Course code: EEE-453
Course title: Numerical Method
Lecture on
Numerical Integration

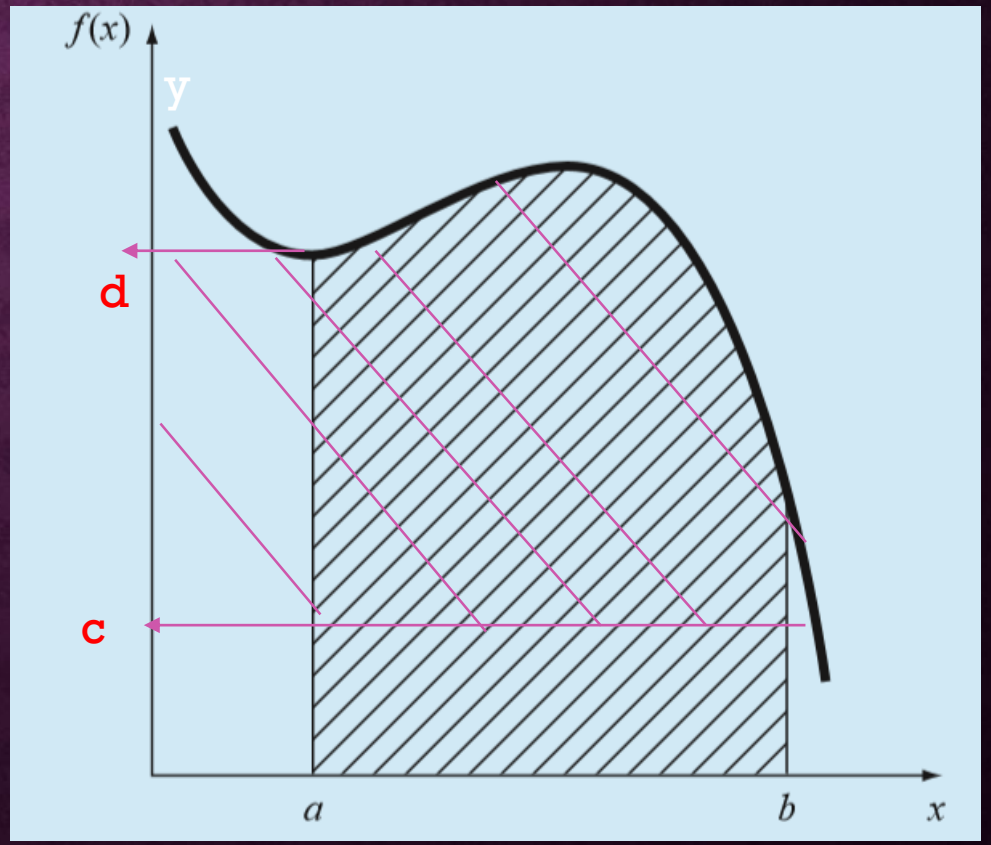
Numerical Integration

Numerical Integration

According to the dictionary definition, to integrate means "to bring together, into a whole; to unite; to indicate the total amount area. . . ." Mathematically, integration is represented by

$$I = \int_a^b f(x)dx \text{ or } I = \int_c^d f(y)dy$$

which stands for the integral of the function f(x) with respect to the independent variable x, evaluated between the limits x=a to x=b. The function f(x) is referred to as the integrand.



