Incremental (Combinatorial) Voronoi Diagrams

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Modern Methods in Computer Science

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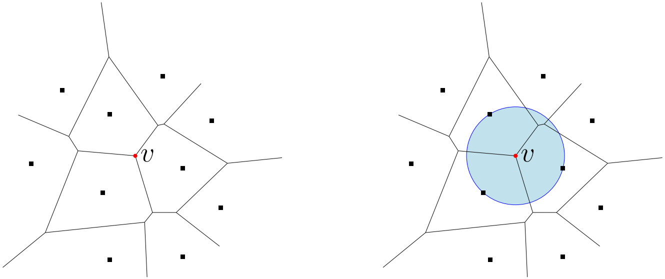
Voronoi Diagrams

Definition

Voronoi diagram of set $S \subset \mathbb{R}^2$ of n points is the subdivision of the plane into n *cells*, one for each site in S, with the property that a point q lies in the cell corresponding to a site s_i if and only if

$$\mathrm{dist}(q, s_i) < \mathrm{dist}(q, s_j)$$

for each $s_j \in S$ with $j \neq i$.



Algorithms for Voronoi Diagram

1. Calculate whole Voronoi diagram — $O(n \log n)$: sweep line, divide-and-conquer.
2. (And no better: sorting reduces to Voronoi.)
3. Update the diagram when a new site came — $O(n)$ — *literally any* possible way.
4. (And no better: *should draw en example here.* There can be that many changes.)
5. But what if we consider *the graph* of a diagram...

Results today

1. $O(\sqrt{n})$, $\Omega(\sqrt{n})$ edge insertions / removals — even if the diagram is a tree.

   This is in contrast to $O(n)$ geometric changes and $O(\log n)$ (amortised, existential) combinatorial changes in case of inserting in clockwise order.

2. *Algorithm* for insertion of sites in convex position, in arbitrary order — $O(\sqrt{n}$ polylog$)$.

Links and Cuts

Link is the addition of an edge, *cut* is the removal of an edge. We count only them. All other operations are thought to have no cost. For example, addition of a vertex.

How many links / cuts are there in our example?

Chan's structure

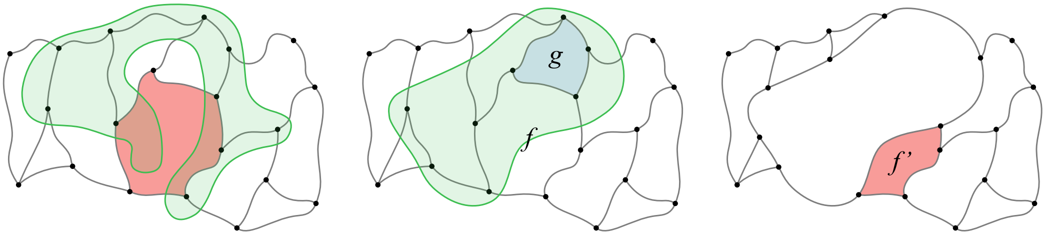
Can be used for:

1. Search of nearest neighbor;
2. Reporting extreme point in given direction.

Makes use of *shallow cuttings.* (Was presented at CG seminar.) Time complexity:

1. Insertion: $O(\log^3 n)$;
2. Deletion: $O(\log^7 n)$ (improved to ^{5});
3. Query: $O(\log^2 n)$.

Flarb



(a) This curve is not flarbable

(b) This curve is flarbable

(c) Result of applying flarb operation

Fleeq-edges: those intersecting $\mathcal{C}$. Note that in a real Voronoi diagram there can be no such cells as g.

Flarb: Results

Theorem

$\mathcal{G}(G, \mathcal{C})$ has at most two more vertices than G (3-regular graph inside).

Theorem

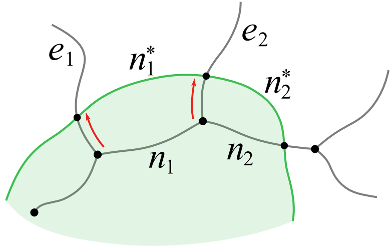
$\mathcal{G}(G, \mathcal{C})$ contains one more face than G (if it comes to Voronoi diagrams).

Theorem

For any new point, there exists a curve such that the changes the graph of V. D. undergoes can be represented by Flarb.

Preserving operation

Sometimes no links and cuts are needed.



$\mathcal{P}(G,\mathcal{C})$ — the set of *preserved* faces. $\mathcal{B}(G,\mathcal{C})$ — faces wholly inside $\mathcal{C}$.
$\mathcal{A}(G,\mathcal{C})$ — augmented, $\mathcal{S}(G,\mathcal{C})$ — shrinking.

Thank you for your attention

Some contents here maybe