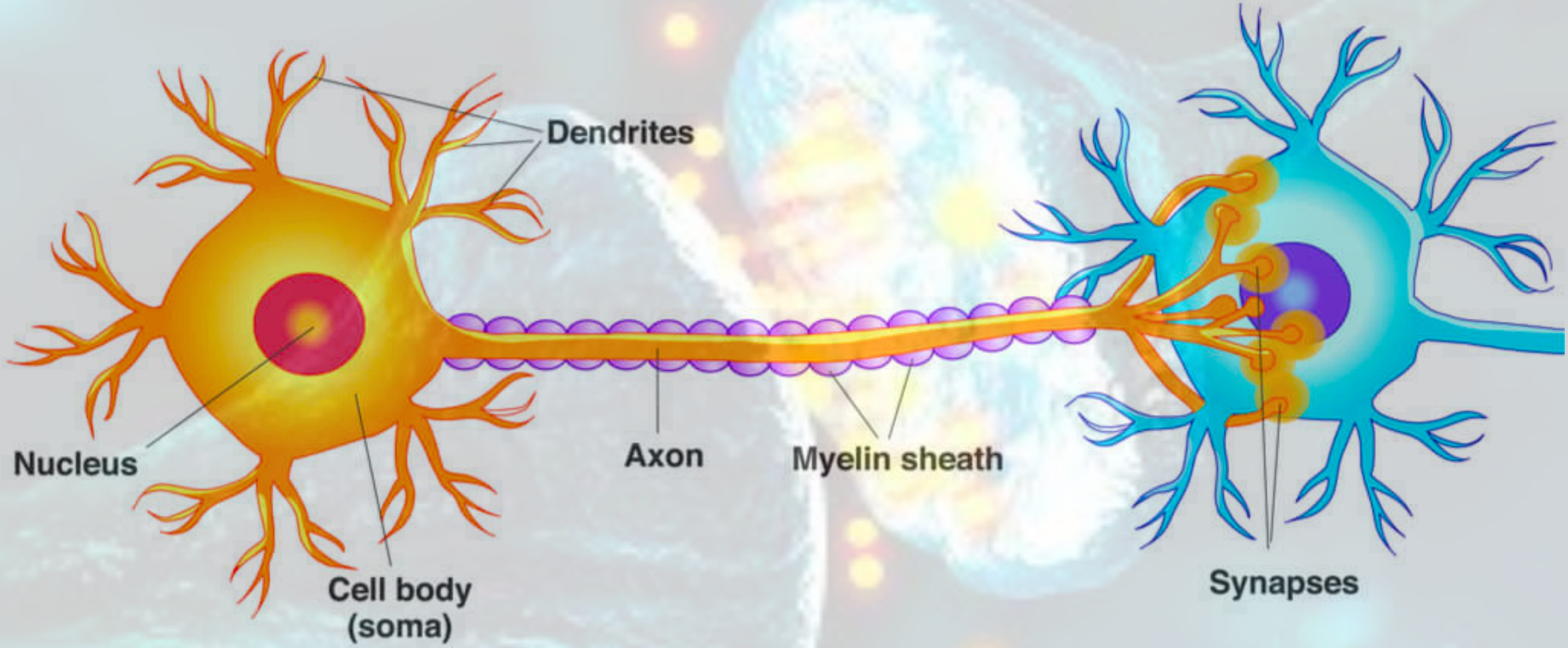
Neuron: ~10^6 ion channels, complex interactions between various intracelullar actors….



