Solving Empirical Bioeconomic Models: A Rangeland Management Application

Richard B. Standiford and Richard E. Howitt

An empirical bioeconomic model was developed for private ranches conducting firewood, cattle, and hunting enterprises on California's hardwood rangelands. Hunting, forage, cattle, and oak firewood production functions were derived. Nonlinear optimal control was used to solve for two state and four control variables to give optimal time pathways for oak density and cattle stocking. Risk was considered through use of chance constraints, rainfall variability, and price expectations. Commercial hunting is shown to be the dominant economic value on these rangelands. Inclusion of stochastics shifts production away from cattle to less risky firewood and hunting enterprises.

Key words: bioeconomics, optimal control, range economics.

Bioeconomic models that integrate biological dynamics and the resulting economic behavioral response have been widely used in theoretical analysis of natural resource problems (Clark, Mangel, Fisher). However, empirical applications of these models have been less common. As use of bioeconomic models moves from theory to policy analysis, empirical problems in solving these models come to the fore.

This paper develops an approach to solving a policy-relevant bioeconomic model characterized by four types of problems. First, the number of state and control variables preclude easy solution by traditional numerical dynamic programming methods. Second, the model has variables with different time steps; that is, different growing seasons have to be nested with single-year decisions. Third, some variables are subject to inequality constraints. Dual values of these constraints are often of considerable policy significance. Fourth, the chance that economic and biological data sets used for parameter estimation will cover the same time interval is remote. In the latter case, the two parameter sets probably will need to be calibrated into a single model using a smaller common data set.

The above problems are found in a bioeconomic model of a ranch in California's hardwood range region. Solution of the resulting control model is achieved using a nonlinear optimization approach.

The California hardwood rangeland region was chosen because of the dynamic interactions between diverse resource values. The 7.4 million acres of hardwood rangeland (Bolsinger) are composed of a woodland and savannah tree overstory of various oak species (*Quercus spp.*) and an annual grassland understory. Historically, these predominantly privately owned areas have been managed for livestock. Hardwood rangelands also provide important wildlife habitat, high quality water supply, outdoor recreation, and aesthetic values.

To date, empirical dynamic range economics models have used dynamic programming solution methods (Burt; Karp and Pope; Garoian, Mjelde, and Connor). The solution of nonlinear control problems by optimization approaches has been established conceptually for some time (Canon, Cullum, and Polok) but not widely used in economics. Our study uses GAMS/MINOS (Brooke, Kendrick, and Meeraus), which enables nonlinear control problems with realistic numbers of states and controls to be solved on personal computers. The hardwood range model uses estimated relationships between annual grasslands, oaks, cattle, and their controls to

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analyze the effect of hunting revenues on the optimal oak density over time.

Detailed description of the range model construction is useful for those concerned with range problems, but more importantly as a template to model a wide range of empirical bioeconomic problems.

Hardwood Rangeland Policy Issues

The study was prompted by several policy issues. Hardwood rangelands have decreased by at least 1.2 million acres in California since 1945 (Bolsinger). Poor regeneration of some oak species has caused concern about the ecological sustainability of this resource (Bolsinger). Low profitability from livestock operations raises questions about the economic sustainability of extensively managed hardwood rangelands.

A policy of regulating oak tree cutting has been suggested to maintain this resource (State Board of Forestry 1982, 1983). As a precursor to regulation, our study is designed to evaluate the optimum level of oak retention for hardwood range landowners operating under strictly private property incentives.

Two major economic forces influencing oak cutting in the state's hardwood range areas are firewood harvesting and oak tree removal for range improvement. Recent increases in demand for private recreational hunting also influence the stock of oak trees on hardwood rangelands, since the principal upland game species in the state—deer, quail, turkey, and feral pig—are all enhanced by oak stands (Barrett). Landowners who market hunting rights may be able to capture private economic benefits from increased oak stocks.

An Empirical Optimal Control Framework

The behavior of hardwood range owners is modeled as an optimal control problem, empirical solution of which is usually obtained through numerical dynamic programming. An alternative method is to employ nonlinear optimization. Nonlinear optimization methods require simplifying assumptions on the stochastic specification, while dynamic programming usually requires simplification of the problem structure by reducing the number of states and controls or reducing the number of discrete levels at which they can be set. While both methods have advantages, we feel nonlinear optimization can adequately model the stochastic aspects of the present problem. On the economic side, this involves specifying chance constraints and expected prices; on the biological side, it involves perturbing the control model with an exogenous sequence of historical rainfall data. Detailed specification of dynamic growth functions permitted by the optimization solution enable us to solve quickly for the initial conditions of alternative locations.

Management decisions determine the optimum trajectory of controls and linked stocks over time. Equation (1) below defines the problem in continuous time. The rancher's objective function is to maximize net present value from firewood harvesting, livestock production, and commercial hunting. The state variables are the stocks of oak trees and livestock, while control variables are the amount of oak firewood cut and livestock sold (or bought). The ranch manager makes annual decisions about the level of these control variables. The appendix defines all variables used in this paper.

$$(1) \quad \max \text{NPV} = \int_{t=0}^{T} e^{-rt}\{WR_t(WDSEL_t) + HR_t(WD_t, HRD_t, \text{exog.}) + LR_t[HRD_t, CS_t, FOR_t(WD_t, \text{exog.})]\}$$

subject to $\dot{WD} = F(WD_t, \text{exog.}) - WDSEL_t$ [Equation of motion for oaks]

$\dot{HRD} = G(HRD_t, \text{exog.}) - CS_t$ [Equation of motion for livestock]

$WD_0 = INITWD$ [Initial stock of wood]

$HRD_0 = INITHRD$ [Initial stock of livestock]

$WDSEL_t \geq 0$ [Wood cutting nonnegativity constraint].

The firewood revenue function (WR_t) depends on the firewood harvest control variable, because gross revenue and harvesting and processing costs are affected by quantity of firewood sold. Hunting revenue (HR_t) is a function of the stock of oak trees and livestock. Oaks provide habitat for game species, while the cattle operation competes with the hunting enterprise for both forage and labor. The structure of this function shows that managers choose optimum hunting level by controlling firewood harvesting and livestock density.

Livestock revenue (LR_t) is a function of the number of cattle sold and the total stock of cattle and oaks. Oak trees affect the livestock revenue function by their effect on forage production. Herd size influences variable costs and forage needs. The two equations of motion define changes in state variables over the optimization period. The instantaneous rate of change of state variables is the difference between biological growth and amount of firewood or cattle sold.

