Fictionalism in philosophy of mathematics

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Theses and basics

Ontological thesis

A view that the things that are discussed do not exist physically, yet obey certain rules, and we can therefore describe them rightfully and completely.

Linguistic thesis

An opposing view: the statements that are discussed do not have the purpose to reflect factial truth, but are rather useful fictions.

A basic description of fictionalism

All the claims that are made are technically false,
still they are worth considering, since these exact claims
might be needed for some theoretical purposes.

Simply said, fictionalism aligns with the linguistic thesis.

Fictionalism in mathematics: departure points

Fictionalism in mathematics

There are no such objects physically as numbers, rings, fields, manifolds, etc. Therefore any statements about them are untrue, because the objects are made up. It does not mean though that mathematics is pure fiction.

Countributors to Fictionalism

- Field (1980, ..., 2016)
- Balaguer (1996, ..., 2009)
- Leng (2005, 2010)
- Rosen (2001)
- Hoffman (2004)

Quite recent isn't it?..

An inference of fictionalism

Face value

Mathematical sentences should be read at face value,
they are making straightforward claims
about the nature of certain objects.

Existence of underlying objects

If the sentence “4 is even” is true, then the object it is about must exist.

Possible fix: if 4 exists, it evenness does not contradict any axioms we have introduced.

Existence of mathematical objects

If sentences like “4 is even” are true, then there exist mathematical objects.

Existence of abstract objects

If there are mathematical objects, then there exist *abstract* objects — ones that do not occupy a point in space or time.

Abstract objects do not exist

- There are no such things as abstract objects,
- Therefore, there are no such things as mathematical objects,
- Therefore, sentences like “4 is even” are not true.

Alternatives to fictionalism

Why the aforementioned inference is important

The logical tools used for deriving the statements are indeed valid. Therefore, to disagree with a fictionalist, one has to disagree with one of the premises we used.

Alternatives to fictionalism

- Mathematical statements are not straightforward claims (8) — **paraphrase nominalists;**
- Truth of a statement does not imply the existence of the underlying object (9) — **deflationary-truth nominalists;**

Alternatives to fictionalism

- The existence of mathematical objects
  does not imply the existence of abstract objects (11) —
  physicalists or **psychologists;**

- There exist abstract objects (12) —
  platonists.

Objections to fictionalism

Indispensability argument

Mathematical statements help other theories like physics,
and the description of the world given by physics is pretty
accurate, therefore we have quite a good reason
to distinguish true mathematical statements.

Objectivity

Among mathematical statements there are some of different flavors, like “4 is even” seems true, and “3 is composite” seems false. Seems — aligns with the theory. However, fictionalists are obliged to say that both statements are false.

There are fictionalist responses to this of course.

Similarity to fiction

Consistency is an important criterion for mathematical statements, which is not the case for a set of any made-up ones.

Fictionalism does not involve the claim that there are no important disanalogies between mathematics and fiction.

Accepting and believing

Fictionalists acknowledge the correctness of mathematical statements, but do not approve of their truth. Does it mean that they accept them without believing them?

Why not then say that truth=acceptance by fictionalists?

Conclusion

Conclusion

Fictionalism is a concept based on that mathematics needs the existence of any object it is talking about and imply it directly — therefore its statements can not be considered true.

Mathematics, though, usually say something like “would an object exist that obeys certain axioms, the following would hold for it”.

Thanks!