MATH-2105

List of Questions

1. Define diagonal matrix, Scalar matrix, Hermitian matrix, Idempotent matrix and nil potent matrix.
2. Define diagonal and tri-diagonal matrix with examples.
3. Show that the matrices $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$ are the inverses of each other.
4. If $A = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 2 \\ 1 & 0 \end{bmatrix}$, then find AB. Is BA exist? Create an argument.
5. Define Column matrix, Row matrix, Inverse matrix, Square matrix and Transpose of a matrix.
6. Determine whether the matrix $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ is idempotent or not.
7. Find whether the matrix $A = \begin{bmatrix} 2 & 2-3i & 3+5i \\ 2+3i & 3 & i \\ 3-5i & -i & 5 \end{bmatrix}$ is Hermitian matrix or not.
8. Solve the following equations for A and B
$$2A - B = \begin{bmatrix} 3 & -3 & 0 \\ 3 & 3 & 2 \end{bmatrix}$$
$$2B + A = \begin{bmatrix} 4 & 1 & 5 \\ -1 & 4 & -4 \end{bmatrix}$$
9. If the matrix $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric, find the value of x and hence find the matrix A.
10. Find x, y, z, t using the concept of equality of matrices, where
$$3\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+t & 3 \end{bmatrix}$$
11. Given $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$, find the matrix C, such that $A + 2C = B$.
12. Determine whether the following vectors are linearly dependent or linearly independent. U = (1, 2, 5), v = (0, 2,4) and w = (-1, 1, 0).
13. Determine the value of a so that the following system in unknowns x, y, z has:
(i) no solution (ii) more than one solution (iii) a unique solution.
x + y - z = 1
2x + 3y + az = 3
x + ay + 3z = 2

14. What is the rank of a matrix? Find the rank of the matrix $X = \begin{bmatrix} 1 & -3 & 2 \\ -2 & 2 & 0 \\ -6 & 9 & -3 \end{bmatrix}$.
15. What is rank of a matrix? Reduce the following matrix A into its Echelon form to find the rank, where $A = \begin{bmatrix} 8 & 6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.
16. Find the rank of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 1 & 2 \end{pmatrix}$
17. Solve the following system of linear equations by
$$x + y + z = 6$$
$$x - y + z = 2$$
$$2x + y - z = 1$$
    i. using Cramer rule.
    ii. using Gauss elimination.
    iii. using matrix method

    Which method involves fewer computations?
18. Solve the following system of linear equations using the matrix method:
$$x + 9y - z = 27$$
$$x - 8y + 16z = 10$$
$$2x + y + 15z = 37$$
19. Solve the following equations by matrix method
    x + 2y +3z = 14
    3x + y + 2z = 11
    2x + 3y + z = 11
20. Find the characteristic equation and all the characteristic roots of the matrix $A = \begin{bmatrix} 8 & 6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.
21. Find the characteristic equation and all the characteristic roots of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$.
22. Diagonalize the matrix $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$
23. State the Cayley Hamilton theorem. Verify the theorem for the matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ and hence find A^{-1}.

24. Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$ and hence find the inverse of A.

25. Show the following system of linear equations as its equivalent matrix form and as linear combination of vectors:

$$x_1 + 2x_2 - 4x_3 + 7x_4 = 4$$

$$3x_1 + x_2 + 6x_3 - 8x_4 = 12$$

$$4x_1 - 5x_2 - 3x_3 + 7x_4 = 8$$

26. Give the matrix and vector representation of the following system of linear equations:

(i)
$$\begin{aligned} x_1 + 2x_2 - 4x_3 + 7x_4 &= 4 \\ 3x_1 - 5x_2 + 6x_3 - 8x_4 &= 8 \\ 4x_1 - 3x_2 - 2x_3 + 6x_4 &= 11 \end{aligned}$$

(ii)
$$\begin{aligned} x_1 + 2x_2 &= 40 \\ -x_1 - 2x_2 + 6x_3 &= 11 \\ -5x_2 + 6x_3 - 8x_4 &= 11 \\ -2x_3 + 6x_4 - x_5 &= 11 \\ 2x_4 - 7x_5 &= 200 \end{aligned}$$

27. Define equal vector and null vector. Find the scalar product of the vectors (2, 3, 1) and (3, 1, -2). Also find the angle between them.

28. If $A = \begin{bmatrix} 1 & 3 \\ 4 & -3 \end{bmatrix}$, find a non-zero column vector $u = \begin{bmatrix} x \\ y \end{bmatrix}$ such that $Au = 3u$. Describe all such vectors.

29. Apply the concept of vector cross product to find the area of the parallelogram constructed by the vectors $\vec{u} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$.

