Solutions of systems of linear equations

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1. **13.1 1)**
   Data:

   $$\begin{cases} x_1 = 4 + 2x_2 = 4 + 3x_3 = 4 \\ 3x_1 = 13 - 2x_2 = 13 + 5x_3 = 13 \\ x_1 = 4 + 2x_2 = 4 - 10x_3 = 4 \\ 2x_1 = 25 - 9x_2 = 25 + 8x_3 = 25 \end{cases}$$

   Solution by Gauss method:

   $$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & -2 & 5 & -13 \\ 1 & -2 & 10 & 4 \\ 2 & -9 & 8 & -25 \end{bmatrix} \Rightarrow$$

   Answers: No solutions

   Solution by Kramer method:

   No solution by Kramer method

2. **13.1 2)**
   Data:

   $$\begin{cases} x_1 = -2 + x_2 = -2 - x_3 = -2 \\ x_1 = 3 + 3x_2 = 3 + x_3 = 3 \\ x_1 = 3 - x_2 = 3 - 4x_3 = 3 \\ 3x_1 = -13 - 0x_2 = -13 + 8x_3 = -13 \end{cases}$$

   Solution by Gauss method:

   $$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 1 & 3 & -1 & 3 \\ 1 & 1 & -4 & -3 \\ 3 & 0 & 8 & 13 \end{bmatrix} \Rightarrow$$

   Answers: No solutions

   Solution by Kramer method:

   No solution by Kramer method

3. **13.1 3)**
   Data:

$$\begin{cases} 7x_1 = -4 + 6x_2 = -4 + 9x_3 = -4 + 4x_4 = -4 \\ x_1 = 6 + 0x_2 = 6 + 2x_3 = 6 - 6x_4 = 6 \\ 7x_1 = -6 + 8x_2 = -6 + 9x_3 = -6 + x_4 = -6 \\ x_1 = 5 + x_2 = 5 - 2x_3 = 5 - 4x_4 = 5 \\ 7x_1 = -2 - 0x_2 = -2 + 9x_3 = -2 - 2x_4 = -2 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 7 & 6 & 9 & 4 & -4 \\ 1 & 0 & -2 & 6 & 6 \\ 7 & 8 & 9 & -1 & -6 \\ 1 & -1 & -2 & 4 & 5 \\ 7 & 0 & 9 & -2 & 2 \end{bmatrix} \Rightarrow$$

Answers: No solutions

Solution by Kramer method:

No solution by Kramer method

4. **13.1 4)**

Data:

$$\begin{cases} 4x_1 = 8 + 4x_2 = 8 + x_3 = 8 - 0x_4 = 8 + x_5 = 8 \\ 2x_1 = 3 + 3x_2 = 3 + 7x_3 = 3 + 5x_4 = 3 + 2x_5 = 3 \\ 3x_1 = 2 + 2x_2 = 2 + 5x_3 = 2 + 7x_4 = 2 + 3x_5 = 2 \\ x_1 = 2 + 2x_2 = 2 + 2x_3 = 2 + x_4 = 2 + x_5 = 2 \\ x_1 = 7 + 7x_2 = 7 + 6x_3 = 7 + 6x_4 = 7 + 5x_5 = 7 \\ 2x_1 = 1 + x_2 = 1 + x_3 = 1 + 2x_4 = 1 + 2x_5 = 1 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 4 & 4 & -1 & 0 & -1 & 8 \\ 2 & 3 & 7 & 5 & 2 & 3 \\ 3 & 2 & 5 & 7 & 3 & 2 \\ 1 & 2 & 2 & 1 & 1 & 2 \\ 1 & 7 & 6 & 6 & 5 & 7 \\ 2 & 1 & 1 & 2 & 2 & 1 \end{bmatrix} \Rightarrow$$

Answers: No solutions

Solution by Kramer method:

No solution by Kramer method

5. **13.1 5)**

Data:

$$\begin{cases} x_1 = -4 + 5x_2 = -4 + 3x_3 = -4 + 5x_4 = -4 \\ 3x_1 = 8 + x_2 = 8 + 2x_3 = 8 + 9x_4 = 8 \\ x_1 = -9 - 7x_2 = -9 + 3x_3 = -9 - 8x_4 = -9 \\ 3x_1 = 7 + 4x_2 = 7 + 2x_3 = 7 + 4x_4 = 7 \\ x_1 = 5 + 8x_2 = 5 + 3x_3 = 5 + 3x_4 = 5 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 1 & 5 & 3 & 5 & -4 \\ 3 & 1 & 2 & 9 & 8 \\ 1 & -7 & 3 & -8 & 9 \\ 3 & 4 & 2 & 4 & 7 \\ 1 & 8 & 3 & 3 & 5 \end{bmatrix} \Rightarrow$$

