2.6.29 $(y^2+2xy)dx - x^2dy = 0$ it's a mess o_o

ODE has int. fact

that is a func of only y

$\mu(y)((y^2+2xy)dx - x^2dy) = 0$

diff dy w/ respect to y and dx w/ respect to x

$(y^2\mu(y) + 2xy\mu(y))dx - x^2\mu(y)dy = 0$

$2y\mu(y) + y^2\mu'(y) + 2x\mu(y) + 2xy\mu'(y) = -2x\mu(y)$

$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

$(y^2+2xy)\mu'(y) = \mu(y)(-4x-2y)$

$\frac{\mu'(y)}{\mu(y)} = \frac{(-4x-2x)}{(y^2+2xy)} = \frac{-2(2x+y)}{y(2x+x)} = -\frac{2}{y}$

$\ln|\mu(y)| = -2\ln|y| = \ln\left(\frac{1}{y^2}\right)$

$\mu(y) = \frac{1}{y^2}$

$(1+\frac{2x}{y})dx - \frac{x^2}{y^2}dy = 0$

check: $\frac{\partial P}{\partial y} = -\frac{2x}{y^2}$ $\frac{\partial Q}{\partial x} = -\frac{2x}{y^2}$

$F = \int 1+\frac{2x}{y}\, dx = x + \frac{x^2}{y} + g(x)$

$\frac{\partial F}{\partial y} = -\frac{x^2}{y^2} + g'(x) = -\frac{x^2}{y^2} = Q$

$g'(x) = 0$

$g(x) = c$, we pick 0

answer: $\boxed{x + \frac{x^2}{y} = c}$

2.7.31 $x' = x - t^2 + 2t$ $x(0) = 1$ show $x(t) > t^2$ for all t

$x_1(t) = t^2$ $2t = t^2 - t^2 + 2t$

$x_1'(t) = 2t$

so $x_1(t) \equiv t^2$ is a solution

assume there is a t_0 st $x(t_0) \le t_0^2$

By IVT, there must be a point $t_1 \in [0, t_0]$ or $[t_0, 0]$ s.t.

$x(t_1) = t_1^2$

however, by U/E, that means $x(t) = x_1(t) \equiv t^2$, which is not

true b/c $0^2 \neq 1$

x and g satisfy the same ODE w/ the same initial condition

$x(t) = t^2$ and $g(t) = t^2$, which contradicts Uniqueness Theorem

$y'' + 4y' + 4y = 4 - t$

$y_h'' + 4y_h' + 4y_h = 0$

$(\lambda+2)^2 = 0$

$\lambda = -2$

$y_h = e^{-2t}$ $y_h = te^{-2t}$

$$\boxed{y = c_1 e^{-2t} + te^{-2t} + -\frac{1}{4}t + \frac{5}{4}}$$

$y_p = at + b$

memorize

$y_p' = a$

$y_p'' = 0$

$4a + 4at + 4b = 4 - t$

$a = -\frac{1}{4}$

$b = \frac{5}{4}$

$y_p = -\frac{1}{4}t + \frac{5}{4}$

---

variation of params

$y'' - y = t + 3$

$\lambda^2 - 1 = 0$

$\lambda = \pm 1$

$y_1 = e^t$ $y_2 = e^{-t}$

$y_p = v_1 y_1 + v_2 y_2$

MEMORIZE

$$v_1' = \frac{-y_2(t+3)}{W(y_1, y_2)}$$

$$v_2 = \frac{y_1(t+3)}{W(y_1, y_2)}$$

TIPS:

- don't leave anthing blank
- show your work
- check your work
  - plug your solution into original