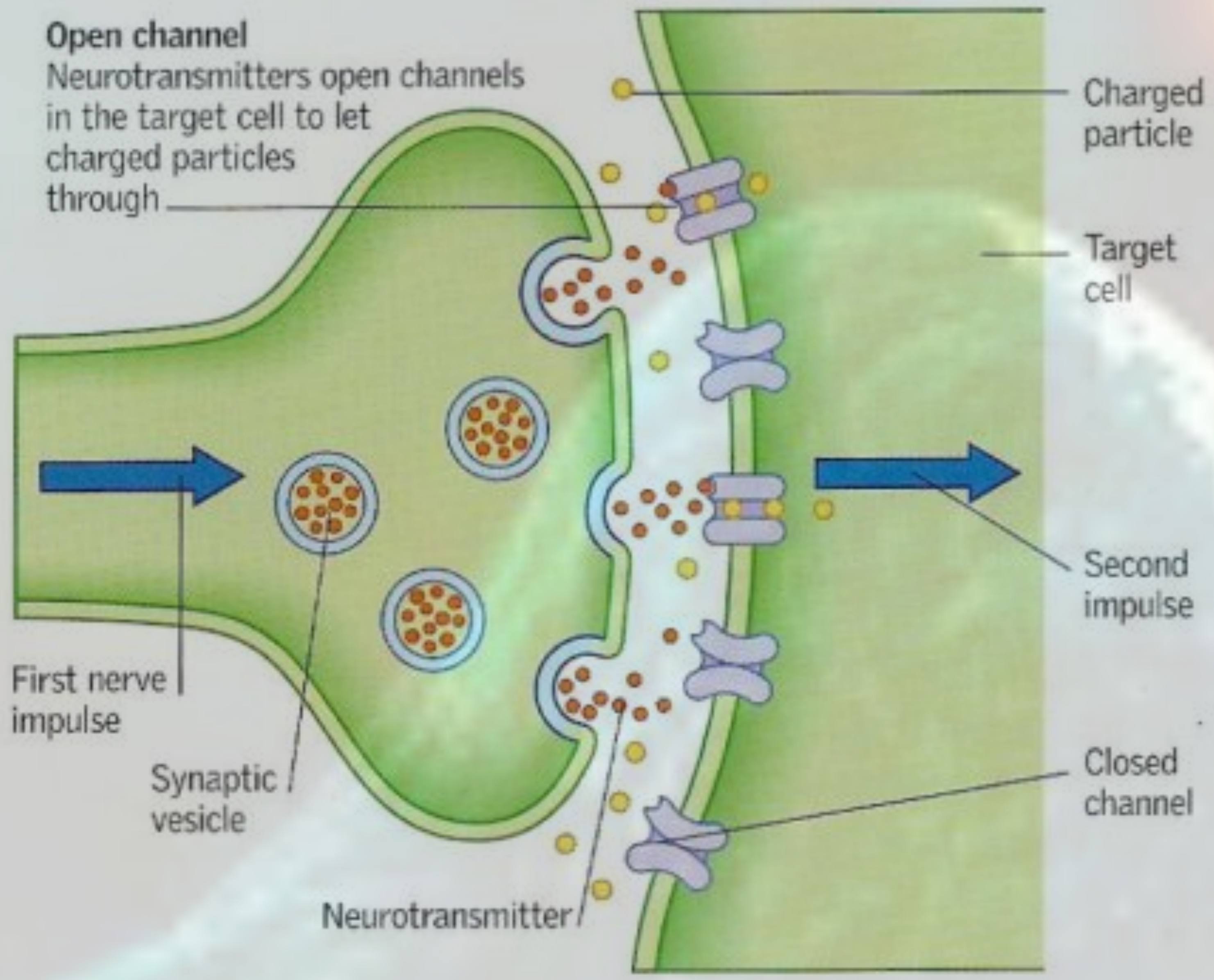


