

2.76

(a) If  $y(n) = x(n) * h(n)$ . Show that  $\sum_i y = \sum_i x \sum_i h$ .

where  $\sum_i x = \sum_{n=-\infty}^{\infty} x(n)$ .

Soln:

$$\begin{aligned} y(n) &= \sum_k h(k) \cdot x(n-k) \\ &= \sum_n \sum_k h(k) x(n-k) \\ &= \sum_k h(k) \sum_{n=-\infty}^{\infty} x(n-k) \\ &= \left( \sum_k h(k) \right) \cdot \left( \sum_n x(n) \right) \end{aligned}$$

(b) Compute the convolution  $y(n) = x(n) * h(n)$  of the following signals and check the correctness of the result by using the test in (a).

Soln:

(1)  $x[n] = [1, 2, 4]$ ,  $h[n] = [1, 1, 1, 1, 1]$

$$\begin{aligned} y[n] &= h[n] * x[n] \\ &= \{1, 3, 7, 7, 7, 6, 4\} \end{aligned}$$

$$\sum_n y(n) = 35, \quad \sum_k h(k) = 5, \quad \sum_k x(k) = 7$$

(2)  $x(n) = [1, 2, -1]$ ,  $h(n) = x(n)$

$$y(n) = \{1, 4, 2, -4, 1\}$$

$$\sum_n y(n) = 4, \quad \sum_k h(k) = 2, \quad \sum_k x(k) = 2.$$

$$(3) x(n) = \{0, 1, -2, 3, -4\}, h(n) = \left\{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\right\}$$

$$y(n) = \left\{0, -\frac{1}{2}, -\frac{1}{2}, \frac{3}{2}, -2, 0, -\frac{5}{2}, -2\right\}$$

$$\sum_n y(n) = -5, \quad \sum_n h(n) = 2.5, \quad \sum_n x(n) = -2.$$

$$(4) x(n) = \left\{\begin{array}{l} 1 \\ -2 \\ 3 \end{array}\right., h(n) = \left\{0, 0, 1, 1, 1, 1\right\}$$

$$y(n) = \left\{0, 0, 1, -1, 2, 2, 1, 3\right\}$$

$$\sum_n y(n) = 8, \quad \sum_n h(n) = 4, \quad \sum_n x(n) = 2.$$

$$(5) x(n) = \{1, 2, 3, 4, 5\}, h(n) = \{4\}$$

$$y(n) = \{1, 2, 3, 4, 5\}$$

$$\sum_n y(n) = 15, \quad \sum_n h(n) = 1, \quad \sum_n x(n) = 15.$$

$$(6) x(n) = \left\{\begin{array}{l} 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 1 \end{array}\right., h(n) = \left\{1, -\frac{2}{1}, 3\right\}$$

$$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$$

$$\sum_n y(n) = 8, \quad \sum_n h(n) = 2, \quad \sum_n x(n) = 4$$

$$(7) x(n) = \left\{\begin{array}{l} 0 \\ 1 \\ 4 \\ -3 \end{array}\right., h(n) = \left\{1, 0, -1, -1\right\}$$

$$y(n) = \{0, 1, 4, -4, -5, -1, 3\}$$

$$\sum_n y(n) = -2, \quad \sum_n h(n) = -1, \quad \sum_n x(n) = 2.$$

$$(8) x(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}, h(n) = u(n)$$

$$y(n) = u(n) + u(n-1) + 2u(n-2)$$

$$\sum_n y(n) = \infty, \quad \sum_n h(n) = \infty, \quad \sum_n x(n) = 4$$

$$(9) x(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}, h(n) = \begin{cases} 1, & n \geq 0 \\ -2, & n = -1 \\ -3, & n = -2 \\ 4, & n = -3 \end{cases}$$

$$y(n) = \begin{cases} 1, & n = 0 \\ -1, & n = -1 \\ -5, & n = -2 \\ 2, & n = -3 \\ 3, & n = -4 \\ -5, & n = -5 \\ 1, & n = -6 \\ 4, & n = -7 \end{cases}$$

$$\sum_n y(n) = 0, \quad \sum_n h(n) = 0, \quad \sum_n x(n) = 4$$

$$(10) x(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}, h(n) = x(n)$$

$$y(n) = \begin{cases} 1, & n = 0 \\ 4, & n = 1 \\ 4, & n = 2 \\ 4, & n = 3 \\ 10, & n = 4 \\ 4, & n = 5 \\ 4, & n = 6 \\ 4, & n = 7 \end{cases}$$

$$\sum_n y(n) = 36, \quad \sum_n h(n) = 6, \quad \sum_n x(n) = 6$$

$$(11) x(n) = \left(\frac{1}{2}\right)^n \cdot u(n), \quad h(n) = \left(\frac{1}{4}\right)^n \cdot u(n).$$

$$y(n) = \left[2 \cdot \left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right] u(n)$$

$$\sum_n y(n) = \frac{8}{3}, \quad \sum_n h(n) = \frac{4}{3}, \quad \sum_n x(n) = 2$$

