IN4060 - Oblig 5

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1 Model Semantics

Interpretation requirements:

- a set $\triangle^{\mathcal{I}}$ - The domain
- For each individual URI i, an element $i^{\mathcal{I}} \in \triangle^{\mathcal{I}}$
- For each class URI C, a subset $C^{\mathcal{I}} \subseteq \triangle^{\mathcal{I}}$
- for each property URI r, a relation $r^{\mathcal{I}} \subseteq \triangle^{\mathcal{I}} \times \triangle^{\mathcal{I}}$

1.1 Interpretation

1. Interpretation $\mathcal{I}_1$, such that $\mathcal{I}_1 \models \Gamma_1$

   $\triangle^{\mathcal{I}_1} = \{$Tweety, JollyJumper, Bruce, $b_1\}$

   :Tweety$^{\mathcal{I}_1}$ = Tweety

   :JollyJumper$^{\mathcal{I}_1}$ = JollyJumper

   :Bruce$^{\mathcal{I}_1}$ = Bruce

   Let

   $\beta(b_1) = b_1$

   Classes

   :Animal$^{\mathcal{I}_1} = \triangle^{\mathcal{I}_1}$

   :food$^{\mathcal{I}_1}$ = {Bruce}

   :Penguin$^{\mathcal{I}_1}$ = :Bird$^{\mathcal{I}_1}$ = {Tweety}

   :Fish$^{\mathcal{I}_1}$ = :Food$^{\mathcal{I}_1}$ = {Bruce}

   :Horse$^{\mathcal{I}_1}$ = {JollyJumper}

   :Vegetable$^{\mathcal{I}_1} = \emptyset$

Properties

$:eats^{\mathcal{I}_1} = \{\langle Tweety, Bruce\rangle, \langle JollyJumper, b1\rangle\ \}$

$:favouriteFood = \{\langle Tweety, Bruce\rangle, \langle JollyJumper, b1\rangle\}$

$:likes^{\mathcal{I}_1} = \{\langle JollyJumper, Tweety\rangle\}$

$:hasNickname = \{\langle JollyJumpet, "JJ"\rangle, \langle Bruce, "Alonso"\rangle\}$

2. Interpretation $\mathcal{I}_2$ such that $\mathcal{I}_1 \not\models \Gamma_2$

$\triangle^{\mathcal{I}_1} = \{Tweety, JollyJumper, Bruce, b_1\}$

$:Tweety^{\mathcal{I}_1} = Tweety$

$:JollyJumper^{\mathcal{I}_1} = JollyJumper$

$:Bruce^{\mathcal{I}_1} = Bruce$

Let

$\beta(b_1) = b_1$

Classes

$:Animal^{\mathcal{I}_1} = \triangle^{\mathcal{I}_1}$

$:food^{\mathcal{I}_1} = \emptyset$

$:Penguin^{\mathcal{I}_1} = :Bird^{\mathcal{I}_1} = \{Tweety\}$

$:Fish^{\mathcal{I}_1} = :Food^{\mathcal{I}_1} = \{Bruce\}$

$:Horse^{\mathcal{I}_1} = \{JollyJumper, Bruce\}$

$:Vegetable^{\mathcal{I}_1} = \{JollyJumper\}$

Properties

$:eats^{\mathcal{I}_1} = \{\langle Tweety, Bruce\rangle, \langle JollyJumper, b1\rangle\ \}$

$:favouriteFood = \{\langle Tweety, Bruce\rangle, \langle JollyJumper, b1\rangle\}$

$:likes^{\mathcal{I}_1} = \{\langle JollyJumper, Tweety\rangle\}$

$:hasNickname = \{\langle JollyJumpet, "JJ"\rangle, \langle Bruce, "Alonso"\rangle\}$

What is wrong:

- JollyJumber should NOT be a Vegetable.
- Bruce should NOT be a Horse.
- Bruce is not defined as food.

