Statistics I: Technology help — Python

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Description: Python code for an introductory applied statistics course. Work in progress.

Module 1: Data and decisions

To import a text file with data in Python, you first have find the path to the file. The data file students.csv used un the example is stored on my web site: https://luc-hens.github.io/students.csv Usually, you will have your data file stored on your computer. In that case you have to give the path to that file, which looks something like: /Users/luchens/Documents/Data/students.csv
This is for macOS; I think Windows uses backslashes: \ (check).

Note: it is good practive to not have spaces in a csv file. The pandas command read_csv has an option skipinitialspace=True ("Skip spaces after delimiter") but that does not solve all the problems caused by blank spaces in a csv file.

In [45]:
```
import pandas as pd
df = pd.read_csv('https://luc-hens.github.io/students.csv', sep=',', skipir
print(df)
display(df)
```

	case	sex	height	weight	major
0	1	Female	172	63	Business
1	2	Female	170	70	International Affairs
2	3	Female	170	52	Other
3	4	Female	171	52	Communications
4	5	Male	186	90	Business
5	6	Male	183	79	Business
6	7	Male	170	66	Communications
7	8	Female	169	56	Business
8	9	Male	175	75	International Affairs
9	10	Female	175	65	Communications
10	11	Male	195	94	Business
11	12	Female	176	51	International Affairs
12	13	Male	188	76	International Affairs
13	14	Male	192	82	Business
14	15	Male	172	70	International Affairs
15	16	Female	169	53	Business
16	17	Female	172	52	International Affairs
17	18	Male	178	85	Business
18	19	Female	177	59	Communications
19	20	Male	178	72	International Affairs
20	21	Female	160	54	Business
21	22	Female	175	54	International Affairs
22	23	Male	190	70	International Affairs
23	24	Male	178	85	Business

```
24     25  Female     163       55               Business
25     26  Female     161       59               Business
26     27  Female     162       44          Communications
27     28  Female     170       54               Business
28     29  Female     154       52               Business
29     30  Female     170       65               Business
```

	case	sex	height	weight	major
0	1	Female	172	63	Business
1	2	Female	170	70	International Affairs
2	3	Female	170	52	Other
3	4	Female	171	52	Communications
4	5	Male	186	90	Business
5	6	Male	183	79	Business
6	7	Male	170	66	Communications
7	8	Female	169	56	Business
8	9	Male	175	75	International Affairs
9	10	Female	175	65	Communications
10	11	Male	195	94	Business
11	12	Female	176	51	International Affairs
12	13	Male	188	76	International Affairs
13	14	Male	192	82	Business
14	15	Male	172	70	International Affairs
15	16	Female	169	53	Business
16	17	Female	172	52	International Affairs
17	18	Male	178	85	Business
18	19	Female	177	59	Communications
19	20	Male	178	72	International Affairs
20	21	Female	160	54	Business
21	22	Female	175	54	International Affairs
22	23	Male	190	70	International Affairs
23	24	Male	178	85	Business
24	25	Female	163	55	Business
25	26	Female	161	59	Business
26	27	Female	162	44	Communications
27	28	Female	170	54	Business
28	29	Female	154	52	Business
29	30	Female	170	65	Business

To display just the first couple of lines of the data frame called df:

In [46]:
```
df.head()
```

Out[46]:

	case	sex	height	weight	major
0	1	Female	172	63	Business
1	2	Female	170	70	International Affairs
2	3	Female	170	52	Other
3	4	Female	171	52	Communications
4	5	Male	186	90	Business

To display the last couple of lines of the data frame called df:

In [47]:
```
df.tail()
```

Out[47]:

	case	sex	height	weight	major
25	26	Female	161	59	Business
26	27	Female	162	44	Communications
27	28	Female	170	54	Business
28	29	Female	154	52	Business
29	30	Female	170	65	Business

To display the 10 first lines of the data frame:

In [48]:
```
df.head(10)
```

Out[48]:

	case	sex	height	weight	major
0	1	Female	172	63	Business
1	2	Female	170	70	International Affairs
2	3	Female	170	52	Other
3	4	Female	171	52	Communications
4	5	Male	186	90	Business
5	6	Male	183	79	Business
6	7	Male	170	66	Communications
7	8	Female	169	56	Business
8	9	Male	175	75	International Affairs
9	10	Female	175	65	Communications

Show the column names (variable names) (this is useful to check whether there are no blank spaces in the variable names):

In [49]:
```
list(df)
```

Out[49]:
```
['case', 'sex', 'height', 'weight', 'major']
```

Inspect the data types in the dataframe called df:

In [50]:
```
df.dtypes
```

Out[50]:
```
case       int64
sex       object
height     int64
weight     int64
major     object
dtype: object
```

To display just one of the variables:

In [51]:
```
print(df.height)
```

```
0     172
1     170
2     170
3     171
4     186
5     183
6     170
7     169
8     175
9     175
10    195
11    176
12    188
13    192
14    172
15    169
16    172
17    178
18    177
19    178
20    160
21    175
22    190
23    178
24    163
25    161
26    162
27    170
28    154
29    170
Name: height, dtype: int64
```

In [ ]:
```

```

Module 2: Displaying and describing categorical data

Bar chart and pie chart

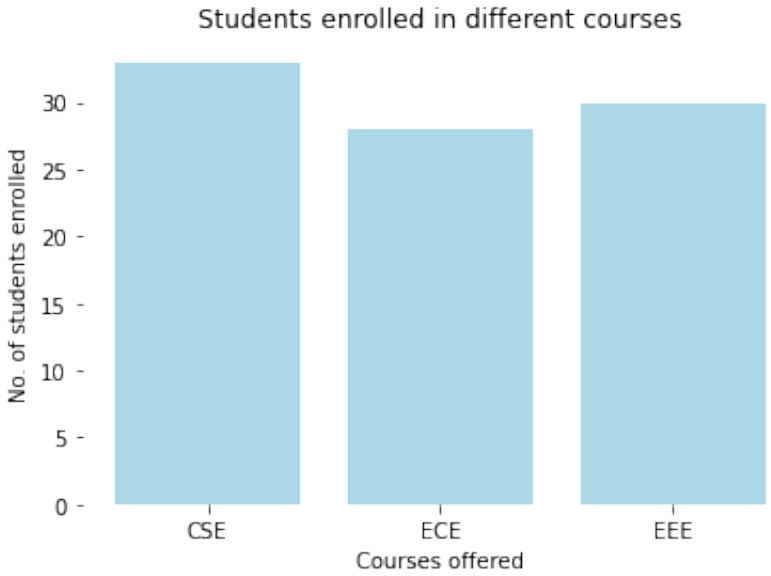
Generate a **bar chart** showing enrolment in three classes with course codes CSE (33 students), ECE (28 students), EEE (30 students):

(Documentation: https://matplotlib.org/stable/api/_as_gen/matplotlib.pyplot.bar.html)

(Example is taken from https://www.analyticsvidhya.com/blog/2021/08/understanding-bar-plots-in-python-beginners-guide-to-data-visualization/)

In [52]:

```
import numpy as np
import matplotlib.pyplot as plt
# Dataset generation
data_dict = {'CSE':33, 'ECE':28, 'EEE':30}
courses   = list(data_dict.keys())
values    = list(data_dict.values())
fig = plt.figure()    # add option `figsize = (10, 5)` to control size
#  Bar plot
plt.box(False)              # get rid of the box
plt.bar(courses, values, color ='lightblue')
plt.xlabel("Courses offered")
plt.ylabel("No. of students enrolled")
plt.title("Students enrolled in different courses")
plt.show()
```

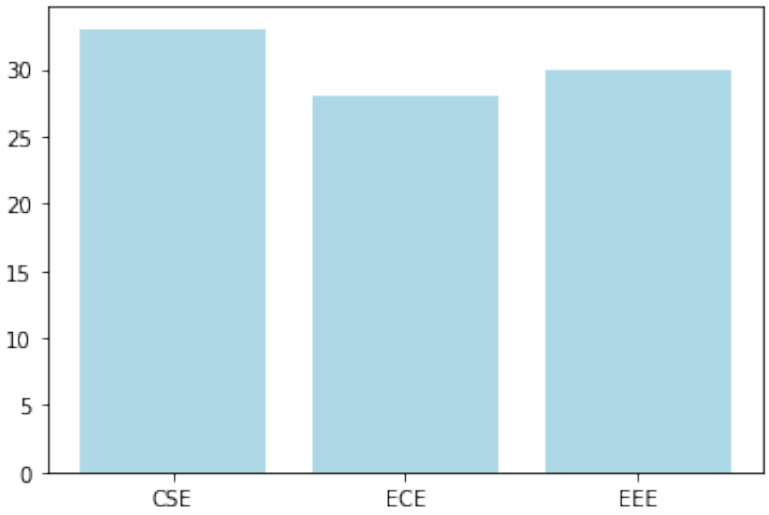


Save a plot to a file (.png or .pdf):

