Digital Signal Processing

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November, 2024

Contents

1 What is DSP?

Digital Signal Processing (DSP) refers to various techniques for improving the accuracy and reliability of digital communication. [1]

2 Signals

Signals are detachable physical quantities or variables by which message or information can be transmitted.
Signals are represented mathematically as a function of one or more variable.

$$y = f(t) \tag{1}$$

$t =$ independent variable, $y =$ dependent variable
Example : Human voice, Television, Picture etc.

2.1 Continuous and Discrete Time Signals

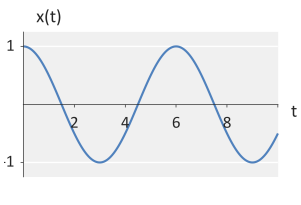
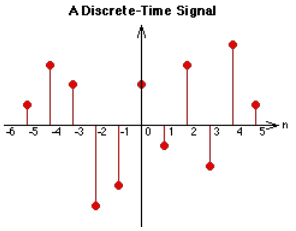
Continuous Time Signal	Discrete Time Signal
A signal is said to be continuous when it is defined for all instance of time.	A signal is said to be discrete when it is defined at only discrete instance of time.
Continuous in both time and amplitude	Discrete in time; amplitude can be continuous or quantized
Analog Signals: Sine, speech	Digital signals: sample audio, digital images
	

Table 1: Continuous and Discrete Time Signals

2.2 Even and Odd Signal

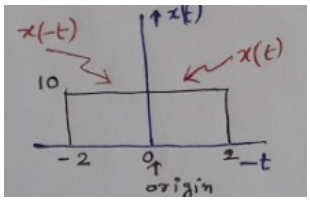
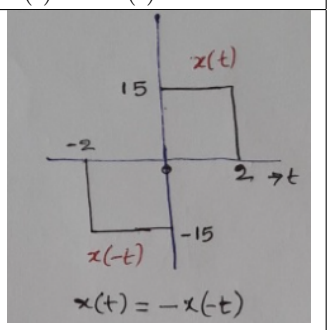
Even Signal	Odd Signal
A signal $x(t)$ is even if $x(t) = x(-t)$ for all values of t	A signal $x(t)$ is odd if $x(t) = -x(-t)$ for all values of t
Identical about the origin	Not identical about the origin. It is identical to its negative.
$x(t) = cos(t)$ is even	$x(t) = sin(t)$ is odd
	

Table 2: Even and Odd Signal

2.3 Periodic and Non-Periodic

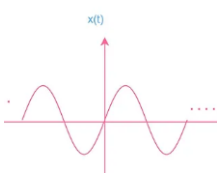
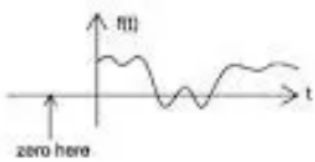
Periodic Signal	Non-Periodic Signal
A signal repeats itself after a fixed time period is called as a periodic signal. Conditions: • $x(t) = x(t+T)$ • $-\infty \leq t \leq \infty$	A signal which is not satisfy the condition $x(t)! = x(t+T)$ Fundamental Time Period, $T = \frac{2\pi}{\omega} = \frac{1}{f}$
	

Table 3: Properties of Signal

2.4 Causality

Causal Signals: Causal signals are signals that are zero for all negative time.

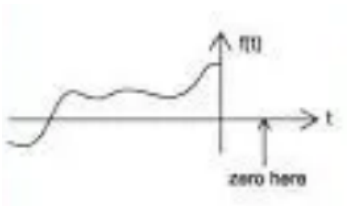
$$x(t) = \begin{cases} x(t) > 0 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



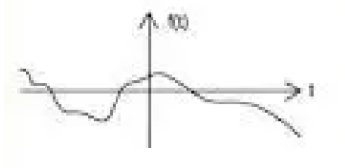
[1] https://www.techtarget.com/whatis/definition/digital-signal-processing-DSP

Anti-Causal Singal: A signal which posses zero value for all positive value of time, but has amplitude which is greater than zero for all negative value of time.

$$x(t) = \begin{cases} x(t) > 0 & t \leq 0 \\ 0 & t > 0 \end{cases}$$

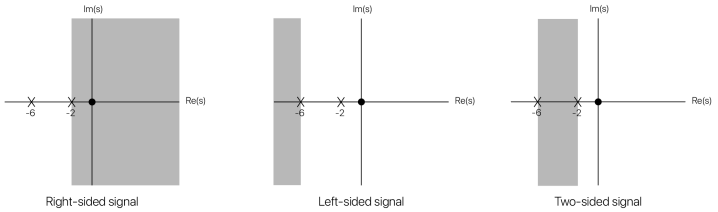


Non-Causal: A signal that has value of amplitude which is greater than zero for both positive and negative instances of time is a non-causal signal.



