MATH 60210: Homework #1

Professor Anthony Sanford

Due: March 13, 2024 by 11:55pm on ZoneCours

Instructions: For this assignment, you are to work in groups of **three**. I will **not** make the groups. It is your responsibility to find team members for your homework. If you have trouble finding a team, you can reach out to me via email and I will try to match you with other classmates. This assignment includes two deliverables: 1) your write-up and 2) your code. Failure to submit one of these **two** components will be considered an incomplete submission. Your code must by written in Python. Your code should run smoothly and be clearly annotated. If you used resources outside of what is assigned for this course, you **must** give credit to the source. For example, if you googled code and used whatever you found, reference the source. Otherwise, you are cheating. Your write-up is meant to analyze, discuss, and interpret your results. Simply submitting a table without clearly discussing that table is not an answer to the question. Consider what I do in class – I present something and discuss it. Think of your write-up in this exact way. You are interpreting your findings for someone who is reading your report. Recall that HEC Montréal has rules regarding plagiarism apply to both your written answers and your computer code.

There are 4 multi-part questions, points for each question/part is written below. For each part, there are four possible grades:

- E: This is the grade if you literally write nothing. Worth 0%
- C: This is the grade if you really don't understand what you're doing...but you wrote something. Worth 50%.
- B: This is the grade if you got the answer mostly correct. Worth 80%. This will likely be the most common grade.
- A: This is the grade if your answer is as good (or better!) than mine. Worth 100%.

Note: Comments in your code that allow me to easily understand what you were trying to do may improve your grade. Particularly ugly or inelegant python code may reduce your grade by 10% (e.g. if you're cutting and pasting a lot because you don't know how to use loops or functions or array operations effectively). You may also lose points if your code is not running correctly or does not provide the same solution as your write-up answer.

Problem 1 [10 Pts]

This question tests your ability to follow instructions. If you follow the instructions for this assignment, you will receive full credit for this question.

- **a.** List the names of your group [2 pts]
- **b.** Type your answers (meaning you did not submit your assignment written by hand) [2 pts]
- **c.** Format your assignment in the form of a write-up, not simply code that you submit (including proper formatting and labelling of tables and graphs) [2 pts]
- **d.** Provide the supporting Python code for your answers [2 pt]
- **e.** Your Python code actually works on its own and provides the answers that you submitted in your write-up [2 pts]

Problem 2 [35 Pts], Value-at-Risk

For this question, we will be looking at some real-world data and calculating some risk measures. To begin, get return data for the value-weighted market return with dividend distributions along with stocks of your choice that each start with the first letter of your team members names. For example, if you had a team whose members included Emily, John, and Jack you would need to get data for three stocks whose tickers start with E, J, and J. Your data should be daily and range from December 2nd 2013 to December 2nd 2022. Your best source for this data will be WRDS, nut you will need access, which can take a couple of days. Answer the following questions:

- **a.** Provide descriptive statistics and time-series graphs for your data. Make sure to label your graphs correctly and discuss the results (both the descriptive statistics and the graphs).[5 pts]
- **b.** Write a function that consists of two inputs: 1) a 1-D array-like object of returns data 2) a real value α that is bounded by zero and one and that will be used to estimate the VaR_α and the ES_α of the 1-D array. Get some results from your function and provide their interpretations. [5 pts]
- **c.** Write a function that will calculate the VaR for the OLS fitted values assuming iid normal errors: $\epsilon_t \sim i.i.d.\ N(0, \widehat{\sigma_{OLS}})$. Make sure that your function accepts an object of type RegressionResults and a real value p that is bounded by zero and one. Get some results from your function and provide their interpretations. [5 pts]
- **d.** Calculate the VaR just like you did in part (c), but this time, we will make the assumption that the regression errors are defined as: $\sigma_t^2 = \frac{\widehat{\sigma_{OLS}^2}}{2} + \frac{\sigma_{t-1}^2}{2}$ and $\sigma_{t=0}^2 = \widehat{\sigma_{OLS}^2}$. Interpret what you find. What type of model is this? How are the results different from what you found in part (c)? [10 pts]
- **e.** Write a function that will allow you to test the null hypothesis that the value-at-risk has coverage rate p (note that p should be bounded by zero and one) for the return array. You can use the formula for s_1 in the slides for lecture 2 for the test statistic and the exact Binomial distribution for the p-value. Interpret your results.[5 pts]
- **f.** Write a function that calculates the Wald-Wolfowitz statistic as well as its p-value. To calculate the p-value, use the large sample approximation; see example 15-14 in Mann's chapter. What do you conclude based on what you found? Interpret your result.[5 pts]

Problem 3 [35 Pts], Simulations

For this question, in order to get credit, you will need to 1) write the functions and 2) interpret the results obtained from the functions you wrote. Make sure to provide context for your interpretation of the result: meaning just writing "I got an answer of 1.21029" will not get you any credit for your interpretation. You need to tell me what the result means in the context of what we discuss in this course.

a. Write a function that accepts [10 pts]:

- an object of type *BitGenerator*, *BG*
- a positive integer *degrees*

and returns

- a 1-D array of 200 i.i.d. draws from a studentt distribution with *degrees* degrees of freedom. **Do not seed** the BitGenerator in your function.

b. Write a function that accepts [5 pts]:

- a BitGenerator *bg*
- a 1-D array, a
- a boolean, r

and returns

- a 1-D array of the same size as a that contains values randomly drawn from a
- those values are randomly drawn with replacement if $replace = True$ and otherwise are drawn without replacement (i.e. they are "shuffled.")

c. Write a function that accepts a 1-D array s and produces a probability plot comparing its values to a $N(0, 1)$ distribution. [5 pts]

d. Write a function that accepts [5 pts]:

- a BitGenerator *bg*
- a 1-D array of real values a
- a positive integer T

and then

- bootstraps (samples with replacement) from a
- calculates the Kolmogorov-Smirnov test statistic for the H_0 that the bootstrapped values follow a $N(0, 1)$ distribution
- repeats the above two steps T times.

The function then returns

- p a real value scalar $0 \leq p \leq 1$, the fraction of the T simulations where we reject H_0 at the 5% significance level.

SUGGESTION: You could use Q2_Sample() to do the bootstrapping

e. Explain briefly for each of the following [10 pts]:

- What do your results from part (d) tell you about the size and power of the KS test?
- Are your results from part (d) consistent with your results from part (c)?

Problem 4 [20 Pts], Combining Simulations and Value-at-Risk

Here we will combine what we did in questions 2 and 3.

a. Write a function that will calculate a bootstrapped VaR using 10,000 draws. Interpret the resulting VaR value. [5 pts]

b. Graph the results in part (a) and discuss what you find. [5 pts]

c. What is the 95% confidence interval for your VaR? [5 pts]

d. Compare the results of what you found in this question to the results of what you found in question 2. Are they the same? Why or why not. [5 pts]