In [53]:

```
fig = plt.figure()
plt.bar(courses, values, color ='lightblue')
fig.savefig('saved_figure-1000dpi.png', dpi = 1000, transparent=True)
# the plot is saved to the current working directory (cwd)
# to find out what the current working directory (cwd) is:
import os
print('The file is saved to the current working directory: ',os.getcwd())
```

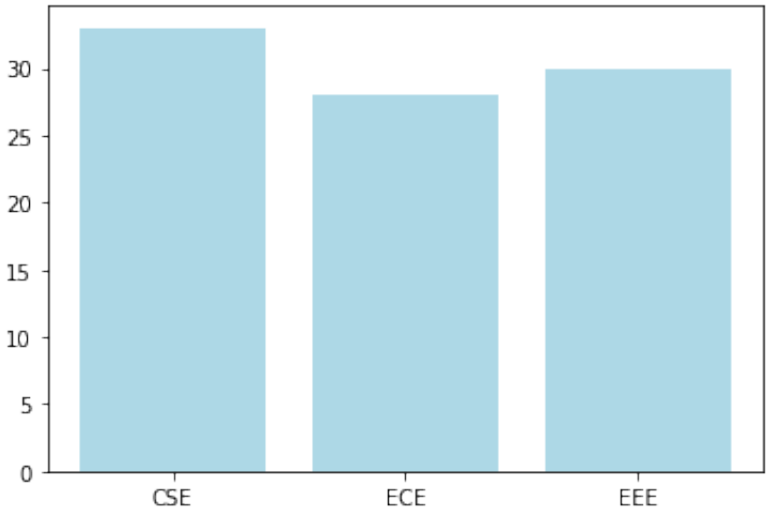
```
The file is saved to the current working directory:  /Users/luchens/Documen
ts/jupyter-notebooks
```



To save the plot as a .pdf:

In [54]:

```
fig = plt.figure()
plt.bar(courses, values, color ='lightblue')
fig.savefig('saved_figure-1000dpi.pdf', dpi = 1000, transparent=True)
```

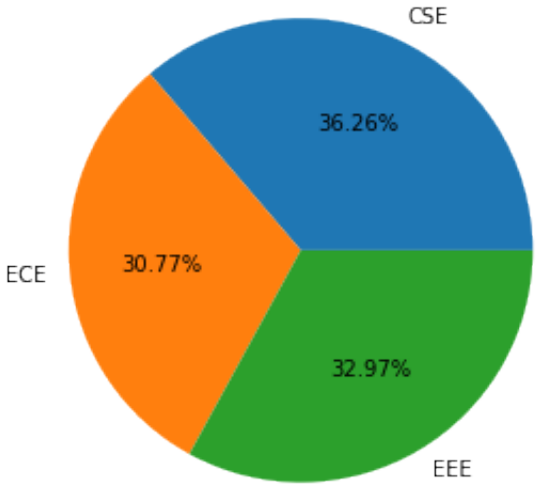


Generate a **pie chart** for the same data (pie charts are usually a poor way to display data):

(documentation: https://matplotlib.org/stable/api/_as_gen/matplotlib.pyplot.pie.html)

In [55]:

```
import numpy as np
import matplotlib.pyplot as plt
# Dataset generation
data_dict = {'CSE':33, 'ECE':28, 'EEE':30}
courses   = list(data_dict.keys())
values    = list(data_dict.values())
fig = plt.figure(figsize = (10, 5))
# Pie chart:
plt.pie(values, labels=courses,autopct='%1.2f%%')   # option: autopct to sh
plt.show()
```



Generate a bar chart and a pie chart from an imported data set (students.csv):

Use `value_counts()` to count the how many times each of the values of the variable 'sex' in the dataframe 'df' occurs:

In [56]:
```
a = df.sex.value_counts()
print(a)
```

```
Female    18
Male      12
Name: sex, dtype: int64
```

In [57]:
```
a.values
```

Out[57]:
```
array([18, 12])
```

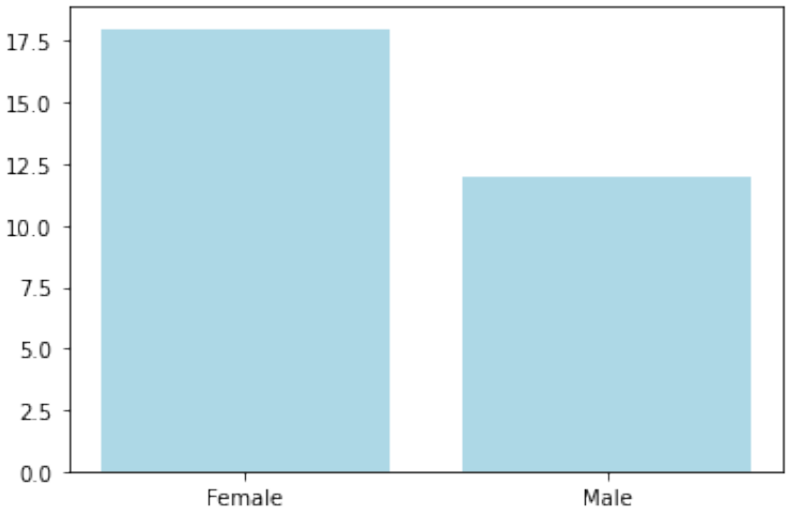
In [58]:
```
a.index
```

Out[58]:
```
Index(['Female', 'Male'], dtype='object')
```

Then use `a` as input for the `bar()` function:

In [59]:
```
plt.bar(a.index, a.values, color ='lightblue')
```

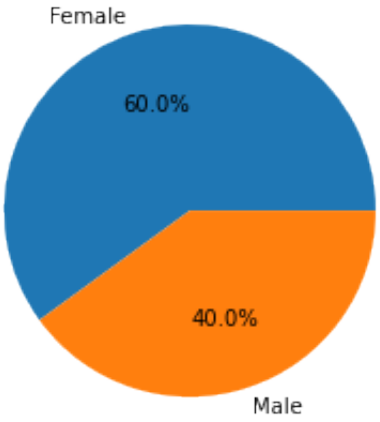
Out[59]:
```
<BarContainer object of 2 artists>
```



Or use `a` as input for the `pie()` function:

In [60]:
```
plt.pie(a.values,labels=a.index,autopct='%1.1f%%')
```

Out[60]:
```
([<matplotlib.patches.Wedge at 0x7fecf0d3db50>,
  <matplotlib.patches.Wedge at 0x7fecf0d4b310>],
 [Text(-0.3399187721714582, 1.046162142464278, 'Female'),
  Text(0.3399188701202255, -1.0461621106387813, 'Male')],
 [Text(-0.18541023936624992, 0.5706338958896061, '60.0%'),
  Text(0.18541029279285026, -0.5706338785302443, '40.0%')])
```



Contingency table

To create a contingency table use `pandas.crosstab(index, columns)` . For an example see: https://www.statology.org/contingency-table-python/ Here is how to create a contingency table for the categorical variables (sex, major) from the students data file stored in the datraframe called df:

In [61]:
```
import pandas as pd
pd.crosstab(index=df['major'], columns=df['sex'],margins=True)
```

Out[61]:

sex	Female	Male	All
major			
Business	9	6	15
Communications	4	1	5
International Affairs	4	5	9
Other	1	0	1
All	18	12	30

To express all frequencies as relative frequencies, divide by number of observations:

In [62]:

```
n = len(df)     # number of observations
pd.crosstab(index=df['major'], columns=df['sex'],margins=True)/n
```

Out[62]:

sex	Female	Male	All
major			
Business	0.300000	0.200000	0.500000
Communications	0.133333	0.033333	0.166667
International Affairs	0.133333	0.166667	0.300000
Other	0.033333	0.000000	0.033333
All	0.600000	0.400000	1.000000

... and multiply by 100 (percent) to get percentages:

In [63]:

```
100*pd.crosstab(index=df['major'], columns=df['sex'],margins=True)/n
```

Out[63]:

sex	Female	Male	All
major			
Business	30.000000	20.000000	50.000000
Communications	13.333333	3.333333	16.666667
International Affairs	13.333333	16.666667	30.000000
Other	3.333333	0.000000	3.333333
All	60.000000	40.000000	100.000000

Summary statistics (qualitative variables only):

In [64]:

```
df.describe(include=['object'])
```

Out[64]:

	sex	major
count	30	30
unique	2	4
top	Female	Business
freq	18	15

Summary statistics of one categorical variable (in this case: sex)

In [65]:
```
df['sex'].describe()
```

Out[65]:
```
count          30
unique          2
top        Female
freq           18
Name: sex, dtype: object
```

Module 3: Displaying and describing quantitative variables

Descriptive statistics (mean, median, standard deviation,...)

