HWK # 6
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George Mason University

Boris Reinosa

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3.2 **– Laplace random variable. let X have PDF $f_X(x) = \frac{\lambda}{2}e^{-\lambda|x|}$ where λ is a positive scalar. verify that f_X satisfies the normalization condition and evaluate the mean and variance of X.**

a) for f_X to satisfy the normalization condition the following must be true, $\int_{-\infty}^{\infty} f_X(x)dx = 1$

$\frac{\lambda}{2}\int_{-\infty}^{\infty} e^{-\lambda|x|}dx = \frac{\lambda}{2}\left[\int_{-\infty}^{\infty} e^{-\lambda - x}dx + \int_{-\infty}^{\infty} e^{-\lambda x}dx\right]$

$= \frac{\lambda}{2}\left[\frac{1}{\lambda}e^{\lambda x}|_{-\infty}^{0} + \frac{1}{-\lambda}e^{-\lambda x}|_{0}^{\infty}\right]$

$= \frac{\lambda}{2}\left[\frac{1}{\lambda}(e^{\lambda 0} - lim_{x\to -\infty}e^{\lambda x}) + \frac{1}{-\lambda}(lim_{x\to\infty}e^{-\lambda x} - e^{\lambda 0})\right]$

$= \frac{\lambda}{2}\left[\frac{1}{\lambda}(1-0) + \frac{1}{-\lambda}(0-1)\right]$

$= \frac{\lambda}{2}\left[\frac{2}{\lambda}\right] = 1$

b) Evaluate E[X]= $\int_{-\infty}^{\infty} xf_X(x)dx = \frac{\lambda}{2}\int_{-\infty}^{\infty} xe^{-\lambda|x|}dx$

$= \frac{\lambda}{2}\left[\int_{-\infty}^{\infty} xe^{-\lambda - x}dx + \int_{-\infty}^{\infty} xe^{-\lambda x}dx\right]$

$= \frac{\lambda}{2}\left[\frac{1}{\lambda^2}e^{\lambda x}(\lambda x - 1)|_{-\infty}^{0} + \frac{1}{-\lambda^2}e^{-\lambda x}(\lambda x + 1)|_{0}^{\infty}\right]$

$= \frac{\lambda}{2}\left[\frac{1}{\lambda^2}(1)(-1) + \frac{1}{-\lambda^2}(-1)(1)\right] = \frac{\lambda}{2}[0] = 0$

c)Var(X)=E[X^2]-E[X]^2
E[X]^2=0
E[X^2]= $\frac{\lambda}{2}\left[\int_{-\infty}^{\infty} x^2(e^{-\lambda - x} + e^{-\lambda x})dx\right]$

$= \frac{\lambda}{2}\left[e^{\lambda x}\frac{(\lambda^2x^2 - 2\lambda x + 2)}{\lambda^3}|_{-\infty}^{0} + e^{-\lambda x}\frac{(\lambda^2x^2 + 2\lambda x + 2)}{\lambda^3}|_{0}^{\infty}\right]$

$= \frac{\lambda}{2}\left[\frac{2}{\lambda^3} + \frac{2}{\lambda^3}\right] = \frac{\lambda}{2}\left[\frac{4}{\lambda^3}\right] = \frac{2}{\lambda^2}$

$Var(X) = \frac{2}{\lambda^2} - 0$

3.5 **– Consider a triangle and a point chosen within the traingle according to the uniform probability law. let X be the distance from point to the base of the triangle. given the height of the triangle, find CDF and PDF of X.**

CDF of X is P(X$\leq x$)which is the area formed below the line x divided by the entire area of the triangle.
$F_X = \frac{A_1(x)}{A}$
if $A_2(x)$ is known then $F_X = 1 - \frac{A_2(x)}{A}$
$A_2(x) = \frac{b_2h_2}{2}$
$b_2 = b(h-x)$
$h_2 = h - x$
$A_2(x) = \frac{b(h-x)(h-x)}{2}$
$F_X = 1 - P(X \geq x) = 1 - \frac{A_2}{A} = 1 - \frac{b(h-x)^2}{2}\frac{2}{bh} = 1 - \frac{(h-x)^2}{h}$
$f_x = \frac{d(F_X)}{dx} = \frac{-2(h-x)(-1)}{h} = \frac{2(h-x)}{h}$

