University of Asia Pacific

Department of EEE

Course Code: EEE 453

Course Title: Numerical Method

Lecture On

Introduction of Numerical Method

Introduction of Numerical Method

What is Numerical Method

Numerical methods are techniques by which mathematical problems are formulated so that they can be solved with arithmetic operations.

Numerical Method

- ✓ No mathematical formulation
- ✓ Some error will always occur

Analytical Method

- ✓ Concrete mathematical formulation
- ✓ Error free result

Introduction of Numerical Method

Why you should study numerical methods

1. Numerical methods are extremely powerful problem-solving tools, can handle large systems of equations, nonlinearities, and complicated geometries that are often impossible to solve analytically
2. You may often have occasion to use commercially available prepackaged computer programs that involve numerical methods.
3. You can design your own programs to solve problems without having to buy or commission expensive software.
4. Numerical methods are an efficient vehicle for learning to use computers. Because numerical methods are for the most part designed for implementation on computers.
5. Numerical methods provide a vehicle for you to reinforce your understanding of mathematics

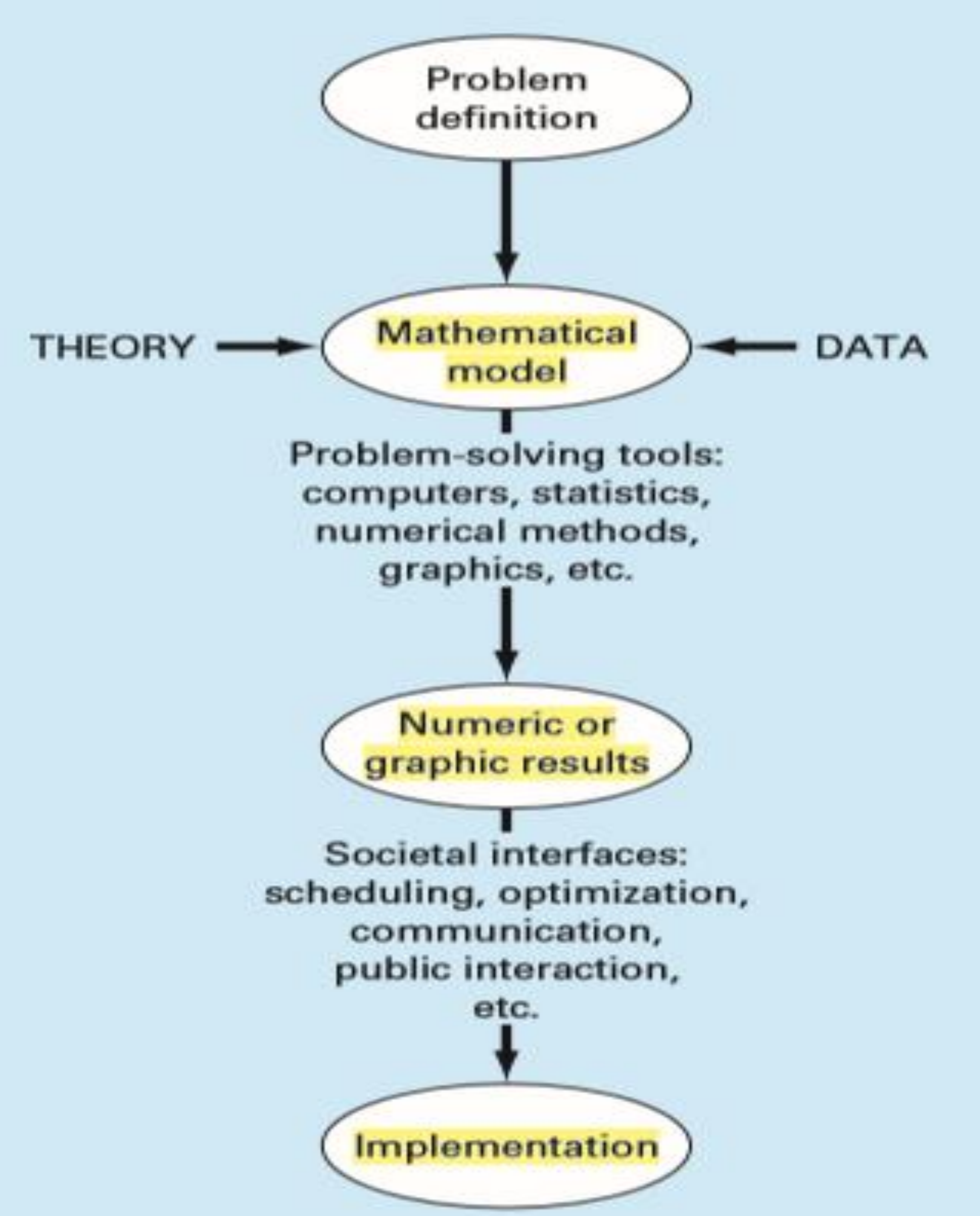
Introduction of Numerical Method

The engineering problem solving process

A mathematical model can be defined as a formulation or equation that expresses the system or process in mathematical terms. It can be represented as

