10/15

just a practice problem

2.7.29 $y' = (y^2-1)e^{xy}$, $y(1)=0$; $y' = f(x,y)$

when $y = \pm 1$, $y' = 0$, so $y=-1$ and $y=1$ are unique solutions to the equation

$-1 < y = 0 < 1$, and we know by E&U that

1) a solution exists b/c continuous

2) the solution cannot intersect w/ $y=-1$ or $y=1$

NEED $f(x,y) = (y^2-1)e^{xy}$

$\frac{\partial f}{\partial y} = 2ye^{xy} + xy^2e^{xy} - xe^{xy}$

are both continuous to show existence

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vector spaces - non empty set (contains $\vec{0}$)

closed under addition & subtraction

closed under scalar multiplication ($\mathbb{R}$)

ex: line through origin, plane through origin

ex: polynomials of degree ≤ 3

$P_3 = \{a_3x^3 + a_2x^2 + a_1x + a_0 : a_3, a_2, a_1, a_0 \in \mathbb{R}\}$

$(4x^3 + 2x^2 - 7x + 5) \in P_3$

$+(2x^3 - x^2 + 1) \in P_3$

$6x^3 + x^2 - 7x + 6 \in P_3$

dim: 4

ex: set of continuous functions: $\mathbb{R} \to \mathbb{R}$

$f_1(x) = e^{2x}$

$f_2(x) = \cos(x^2)$

dim: ∞
b/c functions aren't always lin combs

def - linear combination of vectors $v_1, v_2, \dots, v_n$ is a vector of form

$c_1v_1 + c_2v_2 + \dots + c_nv_n$ where $c_1, c_2, \dots, c_n \in \mathbb{R}$

ex: $f_3(x) = \underset{c_1}{4}\overset{v_1}{\overline{e^{2x}}} \underset{c_2}{-3}\overset{v_2}{\overline{\cos(x^2)}}$

def - the dimension of a vector space is the smallest number of vectors you need to generate the whole space through linear combinations

affine space A (w/ respect to vector space V) is a "translation" of vector space V, so every element of A is in the form

$t + v$; $v \in V$, $t \in A$

not closed under addition, but every $a \in V$ can be written as

$a + v$; $a \in A$, $v \in V$

note: ODE stands for ordinary differential equation (as opposed to partial)

def - a linear (nth order) ODE is an ODE of the form

$$y^{(n)} + f_{n-1}(x)y^{(n-1)} + \dots + f_2(x)y'' + f_1(x)y' + f_0(x)y = g(x)$$

def - a homogenous linear ODE is an ODE of the form

$$y^{(n)} + f_{n-1}(x)y^{(n-1)} + \dots + f_2(x)y'' + f_1(x)y' + f_0(x)y = 0$$

note: homogenous here is different from the $G(tx,ty) = t^n G(x,y)$ definition

thm - the solutions to a linear nth order homogenous ODE form an n-dimensional vector space

ex: suppose

$$\left.\begin{aligned} &y_1'' + f_1(x)y_1' + f_0(x)y_1 = 0 \\ &y_2'' + f_1(x)y_2' + f_0(x)y_2 = 0 \\ &(y_1+y_2)'' + f_1(x)(y_1+y_2)' + f_0(x)(y_1+y_2)' = 0 \end{aligned}\right] \text{scalar addition}$$

$$\left.\begin{aligned} &c(y_1'' + f_1(x)y_1' + f_0(x)y_1 = 0) \\ &(cy_1)'' + f_1(x)(cy_1)' + f_0(x)(cy_1) = 0 \end{aligned}\right] \text{scalar multiplication}$$

$$y = 0 \text{ is a solution} \quad \Big] \text{not empty}$$

thm - solutions to a linear nth order nonhomogenous ODE form an affine space with respect to the vector space of solutions to the homogenous version of the ODE

ex:

$$\left.\begin{aligned} &y_1 \in A && y_1'' + f_1(x)y_1' + f_0(x)y_1 = g(x) \\ &y_2 \in V && y_2'' + f_1(x)y_2' + f_0(x)y_2 = 0 \\ &\text{then} && (y_1+y_2)'' + f_1(x)(y_1+y_2)' + f_0(x)(y_1+y_2) = g(x) \\ &\text{so} && (y_1+y_2) \in A \end{aligned}\right] \text{affine, sort of proof}$$

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ex: $y'' + 5y' + 6y = 0$ is homogenous
given solutions $y = e^{-3x}$, $y = e^{-2x}$
we can find that all solutions must be of the form

$$y = c_1e^{-3x} + c_2e^{-2x}$$

ex: $y'' + 5y' + 6y = 180x$ is nonhomogenous
given $y = 30x - 25$ is a solution
we can find that all solutions must be of the form

$$y = c_1e^{-3x} + c_2e^{-2x} + 30x - 25$$

remember variation of parameters?

DE: $y' = ay + f \rightarrow y' - ay = f$

this is homogenous if $f = 0$; otherwise it is not

also, this theorem:

the solutions of DE: $y' = ay + f$ are of the form $y = y_h + y_p$ where y_h is a nonzero solution of the homogenous portion and y_p is a particular solution