09/26 - Discussion

L'Hopital - only works if $\frac{\lim f(x)}{\lim g(x)} = \frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$

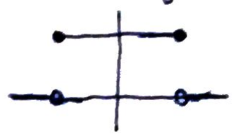
Integrals - any function w/ a <u>countable</u> number of discontinuities is integrable

- <u>countable</u> - can be assigned a natural number

FTC - $f(x)$ must be differentiable, $f'(x)$ must be continous

<u>Fun facts</u>

$$\mathbb{1}_{[0,1]}(x) = \begin{cases} 1 & ; x \in [0,1] \\ 0 & ; x \notin [0,1] \end{cases}$$



$\mathbb{N}_0$ = natural numbers from 0

$\mathbb{N}_1$ = start from 1

<u>If I'm bored</u>

prove L'Hopitals

why do integrals exist

(why limit of sums converge)?

10/01 - Discussion

separation of variables / separable differential equations

$$y' = \frac{f(x)}{g(y)} \rightarrow \underbrace{\frac{dy}{dx} = \frac{f(x)}{g(y)} \rightarrow g(y)dy = f(x)dx \rightarrow \int g(y)dy = \int f(x)dx}_{\text{shorthand notation}}$$

$$\downarrow$$

$$g(y)y' = f(x)$$

$$\downarrow$$

$u = y(x)$

$du = y'(x)du$

$$\underbrace{g(y(x))y'(x) = f(x) \rightarrow \int g(y(x))y'(x)\,dx = \int g(u)du = \int f(x)dx}_{\text{what we're actually doing}}$$

shorthand is good enough for tests and hw and stuff, just keep it in mind

ex: $\frac{dy}{6y^2} = x\,dx$

given $y(1) = \frac{1}{25}$

$-\frac{1}{6y} = \frac{1}{2}x^2 + c$

$y = -\frac{1}{3x^2 + 6c}$

$\frac{1}{25} = -\frac{1}{3 + 6c}$

$c = \frac{25 + 3}{-6}$

$c = -\frac{28}{6}$

$$\boxed{y = -\frac{1}{3x^2 - 28}}$$

$x \neq \pm\sqrt{\frac{28}{3}}$

$-\sqrt{\frac{28}{3}} < 1 < \sqrt{\frac{28}{3}}$

$\boxed{\text{interval of existence is } \left(-\sqrt{\tfrac{28}{3}}, \sqrt{\tfrac{28}{3}}\right)}$

ex. $\frac{dy}{dx} - 4e^{-y} = 2xe^{-y}$; $y(5) = 0$

$\frac{dy}{dx} = e^{-y}(2x + 4)$

$e^y dy = (2x + 4)dx$

$\int e^y dy = \int 2x + 4\,dx$

$e^y = x^2 + 4x + c$

$1 = 25 + 20 + c$

$c = -44$

$e^y = x^2 + 4x - 44$

$$\boxed{y = \ln(x^2 + 4x - 44)}$$

check that $5^2 + 4(5) - 44 > 0$

$45 - 44 = 1 > 0$

$x^2 + 4x - 44 > 0$

$\frac{-4 \pm \sqrt{16 + 176}}{2} = \frac{-4 \pm \sqrt{192}}{2} = -2 \pm \underbrace{\sqrt{48}}_{<7}$

interval of existence is $(-2 + \sqrt{48}, \infty)$

Class Preview

reverse product rule? $f(x) = -2$

ex: $2y' = x - 4y$

$(e^{2x})y' + 2y = \frac{1}{2}x(e^{2x})$

$e^{2x}y' + 2e^{2x}y = \frac{1}{2}xe^{2x}$ ↳ don't ask where it came from yet

$(e^{2x}y)' = \frac{1}{2}xe^{2x}$

$e^{2x}y = \frac{1}{2}\int xe^{2x}\,dx$ $\quad u = x \quad v = \frac{1}{2}e^{2x}$ $\quad du = dx \quad dv = e^{2x}$

$= \frac{1}{4}xe^{2x} - \frac{1}{4}\int e^{2x}\,dx$ ↳ integration by parts

$= \frac{1}{4}xe^{2x} - \frac{1}{8}e^{2x} + C$ $\quad \int u\,dv = uv - \int v\,du$

$$\boxed{y = \frac{1}{4}x - \frac{1}{8} + Ce^{-2x}}$$

standard linear equation

$y' = f(x)y + g(x)$

$y' - f(x)y = g(x)$

magic quantity - <u>integrating factor</u>

$$\boxed{e^{-\int f(x)dx}}$$

$e^{-\int f(x)dx}(y' - f(x)y) = e^{-\int f(x)dx}g(x)$

$e^{-\int f(x)dx}y - e^{-\int f(x)dx}f(x)y = e^{-\int f(x)dx}g(x)$

$(e^{-\int f(x)dx}y)' = e^{-\int f(x)dx}g(x)$

$e^{-\int f(x)dx}y = \int e^{-\int f(t)dt}g(x)\,dx$] change inner integral to t

$$\boxed{y = e^{\int f(x)dx}\int e^{-\int f(t)dt}g(x)\,dx}$$