Empirical Methodology—Discrete Time Optimal Control

Given this general framework, a discrete time model was developed using empirical estimates of the production process. Three seasons within the year were selected to model range forage production, and yearly time increments were used for hunting and wood volume production. The difference in time steps is to be expected in multiple-output bioeconomic models, where there may be different time intervals between the biological states or between biological and behavioral variables. Annual grassland forage available has three interdependent seasons per year, each with their own equations of motion, but also have to nest into the annual state equations and controls, such as the cattle stocking rate or quantity of oaks cut. This substantially expands the model's dimensions. The ability to solve optimally for controls over different nested time steps is another advantage of an optimization solution to the control problem.

Objective Function

Ranchers are hypothesized to base current price expectations on a weighted average of past observations. A stationary autoregressive process was assumed to represent real price expectations for cattle, hay, and firewood prices. An autocorrelation function was calculated for each price variable. Inspection of the autocorrelations determined the number of years ranchers might use to weight their price expectation. Autoregression functions for hay and cattle prices converged to zero and were stationary. The firewood price series was transformed by differencing once to give a stationary time series. Box-Jenkins analysis was used to estimate coefficients of the ARIMA models as well as variances of estimates. A covariance matrix was calculated for the three cattle classes included in the model; cull cows, steers, and heifer calves. Firewood, hay, and livestock prices were found to be mutually independent. Use of the covariance matrix in the chance constrained analysis will be described later.

The objective function is to maximize discounted net revenue from firewood harvesting, hunting revenue, and livestock revenue:

$$(2) \quad \max \sum_{t=1}^{T} \{DF_t \cdot (WR_t + LR_t - LC_t + HR_t - HC_t) + TV_T\}.$$

Firewood revenue. Equation (3) shows revenue from firewood in time t. Net firewood price is determined as the market firewood price minus a harvesting cost relationship. Since there are no firewood processing studies in California, firewood harvest cost was estimated from data on costs of harvesting, processing, and transporting firewood in hardwood stands in New Hampshire (Dammann and Andrews). We derived firewood price expectations from an ARIMA model drawn with data in Doak and Stewart. Quantity of wood sold, $WDSEL_t$, is a control variable in this system.

$$(3) \quad WR_t = [E(PWD_t) - 1629.45 \cdot (0.01 + WDSEL_t)^{-0.139}] \cdot WDSEL_t.$$

Livestock revenue. The livestock enterprise is assumed to be a cow-calf operation, the predominant type on hardwood rangelands in California. Livestock revenue in time t is composed of sale of feeder calves (steers and heifers) and sale of cull cattle. ARIMA models for cull cows, feeder steers, and feeder heifers, derived from published data (USDA, CDFA), were used for the livestock price expectations.

Number of steers sold is a function of the herd's calving percentage and herd size (HRD_t), a state variable for this dynamic system. Calving percentage is assumed to be exogenous. Cattle performance is determined from *COWFLOW* (Bell), based on livestock weights and target rates of gain, with nutritional needs being fully met as described in a later section.

$$(4) \quad STSEL_t = HRD_t \cdot (CLF/2).$$

Number of heifer calves sold is the difference between number of heifer calves born and number used as replacement heifers, REP_t. The number of replacement heifers added to the herd is a control variable for herd size and is determined in solution of the control model:

$$(5) \quad HRD_t \cdot (CLF/2) = HFSEL_t + REP_t.$$

The overall livestock revenue equation for this system, shown in (6), incorporates sale of feeder calves as discussed above as well as any cull cattle sold in time t. We assume a 20% herd replacement, meaning cows are culled every 5 years after 1 year as a replacement heifer. Weights for the different livestock classes are determined exogenously.

$$(6)\quad LR_t = [STSEL_t \cdot WTST \cdot E(PRST_t)] + [HFSEL_t \cdot WTHF \cdot E(PRHF_t) + REP_{t-6} \cdot WTCC \cdot E(PRCC_t)].$$

Costs of the livestock enterprise, shown in (7), include variable and feed costs. Variable costs are based on herd size and were taken from Van Riet and Drake. Feed costs are based on the amount of feed purchased per season, a control variable estimated in the course of optimization.

$$(7)\quad LC_t = HRD_t \cdot VC + \sum_{j=1}^{3} E(PFEED_t) \cdot (FED_{j,t}).$$

Hunting revenue. Hunting revenue and cost is estimated from sixty personal interviews of ranchers with hunting programs on hardwood rangelands. Survey details are provided in Loomis and Fitzhugh, and Standiford.

Total revenue- and cost-per-acre functions for deer clubs were estimated from a hedonic regression (Rosen), which decomposes the costs and revenues into the various physical and biological attributes of the hunting club. Equations (8) and (9) show results of the hedonic regression analysis. These functional forms are based on non-nested hypothesis tests comparing linear and Cobb-Douglas forms (Davidson and MacKinnon). Oak crown cover percentage is a state variable for this system, and the number of animal unit months (*AUMs*) allocated to deer is a control variable.

$$(8)\quad HR_t = HUNT \cdot 17.942 \cdot CC_t^{0.0839} \cdot SCEN^{0.726} \cdot HINC^{0.126} \cdot DRTRP^{0.0457} \cdot (DRAUM_t/ACRE)^{0.131} \cdot PPIG^{0.138} \cdot ACRE^{-0.234} \cdot ADVA_t^{0.108} \cdot ACRE.$$

$$(9)\quad HC_t = HUNT \cdot (0.71185 + 2.2893 \cdot ADVA_t - 0.3818 \cdot GUIDE + 0.6787 \cdot TAG) \cdot ACRE.$$

Terminal value. Every discrete time control problem requires a terminal value function to yield the transversality condition. The effect of the terminal value function on controls depends on length of the control horizon, discount rate, and relative value of the function. Future firewood harvest is assumed to occur as an infinite series of 20-year cutting cycles with harvest equal to the 20-year oak growth. The future earning stream for hunting and livestock is assumed to be an infinite series of annual returns equivalent to the value in period T. Equation (10) shows the terminal value equation derived from equations (3) through (9):

$$(10)\quad TV_T = DF_T \cdot [(HR_T - HC_T + E(LR_T) - E(LC_T))/i + ((E(PWD_T) - 1629.45 \cdot (0.01 + 20 \cdot PAI_T)^{-0.139}) \cdot 20 \cdot PAI_T)/((1+i)^{20} - 1)].$$

