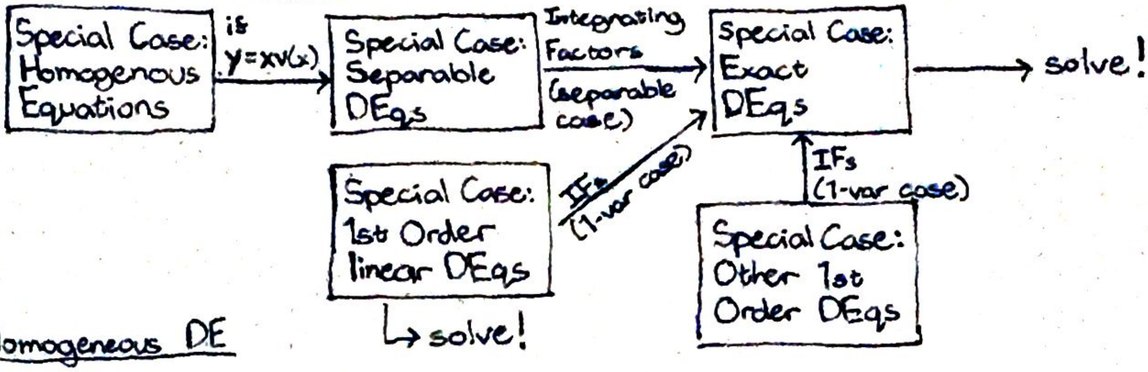
10/11

Review - How to solve 1st order diff eqs? Probably what he drew?

General 1st Order DEs



Homogeneous DE

def - a function $G(x,y)$ is homogeneous of degree n (integer)

if $G(tx, ty) = t^n G(x,y)$ for all $t>0$, $x \neq 0$, $y \neq 0$

ex: x^2+y^2, xy, $\frac{x}{y}$

def - a DE $P(x,y)dx + Q(x,y)dy = 0$ is homogeneous.

thm - if P and Q are homogeneous to the same degree

$\Leftrightarrow$ $\frac{dy}{dx} = -\frac{P(x,y)}{Q(x,y)} = f(x,y)$, $f(x,y)$ is homogeneous of degree 0

Q - How to solve homogeneous equation

A - We let $v(x) = \frac{y(x)}{x}$ $\longrightarrow$ $y = v \cdot x$

$dy = v dx + x dv$ (H)

$P(x, xv) dx + Q(x, xv)(v dx + x dv) = 0$

$x^n P(1,v) dx + x^n Q(1,v)(v dx + x dv) = 0$

$(P(1,v) + Q(1,v)v) dx + Q(1,v) x\, dv = 0$

$$\frac{dv}{dx} = -\frac{P(1,v) + Q(1,v)v}{Q(1,v)x}$$

ex: $(x^2+y^2)dx + xy\,dy = 0$ is homogeneous

$y = xv$ $\quad dy = x dv + v dx$

$(x^2 + (xv)^2) dx + x xv(x dv + v dx) = 0$

$(1+v^2) dx + v(x dv + v dx) = 0$

$(1+2v^2) dx + (vx) dv = 0$

$\frac{dv}{dx} = -\frac{(1+2v^2)}{vx}$

$\frac{v\,dv}{(1+2v^2)} = -\frac{dx}{x}$

$\frac{1}{4} \ln|1+2v^2| = -\ln|x| + c$

$\ln|1+2v^2| = \ln x^{-4} + 4c$

$1+2v^2 = e^{4c} x^{-4}$

$(1+2v^2)x^4 = e^{4c} = c^*$

$x^4 + 2(xv)^2 x^2 = c^*$

$x^4 + 2x^2y^2 = c^*$

Q38 $\frac{dy}{dx} = \frac{y(x^2+y^2)}{xy^2-2x^3}$ homogeneous of degree 0

let $v = \frac{y}{x}$ $\quad dy = v\,dx + x\,dv$

$LHS = \frac{dy}{dx} = \frac{v\,dx + x\,dv}{dx} = v + x\frac{dv}{dx}$

$RHS = \frac{y(x^2+y^2)}{xy^2-2x^3} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \frac{\frac{y}{x}\left(1+\left(\frac{y}{x}\right)^2\right)}{\left(\frac{y}{x}\right)^2 - 2} = \frac{v(1+v^2)}{v^2-2}$

$v + x\frac{dv}{dx} = \frac{v(1+v^2)}{v^2-2}$

$x\frac{dv}{dx} = v\left(\frac{1+v^2}{v^2-2} - 1\right)$

$\frac{x}{dx}dv = v\left(\frac{3}{v^2-2}\right)$ $\quad u = v^2$

$du = 2v\,dv$

$dv = \frac{du}{2v}$

$\frac{dv(v^2-2)}{3v} = \frac{dx}{x}$

weird notation

$\hookrightarrow \frac{1}{2v}\frac{(v^2-2)\,dv^2}{3v} = \frac{(u-2)\,du}{6u} = \left(\frac{1}{6} - \frac{2}{3u}\right)du = \frac{dx}{x}$

$\frac{1}{6}u - \frac{1}{3}\ln|u| = \ln|x| + c$

$\frac{1}{6}v^2 - \frac{1}{3}\ln|v^2| = \ln|x| + c$ $\quad$ what are we doing again?

$\frac{1}{6}v^2 - \frac{2}{3}\ln|v| = \ln|x| + c$