30. If $A = [a_{ij}]$, where $a_{ij} = \begin{cases} 0, & when\ i \neq j \\ C, & when\ i = j \end{cases}$

    Construct a 3×3 order matrix and identify the type of matrix, where C is the sum of the 1st digit and the last digit of your ID. Also test the matrix A is

    i. orthogonal or not

    ii. singular or not

31. If $A = [a_{ij}]$ where $a_{ij} = \begin{cases} 0, & when\ i < j \\ i + j, & when\ i = j \\ 2i - j, & when\ i > j \end{cases}$

    Construct a 3×3 matrix and identify the type of the matrix A. Also check whether it is singular or not.

32. Define Augmented matrix. Use Augmented matrix to solve the system of linear equations

$$2x + y - 2z = 10$$
$$3x + 2y + 2z = 1$$
$$5x + 4y + 3z = 4$$

33. Find a unit vector perpendicular to each of the vectors $r_1 = 3i + 2j - 4k$ and $r_2 = i + j + 2k$
34. Find a unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$.
35. Determine whether the force field $\vec{F}(x, y, z) = x^2y\hat{\imath} + xyz\hat{\jmath} - x^2y^2\hat{k}$ is a conservative or not.
36. Find the angle between the surfaces $x^2 + y^2 + z^2 = 49$ and $x^2 + y^2 - z = 43$ at (6, 3, -2).
37. Show that the vector $\vec{F} = (6xy + z^3)\underline{i} + (3x^3 - z)\underline{j} + (3xz^2 - y)\underline{k}$ is irrotational.
38. Evaluate $\oint_c (5x^2 + 3y)dx + 11yzdy + 10xz^3dz$ along the following paths c:
    i. the straight lines from (0, 0, 0) to (0, 0, 1) then to (0, 1, 1) and then to (1, 1, 1)
    ii. the straight line joining from (0, 0, 0) to (3, 9, 27)
39. Write down three vector operators gradient, divergence and curl.
40. Show that the divergence of the curl of a vector field *A* is zero.
41. Let $\vec{A} = xy^2\,\underline{i} - 3x^2y\,\underline{j} + 2yz^2\,\underline{k}$. Now find curlcurl of $\vec{A}$ at (1, 0, - 4).
42. Verify the relation $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$ for the vector $A = x^2yi + 2xyzj + 3y^2zk$
43. Verify Green's theorem in the plane for $\oint_c (xy + y^2)dx + x^2dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.
44. State and prove Green's theorem.
45. Apply Green's theorem find $\oint_c (x^2ydx + x^2dy)$, where c is the boundary of the region enclosed by the line y = x and the curve $y = x^2$
46. Give the statement of the Divergence theorem. Verify Divergence theorem for the vector field $\vec{F} = (2xy + z)\underline{i} + y^3\,\underline{j} - (x + 3y)\underline{k}$
47. Find $\iint \vec{F} \bullet \hat{n}\, dA$, where $\vec{F} = x^2\,\underline{i} + 3y^2\,\underline{k}$ and S is the portion of the planes x + y + z = 1 in the first octant.
48. Evaluate $\oint_c y^2dx - x^2dy$, where c is the triangle whose vertices are (1, 0), (0, 1), (-1, 0).
49. Determine the angles α, β, γ which the vector $\vec{A} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}$ makes with the positive directions of the coordinate axes. Also show that $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$.
50. Find a unit vector
    i. in the direction to the vector $\vec{A} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$.
    ii. parallel to the resultant of the vectors $\vec{B} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\vec{C} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

iii. perpendicular to the plane constructed by the vectors $\vec{D} = 3\hat{\imath} + \hat{\jmath}$ and $\vec{E} = -\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$.

51. Find whether the vectors $\vec{A} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$, $\vec{B} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{C} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$ are coplanar.
52. What is inner product of vectors? Apply the Gram-Schmidt orthonormalization algorithm to the set of vectors $v_1 = (1,0,1)$, $v_2 = (1,0,-1)$ and $v_3 = (0,3,4)$ to obtain an orthonormal basis. Justify your results.
53. Explain direction cosines of a line. If the angle between two straight lines is θ and their direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2 then show that
$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2.$$
Hence develop this relation for $\sin\theta$.
54. Explain shortest distance. Find the equation of the line of shortest distance and evaluate the length of the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-5}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$
55. Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ are coplanar. Find their intersection point and the equation of the plane in which they lie.
56. Find the equation of the straight line that intersect the lines $4x + y - 10 = 0 = y + 2z + 6$ and $3x - 4y + 5z + 5 = 0 = x + 2y - 4z + 7$ and passing through the point $(-1, 2, 2)$.