10/16 - Autonomous Equations, Stabilities (Sec 2.9)

def - autonomous DE $\iff$ $x' = f(x)$ $\quad (x = x(t))$

if x_0 is a root of $f(x)$, i.e. $f(x_0) = 0$, then $f(x) \equiv x_0$ is a solution

x_0 is called equilibrium point, $x(t) = x_0$ is called an equilibrium solution

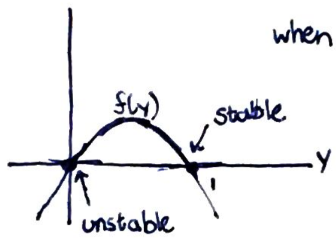
remark - if $f'(x) = \frac{\partial f}{\partial x}$ is continuous, then the Initial Value Problem has a unique solution

might be wrong, my notes got erased here

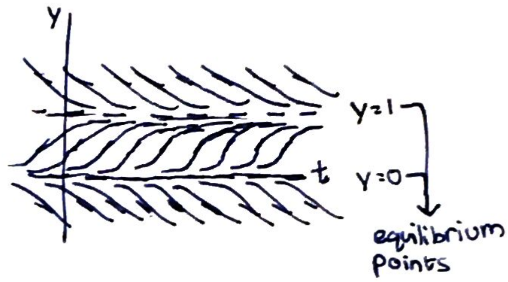
ex: $y' = y(1-y)$ he switches to y now, don't know why

1) sketch the directional fields
2) find equilibrium
3) sketch the solutions
4) prove that if $y(0) = \frac{1}{2}$, then $0 < y(t) < 1$ for all t in the domain of $y(t)$

ans: 1-3) slope $= \tan(\theta) = y' = f(y) = y(1-y)$ $\quad \theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$



when $y>1$	$f(y)<0$	$\theta<0$
$y=1$	$f(y)=0$	$\theta=0$
$y<1$	$f(y)>0$	$\theta>0$
$y=0$	$f(y)=0$	$\theta=0$
$y<0$	$f(y)<0$	$\theta<0$



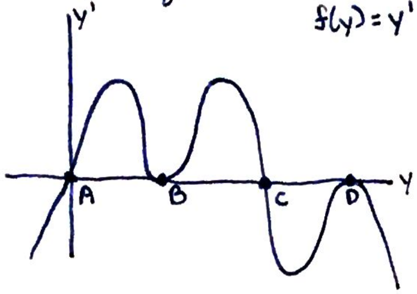
4) WTS (want to solve?) $y(t) < 1$ $\quad (y(0) = \frac{1}{2})$

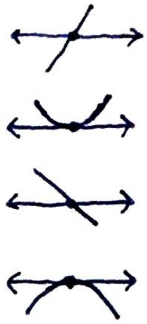
proof by contradiction

- assume at t_1, $y(t_1) \geq 1$, without loss of generality (WLOG) we also assume $t_1 > 0$
- solution of DE is continuous because $f(y) = y(1-y)$ is continuous and differentiable and $\frac{\partial f}{\partial y}$ is continuous
- by IVT, $\exists\, t_0 \in (0, t_1]$ such that $y(t_0) = 1$
  - IVT: if $f(\alpha) = a$ and $f(b) = b$, and $f(x)$ is cont on $[\alpha, \beta]$ then for any $c \in [a,b]$ $\exists\, \gamma \in [\alpha, \beta]$
- we know that $y(t) \equiv 1$ is a solution
- we assumed the existence of a solution to IVP w/ IC $y(0) = \frac{1}{2}$ for which $y(t_0) = 1$
- these two solutions are not the same, so by E&U, there is a contradiction, so if $y(0) = \frac{1}{2}$, $0 < y(t) < 1$

classify equilibrium

$f(y) = y'$





A = unstable

B = semi-stable

C = stable

D = semi-stable

Guess:

unstable means as t goes to infinity solutions diverge

stable means solutions converge?

semi means pass through but like, flat

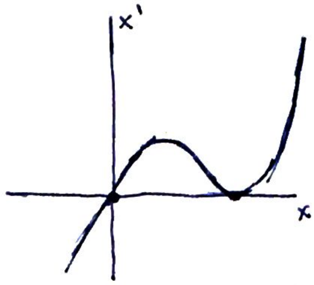
ex: $x' = x^3 - 2x^2 + x = f(x)$

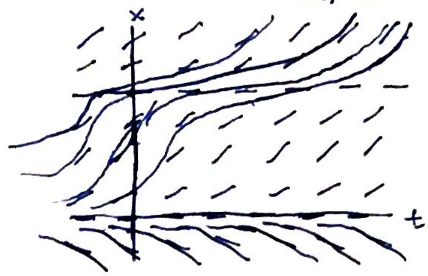
$= x(x^2 - 2x + 1)$

$= x(x-1)^2$

$x' = 0$ @ $x = 0, 1$

$(x-1)^2$ is always positive





x=1 is semi stable

x=0 is unstable

↳ how is he so good at drawing these?

EMPTY SPACE

[Baat!]

#orvhalloweek