Dependent variable =f(independent variables, parameters, forcing functions)

$$y=f(x)= 1/(1-3x)$$



Introduction of Numerical Method

Contrast between Numerical Methods and Analytic Methods of Solving a Mathematical Problem

Problem Statement:

A parachutist of mass 68.1 kg jumps out of a stationary hot air balloon. Compute velocity prior to opening the chute using the equation $V(t)=\frac{gm}{c}(1- e^{-(c/m)t})$. The drag coefficient(c) is equal to 12.5 kg/s.

Solution:

Analytic method:

If the parachutist is initially at rest (v = 0 at t = 0),

$V(t)=\frac{gm}{c}(1- e^{-(c/m)t})$ can be used to calculate the velocity.

Introduction of Numerical Method

Here

C= 12.5 kg/s, m= 68.1 kg and g= 9.8 m/s^2

$V(t)=\frac{gm}{c}(1- e^{-(c/m)t})= 53.39(1- e^{-0.18355t})$

A velocity of 44.87 m/s (100.4 mi/h) is attained after 10 s. After a sufficiently long time, a constant velocity, called the terminal velocity, of 53.39 m/s (119.4 mi/h) is reached. This velocity is constant because, eventually, the force of gravity will be in balance with the air resistance. Thus, the net force is zero and acceleration has ceased.

t(s)	V(m/s)
0	0.00
2	16.40
4	27.77
6	35.64
8	41.10
10	44.87
12	47.49
a	53.39

Numerical Method:

In numerical method mathematical problem can be solved by arithmetic operations.

So, At the start of the computation (ti = 0), the velocity of the parachutist is zero(Vti=0). Using this information and the parameter values velocity at ti+1 = 2 s can be calculated.

[V=U+at]