Numerical Integration

Different rule/scheme in Numerical Integration

1. Trapezoidal Rule
2. Simpson's 1/3 Rule
3. Simpson's 3/8 Rule etc.

Trapezoidal Rule

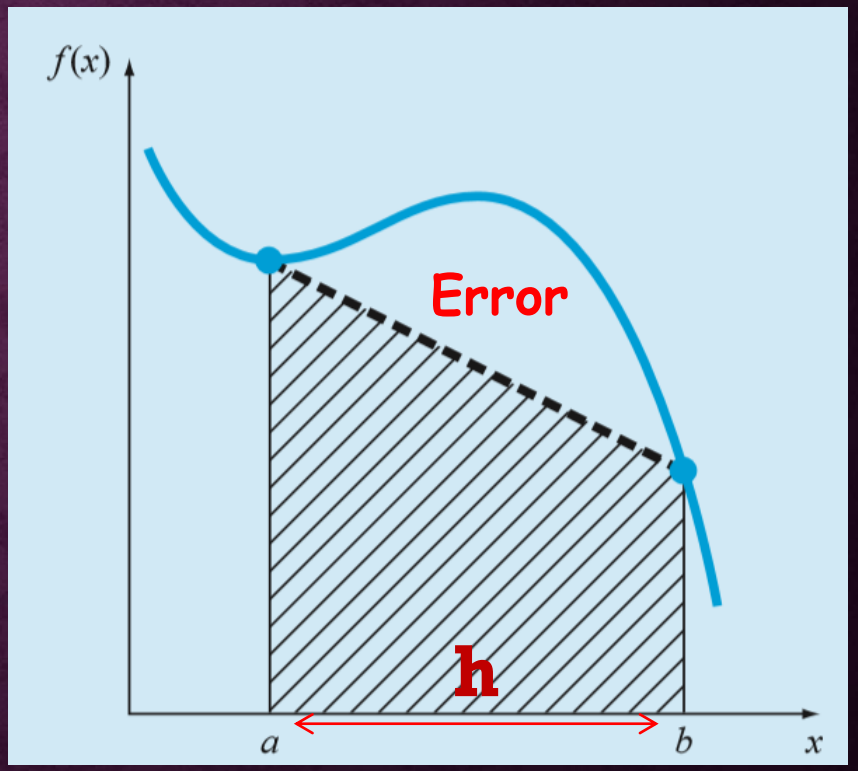
Feature of single segment Trapezoid rule

1. In this method complete area is considered as trapezium.
2. Then determine the area of trapezium to evaluate the integration

$$I = \int_a^b f(x)dx$$

3. Here n= trapezoid/segment number
   h= trapezoid/segment length
4. The formula for the process single Segment Trapezoid rule is