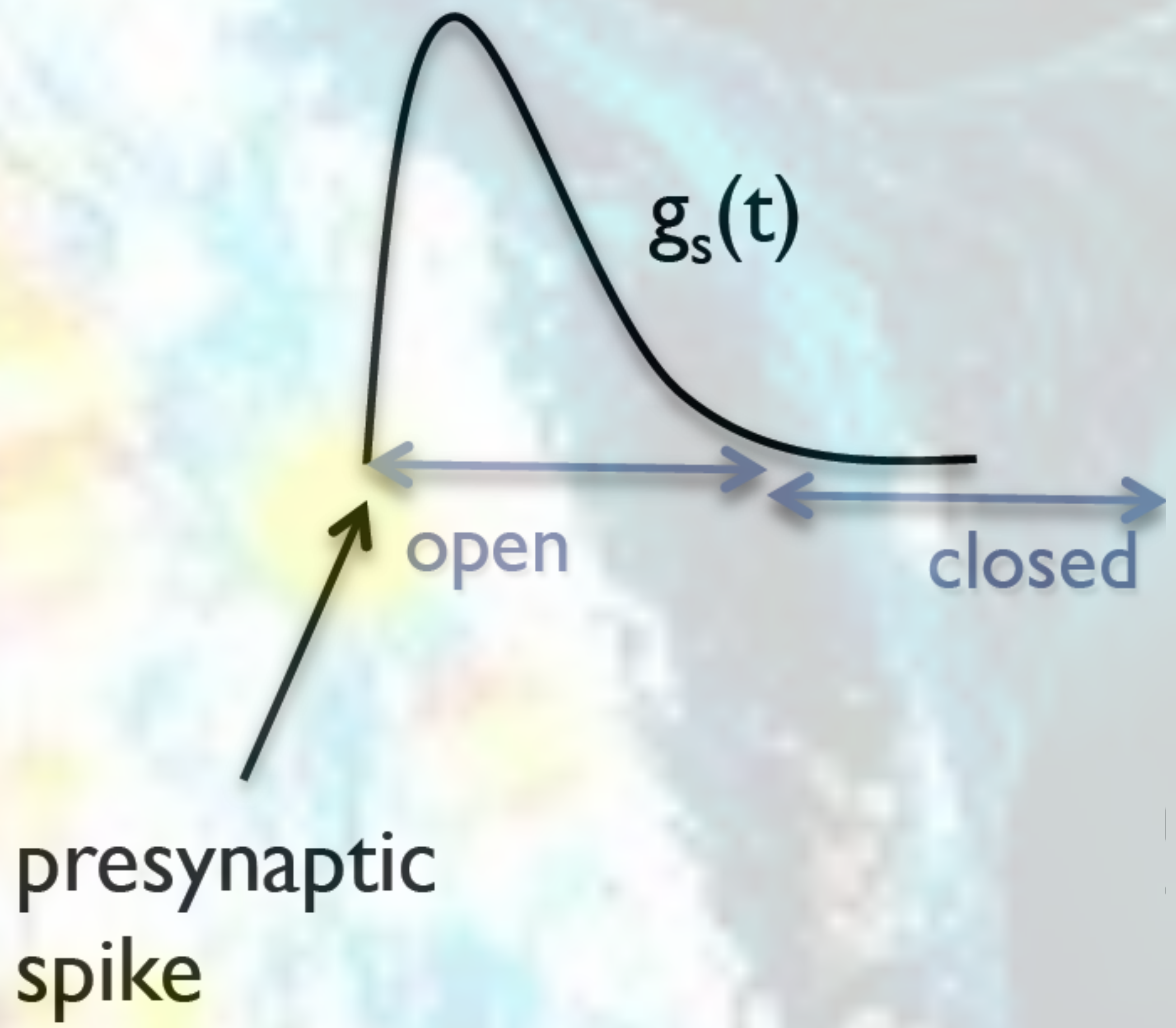
Need to Simplify & Reduce!

