

# **Portfolio**

## **English exam**

**MCS Ph. D. Programme**

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## **ABSTRACT 1**

Edge-vertex resolution of a planar drawing is an important measure that reflects a minimal separation between a vertex and an edge, allowing the vertices to be objects of non-zero size rather than points.

The authors present an algorithm that for any planar 3-connected graph computes its drawing with the edge-vertex resolution equal to  $1/2$  (that is, a disk-link drawing).

What is more, the drawing is computed in time linear in the number of vertices of the graph, and the vertices of the drawing are located on a linear-size grid (which is proved to be optimal).

The algorithm is based on the earlier result of Chrobak and Kant who had a similar problem but did not obey the requirement for the drawing to be disc-link.

## **ABSTRACT 2**

Unit Disc Cover (UDC) problem asks for the minimal number of unit disks covering a given set of points and the placements of these circles.

The problem is proved to be NP-hard; however, there exist plenty of polynomial and pseudopolynomial approximation algorithms solving it.

The authors implement several known algorithms for UDC and compare their efficiency. They also present a new 7-approximation algorithm running in linear time on average and two heuristics that help speed it up.

The algorithm devised by the authors covers the query set with circles aligned to the  $\sqrt{2}$ -sized grid. Then it is proved that the algorithm outputs not more than seven times the number of circles contained in the optimal answer.

The authors conclude that the algorithm by Liu and Lu (2014) is optimal for practical applications, yet their algorithm overperforms it in certain cases.

## **ABSTRACT 3**

The authors present an algorithm that enumerates and classifies all edge-to-edge gluings of unit squares that correspond to convex polyhedra.

The authors show that the number of such gluings of  $n$  squares is polynomial in  $n$ , and the algorithm runs in time polynomial in  $n$  (pseudopolynomial if  $n$  is considered the only input).

Their technique can be applied in several similar settings, including gluings of regular hexagons and triangles.

1. Penumbra of aggravations — countless annoying little problems related to a big problem
2. Coherent argument — a well-organized reasoning without any self-contradictions
3. Substantive contribution — an important piece of study, essential for the field
4. Diffuse activity — the skill in which is spread between researchers by itself
5. Credible resources — ones to which respect is paid
6. Prowess in a discipline — expertise
7. Skills inculcated — taught thoroughly
8. By-product approach — developing something not as a main result of a process
9. Doctoral instruction — the process of writing thesis and receiving knowledge under supervision
10. Humanities and social sciences — гуманитарные науки
11. Set out to write a book — decide to do something huge
12. Grapple with — deal or understand a difficult problem
13. Shelf-bending thesis — one that is never published
14. Apprentice researcher — working for low payment for a more skilled person
15. Dubious kudos — praise that is not certainly there
16. Taught Ph. D
17. Classical Ph. D
18. Professional doctorate
19. Leveled planar graph — a graph drawn on the plane whose vertices are on several parallel lines
20. Layered pathwidth — the maximum size of the intersection between a layer and a bag in a decomposition of a graph
20. Outerplanar graph — a planar graph drawn in a way such that all the vertices are in the outer (unbounded) face
21. Squaregraphs — 4-faces, each vertex either in outer face or has degree 4+
22. Breadth first layering — each vertex is marked by its distance to the source vertex
23. Specify an ordering — assign consequent natural (positive integer) numbers to objects
24. Net — a simple non-overlapping unfolding of a polyhedron
25. Unit-speed parametrization — parametrization by the distance to the origin

26. Osculating circle — the one with the same curvature
27. Unit normal vector — perpendicular, of length 1
28. Principal curvatures — the highest and the lowest curvatures of a surface at a point
29. A flaw of sorts — same things having different properties
30. Subtended — bounded, contracted
31. Intrinsic — defined from an internal view
32. Geodesic — measured inside surface
33. Elliptic, hyperbolic points — with respect to curvature
34. Angle deficit, Gaussian curvature — the difference between  $2\pi$  and the facial angle at a point
35. Meager evidence — sparse, insufficient
36. Lament — to complain
37. Paucity — sparsity, poverty
38. Star unfolding, source unfolding — methods of unfolding a polyhedron that (supposedly) produce a non-overlapping net
39. Overlap penetration — the ratio of a penetration to the diameter
40. Follow suit — to act conformly
41. Convex hull — a minimal convex polygon containing a given set of points
42. Vertex-edge resolution — a minimal separation between a vertex and a non-incident edge in a drawing
43. Disk-link drawing — a drawing with vertex-edge resolution of at least  $1/2$
44. Gluing — a pair of a set of polygons and rules specifying how to glue their boundaries
45. Polyhedron — a union of convex polyhedra, which are intersections of half-spaces
46. UDC Problem — a problem of covering a given set of points with the minimal number of unit disks
47. Erdős-Rényi graph — a randomly generated graph with sufficient connectivity
48. Prisoner's dilemma — a noncooperative game about betraying or keeping silent
49. Hawk-dove game — a noncooperative game about showing off or retreating
50. MAXQ — a method of reinforcement learning that takes the next action depending on the output of max-nodes and q-nodes