$$I = h/2[f(a)+f(a+h)]$$



Note

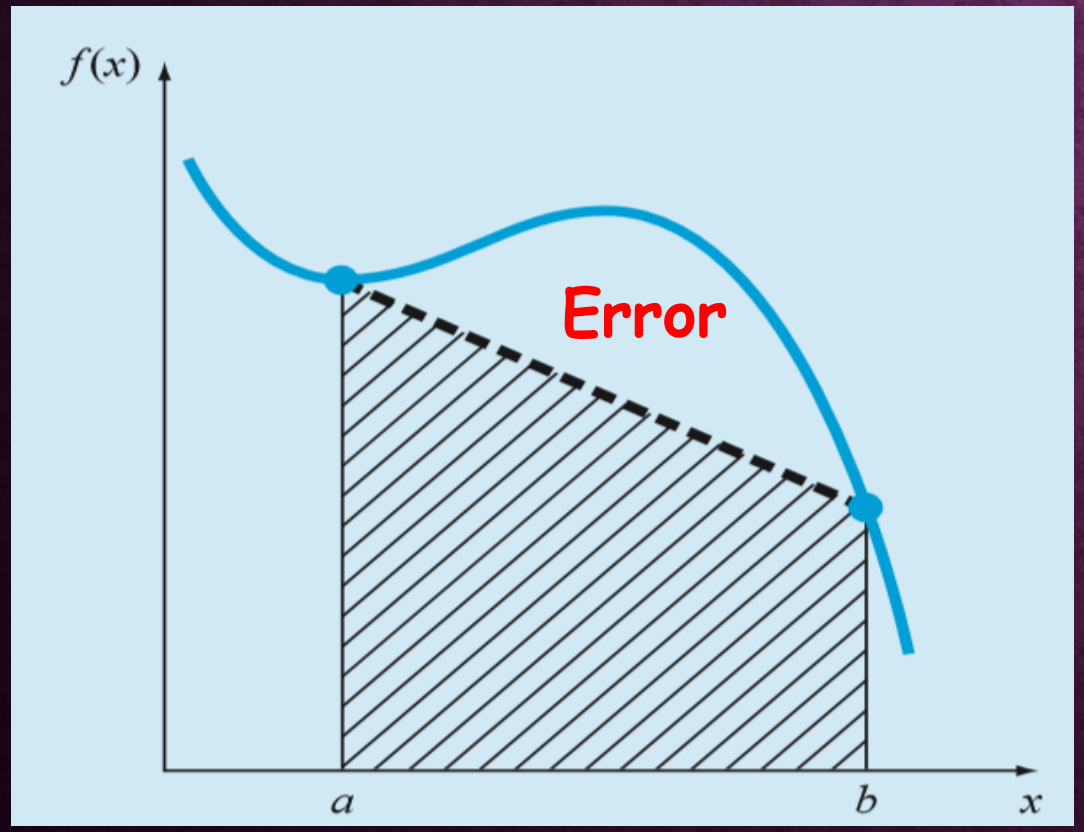
Area of trapezoid= $\int_a^b f(x)dx$

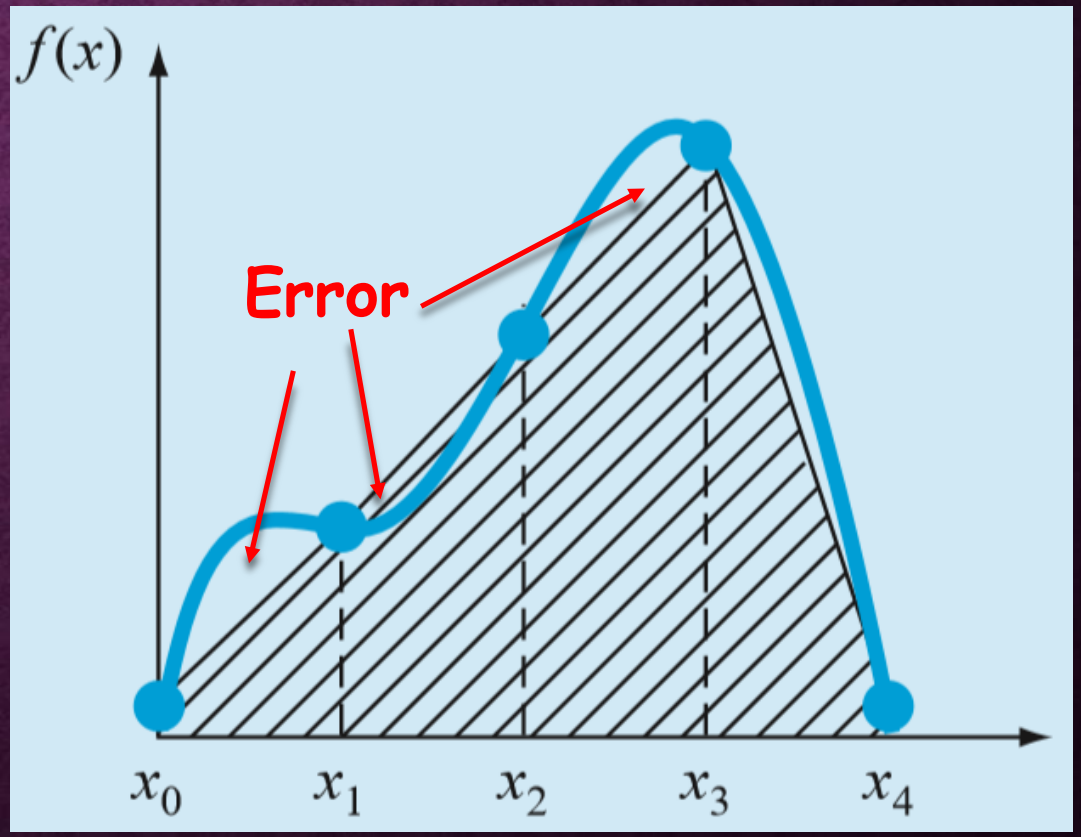
$$= 1/2*(b-a)(f(a)+f(b))$$

Trapezoidal Rule

Problems of single segment Trapezoid rule

There are more error in single segment Trapezoid rule than the multi segment Trapezoid rule