3.6– **Calamity Jane goes to the bank to make a withdrawal and is equally likely to find 0 or 1 customers around of her. the service time of the customers ahead, if present is exponentially distributed with parameter λ. what is the CDF of janes waiting time.**

Let X be the number of customers ahead of jane, and Y the service time of costumers ahead The probability that waiting time will be less than any given y when there is no one ahead is 1 then $P(Y \leq y|X = 0) = 1$
$P_X(x) = 1/2, x \in \{0, 1\}$
$P(Y \geq y|X = 1) = e^{-\lambda y}$
$F_Y(y) = P(Y \leq y) = P(Y \leq y|X = 0)P(X = 0) + P(Y \leq y|X = 1)P(X = 1)$
$P(Y \leq y) = (1)\frac{1}{2} + (1 - e^{-\lambda y})\frac{1}{2} = \frac{1}{2}(2 - e^{-\lambda y})$

$$F_Y(y) = \begin{cases} \frac{1}{2}(2-e^{-\lambda y}), y \geq 0 \\ 0, otherwise \end{cases}$$

***3.21*– We start with a stick length l. we break it at a point which is chosen according to a uniform distribution and keep a piece of length Y, that contains the left end of the stick. we then repeat the same process on the piece that we were left with, and let X be the length of the remaining piece after breaking the second time.**

a) find PDF of Y and X

$F_Y = P(Y \leq y) = \frac{y}{l}, [0, l]$ then

$f_y = \frac{d(F_Y)}{dy} = \frac{1}{l}$

$F_{X|Y} = P(X \leq x|Y = y) = \frac{x}{y}, [0, y]$

$f_{X|Y} = \frac{1}{y}, 0 \leq x \leq y$

$$f_{X,Y}(x,y) = f_Y(y)f_{X|Y}(x,y) = \frac{1}{l}\frac{1}{y} = \begin{cases} \frac{1}{ly}, 0 \leq x \leq \ y \leq l \\ 0, otherwise \end{cases}$$

b) find marginal PDF of X

$f_X(x) = \int_x^l \frac{1}{yl}dy = \frac{1}{l}ln(y)|_x^l = \frac{1}{l}log(l/x), 0 \leq x \leq l$

c) find E[X] $E[X] = \int_0^l xf_X(x)dx = \int_0^l \frac{x}{l}log(l) - xlog(x)dx$

$= \frac{log(l)}{l}\int_0^l xdx - \frac{1}{l}\int_0^l xlog(x)dx$

$= \frac{1}{l}\left[\frac{log(l)x^2}{2}|_{x=l} - \frac{1}{4}x^2(2log(x)-1)|_{x=l}\right] = \frac{1}{l}\left[\frac{log(l)l^2}{2} - \frac{1}{4}l^2(2log(l)-1)\right]$

$= \frac{1}{2l}\left[log(l)l^2 - log(l)l^2 + \frac{l^2}{2}\right]$

$= \frac{1}{2l}\left[\frac{l^2}{2}\right] = \frac{l}{4}$

d) evaluate E[X] by exploiting X=Y(X/Y) $E[X] = E[Y]E[\frac{X}{Y}]$

$E[Y] = \int_0^l y\frac{1}{l}dy = \frac{y^2}{2l}|_{y=l} = \frac{l^2}{2l} = \frac{l}{2}$

$E[X/Y] = \int_0^l \int_0^y \frac{x}{y^2 l}dxdy = \int_0^l \frac{y^2}{2y^2 l}dy = \frac{y}{2l}|_{y=l} = \frac{1}{2}$

$E[X] = \frac{l}{2}\frac{1}{2} = \frac{l}{4}$