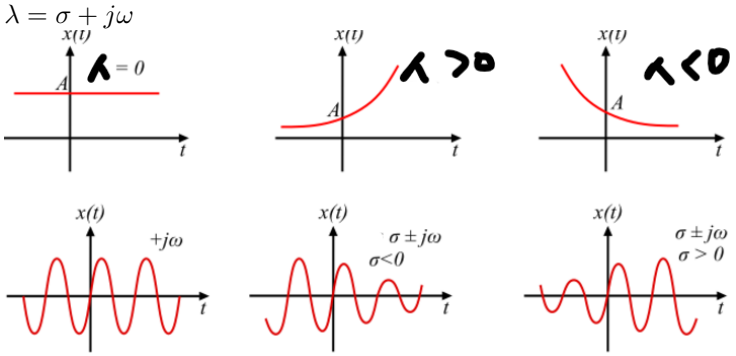
2.5 Sided Signals

Left Sided Signal: A signal is called left-sided if it is non-zero only for $t \leq T$, where T is a finite time.
Right Sided Signal: A signal is called right-sided if it is non-zero only for $t \geq T$, where T is a finite time.
Two Sided Signal: A signal is called two-sided if it is non-zero for both $t \leq T_1$ and $t \geq T_2$, where T_1, T_2 are finite times.

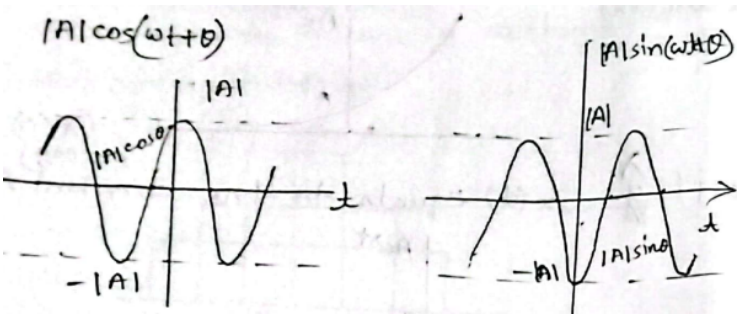


2.6 Complex Exponential Signal

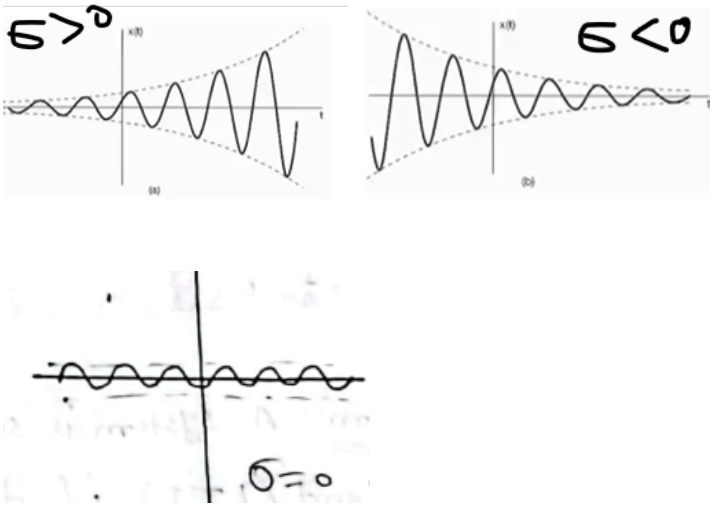
An exponential signal whose samples are complex numbers is known as a complex exponential signal.
Continuous-Time Complex Exponential Signal : A CT Complex exponential signal is the one that is defined for every instance of time.
$x(t) = Ae^{\lambda t}$ Where, A is amplitude and s is a complex variable.
$\lambda = \sigma + j\omega$



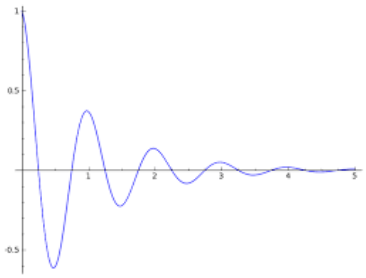
Complex Sinusoid : A complex sinusoid signal is a special case of a complex exponential $x(t) = A^{\lambda t}$, where A is complex and λ is pure imaginary.
$x(t) = Ae^{j\omega t}$, where ω is real.



By expressing A in polar form as, $A = |A|e^{j\theta}$, where θ is real and using Euler's relation, $x(t) = |A|\cos(\omega t + \theta) + j|A|\sin(\omega t + \theta)$
Most general case of a complex exponential, A and λ both are complex, letting $A = |A|e^{j\theta}$ and $\lambda = \sigma + j\omega$ and now using Euler's relation,
$x(t) = |A|e^{\sigma t}\cos(\omega t + \theta) + j|A|e^{\sigma t}\sin(\omega t + \theta)$

