10/18 - Second Order Linear DE

recall $F(t,y,y',y'')=0$ (eg. $y+y'+y''=t$)

$y''=f(t,y,y')$ (normal form)

note: changed my β to β

<u>def</u> - 2nd order linear DE $\Leftrightarrow$ $y''+p(t)y'+q(t)y=\underline{g(t)}$ ↳ forcing term

when $g(t)=0$, DE is homogenous

<u>thm</u> - Existence & Uniqueness for 2nd order linear DE

suppose $p(t), q(t)$ are continuous on interval (α,β)

then for initial condition (y_1, y_2)

there is one and only one solution $y(t)$ to

$y''+p(t)y'+q(t)y=g(t)$; $y_1=y(t_0)$, $y_2=y'(t_0)$, $t_0\in(\alpha,\beta)$

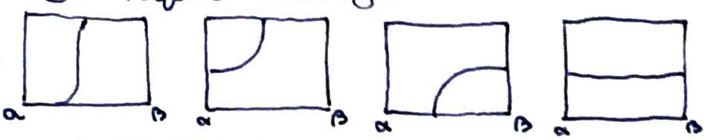
<u>remark</u> - $y''=-p(t)y'-q(t)y+g(t)$

$=f(y',y,t)$ ⟵ why isn't it $f(t,y,y')$

$\frac{\partial f}{\partial(y')}=-p(t)$ $\frac{\partial f}{\partial y}=-q(t)$

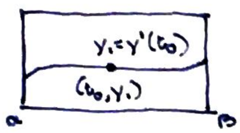
note, we no longer require rectangle

1st Order



need to define a height

2nd Order



<u>thm</u> - 1st order linear DE $y=y_p+cy_h$

2nd order linear DE $y=y_p+c_1y_{h_1}+c_2y_{h_2}$

(where y_{h1} and y_{h2} are linearly independent solutions to $y''+p(t)y'+q(t)y=0$)

<u>def</u> - two functions $u(x)$ and $v(x)$ are linearly independent on (α,β) if $u(x)\not\equiv cv(x)$ and $v(x)\not\equiv cu(x)$; $c\in\mathbb{R}$

<u>prop</u> - suppose $u(t), v(t)$ are 2 solutions to the homogeneous equation

1) u and v are linearly independent $\Leftrightarrow$ $W(x)=\begin{vmatrix} u & u' \\ v & v' \end{vmatrix}\not\equiv 0$ on (α,β)

2) u and v are linearly dependent $\Leftrightarrow$ $W(x)\equiv 0$ on (α,β)

thm - 2nd order linear homogeneous DE $y'' + py' + qy = 0$, then general solution is $y = C_1 y_{h1} + C_2 y_{h2}$ (y_{h1} and y_{h2} are linearly independent)

proof - 1) if y_{h1}, y_{h2} are solutions $\Rightarrow C_1 y_{h1} + C_2 y_{h2}$ is a solution ✓

2) b/c y_{h1} and y_{h2} are linearly independent, $W(x) \neq 0$ on (α, β)

let $y(t)$ be a solution of DE, $y(t_0) = y_0$, $y(t_1) = y_1$

now consider

$$\left.\begin{aligned} C_1 y_{h1}(t_0) + C_2 y_{h2}(t_0) &= y_0 \\ C_1 y_{h1}'(t_1) + C_2 y_{h2}'(t_1) &= y_1 \end{aligned}\right] \text{might be what he wrote}$$

$\exists$ some C_1, C_2 satisfy the linear system

we let $y_3 = C_1 y_{h1} + C_2 y_{h2} \Rightarrow y_3(t_0) = y_0 = y(t_0)$

$y_3' = C_1 y_{h1}' + C_2 y_{h2}' \Rightarrow y_3'(t_1) = y_1 = y'(t_1)$

therefore, y_3 must be equal to y by E/U