Exploiting Structure and Uncertainty of Bellman Updates in Markov Decision Processes

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Problem

- Learning is difficult in highly stochastic environments
- Uncertainty in action-value function estimates propagates
- Some algorithms face this problem focusing on the bias of the estimate

Contributions

1. Split the estimate in two components:
   - The expected reward $\widetilde{R}(x,u)$
   - The expected next state value function $\widetilde{Q}(x,u)$
2. Use different learning rates for the two components
3. We provide empirical results showing the effectiveness of our approach

RQ-Learning Algorithm

Idea

Split the action-value function in two components:

- $\widetilde{R}(x,u) = \mathop{\mathbb{E}}_{x'\sim\mathcal{P}(x'|x,u)}\left[r(x,u,x')\right]$
- $\widetilde{Q}(x,u) = \mathop{\mathbb{E}}_{x'\sim\mathcal{P}(x'|x,u)}\left[\max_{u'} Q^*(x',u')\right]$
- $Q^*(x,u) = \widetilde{R}(x,u) + \gamma\widetilde{Q}(x,u)$

Compute the update as follows:

- $\widetilde{R}_{t+1}(x,u) \leftarrow \widetilde{R}_t(x,u) + \alpha_t(R(x,u,x') - \widetilde{R}_t(x,u))$
- $\widetilde{Q}_{t+1}(x,u) \leftarrow \widetilde{Q}_t(x,u) + \beta_t(\max_{u'} Q_t(x',u') - \widetilde{Q}_t(x,u))$

Different effects on the choiche of α and β:

Q-Learning $\alpha_t = \beta_t$

RQ$_\delta$-Learning $\beta_t = \alpha_t\delta_t$

RQ-Learning $\beta_t \neq \alpha_t$

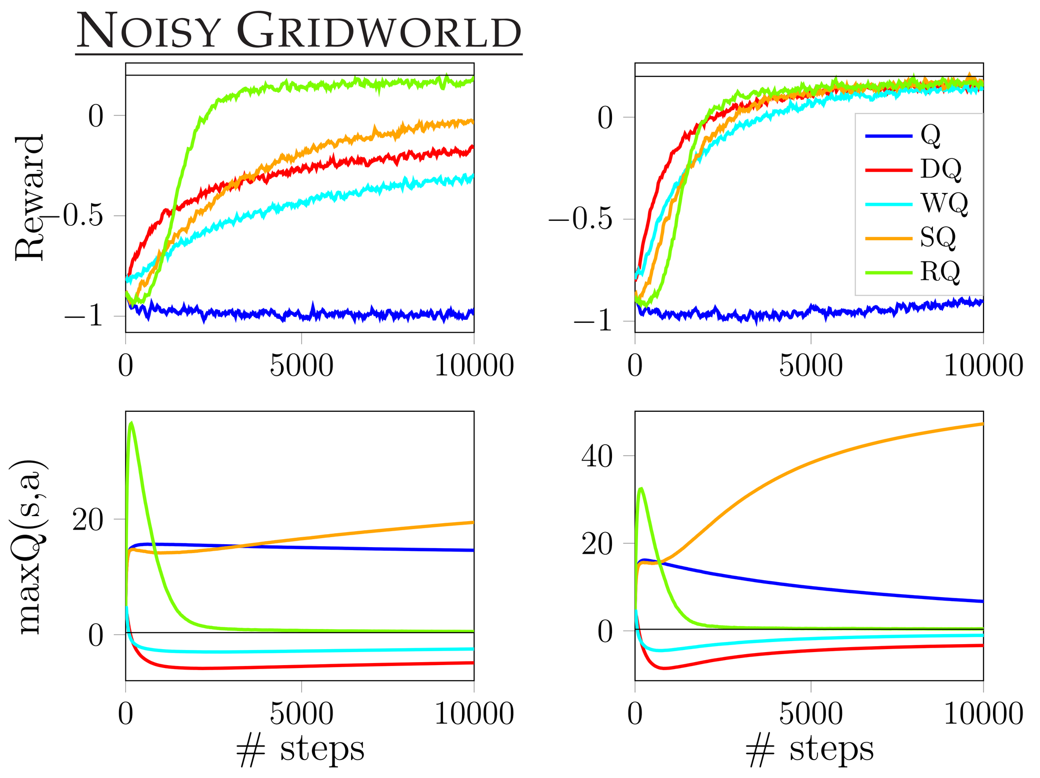
Learning rate on the variance of the estimation

