12/06 - Review

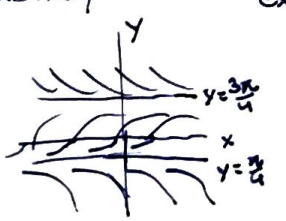
1) 1st Order DE
2) 2nd Order DE
3) Linear Systems

Types of questions

1) Solving 1st Order Linear DE $y'(x) = a(x)y + f$
   - integrating factors, variation of parameters
2) Solve (Exact) DE $P(x,y)dx + Q(x,y)dy = 0$
   if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \Rightarrow$ exact
   - integrating factors $\mu(x)$, $\mu(y)$
   - $P(x,y)$, $Q(x,y)$ are homogeneous to the same degree
     $P(tx, ty) = t^n P(x,y)$
     $y = vx \quad v = \frac{y}{x} \quad dy = v\,dx + x\,dv$
3) Autonomous Eqs
   $y' = \Phi(y)$
   $y' = y(1-y)$
   $\frac{dy}{y(1-y)} = dt$

   find equilibrium of $y' = \Phi(y)$
   find when $y' = 0$
   then find stability

   ex: $y' = \sin y + \cos y$
   $\sin y + \cos y = 0$
   $\tan y + 1 = 0$
   $y = k\pi + \frac{3\pi}{4}$; $k \in \mathbb{Z}$



4) Existence and Uniqueness
   proof by contradiction ($f(x,y)$ and $\frac{\partial f}{\partial y}$ are continuous)
   assume solution has points above & below equilibrium
   by IVT, this solution must intersect w/ equilibrium
   by E/U, this solution cannot exist

5) 2nd Order Linear DE w/ const coefficients $y'' + py' + qy = f$
   solve homogeneous eq $y_h'' + py_h' + qy = 0$
   $\lambda^2 + p\lambda + q = 0$

   find y_p
   - undetermined coefficient
   - variation of parameters

   if $e^{\lambda t}$ doesn't work
   try $te^{\lambda t}$

   note: can split $y'' + py' + qy = f_1 + f_2$

   $y = y_h + y_p = c_1 y_1 + c_2 y_2 + y_p$

6) 2×2 system. phase plane portraits

7), 8) higher dimensional systems

$\vec{y}' = A\vec{y} \longrightarrow \vec{y}_1, \vec{y}_2, \vec{y}_3$

$\vec{y}' = B\vec{y} \longrightarrow \vec{y}_4, \vec{y}_5$

$$\vec{y}' = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} \vec{y} \; : \; A_{3\times3}, B_{2\times2}$$

$$\begin{bmatrix} \vec{y}_1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \vec{y}_2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \vec{y}_3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vec{y}_4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vec{y}_5 \end{bmatrix}$$