Equations of Motion

Oak equation of motion. The equation of motion for oak volume is derived from data taken on 81 sample plots in hardwood rangelands throughout the state (Standiford and Howitt). Equation (11) shows that the total wood volume in a given year includes the volume in the previous year plus annual growth less volume harvested. Oak wood volume is a state variable for this model and quantity of wood sold is a control variable:

$$(11)\quad WD_t = WD_{t-1} + (0.00195 \cdot WD_{t-1}^{\ 0.61157} \cdot SITE^{1.209}) - WDSEL_t.$$

Equation (12) shows the identity which converts wood volume to oak crown canopy based on the study described in Standiford and Howitt. This allows for interaction of oak cover with forage production and hunting revenue:

$$(12)\quad CC_t = 0.00152 \cdot WD_t^{0.42681} \cdot SITE^{0.83366}.$$

An additional constraint is added to ensure that more wood cannot be sold than exists on the site:

$$(13)\quad WD_t - WDSEL_t \geq 0.$$

Cattle and forage equation of motion. The cattle equation of motion is based upon the stock of cattle in the previous time period plus the number of cows added as replacement heifers, less the number of cattle culled and lost to mortality:

$$(14)\quad HRD_t = (1 - MORT) \cdot HRD_{t-1} + REP_t - REP_{t-6}.$$

The seasonal forage equation of motion assumes that forage growth on the annual grassland range occurs only in the first two seasons (i.e., September 1 through May 31), with residual forage left in the summer available as low-quality dry forage. A data set of 142 observations of seasonal forage production in the open and under several different densities of oak canopy was collected from four different studies, representing time series ranging from 2 to 22 years (Jensen; McClaren and Bartolome; Kay; Heady and Pitt). Since the literature indicates that the effect of overstory varies by rainfall zone (Holland; Kay), seasonal rainfall was included. A combined cross-section analysis for the sample sites and time series analysis for the various data years was used to predict forage yield as a function of overstory oak tree density and climatic variables. A nonnested hypothesis test (Davidson and MacKinnon) on two hypothesized functional forms for the forage/tree cover relationship showed that a nonlinear form should be used for the forage equation of motion during the winter and spring, as shown in (15). Available forage is expressed in terms of total *AUMs* available for the season:

$$(15)\quad FOR_{j=1,2,t} = AVL_j \cdot (\exp[(-0.057 + 0.0189 \cdot RAIN_{j,t} - 0.000185 \cdot RAIN_{j,t}^2) \cdot \ln(100 - CC_t) + 4.3314 + SIINT + SISLP \cdot RAIN_j]) \cdot ACRE/FORQUNT_j.$$

Available summer forage is the difference between total production at the end of spring (peak standing crop) and forage consumed in winter and spring by wildlife and livestock. Summer residual dry matter in the annual range is degraded from effects of shattering and nutrient leaching:

$$(16)\quad FOR_{j=3,t} = AVL_j \cdot (1 - DEG_j) \cdot (FOR_{j-1,t} - CAUM_{j-1,t} - DERAUM_{j-1,t} + FED_{j-1,t} + FED_{j-2,t}).$$

Livestock nutritional requirements were determined exogenously using *COWFLOW* (Bell). Managers set livestock weights and target rates of gain to calculate seasonal *AUMs*. These *AUMs* include cows, calving percentage, and proportion of bulls in the herd. Additional complexity could have been introduced by including optimum sale weight and a livestock growth function. However, a sale-weight decision variable would have made solution much more difficult. Equation (17) shows determination of livestock *AUMs*:

$$(17)\quad CAUM_{j,t} = HRD_t \cdot NUT_j.$$

A forage availability constraint balances forage production and quantity of supplemental feed against livestock nutritional needs. Quantity of feed in season *j* and allocation of *AUMs* to the hunting enterprise are control variables:

$$(18)\quad FOR_{j,t} + FED_{j,t} - CAUM_{j,t} - DERAUM_{j,t} \geq 0.$$

Annual *AUMs* deferred from cattle production and allocated to deer production contribute to the hunting revenue function in (8). This value is determined from (19), which balances seasonal fluctuations in allocations to deer:

$$(19)\quad DERAUM_{j,t} \geq DRAUM_t.$$

Initial conditions for the stock of cattle and wood volume are specified as

$$(20)\quad HRD_0 = INITHRD \quad \text{(initial stock of livestock);}$$

$$(21)\quad WD_0 = INITWD_0 \quad \text{(initial stock of wood).}$$

Nonnegativity constraints are imposed on all state and control variables.

Incorporating Uncertainty—Chance-Constrained Approach

A chance-constrained approach to uncertainty was chosen to incorporate producer price uncertainty (Charnes and Cooper). The lexicographic utility function underlying this safety-first specification (Robison) was selected as best representing the risk attitudes of hardwood range managers. Incorporating time-varying inequality constraints on state and control variables made empirical implementation of this risk specification straightforward. If empirical estimates of the monetary cost of risk in terms of Arrow-Pratt measures were available, these also could have been incorporated..

The general chance constrained problem is

$$(22)\quad \text{Prob}[GM_t \geq 0] \geq \gamma \text{ where } GM_t = WR_t + LR_t - LC_t + HR_t - HC_t,$$

where γ is the probability that net cash flow each year is positive. For example, suppose a producer's indebtedness and interest rate dictate that a net loss can occur only in one year out of ten. The chance-constrained approach then is to set at 0.90 the probability that total revenue is greater than or equal to zero. As γ increases, the less able is a producer to take a loss, representing a more risk-averse individual.

Gross margin has a normal distribution:

(23) $$GM_t \sim N[\mathrm{E}(GM_t), \mathrm{var}(GM_t)].$$

A standard normal variable, $z_{1-\gamma}$ is chosen so that

(24) $$\gamma = \mathrm{Prob}[-GM_t \leq 0] = \mathrm{Prob}[(-GM_t + \mathrm{E}(GM_t))/(\mathrm{var}(GM_t)^{.5})] \leq z_{1-\gamma}.$$

Equation (24) is transformed to give (25), a deterministic equation which fully captures the probabilistic elements of (22):

(25) $$\mathrm{E}(GM_t) - z_{1-\gamma} \cdot \mathrm{var}(GM_t)^{.5} \geq 0.$$

Total revenue for the hardwood range ranch is the weighted sum of a vector of five revenues and three costs. One revenue source—hunting—and three costs are specified as deterministic. Selling price of wood and purchase price of hay are stochastic but independent of each other and the three cattle prices. Distribution of wood and hay prices, and the joint distribution of steer, heifer, and cull cow prices, are determined from time series data.

