Course code: EEE-453
Course title: Numerical Method
Lecture on
Numerical Methods of Ordinary Differential Equation

Numerical Differentiation

Differential Equation

i. Ordinary Differential Equation

Ex---- $f(x)=3x\frac{d^2y}{dx^2} + 4\sin(x)\frac{dy}{dx} + x - 3$

Order---2(max. derivatives)

Degree----1(max. power)

ii. Partial Differential Equation

Ex-----$f(x,y)= 4$

$\frac{\partial^2 u}{\partial x^2} + x\frac{\partial u}{\partial y} = 3\sin(x)e^x + 3y = 4$

Numerical Differentiation

Numerical Methods Of Solving Ordinary Differential Equation(ODE)

To solve a differential equation some conditions(no. of condition must be equal to the order of the equation) are required. Based on these conditions ODE can be classified as

i. Initial value problem(IVP)
ii. Boundary value problem(BVP)

Numerical Differentiation

IVP Vs BVP

IVP	BVP
1.All condition will be given for the stating point of solution	1.Condition will be given for the terminal point of solution concerned
2. Ex----$3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} = 3$ Limit--- x= 0 to 10 Condition---Y(x=0)=5 $\frac{dy}{dx}(0)=Y'(x=0)=0$	2. Ex-----$3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} = 3$ Limit--- x= 0 to 10 Condition---Y(x=0)=5 $\frac{dy}{dx}(x=10)=Y'(x=10)=5$

Initial value problem(IVP)

Initial value problem(IVP):1st order problem

- ✓ $\frac{dy}{dx} = f(x,y)$
- ✓ Initial condition $(x_i, y_i),(x_{i+1}, y_{i+1})$............
- ✓ Limit of X-------xi:h:xup=0:h:10
- ✓ Xi=initial value or lower limit=0
- ✓ Xup=upper limit=10
- ✓ h=interval of x
- ✓ $y_{(i+1)}=y_i+\varphi h$
- ✓ φ= m=slope

Initial value problem(IVP)

Initial value problem(IVP) solving method

i. 1st order Range Kutta (R.K.) Method or Euler method
ii. 2nd order Range Kutta (R.K.) Method or Heun's method
iii. 3rd order Range Kutta (R.K.) Method

Initial value problem(IVP)

Determination of φ in Initial value problem(IVP) solving method

Method	Determination of φ
1.Euler method	$\varphi = f(x_i, y_i)$ [If result linear , error=0]
2.Heun's method	$\varphi = \frac{1}{2}K_1 + \frac{1}{2}K_2$ $K_1 = f(x_i, y_i)$ $K_2 = f(x_i + h, y_i + K_1 * h)$ [If result quadratric, error=0]

Initial value problem(IVP)

Determination of φ in Initial value problem(IVP) solving method

Method	Determination of φ
3. 3rd order Range Kutta (R.K.) Method	$\varphi = \frac{1}{6}K1 + \frac{4}{6}K2 + \frac{1}{6}K3$ $= \frac{1}{6}(K_1 + 4K_2 + K_3)$ $K_1 = f(xi, yi)$ $K_2 = f(xi + \frac{1}{2}*h,\ yi + \frac{1}{2}*K_1*h)$ $K_3 = f(xi+h,\ yi - K_1*h + 2*K_2*h)$ [If result cubic , error=0]

Initial value problem(IVP)

Problem-01

Solve $\sin(x)\cos(y)dx=(1+\cos(x)\sin(Y))dy$,$y(x=0.1)=-2$ from $x=0.1$ to 0.4 with regular interval of 0.1. Use

i. 1st order Range Kutta (R.K.) Method or Euler method

ii. 3rd order Range Kutta (R.K.) Method

Initial value problem(IVP)

Solution

$$\sin(x)\cos(y)dx=(1+\cos(y)\sin(x))dy$$

$$\frac{dy}{dx}=\frac{\sin(x)\cos(y)}{(1+\cos(y)\sin(x)}$$

$$f(x,y) = \frac{\sin(x)\cos(y)}{(1+\cos(y)\sin(x)}$$

I. Euler method

$\varphi= f(xi,yi)$

$x=0.1$ to 0.4

$h=0.1$

Initial value problem(IVP)

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Euler method

$\varphi = f(x_i, y_i)$

x_i	y_i	$\varphi = f(x_i, y_i)$	$y_{(i+1)} = y_i + \varphi h$
0.1	-2	-0.4362	-2.0436
0.2	-2.0436	-0.7098	-2.1146
0.3	-2.1146	-0.8379	-2.1984
0.4	-2.1934		

Ans—(x,y)=(…,…),(…,…),(…,…),(…,…)

Initial value problem(IVP)