Trapezoidal Rule

Feature of multi segment Trapezoid rule

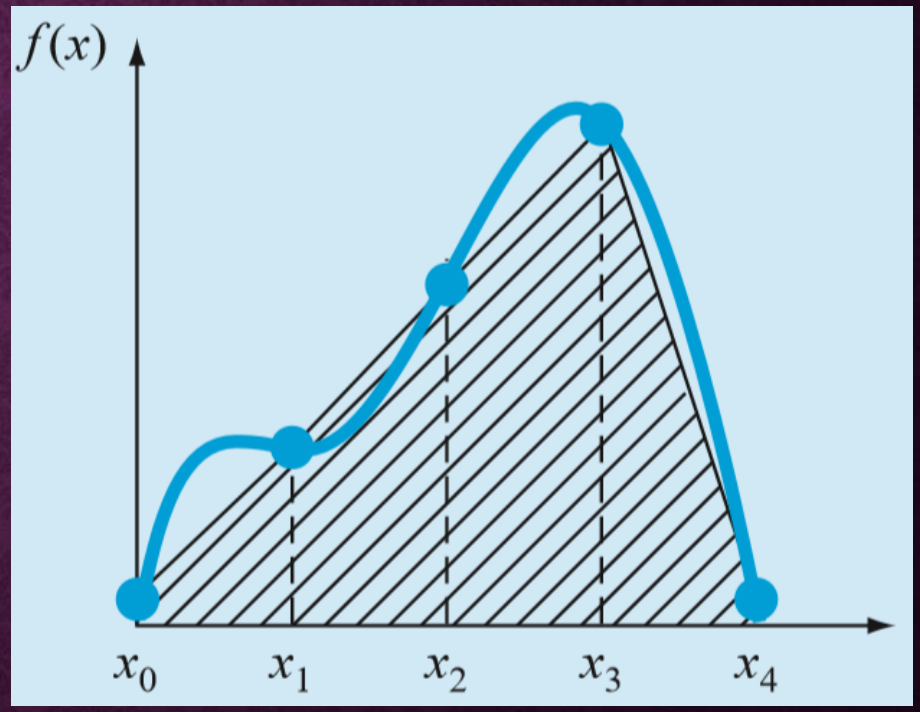
1. In this method complete area is considered as multiple trapeziums.
2. Then determine the area of trapezium to evaluate the integration

$$I = \int_a^b f(x)dx$$

3. Here n= trapezoid number=4(for figure)
4. The formula for the process single Segment Trapezoid rule is

$$I1 = h/2[f(a)+f(a+h)]$$
$$I2 = h/2[f(a+h)+f(a+2h)]$$
$$I3 = h/2[f(a+2h)+f(a+3h)]$$
$$I4 = h/2[f(a+3h)+f(a+4h)]$$

So $I = I1+I2+I3+I4$



Trapezoidal Rule

Problem 01

Evaluate $\int_{3\Pi/2}^{\Pi/4} \frac{sinx}{x} dx$ using Trapezoidal rule where the function is segmented in three part.

Solution

Here

$$f(x)= \frac{sinx}{x}$$

a= $3\Pi/4$=4.7124

b= $\Pi/4$ =0.7854

n= 3

h=(b-a)/n =-1.309

I=1*h/2[f(a)+f(a+h)]

Trapezoidal Rule

Notes

1. Calculator should be in radian mode
2. f(x)= sin(x)/x

	a	a+h	a+2h	a+3h
X	4.7124	3.4034	2.0944	0.7854
f(X)	-0.2122	-0.0760	0.4135	0.9003

I1=1*h/2[f(a)+f(a+h)] =1*h/2[-0.2122+-0.0760] = 0.1886

Similarly I2= -0.2209 and I3= -0.8599

So, I=I1+I2+I3= -0.8922

Assignment

Simpson's 1/3 Rule

Feature of Simpson's 1/3 rule

1. It use three points on the function and determine a quadratic Interpolation like $Y=f(x)=a_0+a_1x+a_2x^2$
2. Then determine the area to evaluate the integration