Summary statistics (all variables) using the `describe()` function from pandas:

In [66]:
```
df.describe(include='all')
```

Out[66]:

	case	sex	height	weight	major
count	30.000000	30	30.000000	30.000000	30
unique	NaN	2	NaN	NaN	4
top	NaN	Female	NaN	NaN	Business
freq	NaN	18	NaN	NaN	15
mean	15.500000	NaN	174.033333	65.133333	NaN
std	8.803408	NaN	9.625696	13.356474	NaN
min	1.000000	NaN	154.000000	44.000000	NaN
25%	8.250000	NaN	170.000000	54.000000	NaN
50%	15.500000	NaN	172.000000	64.000000	NaN
75%	22.750000	NaN	178.000000	74.250000	NaN
max	30.000000	NaN	195.000000	94.000000	NaN

Summary statistics of one variable (in this case: height) using the `describe()` function from pandas

In [67]:
```
df['height'].describe()
```

Out[67]:
```
count     30.000000
mean     174.033333
std        9.625696
min      154.000000
25%      170.000000
50%      172.000000
75%      178.000000
max      195.000000
Name: height, dtype: float64
```

Summary statistics (quantitative variables only) using the `describe()` function from pandas:

In [68]:
```
df.describe()
```

Out[68]:

	case	height	weight
count	30.000000	30.000000	30.000000
mean	15.500000	174.033333	65.133333
std	8.803408	9.625696	13.356474
min	1.000000	154.000000	44.000000
25%	8.250000	170.000000	54.000000
50%	15.500000	172.000000	64.000000
75%	22.750000	178.000000	74.250000
max	30.000000	195.000000	94.000000

Mean and **standard deviation** of one of the variables ('height') from the 'df' dataframe:

In [69]:
```
import pandas as pd
df.height.mean()                              # height.mean() does not work: Name
```

Out[69]:
```
174.03333333333333
```

In [70]:
```
df.height.std()
```

Out[70]:
```
9.625695974240289
```

Other descriptive statistics:

In [71]:
```
df.height.median()    # simularly: min() ; max() ; sum() ; count() ; quanti
```

Out[71]:
```
172.0
```

In [72]:
```
df.height.quantile(q=0.50)    # the 50th percentile is the same as the media
```

Out[72]:
```
172.0
```

In [73]:
```
df.height.quantile(q=0.25)  # the 25th percentile (the first quartile)
```

Out[73]:
```
170.0
```

In [74]:
```
df.height.quantile(q=0.75)  # the 75th percentile (the third quartile)
```

Out[74]:
```
178.0
```

In [75]:
```
df.height.quantile(q=0.75)-df.height.quantile(q=0.25) # the interquartile r
```

Out[75]:
```
8.0
```

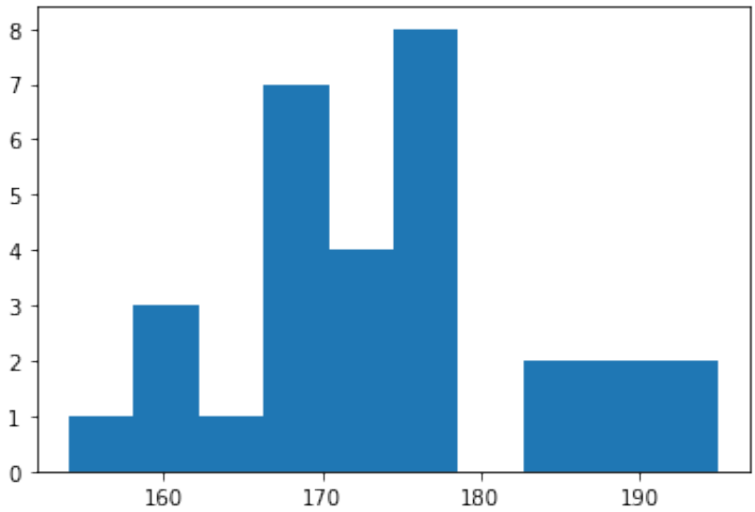
Histogram

To draw a **histogram** use `hist()` from the `maplotlib.pyplot` package.

Frequency histogram (vertical axis shows counts, absolute frequencies):

In [76]:
```
import matplotlib.pyplot as plt
plt.hist(df.height)    # the default is a frequency histogram: vertical axi
```

Out[76]:
```
(array([1., 3., 1., 7., 4., 8., 0., 2., 2., 2.]),
 array([154. , 158.1, 162.2, 166.3, 170.4, 174.5, 178.6, 182.7, 186.8,
        190.9, 195. ]),
 <BarContainer object of 10 artists>)
```

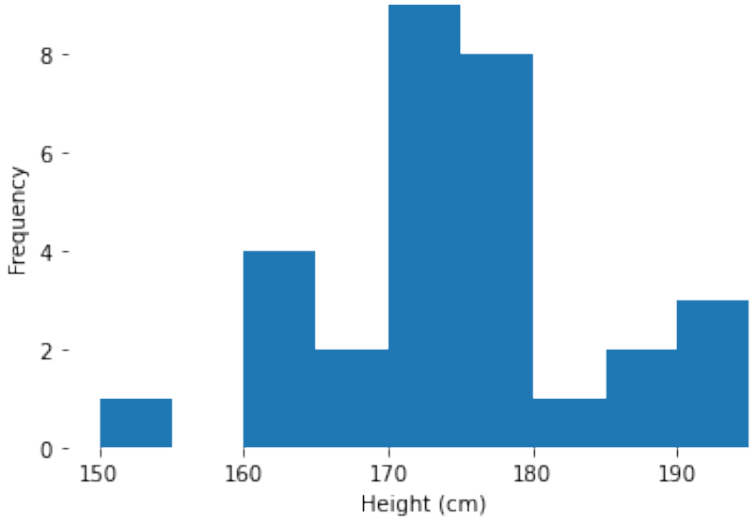


In the output, the first array gives gives the counts (absolute frequencies) for each of the classes (bins). The second array gives the edges of the bins (https://matplotlib.org/stable/api/_as_gen/matplotlib.pyplot.hist.html).

Get rid of the box, add labels to the axes, and let the bins start at 150, 155, 160,...:

In [77]:
```
import matplotlib.pyplot as plt
plt.box(False)               # get rid of the box
plt.xlabel('Height (cm)')    # add label on x-axis
plt.ylabel('Frequency')      # add label on x-axis
plt.hist(df.height,bins=[150,155,160,165,170,175,180,185,190,195])
```

Out[77]:
```
(array([1., 0., 4., 2., 9., 8., 1., 2., 3.]),
 array([150, 155, 160, 165, 170, 175, 180, 185, 190, 195]),
 <BarContainer object of 9 artists>)
```



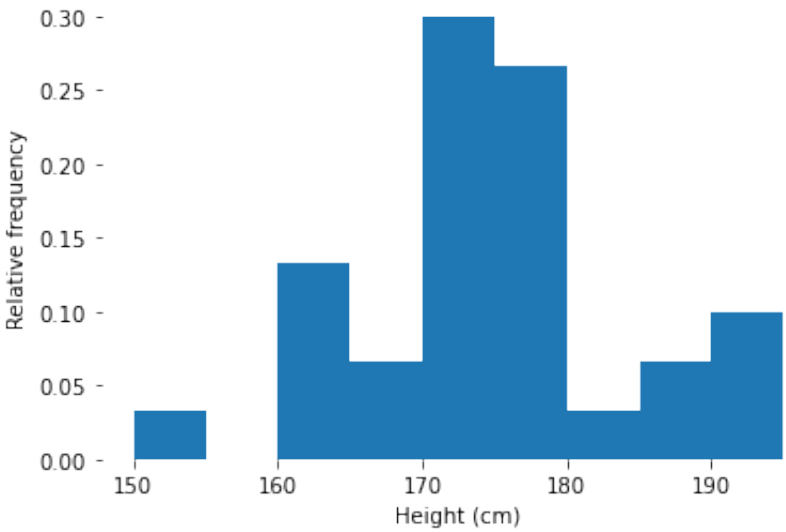
Relative frequency histogram (vertical axis shows relative frequencies):