1.2 Entailment

1. Tweety is an animal

   Yes, with the following deriation:

   (a) $: Tweety\ rdf : type\ : Penguin.$ - P
   (b) $: Penguin\ rdfs : subClassOf\ : Animal.$ - P
   (c) $: Tweety\ rdf : type\ : Animal$ - rdfs9 on (a) and (b)

2. :Tweety likes :JollyJumper

   No, this cannot be true in Γ_1.

   Using the interpretation $\mathcal{I}_1$, it follows that Γ_1 is true.

   In this interpretation, there is no pair $\langle$Tweety, JollyJumper$\rangle \in likes^{\mathcal{I}_1}$, therefore, it cannot be true that Tweety likes JollyJumper.

3. :Food is the range of :favouriteFood

   Yes, this is true.

   We know:

   (a) $: favouriteFood\ rdfs : subPropertyOf\ : eats.$ - P
   (b) $: eats\ rdfs : range\ : Food.$ - P

   It cannot be derived directly, but because subproperties inherit properties of its superproperty, this includes the range.

   Meaning, because $: eats$ has $rdfs : range$ and $: favouriteFood$ is $rdfs : subPropertyOf$, therefore $: favouriteFood\ rdfs : range\ : Food.$

4. :Bruce has some favourite food

   No, this cannot be true in Γ_1.

   Using the interpretation $\mathcal{I}_1$, it follows that Γ_1 is true.

   In this interpretation, there is no pair $\langle$Bruce, y$\rangle \in$ favouriteFood, therefore Bruce cannot have any favourite food.

5. :Bruce is a vegetable

   No, this cannot be true in Γ_1.

   Using the interpretation $\mathcal{I}_1$, it follows that Γ_1 is true.

   In the interpretation, vegetable is only defined and not assigned.
   Because we interpret $:Vegetable$ in the interpretation as the empty set, no one or no thing is a vegetable. Therefore, Bruce cannot be a vegetable.

6. :Bruce is a horse

   No, this cannot be true in Γ_1.

   Using the interpretation $\mathcal{I}_1$, it follows that Γ_1 is true.

   If Bruce were to be a horse, the subset $:\text{Horse}^{\mathcal{I}_1}$ must also contain Bruce.
   This is not true, and therefore Bruce cannot be a horse.

7. :Bruce is a fish

   Yes, this fact is defined in Γ_1 and therefore cannot be derived:

   $:Bruce\ rdf:type\ :Fish.$

2 RDF(S) formal semantics)

2.1 Difference in RDF(S) semantics from web and lecture

"Foundations of Semantic Web Technologies" introduces the term "Simple interpretations", a specialized version of interpretation used for giving meaning to RDF graphs specifically.
Simple interpretations contain elements such as a set of resources (IR), a set of properties (IP), and various functions to map elements together, such as the relationship between elements and properties ($\mathbf{I}_{\text{EXT}}$).

Additionally, axiomatic triples are foundational predefined statements that establish concepts such as asserting that rdf:type is a property (rdf:type rdf:type rdf:Property). This is important since rdf:type serves as a predicate to classify individuals or other resources.

Simple entailment is applied in both RDF and RDFS where various terms are used to give meaning. For example, the simple interpretation for RDFS introduces more sets and functions, such as the set of classes (IC). In RDF the function I_{EXT} is a function that maps properties to pairs of resources in the domain, while in
RDFS, the function I_{CEXT} serves as a class extension to map a resource to a set of resources, essentially categorizing a resource to a set of other resources.