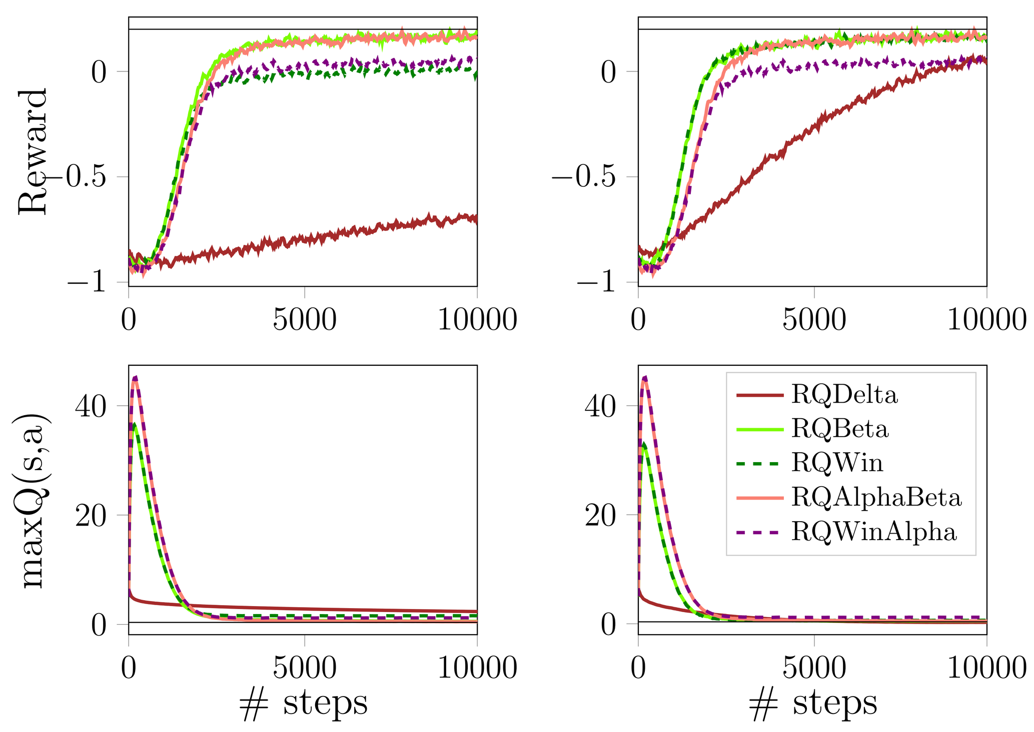
Exploit the variance of estimation to set the learning rate:

1. Estimate the variance of the estimator $\widetilde{Q}$, using the sample variance of the target:
   $\mathrm{Var}\left[\widetilde{Q}\right] = S_t^2\omega_t$
   $\omega_{t+1} = (1-\beta_t)^2\omega_t + \beta_t^2$
2. Compute the learning rate
   - Select a β that decreases when the estimate precision increases:
     $\beta_t = \dfrac{\sigma_e^2(t)}{\sigma_e^2(t)+\eta}$
   - Or, select a δ that increases when the estimate precision increases:
     $\delta_t = e^{\frac{\sigma_e^2}{\eta}\log\frac{1}{2}}$