51. Schnyder labeling — an orientation on a maximal planar graph where each vertex has 3 outgoing edges
52. Canonical ordering of a planar graph — an ordering that helps draw the graph: each next vertex is in the outer face connected with a continuous segment of previous vertices
53. LR-partition — a partition of all the back-edges with respect to a DFS-ordering into two sets. It helps test planarity of a graph
54.  $n$ -connected graph — a graph that remains connected after removal of  $n-1$  vertices
55. Closed rectangle-of-influence drawing — a drawing such that no vertex lies in the axis-parallel rectangle defined by two ends of every edge
56. Bold drawing — vertices are drawn as disks of radius  $r$  and edges as rectangles of width  $w$
57. Planar embedding — an equivalence class of topologically-equivalent planar drawings
58. Straight-line drawing — a drawing where all the arcs are straight-line segments
59. Grid drawing — straight-line drawing where vertices have integer coordinates
60. Point density — ratio of  $|P|$  to the area of its bounding box
61. Track layout — partition of a graph into tracks, which are independent sets, and edges between them do not cross
62. Tree decomposition — an assignment of bags such that endpoints of any edge share a bag, and each vertex is in a continuous subtree of bags
63. Path decomposition — a tree decomposition but the tree is a path
64. Treewidth — minimal size of a maximal bag  $-1$
65. Minor — a graph obtained by deleting edges / vertices and contracting edges
66. Euler characteristic —  $V-E+F$
67. Euler genus —  $1 - \chi/2$
69. General position — assumptions about data that hold almost often and help evade degenerate cases
70. Hull signature — an ordered list of vertices of the convex hull of a set
71. Dynamic nearest neighbor structure — a structure supporting insertion of a site and the query of the nearest site to a given point
72. Voronoi circle — a circle passing through three adjacent Voronoi sites
73. Sweep line — a line that is moved through the plane used to construct several geometric structures

74. Combinatorial change — a change in the graph of a structure, not only in the location of its vertices
75. Affirm — show that one has obtained a message and has no objections
76. Visibility center — a point such that the maximum geodesic distance from it to \_see\_ any point in the set is minimized
77. Book embedding — a drawing of a graph on several half-planes glued along their boundary
78. Abuse of notation — using the same denotation for objects of different generality
79. Winding number — the number of counterclockwise rotations of a curve around a point
80. Young diagram — a way to demonstrate integer partitions
81. Bipartite graph — a graph colored in black and white in a way that each edge has endpoints of distinct colors
82. Spanning tree — a tree obtained by deletion of several edges from a graph, but on the same set of vertices
83. Building greedily — taking a maximal possible set on each step
84. Minor-closed class — a class such that if  $G$  is in it, every minor of  $G$  is in it
85. Dual graph — a graph whose vertices are faces of  $G$  and whose edges are edges incident to these faces
86. Geodesic center — a point that minimizes the maximum geodesic distance from it to any point of the set
87. Chord — a segment inside a polygon intersecting its boundary only at endpoints
88. Geodesically convex subset — a subset such that if two points lie in it, so does the geodesic shortest path between them
89. Pareto optimality — a situation where no individual or preference criterion can be better off without making at least one individual or preference criterion worse off
90. Nash equilibrium — each player is assumed to know the equilibrium strategies of the other players, and no one has anything to gain by changing only one's own strategy
91. Oracle — a thing that is somehow able to solve a complicated problem in constant time
92. Winning strategy — a sequence of actions that depends on a position in a game that makes a player win the game
93. DFS — graph traversal that first goes as deep as possible along the edges
94. BFS — graph traversal that considers all the vertex's neighbors immediately after this vertex
95. Manifold — a topological space with continuous base that is locally homeomorphic to Euclidean space