2.2 RDFS-interpretation for Γ_2

IR = { ty, sc, pr, dom, rng, li, cls, an, pen, tt, jj }

IP = { ty, sc, dom, rng, li }

IC = { cls, an, pen, pr}

$$
\mathbf{I_S} = \left\{
\begin{array}{lcl}
\text{rdf:type} & \mapsto & \text{ty} \\
\text{rdfs:subclassOf} & \mapsto & \text{sc} \\
\text{rdfs:Property} & \mapsto & \text{pr} \\
\text{rdfs:domain} & \mapsto & \text{dom} \\
\text{rdfs:range} & \mapsto & \text{rng} \\
\text{rdfs:Class} & \mapsto & \text{cls} \\
\text{:likes} & \mapsto & \text{li} \\
\text{:Animal} & \mapsto & \text{an} \\
\text{:Penguin} & \mapsto & \text{pen} \\
\text{:Tweety} & \mapsto & \text{tt} \\
\text{:JollyJumper} & \mapsto & \text{jj}
\end{array}
\right\}
$$

$$
\mathbf{I_{EXT}} = \left\{
\begin{array}{lcl}
\text{ty} & \mapsto & \{\langle an, cls\rangle, \langle pen, cls\rangle, \langle li, pr\rangle, \langle tt, pen\rangle\} \\
\text{sc} & \mapsto & \{\langle pen, an\rangle\} \\
\text{dom} & \mapsto & \{\langle li, an\rangle\} \\
\text{rng} & \mapsto & \{\langle li, an\rangle\} \\
\text{li} & \mapsto & \{\langle jj, tt\rangle\}
\end{array}
\right\}
$$

$$
\mathbf{I_{CEXT}} = \left\{
\begin{array}{lcl}
\text{cls} & \mapsto & \{ cls, an, pen \} \\
\text{an} & \mapsto & \{ tt \} \\
\text{pen} & \mapsto & \{ tt \} \\
\text{pr} & \mapsto & \text{IP}
\end{array}
\right\}
$$

2.3 Counter-model for Γ_3

IR = { ty, sc, oom, ooa, oob, el, ea, bojo }

IP = { ty, sc }

IC = { oom, ooa, oob, el, ea }

$$
\mathbf{I_S} = \left\{
\begin{array}{lcl}
\text{rdf:type} & \mapsto & \text{ty} \\
\text{rdfs:subclassOf} & \mapsto & \text{sc} \\
\text{:OrderOfMammals} & \mapsto & \text{oom} \\
\text{:OrderOfAnimals} & \mapsto & \text{ooa} \\
\text{:OrderOfBirds} & \mapsto & \text{oob} \\
\text{:Elephant} & \mapsto & \text{el} \\
\text{:Eagle} & \mapsto & \text{ea} \\
\text{:Bobo} & \mapsto & \text{bojo} \\
\text{:Joe} & \mapsto & \text{bojo}
\end{array}
\right\}
$$

$$
\mathbf{I_{EXT}} = \left\{
\begin{array}{lcl}
\text{ty} & \mapsto & \{\langle el, oom\rangle, \langle ea, oob\rangle, \langle bojo, el\rangle, \langle bojo, ea\rangle\} \\
\text{sc} & \mapsto & \{\langle oom, ooa\rangle, \langle oob, ooa\rangle\}
\end{array}
\right\}
$$

$$
\mathbf{I_{CEXT}} = \left\{
\begin{array}{lcl}
\text{oom} & \mapsto & \{ el, bojo \} \\
\text{ooa} & \mapsto & \{ oom, oob \} \\
\text{oob} & \mapsto & \{ ea, bojo\} \\
\text{el} & \mapsto & \{ bojo \} \\
\text{ea} & \mapsto & \{ bojo \}
\end{array}
\right\}
$$

In this counter-model, the Γ_3 graph is a valid consequence, as well as an interpretation where :Bobo and :Joe are interpreted as the same individual.

Because of this, we can see :Joe and :Bobo are defined as both :Elephant and :Eagle, possible due to the subclass hierarchy.

Since bojo is in both el and in ea, $bojo \in I_{CEXT}(el) \cap I_{CEXT}(ea)$, the individual belongs in both classes, therefore showing the contradiction,