11/18 - Phase Plane Portraits (cont)

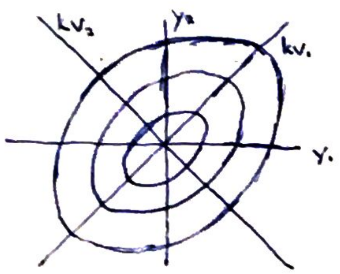
<u>case 2</u>: complex eigenvalues $\lambda = \alpha \pm \beta i$

center	spiral source	spiral sink
$\alpha = 0$	$\alpha > 0$	

suppose $\vec{w}$ eigenvector corresponds to $\lambda = \alpha + \beta i$
$\vec{w} = \vec{v}_1 + i\vec{v}_2$

then $y(t) = c_1 e^{\alpha t}(\cos\beta t\, \vec{v}_1 - \sin\beta t\, \vec{v}_2) + c_2 e^{\alpha t} i(\sin\beta t\, \vec{v}_1 + \cos\beta t\, \vec{v}_2)$

center
$\alpha = 0$ $\quad \|\vec{v}_1\| > \|\vec{v}_2\|$



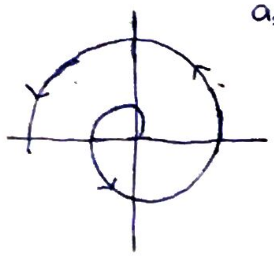
direction of solutions
check $y'(t)$ at $(1,0)$

$A\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}a_{11}\\a_{21}\end{bmatrix}$

when $a_{21} > 0$, counterclockwise
when $a_{21} < 0$, clockwise

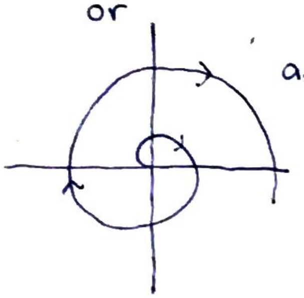
note, length of ellipse depends on magnitude of $\vec{v}_1$ and $\vec{v}_2$

spiral source
$\alpha > 0$



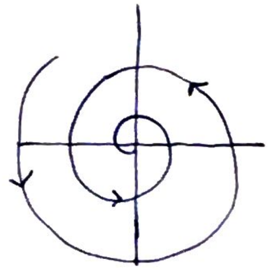
$a_{21} > 0$

or



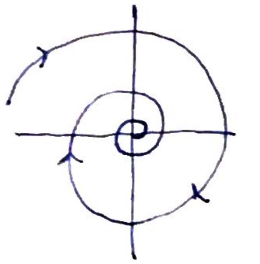
$a_{21} < 0$

spiral sink
$\alpha < 0$



$a_{21} > 0$

or

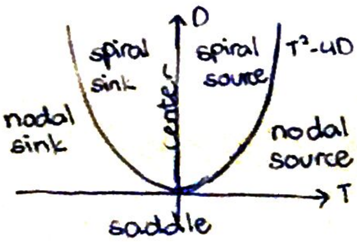


$a_{21} < 0$

<u>case 3</u>: repeated root (not required)

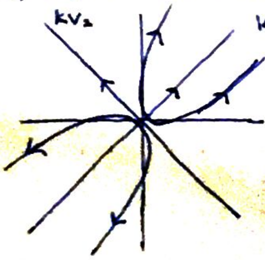
<u>case 4</u>: degenerate case (not required)
at least one eigenvalue is 0

<u>Trace Det Plane</u> (T, D)
$\;\;\llcorner$ $\to$ det(A)
$\llcorner$ $\to$ tr(A)



ex: $\lambda_1 = 2 \quad \vec{v}_1 = [1,1]^T$
$\lambda_2 = 1 \quad \vec{v}_2 = [-1,1]^T$

$\lambda_1 > \lambda_2 > 0$ so nodal source



$|\lambda_1| > |\lambda_2|$
so follow kv_1

ex: $A = \begin{bmatrix}2 & -1\\2 & 0\end{bmatrix} \Rightarrow \lambda = 1 \pm i \Rightarrow \alpha > 0$
spiral source

$a_{21} = 2 > 0$ so ↺