96. Base — a set of sets such that any open set of the topology can be represented as a union of the sets in the base
97. Curry—Howard isomorphism — a mapping between habitable types and provable statements
98. Unit-disk graph — a graph for a set of vertices in the plane that only has edges of unit length
99. Series-parallel graph — a graph with two terminal vertices that is formed by application of composition or parallel composition
100. Tree-apex graph — a graph that is a tree with a vertex added that is connected to each leaf
101. Apex graph — a graph that can be made planar by removing a single vertex
102. Dent — an unplanned piece of curvature on a metal sheet left behind by a miscalculated impact
103. Primary root — a generator of the multiplication group of residues modulo  $p$  (which is, in fact, cyclic)
104. Quadratic residue — a residue congruent to the square of an integer number modulo  $n$
105. Euclidean algorithm — an algorithm to find the greatest common divisor of a pair of numbers or polynomials that does not rely on the prime decomposition
106. Determinant — a number assigned to a matrix that behaves in a certain way under addition or permutation of rows
107. Permutation — a bijection between a finite set and itself
108. Sign — a homomorphism from a group to  $\{-1,1\}$
109. Homomorphism — a linear map, i. e. a map preserving the structure
110. Arrangement — the subdivision of the plane into cells, edges and vertices induced by a given set of lines
111. Heavy path decomposition — a decomposition of a tree into paths that is used to implement link-cut tree
112. Splay tree — a tree where each vertex is raised to the root before any operation is carried out on it
113. B-tree — a balanced search tree with a varying number of descendants of each vertex
114. AVL-tree — the default balanced binary search tree that demands rotations at certain vertices in order to stay balanced
115. Range searching — asking to return all pieces of data in a given interval.
116. Output-sensitive algorithm — an algorithm such that one has to take into consideration its output to estimate its running time.
117. Kd-tree — a tree used for range searching in which adjacent nodes split the data according to the dimensions of the space that are alternating

- 118. Amortised — evaluated on average during large number of consecutive queries
- 119. Sublinear — estimated by  $o(n)$
- 120. Median — a number in a set such that exactly a half of the set's numbers are smaller than it
- 121. Recurrence — a way to introduce a function such that the value  $f(n)$  is calculated using values on smaller arguments
- 122. Canonical subset of  $v$  — the subset of points stored in the leaves of the subtree rooted at a node  $v$
- 123. Fractional cascading — a technique to improve the running time of a range searching tree
- 124. Squaring search — a way to find an upper bound for the answer by considering  $2^{2^k}$  at each step. This way logarithms of the query double at each step.
- 125. Kuratowski's problem — a problem of determining how many different sets one can obtain using  $\cap$  and  $\cup$  operations
- 126. Composite-number space — a space in which pairs of points are sorted lexicographically
- 127. Point location query — given a map and a query point specified by its coordinates, find the region of the map containing it
- 128. Planar subdivision — a structure of cells, edges and vertices covering the plane.
- 129. Vertical slab — a stripe inside which there are no vertices
- 130. Trapezoidal map — a subdivision whose cells are trapezoids with vertical bases or triangles
- 131. Incremental algorithm — an algorithm constructing a structure adding one site after another, rather than considering all sites at once. It is usually slower than an offline algorithm.
- 132. Topology — a collection of sets that are to be called open. It satisfies certain properties: any union of open sets is an open set.
- 133. Advisory board — a group of people making decisions.
- 134. Voronoi assignment model — the model where every point is assigned to the nearest site
- 135. Cartesian coordinates — a system assigning a pair of numbers to each point in the plane
- 136. Metric — a symmetric function that is zero on coinciding elements and satisfies the triangle inequality
- 137. Metric space — a set equipped with a metric
- 138. Triangulation — a decomposition of an object into a set of triangles or simplices
- 139. Face — a polygon of the highest possible dimension that belongs to a graph / to a surface
- 140. Edge — an intersection of two faces