In [78]:
```
df.height.size  # size gives the number of observations of the variable hei
```

Out[78]:
```
30
```

In [79]:
```
import matplotlib.pyplot as plt
import numpy as np
plt.box(False)              # get rid of the box
plt.xlabel('Height (cm)')   # add label on x-axis
plt.ylabel('Relative frequency')     # add label on x-axis
plt.hist(df.height, weights=np.zeros_like(df.height) + 1. / df.height.size,
         bins=[150,155,160,165,170,175,180,185,190,195])
```

Out[79]:
```
(array([0.03333333, 0.        , 0.13333333, 0.06666667, 0.3       ,
        0.26666667, 0.03333333, 0.06666667, 0.1       ]),
 array([150, 155, 160, 165, 170, 175, 180, 185, 190, 195]),
 <BarContainer object of 9 artists>)
```

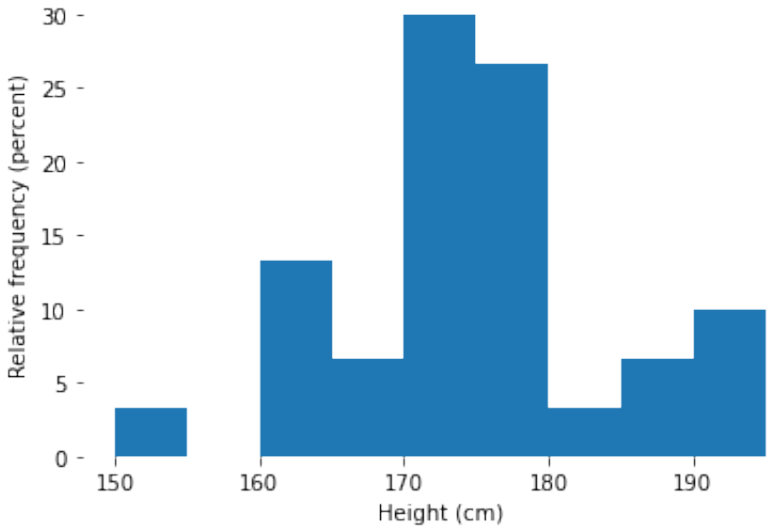


In the output, the first array gives gives the relative frequencies for each of the classes (bins). The second array gives the edges of the bins (https://matplotlib.org/stable/api/_as_gen/matplotlib.pyplot.hist.html).

To get a relative frequency histogram with relative frequencies expressed as percentages, multiply the weights by 100:

In [80]:
```
import matplotlib.pyplot as plt
import numpy as np
df.height.size  # size gives the number of observations of the variable hei
plt.box(False)              # get rid of the box
plt.xlabel('Height (cm)')   # add label on x-axis
plt.ylabel('Relative frequency (percent)')     # add label on x-axis
plt.hist(df.height, weights=100*(np.zeros_like(df.height) + 1. / df.height.
         bins=[150,155,160,165,170,175,180,185,190,195])
```

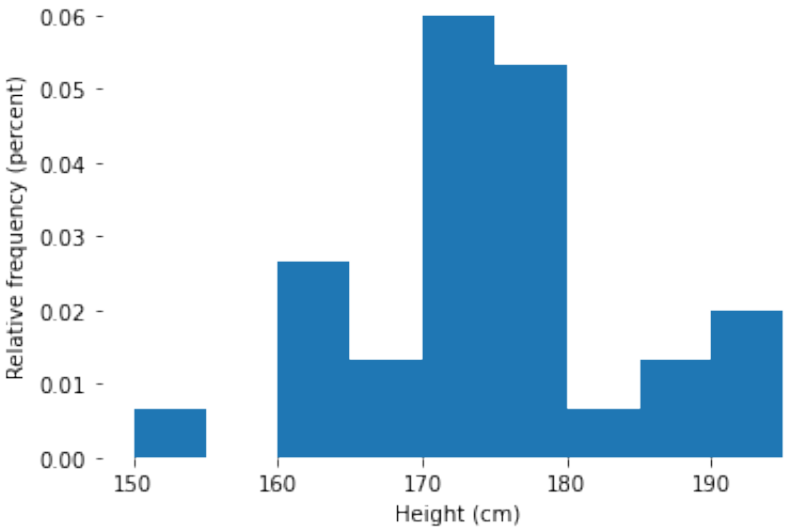
Out[80]:
```
(array([ 3.33333333,  0.        , 13.33333333,  6.66666667, 30.        ,
        26.66666667,  3.33333333,  6.66666667, 10.        ]),
 array([150, 155, 160, 165, 170, 175, 180, 185, 190, 195]),
 <BarContainer object of 9 artists>)
```



Density histogram (vertical axis shows densities):

In [81]:
```
import matplotlib.pyplot as plt
df.height.size  # size gives the number of observations of the variable hei
plt.box(False)              # get rid of the box
plt.xlabel('Height (cm)')   # add label on x-axis
plt.ylabel('Relative frequency (percent)')     # add label on x-axis
plt.hist(df.height, density=True, bins=[150,155,160,165,170,175,180,185,190
```

Out[81]:
```
(array([0.00666667, 0.        , 0.02666667, 0.01333333, 0.06      ,
        0.05333333, 0.00666667, 0.01333333, 0.02      ]),
 array([150, 155, 160, 165, 170, 175, 180, 185, 190, 195]),
 <BarContainer object of 9 artists>)
```



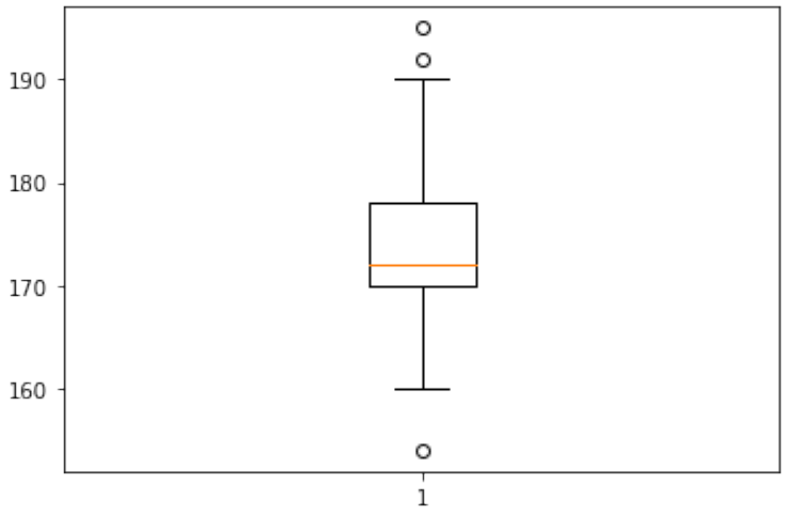
In the output, the first array gives gives the densities for each of the classes (bins). The second array gives the edges of the bins (https://matplotlib.org/stable/api/_as_gen/matplotlib.pyplot.hist.html).

