

**Step 1:** Convert the axioms to first order predicate logic.

Note that we may or may not use quantifiers ( $\forall, \exists$ ) and the (letter) case doesn't matter here.

1. company(ABC)  $\wedge$  has\_employees(ABC, 500).
2. employee(x, y)  $\wedge$  company(y)  $\wedge$  earn(x, z)  $\wedge$  more(z, 250000)  $\rightarrow$  pays\_tax(x).  
Quantifiers are not necessary. We can also write it as:  
 $\forall x \forall y \forall z (\text{employee}(x, y) \wedge \text{company}(y) \wedge \text{earn}(x, z) \wedge \text{more}(z, 250000) \rightarrow \text{pays\_tax}(x))$ .
3. manager(Jamal, ABC).
4. manager(x, ABC)  $\rightarrow$  earn(x, 500000).
5. manager(x, y)  $\rightarrow$  employee(x, y). [additional axiom]
6. more(500000, 250000). [additional axiom]

In axiom 1, the fact that ABC company has 500 employees is not necessary for proving our goal. So we can ignore it.

**Step 2:** Convert the logic into Conjunctive Normal Form (CNF)

In CNF  $P \rightarrow Q$  becomes  $\neg P \vee Q$

1. company(ABC) [as it is, ignoring has\_employees(ABC, 500)]
2.  $\neg(\text{employee}(x, y) \wedge \text{company}(y) \wedge \text{earn}(x, z) \wedge \text{more}(z, 250000)) \vee \text{pays\_tax}(x)$ .  
 $\neg \text{employee}(x) \vee \neg \text{company}(y) \vee \neg \text{earn}(x, z) \vee \neg \text{more}(z, 250000) \vee \text{pays\_tax}(x)$
3. manager(Jamal, ABC).
4.  $\neg \text{manager}(x, ABC) \vee \text{earn}(x, 500000)$
5.  $\neg \text{manager}(x, y) \vee \text{employee}(x, y)$ .
6. more(500000, 250000). [as it is]

**Step 3:** The goal is to prove:  $\text{pays\_tax}(\text{Jamal})$ .

We can prove this by contradiction. We will try to assert the negation of the goal, i.e.  $\neg \text{pays\_tax}(\text{Jamal})$ . If we fail, it will prove that our goal is true.

In each step, we match our current goal with an axiom to get rid of a part of the goal.

So in this example, in the first step we match our goal with axiom 2. So  $\neg \text{pays\_tax}(\text{Jamal})$  gets crossed out by  $\text{pays\_tax}(x)$  where we assume x is Jamal.

- $\neg \text{pays\_tax}(\text{Jamal})$
- $\neg \text{employee}(\text{Jamal}, y) \vee \neg \text{company}(y) \vee \neg \text{earn}(\text{Jamal}, z) \vee \neg \text{more}(z, 250000)$  [Matching with 2]
- $\neg \text{manager}(\text{Jamal}, y) \vee \neg \text{company}(y) \vee \neg \text{earn}(\text{Jamal}, z) \vee \neg \text{more}(z, 250000)$  [Matching with 5]
- $\neg \text{company}(\text{ABC}) \vee \neg \text{earn}(\text{Jamal}, z) \vee \neg \text{more}(z, 250000)$  [Matching with 3]
- $\neg \text{earn}(\text{Jamal}, z) \vee \neg \text{more}(z, 250000)$  [Matching with 1]
- $\neg \text{manager}(\text{Jamal}, \text{ABC}) \vee \neg \text{more}(500000, 250000)$  [Matching with 4]
- $\neg \text{manager}(\text{Jamal}, \text{ABC})$ . [Matching with 6]

This contradicts with our axiom 3.

(In step 6) we could have matched with axiom 3 [ $\text{manager}(\text{Jamal}, \text{ABC})$ ], then it would contradict with axiom 6 [ $\text{more}(500000, 250000)$ ].

So  $\neg \text{pays\_tax}(\text{Jamal})$  is asserted false.

Thus  $\text{pays\_tax}(\text{Jamal})$  is true.