Given the vector of expected prices $\mathrm{E}(P_t)$, the matrix of variance and covariances Σ_p, and the normal distribution of gross margin (23), gross margin is $\mathrm{E}(P_t)'x_t$, where x_t is a vector of outputs (firewood sold, cattle sold) and inputs (hay bought). Variance of gross margin can be expressed as $x_t'\Sigma_p x_t$.

Given the simplification of the variance covariance matrix, (25) can be rewritten as

(26) $$\mathrm{E}(PWD_t) \cdot WDSEL_t + \mathrm{E}(PRST_t) \cdot WTST \cdot STSEL_t + \mathrm{E}(PRHF_t) \cdot WTHF \cdot HFSEL_t + \mathrm{E}(PRCC_t) \cdot WTCC \cdot CULL_t - \mathrm{E}(PFEED_t) \cdot FED_{j,t} - HRD_t \cdot VC + HR_t - HC_t - z_{1-\gamma} \cdot [WDSEL_t^2 \cdot \mathrm{var}(PWD) + (WTST \cdot STSEL_t)^2 \cdot \mathrm{var}(PRST) + (WTHF \cdot HFSEL_t)^2 \cdot \mathrm{var}(PRHF) + (WTCC \cdot CULL_t)^2 \cdot \mathrm{var}(PRCC) + 2 \cdot (WTST \cdot STSEL_t \cdot WTHF \cdot HFSEL_t) \cdot \mathrm{cov}(PRST, PRHF) + 2 \cdot (WTCC \cdot CULL_t \cdot WTST \cdot STSEL_t) \cdot \mathrm{cov}(PRCC, PRST) + 2 \cdot (WTHF \cdot HFSEL_t \cdot WTCC \cdot CULL_t) \cdot \mathrm{cov}(PRHF, PRCC) + FED_t^2 \cdot \mathrm{var}(PFEED)]^{.5} \geq 0.$$

The chance constraint on annual gross margin is incorporated as an inequality constraint in each time period. Analytic solution of the discrete time, inequality constrained control problem shows that binding constraints are reflected by changes in current costate values, hence all previous costates and control actions (Chow). In this way, a binding chance constraint in the future will modify current optimal control actions. Thus, while specified as single-period constraints, the chance constraints are reflected in the rancher's earlier actions. Analytic recursive methods of empirically solving the control problem cannot reflect the constraint values in previous time periods, whereas solution by optimization that does not rely on recursive properties can incorporate such a constraint.

Solution Techniques

Equation (2) is the nonlinear control objective function. The problem is constrained by nineteen equations, (3) through (21). The number of equations could be reduced slightly by substitution, but the need for a six-year lag on the effect of current reproduction on cow culling stock coupled with seasonal forage equations results in the set of nineteen linkages, equations of motion, and inequality constraints. The control problem is collapsed into an optimization format by defining the state and control variables in each time period as activities. The equations of motion are then specified as nonlinear constraints in each time period that link one or more time periods.

This system was solved using GAMS/MINOS (Brooke, Kendrick, and Meeraus) for nonlinear optimization on a personal computer. GAMS/MINOS uses a reduced gradient algorithm (Wolfe), combined with a quasi-newton optimization method (Davidson) and sequential linearization of the nonlinear constraints. Given that second order conditions hold, that is, that the objective function is concave and the constraint set is convex with respect to controls and states, optimal solutions should converge. Skillful scaling of the hessian of the nonlinear problem is important to guarantee a solution, but general rules for this are not yet available (Gill, Murray, and Wright).

The optimal control model was solved for the certainty equivalent case for the four control variables—namely, forage allocation to hunting ($DRAUM_t$), supplemental feed purchased ($FED_{j,t}$), number of cattle to hold as replacement heifers (REP_t), and quantity of firewood sold ($WDSEL_t$)—and for two state variables—number of cow-calf pairs (HRD_t) and standing volume of oak trees (WD_t). The chance-constrained case was solved by adding (26) as a constraint. Forage variability was considered by including the actual time series of seasonal rainfall for the areas evaluated.

The problem was solved over a 20-year planning horizon ($T = 20$). The first six years of the solution served as an initialization period to set up the necessary lags. Price expectations and actual rainfall series used following the initialization period corresponded to a fourteen-year control period. Solution time on a 386 personal computer was under 15 minutes. The GAMS program used for this bioeconomic model is available from the authors.

Results

Preliminary optimization runs were evaluated to test the use of the model for policy analysis. These were compared to actual oak firewood harvest data in Bolsinger to determine if calculated optimum oak harvest was within the historical range. For all oak volumes evaluated, except the lightest, the normative model showed that the oak trees should be clearcut to maximize net present value. Oak clearcutting at all but the lowest stand volume does not correspond to actual practice in which harvest over a 14-year period occurred on only 14% of average blue oak stands and 22% of live oak stands.

These preliminary control runs indicate our firewood harvest specification is a "bang-bang" control. The control is either at its minimum (no wood cut) or its maximum (all wood clearcut). Since firewood harvest costs in (3) show a monotonic decrease as wood harvest increases, net value of wood harvest is a maximum when trees are clearcut. The model finds the combination of retail firewood price expectation and future tree growth to maximize this net revenue.

Calibration Using Hedonic Firewood Costs

In its "bang-bang" response, the model specification based on the wood revenue equation, (3), does not allow partial oak harvest. Since actual wood harvest data show that partial harvesting is the most typical method, the normative specification is unsuitable for empirical policy analysis. Such a shortcoming is undoubtedly because of failure to accurately account for the rancher's marginal utility of maintaining a given tree stock. Ranchers may value a stock of oak trees for the benefits they add for future subdivision, for their personal aesthetics, or for personal value of game and nongame wildlife species, even in the absence of a commercial hunt club. These utilities were not adequately represented in the specified model. There may also be disutility of pressure from neighbors who are concerned with clearcutting oaks on hardwood rangelands. Monotonically increasing net firewood revenue as the wood harvest control level increases is probably inaccurate in a utility sense.

We hypothesize the discrepancy between results based on the engineering wood revenue function (3) and observed rancher behavior can be reconciled through a hedonic cost function for oak wood harvesting (Rosen). Our approach was to estimate the hedonic cost which would reflect the net value of the wood harvest to the rancher by calibrating the model to actual oak harvest data.