- 142. Incident — two objects of different priorities next to each other
- 143. Adjacent — two objects of equal priorities next to each other
- 144. Quadrilateral — a polygon with four sides
- 145. Tight bound — a bound that is reachable and thus can not be improved
- 146. Iterative — a process that is repeated several times
- 148. Nearest neighbor — a site such that the distance from it to the query point is minimal
- 149. Residue — a number between 0 and  $n-1$  that is congruent to the given number modulo  $n$
- 150. Power series — a formal sum of powers of a variable with various coefficients
- 151. Congruence — any sensible equivalence relation
- 152. Convex domain — a domain such that any segment connecting two its points lies inside it
- 153. Connected space — a space that cannot be represented as a union of two open sets
- 154. Path connected space — a space where there exists a path between any two points
- 155. Simply connected space — a space where every loop can be contracted
- 156. A loop — a path from a point to itself
- 157. Primality test — an algorithm testing if a given number is prime
- 158. Turing machine — a model with a finite number of states, an infinite tape and a set of rules describing transition between states
- 159. Cellular automaton — a set of cells where the state of a cell changes at each step in accordance with the states of its neighbors
- 160. Markov's algorithm — a Turing-complete model comprised only of replacement rules
- 161. Winning position — a position from which a turn can be taken into a losing position
- 163. Losing position — a position from which any turn leads to a winning position
- 164. Voronoi diagram — a subdivision of the plane according to the nearest neighbor queries.
- 165. Flarb — a combinatorial operation of adding a new face to a graph.
- 166. Tree — a connected graph with no cycles.
- 167. Pendant vertex — a vertex of degree 1.
- 168. Euler cycle — a cycle that visits each edge of a graph exactly once.
- 169. Hamiltonian cycle — a cycle that visits each vertex of a graph exactly once.
- 170. Search — a problem of outputting the piece of data assigned to a given key

171. Sorting — a problem of placing several numbers or keys in the ascending order.
172. RAM machine — a model that can access any memory cell in constant time
173. Forest — a possibly disconnected graph with no cycles.
174. Dual graph — a graph whose vertices are faces of  $G$  and whose edges are edges incident to these faces
175. Language — a set of strings that one usually wants to be able to distinguish
176. Computational hierarchy — a separation of problems into levels according to the number of quantifiers preceding a computable problem
177. Motion planning — a problem of assigning a strategy to several moving robots such that no collisions happen and total traveled distance is minimal
178. Geodesic center — a point of the set such that maximal geodesic distance from it to a point of the set is minimised
179. Noncooperative game — a game where players take their turns simultaneously, and the prize is split immediately afterwards
180. Network — a graph with a large number of vertices and a set of rules that apply to its connectivity
181. PCB — a printed contact board that is used for connecting several electronic devices. It is comprised of several sheets on which tracks are laid
182. Delaunay triangulation — a triangulation in the plane such that the circumcircle of any triangle does not contain any vertices but of this triangle
183. Euclidean shortest path problem — a problem of connecting two points in a Euclidean space (with polyhedral obstacles) by a shortest path
184. Mesh — a subdivision of an area into cells of simple shapes
185. Ray tracing problem — given a set of objects in space, produce a data structure that efficiently outputs which object a query ray intersects first
186. Flipping — an operation replacing one diagonal of a quadrilateral in a triangulation with the other diagonal
187. Divide-and-conquer algorithm — an algorithm whose initial step is splitting the query set into two roughly equal parts
188. Farthest-first traversal — a sequence of points where the first point is selected arbitrarily and each successive point is as far as possible from the set of previously-selected points
189. Prefix — a part of the sequence from the beginning to a certain point
190. Suffix — a part of the sequence from a certain point to the end

191. Delaunay refinement — adding vertices to a triangulation in a way such that the triangulation obeys certain quality requirements afterwards.
192. Pigeonhole principle — a statement concerning not being able to fit  $n+1$  pigeons into  $n$  holes
193. Osculating circle — the circle passing through the point of a curve that has the same curvature as the curve at this point
194. Source vertex — a vertex in an oriented graph with no incoming edges
195. Sink vertex — a vertex in an oriented graph with no outgoing edges
196. Triangle strip — a sequence of triangles where each next triangle shares an edge with the previous one
197. Subdivision of a graph — a graph obtained from the given one by adding vertices of degree 2 in the interior of the edges
198. Regular graph — a graph in which all the vertices have the same degree
199. Flow on a graph — an assignment of a number and an orientation to each edge of the graph such that the sum of the numbers at each vertex is zero
200. Hashing — assigning certain numbers to strings in such a way that it becomes easy to check if two long strings differ