NOTE'S DATE:

Q-1:- prove that a binary source can produce ~~max~~ maximum entropy when both symbol are equiprobable.

Soln:-

∵ Binary source produces 0 or 1.

∴ value of i varies from 1 to 2

$$H(X) = -\sum_{i=1}^{2} P(x_i) \log_2 P(x_i)$$

$$= -\Big[P(x_1) \log_2 P(x_1) + P(x_2) \log_2 P(x_2)\Big]$$

Let $P(x_1) = P$

$P(x_2) = 1 - P$

NOTE'S DATE:

$$H(x) = -\left[p\log_2 p + (1-p)\log_2(1-p)\right]$$

$$\Rightarrow H(x) = -\left[p\frac{\log_e p}{\log_e 2} + (1-p)\frac{\log_2(1-p)}{\log_e 2}\right] \quad \left\{ \log_a b = \frac{\log_e b}{\log_e a} \right.$$

$$= -\frac{1}{\log_e 2}\left[p\log_e p + (1-p)\log_e(1-p)\right]$$

Diff. both sides w.r.t. p

$$\frac{d}{dp}H(x) = -\frac{1}{\log_e 2}\left[p\cdot\frac{1}{p} + \log_e p\,(1) + (1-p)\cdot\frac{1}{(1-p)}\cdot -1 + (-1)\log_e(1-p))\right]$$

NOTE'S DATE:

$$\Rightarrow \frac{dH(x)}{dp} = -\frac{1}{\log_e 2}\left[(1+\log_e p) + (-1-\log_e(1-p))\right]$$

$$= -\frac{1}{\log_e 2}\left[\log_e p - \log_e(1-p)\right]$$

$$= -\frac{1}{\log_e 2}\left[\log_e\left(\frac{p}{1-p}\right)\right]$$

$\because$ $H(x) = 1$, Assumed.

$$\frac{d}{dp}H(x) = 0$$

$$\Rightarrow 0 = -\frac{1}{\log_e 2}\left[\log_e\left(\frac{p}{1-p}\right)\right]$$

NOTE'S DATE:

$\Rightarrow 0 = \log_e\left(\frac{P}{1-P}\right)$

$\because \log_e 1 = 0$

$\therefore \log_e 1 = \log_e\left(\frac{P}{1-P}\right)$

$\Rightarrow 1 = \frac{P}{1-P}$

$\Rightarrow 1-P = P$

$\Rightarrow \boxed{P = \frac{1}{2}}$

$\therefore P(x_1) = P = \frac{1}{2}$

$P(x_2) = (1-P) = \left(1-\frac{1}{2}\right)$

$= \frac{1}{2}$

NOTE'S DATE:

$P(x_1) = P(x_2) = 1/2$

x_1 and x_2 are equiprosble.

Condition 2

$H(x) = 0$ when

Probability is 0 or 1

$\because H(x) = \sum_{i=1}^{n} P(x_i)\log_2\left(\frac{1}{P(x_i)}\right)$

Case-I

if $P(x_i) = 0$

$H(x) = 0$

Case II

if $P(x_i) = 1$

$H(x) = \sum_{i=1}^{n} 1 \log_2(1)$

$H(x) = 0$

NOTE'S

DATE :

~~A prob~~

$m_1 = 1/2$

$m_2 = 1/4$

$m_3 = 1/8$

$m_4 = 1/8$

$H(m) =$

$I(m_1)$

$I(m)$

$I(m_2)$

$H(x) = 2.223$ bit/symbol