Recent research achievements

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Advanced Mathematics

Friday, December 20

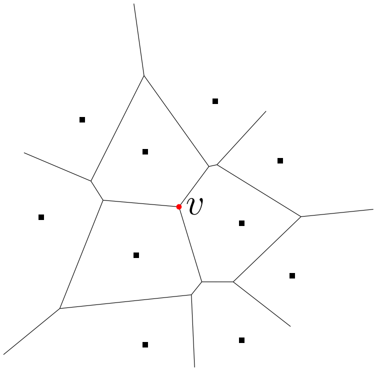
Part 1

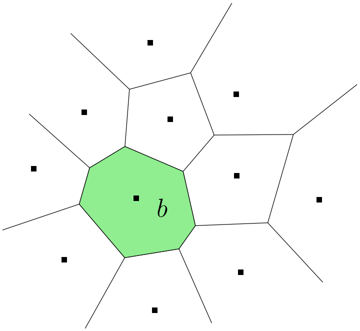
Sublinear Explicit Incremental Voronoi Diagrams

Voronoi Diagram: basics

Voronoi diagram of $S = \{s_1, \ldots, s_N\}$: subdivision of the Euclidean plane, for each q inside the *cell* of s_i

$$\mathrm{dist}(q, s_i) < \mathrm{dist}(q, s_j), \quad j \neq i.$$





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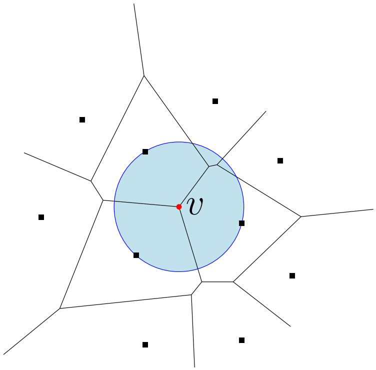

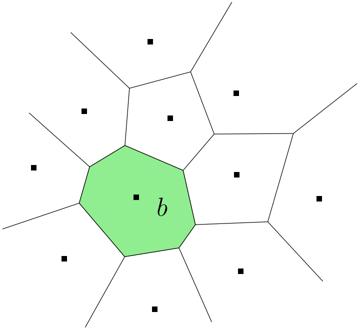

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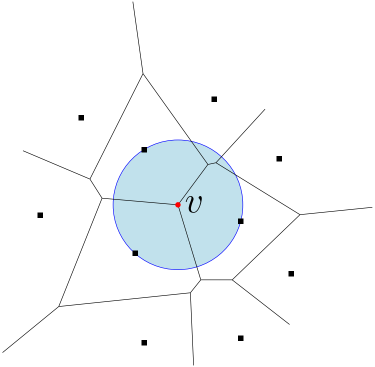

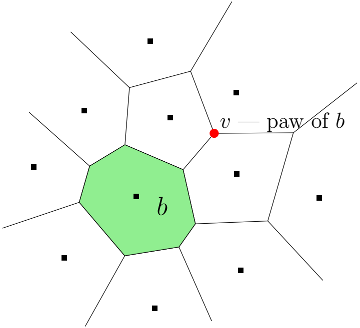

Dynamic VD: setting

Incremental Voronoi Diagram

Maintain initially empty Voronoi diagram under insertion of new sites

There is a linear-time solution, no faster solutions possible: there may be $O(N)$ changes per insertion.

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Incremental Combinatorial Voronoi Diagram

Maintain the graph of initially empty Voronoi diagram under insertion of new sites

There is a naive linear-time algorithm. Can we find a faster solution?

Dynamic VD: combinatorial changes

Theorem (Allen et al. 2017)

The number of cells with combinatorial changes is $O(N^{\frac{1}{2}})$ amortized, there are a constant number of combinatorial changes per cell, cells with changes are connected.

