11/18 - Review

Types of Questions

1: Solve 2x2 systems $\vec{y}' = A\vec{y}$

$p(\lambda) = \lambda^2 - tr(A)\lambda + det(A)$

case 1: two real roots λ_1, λ_2

solve $(A-\lambda_1 I)\vec{v}_1 = 0$ and $(A-\lambda_2 I)\vec{v}_2 = 0 \Rightarrow$ ☆ $\vec{y} = c_1 e^{\lambda_1 t}\vec{v}_1 + c_2 e^{\lambda_2 t}\vec{v}_2$

case 2: two complex roots $\lambda = \alpha + \beta i$ and $\bar{\lambda} = \alpha - \beta i$

you can just pick λ or $\bar{\lambda}$ and solve for that b/c the other sign will get you the conjugate of your solution

solve $(A - \lambda I)\vec{w} = 0$

$\vec{w} = \vec{v}_1 + i\vec{v}_2$

$\vec{z} = e^{\lambda t}\vec{w} = e^{(\alpha+\beta i)t}\vec{w} = e^{\alpha t}(\cos\beta t + \sin\beta t)(\vec{v}_1 + i\vec{v}_2)$

$\bar{\vec{z}} = e^{\bar{\lambda} t}\bar{\vec{w}} = e^{(\alpha-\beta i)t}\bar{\vec{w}} = e^{\alpha t}(\cos\beta t - \sin\beta t)(\vec{v}_1 - i\vec{v}_2)$

☆ $\vec{z} = Real(\vec{z}) + i\, Im(\vec{z}) \Rightarrow \vec{y} = c_1 Real(\vec{z}) + c_2 Im(\vec{z})$

case 3: repeated roots

case 3.1: dim(eigenspace) = 2 (only for diagonal matrices)

☆ $\vec{y} = c_1 e^{\lambda t}\begin{bmatrix}1\\0\end{bmatrix} + c_2 e^{\lambda t}\begin{bmatrix}0\\1\end{bmatrix}$

case 3.2: dim(eigenspce) = 1

solve $(A-\lambda I)\vec{v}_1 = 0$

pick a $\vec{w}$ that is lin ind to $\vec{v}$

solve $(A-\lambda I)\vec{w} = k\vec{v}_1$ for k

then $\vec{v}_2 = \frac{1}{k}\vec{w}$

$\vec{y}_1 = e^{\lambda t}\vec{v}_1 \quad \vec{y}_2 = e^{\lambda t}(\vec{v}_2 + t\vec{v}_1)$

☆ $\vec{y} = c_1 e^{\lambda t}\vec{v}_1 + c_2 e^{\lambda t}(\vec{v}_2 + t\vec{v}_1)$

and then phase plane portraits I guess?