The behavioral model was determined by solving the entire optimal control model, constrained by actual firewood harvested over the 14-year period, for different hardwood range forest types (Bolsinger). Shadow price of the actual harvest calibration constraint represents the additional marginal cost of harvesting oak trees. This hedonic cost can be thought of as composed of the missing elements of the true costs omitted from the preliminary normative model. Marginal cost was obtained as shown below.

A quadratic cost curve is hypothesized:

$$\text{(27)}\quad \text{TOTAL FIREWOOD ADJUSTMENT COST} = ALPHA \cdot WDSEL^2,$$

and the marginal firewood cost is

$$\text{(28)}\quad \text{MARGINAL COST} = \text{Shadow value} = ALPHA \cdot 2 \cdot WDSEL.$$

Since the shadow value is determined through several calibration optimal control runs for different known amounts of firewood cut ($WDSEL$), the $ALPHA$ can be calculated as

$$\text{(29)}\quad ALPHA = \text{COSTATE}/(2 \cdot WDSEL).$$

Adjustment cost will vary depending on initial wood volume, amount of wood cut, and presence of stochasticity constraints. Therefore, adjustment cost was calibrated for a variety of conditions, and a set of $ALPHA$ values for adjustment cost were determined for two range sites and for stochastic and deterministic cases.

The rancher's net firewood return is derived by modifying (3) to include this adjustment cost:

$$\text{(30)}\quad WR_t = [\text{E}(PWD_t) - 1629.45 \cdot (.01 + WDSEL_t)^{-0.139} - ALPHA \cdot WDSEL_t] \cdot WDSEL_t.$$

Such an adjustment enables the oak stand

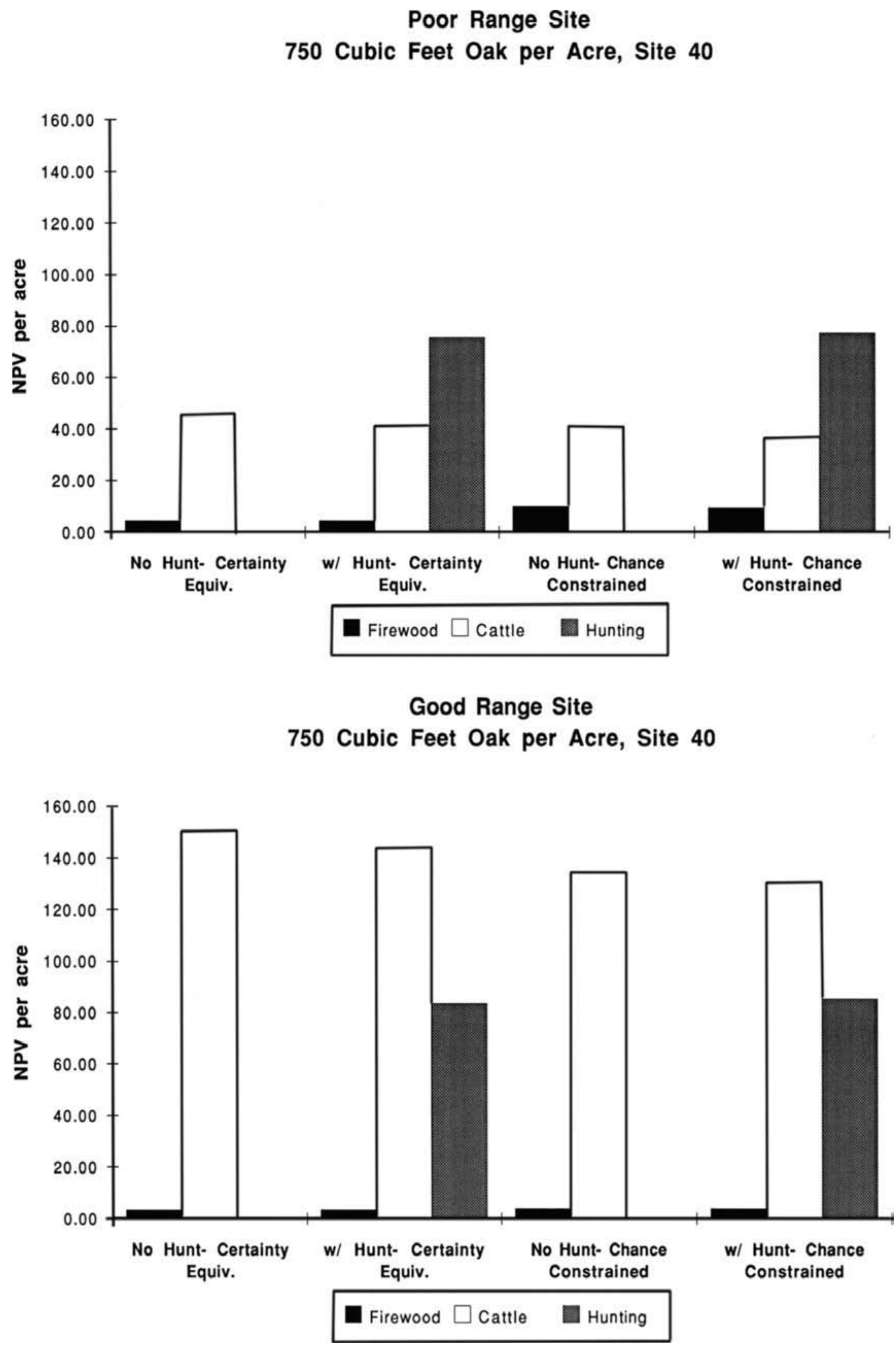

Figure 1. Hunting, firewood, and cattle production effect on net present value

model, economic ranch model, and normative tree harvest cost function to calibrate against observed behavior to produce an integrated and consistent bioeconomic model.

Policy Scenarios Evaluated

Using the behavioral bioeconomic control model specified above, a number of scenarios were considered. Forage production equations for a low and high quality range site were evaluated. Initial conditions for livestock were tied to range productivity. Three alternative oak site indexes represented poor, medium, and good oak stand growth conditions. Four different initial oak volume levels (*INITWD*) were represented, namely 250, 500, 750, and 1000 cubic feet per acre. Effect of hunting was evaluated by solving the optimal control for average- and good-quality hunting conditions, and for no hunting. Average-quality hunting was simulated by setting the exogenous variables in hunting revenue function (8) equal to mean values from our survey of hunt club operators. High-quality hunting conditions were represented by setting the exogenous variables one standard error above the mean survey values. The entire set of policy runs was solved for both the certainty equivalent and chance-constrained case. Chance-constrained runs assumed a probability of 0.90 of not losing money. This range of scenarios resulted in 144 optimal solutions with trajectories for the two state variables (WD_t and HRD_t), and four control variables ($DRAUM_t$, $FED_{j,t}$, REP_t, and $WDSEL_t$).