Answers: No solutions

Solution by Kramer method:

No solution by Kramer method

6. **13.1 6)**

Data:

$$\begin{cases} 5x_1 = 16 - 7x_2 = 16 - 2x_3 = 16 - 2x_4 = 16 + 2x_5 = 16 \\ 0x_1 = 0 + 0x_2 = 0 + 4x_3 = 0 + 0x_4 = 0 + 5x_5 = 0 \\ 2x_1 = 0 + 0x_2 = 0 + 2x_3 = 0 - 0x_4 = 0 + 2x_5 = 0 \\ 6x_1 = -5 + 4x_2 = -5 + 6x_3 = -5 + x_4 = -5 - 15x_5 = -5 \\ 5x_1 = 6 + 4x_2 = 6 - 10x_3 = 6 + x_4 = 6 + 14x_5 = 6 \\ 3x_1 = 0 + 0x_2 = 0 + 2x_3 = 0 - 0x_4 = 0 + 3x_5 = 0 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 5 & 7 & 2 & -2 & 2 & -16 \\ 0 & 0 & 4 & 0 & -5 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 6 & 4 & 6 & -1 & 15 & -5 \\ 5 & -4 & 10 & 1 & 14 & 6 \\ 3 & 0 & -2 & 0 & 3 & 0 \end{bmatrix} \Rightarrow$$

Answers: No solutions

Solution by Kramer method:

No solution by Kramer method

7. **13.1 7)**

Data:

$$\begin{cases} 1001x_1 = 1004 + 1002x_2 = 1004 + 1003x_3 = 1004 \\ 1002x_1 = 1002 + 1003x_2 = 1002 + 1001x_3 = 1002 \\ 1001x_1 = 999 + 1001x_2 = 999 + 1001x_3 = 999 \\ 1001x_1 = 999 + 1000x_2 = 999 + 998x_3 = 999 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 1001 & 1002 & 1003 & 1004 \\ 1002 & 1003 & 1001 & 1002 \\ 1001 & 1001 & 1001 & 999 \\ 1001 & 1000 & 998 & 999 \end{bmatrix} \Rightarrow$$

Answers: No solutions

Solution by Kramer method:

No solution by Kramer method

8. **13.1 8)**
   Data:

$$\begin{cases} 27x_1 = 55 + 44x_2 = 55 + 40x_3 = 55 \\ 20x_1 = 40 + 64x_2 = 40 + 21x_3 = 40 \\ 13x_1 = 24 + 20x_2 = 24 - 13x_3 = 24 \\ 46x_1 = 84 + 45x_2 = 84 + 55x_3 = 84 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 27 & 44 & 40 & 55 \\ 20 & 64 & 21 & 40 \\ 13 & -20 & -13 & 24 \\ 46 & 45 & -55 & 84 \end{bmatrix} \Rightarrow$$

Answers: No solutions

Solution by Kramer method:

No solution by Kramer method

9. **13.1 9)**
   Data:

$$\begin{cases} 30x_1 = 12 + 20x_2 = 12 + 15x_3 = 12 \\ 20x_1 = 15 + 15x_2 = 15 + 12x_3 = 15 \\ 15x_1 = 20 + 12x_2 = 20 + 15x_3 = 20 \\ 12x_1 = 30 + 15x_2 = 30 + 20x_3 = 30 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 30 & 20 & 15 & 12 \\ 20 & 15 & 12 & 15 \\ 15 & 12 & 15 & 20 \\ 12 & 15 & 20 & 30 \end{bmatrix} \Rightarrow$$

Answers: No solutions

Solution by Kramer method:

No solution by Kramer method

10. **13.1 10)**
    Data:

$$\begin{cases} x_1 = \frac{1}{1} + x_2 = \frac{1}{1} + x_3 = \frac{1}{1} \\ x_1 = \frac{1}{2} + x_2 = \frac{1}{2} + x_3 = 1 \\ x_1 = \frac{1}{3} + x_2 = 1 + x_3 = \frac{1}{3} \\ x_1 = 1 + x_2 = \frac{1}{2} + x_3 = \frac{1}{2} \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 3 & 2 & 3 & 6 \\ 2 & 3 & 6 & 3 \\ 3 & 6 & 3 & 2 \\ 6 & 3 & 2 & 3 \end{bmatrix} \Rightarrow$$