The problem


Dendrites
Nucleus
Cell body
(soma)
Axon
Myelin sheath
Synapses

The synapse





$$I_s = g_s(E_s - V_m)$$

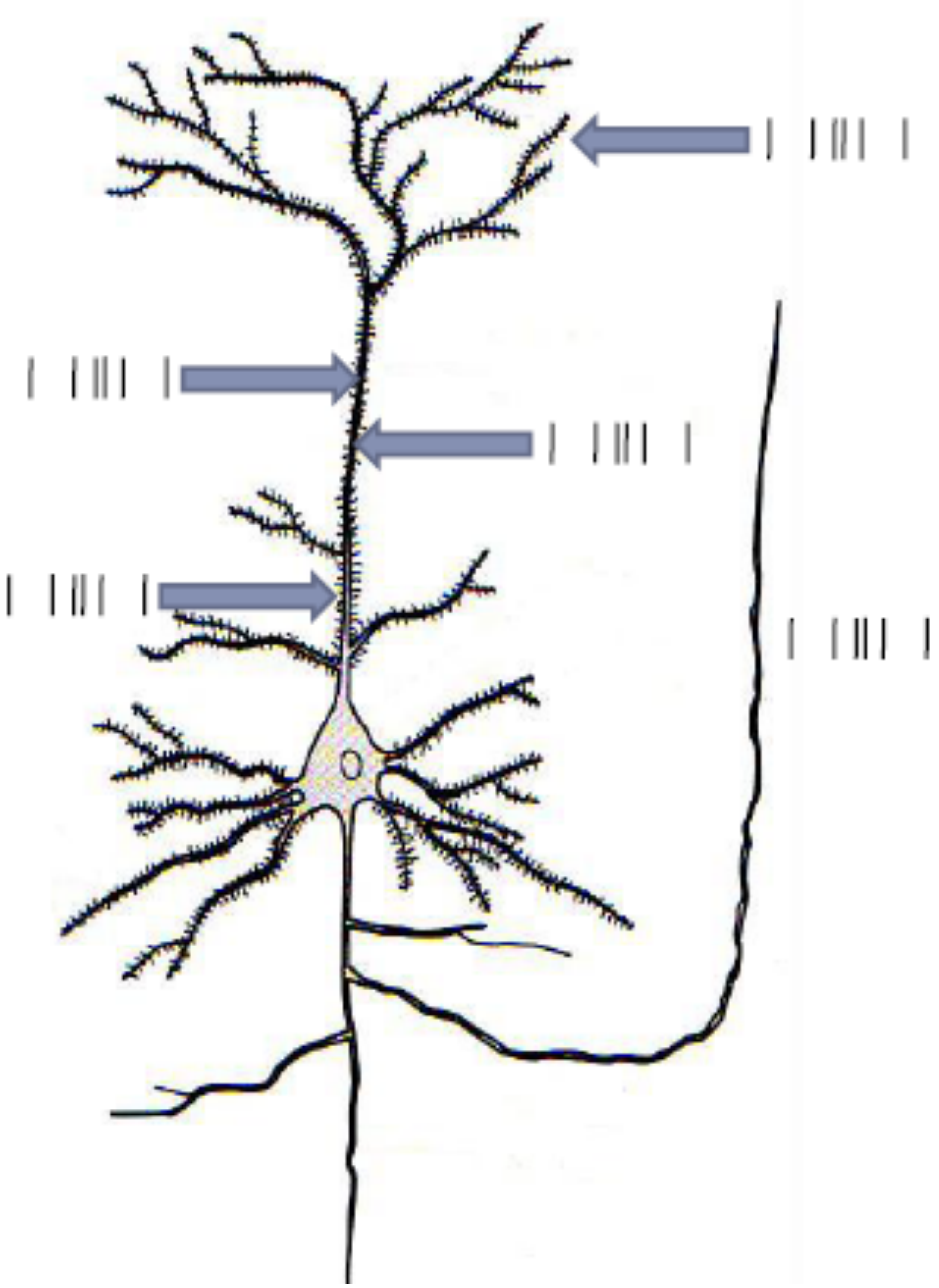
If:

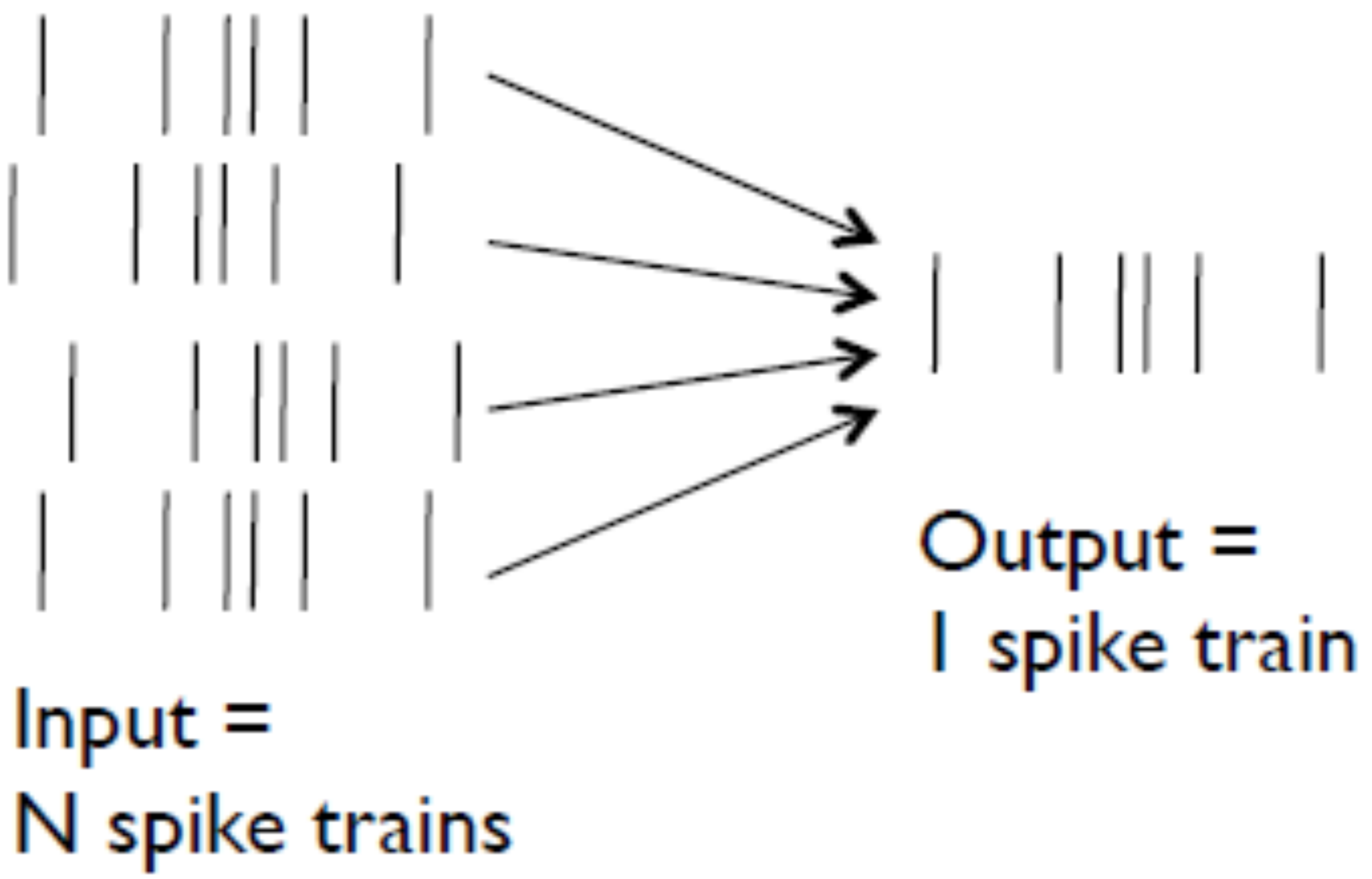
- Es is « far » from resting and threshold pot (Vth-Vrest << Es-V = ΔV)
- The maximum conductance is the same for each spike (Gmax=cst)