Effect of Hunting on Total Return

Figure 1 shows net present value (NPV) of poor and good quality hardwood range sites with an initial condition of 750 cubic feet of oak per acre. The figure shows the major impact that hunting has on total economic value. On the poor hardwood range site, total NPV is increased by 144% when average quality hunting is included for the certainty equivalent case (from \$50 to \$122 per acre), and hunting is the dominant economic enterprise. The same effect is observed for the chance constrained case. The results show that on poor range sites, hunting and firewood returns are higher and livestock returns are lower for the chance constrained case than the certainty equivalent case. On the good range site, hunting increases NPV by 50% over the no hunting case (from \$154 to \$231 per acre), although cattle production is the dominant economic value.

Optimal Wood Harvest Levels

Table 1 shows cumulative firewood harvest over the 14-year control period by oak site index, initial oak volume, and range productivity class with and without hunting. The effect of including risk is also shown. Our hypothesis was that selling

Table 1. Cumulative Firewood Harvest in Cubic Feet Per Acre Over 14 Years

Range production	Method of uncertainty	Initial oak Volume (cu.ft./ac)	Oak site class 30 No hunt	30 Hunt	40 No hunt	40 Hunt	50 No hunt	50 Hunt
Poor range site	Certainty equivalent	250	0.00	0.00	0.00	0.00	0.00	0.00
		500	0.00	0.00	71.43	64.88	73.06	67.00
		750	116.42	110.86	117.87	112.62	120.21	115.54
		1000	229.25	222.57	231.48	225.26	235.54	230.29
	Chance constrained	250	0.00	0.00	0.00	0.00	0.00	0.00
		500	0.00	0.00	0.00	0.00	0.00	0.00
		750	281.05	260.16	122.81	114.28	0.00	0.00
		1000	258.48	249.58	232.05	223.91	204.86	197.91
Good range site	Certainty equivalent	250	0.00	0.00	0.00	0.00	0.00	0.00
		500	0.00	0.00	0.00	65.69	0.00	0.00
		750	114.46	104.49	117.62	112.58	123.81	118.43
		1000	225.71	219.19	231.64	224.66	241.33	234.86
	Chance constrained	250	0.00	0.00	46.81	0.00	67.92	0.00
		500	68.95	62.56	71.51	65.02	102.27	68.69
		750	114.46	108.86	116.98	111.93	122.31	117.39
		1000	223.42	217.41	228.81	222.22	237.09	228.17

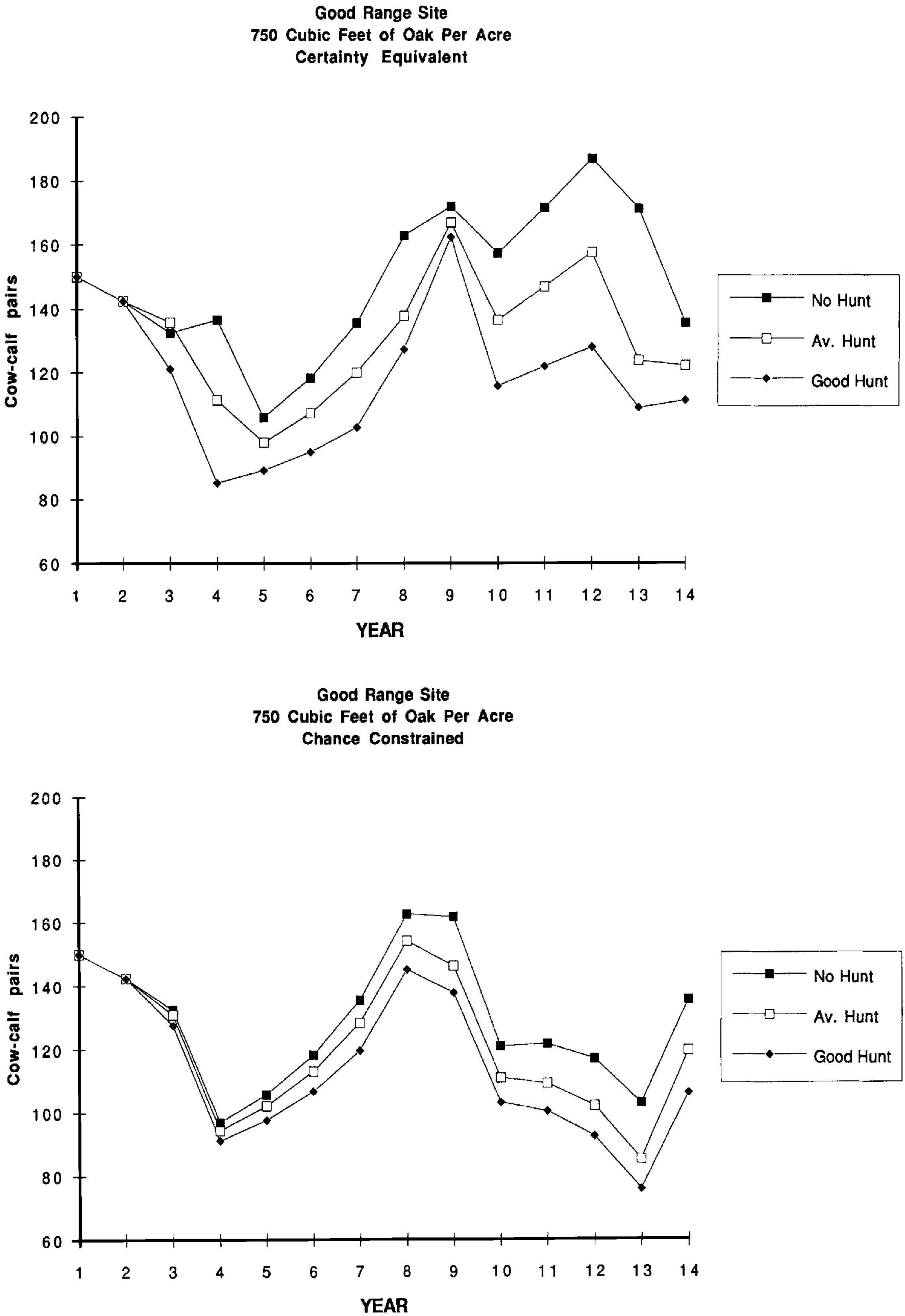

Figure 2. Hunting and risk effects on cow-calf pair trajectory

hunting rights would provide a market mechanism for conserving oaks because of their wildlife habitat value. Results show that slightly less oak firewood harvest occurs when hunting revenues are received, especially in stands with 750 to 1,000 cubic feet per acre. This result indicates that the marginal decrease in hunting revenue from oak canopy changes is greater than the marginal revenue from firewood harvest. Although hunting apparently does provide an incentive for hardwood range managers to conserve oaks, the magnitude of the harvest differences between hunting and no hunting is relatively small. The adjustment cost calculated for firewood harvest may reflect enhanced rancher utility from game habitat, nongame wildlife, and the aesthetic value of oaks even without a hunting club.