$$V(ti+1) = V(ti)+[g-\frac{c}{m}V(ti)](ti+1 - ti)$$

V(2s) = 0+[9.8- 12.5/68.1(0)] (2-0) = 19.60 m/s

V(4s) = 19.60+[9.8- 12.5/68.1(19.60)] (4-2) = 32.00 m/s

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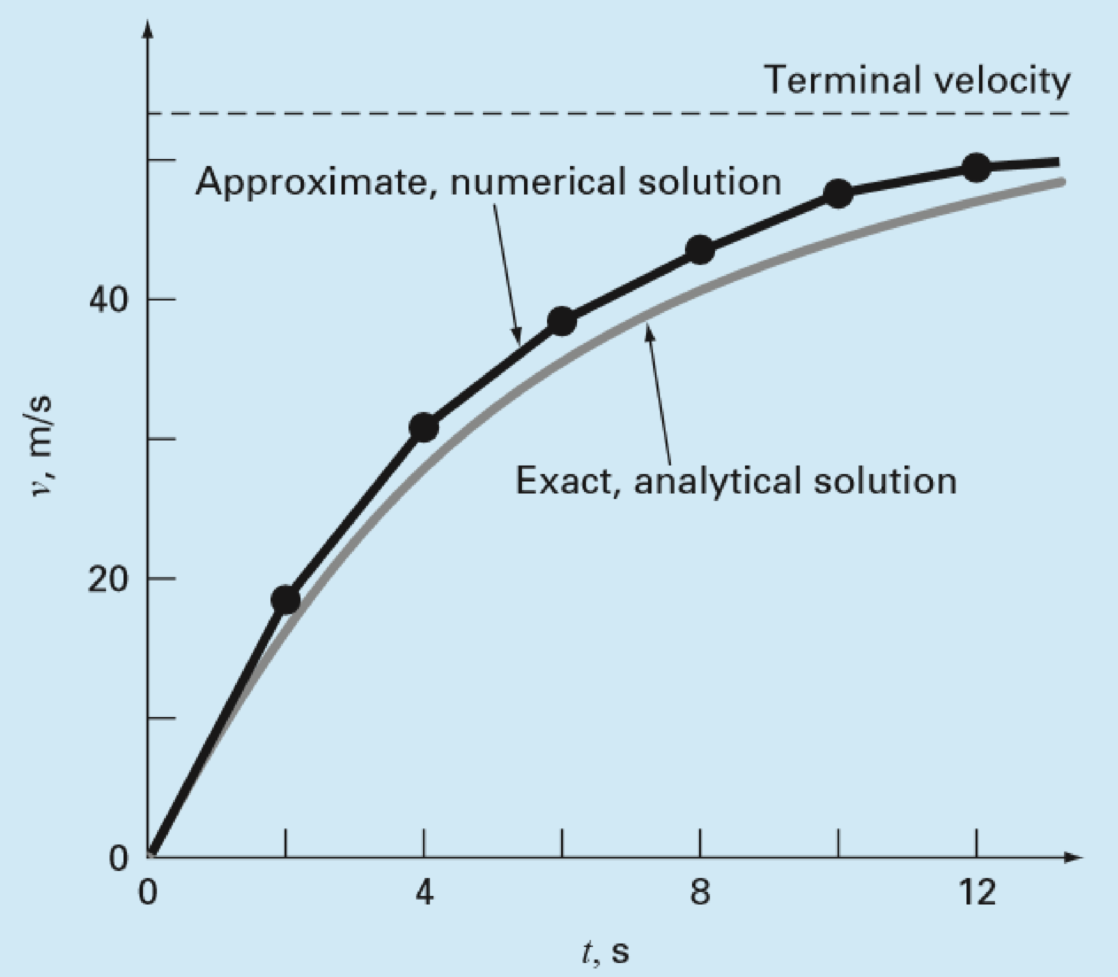
Analytic Vs Numerical Method