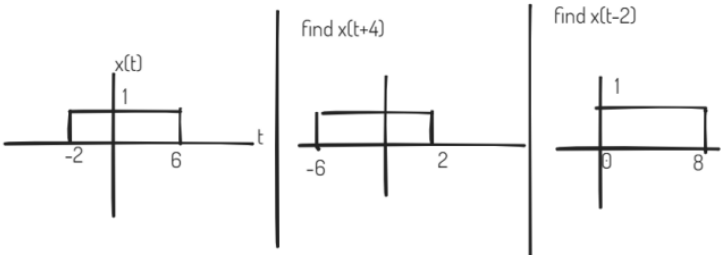


Exponentially Sinusoid Dumped Signal: A dumped signal is sinusoidal signal which amplitude approaches zero as times increases.
$x(t) = Ae^{-\lambda t}\sin(\omega t + \Phi)$

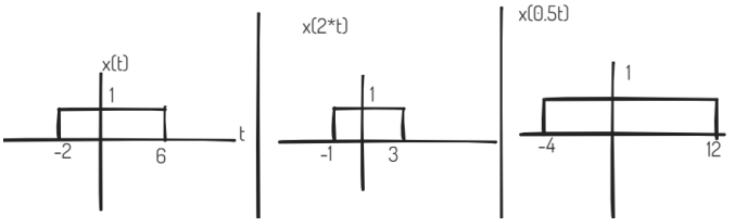


2.7 Operations on Signals

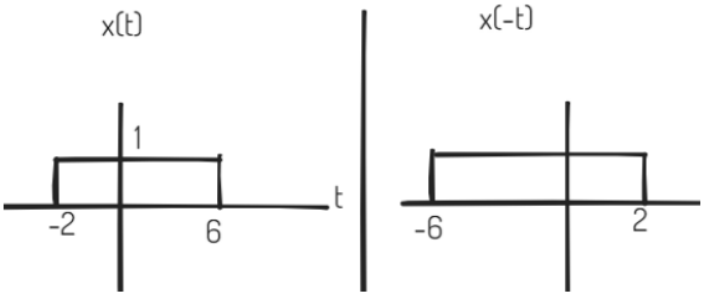
Time Shifting Operation : $x(t \pm t_0)$



Time Scaling Operation $x(\alpha t)$



Time reverse or folding Operation $x(-t)$



2.8 Standard Test Signals

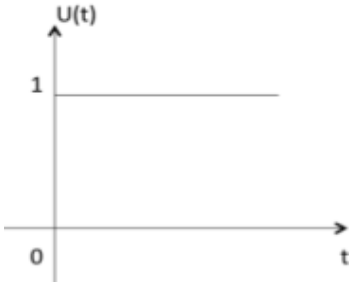
They are used to check the control systems performance using time response of the output.

Unit Step Signal

The step signal or step function is that type of standard signal which exists only for positive time and it is zero for negative time. If a step signal has unity magnitude, then it is known as unit step signal.

$$u(t) = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}$$

Amplitude = Coefficient of $u(t)$

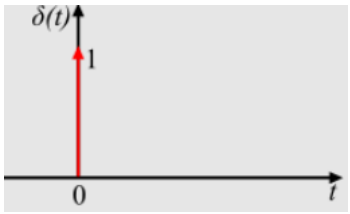


Impulse Signal

An Ideal impulse signal is a signal that is zero everywhere but at the origin $(t = 0)$, it is infinitely high.

$$\delta(t) = \begin{cases} 1 & \text{if } t = 0 \\ 0 & \text{otherwise} \end{cases}$$

$A \cdot \delta(t), A =$ A is the area

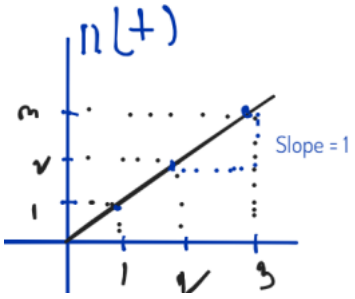


Ramp Signal

A ramp signal is a type of standard signal which starts at $t = 0$ and increase linearly with time.

$$r(t) = \begin{cases} t & \text{if } t \leq 0 \\ 0 & \text{if } t < 0 \end{cases}$$

Slope = Coefficient of r(t)

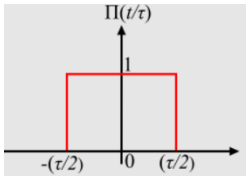


Rectangular Single

A signal that produces a rectangular shaped pulse with a width of τ (where $\tau = 1$ for unit rectangular function) centered at $t = 0$ is known as rectangular signal.

$$rec(t) = \begin{cases} 1 & \text{if } |t| \leq \frac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$$

$\tau = 1$ for unit rectangular



2.9 Energy and Power Signal

Energy Signal: A signal is said to be an energy signal if and only if its total energy E is finite, $0 < E < \infty$.

$E = \lim_{T\to\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$

Power Signal: A signal is said to be a power signal if its average P is finite, $0 < P < \infty$.

$P = \lim_{T\to\infty} \frac{E}{T} = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$

A signal cant be Energy and Power Signals together. If E_x is finite, then P_x is zero, vice-versa.

3 System

System is a interconnection of different physical components which is used to convert one form of signal to others.