Answers: No solutions

Solution by Kramer method:

No solution by Kramer method

11. **13.1 11)**
Data:

$$\begin{cases} x_1 = 100000 + 10x_2 = 100000 + 100x_3 = 100000 + 1000x_4 = 100000 + 10000x_5 = 100000 \\ x_1 = \frac{60000}{1} + 2x_2 = 60000 + 30x_3 = 60000 + 400x_4 = 60000 + 5000x_5 = 60000 \\ 0x_1 = 15000 + x_2 = \frac{15000}{1} + 3x_3 = 15000 + 60x_4 = 15000 + 1000x_5 = 15000 \\ 0x_1 = 2000 + 0x_2 = 2000 + x_3 = \frac{2000}{1} + 4x_4 = 2000 + 100x_5 = 2000 \\ 0x_1 = 150 + 0x_2 = 150 + 0x_3 = 150 + x_4 = \frac{150}{1} + 5x_5 = 150 \\ 0x_1 = 6 + 0x_2 = 6 + 0x_3 = 6 + 0x_4 = 6 + x_5 = \frac{6}{1} \end{cases}$$

Solution by Gauss method:

$$\left[\begin{array}{cccccc} 1 & 10 & 100 & 1000 & 10000 & 100000 \\ 1 & 20 & 300 & 4000 & 50000 & 600000 \\ 0 & 1 & 30 & 600 & 10000 & 150000 \\ 0 & 0 & 1 & 40 & 1000 & 20000 \\ 0 & 0 & 0 & 1 & 50 & 1500 \\ 0 & 0 & 0 & 0 & 1 & 60 \end{array}\right] \Rightarrow$$

Answers: No solutions

Solution by Kramer method:

No solution by Kramer method

12. **13.1 12)**
Data:

$$\begin{cases} 4x_1 = 5 + 2x_2 = 5 - 0x_3 = 5 \\ 3x_1 = 1 + 2x_2 = 1 + 2x_3 = 1 \\ 2x_1 = -1 - x_2 = -1 + 3x_3 = -1 \\ 2x_1 = -3 + 3x_2 = -3 + 6x_3 = -3 \end{cases}$$

Solution by Gauss method:

$$\left[\begin{array}{cccc} 4 & -2 & 0 & 5 \\ 3 & 2 & -2 & 1 \\ 2 & -1 & -3 & 1 \\ 2 & 3 & -6 & -3 \end{array}\right] \Rightarrow$$

Answers: No solutions

Solution by Kramer method:

No solution by Kramer method

13. **13.1 13)**
Data:

$$\begin{cases} 4x_1 = 5 + 3x_2 = 5 + 3x_3 = 5 \\ 3x_1 = 2 + 4x_2 = 2 + 3x_3 = 2 \\ 3x_1 = 4 + 2x_2 = 4 + 5x_3 = 4 \\ 2x_1 = 3 + 4x_2 = 3 + 2x_3 = 3 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 4 & 3 & 3 & 5 \\ 3 & 4 & 3 & 2 \\ 3 & 2 & 5 & 4 \\ 2 & 4 & 2 & 3 \end{bmatrix} \Rightarrow$$

Answers: No solutions

Solution by Kramer method:

No solution by Kramer method

14. **13.1 14)**
Data:

$$\begin{cases} 3x_1 = 5 + 2x_2 = 5 + 4x_3 = 5 \\ 4x_1 = -4 + 3x_2 = -4 - 2x_3 = -4 \\ 5x_1 = -7 + 2x_2 = -7 - 3x_3 = -7 \\ 3x_1 = 9 - 4x_2 = 9 + 2x_3 = 9 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 3 & 2 & 4 & 5 \\ 4 & -3 & 2 & -4 \\ 5 & -2 & -3 & -7 \\ 3 & -4 & -2 & -9 \end{bmatrix} \Rightarrow$$

Answers: No solutions

Solution by Kramer method:

No solution by Kramer method

15. **13.1 15)**
Data:

$$\begin{cases} 14x_1 = -13 + 13x_2 = -13 + 3x_3 = -13 \\ 7x_1 = 10 - 4x_2 = 10 - 2x_3 = 10 \\ 21x_1 = -23 + 23x_2 = -23 + 0x_3 = -23 \\ 7x_1 = -6 + 12x_2 = -6 + 2x_3 = -6 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 14 & 13 & 3 & -13 \\ 7 & 4 & -2 & -10 \\ 21 & 23 & 0 & -23 \\ 7 & 12 & -2 & -6 \end{bmatrix} \Rightarrow$$

