

Reducing Uncertainty Propagation in Markov Decision Processes

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Abstract—In many real-world problems stochasticity is a critical issue for the learning process. The sources of stochasticity come from the transition model, the explorative component of the policy or, even worse, from noisy observation of the reward function. For a finite number of samples, traditional reinforcement learning methods provide biased estimates of the action-value function. The presence of the bias leads to a poor estimation of the action-value function that is propagated to other action-values by the application of the Bellman operator. We propose an approach that significantly mitigates this issue avoiding the propagation of bad estimates of the action-value function.

I. INTRODUCTION

It is well known that a key issue of reinforcement learning problems is the accuracy of the estimation of the action-value with a limited number of samples. While most algorithms guarantee the convergence of the estimates to the optimal action-value, in practice the presence of stochastic components lead to poor performance. In fact, the majority of real-world problems have significant sources of stochasticity: the environment could be have stochastic transition and this complicates the estimation of the effectiveness of an action; most of the times it is necessary to use stochastic policies to guarantee that all states are visited infinitely many times, that is required to guarantee the convergence of the algorithm; the policy could change during the learning process resulting in very different behaviors; the reward function is often corrupted by noisy observations and, in other cases, the reward function is stochastic itself. Moreover, it usually happens that some deterministic environments are partially observable and, thus, are perceived by the agent as stochastic decision processes (e.g. Blackjack).

Since Monte-Carlo estimates of action-values are affected by high variance of the returns, the most successful reinforcement learning algorithms are based on bootstrapping (e.g. Q-Learning [1]), that trades off the variance of the estimation with a consistent but biased estimator. However with a finite number of samples the bias of the estimation could be significantly relevant when propagating the action-values to the next state and, recursively, it propagates to all the other states. Recent works tried to deal with this issue, in particular focusing on the estimation of the maximum expected value. It is well known [2], [3] that the maximum operator (used in the Q-Learning update) is positively biased, thus it overestimates the optimal action-value. In highly stochastic

environments, this overestimation leads to unstable learning and poor convergence rates. To avoid the presence of the positive bias, the Double Q-Learning algorithm [4] has been proposed. This algorithm uses the double estimator [5] that provides a negatively biased estimation of the action-values (i.e. it underestimates the optimal action-value) and this improves the performance when stochasticity is an issue. Another recently proposed approach is the Weighted Q-Learning [6] that balances between underestimation and overestimation.

However, the correlation between bias of the estimation and performance is still unclear; indeed, as also shown in the empirical section of this paper, Speedy Q-Learning algorithm [7] has very good performance despite its extremely poor estimation of the action-value. This is due to the fact that most of the policies are not dependent on the accuracy of the action-values, but instead they rely on their ordering. Starting from this consideration, we try to address this problem from another point of view. Indeed, we do not focus on the estimation of the maximum expected value, but we care about avoiding the propagation of the action-value estimates. In fact, we propose this work starting from the consideration that it is not relevant whether the approximations of the point estimates of the maximum expected value are good or not, but how these information is propagated to the other states. Our idea is to propagate the information about the value of the best action only when there is sufficiently certainty about the estimation.

The interesting aspect of this approach is that it is possible to use any maximum expected value estimator and, moreover, it can be easily extended in an on-policy scenario. Since we believe that in our approach the choice of the estimator is not a relevant issue, we choose the maximum operator as it is the simplest one.

II. PRELIMINARIES

—TODO preso dal mio paper, va parafrasato, solo modifiche minori— A Markov Decision Process (MDP) is defined by $\mathcal{M} = \langle \mathcal{X}, \mathcal{U}, \mathcal{P}, \gamma, \mathcal{R}, D \rangle$, where \mathcal{X} is the state space, \mathcal{U} is the action space, \mathcal{P} is a Markovian transition model where $\mathcal{P}(x'|x, u)$ defines the transition density between state x and x' under action u , $\gamma \in [0, 1)$ is the discount factor, $\mathcal{R}(r|x, u, x')$ defines the distribution of the reward, and D is the distribution of the initial state. A stochastic policy is defined by a density distribution $\pi(\cdot|x)$ that specifies for each state x the density

distribution over the action space \mathcal{U} . — Given a policy π , we define the action value function as:

$$Q^\pi(x, u) = \mathbb{E}_{\substack{(x', u') \sim \\ \mathcal{P}(x'|x, u)\pi(u'|x')}} [r(x, u, x') + \gamma Q^\pi(x, u)(x', u')] \quad (1)$$

The optimal policy π^* is the policy with the highest expected return in the MDP. It has been proven that the optimal policy can be always represented by a deterministic policy. The optimal action value function $Q^*(x, u)$ is the action-value function of the optimal policy. It has been shown that the optimal policy in any MDP can be always a deterministic policy that at each state chooses the action with the highest $Q^*(x, u)$ value.

