Homework 6

David Abeiku Saah

MATH212: Linear Algebra

Dr Ayawoa Dagbovie

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Problem 1

$$\lambda = 3$$

$$\text{Let } A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

To find the eigen vector,

$$(A - \lambda I)x = 0$$

$$\left(\begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 & 2 \\ 3 & -5 & 1 \\ 0 & 1 & -2 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Augmenting the matrix with $\mathbf{0}$,

$$\implies \begin{bmatrix} -2 & 2 & 2 & 0 \\ 3 & -5 & 1 & 0 \\ 0 & 1 & -2 & 0 \end{bmatrix}$$

$$R_1 \to -\frac{1}{2}R_1$$

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 3 & -5 & 1 & 0 \\ 0 & 1 & -2 & 0 \end{bmatrix}$$

$$R_2 \to -3R_1 + R_2$$

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & -2 & 4 & 0 \\ 0 & 1 & -2 & 0 \end{bmatrix}$$

$$R_2 \to -\frac{1}{2}R_2$$

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 1 & -2 & 0 \end{bmatrix}$$

$$R_3 \to -R_2 + R_3$$

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since a free variable exists, $(A - \lambda I)x = 0$ has non-trivial solutions. This confirms that $\lambda = 3$ is an eigen value of A.

$$R_1 \to R_1 + R_2$$

$$\begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

General solution is $\begin{cases} x_1 = 3x_3 \\ x_2 = 2x_3 \\ x_3 = x_3 \end{cases}$

$$\implies x = \begin{bmatrix} 3x_3 \\ 2x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$\therefore$ The corresponding eigen vector is $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$.

Problem 2

$$A = \begin{bmatrix} -6 & 28 & 21 \\ 4 & -15 & -12 \\ -8 & a & 25 \end{bmatrix}$$

a	Characteristic polynomial	Eigen values
32	$\lambda^3 - 4\lambda^2 + 5\lambda - 2$	2, 1, 1
31.9	$\lambda^3 - 4\lambda^2 + 3.8\lambda - 0.8$	0.2958, 1, 2.7042
31.8	$\lambda^3 - 4\lambda^2 + 2.6\lambda + 0.4$	-0.1279, 1, 3.1279
32.1	$\lambda^3 - 4\lambda^2 + 6.2\lambda - 3.2$	1.5 - 0.97i, 1.0 - 2.97$e^{-64}i$, 1.5 + 0.97i
32.2	$\lambda^3 - 4\lambda^2 + 7.4\lambda - 4.4$	1.5 - 1.47i, 1.0 + 4.18$e^{-64}i$, 1.5 + 1.47i

