

Homework 1

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MATH212: Linear Algebra

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Problem 1

Finding the general solution of the system whose augmented matrix is:

$$\begin{bmatrix} 1 & 0 & -5 & 0 & -8 & 3 \\ 0 & 1 & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Converting to a reduced row echelon form:

$$R_1 \rightarrow 8R_3 + R_1$$

$$= \begin{bmatrix} 1 & 0 & -5 & 0 & 0 & 3 \\ 0 & 1 & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

There are 5 variables because the augmented matrix has 6 columns.
Associated system of equations:

$$\begin{aligned} x_1 - 5x_3 &= 3 \\ x_2 + 4x_3 - x_4 &= 6 \\ x_5 &= 0 \\ 0 &= 0 \end{aligned}$$

$$\therefore \text{The general solution of the system is: } \begin{cases} x_1 = 3 + 5x_3 \\ x_2 = 6 - 4x_3 + x_4 \\ x_3 \text{ is free} \\ x_4 \text{ is free} \\ x_5 = 0 \\ x_6 \text{ is free} \end{cases}$$

Problem 2

System of equations:

$$\begin{aligned}x_1 - 3x_2 &= 1 \\ 2x_1 + hx_2 &= k\end{aligned}$$

Augmented Matrix:

$$\begin{bmatrix} 1 & -3 & 1 \\ 2 & h & k \end{bmatrix}$$

Finding the echelon form of the augmented matrix:

$$\begin{aligned}R_2 &\rightarrow -2R_1 + R_2 \\ &= \begin{bmatrix} 1 & -3 & 1 \\ 0 & h+6 & k-2 \end{bmatrix}\end{aligned}$$

(a)

The system has no solution when any of its equations results in an indeterminate form. This occurs when $h+6=0$ and $k-2 \neq 0$.

\therefore The system has no solution when $h = -6$ and $k \in \mathbb{R}, k \neq 2$.

(b)

The system has a unique solution when all the columns (except the rightmost) are pivot columns. This occurs when $h+6 \neq 0$ and $k-2$ can result to any real number.

\therefore The system has a unique solution when $h \neq -6$ and $k \in \mathbb{R}$.

(c)

The system has infinitely many solutions when there is at least one free variable. This occurs when $h+6=0$ and $k-2=0$.

\therefore The system has infinitely many solutions when $h = -6$ and $k = 2$.