Given that we can always write the optimal action-value function as:

$$Q^*(x, u) = \mathbb{E}_{x' \sim \mathcal{P}(x'|x, u)} [r(x, u, x') + \gamma \max_{u'} Q^*(x', u')] \quad (2)$$

As the expected value is a linear operator, we can always write 2 as:

$$Q^*(x, u) = \mathbb{E}_{x' \sim \mathcal{P}(x'|x, u)} [r(x, u, x')] + \mathbb{E}_{x' \sim \mathcal{P}(x'|x, u)} [\gamma \max_{u'} Q^*(x', u')] \quad (3)$$

We now introduce two functions, \tilde{R} and \tilde{Q} , defined as:

$$\begin{aligned} \tilde{R}(x, u) &= \mathbb{E}_{x' \sim \mathcal{P}(x'|x, u)} [r(x, u, x')] \\ \tilde{Q}(x, u) &= \mathbb{E}_{x' \sim \mathcal{P}(x'|x, u)} [\gamma \max_{u'} Q^*(x', u')] \end{aligned} \quad (4)$$

We can give an interpretation of this two functions. $\tilde{R}(x, u)$ is the expected immediate reward of the action u in the state x . $\tilde{Q}(x, u)$ is the expected discounted return of the states reached after performing action u in state x , i.e. the expected gain of the reached state.

We can now write the optimal value function as:

$$Q^*(x, u) = \tilde{R}(x, u) + \tilde{Q}(x, u) \quad (5)$$

Our approach shift the focus of the reinforcement learning task from finding a good estimator for the optimal action-value function, to the task of finding good estimators for the \tilde{R} and \tilde{Q} functions. The main motivation is that the sources of uncertainty of the two components of the action-value function are different: the \tilde{R} function only depends on the transition and reward models, while \tilde{Q} also depends on the optimal policy.

III. THE PROPOSED METHOD

A. Decomposition of the TD error

Decompose Q function:

$$\begin{aligned} Q(x, u) &= \mathbb{E} [R(x, u, x') + \gamma Q(x', \pi(x'))] \\ &= \mathbb{E} [R(x, u, x')] + \gamma \mathbb{E} [Q(x', \pi(x'))] \\ &= \tilde{R}(x, u) + \gamma \tilde{Q}(x, u) \end{aligned} \quad (6)$$

Decomposed TD update:

$$\tilde{R}(x, u) \leftarrow \tilde{R}(x, u) + \alpha(R(x, u, x') - \tilde{R}(x, u)) \quad (7)$$

$$\tilde{Q}(x, u) \leftarrow \tilde{Q}(x, u) + \beta(Q(x', \pi(x')) - \tilde{Q}(x, u)) \quad (8)$$

Update of the Q function:

$$\begin{aligned} Q(x, u) &\leftarrow \tilde{R}(x, u) + \alpha(R(x, u, x') - \tilde{R}(x, u)) \\ &\quad + \gamma (\tilde{Q}(x, u) + \beta(Q(x', \pi(x')) - \tilde{Q}(x, u))) \\ &= Q(x, u) + \alpha(R(x, u, x') - \tilde{R}(x, u)) \\ &\quad + \gamma \beta(Q(x', \pi(x')) - \tilde{Q}(x, u)) \end{aligned} \quad (9)$$

B. Analysis of the decomposed update

If $\alpha = \beta$

$$Q(x, u) \leftarrow Q(x, u) + \alpha(R(x, u, x') + \gamma Q(x', \pi(x')) - Q(x, u)) \quad (10)$$

That is the classical Q-Learning update

If $\beta = \delta\alpha$

$$\begin{aligned} Q(x, u) &\leftarrow Q(x, u) + \alpha(R(x, u, x') + \gamma\delta Q(x', \pi(x')) \\ &\quad - (\tilde{R}(x, u) + \gamma\delta\tilde{Q}(x, u))) \\ &= Q(x, u) + \alpha(R(x, u, x') + \gamma'\delta Q(x', \pi(x')) \\ &\quad - (\tilde{R}(x, u) + \gamma'\delta\tilde{Q}(x, u))) \\ &= Q(x, u) + \alpha((R(x, u, x') + \gamma'\delta Q(x', \pi(x')) \\ &\quad - Q'(x, u)) \end{aligned} \quad (11)$$

With $\gamma' = \gamma\delta$. Notice that $Q'(x, u)$ is the current Q function with a different learning rate.

C. Variance dependent learning rate

$$\alpha = \frac{\sigma^2}{\sigma^2 + 1} \quad (12)$$

IV. EXPERIMENTAL RESULTS

V. CONCLUSION

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