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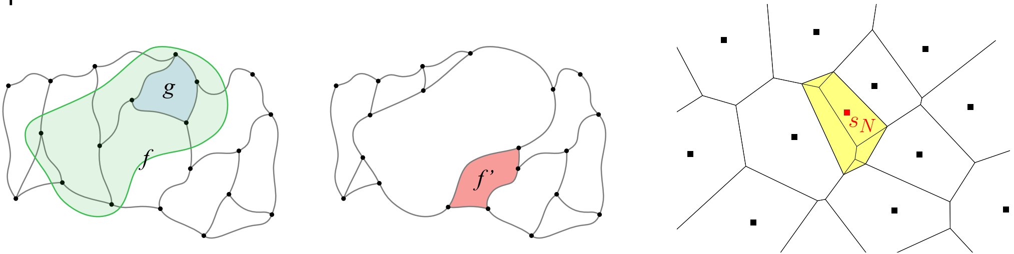
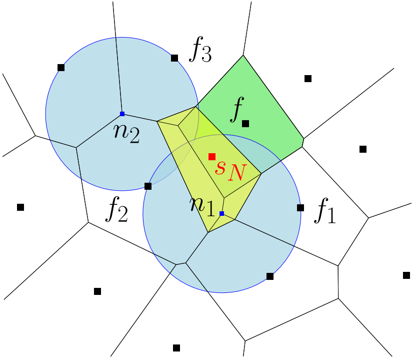
When we are inserting new site s_N, the graph of the VD undergoes operation called *flarb*.



(a) Graph before flarb (b) Graph after flarb (c) Flarb on a VD

Identifying changes

Theorem: Let g be a cell adjacent to f. Cell g needs to undergo changes $\iff$ circle of either of its vertices that are paws of f encloses s_N. Allen et al. 2017



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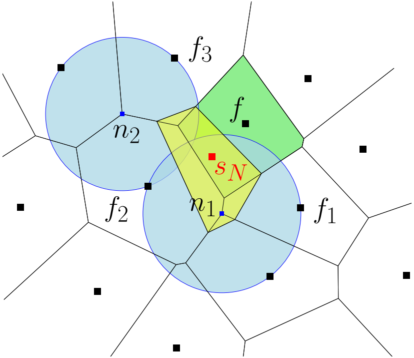

Dynamic circle reporting structure (Chan 2010; Allen et al. 2017)

Returns all k circles enclosing given point in $\tilde{O}(k)$. Addition and deletion of a circle in $\tilde{O}(1)$.

Description of data structure

Definition

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(1) Graph of VD in a form of adjacency list,

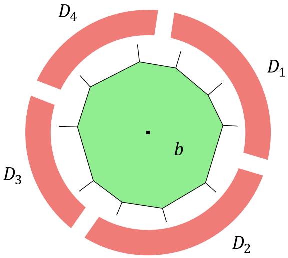
(2) *Dynamic nearest neighbor* structure for sites,

(3) Graph of big cells stored as adjacency list,

(4) For each big cell:

– linked list of DCRs,

– binary search tree of vertices in circular order.



Handling insertion

Initialize queue; look at cells one-by-one, on each step add to it unprocessed cells with changes, then implement changes.

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Time complexity: $$\tilde{O}\left(sN^{\frac{1}{4}} + \sum_{i=1}^{|B|}\left(\left\lceil \frac{|b_i|}{N^{\frac{1}{4}}} \right\rceil + \ell_i\right) + N^{\frac{3}{4}} + sN^{\frac{1}{4}}\right).$$

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This is $\tilde{O}(N^{\frac{3}{4}})$ amortized.

Ongoing research:

Õptimal-Time Incremental Voronoi Diagrams

Allen, S. R., Barba, L., Iacono, J., and Langerman, S. (2017). Incremental voronoi diagrams. *Discrete & Computational Geometry*, 58(4):822–848.

Chan, T. M. (2010). A dynamic data structure for 3-d convex hulls and 2-d nearest neighbor queries. *J. ACM*, 57(3):16:1–16:15.