Answers: No solutions

Solution by Kramer method:

No solution by Kramer method

16. **13.1 16)**
Data:

$$\begin{cases} 6x_1 = -4 + 3x_2 = -4 + 8x_3 = -4 \\ 5x_1 = 2 + 6x_2 = 2 + 4x_3 = 2 \\ 0x_1 = 2 + 3x_2 = 2 + 4x_3 = 2 \\ 4x_1 = 6 + x_2 = 6 + 4x_3 = 6 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 6 & 3 & 8 & -4 \\ 5 & 6 & 4 & 2 \\ 0 & 3 & 4 & 2 \\ 4 & 1 & -4 & 6 \end{bmatrix} \Rightarrow$$

Answers: No solutions

Solution by Kramer method:

No solution by Kramer method

17. **13.1 17)**
Data:

$$\begin{cases} 2x_1 = -5 + 4x_2 = -5 + 6x_3 = -5 \\ x_1 = 4 + 6x_2 = 4 + 5x_3 = 4 \\ 3x_1 = 6 - 2x_2 = 6 + 4x_3 = 6 \\ 4x_1 = 3 + 5x_2 = 3 + 2x_3 = 3 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 2 & 4 & 6 & -5 \\ 1 & 6 & 5 & 4 \\ 3 & -2 & -4 & -6 \\ 4 & 5 & 2 & 3 \end{bmatrix} \Rightarrow$$

Answers: No solutions

Solution by Kramer method:

No solution by Kramer method

18. **2**
Data:

$$\begin{cases} 2x_1 = 8 + 5x_2 = 8 + 8x_3 = 8 \\ 4x_1 = 9 + 3x_2 = 9 + 9x_3 = 9 \\ 2x_1 = 7 + 3x_2 = 7 + 5x_3 = 7 \\ x_1 = 12 + 8x_2 = 12 + 7x_3 = 12 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 2 & 5 & -8 & 8 \\ 4 & 3 & -9 & 9 \\ 2 & 3 & -5 & 7 \\ 1 & 8 & -7 & 12 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & 5 & -8 & 8 \\ 0 & 1 & -1 & 1 \\ 0 & 2 & -3 & 1 \\ 0 & 11 & -6 & 16 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & 5 & -8 & 8 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & 5 & -8 & 8 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Answers: $x1 = 3$, $x2 = 2$, $x3 = 1$

Solution by Kramer method:

No solution by Kramer method

19. **1**

Data:

$$\begin{cases} 6x_1 = 1 + 4x_2 = 1 + 5x_3 = 1 + 2x_4 = 1 + 3x_5 = 1 \\ 3x_1 = -7 + 2x_2 = -7 + 2x_3 = -7 - x_4 = -7 + 0x_5 = -7 \\ 9x_1 = 2 + 6x_2 = 2 + x_3 = 2 + 3x_4 = 2 + 2x_5 = 2 \\ 3x_1 = 3 + 2x_2 = 3 + 4x_3 = 3 + x_4 = 3 + 2x_5 = 3 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 6 & 4 & 5 & 2 & 3 & 1 \\ 3 & 2 & -2 & 1 & 0 & -7 \\ 9 & 6 & 1 & 3 & 2 & 2 \\ 3 & 2 & 4 & 1 & 2 & 3 \end{bmatrix} \Rightarrow$$

Answers: Infinity of solutions

Solution by Kramer method:

No solution by Kramer method

20. **Test**

Data:

$$\begin{cases} x_1 = 1 + x_2 = 1 + x_3 = 1 + x_4 = 1 \\ 2x_1 = 2 + 2x_2 = 2 + 2x_3 = 2 + 2x_4 = 2 \\ 3x_1 = 3 + 3x_2 = 3 + 3x_3 = 3 + 3x_4 = 3 \\ 4x_1 = 5 + 4x_2 = 5 + 4x_3 = 5 + 4x_4 = 5 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 4 & 4 & 4 & 4 & 5 \end{bmatrix} \Rightarrow$$

Answers: No solutions

Solution by Kramer method:

Answers: No solution

21. **353**

Data:

$$\begin{cases} 2x_1 = 0 + 3x_2 = 0 - 4x_3 = 0 + 3x_4 = 0 \\ 3x_1 = 0 + x_2 = 0 - 11x_3 = 0 + 13x_4 = 0 \\ 4x_1 = 0 + 5x_2 = 0 + 7x_3 = 0 - 2x_4 = 0 \\ 13x_1 = 0 + 25x_2 = 0 - x_3 = 0 + 11x_4 = 0 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 2 & -3 & 4 & -3 & 0 \\ 3 & -1 & 11 & -13 & 0 \\ 4 & 5 & -7 & -2 & 0 \\ 13 & -25 & 1 & 11 & 0 \end{bmatrix} \Rightarrow$$

Answers: Infinity of solutions

Solution by Kramer method:

Answers: No solution

22. **335**

Data:

$$\begin{cases} 2x_1 = 4 + x_2 = 4 - x_3 = 4 \\ 3x_1 = 11 + 4x_2 = 11 + 2x_3 = 11 \\ 3x_1 = 11 + 2x_2 = 11 - 4x_3 = 11 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 2 & -1 & -1 & 4 \\ 3 & 4 & -2 & 11 \\ 3 & -2 & 4 & 11 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & -1 & -1 & 4 \\ 0 & 11 & -1 & 10 \\ 0 & 1 & -11 & -10 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & -1 & -1 & 4 \\ 0 & 11 & -1 & 10 \\ 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & -1 & -1 & 4 \\ 0 & 11 & -1 & 10 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Answers: $x1 = 3, \quad x2 = 1, \quad x3 = 1$

Solution by Kramer method:

9

$$\Delta : \begin{vmatrix} 2 & -1 & -1 & 4 \\ 3 & 4 & -2 & 11 \\ 3 & -2 & 4 & 11 \end{vmatrix} =$$

$$\overline{(3)*(2)*(2)} \begin{vmatrix} 6 & -3 & -3 \\ 0 & 11 & -1 \\ 0 & -1 & 11 \end{vmatrix} =$$

$$\overline{(3)*(2)*(2)*(-11)} \begin{vmatrix} 6 & -3 & -3 \\ 0 & 11 & -1 \\ 0 & 0 & -120 \end{vmatrix} =$$

60

$$\Delta_1 : \frac{(3)*(3)}{(11)*(4)*(4)} \begin{vmatrix} 44 & -11 & -11 \\ 0 & 9 & 1 \\ 0 & 1 & 9 \end{vmatrix} =$$

$$\frac{(3)*(3)*(80)}{(11)*(4)*(4)*(9)} \begin{vmatrix} 44 & -11 & -11 \\ 0 & 9 & 1 \\ 0 & 0 & 1 \end{vmatrix} =$$

180

$$x_0 = \frac{\Delta_0}{\Delta} = 3$$

$$\Delta_2 : \frac{1}{(3)*(2)*(2)} \begin{vmatrix} 6 & 12 & -3 \\ 0 & 10 & -1 \\ 0 & 10 & 11 \end{vmatrix} =$$

$$\frac{(12)}{(3)*(2)*(2)} \begin{vmatrix} 6 & 12 & -3 \\ 0 & 10 & -1 \\ 0 & 0 & 1 \end{vmatrix} =$$

60

$$x_1 = \frac{\Delta_1}{\Delta} = 1$$

$$\Delta_3 : \frac{(3)}{(3)*(2)*(2)} \begin{vmatrix} 2 & -1 & 4 \\ 0 & 11 & 10 \\ 0 & -1 & 10 \end{vmatrix} =$$

$$\frac{(3)}{(3)*(2)*(2)*(-11)} \begin{vmatrix} 2 & -1 & 4 \\ 0 & 11 & 10 \\ 0 & 0 & -120 \end{vmatrix} =$$

60

$$x_2 = \frac{\Delta_2}{\Delta} = 1$$

Answers: $x1 = 3, \quad x2 = 1, \quad x3 = 1$

23. **336**

Data:

$$\begin{cases} x_1 = -1 + x_2 = -1 + 2x_3 = -1 \\ 2x_1 = -4 + x_2 = -4 - 2x_3 = -4 \\ 4x_1 = -2 + x_2 = -2 + 4x_3 = -2 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 1 & 1 & 2 & -1 \\ 2 & -1 & 2 & -4 \\ 4 & 1 & 4 & -2 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 2 & -1 \\ 0 & 3 & 2 & 2 \\ 0 & 3 & 4 & -2 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 2 & -1 \\ 0 & 3 & 2 & 2 \\ 0 & 0 & 1 & -2 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 2 & -1 \\ 0 & 3 & 2 & 2 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

