2.16

(a) If $y(n) = x(n) * h(n)$. Show that $\sum y = \sum x \sum h$.
where $\sum x = \sum_{n=-\infty}^{\infty} x(n)$.

Soln:

$$y(n) = \sum_{k} h(k) \cdot x(n-k)$$

$$= \sum_{n} \sum_{k} h(k)\, x(n-k)$$

$$= \sum_{k} h(k) \sum_{n=-\infty}^{\infty} K(n-K)$$

$$= \left(\sum_{k} h(k)\right) \cdot \left(\sum_{n} x(n)\right)$$

(b) Compute the convolution $y(n) = x(n) * h(n)$ of the following signals and check the correctness of the result by using the test in (a).

Soln:

(1) $x[n] = [1, 2, 4]$, $h[n] = [1, 1, 1, 1, 1]$

$$y[n] = h[n] * x[n]$$

$$= \{1, 3, 7, 7, 7, 6, 4\}$$

$\sum_{n} y(n) = 35$, $\sum_{k} h(k) = 5$, $\sum_{k} x(k) = 7$

(2) $x(n) = [1, 2, -1]$, $h(n) = x(n)$

$$y(n) = \{1, 4, 2, -4, 1\}$$

$\sum_{n} y(n) = 4$, $\sum_{k} h(k) = 2$, $\sum_{k} x(k) = 2$.

(3) $x(n) = \{0, 1, -2, 3, -4\}$, $h(n) = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$

$y(n) = \{0, \frac{1}{2}, -\frac{1}{2}, \frac{3}{2}, -2, 0, -\frac{5}{2}, -2\}$

$\sum_n y(n) = -5$, $\sum_n h(n) = 2.5$, $\sum_n x(n) = -2$.

(4) $x(n) = \{\underset{\uparrow}{1}, -2, 3\}$ $h(n) = \{0, 0, 1, 1, 1, 1\}$

$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$

$\sum_n y(n) = 8$, $\sum_n h(n) = 4$, $\sum_n x(n) = 2$.

(5) $x(n) = \{1, 2, 3, 4, 5\}$ $h(n) = \{1\}$

$y(n) = \{1, 2, 3, 4, 5\}$

$\sum_n y(n) = 15$, $\sum_n h(n) = 1$, $\sum_n x(n) = 15$.

(6) $x(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$, $h(n) = \{1, \underset{\uparrow}{-2}, 3\}$

$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$

$\sum_n y(n) = 8$, $\sum_n h(n) = 2$, $\sum_n x(n) = 4$

(7) $x(n) = \{\underset{\uparrow}{0}, 1, 4, -3\}$, $h(n) = \{\underset{\uparrow}{1}, 0, -1, -1\}$

$y(n) = \{0, 1, 4, -4, -5, -1, 3\}$

$\sum_n y(n) = -2$, $\sum_n h(n) = -1$, $\sum_n x(n) = 2$.

(8) $x(n) = \{\underset{\uparrow}{1}, 1, 2\}$, $h(n) = u(n)$

$y(n) = u(n) + u(n-1) + 2u(n-2)$

$\sum_n y(n) = \infty$, $\sum_n h(n) = \infty$, $\sum_n x(n) = 4$.

(9) $x(n) = \{1, 1, 0, 1, 1\}$, $h(n) = \{1, -2, -3, 4\}$

$y(n) = \{1, -1, -5, 2, 3, -5, 1, 4\}$

$\sum_n y(n) = 0$, $\sum_n h(n) = 0$, $\sum_n x(n) = 4$

(10) $x(n) = \{1, 2, \underset{\uparrow}{0}, 2, 1\}$, $h(n) = x(n)$

$y(n) = \{1, 4, 4, 4, 10, 4, 4, 4\}$

$\sum_n y(n) = 36$, $\sum_n h(n) = 6$, $\sum_n x(n) = 6$.

(11) $x(n) = (\frac{1}{2})^n \cdot u(n)$, $h(n) = (\frac{1}{4})^n u(n)$.