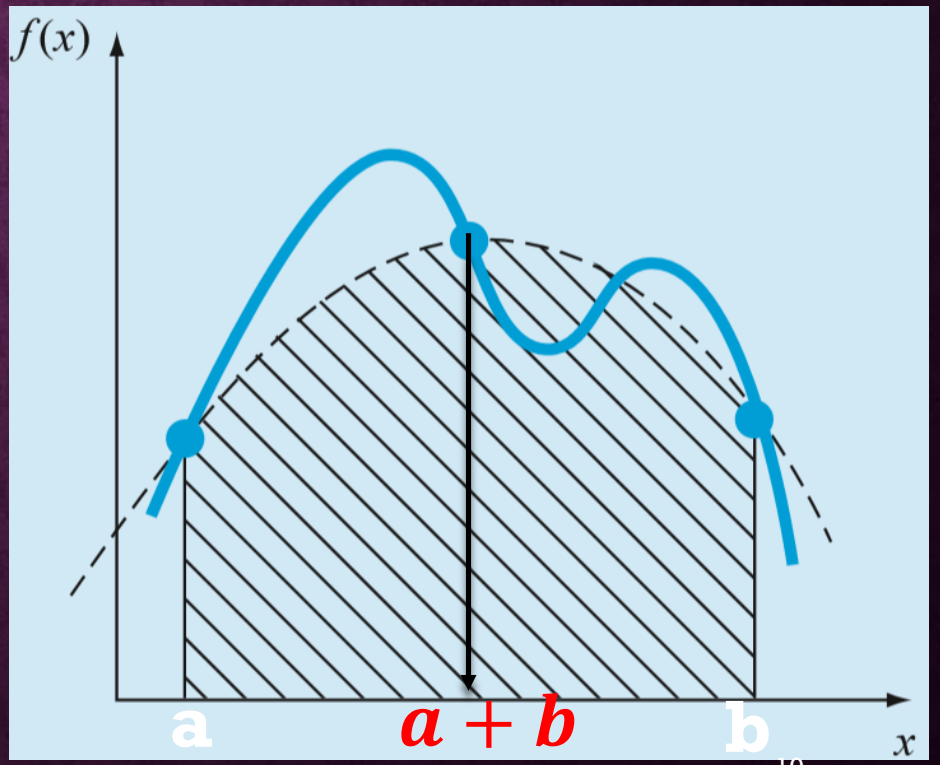
$$I=\int_a^b f(x)dx$$

3. It requires even no. of segment (n=2,4,6…) and minimum is 2.
4. The formula for the process Simpson's 1/3 rule is

$$I=h/3[f(x1)+4f(x2)+f(x3)]$$

5. For two segment x1=a, x3=b and

$$x2=\frac{a+b}{2}$$



Simpson's 1/3 Rule

Problem 02

Evaluate $\int_{3\Pi/2}^{\Pi/4} \frac{sinx}{x} dx$ using Simpson's 1/3 rule where the function is segmented in four part.

Solution

Here

$$f(x)= \frac{sinx}{x}$$

a= $3\Pi/2$=4.7124

b= $\Pi/4$ =0.7854

n= 4

h=(b-a)/n =-0.9818

I=h/3[f(x1)+4f(x2)+f(x3)]

Simpson's 1/3 Rule

Notes

1.Calculator should be in radian mode

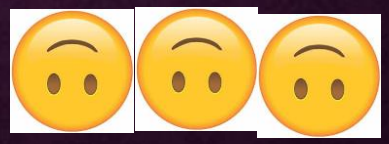
2.f(x)= sin(x)/x

	a	a+h	a+2h	a+3h	a+4h
X	4.7124	3.7306	2.7488	1.767	0.7852
f(X)	-0.2122	-0.1489	0.1392	0.5551	0.9003

I1=h/3[f(x1)+4f(x2)+f(x3)] and I2= h/3[f(x1)+4f(x2)+f(x3)]

=0.2188 = -1.0669

I=I1+I2= -0.8481(approx.)



Simpson's 1/3 Rule

Assignment

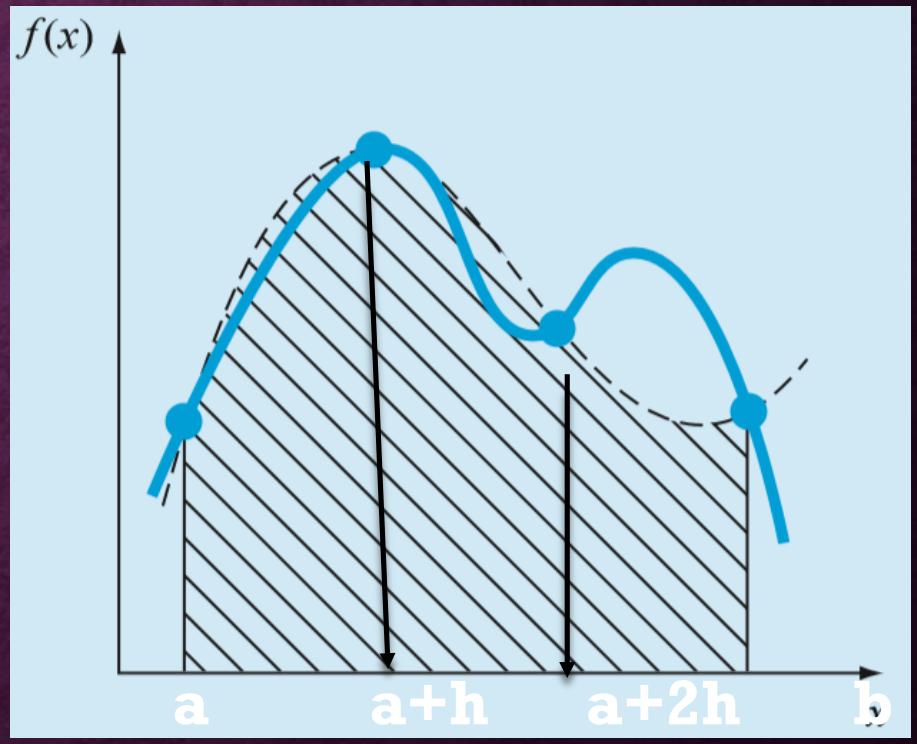
Simpson's 3/8 Rule

Feature of Simpson's 3/8 rule

1. It use multiple points on the function and determine a cubic Interpolation.
2. Then determine the area to evaluate the integration

