

**Step 1:** Convert the axioms to first order predicate logic.

*Note that we may or may not use quantifiers ( $\forall$ ,  $\exists$ ) and the (letter) case doesn't matter here.*

1.  $\text{company}(\text{ABC}) \wedge \text{has\_employees}(\text{ABC}, 500)$ .
2.  $\text{employee}(x, y) \wedge \text{company}(y) \wedge \text{earn}(x, z) \wedge \text{more}(z, 250000) \rightarrow \text{pays\_tax}(x)$ .  
Quantifiers are not necessary. We can also write it as:  
 $\forall x \forall y \forall z (\text{employee}(x, y) \wedge \text{company}(y) \wedge \text{earn}(x, z) \wedge \text{more}(z, 250000) \rightarrow \text{pays\_tax}(x))$ .
3.  $\text{manager}(\text{Jamal}, \text{ABC})$ .
4.  $\text{manager}(x, \text{ABC}) \rightarrow \text{earn}(x, 500000)$ .
5.  $\text{manager}(x, y) \rightarrow \text{employee}(x, y)$ . [additional axiom]
6.  $\text{more}(500000, 250000)$ . [additional axiom]

In axiom 1, the fact that ABC company has 500 employees is not necessary for proving our goal. So we can ignore it.

**Step 2:** Convert the logic into Conjunctive Normal Form (CNF)

*In CNF  $P \rightarrow Q$  becomes  $\neg P \vee Q$*

1.  $\text{company}(\text{ABC})$  [as it is, ignoring  $\text{has\_employees}(\text{ABC}, 500)$ ]
2.  $\neg (\text{employee}(x, y) \wedge \text{company}(y) \wedge \text{earn}(x, z) \wedge \text{more}(z, 250000)) \vee \text{pays\_tax}(x)$ .  
 $\neg \text{employee}(x) \vee \neg \text{company}(y) \vee \neg \text{earn}(x, z) \vee \neg \text{more}(z, 250000) \vee \text{pays\_tax}(x)$
3.  $\text{manager}(\text{Jamal}, \text{ABC})$ .
4.  $\neg \text{manager}(x, \text{ABC}) \vee \text{earn}(x, 500000)$
5.  $\neg \text{manager}(x, y) \vee \text{employee}(x, y)$ .
6.  $\text{more}(500000, 250000)$ . [as it is]

**Step 3:** The goal is to prove:  $\text{pays\_tax}(\text{Jamal})$ .

We can prove this by contradiction. We will try to assert the negation of the goal, i.e.  $\neg \text{pays\_tax}(\text{Jamal})$ . If we fail, it will prove that our goal is true.

In each step, we match our current goal with an axiom to get rid of a part of the goal.

So in this example, in the first step we match our goal with axiom 2. So  $\neg \text{pays\_tax}(\text{Jamal})$  gets crossed out by  $\text{pays\_tax}(x)$  where we assume  $x$  is Jamal.

$\neg \text{pays\_tax}(\text{Jamal})$   
 $\neg \text{employee}(\text{Jamal}, y) \vee \neg \text{company}(y) \vee \neg \text{earn}(\text{Jamal}, z) \vee \neg \text{more}(z, 250000)$  [Matching with 2]  
 $\neg \text{manager}(\text{Jamal}, y) \vee \neg \text{company}(y) \vee \neg \text{earn}(\text{Jamal}, z) \vee \neg \text{more}(z, 250000)$  [Matching with 5]  
 $\neg \text{company}(\text{ABC}) \vee \neg \text{earn}(\text{Jamal}, z) \vee \neg \text{more}(z, 250000)$  [Matching with 3]  
 $\neg \text{earn}(\text{Jamal}, z) \vee \neg \text{more}(z, 250000)$  [Matching with 1]  
 $\neg \text{manager}(\text{Jamal}, \text{ABC}) \vee \neg \text{more}(500000, 250000)$  [Matching with 4]  
 $\neg \text{manager}(\text{Jamal}, \text{ABC})$ . [Matching with 6]

This contradicts with our axiom 3.

*(In step 6) we could have matched with axiom 3 [ $\text{manager}(\text{Jamal}, \text{ABC})$ ], then it would contradict with axiom 6 [ $\text{more}(500000, 250000)$ ].*

So  $\neg \text{pays\_tax}(\text{Jamal})$  is asserted false.

Thus  $\text{pays\_tax}(\text{Jamal})$  is true.