$y(n) = [2 \cdot (\frac{1}{2})^n - (\frac{1}{4})^n] u(n)$

$\sum_n y(n) = \frac{8}{3}$, $\sum_n h(n) = \frac{4}{3}$, $\sum_n x(n) = 2$

2.19

Compute the convolution $y(n)$ of the signals.

$$x(n) = \begin{cases} \alpha^{n}, & -3 \leq n \leq 5 \\ 0, & \text{otherwise.} \end{cases}$$

$$h(n) = \begin{cases} 1, & 0 \leq n \leq 4 \\ 0, & \text{elsewhere} \end{cases}$$

Soln:

$$y(n) = \sum_{k=0}^{4} h(k)\, x(n-k)$$

$$x(n) = \{\alpha^{-3}, \alpha^{-2}, \alpha^{-1}, \underset{\uparrow}{1}, \alpha, \dots, \alpha^{5}\}$$

$$h(n) = \{1, 1, 1, 1, 1\}$$

$$y(n) = \begin{cases} \sum_{k=0}^{4} x(n-k), & (-3, \leq x \leq 9) \\ 0, & \text{otherwise.} \end{cases}$$

Therefore,

$$y(-3) = \alpha^{-3}$$

$$y(-2) = x(-3) + x(-2) = \alpha^{-3} + \alpha^{-2}$$

$$y(-1) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} \cancel{+1}$$

$$y(0) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1$$

$$y(1) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha$$

$y(2) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha + \alpha^{2}$

$y(3) = \alpha^{-1} + 1 + \alpha + \alpha^{2} + \alpha^{3}$

$y(4) = \alpha^{4} + \alpha^{3} + \alpha^{2} + \alpha + 1$

$y(5) = \alpha + \alpha^{2} + \alpha^{3} + \alpha^{4} + \alpha^{5}$

$y(6) = \alpha^{2} + \alpha^{3} + \alpha^{4} + \alpha^{5}$

$y(7) = \alpha^{3} + \alpha^{4} + \alpha^{5}$

$y(8) = \alpha^{4} + \alpha^{5}$

$y(9) = \alpha^{5}$

2.25

Determine the zero-input response of the the the second-order system described by the difference equation.

$$x(n) - 3y(n-1) - 4y(n-2) = 0$$

Soln: With $x(n) = 0$, we have,

$$y(n-1) + \frac{4}{3}y(n-1) = 0$$

$$\Rightarrow y(-1) = -\frac{4}{3}y(-2)$$

$$y(0) = \left(-\frac{4}{3}\right)^{2} y(-2)$$

$y(1) = \left(-\frac{4}{3}\right)^3 y(-2)$

$\vdots$

$y(K) = \left(-\frac{4}{3}\right)^{K+1} y(-1)$

So, the zero-input response of the system is $y(K) = \left(-\frac{4}{3}\right)^{K+1} \cdot y(-1)$

(a) Write the properties of linear - FIR filter.

Soln:

An FIR-filter is a digital filter whose impulse response is finite duration, because it settlers to zero in finite time.

$$y[n] = \sum_{i=0}^{N} b_i \cdot x[n-i]$$

Properties of FIR:

(1) Impulse response $h(n)$ of an filter has a finite limit length (M).

$$h_n = \begin{cases} b_n, & 0 \leq n \leq M-1 \\ 0, & \text{otherwise.} \end{cases}$$

(2) Impulse response $H(z)$ of an FIR filter is determined by its impulse response and is always bounded. ~~by~~

$$H(z) = b_0 + b_1 z^{-1} + \dots + b_{M-1} z^{-(M-1)}$$

$$= b_0 \sum_{K=1}^{K} (1 + B_{K,1} z^{-1} + B_{K,2} z^{-2}).$$

(3) Symmetric and anti-symmetric impulse responses allow FIR filters to exhibit linear phase.

Symmetric : $h(n) = h(M-1-n)$

Anti-Symmetric : $h(n) = -h(M-1-n)$.

(4) FIR filters can ~~have~~ be designed to have a linear phase characteristics which means the phase shift introduced by the filters is constant. It preserves waveform shape.

$\theta(\omega) = -\alpha\omega$, where α is a constant phase delay.

(5) FIR filters are always ~~stable~~ their impulse is finite and absolutely summable.

$$\sum_{n=0}^{M-1} |h(n)| < \infty$$

(6) Ease of design.

(b) Design an FIR digital filter to approx an ideal LPF with passband gain of unity, cut-off frequency of 750 Hz and working at sampling frequency of 4750 Hz. The length of impulse response should be 7. Use rectangular window technique.

Soln:

Cut-off frequency, F_c = 750 Hz.

