# IN4060 - Oblig 5

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## 1 Model Semantics

Interpretation requirements:

- a set $\triangle^{\mathcal{I}}$ - The domain
- For each individual URI $i$, an element $i^{\mathcal{I}} \in \triangle^{\mathcal{I}}$
- For each class URI $C$, a subset $C^{\mathcal{I}} \subseteq \triangle^{\mathcal{I}}$
- for each property URI $r$, a relation $r^{\mathcal{I}} \subseteq \triangle^{\mathcal{I}} \times \triangle^{\mathcal{I}}$

### 1.1 Interpretation

1. Interpretation $\mathcal{I}_1$, such that $\mathcal{I}_1 \models \Gamma_1$

$\triangle^{\mathcal{I}_1} = \{$Tweety, JollyJumper, Bruce, $b_1\}$

:Tweety$^{\mathcal{I}_1}$ = Tweety

:JollyJumper$^{\mathcal{I}_1}$ = JollyJumper

:Bruce$^{\mathcal{I}_1}$ = Bruce

**Let**

$\beta(b_1) = b_1$

**Classes**

:Animal$^{\mathcal{I}_1} = \triangle^{\mathcal{I}_1}$

:food$^{\mathcal{I}_1} = \{$Bruce$\}$

:Penguin$^{\mathcal{I}_1}$ = :Bird$^{\mathcal{I}_1} = \{$Tweety$\}$

:Fish$^{\mathcal{I}_1}$ = :Food$^{\mathcal{I}_1} = \{$Bruce$\}$

:Horse$^{\mathcal{I}_1} = \{$JollyJumper$\}$

:Vegetable$^{\mathcal{I}_1} = \emptyset$

**Properties**

$:\text{eats}^{\mathcal{I}_1} = \{\langle\text{Tweety, Bruce}\rangle, \langle\text{JollyJumper, b1}\rangle\ \}$

$:\text{favouriteFood} = \{\langle\text{Tweety, Bruce}\rangle, \langle\text{JollyJumper, b1}\rangle\}$

$:\text{likes}^{\mathcal{I}_1} = \{\langle\text{JollyJumper, Tweety}\rangle\}$

$:\text{hasNickname} = \{\langle\text{JollyJumpet, "JJ"}\rangle, \langle\text{Bruce, "Alonso"}\rangle\}$

2. Interpretation $\mathcal{I}_2$ such that $\mathcal{I}_1 \not\models \Gamma_2$

$\triangle^{\mathcal{I}_1} = \{\text{Tweety, JollyJumper, Bruce, } b_1\}$

$:\text{Tweety}^{\mathcal{I}_1} = \text{Tweety}$

$:\text{JollyJumper}^{\mathcal{I}_1} = \text{JollyJumper}$

$:\text{Bruce}^{\mathcal{I}_1} = \text{Bruce}$

**Let**

$\beta(b_1) = b_1$

**Classes**

$:\text{Animal}^{\mathcal{I}_1} = \triangle^{\mathcal{I}_1}$

$:\text{food}^{\mathcal{I}_1} = \emptyset$

$:\text{Penguin}^{\mathcal{I}_1} = :\text{Bird}^{\mathcal{I}_1} = \{\text{Tweety}\}$

$:\text{Fish}^{\mathcal{I}_1} = :\text{Food}^{\mathcal{I}_1} = \{\text{Bruce}\}$

$:\text{Horse}^{\mathcal{I}_1} = \{\text{JollyJumper, Bruce}\}$

$:\text{Vegetable}^{\mathcal{I}_1} = \{\text{JollyJumper}\}$

**Properties**

$:\text{eats}^{\mathcal{I}_1} = \{\langle\text{Tweety, Bruce}\rangle, \langle\text{JollyJumper, b1}\rangle\ \}$

$:\text{favouriteFood} = \{\langle\text{Tweety, Bruce}\rangle, \langle\text{JollyJumper, b1}\rangle\}$

$:\text{likes}^{\mathcal{I}_1} = \{\langle\text{JollyJumper, Tweety}\rangle\}$

$:\text{hasNickname} = \{\langle\text{JollyJumpet, "JJ"}\rangle, \langle\text{Bruce, "Alonso"}\rangle\}$

What is wrong:

- JollyJumber should NOT be a Vegetable.
- Bruce should NOT be a Horse.
- Bruce is not defined as food.