On the good range site and at 500 cubic feet per acre initial volume, more firewood is cut in the presence of risk than without risk. On these good range sites with higher cattle numbers and risk associated with cattle price fluctuations, higher wood harvest helps to ensure the constraint for a positive net revenue is met with the required probability.

Optimal Livestock Density

Figure 2 shows an example of the optimal trajectory for cow-calf pairs given a good range site and an initial oak volume of 750 cubic feet per acre. As hunting quality increases, livestock density decreases. The low inventory of livestock can be thought of as allocation of forage to wildlife and allocation of management effort to the hunting enterprise. Allocation of *AUMs* to either the hunting or livestock operation is optimal when hunting revenue from the last *AUM* added to the hunting enterprise equals the associated decrease in livestock revenue.

When risk is included, livestock numbers do not show as much annual fluctuation as in the certainty-equivalent case because, in the chance-constrained model, costs of carrying the extra cattle stock in years following peak prices would exceed the positive net revenue constraint.

General Conclusions

The optimal control methodology used in this study allowed complex analyses of nonlinear relationships for two state variables and four control variables. Influence of stochastic variables on management was reflected using chance constraints, price expectations, and rainfall variability. This study also evaluated intangible factors in a rancher's utility function for oak trees, and included them in policy analysis by calibrating the model for actual firewood harvested over time and deriving hedonic costs for firewood harvest.

Our research revealed some interesting empirical results useful for hardwood rangeland policy makers. Commercial hunting was shown to be a dominant economic enterprise, illustrating the importance of including multiple products in bioeconomic modeling. Consideration of cattle alone would have grossly underestimated hardwood rangeland values. The interrelationship among alternative products was demonstrated inasmuch as hunting resulted in decreased cattle stocking and firewood harvest. The calibration procedure used represented the actual practice of partial oak harvesting better than did a normative model we tested. Inclusion of risk resulted in decreased cattle stocking, increased hunting intensity, and increased firewood harvest.

Our bioeconomic model of California's hardwood rangelands shows considerable promise for empirical analysis of rangeland and other agricultural resource issues. As public scrutiny of natural resource management increases, dynamic analysis of tradeoffs among multiple uses, including wildlife, trees, and livestock, will become increasingly important. Demands for nontraditional goods and restrictions on production methods of privately owned lands will escalate as urban dwellers move closer to agricultural and wildland areas.

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References

Barrett, R. H. "Mammals of California Oak Habitats—Management Implications." In: Proceedings of the Symposium on the Ecology, Management, and Utilization of California Oaks, June 26–28, 1979. *USDA Forest Service General Technical Report* PSW-44:275–291, 1980.

Bell, M. "COWFLOW: A Cow-Calf Monthly Inventory and Production Calendar." A Spreadsheet Copyrighted by University of California Cooperative Extension, 1988.

Bolsinger, C. L. "The Hardwoods of California's Timberlands, Woodlands, and Savannas." *USDA Forest Service General Resource Bulletin* PNW-RB-148, 1988.

Brooke, A., D. Kendrick, and A. Meeraus. *GAMS: A user's guide*. The Scientific Press, Redwood City, CA, 1988.

Burt, O. R. "A Dynamic Economic Model of Pasture and Range Improvements." *Amer. J. Agric. Econ.* 53(1971):197–205.

Canon, M. D., Cullum, J. R., and E. Polok. *Theory of Optimal Control and Mathematical Programming*. McGraw-Hill, 1970.

CDFA (California Dept. of Food and Agric.). "Alfalfa hay price report." Sacramento, CA, 1960–1986.

Charnes, A. and W. W. Cooper. "Chance-Constrained Programming." *Management Science* 6(1959):70–79.

Chow, G. C. *Analysis and Control of Dynamic Economic Systems*. John Wiley, New York, 1970.

Clark, C. W. *Mathematical Bioeconomics*. Wiley, New York, 1976.

Dammann, J. C. and R. A. Andrews. "Economies in wood supply firms in New Hampshire." *New Hampshire Agric. Expt. Station Res. Report* 76, 1979.

Davidson, R. "Variable metric methods for minimization." *A.E.C. Res. and Devel. Report* ANL-5990, Argonne National Laboratory, Argonne, Illinois, 1959.

Davidson, R. and J. G. MacKinnon. "Several Tests for Model Specification in the Presence of Alternative Hypotheses." *Econometrica* 49(1981):781–793.

Doak, S. C. and B. Stewart. "A Model of Economic Forces Affecting California's Hardwood Resource: Monitoring and Policy Implications." Univ. of Calif. Dept. of Forestry and Res. Mgmt. Report submitted to California Department of Forestry in partial fulfillment of contract #8CA42151, 1986.

Drake, D. "Siskiyou County Cow/Calf Costs." unpublished UC Cooperative Extension mimeo, 1987.

Fisher, A. C. *Resource and Environmental Economics*. Cambridge University Press, Cambridge, 1981.

Garoian, L., J. W. Mjelde, and J. R. Connor. "Optimal Strategies for Marketing Calves and Yearlings from Rangeland." *Amer. J. Agr. Econ.* 72(1990):604–613.

Gill, P. E., W. Murray and M. H. Wright. *Practical Optimization*. Academic Press, New York, 1981.

Heady, H. F. and M. D. Pitt. "Reactions Of Northern California Grass-Woodland To Vegetational Type Conversions." *Hilgardia* 47(1979):51–73.

Holland, V. L. "Effect Of Blue Oak On Rangeland Forage Production In Central California." In: "Proceedings of the symposium on the ecology, management, and utilization of California oaks," June 26–28, 1979, *USDA Forest Service General Technical Report* PSW-44:314–318, 1980.

