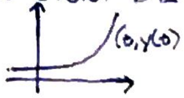
11/15 - Phase Plane Portraits

1) Motivation (not in textbook)

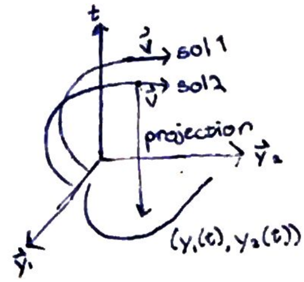
1st Order DE: $y' = f(t,y)$, solution $y = y(t)$



we have directional fields and graph of sols $t \to (t, y(t))$

2×2 System: solution $\vec{y} = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$

have a graph that is a space curve



for a system w/ constant A

$y' = Ay$, A does not depend on t

suppose

IVP1: $\vec{y}' = A\vec{y}$ $\quad \vec{y}'(t_1, y_1, y_2) = \vec{v}$ $\quad$ sol 1

IVP2: $\vec{y}' = A\vec{y}$ $\quad \vec{y}'(t_2, y_1, y_2) = \vec{v}$ $\quad$ sol 2

if you graph sol 1 and sol 2, they will have the same projection on the y_1-y_2 plane (Phase Plane!)

Describe the projection of sol curves on Phase Plane

Summary: <u>Case 1</u>: A has two distinct real eigenvalue (non-zero)

generic

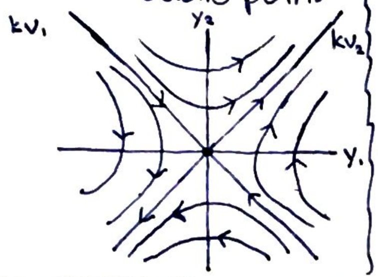
saddle point	nodal source	nodal sink
$\lambda_1 < 0 < \lambda_2$	$0 < \lambda_1 < \lambda_2$	$\lambda_1 < \lambda_2 < 0$

$\lambda_1 \to \vec{v}_1$

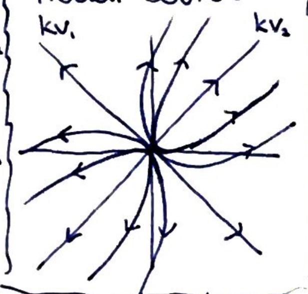
$\lambda_2 \to \vec{v}_2$

$\vec{y} = c_1 e^{\lambda_1 t} \vec{v}_1 + c_2 e^{\lambda_2 t} \vec{v}_2 = c_1 y_1 + c_2 y_2$

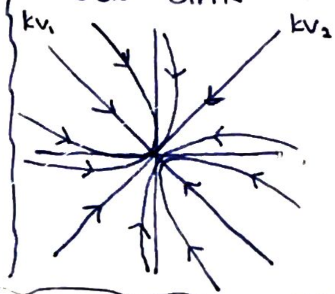
saddle point



nodal source



nodal sink



<u>Case 2</u>: A has two complex eigenvalues, $\lambda = a + bi$

center	spiral source	spiral sink
$a = 0$	$a > 0$	$a < 0$
not generic	generic	generic

To Be Continued