## 1.2 Entailment

1. Tweety is an animal

   **Yes**, with the following deriation:

   (a) $:Tweety\ rdf:type\ \ :Penguin.$ - P
   (b) $:Penguin\ rdfs:subClassOf\ \ :Animal.$ - P
   (c) $:Tweety\ rdf:type\ \ :Animal$ - rdfs9 on (a) and (b)

2. :Tweety likes :JollyJumper

   **No**, this cannot be true in $\Gamma_1$.

   Using the interpretation $\mathcal{I}_1$, it follows that $\Gamma_1$ is true.

   In this interpretation, there is no pair $\langle$Tweety, JollyJumper$\rangle \in likes^{\mathcal{I}_1}$, therefore, it cannot be true that Tweety likes JollyJumper.

3. :Food is the range of :favouriteFood

   **Yes**, this is true.

   We know:

   (a) $:favouriteFood\ rdfs:subPropertyOf\ \ :eats.$ - P
   (b) $:eats\ rdfs:range\ \ :Food.$ - P

   It cannot be derived directly, but because subproperties inherit properties of its superproperty, this includes the range.

   Meaning, because $:eats$ has $rdfs:range$ and $:favouriteFood$ is $rdfs:subPropertyOf$, therefore $:favouriteFood\ rdfs:range\ \ :Food.$

4. :Bruce has some favourite food

   **No**, this cannot be true in $\Gamma_1$.

   Using the interpretation $\mathcal{I}_1$, it follows that $\Gamma_1$ is true.

   In this interpretation, there is no pair $\langle$Bruce, y$\rangle \in$ favouriteFood, therefore Bruce cannot have any favourite food.

5. :Bruce is a vegetable

   **No**, this cannot be true in $\Gamma_1$.

   Using the interpretation $\mathcal{I}_1$, it follows that $\Gamma_1$ is true.

   In the interpretation, vegetable is only defined and not assigned.
   Because we interpret $:Vegetable$ in the interpretation as the empty set, no one or no thing is a vegetable. Therefore, Bruce cannot be a vegetable.

6. :Bruce is a horse

   **No**, this cannot be true in $\Gamma_1$.

   Using the interpretation $\mathcal{I}_1$, it follows that $\Gamma_1$ is true.

   If Bruce were to be a horse, the subset :Horse$^{\mathcal{I}_1}$ must also contain Bruce.
   This is not true, and therefore Bruce cannot be a horse.

7. :Bruce is a fish

   **Yes**, this fact is defined in $\Gamma_1$ and therefore cannot be derived:

   $:Bruce\ rdf:type\ \ :Fish.$

# 2 RDF(S) formal semantics)

## 2.1 Difference in RDF(S) semantics from web and lecture

"Foundations of Semantic Web Technologies" introduces the term "Simple interpretations", a specialized version of interpretation used for giving meaning to RDF graphs specifically.
Simple interpretations contain elements such as a set of resources (IR), a set of properties (IP), and various functions to map elements together, such as the relationship between elements and properties ($\mathbf{I}_{\mathrm{EXT}}$).

Additionally, axiomatic triples are foundational predefined statements that establish concepts such as asserting that rdf:type is a property (rdf:type rdf:type rdf:Property). This is important since rdf:type serves as a predicate to classify individuals or other resources.

Simple entailment is applied in both RDF and RDFS where various terms are used to give meaning. For example, the simple interpretation for RDFS introduces more sets and functions, such as the set of classes (IC). In RDF the function $I_{EXT}$ is a function that maps properties to pairs of resources in the domain, while in
RDFS, the function $I_{CEXT}$ serves as a class extension to map a resource to a set of resources, essentially categorizing a resource to a set of other resources.