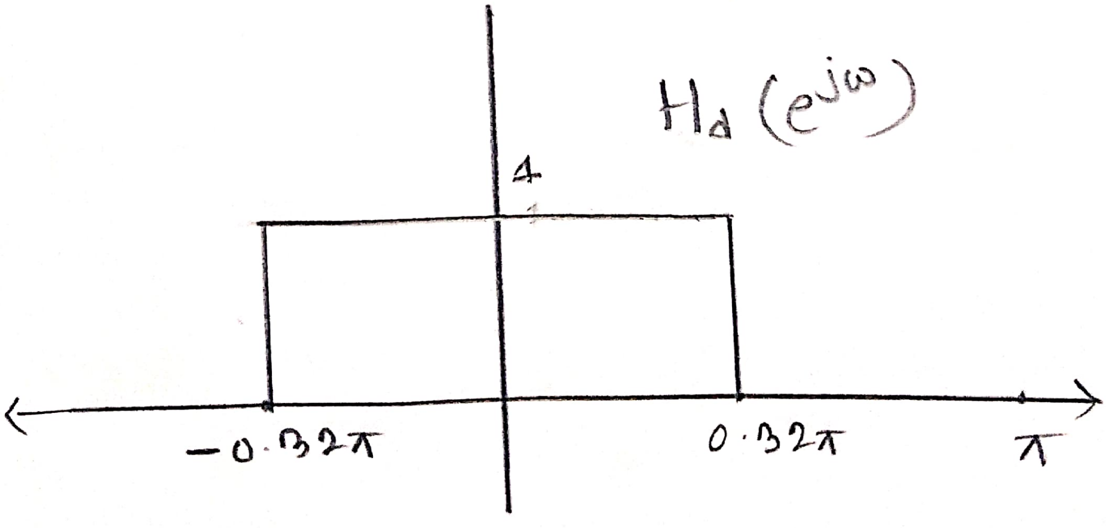
Sampling frequency, F_s = 4750 Hz.

Normalized cut-off frequency, $f_c = \frac{F_c}{F_s}$

$= 0.16$ Hz.

Normalized angular cut-off frequency, $\omega_c = 2\pi f_c$

$= 0.32\pi$ rad



Frequency response of LPF

$$H_d(\omega) = \begin{cases} e^{-j\omega t} & \text{for } -\omega_c \leq \omega \leq \omega_c \\ 0 & \text{otherwise.} \end{cases}$$

$\gamma = \frac{M-1}{2} = 3$; M = Length of impulse response.

Now,

$$h_d(n) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{-j\omega\gamma} \cdot e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega(n-\gamma)} \, d\omega$$

$$= \frac{1}{2\pi} \left[\frac{e^{\omega(n-3)j}}{j(n-3)} \right]_{-0.32\pi}^{0.32\pi}$$

$$= \frac{\sin[0.32(n-3)]}{\pi(n-3)}.$$

When, $n \neq 3$,

$$h_d(n) = \frac{\sin[0.32(n-3)]}{\pi(n-3)}$$

$n = 3$,

$$h_d(n) = \frac{0.32\pi}{\cancel{3}\pi} = 0.32$$

Now,

$$h_d(0) = \frac{\sin[0.32(0-3)]}{\pi(0-3)} = \cancel{0.0869} \; 0.8691$$

$$h_d(1) = \frac{\sin[0.32(1-3)]}{\pi(1-3)} = 0.9504.$$

$$h_d(2) = \frac{\sin[0.32(2-3)]}{\pi(2-3)} = 0.1001$$

$$h_d(3) = 0.32$$

$$h_d(4) = \frac{\sin[0.32(4-3)]}{\pi(4-3)} = 0.1001$$

$$h_d(5) = \frac{\sin[0.32(5-3)]}{\pi(5-3)} = 0.9504$$

$$h_d(6) = \frac{\sin[0.32(6-3)]}{\pi(6-3)} = 0.8691$$

$$h_n = [0.8691, 0.9504, 0.1001, 0.32, 0.1001, 0.9504, 0.8691]$$

(c) show the Impulse Invariance Method (IIM) using block diagram for designing IIR filter.

Soln :

Impulse Invariance Method using block diagram:

$$H_a(s) \xrightarrow[\text{Laplace Transform}]{\text{Applying inverse}} h_a(t) \xrightarrow[(t = nT_s)]{\text{Sampling}} h_a(nT_s) \xrightarrow{z\text{-transform}} H(z)$$

$H_a(s)$: Transfer function in analog domain

$h_a(t)$: Transfer function in time domain.

$H(z)$: Transfer function. (digital domain)