t(s)	V(m/s)Ana
0	0.00
2	16.40
4	27.77
6	35.64
8	41.10
10	44.87
12	47.49
α	53.39

t(s)	V(m/s)Num
0	0.00
2	19.60
4	32.00
6	39.85
8	44.82
10	47.97
12	49.96
α	53.39

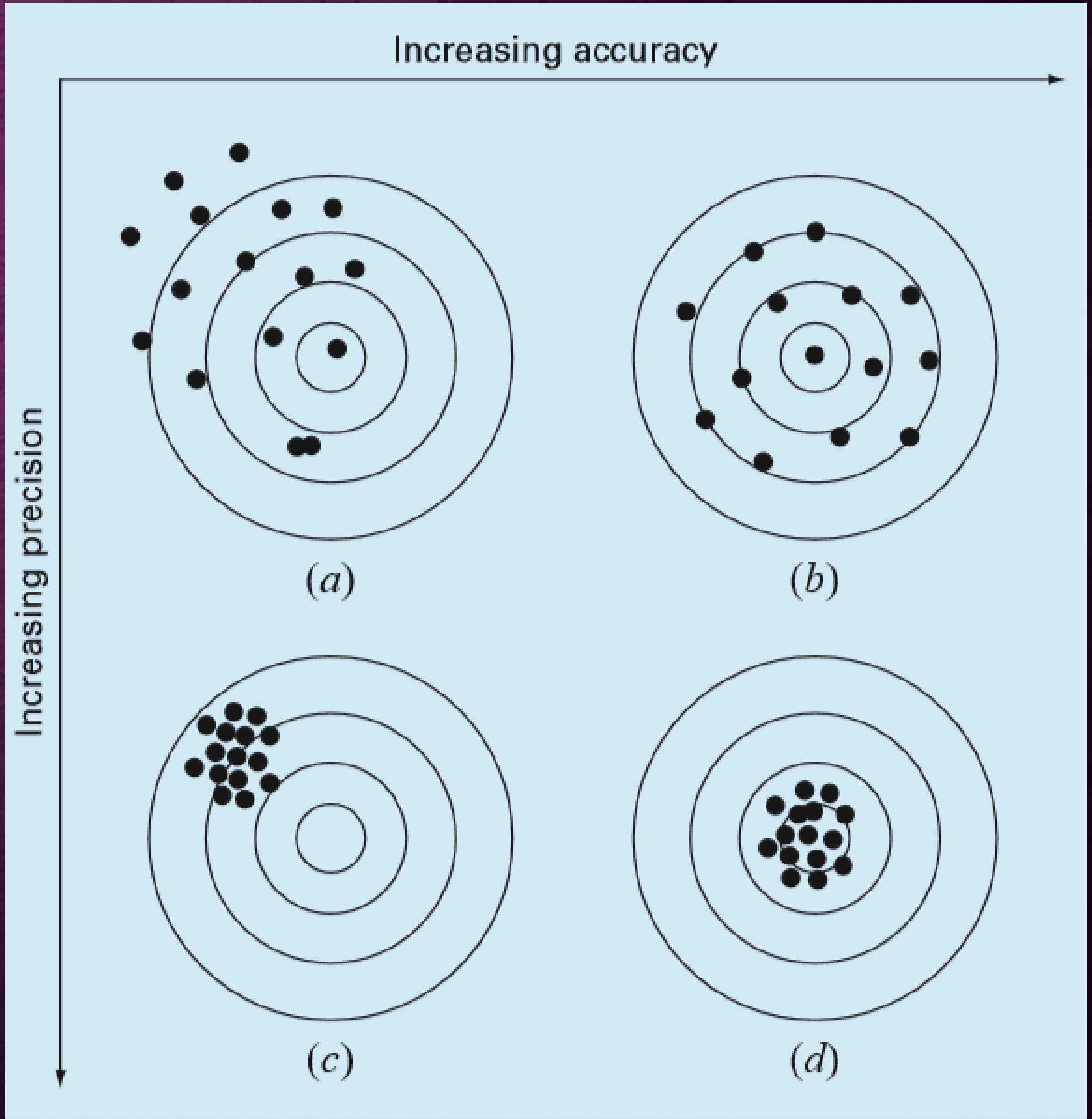


Introduction of Numerical Method

Numerical methods in term of Accuracy and Precision

Accuracy

- ✓ Accuracy refers to how closely a computed or measured value agrees with the true value
- ✓ These concepts can be illustrated graphically using an analogy from target practice.
- ✓ Although the shots in Fig(C) are more tightly grouped than those in Fig(a), the two cases are equally inaccurate because they are both centered on the upper left quadrant of the target