## 2.2 RDFS-interpretation for $\Gamma_2$

$\mathbf{IR} = \{$ ty, sc, dom, rng, li, cls, an, pen, tt, jj $\}$

$\mathbf{IP} = \{$ ty, sc, dom, rng, li $\}$

$\mathbf{IC} = \{$ cls, an, pen $\}$

$$\mathbf{I_S} = \left\{ \begin{array}{lll} \text{rdf:type} & \mapsto & \text{ty} \\ \text{rdfs:subclassOf} & \mapsto & \text{sc} \\ \text{rdfs:Property} & \mapsto & \text{pr} \\ \text{rdfs:domain} & \mapsto & \text{dom} \\ \text{rdfs:range} & \mapsto & \text{rng} \\ \text{rdfs:Class} & \mapsto & \text{cls} \\ \text{:likes} & \mapsto & \text{li} \\ \text{:Animal} & \mapsto & \text{an} \\ \text{:Penguin} & \mapsto & \text{pen} \\ \text{:Tweety} & \mapsto & \text{tt} \\ \text{:JollyJumper} & \mapsto & \text{jj} \end{array} \right\}$$

$$\mathbf{I}_{\mathrm{EXT}} = \left\{ \begin{array}{lll} \text{ty} & \mapsto & \{\langle \text{an, cls} \rangle, \langle \text{pen, cls} \rangle, \langle \text{li, pr} \rangle, \langle \text{tt, pen} \rangle\} \\ \text{sc} & \mapsto & \{\langle \text{pen, an} \rangle\} \\ \text{dom} & \mapsto & \{\langle \text{li, an} \rangle\} \\ \text{rng} & \mapsto & \{\langle \text{li, an} \rangle\} \\ \text{li} & \mapsto & \{\langle \text{jj, tt} \rangle\} \end{array} \right\}$$

$$\mathbf{I}_{\mathrm{CEXT}} = \left\{ \begin{array}{lll} \text{cls} & \mapsto & \{ \text{ cls, an, pen } \} \\ \text{an} & \mapsto & \{ \text{ tt } \} \\ \text{pen} & \mapsto & \{ \text{ tt } \} \end{array} \right\}$$

## 2.3 Counter-model for $\Gamma_3$

$\mathbf{IR} = \{$ ty, sc, oom, ooa, oob, el, ea, bo/jo $\}$

$\mathbf{IP} = \{$ ty, sc $\}$

$\mathbf{IC} = \{$ oom, ooa, oob, el, ea $\}$

$$\mathbf{I_S} = \left\{ \begin{array}{ll} \text{rdf:type} & \mapsto \text{ty} \\ \text{rdfs:subclassOf} & \mapsto \text{sc} \\ \text{:OrderOfMammals} & \mapsto \text{oom} \\ \text{:OrderOfAnimals} & \mapsto \text{ooa} \\ \text{:OrderOfBirds} & \mapsto \text{oob} \\ \text{:Elephant} & \mapsto \text{el} \\ \text{:Eagle} & \mapsto \text{ea} \\ \text{:Bobo} & \mapsto \text{bojo} \\ \text{:Joe} & \mapsto \text{bojo} \end{array} \right\}$$

$$\mathbf{I}_{\text{EXT}} = \left\{ \begin{array}{ll} \text{ty} & \mapsto \{\langle \text{el, oom} \rangle, \langle \text{ea, oob} \rangle, \langle \text{bojo, el} \rangle, \langle \text{bojo, ea} \rangle\} \\ \text{sc} & \mapsto \{\langle \text{oom, ooa} \rangle, \langle \text{oob, ooa} \rangle\} \end{array} \right\}$$

$$\mathbf{I}_{\text{CEXT}} = \left\{ \begin{array}{ll} \text{oom} & \mapsto \{ \text{ el, bojo } \} \\ \text{ooa} & \mapsto \{ \text{ oom, oob } \} \\ \text{oob} & \mapsto \{ \text{ ea } \} \\ \text{el} & \mapsto \{ \text{ bojo } \} \\ \text{ea} & \mapsto \{ \text{ bojo } \} \end{array} \right\}$$

In this counter-model, the $\Gamma_3$ graph is a valid consequence, as well as an interpretation where :Bobo and :Joe are interpreted as the same individual.

Because of this, we can see :Joe and :Bobo are defined as both :Elephant and :Eagle, possible due to the subclass hierarchy.