Then: $$I_s = G_{max}\,\Delta V\,\alpha(t)$$

$$\alpha(t) = \frac{1}{\tau_1 - \tau_2}\left(e^{-(t-t_{spike})/\tau_1} - e^{-(t-t_{spike})/\tau_2}\right)$$

Neuronal integration



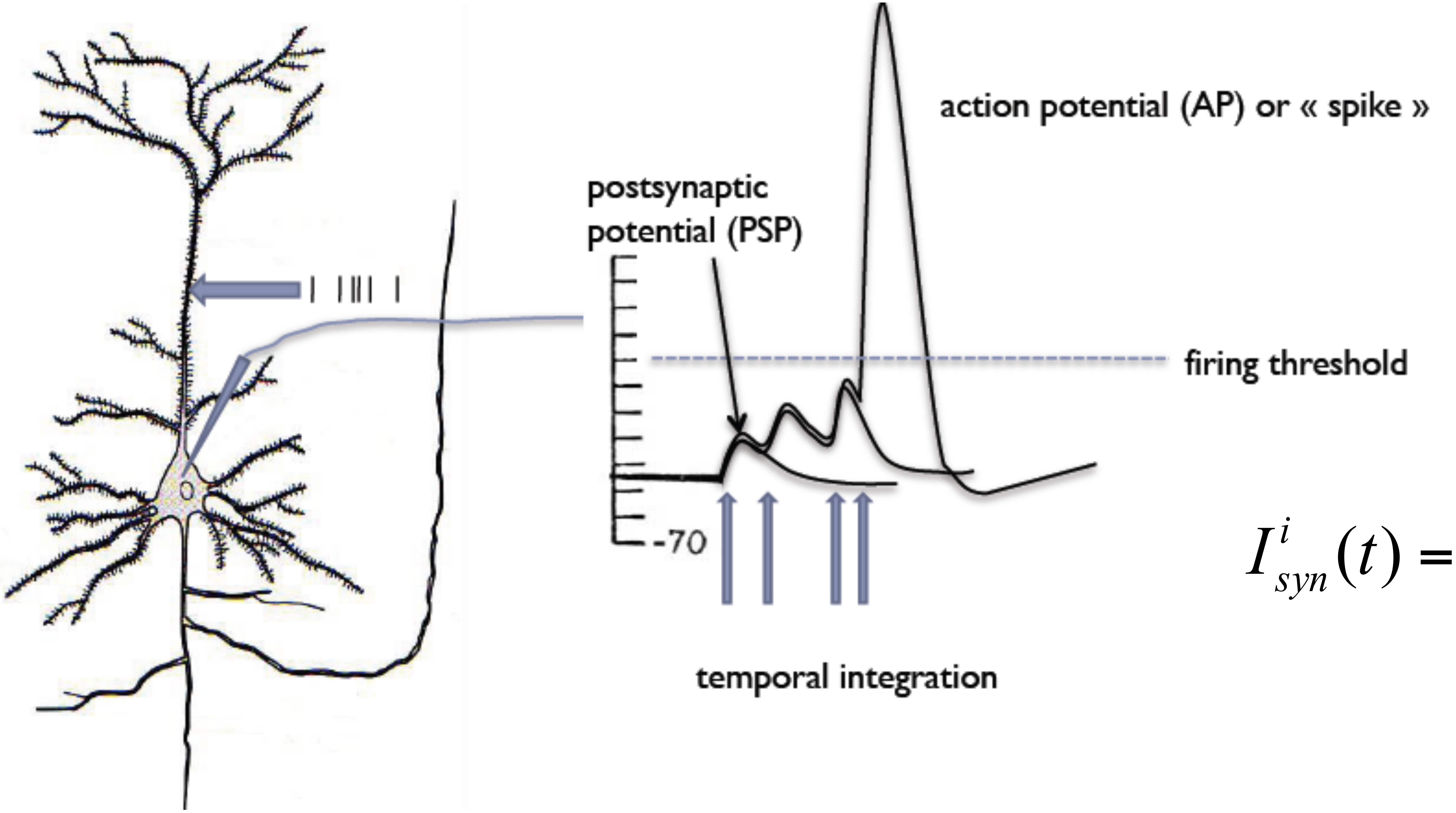


$$I_s = G_{max}\,\Delta V\,\alpha(t) = w_{ij}\,\alpha(t)$$

Total synaptic current in neuron *i* at time *t*:

$$I^i_{syn}(t) = \sum_j w_{ij} \sum_k \alpha\left(t - t_j^{(k)}\right)$$

Neuronal integration



$$I^{i}_{syn}(t) = \sum_j w_{ij} \sum_k \alpha(t - t_j^{(k)})$$

Neuronal integration

Single neuron models:

- Hodgkin&Huxley: description of ion channel dynamics
- Integrate-and-fire: description of membrane potential
- Rate models: description of the average firing rate

Hypothesis: no active currents (g=cste)

$$C\frac{dV}{dt} = g_{Na}(V_{Na} - V) + g_K(V_K - V) + g_L(V_L - V) + I_{ext}$$

$$C\frac{dV}{dt} = g_{Na}V_{Na} + g_K V_K + g_L V_L - (g_{Na} + g_K + g_L)V + I_{ext}$$

