Recent research achievements

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Advanced Mathematics

Friday, December 20

Sublinear Explicit Voronoi Diagrams

Ongoing research:

$\tilde{\text{O}}$ptimal-Time Voronoi Diagrams

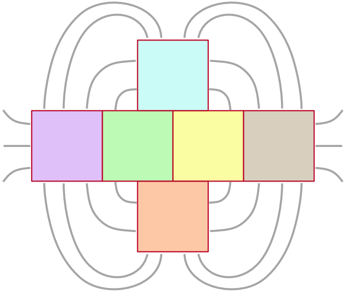
Previous research:

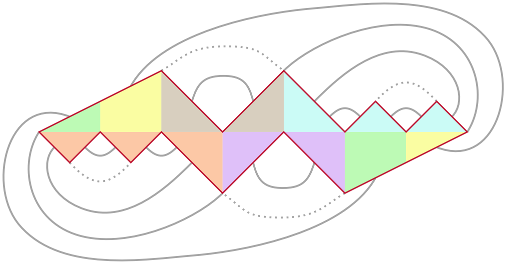
Polyhedra glued from regular pentagons

Gluings

Definition (Alexandrov 1950)

A *gluing* is a set of polygons equipped with a number of rules describing the way edges of these polygons must be glued to each other.





Alexandrov's Theorem

Theorem

If a gluing is homeomorphic to a sphere and the sum of angles at each of its vertices is $\leq 360°$, *there* $\exists!$ *convex polyhedron that corresponds to this gluing.*

Polyhedron Reconstruction

The proof of Alexandrov's theorem is non-constructive.

Each of the following problems is still open:

Alexandrov's Problem

Given a gluing $\mathcal{T}$ satisfying the conditions of Alexandrov's Theorem, find the convex polyhedron corresponding to it.

Cauchy Rigidity Problem (Demaine and O'Rourke 2007, 23.22)

A *poly-time* algorithm that takes as input edge lengths of a triangulated convex polyhedron, and outputs approximate coordinates of its vertices.

Skeleton Reconstruction Problem

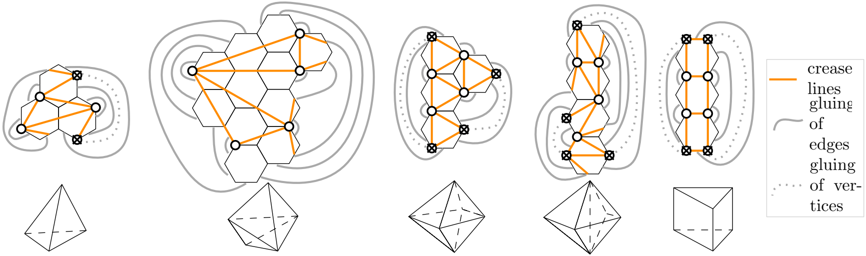
A *poly-time* algorithm that, given a net satisfying the conditions of Alexandrov's Theorem, *finds the skeleton* of the convex polyhedron corresponding to it.

Edge-to-Edge Gluings of Regular Polygons

Maybe Alexandrov's Problem becomes easier, if we start building the gluings from similar simple blocks?

Definition (Demaine and O'Rourke 2007; Lubiw and O'Rourke 1996)