RK-3 method

x=0.1 to 0.4

h=0.1

$$\varphi = \frac{1}{6}K1 + \frac{4}{6}K2 + \frac{1}{6}K_3$$

$$= \frac{1}{6}(K_1 + 4K_2 + K_3)$$

$K_1 = f(xi, yi)$

$K_2 = f(xi + \frac{1}{2} * h,\ yi + \frac{1}{2} * K_1 * h)$

$K_3 = f(xi + h,\ yi - K_1 * h + 2 * K_2 * h)$

Initial value problem(IVP)

xi	yi	$K_1 = f(x_i, y_i)$	K_2	K_3	φ	$y_{(i+1)} = y_i + \varphi h$
0.1	-2.0000	-0.0433	-0.0667	-0.0921	-0.0670	-2.0067
0.2	-2.0067	-0.0916	-.1179	-0.1476	-0.1185	-2.0186
0.3	-2.0186	-0.1467	-0.1775	-0.2134	-0.1784	-2.0364
0.4	-2.0364					

Initial value problem(IVP)

Initial value problem(IVP): Higher order equation

For IVP, the higher order ODEs are first converted to 1^{st} order then the equation are consecutively solved.

For n-th order IVP

n= no of 1^{st} order of ODEs

Initial value problem(IVP)

Initial value problem(IVP): Higher order equation

Example

$$f(x)=3\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = \sin x$$

It's a 4^{th} order ODE

So

Step1---Assumption

Step2----convert main function according to assumption

Given

$X=1$ to 2

$y(1)=4$

$y_2=y'(1)=0$

$y_3=y''(1)=2$

$y_4=y'''(1)=-2$

Initial value problem(IVP)

Assumptions:

No. of assumption = max.order -1

$$\frac{dy}{dx} = y1\text{---------solving this get answer of } y$$

$$\frac{d^2y}{dx^2} = y2 = \frac{d(y1)}{dx}\text{----solving this get answer of } y1$$

$$\frac{d^3y}{dx^3} = y3 = \frac{d(y2)}{dx}\text{----solving this get answer of } y2$$

Initial value problem(IVP)

Now Converting equation

$$3\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = \sin x$$

$$\text{Or, } 3\,\frac{dy_3}{dx} + 2y_2 + y = \sin x$$

Determination of φ in Initial value problem(IVP) solving method

Method	Determination of φ
1.Euler method	$\varphi_1= f(xi,yi,y1_{(i)})$ ----$y1_{(i+1)} = y1_{(i)}+\varphi_1*h$ $\varphi= f(xi,yi,y1_{(i+1)})$ ----$y_{(i+1)}= y1+\varphi*h$
2.Heun's method	$\varphi_1= \frac{1}{2}K_1+\frac{1}{2}K_2$----- $y1_{(i+1)} = y1_{(i)}+\varphi_1*h$ $\varphi= f(xi,yi,y1_{(i+1)})$ ------$y_{(i+1)}= y1+\varphi*h$ $K_1=f(xi,yi,y1_i)$

Initial value problem(IVP):Higher order

Problem-02

Solve $3x^2\dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} + y = \sin x$

with $y(x=1)=3.4$; $y'(x=1)0$ using

i. Euler method

ii. RK-2 method

For x=1 to 3 with h=0.5 interval

Initial value problem(IVP):Higher order

Solution

It's a 2nd order 2^{nd} degree ODE

1 .Assumption

$$\frac{dy}{dx} = y1$$

2.Converting equation

$$3x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + y = \sin x$$

$$\text{Or, } 3x^2\frac{dy1}{dx} + 2xy1 + y = \sin x$$

$$\text{Or, } \frac{dy1}{dx} = \frac{\sin x - 2xy1 - y}{3x^2}$$

Initial value problem(IVP):Higher order

$$f1= \frac{\sin x-2xy1-y}{3x^2}$$

f= y1

Given

Xi=1 then yi=3.4

Xi=1 then y'i=y1i=0

h= 0.5

X=1 to 3

Initial value problem(IVP):Higher order

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1.Euler method

xi	yi	Y1i	φ1=f1(xi,yi,y1i)	y1(i+1)=y1i+φ1*h	φ=f(xi,yi,y1(i+1))	y(i+1)=yi+φh
1	3.4	0	-0.8528	-0.4264	-0.4264	3.1868
1.5	3.1868	-0.4264				
2						
2.5						
3						

Initial value problem(IVP):Higher order

1.Rk-2 method

xi	yi	Y1i	K1=xi,yi,y1i	K2=xi+h,yi,y1i +K1*h	Φ1=1/2(k1+k 2)	y1(i+1)=y1i+ Φ1*h	Φ=f(xi,yi, y1(i+1))	y(i+1)
1	3.4	0	-0.8528	-0.1664	-0.5096	-0.2548	-0.2548	3.2726
1.5	3.2726	-0.2548						
2								
2.5								
3								

Initial value problem(IVP):Higher order

Ans----(…,…),(…,…),(…,…),(…,…)….

Initial value problem(IVP):Higher order

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Initial value problem(IVP):Higher order

Initial value problem(IVP):Higher order

Initial value problem(IVP):Higher order