2.19

Compute the convolution  $y(n)$  of the signals.

$$x(n) = \begin{cases} d^n, & -3 \leq n \leq 5 \\ 0, & \text{otherwise.} \end{cases}$$

$$h(n) = \begin{cases} 1, & 0 \leq n \leq 4 \\ 0, & \text{elsewhere} \end{cases}$$

Soln:

$$y(n) = \sum_{k=0}^4 h(k) x(n-k)$$

$$x(n) = \{d^{-3}, d^{-2}, d^{-1}, \underset{\uparrow}{1}, d, \dots, d^5\}$$

$$h(n) = \{1, 1, 1, 1, 1\}$$

$$y(n) = \begin{cases} \sum_{k=0}^4 x(n-k), & (-3, \leq n \leq 9) \\ 0, & \text{otherwise.} \end{cases}$$

Therefore,

$$y(-3) = d^{-3}$$

$$y(-2) = x(-3) + x(-2) = d^{-3} + d^{-2}$$

$$y(-1) = d^{-3} + d^{-2} + d^{-1} + \cancel{1}$$

$$y(0) = d^{-3} + d^{-2} + d^{-1} + 1$$

$$y(1) = d^{-3} + d^{-2} + d^{-1} + 1 + d$$

$$y(2) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha + \alpha^2$$

$$y(3) = \alpha^{-4} + 1 + \alpha + \alpha^2 + \alpha^3$$

$$y(4) = \alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1$$

$$y(5) = \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$$

$$y(6) = \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$$

$$y(7) = \alpha^3 + \alpha^4 + \alpha^5$$

$$y(8) = \alpha^4 + \alpha^5$$

$$y(9) = \alpha^5$$

2.25

Determine the zero-input response of the system described by the second-order difference equation.

$$x(n) - 3y(n-1) - 4y(n-2) = 0$$

Soln: With  $x(n)=0$ , we have,

$$y(n-1) + \frac{4}{3}y(n-2) = 0$$

$$\Rightarrow y(-1) = -\frac{4}{3}y(-2)$$

$$y(0) = (-\frac{4}{3})^2 y(-2)$$

$$y(4) = \left(-\frac{4}{3}\right)^3 y(-2)$$

⋮  
⋮  
⋮

$$y(k) = \left(-\frac{4}{3}\right)^{k+1} y(-2)$$

So, the zero-input response of the system  
is  $y(k) = \left(-\frac{4}{3}\right)^{k+1} \cdot y(-2)$

(a) Write the properties of linear-FIR filter.

Soln:

An FIR-filter is a digital filter whose impulse response is finite duration, because it settles to zero in finite time.

$$y[n] = \sum_{i=0}^N b_i \cdot x[n-i]$$

Properties of FIR:

(1) Impulse response  $h(n)$  of an filter has a finite limit length ( $M$ ).

$$h_n = \begin{cases} b^n, & 0 \leq n \leq M-1 \\ 0, & \text{otherwise.} \end{cases}$$

(2) Impulse response  $H(z)$  of an FIR filter is determined by its impulse response and is always bounded.

$$H(z) = b_0 + b_1 z^{-1} + \dots + b_{N-1} z^{-(N-1)}$$

$$= b_0 \sum_{k=1}^K (1 + B_{k,1} z^{-1} + B_{k,2} z^{-2})$$

(3) Symmetric and anti-symmetric impulse responses allow FIR filters to exhibit linear phase.

$$\text{Symmetric : } h(n) = h(N-1-n)$$

$$\text{Anti-symmetric : } h(n) = -h(N-1-n).$$

(4) FIR filters can have be designed to have a linear phase characteristics which means the phase shift introduced by the filters is constant. It preserves waveform shape.

$$\theta(\omega) = -\alpha\omega, \text{ where } \alpha \text{ is a constant phase delay.}$$

(5) FIR filters are always stable their impulse is finite and absolutely summable.

$$\sum_{n=0}^{N-1} |h(n)| < \infty$$

(6) Ease of design.

(b) Design an FIR digital filter to approach an ideal LPF with passband gain of unity, cut-off frequency of 750 Hz and working at sampling frequency of 4750 Hz. The length of impulse response should be 7. Use rectangular window technique.