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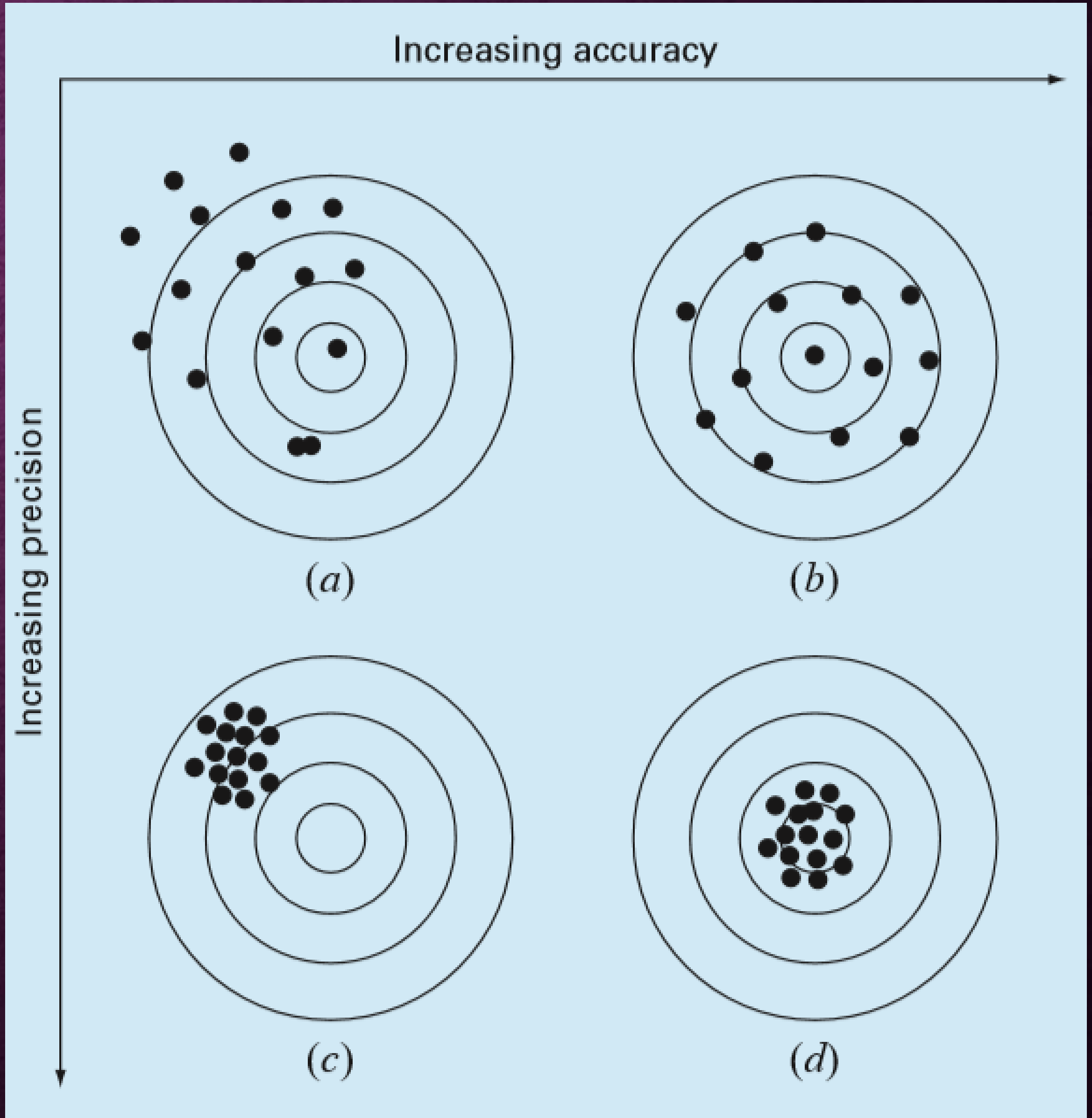
Numerical methods in term of Accuracy and Precision

Precision

- ✓ Precision refers to how closely individual computed or measured values agree with each other.
- ✓ Although the shots in Fig(b) and (d) are equally accurate, the latter is more precise because the shots are tightly grouped.

So,

(a) Inaccurate and imprecise;
(b) accurate and imprecise;
(c) inaccurate and precise;
(d) accurate and precise.



Introduction of Numerical Method

Some basic ideas of Numerical Methods

✓ Source of error

1. Round of error

   It occurs due to limiting the number of digits up to certain Decimal position.

   Example--e^1 =2.718281828

   $e^1 \approx 2.7183$

   Round of Error= 2.7183-2.718281828=+ 0.000018171

   This is called round of error

2. Truncation error

   It occurs for limiting a mathematical formula. Taylor series is of great value in the study of numerical methods

   Example-- e^1 =2.718281828

   Now from series expansion $e^x = 1 + x/1! + x^2/2! + \ldots\ldots a$

Taking only three terms

$$e^1 = 1 + 1/1! + 1^2/2!$$
$$\approx 2.5$$

Truncation error= 2.5-2.718281828= -0.2182818285

This is called truncation error

Problem 01

Show how round off and truncation error occur in numerical method for evaluating ln(2). Take your result data up to three decimal position

Solution

1.Round off error

$\ln(2) = 0.6931471806 \approx 0.693$

2. Truncation error

The expansion of ln(x) is

$\ln(x)=x- x^2/2+ x^3/3 - x^4/4 +\ldots\ldots\ldots\ldots.a$

So $\ln(2) = 2- 2^2/2+ 2^3/3 - 2^4/4 +\ldots\ldots\ldots\ldots.a$

$= 2- 2^2/2+ 2^3/3$

≈ 0.75

Here in two time we got two results. That's how round off and truncation error occur in numerical method .

Note: ln(22)……………………..

