12/04

$\vec{y}' = A\vec{y}$ $\qquad p(\lambda) = \underbrace{(\lambda-\lambda_1)\dots(\lambda-\lambda_k)}_{\text{real}}(\text{quadratic factor})^{r_1}\dots(\text{quad fact})^{r_s}$

$\hookrightarrow (\lambda-\lambda_j)(\lambda-\bar{\lambda}_j)$

suppose $\lambda = \alpha + \beta i$ is a complex eigenvalue w/ alg mult m

then $\bar{\lambda} = \alpha - \beta i$ is also a complex eigenvalue w/ alg mult m

for λ, follow the same strategy as the real case, we will find

$\underbrace{\vec{z}_1, \dots, \vec{z}_m}_{\text{complex vectors}}$, m lin ind solutions associated w/ λ

$\vec{z}_k = \vec{x}_k + i\vec{y}_k \qquad \begin{matrix} Re(\vec{z}_k) = \vec{x}_k \\ Im(\vec{z}_k) = \vec{y}_k \end{matrix}$

conjugations $\bar{\vec{z}}_1, \dots, \bar{\vec{z}}_m$ are m lin ind solutions associated w/ $\bar{\lambda}$

$\left.\begin{matrix} Re(\vec{z}_k) \\ Im(\vec{z}_k) \end{matrix} \quad k=1,\dots,m \right\} = 2m$ lin ind solutions

ex: $\begin{bmatrix} 6 & 6 & -3 & 2 \\ -4 & -4 & 2 & 0 \\ 8 & 7 & -4 & 4 \\ 1 & 0 & -1 & -2 \end{bmatrix}$ $\quad -1 \pm i$ are eigenvalues of alg mut 2

when $\lambda = -1+i$

$A - (-1+i)I = \begin{bmatrix} 7-i & 6 & -3 & 2 \\ -4 & -3-i & 2 & 0 \\ 8 & 7 & -3-i & 4 \\ 1 & 0 & -1 & -1-i \end{bmatrix} \qquad (A-(-1+i)I)^2 = \begin{bmatrix} 2-14i \\ 8i \\ 8-16i \\ -2 \end{bmatrix}$

$\ker(A - (-1+i)I) = \text{span}\begin{bmatrix} 1+i \\ 0 \\ 2+i \\ -1 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 1+2i \\ -2-2i \\ 0 \\ 2 \end{bmatrix}$

$\vec{z}_1 = e^{tA}\vec{v}_1 = e^{(-1+i)t}\begin{bmatrix} 1+i \\ 0 \\ 2+i \\ -1 \end{bmatrix}$

$\vec{z}_2 = e^{tA}\vec{v}_2 = e^{(-1+i)t}(\vec{v}_2 + t(A-(-1+i)I)\vec{v}_2)$

$= e^{(-1+i)t}(\vec{v}_2 + t\vec{v}_1) = e^{(-1+i)t}\begin{bmatrix} t(1+i)+1+2i \\ -2-2i \\ t(2+i) \\ -t+2 \end{bmatrix}$

Q39 $\quad \vec{y}' = A\vec{y} = \begin{bmatrix} 5 & -1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 1 & 1 & -1 & -3 \\ 0 & -1 & 0 & 7 \end{bmatrix}\vec{y}$

$\det(A-\lambda I) = \det\begin{bmatrix} 5-\lambda & -1 & 0 & 2 \\ 0 & 3-\lambda & 0 & 4 \\ 1 & 1 & -1-\lambda & -3 \\ 0 & -1 & 0 & 7-\lambda \end{bmatrix} = (-1-\lambda)\det\begin{bmatrix} 5-\lambda & -1 & 2 \\ 0 & 3-\lambda & 4 \\ 0 & -1 & 7-\lambda \end{bmatrix}$

$= (-1-\lambda)(5-\lambda)\det\begin{bmatrix} 3-\lambda & 4 \\ -1 & 7-\lambda \end{bmatrix} = (-1-\lambda)(5-\lambda)(\lambda-5)^2$

$= (-1-\lambda)(5-\lambda)^3$

$\lambda_1 = -1 \qquad A-\lambda I = A+I = \begin{bmatrix} 6 & -1 & 0 & 2 \\ 0 & 4 & 0 & 4 \\ 1 & 1 & 0 & -3 \\ 0 & -1 & 0 & 8 \end{bmatrix} \qquad (A-\lambda I)\vec{v}_1 = 0 \qquad \vec{v}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 6 \\ 0 \\ 1 \\ 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

$\lambda_2 = 5 \qquad A-5I = \begin{bmatrix} 0 & -1 & 0 & 2 \\ 0 & -2 & 0 & 4 \\ 1 & 1 & -6 & -3 \\ 0 & -1 & 0 & 2 \end{bmatrix} \quad \text{rref} \quad \begin{bmatrix} 1 & 1 & -6 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$a - 6c - d = 0$

$b - 2d = 0$

$\text{rank}(A-5I) = 2$ so $\dim(\ker(A-5I)) = 2$

$a = 6c + d$

$b = 2d$

$\lambda_2 = 5$ (cont)

$$(A-5I)^2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -6 & -6 & 36 & 18 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & -6 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$a + b - 6c - 3d = 0$

find $\vec{v}_2 \in \ker(A-5I)^2 \setminus \ker(A-\lambda_2 I)$

$a = -b + 6c + 3d$ check $b=1 \quad c=0 \quad d=0$

$b=0 \quad c=1 \quad d=0$

$b=0 \quad c=0 \quad d=1$

choose lin ind