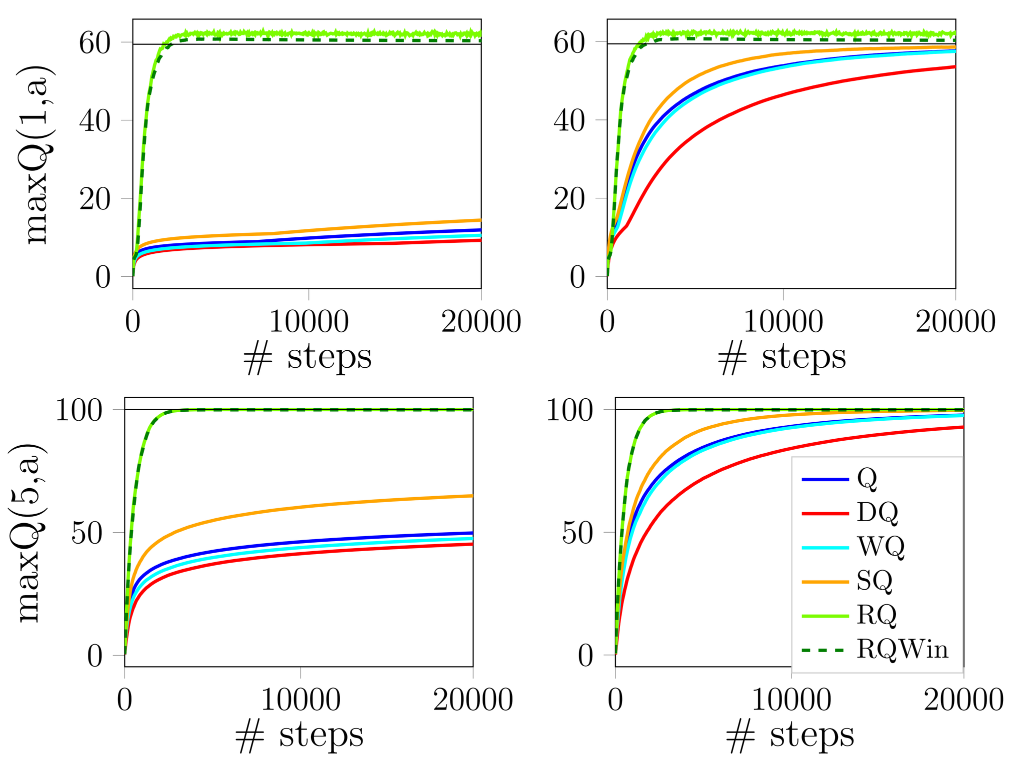
Empirical Results

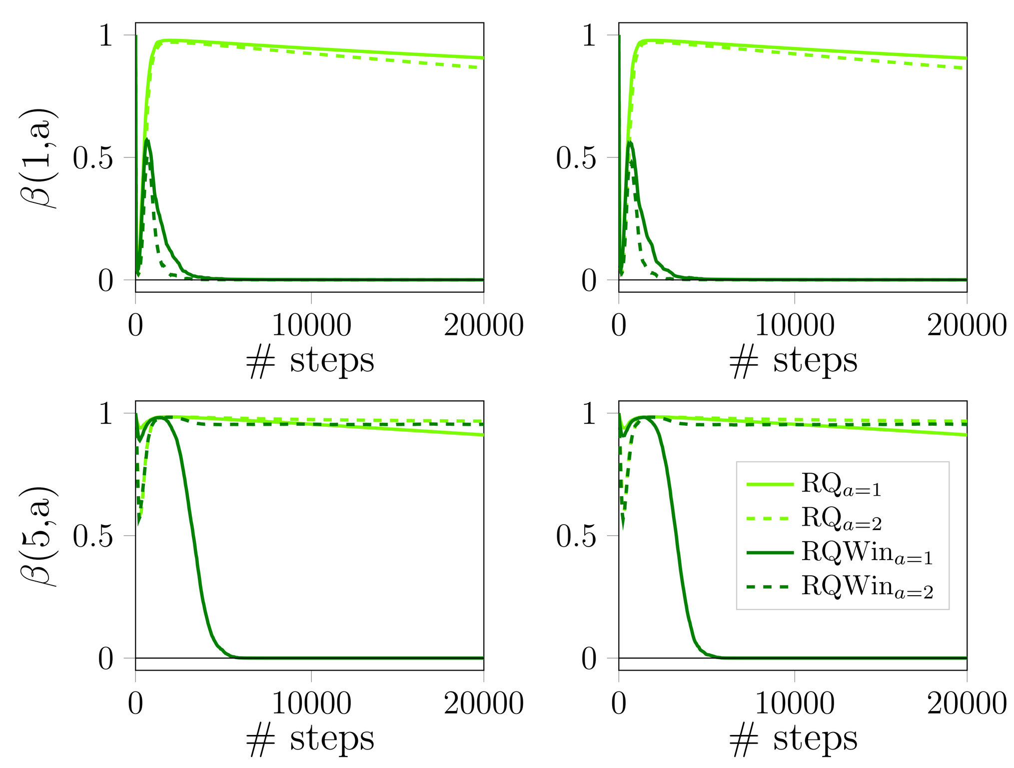
Noisy Gridworld





Double chain





Gridworld with Holes