Midterm Review Guide - 1st Order

General solutions

$\boxed{y = cy_h + y_p}$

1) separable equations
2) integrating factors

☆ for $y' = ay + f$, the IF is $\boxed{e^{-\int a(t)dt}}$

3) variation of parameters

for $y' = ay + f \Rightarrow v'y_{h1} + vy_{h1}' = avy_{h1} + f$

$\boxed{y_p = vy_{h1}}$; y_{h1} is a solution to the homogeneous$_1$ portion (const = 0)

$\boxed{y_p' = v'y_{h1} + vy_{h1}'}$ $\qquad y_{h1}' = ay_{h1}$

4) exact equation test

given $P(x,y)dx + Q(x,y)dy = 0$, it is exact if and only if $\boxed{\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}}$

5) integrating factors for $P(x,y)dx + Q(x,y)dy = 0$ equations

1-var case:

if $h = \frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$ is a function of only x, $\boxed{\mu(x) = e^{\int h(x)dx}}$

if $g = \frac{1}{P}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$ is a function of only y, $\boxed{\mu(y) = e^{-\int g(y)dy}}$

just try to solve for it? need to know if IF is $\mu(x)$ or $\mu(y)$

$\mu(x)(P(x,y)dx + Q(x,y)dy) = 0 \Rightarrow \frac{\partial P}{\partial y} = \mu(x)P_y = \mu'(x)Q + \mu(x)Q_x = \frac{\partial Q}{\partial x}$

same thing but $\mu(y)$

☆ 6) homogeneous$_2$ equations

a function is homogeneous to degree n if $G(tx, ty) = t^n G(x,y)$

a DE $Px + Qy = 0$ is homogeneous if both

P and Q are homogeneous to the same degree

how to solve - $\boxed{\text{assume } y = vx}$ (<u>not</u> variation of parameters!)

$dy = vdx + xdv$

$P(x,xv)dx + Q(x,xv)(vdx + xdv) = x^n P(1,v)dx + x^n Q(1,v)(vdx + xdv) = 0$

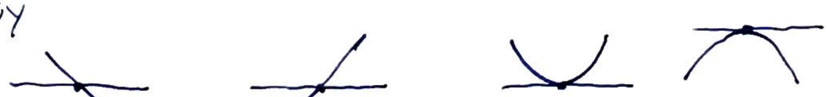
7) Existence and Uniqueness

1 show $f(t)$ is continuous

2. assume a solution x_2, which intersects w/ $x_1 \equiv f(t)$ by IVT

3. if intersect, $x_1 = x_2$, but that's a contradiction

8) Stability



stable $\qquad$ unstable $\qquad$ semi-stable

Midterm Review Guide - 2nd Order

General solution

$\boxed{y = C_1 y_{h_1} + C_2 y_{h_2} + y_p}$

1) Existence and Uniqueness - $y'' + py' + qy = g$

   if p, q, and g are continuous, then for initial condition (y_0, y_1), there is only one solution s.t $y(t_0) = y_0$ and $y'(t_0) = y_1$

2) Wronskian

$$W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} = \begin{cases} 0 & \text{is not linearly indepent} \\ 0 & \text{is linearly independent} \end{cases}$$

☆ 3) Constant Coefficient - $y'' + py' + qy = 0 \Rightarrow \lambda^2 + p\lambda + q = 0$

2 distinct real roots $\lambda_1 \lambda_2$	$y_1 = e^{\lambda_1 t}$ $y_2 = e^{\lambda_2 t}$
2 complex roots $\alpha + \beta i$	$y_1 = e^{\alpha t}\cos\beta t$ $y_2 = e^{\alpha t}\sin\beta t$
1 repeated root λ	$y_1 = e^{\lambda t}$ $y_2 = te^{\lambda t}$

☆ 4) Undetermined Coefficient - $y'' + py' + qy = f(t)$

solve for underlined variables

$f(t) = ce^{at}$	$y_p = \underline{k}e^{at}$
$f(t) = A\cos\omega t + B\cos\omega t$	$y_p = \underline{a}\cos\omega t + \underline{b}\sin\omega t$
$f(t) = a_0 t^n + a_1 t^{n-1} + \ldots + a_{n-1}t + a_n$	$y_p = \underline{b_0} t^n + \underline{b_1} t^{n-1} + \ldots + \underline{b_n}$

note: if $y'' + py' + qy = f + g$

if y_1 is a solution to $y'' + py' + qy = f$

if y_2 is a solution to $y'' + py' + qy = g$

then $y_p = y_1 + y_2$ is a solution to $y'' + py' + qy = f + g$

5) Variation of Parameters

for $y'' + py' + qy = g$

$y_p = v_1 y_1 + v_2 y_2$

$$\boxed{v_1 = \int \frac{-y_2 g}{W} \quad v_2 = \int \frac{y_1 g}{W}}$$ ☆

W is Wronskian

Good luck!