$$I = \int_a^b f(x)dx$$

3. It requires odd no. of segment (n=3,6,9…) minimum segment=3.
4. The formula for the process Simpson's 3/8 rule is



$$I=3h/8[f(x1)+3f(x2)+4f(x3)+f(x4)]$$

Simpson's 3/8 Rule

Problem 03

Evaluate $\int_{3\Pi/2}^{\Pi/4} \frac{sinx}{x} dx$ using Simpson's 3/8 rule where the function is segmented in six part.

Solution

Here

$$f(x)= \frac{sinx}{x}$$

a= $3\Pi/2$=4.7124

b= $\Pi/4$ =0.7854

n= 6

h=(b-a)/n =-0.6545

I=3h/8[f(x1)+3f(x2)+4f(x3)+f(x4)]

Simpson's 3/8 Rule

Notes

1.Calculator should be in radian mode

2.f(x)= sin(x)/x

	a	a+h	a+2h	a+3h	a+4h	a+5h	a+6h
X	4.7124	4.0579	3.4034	2.7489	2.0944	1.4399	0.7854
f(X)	-0.2122	-0.1955	-0.0760	0.1392	0.4135	0.6886	0.9003

I1= 3h/8[f(x1)+3f(x2)+4f(x3)+f(x4)] = 0.2365

and I2= 3h/8[f(x1)+3f(x2)+4f(x3)+f(x4)] = -1.2356

I=I1+I2=-0.9991….(approx.)

Simpson's 3/8 Rule

Problem 04

Evaluate $\int_{x=0}^{1}\int_{y=0}^{\Pi/4} e^x \, siny \, dydx$ using Simpson's 3/8 rule with nx=3 and ny=6.

Solution

f(x)=$e^x \, siny$

nx=3, ax=0, bx=1 and

hx=(bx-ax)/nx = 0.3333

ny=6, ay=0, by= $\Pi/4$ and

hy=(by-ay)/ny = 0.1309

I= 3h/8[f(x1)+3f(x2)+4f(x3)+f(x4)]

So IX_1= 3hx/8[f(x1)+3f(x2)+4f(x3)+f(x4)]

Iy_1= 3hy/8[f(x1)+3f(x2)+4f(x3)+f(x4)]

Simpson's 3/8 Rule

Notes

1. Calculator should be in radian mode
2. $f(x)= e^x siny$ and the table stars from the end

ay+6h=0.7854	0.7071	0.9868	1.3777	1.9221	IX_7=1.3874
ay+5h=0.6545	0.6087	0.8496	1.1861	1.6548	IX_6=1.1945
ay+4h=0.5236	0.5000	0.6978	0.9742	1.3591	IX_5=0.9801
ay+3h=0.3927	0.3827	0.5341	0.7456	1.0402	IX_4=0.7508
ay+2h=0.2618	0.2588	0.3612	0.5042	0.7035	IX_3=0.5078
ay+h=0.1309	0.1305	0.1822	0.2543	0.3548	IX_2=0.2561
ay=0	0.0000	0.0000	0.0000	0.0000	IX_1=0.0000
	ax=0	ax+h=0.3333	ax+2h=0.667	ax+3h=1	

Iy2

Iy1

Simpson's 3/8 Rule

IX1= 3hx/8[0+3*0+4*0+0] = 0

IX2= 3hx/8[0.1305+3*0.2588+4*0.3827+0.5000] =0.2243

Similarly

IX3 =0.4447

IX4=0.6576

IX5=0.8592

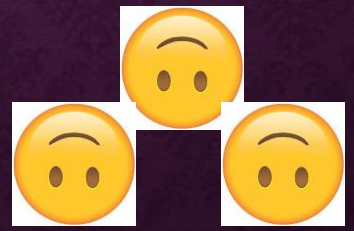
IX6=1.0461

IX7=1.2151

Iy_1= 3hy/8[0+3*0.3548+4f*7035+1.0402] = 0.1743

Similarly Iy_2=0.4838

I=Iy1+Iy2= 0.6581



Assignment