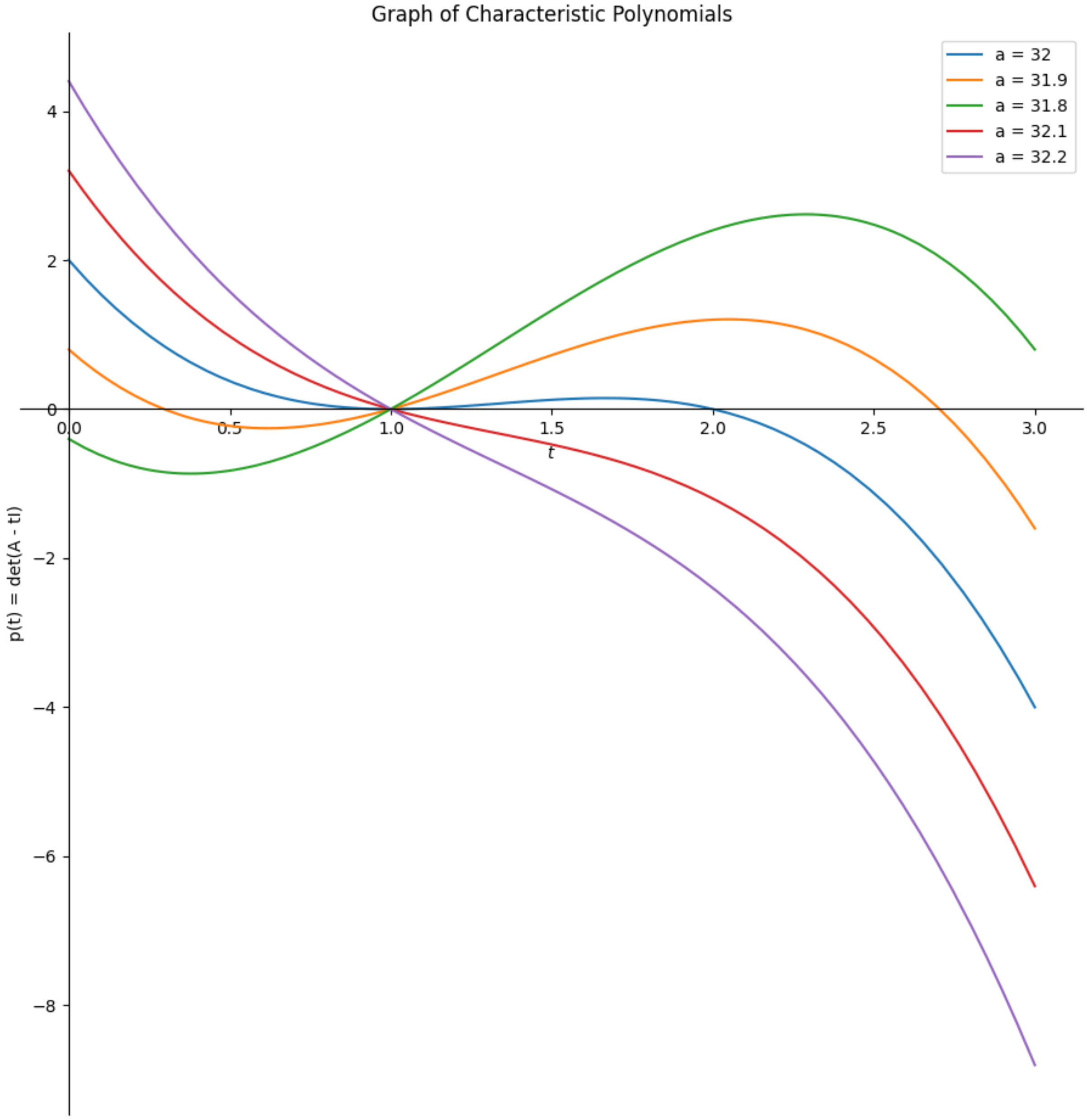

Figure 1: Graph of $p(t) = det(A - tI), 0 \leq t \leq 3$

The graph shows that as a increases, the eigen values of A move further away from each other. This is because the eigen values of A are the roots of the characteristic polynomial of A. As a increases, the roots of the characteristic polynomial move further away from each other.

Problem 3

$$\text{Let A} = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$$

Since A is a triangular matrix, the eigen values of A are the diagonal entries of A.

$\therefore$ The eigen values of A are 1 and -1.

Finding the eigenvalues of A using $(A - \lambda I)x = 0$.

when $\lambda = 1$

$$(A - \lambda I)x = 0$$

$$\left(\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix} - 1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 6 & -2 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Augmenting the matrix with $\mathbf{0}$,

$$\implies \begin{bmatrix} 0 & 0 & 0 \\ 6 & -2 & 0 \end{bmatrix}$$

Interchange R_1 and R_2

$$\begin{bmatrix} 6 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_1 \to \frac{1}{6}R_1$$

$$\begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{General solution is } \begin{cases} x_1 = \frac{1}{3}x_2 \\ x_2 = x_2 \end{cases}$$

$$\implies x = \begin{bmatrix} \frac{1}{3}x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} \frac{1}{3} \\ 1 \end{bmatrix}$$

$\therefore$ The corresponding eigen vector when $\lambda = 1$ is $\begin{bmatrix} \frac{1}{3} \\ 1 \end{bmatrix}$.

When $\lambda = -1$

$$\left(\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix} + 1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 6 & 0 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Augmenting the matrix with $\mathbf{0}$,

$$\implies \begin{bmatrix} 2 & 0 & 0 \\ 6 & 0 & 0 \end{bmatrix}$$

$$R_2 \to -3R_1 + R_2$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_1 \to \frac{1}{2}R_1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{General solution is } \begin{cases} x_1 = 0 \\ x_2 = x_2 \end{cases}$$

$$\implies x = \begin{bmatrix} 0 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

To diagonalise the matrix, we need D and P such that $A = PDP^{-1}$.

$$D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \ \& \ P = \begin{bmatrix} \frac{1}{3} & 0 \\ 1 & 1 \end{bmatrix}$$