(1.2) Determine which of the following sinusoids are periodic and compute their fundamental period.

(a) $\cos 0.01\pi n$ (b) $\cos \pi \cdot \frac{30n}{105}$ (c) $\cos 3\pi n$ (d) $\sin 3n$

(e) $\sin\left(\pi \cdot \frac{62n}{10}\right)$

Soln:

(a) $f = \frac{0.01\pi}{2\pi} = \frac{1}{200}$

It is periodic with $T = 200$.

(b) $f = \frac{30\pi}{105} \times \frac{1}{2\pi} = \frac{1}{7}$

It is periodic with $T = 7$.

(c) $f = \frac{3\pi}{2\pi} = \frac{3}{2}$

It is periodic with $T = 2$.

(d) $f = \frac{3}{2\pi}$

Hence, 2π is an irrational number. It is not periodic.

(e) $f = \frac{62\pi}{10} \cdot \frac{1}{2\pi} = \frac{31}{10}$.

It is periodic with $T = 10$.

(1.3) Determine whether or not each of the following signals is periodic. In case a signal is periodic, specifies the fundamental time period.

(a) $x_a(t) = 3\cos\left(5t + \frac{\pi}{6}\right)$ (b) $x[n] = 3\cos\left(5n + \frac{\pi}{6}\right)$

(c) $x[n] = 2\exp\left[j\left(\frac{n}{6} - \pi\right)\right]$

(d) $x[n] = \cos\left(\frac{\pi}{8}\right) . \cos\left(\frac{\pi n}{8}\right)$

(e) $x[n] = \cos\left(\frac{\pi n}{2}\right) - \sin\left(\frac{\pi n}{8}\right) + 3\cos\left(\frac{\pi n}{4} + \frac{\pi}{3}\right)$

Soln:

(a) $\omega = 5.$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5}$$

This is finite, positive value, meaning the signal repeats itself every $T = \frac{2\pi}{5}$ seconds. Hence, $x_a(t)$ is periodic.

(b) $f = \frac{5}{2\pi}.$

2π is an irrational number. So, it is aperiodic.

(c) $f = \frac{1}{12\pi}$

12π is an irrational number. So, it is non-periodic.

(d) $f = \frac{3}{2\pi}$???

2π is an irrational number. Hence, it is not periodic.

(e) $\cos\left(\frac{\pi n}{2}\right)$ is periodic with $T_1 = 4$

$\cos\left(\frac{\pi n}{4} + \frac{\pi}{3}\right)$ is periodic with $T_2 = 8$

$\sin\left(\frac{\pi n}{8}\right)$ is periodic with, $T_3 = 16$

Hence, $x[n]$ is periodic with, $T = LCM(T_1, T_2, T_3) = 16$.

(1.4)
(a) show that the fundamental time period Np of the signals,

$$S_K(n) = e^{j2\pi Kn/14}, \quad K = 0, 1, 2, 3, \ldots$$

is given by $N_p = N / GCD(K, N)$ where GCD is the greatest common divisor of K and N.

(b) What is the fundamental period of the set for N = 7?

(c) What is it for N = 16?

Soln:

(a) $\omega = \frac{2\pi K}{N}$ implies that $f = \frac{K}{N}$. Let

$\alpha = GCD$ of (K, N) i.e

$K = K'\alpha, \quad N = N'\alpha$

then, $f = \frac{K'}{N'}$ which implies that

$$N' = \frac{N}{\alpha}$$

(b) N = 7

K =	0	1	2	3	4	5	6	7
GCD(K, N) =	7	1	1	1	1	1	1	7
Np =	1	7	7	7	7	7	7	1

(c) $N = 16$

$K = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16$

$GCD(K,N) = 16, 1, 2, 1, \cancel{16}\,4, 1, 2, 1, 8, 1, 2, 1, 4, 1,$
$2, 1, 16$

$N_p = 1, 16, 8, 16, 4, 16, 8, 16, 2, 16, 8, 16, 4,$
$16, 8, 16, 1$

(1.10) A digital ~~connection~~ communication link carries binary-coded words representing samples of an input signal

$$x_a(t) = 3\cos 600\pi t + 2\cos 1800\pi t$$

The link is operated at 10,000 bits/s and each input sample is quantized into 1024 different voltage levels.

(a) What is the sample frequency and the folding frequency? What is the Nyquist rate for the signal $x_a(t)$?

(b) What is the Nyquist rate for the resulting

(c) What are the frequencies in the resulting discrete-time signal $x(n)$?

(d) What is the resolution Δ?

Soln:

(a)
Number of bits/sample $= \log_2 1024 = 10$

$$F_s = \frac{[10{,}000 \text{ bits/sec}]}{[10 \text{ bits/sample}]}$$

$= 1000$ samples/sec

$F_{fold} = 500$ Hz.

(b) $F_{max} = \frac{1800\pi}{2\pi} = 900$ Hz

$F_N = 2F_{max} = 1800$ Hz.

(c) $F_1 = \frac{600\pi}{2\pi}\left(\frac{1}{F_s}\right) = 0.3$

$F_2 = \frac{1800\pi}{2\pi}\left(\frac{1}{F_s}\right) = 0.9$

But, $f_2 > f_1$.

Hence, $x[n] = 3\cos[(2\pi)(0.3)n] + 2\cos[(2\pi)(0.1)n]$

(d) $\Delta = \frac{x_{max} - x_{min}}{n-1}$

$= \frac{5-(-5)}{1023}$

$= \frac{10}{1023}$.