10/28 - Review

2.4.19 Solve $(2x+3)y' = y + (2x+3)^{\frac{1}{2}}$; $y(-1)=0$

$$y' = \frac{y}{(2x+3)} + (2x+3)^{-1/2}$$

$$y_h' = \frac{y_h}{2x+3}$$

$$\frac{dy_h}{y_h} = \frac{dx}{2x+3}$$

$\ln|y_h| = \frac{1}{2}\ln|2x+3| + c \leftarrow$ let $c=0$ for convenience

$$\ln|y_h^2| = \ln|2x+3|$$

$$y_h^2 = 2x+3$$

$$y_h = (2x+3)^{1/2}$$

$$y_h' = (2x+3)^{-1/2}$$

$$y_p = v y_h$$

$$y' = v' y_h + v y_h'$$

$$v' y_h + v y_h' = \frac{v y_h}{(2x+3)} + (2x+3)^{-1/2}$$

$$v'(2x+3)^{1/2} + \cancel{v(2x+3)^{-1/2}} = \cancel{v(2x+3)^{-1/2}} + (2x+3)^{-1/2}$$

$$v' = \frac{(2x+3)^{-1/2}}{(2x+3)^{1/2}}$$

$$v = \int (2x+3)^{-1}$$

$$= \frac{1}{2}\ln|2x+3|$$

$$y_p = \frac{1}{2}\ln|2x+3|(2x+3)^{-1/2}$$

$$\boxed{y = c(2x+3)^{1/2} + \frac{1}{2}\ln|2x+3|(2x+3)^{-1/2}}$$

2.4.37 $(3x+y)\,dx + x\,dy = 0$

It's homogeneous

$\frac{\partial P}{\partial y} = 1$ $\|$ $\frac{\partial Q}{\partial x} = 1$ It's exact

$$w = \int 3x + y\,dx + \Phi(y)$$

$$= \frac{3}{2}x^2 + xy + \Phi(y)$$

$$\frac{\partial w}{\partial y} = Q = x + \Phi(y) = x$$

$$\Phi(y) = 0$$

$$\boxed{w = \frac{3}{2}x^2 + xy + C = 0}$$

method 2

$$y = xv$$

$$dy = x\,dv + v\,dx$$

$$(3x + xv)\,dx + (x)(x\,dv + v\,dx) = 0$$

$$(3x + xv)\,dx + x^2\,dv + xv\,dx = 0$$

$$(3x^2 + 2xv)\,dx + x^2\,dv = 0$$

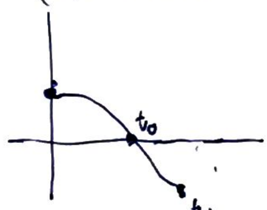
$$\frac{dv}{3x + 2v} + \frac{dx}{x^2} = 0$$

$$\frac{1}{2}\ln|3x + 2v| + \ln|x| = C$$

$$(3x + 2v)x^2 = c$$

2.7.27 $x' = x\cos^2 t$ $x(0) = 1$

show $x(t) > 0$ for all t



proof $x(t) \equiv 0$ is equilibrium

suppose $x(t_1) < 0$ by IVT, there exists a $t \in (0, t_1)$ s.t. $x(t_0) = 0$

$x' = x\cos^2 t$ $x(t_0) = 0$

$x_1(t) \equiv 0$ is a solution

$x_2(t) = x(t)$ is also a solution

however, $x\cos^2 t$ is continuous

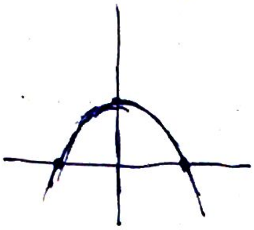
$\frac{\partial f}{\partial x} = \cos^2 t$ is cont

By E/U, solution is unique

2.9.27 $x' = 4 - x^2$ Find equilibrium, stability

$4 - x^2 = 0$

$x = \pm 2$



$x=-2$ is unstable

$x=2$ is stable

4.5.35 $y'' + 4y' + 3y = \cos 2t + 3\sin 2t$

$y_p = A\cos 2t + B\sin 2t$

$y_h'' + 4y_h' + 3y_h = 0$

$(\lambda + 3)(\lambda + 1) = 0$

$\lambda = -1, -3$

$y_{h1} = e^{-t}$ $y_{h2} = e^{-3t}$

$$\boxed{y = c_1 e^{-t} + c_2 e^{-3t} + -\frac{5}{13}\cos 2t + \frac{1}{13}\sin 2t}$$

$y_p' = -2A\sin 2t + 2B\cos 2t$

$y_p'' = -4A\cos 2t - 4B\sin 2t$

$-4A\cos 2t - 4B\sin 2t - 8A\sin 2t$

$+ 8B\cos 2t + 3A\cos 2t + 3B\sin 2t$

$= \cos 2t + 3\sin 2t$

$(-A + 8B)\cos 2t + (-B - 8A)\sin 2t =$

$= \cos 2t + 3\sin 2t$

$-A + 8B = 1$ $-B - 8A = 3$

$-65A = 25$ $-8B - 64A = 24$

$A = -\frac{5}{13}$

$B = \frac{1}{13}$

$y_p = -\frac{5}{13}\cos 2t + \frac{1}{13}\sin 2t$