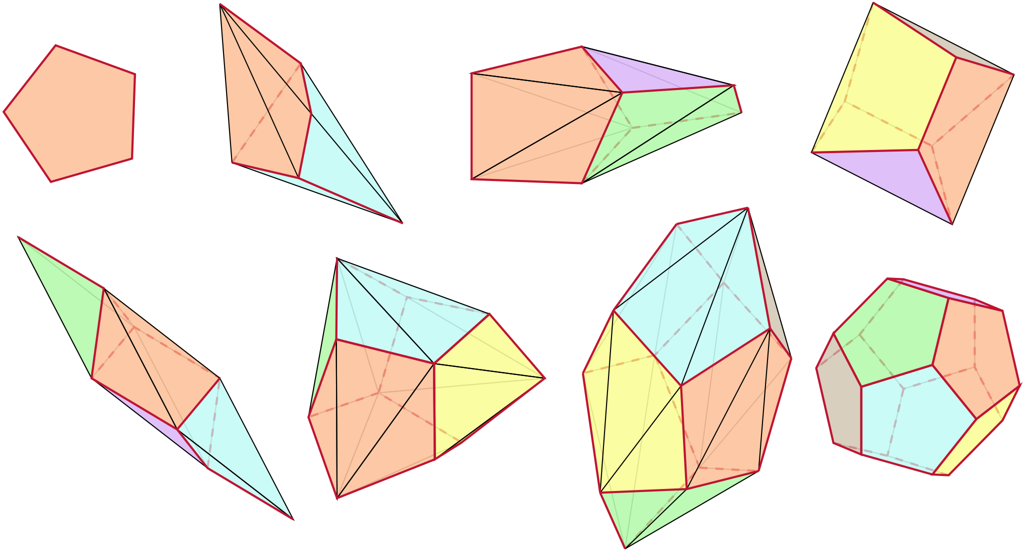
A gluing is *edge-to-edge* if every edge is glued to another entire edge.



Khramtcova and Langerman 2017

Polyhedra glued from regular pentagons

One can enumerate all gluings of pentagons. Here are all the polyhedra that can be obtained:



Precision of vertex location based on the approximation

Theorem

If $\mathcal{D}\gamma < \pi/2$, then each vertex of $\mathcal{P}$ lies within an r-ball centered at the corresponding vertex of P, where

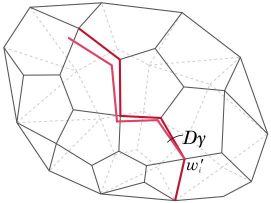
$$r = E^2 \cdot L \cdot 2\sin(\mathcal{D}\gamma/2) + E\mu.$$

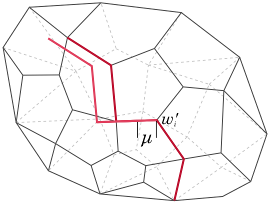
$\mathcal{P}$ — convex polyhedron corresponding to a given net,

P — approximation of $\mathcal{P}$. $\mathcal{D}$ — max degree of a vertex of P.

μ — max edge discrepancy between P and $\mathcal{P}$,

γ — max face angle discrepancy.

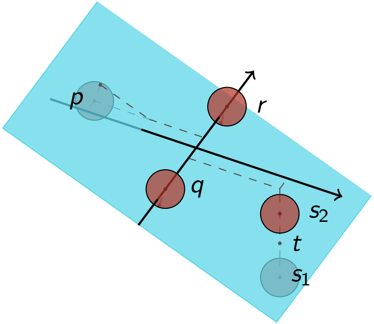




Determining the shape from the gluing

Let pqr, sqr be adjacent faces of P.

Are there edges ps, qr in $\mathcal{P}$?



Assume pqr is not vertical and P lies *below it*.

Consider three planes Π_1, Π_2, Π_3 tangent to $B_r(p)$, $B_r(q)$, $B_r(r)$.

Theorem

If s lies below Π_1, Π_2 and Π_3 and the distance from s to each of the planes Π_1, Π_2 and Π_3 is greater than r, then the edge qr must be in $\mathcal{P}$.

Implemented on *Haskell.*

Geometric methods can also be used.

Demaine, E. and O'Rourke, J. (2007). *Geometric folding algorithms*. Cambridge University Press.

Khramtcova, E. and Langerman, S. (2017). Which convex polyhedra can be made by gluing regular hexagons? In *JCDCG*^{3}.

Lubiw, A. and O'Rourke, J. (1996). When can a polygon fold to a polytope? Technical Report 048, Department of Computer Science, Smith College, Northampton, MA. Presented at AMS Conf., 1996.