Taylor Series

Taylor's theorem and its associated formula, Taylor series is of great value in the study of numerical methods

Introduction of Numerical Method

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2!}h^2 + \frac{f^{(3)}(x_i)}{3!}h^3 + \cdots + \frac{f^{n}(x_i)}{n!}h^n + R_n$$

where the remainder term is now

$$R_n = \frac{f^{(n+1)}(\xi)}{(n+1)!}h^{n+1}$$

When x_i = 0 in Taylor Series, it is called Maclaurin Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$= f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

Maclaurin series

Function	Maclaurin series expansion	Sigma Notation
$\frac{1}{1-x}$	$1 + x + x^2 + x^3 + x^3 + x^4 + \cdots$	$\sum_{n=0}^{\infty} x^n$
$\cos x$	$1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \cdots$	$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$
e^x	$1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^2 \ldots$	$\sum_{n=0}^{\infty} \frac{1}{n!} x^n$
$\ln(1+x)$	$x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots$	$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n} x^n$
$\sin(x)$	$x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \cdots$	$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$

Numerical Differentiation : Another Example of Truncation Error

Forward Divided Difference Method:

First Derivative

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h}$$

Second Derivative

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2}$$

Backward Divided Difference Method:

First Derivative

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h}$$

Second Derivative

$$f''(x_i) = \frac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2})}{h^2}$$

Central Divided Difference Method:

First Derivative

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$$

Second Derivative

$$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$$

Problem 01

Use forward and backward difference approximations of $O(h)$ and a centered difference approximation of $O(h^2)$ to estimate the first derivative of $f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$. Here step size=h=0.5 and x=0.5

Solution: Forward divided difference method

For h=0.5, the function can be employed to determine

X(i−1) = 0,

X(i) = 0.5,

X(i+1) = 1.0,

f(xi−1) = 1.2,

f(xi) = 0.925

f(xi+1) = 0.2

First Derivative

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h}$$

f'(0.5)= (0.2−0.925) /0.5 = -1.45

Again the derivative can be calculated directly as

$f'(x) = -0.4x^3 - 0.45x^2 - 0.1x - 0.25$

$f'(0.5) = -0.9125.$

So, Percentage error, $E_t = \left|\frac{-0.9125-(-1.45)}{-0.9125}\right| * 100 = 58.9\%$

Now do the same for

1. backward difference and a centered difference
2. Also for second derivatives when h=0.25 and x=0.5 for forward, backward and centered divided difference method.

Introduction of Numerical Method

✓ Calculation of error

1. True error, Et

$$Et = |Xr - Xt|$$

Xr= calculated value of X
Xt= True/actual value of X

2. Percentage/ relative error, Et

$$Et = \left|\frac{Xr - Xt}{Xr}\right| * 100$$

It express as percentage.

3. Approximate/apparent error, Ea

$$Ea = \left|\frac{Xr,new - Xr,old}{Xr,new}\right| * 100$$

Xr,new = current value of X
Xr,old = previous value of X

Notes: The problem of true error is that one should know the actual result of the problem

To evaluate Ea, knowledge of actual result is not necessary that's why approximate error is mostly used in numerical method

Introduction of Numerical Method

Some basic terms of Numerical Method

- ✓ **Iteration/Step:**
  - These are the number of attempts made to come to a result.
  - The less the iteration number the faster the method.
- ✓ **Initial guess:**
  - To start a numerical method first you need to guess the result And then start .This initial guesses may single or multiple.
  - Less initial guess better method

Introduction of Numerical Method

- ✓ Progressing of error with iteration

1. Error decreases as step increase
   Converging method (Best)

2. Error increases as step increase
   Diverging method (acceptable)

3. Error follow no order as step increases
   Fluctuating method (Not acceptable)

Introduction of Numerical Method

Use of calculator in Numerical Method

✓ Equation solve

a. Linear

Ex—$2x+4x+6=0$

$2x+6=0$

b. Non-linear

Ex—$x^2+2x+4=0$

$x^3+2x^2+7x+4=0$

c. Exponential

Ex--e^{3x} $-e^x$ $=0$

d. Trigonometric

Ex—$Sin(5x)+10=0$

$2x^2+4x-6=0$------$(x-1)(2x+6)$

$2x^2+3x+6=0$---------$x=(a+ib)(a-ib)$

$x^3+2x^2+7x+4=0$------------x====

e^{3x} $-e^x$ $=0$--------------x======

$\text{Sin}(5x)+10=0$------------x======