Box plot

Use `boxplot()` from `matplotlib` :

In [82]:
```
import matplotlib.pyplot as plt
plt.boxplot(df.height)
```

Out[82]:
```
{'whiskers': [<matplotlib.lines.Line2D at 0x7fecfaba22b0>,
  <matplotlib.lines.Line2D at 0x7fecfaba2610>],
 'caps': [<matplotlib.lines.Line2D at 0x7fecfaba29a0>,
  <matplotlib.lines.Line2D at 0x7fecfaba2d30>],
 'boxes': [<matplotlib.lines.Line2D at 0x7fecfab94ee0>],
 'medians': [<matplotlib.lines.Line2D at 0x7fecfabae100>],
 'fliers': [<matplotlib.lines.Line2D at 0x7fecfabae490>],
 'means': []}
```



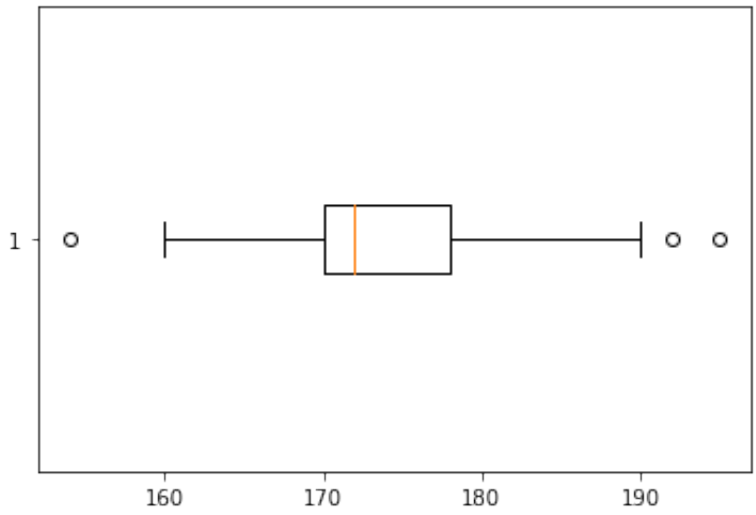
Rotate the boxplot to get a horizontal orientation:

In [83]:

```
import matplotlib.pyplot as plt
plt.boxplot(df.height,vert=False)
```

Out[83]:

```
{'whiskers': [<matplotlib.lines.Line2D at 0x7fecfabc51f0>,
  <matplotlib.lines.Line2D at 0x7fecfabc55b0>],
 'caps': [<matplotlib.lines.Line2D at 0x7fecfabc5940>,
  <matplotlib.lines.Line2D at 0x7fecfabc5cd0>],
 'boxes': [<matplotlib.lines.Line2D at 0x7fecf0a97e20>],
 'medians': [<matplotlib.lines.Line2D at 0x7fecfabd20a0>],
 'fliers': [<matplotlib.lines.Line2D at 0x7fecfabd2430>],
 'means': []}
```

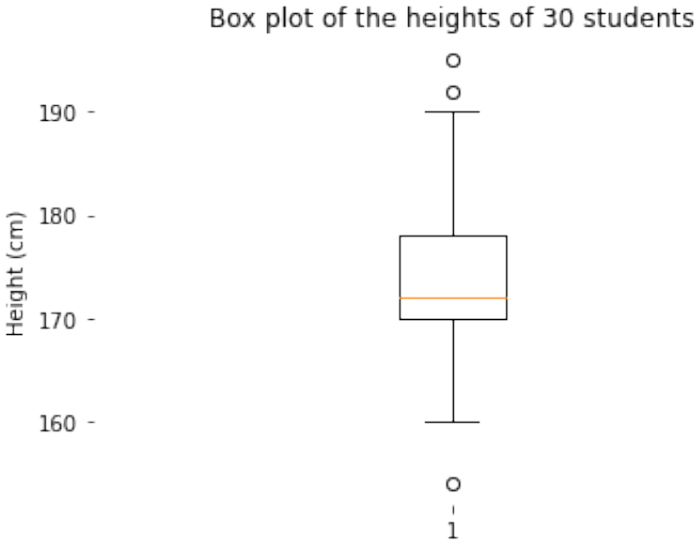


Make the box plot prettier (get rid of the box, label the axes, add title) (still to fix: get rid of the "1" label):

In [84]:

```
import matplotlib.pyplot as plt
plt.box(False)              # get rid of the box
plt.title("Box plot of the heights of 30 students") # add title
plt.xlabel("")  # I want to get rid of the 1 on the x-axis
plt.ylabel("Height (cm)")  # add label to y-axis
plt.boxplot(df.height)
```

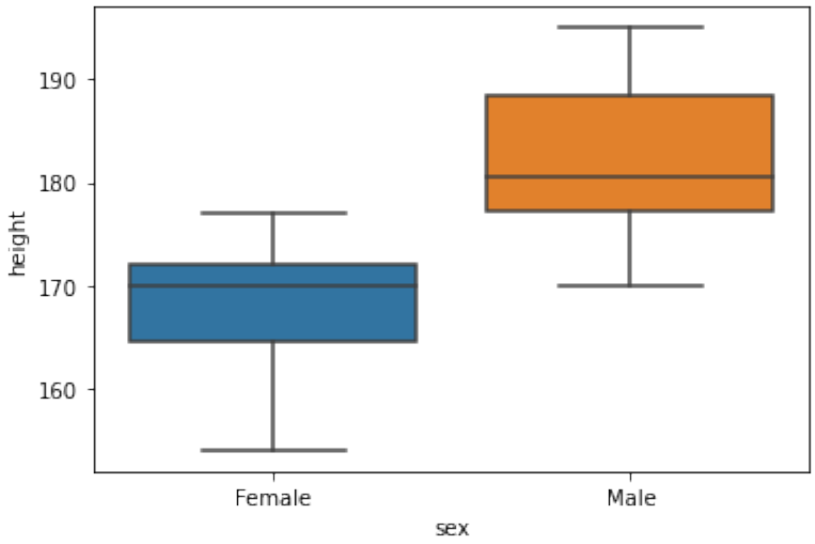
Out[84]:
```
{'whiskers': [<matplotlib.lines.Line2D at 0x7fecd8306520>,
  <matplotlib.lines.Line2D at 0x7fecd83068b0>],
 'caps': [<matplotlib.lines.Line2D at 0x7fecd8306c40>,
  <matplotlib.lines.Line2D at 0x7fecd8306fd0>],
 'boxes': [<matplotlib.lines.Line2D at 0x7fecd83061c0>],
 'medians': [<matplotlib.lines.Line2D at 0x7fecd83133a0>],
 'fliers': [<matplotlib.lines.Line2D at 0x7fecd8313730>],
 'means': []}
```



To compare the heights of men and women, use **side-by-side boxplots** (from the `seaborn` package):

In [85]:
```
import seaborn as sns
sns.boxplot(data=df,x="sex",y='height')
```

Out[85]:
```
<AxesSubplot:xlabel='sex', ylabel='height'>
```



In [86]:
```
df.sex.count()
```

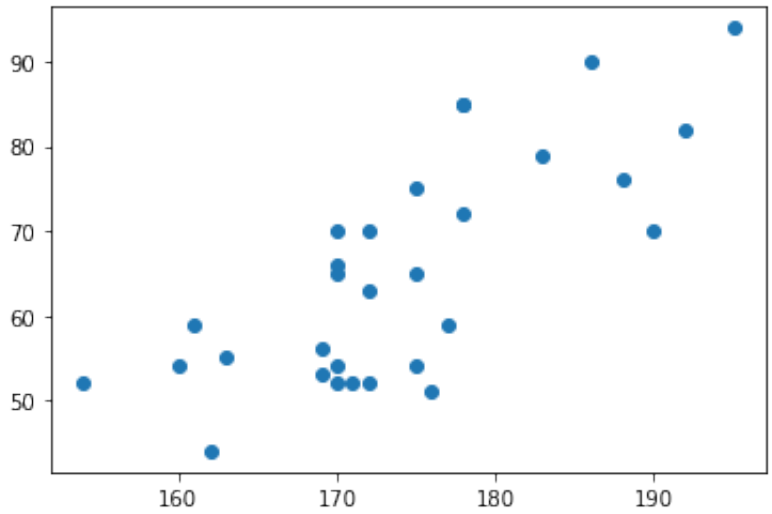
Out[86]:
```
30
```

Module 4: Correlation and Regression

Make a **scatter plot** of heights (horizontal axis) and weights (vertical axis) using `scatter()` from `matplotlib.pyplot`:

In [87]:
```
import numpy as np
import matplotlib.pyplot as plt
plt.scatter(df.height,df.weight)
```

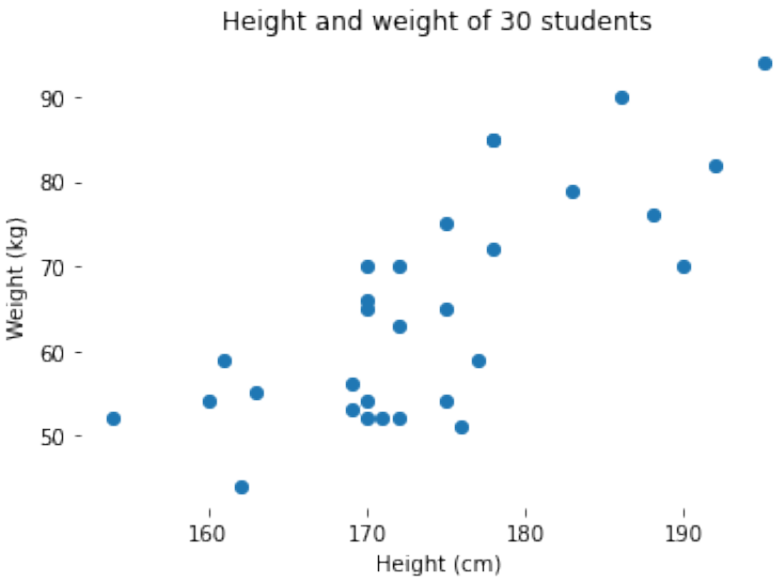
Out[87]:
```
<matplotlib.collections.PathCollection at 0x7fecd8426550>
```



Make the scatter plot prettier (get rid of the box, add labels to the axes):

In [88]:
```
plt.box(False)              # get rid of the box
plt.title("Height and weight of 30 students") # add title
plt.xlabel("Height (cm)")  # add label to x-axis
plt.ylabel("Weight (kg)")  # add label to y-axis
plt.scatter(df.height,df.weight)
```

Out[88]:
```
<matplotlib.collections.PathCollection at 0x7fece8a20b20>
```



Correlation coefficient:

In [89]:
```
# correlation between height and weight:
df.height.corr(df.weight)
```

Out[89]:
```
0.7525668130284301
```

In [90]:
```
# correlation matrix between all quantitative variables of a data frame:
df.corr()
```

Out[90]:

	case	height	weight
case	1.000000	-0.320457	-0.254260
height	-0.320457	1.000000	0.752567
weight	-0.254260	0.752567	1.000000

To get *one* of the correlation coefficients, first convert to matrix:

In [91]:
```
import numpy as np
corr_matrix = np.array(df.corr())
print(corr_matrix)
```

```
[[ 1.          -0.32045686 -0.25426022]
 [-0.32045686  1.           0.75256681]
 [-0.25426022  0.75256681  1.        ]]
```

In [92]:
```
# extract the correlation between height and weight (caution: rows and colu
corr_matrix[1][2]
```

Out[92]:
```
0.7525668130284303
```

To round numbers, use `round()` from the numpy library:

In [93]:
```
import numpy as np
np.round(_,2)   # in Python, _ is the output of the last cell; the second a
```

Out[93]:
```
0.75
```

Find the **line of best fit** (using the ordinary least squares method) using `statsmodels`:

In [94]:
```
import statsmodels.api as sms
import statsmodels.formula.api as smf
# Fit regression model:
results = smf.ols('df.weight ~ df.height', data=df).fit()
# Inspect the results:
print(results.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:              df.weight   R-squared:                       0.566
Model:                            OLS   Adj. R-squared:                  0.551
Method:                 Least Squares   F-statistic:                     36.57
Date:                Wed, 12 Jan 2022   Prob (F-statistic):           1.61e-06
Time:                        15:25:33   Log-Likelihood:                -107.29
No. Observations:                  30   AIC:                             218.6
Df Residuals:                      28   BIC:                             221.4
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept   -116.6011     30.097     -3.874      0.001    -178.252     -54.951
df.height      1.0443      0.173      6.047      0.000       0.691       1.398
==============================================================================
Omnibus:                        2.804   Durban-Watson:                   2.486
Prob(Omnibus):                  0.246   Jarque-Bera (JB):                1.333
Skew:                           0.007   Prob(JB):                        0.514
Kurtosis:                       1.968   Cond. No.                     3.21e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 3.21e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

To get just the coefficients:

In [95]:
```
results.params
```

Out[95]:
```
Intercept   -116.601087
df.height      1.044251
dtype: float64
```

The intercept is the first coefficient:

In [96]:
```
results.params[0]
```

Out[96]:
```
-116.6010867272883
```

The slope coefficient is the second coefficient:

In [97]:
```
results.params[1]
```

Out[97]:
```
1.044250641987867
```

To get the *t*-values of the coefficients (Statistics II):

In [98]:
```
results.tvalues
```

Out[98]:
```
Intercept   -3.874200
df.height    6.047248
dtype: float64
```

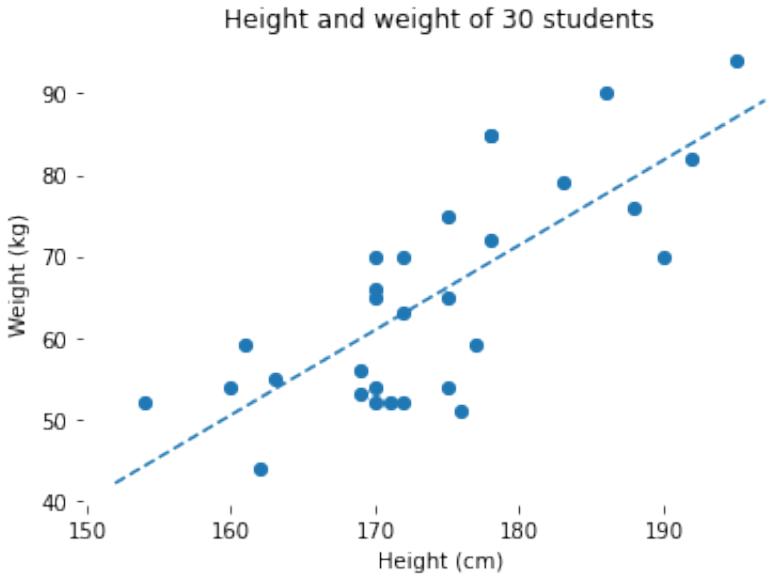
To get the *p*-values of the coefficients (Statistics II):

In [99]:
```
results.pvalues
```

Out[99]:
```
Intercept    0.000588
df.height    0.000002
dtype: float64
```

Add line of best fit to the scatter plot (see: https://stackoverflow.com/questions/7941226/how-to-add-line-based-on-slope-and-intercept-in-matplotlib):

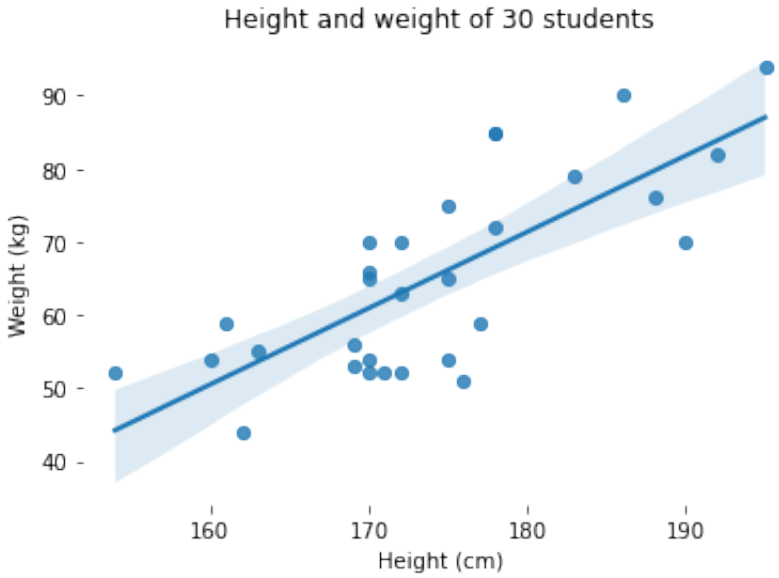
In [100…
```
import matplotlib.pyplot as plt
import numpy as np
import statsmodels.api as sms
import statsmodels.formula.api as smf

# Fit regression model:
results = smf.ols('df.weight ~ df.height', data=df).fit()

def abline(slope, intercept):
    """Plot a line from slope and intercept"""
    axes = plt.gca()
    x_vals = np.array(axes.get_xlim())
    y_vals = intercept + slope * x_vals
    plt.plot(x_vals, y_vals, '--')

plt.box(False)              # get rid of the box
plt.title("Height and weight of 30 students") # add title
plt.xlabel("Height (cm)")  # add label to x-axis
plt.ylabel("Weight (kg)")  # add label to y-axis
plt.scatter(df.height,df.weight)
abline(results.params[1],results.params[0])     # add line y=a*x+b  (a = s
```



The `seaborn` package has more sophisticated ways to visualize data. Add a **line of best fit** to the scatter plot using the `seaborn` package:

In [101…

```
import seaborn as sns
sns.regplot(x=df.height,y=df.weight, data=df) # regplot: regression plot
plt.box(False)              # get rid of the box
plt.title("Height and weight of 30 students") # add title
plt.xlabel("Height (cm)")  # add label to x-axis
plt.ylabel("Weight (kg)")  # add label to y-axis
plt.show()
```



Module 5: Randomness and Probability

(No Python code.)

Module 6: Random variables and probability models

Calculate **binomial probability** (k = number of successes, n = number of trials, p = probability of success) (`pmf` stands for probability mass function—like pdf but for a discrete random variable):

In [102…
```
# calculate binomial probability (k= number of successes, n = number of tri
from scipy.stats import binom
binom.pmf(k=10, n=12, p=0.6)   # pmf: probability mass function (like pdf f
```

Out[102…
```
0.063852281856
```

Calculate cumulative binomial probability:

In [103…
```
from scipy.stats import binom
binom.cdf(k=10, n=12, p=0.6)
```

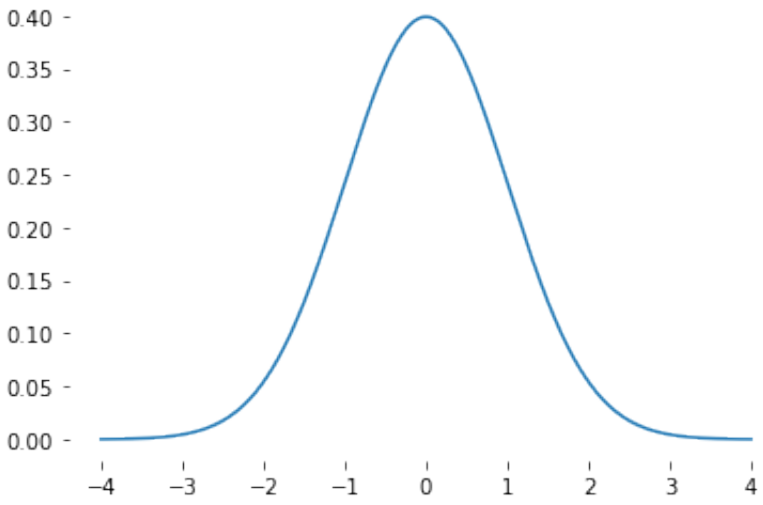
Out[103…
```
0.980408958976
```

Module 7: The Normal distribution

Plot the **probability density function** (pdf) of the normal curve (https://www.statology.org/plot-normal-distribution-python/):

In [104…
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm
# x-axis ranges from -4 and 4 with .001 steps:
x = np.arange(-4, 4, 0.001)

# plot normal distribution with mean 0 and standard deviation 1
plt.box(False) # no box around plot
plt.plot(x, norm.pdf(x, 0, 1))
```

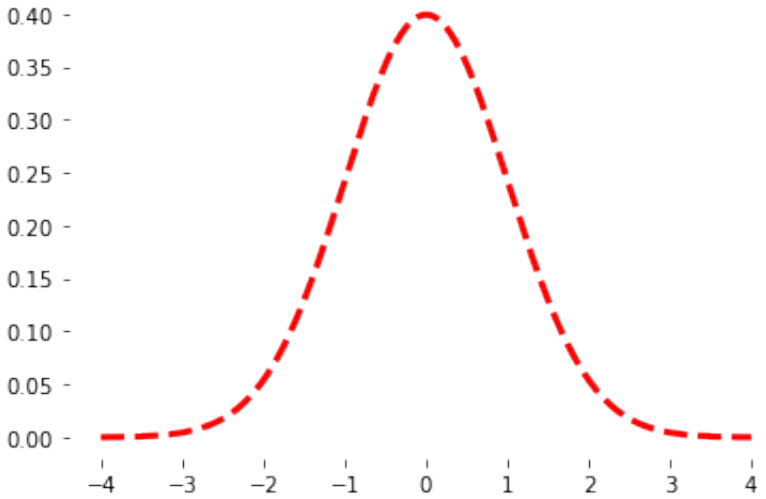
Out[104…
```
[<matplotlib.lines.Line2D at 0x7fecd88ca730>]
```



To change color, linewidth, linestyle:

In [105…
```
plt.box(False) # no box around plot
plt.plot(x, norm.pdf(x, 0, 1), color='red', linewidth=3, linestyle='dashed'
```

Out[105… `[<matplotlib.lines.Line2D at 0x7fecfaf452b0>]`



Area under normal curve:

To find an area under the normal curve, use the **cumulative density function** (cdf) of the normal distribution. (documentation: see: https://docs.scipy.org/doc/scipy/reference/stats.html)

In [106…

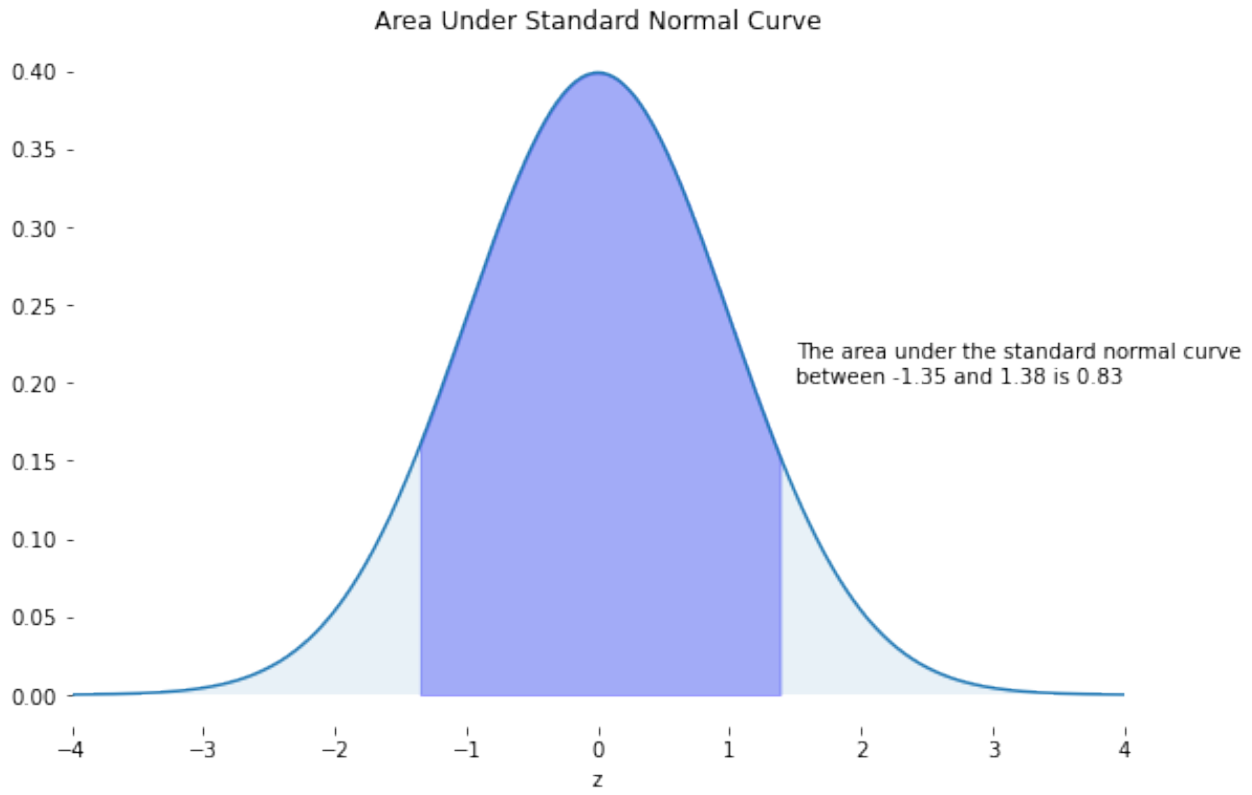
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm
mu    = 0     # mean                (for the standard normal distribution, t
sigma = 1     # standard deviation (for the standard normal distribution, t
x1 = -1.96    # lower boundary
x2 = 1.96   # lower boundary
# area under normal curve between x1 and x2:
area = norm.cdf(x2, loc=mu, scale=sigma)-norm.cdf(x1, loc=mu, scale=sigma)
print('The area under the normal curve is', area)
```

```
The area under the normal curve is 0.950004209703559
```

To **plot** the area under the normal curve (see: https://pythonforundergradengineers.com/plotting-normal-curve-with-python.html)

In [107…

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm
# define constants
mu = 998.8 # mean
sigma = 73.10  # standard deviation
x1 = 900    # lower boundary
x2 = 1100   # lower boundary
# calculate the standardized values:
z1 = ( x1 - mu )/ sigma
z2 = ( x2 - mu) / sigma
x = np.arange(z1, z2, 0.001) # range of x in spec
x_all = np.arange(-10, 10, 0.001) # entire range of x, both in and out of s
# for standard normal distribution, mean = 0, stddev = 1:
y = norm.pdf(x,0,1)
y2 = norm.pdf(x_all,0,1)
# find area under normal curve between x1 and x2:
area = norm.cdf(x2, loc=mu, scale=sigma)-norm.cdf(x1, loc=mu, scale=sigma)
###
# build the plot
fig, ax = plt.subplots(figsize=(9,6))
ax.plot(x_all,y2)
ax.fill_between(x,y,0, alpha=0.3, color='b')
ax.fill_between(x_all,y2,0, alpha=0.1)
ax.set_xlim([-4,4])
ax.set_xlabel('z')
# ax.set_yticklabels([])
ax.set_title('Area Under Standard Normal Curve')
plt.box(False)
plt.text(1.5,0.2, f"The area under the standard normal curve \nbetween {np.
plt.savefig('normal_curve.png', dpi=72, bbox_inches='tight')
plt.show()

print(f"The area under the standard normal curve between {np.round(z1,2)} a
```




```
The area under the standard normal curve between -1.35 and 1.38 is 0.83.
```

Other continuous distributions

Uniform distribution (https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.uniform.html):

In [108…

```
from scipy.stats import uniform
# In the standard form, the distribution is uniform on [0, 1].
# Using the parameters loc and scale, one obtains the uniform distribution
uniform.cdf(0.9)
```

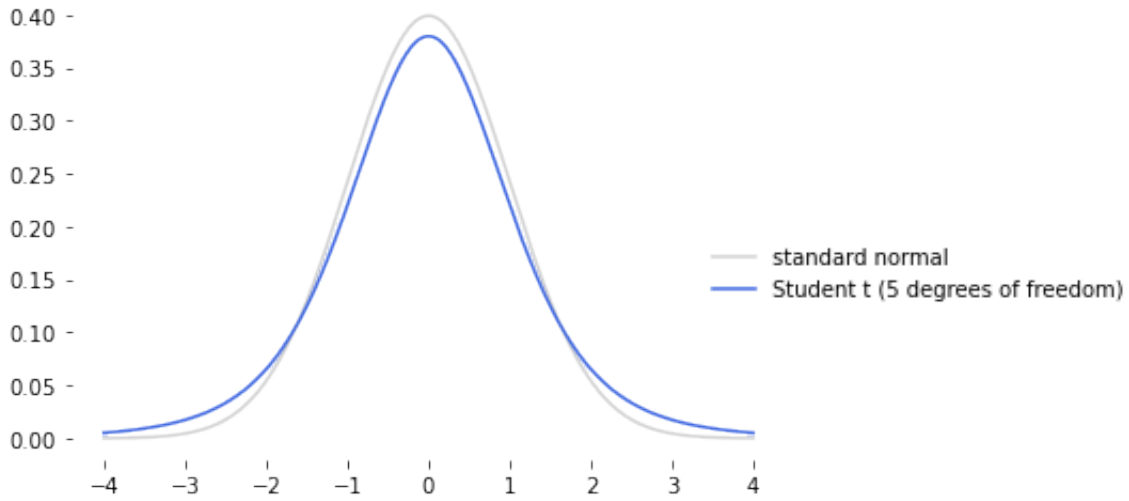
Out[108…

```
0.9
```

Student *t* distribution (covered in Statistics II)

In [109…

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import t
dof = 5   # degrees of freedom of Student t distribution
# x-axis ranges from -4 and 4 with .001 steps:
x = np.arange(-4, 4, 0.001)
plt.figure()
plt.box(False)
plt.plot(x, norm.pdf(x,0,1 ), color="lightgrey")     # plot the standard nc
plt.plot(x, t.pdf(x, dof)     , color="royalblue")   # the second argument
plt.legend(["standard normal" , f"Student t ({dof} degrees of freedom)"], l
# plt.text(3.5,0.35, f"({dof} degrees of freedom)")
plt.show()
```



Plot Student *t* distributions with different degrees of freedom and compare with standard normal distribution (see Haslwanter (2016), p. 110) in an interactive diagram where you can change the degrees of freedom with a slider:

(Normally the `ipywidgets` library is installled in Anacanda. If that is not the case, remove the hashtag at the beginning of the following line and run the line: `pip install ipywidgets` :)

In [110…

```
# pip install ipywidgets
```

In [111…

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm, t
from ipywidgets import interact # import ipywidgets.interact class, this cl
# x-axis ranges from -4 and 4 with .001 steps:
x = np.arange(-4, 4, 0.001)
# use interact decorator to decorate the function, so the function can rece
@interact(dof=(1,60,1))
def plt_t(dof):
    plt.figure()
    plt.box(False)
    plt.plot(x, norm.pdf(x,0,1 ), color="lightgrey")    # plot the standard
    plt.plot(x, t.pdf(x, dof)   , color="royalblue")    # plot Student t di
    plt.legend(["standard normal" , "Student $\it{t}$"], loc="upper right",
    plt.show
```

To find an area under the Student t curve, use the **cumulative density function** (cdf) of the Student t distribution (documentation: see: https://docs.scipy.org/doc/scipy/reference/stats.html):

In [112…

```
import numpy as np
from scipy.stats import t
dof = 30       # degrees of freedom
t1 = -1.00    # lower boundary
t2 =  1.00    # lower boundary
# area under Student t curve between t1 and t2:
area = t.cdf(t2,dof)-t.cdf(t1,dof)
print(f'The area under the Student t curve with {dof} degrees of freedom be
```

```
The area under the Student t curve with 30 degrees of freedom between -1.0
and 1.0 is 0.6746913845739702
```

(To **plot** the area, adapt the code above to plot the area under the normal curve.)

Module 9: Sampling Distributions and Confidence Intervals for Proportions

To find the **confidence interval for a proportion**:
(https://www.statsmodels.org/dev/generated/statsmodels.stats.proportion.proportion_confint

In [113…

```
310 / 1126    # sample proportion = p=hat = number of successes / number of

import statsmodels.api as sm
from statsmodels.stats.proportion import proportion_confint   # Function fo

proportion_confint(count=310,         # count= number of successes
                   nobs=1126,         # nobs = number of trials
                   alpha=(1 - 0.95)) # alpha =  1 - confidence level
```

Out[113…

```
(0.24922129423231776, 0.30140037539468045)
```

Module 10: Sampling Distributions and Confidence Intervals for Means

To find the **confidence interval for a mean** (in this case: the mean height of all students in the dataframe df):

In [114…

```
import numpy as np
import scipy.stats as stats

degrees_of_freedom    = len(df)-1           # degrees of freedom = sample s
sample_mean           = np.mean(df.height)  # sample mean
sample_standard_error = stats.sem(df.height)   # sample standard error of m

# create confidence interval for the population mean:
stats.t.interval(alpha=0.05, df=degrees_of_freedom, loc=sample_mean, scale=
```

Out[114…

```
(173.9221744669485, 174.14449219971817)
```

Hypothesis tests (Statistics II)

For hypothesis tests (covered in Statistics II), see:
https://www.statsmodels.org/stable/stats.html

Interacting with the operating system (changing current working directory etc.)

The `os` package allows you to interact with the operating system using Python code.

The **current working directory** (cwd) is where Python will look for input (such as data files) and where it will store output (such as .png or .pdf figures and tables with results). To find out what the current working directory of Python is:

In [115…

```
import os
print(os.getcwd())
```

```
/Users/luchens/Documents/jupyter-notebooks
```

If you want to change the current working directory, use the chdir command of the os package to do so. Here is how to change the current working directory (the expression in quotes is the path to the new working directory — it will be a different path for you, of course):

In [116…

```
os.chdir('/Users/luchens/Documents/Data/')
print(os.getcwd())
```

```
/Users/luchens/Documents/Data
```

To display the files and directories in the working directory, use `os.listdir()`