MATH-2105

List of Questions

1. Define diagonal matrix, Scalar matrix, Hermitian matrix, Idempotent matrix and nil potent matrix.
2. Define diagonal and tri-diagonal matrix with examples.
3. Show that the matrices $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$ are the inverses of each other.
4. If $A = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 2 \\ 1 & 0 \end{bmatrix}$, then find AB. Is BA exist? Create an argument.
5. Define Column matrix, Row matrix, Inverse matrix, Square matrix and Transpose of a matrix.
6. Determine whether the matrix $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ is idempotent or not.
7. Find whether the matrix $A = \begin{bmatrix} 2 & 2-3i & 3+5i \\ 2+3i & 3 & i \\ 3-5i & -i & 5 \end{bmatrix}$ is Hermitian matrix or not.
8. Solve the following equations for A and B

$$2A - B = \begin{bmatrix} 3 & -3 & 0 \\ 3 & 3 & 2 \end{bmatrix}$$
$$2B + A = \begin{bmatrix} 4 & 1 & 5 \\ -1 & 4 & -4 \end{bmatrix}$$

9. If the matrix $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric, find the value of x and hence find the matrix A.
10. Find x, y, z, t using the concept of equality of matrices, where

$$3\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+t & 3 \end{bmatrix}$$

11. Given $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$, find the matrix C, such that $A + 2C = B$.
12. What is the rank of a matrix? Find the rank of the matrix $X = \begin{bmatrix} 1 & -3 & 2 \\ -2 & 2 & 0 \\ -6 & 9 & -3 \end{bmatrix}$.
13. What is rank of a matrix? Reduce the following matrix A into its Echelon form to find the rank, where $A = \begin{bmatrix} 8 & 6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

14. Find the rank of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 1 & 2 \end{pmatrix}$

15. Solve the following system of linear equations by

$$x + y + z = 6$$
$$x - y + z = 2$$
$$2x + y - z = 1$$

    i. using Cramer rule.
    ii. using Gauss elimination.
    iii. using matrix method

    Which method involves fewer computations?

16. Solve the following system of linear equations using the matrix method:

$$x + 9y - z = 27$$
$$x - 8y + 16z = 10$$
$$2x + y + 15z = 37$$

17. Solve the following equations by matrix method

    x + 2y +3z = 14
    3x + y + 2z = 11
    2x + 3y + z = 11

18. Find the characteristic equation and all the characteristic roots of the matrix $A = \begin{bmatrix} 8 & 6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

19. Find the characteristic equation and all the characteristic roots of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$.

20. State the Cayley Hamilton theorem. Verify the theorem for the matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ and hence find A^{-1}.

21. Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$ and hence find the inverse of A.

22. Show the following system of linear equations as its equivalent matrix form and as linear combination of vectors:

$$x_1 + 2x_2 - 4x_3 + 7x_4 = 4$$

$$3x_1 + x_2 + 6x_3 - 8x_4 = 12$$

$$4x_1 - 5x_2 - 3x_3 + 7x_4 = 8$$

23. Give the matrix and vector representation of the following system of linear equations:

(i) $$\begin{aligned} x_1 + 2x_2 - 4x_3 + 7x_4 &= 4 \\ 3x_1 - 5x_2 + 6x_3 - 8x_4 &= 8 \\ 4x_1 - 3x_2 - 2x_3 + 6x_4 &= 11 \end{aligned}$$

(ii) $$\begin{aligned} x_1 + 2x_2 &= 40 \\ -x_1 - 2x_2 + 6x_3 &= 11 \\ -5x_2 + 6x_3 - 8x_4 &= 11 \\ -2x_3 + 6x_4 - x_5 &= 11 \\ 2x_4 - 7x_5 &= 200 \end{aligned}$$

24. Define equal vector and null vector. Find the scalar product of the vectors (2, 3, 1) and (3, 1, -2). Also find the angle between them.
25. If $A = \begin{bmatrix} 1 & 3 \\ 4 & -3 \end{bmatrix}$, find a non-zero column vector $u = \begin{bmatrix} x \\ y \end{bmatrix}$ such that $Au = 3u$. Describe all such vectors.
26. Apply the concept of vector cross product to find the area of the parallelogram constructed by the vectors $\vec{u} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$.
27. If $A = [a_{ij}]$, where $a_{ij} = \begin{cases} 0, & when\ i \neq j \\ C, & when\ i = j \end{cases}$

    Construct a 3×3 order matrix and identify the type of matrix, where C is the sum of the 1st digit and the last digit of your ID. Also test the matrix A is

    i. orthogonal or not
    ii. singular or not
28. If $A = [a_{ij}]$ where $a_{ij} = \begin{cases} 0, & when\ i < j \\ i + j, & when\ i = j \\ 2i - j, & when\ i > j \end{cases}$

    Construct a 3×3 matrix and identify the type of the matrix A. Also check whether it is singular or not.
29. Find a unit vector perpendicular to each of the vectors $r_1 = 3i + 2j - 4k$ and $r_2 = i + j + 2k$
30. Find a unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$.
31. Determine whether the force field $\vec{F}(x, y, z) = x^2y\hat{\imath} + xyz\hat{\jmath} - x^2y^2\hat{k}$ is a conservative or not.
32. Find the angle between the surfaces $x^2 + y^2 + z^2 = 49$ and $x^2 + y^2 - z = 43$ at (6, 3, -2).
33. Show that the vector $\vec{F} = (6xy + z^3)\underline{i} + (3x^3 - z)\underline{j} + (3xz^2 - y)\underline{k}$ is irrotational.
34. Write down three vector operators gradient, divergence and curl.
35. Show that the divergence of the curl of a vector field *A* is zero.
36. Let $\vec{A} = xy^2\,\underline{i} - 3x^2y\,\underline{j} + 2yz^2\,\underline{k}$. Now find curlcurl of $\vec{A}$ at (1, 0, - 4).

37. Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2 dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.
38. State and prove Green's theorem.
39. Apply Green's theorem find $\oint_c (x^2 y dx + x^2 dy)$, where c is the boundary of the region enclosed by the line y = x and the curve $y = x^2$
40. Determine the angles α, β, γ which the vector $\vec{A} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}$ makes with the positive directions of the coordinate axes. Also show that $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$.
41. Find a unit vector
    i. in the direction to the vector $\vec{A} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$.
    ii. parallel to the resultant of the vectors $\vec{B} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\vec{C} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.
    iii. perpendicular to the plane constructed by the vectors $\vec{D} = 3\hat{\imath} + \hat{\jmath}$ and $\vec{E} = -\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$.
42. Find whether the vectors $\vec{A} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$, $\vec{B} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{C} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$ are coplanar.
43. What is inner product of vectors? Apply the Gram-Schmidt orthonormalization algorithm to the set of vectors $v_1 = (1, 0, 1)$, $v_2 = (1, 0, -1)$ and $v_3 = (0, 3, 4)$ to obtain an orthonormal basis. Justify your results.
44. Explain direction cosines of a line. If the angle between two straight lines is θ and their direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2 then show that
$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2.$$
Hence develop this relation for $\sin\theta$.
45. Explain shortest distance. Find the equation of the line of shortest distance and evaluate the length of the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-5}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$
46. Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ are coplanar. Find their intersection point and the equation of the plane in which they lie.
47. Find the equation of the straight line that intersect the lines $4x + y - 10 = 0 = y + 2z + 6$ and $3x - 4y + 5z + 5 = 0 = x + 2y - 4z + 7$ and passing through the point $(-1, 2, 2)$.