Jensen, H. C. "The Effect Of Blue Oak Removal On Herbaceous Production On a Foothill Site in the Northern Sierra Nevada." In: "Symposium on Multiple Use Management of California's Hardwood Resources," November 12–14, 1986, *USDA Forest Service General Technical Report* PSW-100:343–350, 1987.

Karp, L. and C. A. Pope III. "Range Management Under Uncertainty." *Amer. J. Agr. Econ.* 66(1984):437–446.

Kay, B. L. "Long-term Effects of Blue Oak Removal On Forage Production, Forage Quality, Soil, and Oak Regeneration." In: Symposium on Multiple Use Management of California's Hardwood Resources, November 12–14, 1986, *USDA Forest Service General Technical Report* PSW-100:351–357, 1987.

Loomis, J. B., and E. L. Fitzhugh. "Financial returns to California landowners for providing hunting access: analysis and determinants of returns and implications to wildlife management." *Trans. 54th North American Wildlife and Nat. Res. Conf.* 54(1989):196–201.

Mangel, M. *Decision and Control in Uncertain Resource Systems*. Academic Press, Orlando, 1985.

McClaren, M. P. and J. W. Bartolome. "Effect of *Quercus douglasii* (Fagaceae) on herbaceous understory along a rainfall gradient." *Madrono* 36(1989):141–153.

Robison, L. J. "Risk attitudes: concepts and measurement approaches," in: *Risk Management in Agriculture*, ed. P. J. Barry, Iowa State University Press, Ames, 1984.

Rosen, S. "Hedonic Prices and Implicit Markets: Product Differentiation In Pure Competition." *J. Polit. Econ.* 82(1974):34–55.

Standiford, R. B. "A bioeconomic model of California's hardwood rangelands." unpublished PhD dissertation, University of California, Davis, 1989.

Standiford, R. B. and R. E. Howitt. "Oak stand growth on California's hardwood rangelands." *California Agriculture* 42(1988):23–24.

State Board of Forestry. "California's Hardwood Regulation In California Hardwood Types." Report of the Study Committee to the State Board of Forestry, Sacramento, CA, 1982.

———. "California's Hardwood Resource." Preliminary Report of the Hardwood Task Force, Sacramento, CA, 1983.

USDA (U.S. Dept. of Agric.). "Livestock and meat prices and markets." *Federal-State Market News Serive*, 1960–1986.

Van Riet, W. "COWBUDG: Beef Breeding Herd Income & Costs—Stanislaus & San Joaquin Co's." A Spreadsheet Copyrighted by University of California Cooperative Extension, 1988.

Wolfe, P. "The reduced-gradient method." Unpublished manuscript, RAND Corporation, 1962.

Appendix

Variable Descriptions

$ACRE$ = acres in parcel
$ADVA_t$ = advertising spent per acre (exogenous)
AVL_j = percent forage available in season j
$CAUM_{j-1,t}$ = AUMs allocated to cattle in previous season (spring)
CC_t = oak crown canopy percent (state variable)
CLF = calving percent
CS_t = Vector of different classes of livestock sold
DEG_j = forage degrade in season $j = 3$
$DERAUM_{j-1,t}$ = AUMs allocated to deer in previous season (spring)
DF_t = discount factor for time t
$DRAUM_t$ = AUMs allocated to deer on parcel (control variable)
$DRTRP$ = percent trophy deer (exogenous)

e^{-rt} = continuous time discount factor
exog. = exogenous range productivity factors
E($PFEED_t$) = expectation of price of supplemental feed in year t
E($PRCC_t$) = expectation of price of cull cows in time t
E($PRHF_t$) = expectation of price per pound for heifer calves in time t
E($PRST_t$) = expectation of price per pound for steers in time t
E(PWD_t) = delivered firewood price expectation
$F(WD_t$, exog.) = Oak growth at time t as function of oak stock and exogenous site factors
$FED_{j,t}$ = quantity of seasonal supplementary feed in season j in AUMs
$FOR_{j,t}$ = forage quantity available in AUMs in season j, year t
$FOR_t(WD_t$, exog.) = Forage production expressed as a function of tree canopy and
$FORQUNT_j$ = pounds of forage required in season j per AUM
$G(HRD_t$, *exog.*) = Livestock growth at time t as function of livestock density and exogenous factors (i.e. cattle breed)
GM_t = gross margin in time t
GUIDE = dummy variable for whether guide services are provided (exogenous)
HC_t = hunting cost in time t
$HFSEL_t$ = number of heifer calves sold in time t
HINC = percent of hunters in high income category (exogenous)
HR_t = hunting revenue in time t
$HR_t(WD_t, HRD_t$, exog.) = Hunting revenue at time t as function of tree stock and livestock and exogenous variables (i.e. location, wildlife population, etc.)
HRD_t = herd size in cows at time t (state variable)
$\dot{HRD}$ = instantaneous change of the stock of livestock
HUNT = hunting dummy variable (1 = hunting, 0 = no hunting)
LC_t = livestock costs in time t
LR_t = livestock revenue in time t
$LR_t(HRD_t, CS_t, FOR_t(WD_t$, *exog.*)) = Livestock revenue at time t as function of the stock of livestock, forage production, and the number of livestock sold.
MORT = percent of herd lost to mortality (exogenous)
NUT_j = seasonal AUMs required per cow (exogenous)
PAI_T = Periodic annual increment on oak tree stock (Standiford and Howitt, 1988).
PPIG = percent of area with pig hunting (exogenous)
$RAIN_{j,t}$ = cumulative seasonal rainfall in season j, year t,
REP_t = number of replacement heifers added to herd in time t (control variable)
REP_{t-6} = number of cull cows sold in time t (control variable)
SCEN = scenery quality (exogenous)
SIINT = range production intercept for site quality
SISLP = range production slope for site quality
SITE = oak woodland site index (Standiford and Howitt, 1988)
$STSEL_t$ = number of steers sold in time t
T = terminal time period
TAG = dummy variable for whether deer tags are provided (exogenous)
TV_T = terminal value of system at time T
VC = variable costs per cow
WD_t = wood volume in thousand cubic feet per 1000 acres (state variable)
$WDSEL_t$ = volume of wood sold in MCF per 1000 acres (control variable)
WTCC = weight of cull cows
WTHF = weight of heifer calves sold
WTST = weight of steer calves sold
WD_t = Stock of oak trees at time t
$\dot{WD}$ = instantaneous change of the stock of oak trees
WR_t = firewood net revenue in time t
$WR_t(WDSEL_t)$ = firewood revenue as a function of firewood cut in time t