$$C\frac{dV}{dt} = G_{tot}(V_0 - V) + \tilde{I}_{ext}$$

$$\boxed{\tau\frac{dV}{dt} = (V_0 - V) + \frac{\tilde{I}_{ext}}{G_{tot}}}$$

$$\tau = \frac{C}{G_{tot}}$$

Single neuron models: Integrate-and-fire

$$\tau \frac{dV}{dt} = (V_0 - V) + \frac{\tilde{I}_{ext}}{G_{tot}}$$

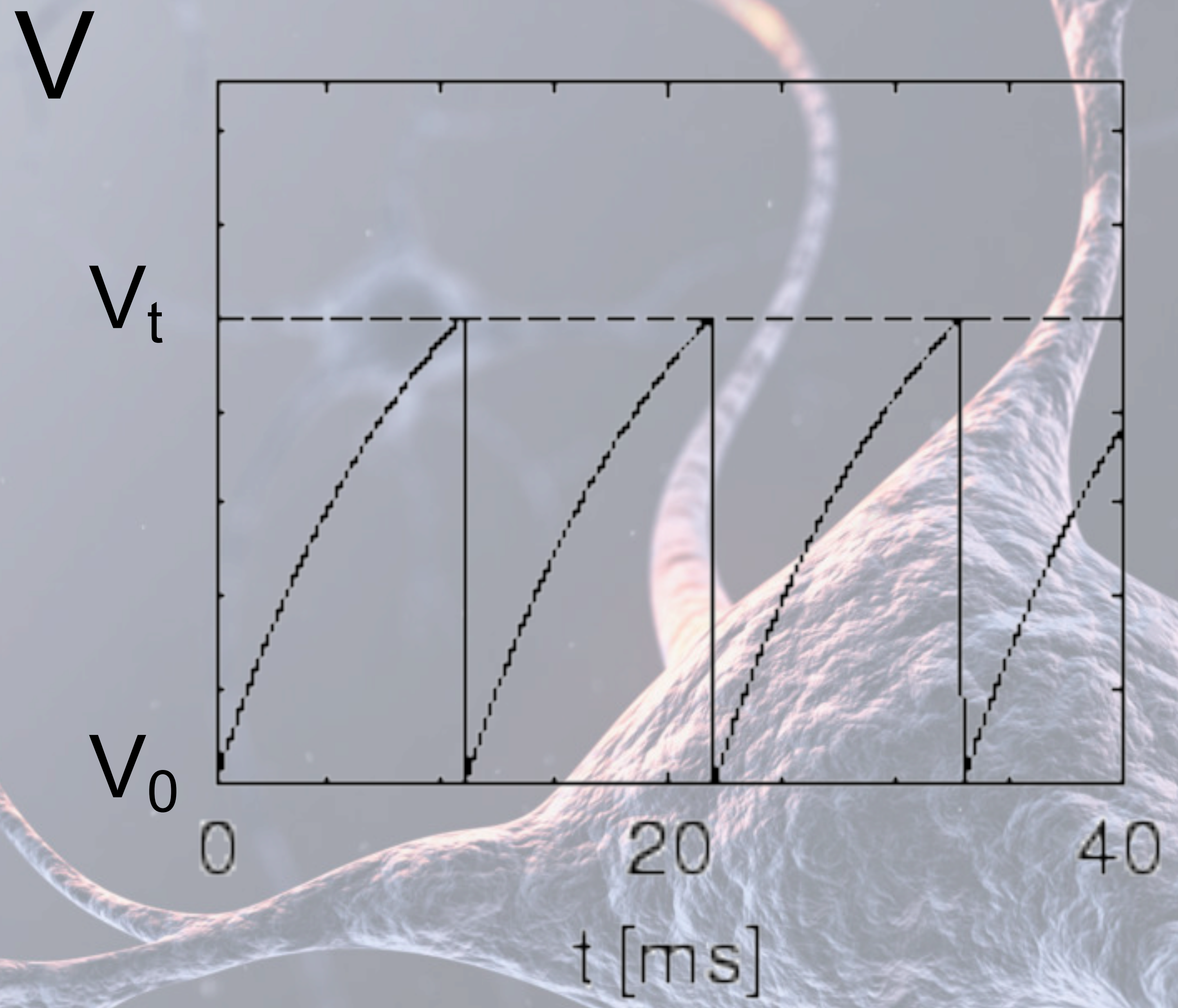
- V_0 : resting potential
- τ membrane time constant
- I : external current (synaptic)
- G_{tot} : total conductance

$$V = V_0 + \frac{I_0}{G_{tot}}\left(1 - e^{-\frac{t-t0}{\tau}}\right)$$

- If V=Vt (threshold), neuron spikes and V $\rightarrow V_0$

Single neuron models: Integrate-and-fire

Voltage as a function of time



$$V_t - V_0 = \frac{I_0}{G_{tot}}\left(1 - e^{-\frac{T}{\tau}}\right)$$

$$T = \tau \ln\left(\frac{I_0}{I_0 - G_{tot}(V_t - V_0)}\right)$$

With a refractory period, the firing rate is:

$$\nu = 1 \Big/ \left(D_{ref} + \tau \ln\left(\frac{I_0}{I_0 - G_{tot}(V_t - V_0)}\right)\right)$$

How to build the network model

Important anatomical and physiological data:

- Time constant of connections and integration: synaptic delays, synaptic time constants, neuronal time constants
- Pattern of connectivity: between neuronal populations (circuit), within neuronal populations (recurrent), within each interconnection (connectivity pattern)
- External inputs: sensory information, inputs from other brain areas.
- Sources of variability: synaptic noise, random connectivity, heterogeneities.
- Neuromodulation?

How to build the network model

Step 1: a simplified network for mathematical analysis

- Simple neuron model (linear rate model or Integrate-and-Fire)
- All-to-all connectivity or simple connectivity pattern (Gaussian?)
- No noise, no heterogeneities

Step 2: numerical simulations in a realistic model

- Realistic neuron model (non-linear input-output, H&H, conductance based…)
- Realistic connectivity pattern (with some randomness)
- Synaptic noise
- Heterogeneities in single neuron parameters (threshold, gain)