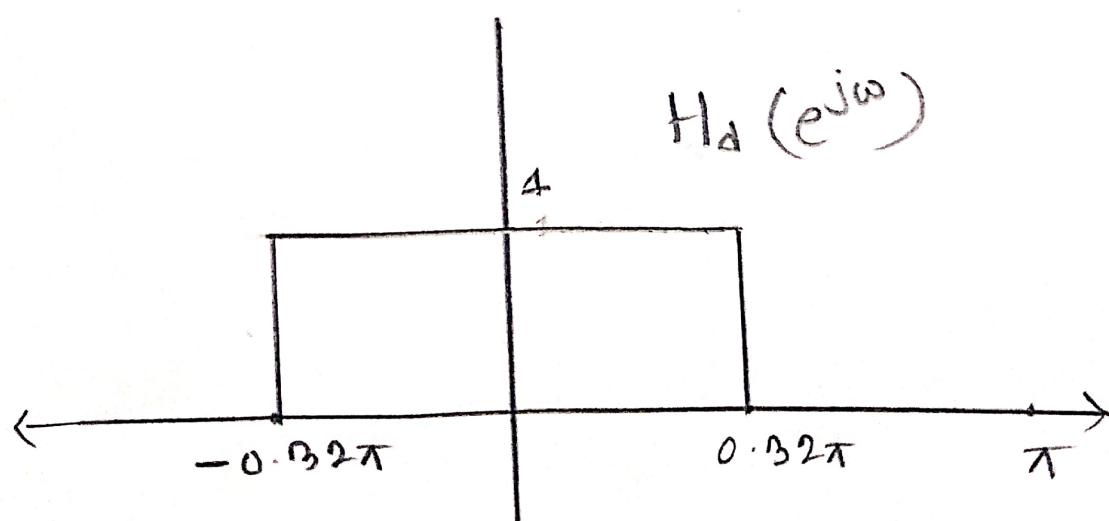
Soln:

Cut-off frequency,  $F_c = 750 \text{ Hz}$ .

Sampling frequency,  $f_s = 4750 \text{ Hz}$ .

$$\text{Normalized cut-off frequency. } f_c = \frac{F_c}{f_s} \\ = 0.16 \text{ Hz.}$$

$$\text{Normalized angular cut-off frequency. } \omega_c = 2\pi f_c \\ = 0.32\pi \text{ rad}$$



Frequency response of LPF

$$H_d(\omega) = \begin{cases} e^{-j\omega t}, & \text{for } -\omega_c \leq \omega \leq \omega_c \\ 0, & \text{otherwise.} \end{cases}$$

$\gamma = \frac{M-1}{2} = 3$ ;  $M$  = Length of impulse response.

Now,

$$\begin{aligned} h_d(n) &= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{-j\omega\gamma} \cdot e^{j\omega n} d\omega \\ &= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega(n-\gamma)} d\omega \\ &= \frac{1}{2\pi} \left[ \frac{e^{j\omega(n-\gamma)}}{j(n-\gamma)} \right]_{-0.32\pi}^{0.32\pi} \\ &= \frac{\sin [0.32(n-\gamma)]}{\pi(n-\gamma)}. \end{aligned}$$

When,  $n \neq 3$ ,  $\frac{\sin [0.32(n-3)]}{\pi(n-3)}$

$$h_d(n) = \frac{0.32\pi}{\cancel{0.32\pi}} = 0.32$$

Now,

$$h_d(0) = \frac{\sin [0.32(0-3)]}{\pi(0-3)} = \cancel{0.0864} \quad 0.08691$$

$$h_d(1) = \frac{\sin [0.32(1-3)]}{\pi (1-3)} = 0.0504.$$

$$h_d(2) = \frac{\sin [0.32(2-3)]}{\pi (2-3)} = 0.1001$$

$$h_d(3) = 0.32$$

$$h_d(4) = \frac{\sin [0.32(4-3)]}{\pi (4-3)} = 0.1001$$

$$h_d(5) = \frac{\sin [0.32(5-3)]}{\pi (5-3)} = 0.0504$$

$$h_d(6) = \frac{\sin [0.32(6-3)]}{\pi (6-3)} = 0.8691$$

$$h_n = [0.8691, 0.0504, 0.1001, 0.32, 0.1001, 0.0504, 0.8691]$$

(c) show the Impulse Invariance Method (IIM)  
 using block diagram for designing IIR filter.

Sol<sup>n</sup>:

## Impulse Invariance Method using block diagram:

