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1 Definitions

Definition 1. A *Voronoi diagram* is a subdivision of a $n \times m$ table equipped with a greedy braid into regions such that

- 1) Left boundary of each region is composed of a single braid strand and possibly a piece of the boundary;
- 2) Right boundary of each region is composed of several pieces e_1, e_2, \dots, e_k of braid strands, such that
 - (a) the pieces e_1, e_2, \dots, e_k cross no strand from left to right,
 - (b) the strand containing e_i crosses the strand containing e_{i+1} from left to right at the point where e_i meets e_{i+1} , $i = 1, \dots, k - 1$.

Lemma 1. *The following are equivalent:*

- 1) Point p belongs to the Voronoi cell f_i of a Voronoi site s_i ;
- 2) Site s_i is the leftmost site such that there is a path from s_i to p that crosses no braid strand from left to right.

2 Notation

- 1) \mathcal{B} for the greedy braid of the $\frac{n}{2} \times m$ table;
- 2) VD for the Voronoi diagram of \mathcal{B}
- 3) \mathcal{B}^* for the upward $\frac{n}{2} \times m$ greedy braid;
- 4) \mathcal{B}^{-h} for the $(\frac{n}{2} + h) \times m$ greedy braid that starts h rows above the middle line;
- 5) VD^{-h} for the Voronoi diagram corresponding to \mathcal{B}^{-h} ;
- 6) s_0, \dots, s_{m+n} for the sites of the Voronoi diagram;
- 7) f_0, \dots, f_{m+n} ; $f_0^{-h}, \dots, f_{m+n}^{-h}$ for the Voronoi cells of VD and VD^{-h} correspondingly;
- 8) c_0, \dots, c_{m+n} ; $c_0^{-h}, \dots, c_{m+n}^{-h}$ for the lower right corners of the Voronoi cells of VD and VD^{-h} correspondingly.

3 Query

Lemma 2. *The following are equivalent:*

- 1) *Point p belongs to the Voronoi cell f_i^{-h} of VD^{-h} ;*
- 2) *There is a path from s_i to c_i^{-h} passing through p that crosses no strand of \mathcal{B} twice.*