Answers: $x1 = 1, \quad x2 = 2, \quad x3 = -2$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 1 & 1 & 2 & -1 \\ 2 & -1 & 2 & -4 \\ 4 & 1 & 4 & -2 \end{vmatrix} =$$

$$\frac{(4)}{(4)*(2)} \begin{vmatrix} 1 & 1 & 2 \\ 0 & -6 & -4 \\ 0 & -3 & -4 \end{vmatrix} =$$

$$\frac{(4)}{(4)*(2)*(2)} \begin{vmatrix} 1 & 1 & 2 \\ 0 & -6 & -4 \\ 0 & 0 & -4 \end{vmatrix} =$$

6

$$\Delta_1 : \frac{(4)}{(4)*(2)} \begin{vmatrix} -1 & 1 & 2 \\ 0 & -5 & -6 \\ 0 & -2 & 0 \end{vmatrix} =$$

$$\frac{(4)*(12)}{(4)*(2)*(2)*(5)} \begin{vmatrix} -1 & 1 & 2 \\ 0 & -10 & -12 \\ 0 & 0 & 1 \end{vmatrix} =$$

6

$$x_0 = \frac{\Delta_0}{\Delta} = 1$$

$$\Delta_2 : \frac{(4)}{(4)*(2)} \begin{vmatrix} 1 & -1 & 2 \\ 0 & -4 & -4 \\ 0 & 2 & -4 \end{vmatrix} =$$

$$\frac{(4)*(12)}{(4)*(2)*(-2)} \begin{vmatrix} 1 & -1 & 2 \\ 0 & -4 & -4 \\ 0 & 0 & 1 \end{vmatrix} =$$

12

$$x_1 = \frac{\Delta_1}{\Delta} = 2$$

$$\Delta_3 : \frac{}{(4)*(2)} \begin{vmatrix} 4 & 4 & -4 \\ 0 & -6 & -4 \\ 0 & -3 & 2 \end{vmatrix} =$$

$$\frac{(8)}{(4)*(2)*(2)} \begin{vmatrix} 4 & 4 & -4 \\ 0 & -6 & -4 \\ 0 & 0 & 1 \end{vmatrix} =$$

-12

$x_2 = \frac{\Delta_2}{\Delta} = -2$

Answers: $x1 = 1, \quad x2 = 2, \quad x3 = -2$

24. **337**

Data:

$$\begin{cases} 3x_1 = 5 + 2x_2 = 5 + x_3 = 5 \\ 2x_1 = 1 + 3x_2 = 1 + x_3 = 1 \\ 2x_1 = 11 + x_2 = 11 + 3x_3 = 11 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 3 & 2 & 1 & 5 \\ 2 & 3 & 1 & 1 \\ 2 & 1 & 3 & 11 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 3 & 2 & 1 & 5 \\ 0 & 5 & 1 & -7 \\ 0 & 1 & -7 & -23 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 3 & 2 & 1 & 5 \\ 0 & 5 & 1 & -7 \\ 0 & 0 & 1 & 3 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 3 & 2 & 1 & 5 \\ 0 & 5 & 1 & -7 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

Answers: $x1 = 2, \quad x2 = -2, \quad x3 = 3$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 3 & 2 & 1 & 5 \\ 2 & 3 & 1 & 1 \\ 2 & 1 & 3 & 11 \end{vmatrix} =$$

$$\frac{(2)}{(2)*(3)*(3)} \begin{vmatrix} 3 & 2 & 1 \\ 0 & 5 & 1 \\ 0 & -1 & 7 \end{vmatrix} =$$

$$\frac{(2)}{(2)*(3)*(3)*(-5)} \begin{vmatrix} 3 & 2 & 1 \\ 0 & 5 & 1 \\ 0 & 0 & -36 \end{vmatrix} =$$

12

$$\Delta_1 : \frac{(11)*(11)}{(11)*(55)*(5)} \begin{vmatrix} 5 & 2 & 1 \\ 0 & 13 & 4 \\ 0 & -17 & 4 \end{vmatrix} =$$

$$\frac{(11)*(11)*(17)}{(11)*(55)*(5)*(17)*(-13)} \begin{vmatrix} 5 & 2 & 1 \\ 0 & 13 & 4 \\ 0 & 0 & -120 \end{vmatrix} =$$

24

$$x_0 = \frac{\Delta_0}{\Delta} = 2$$

$$\Delta_2 : \frac{(2)}{(2)*(3)*(3)} \begin{vmatrix} 3 & 5 & 1 \\ 0 & -7 & 1 \\ 0 & 23 & 7 \end{vmatrix} =$$

$$\frac{(2)*(23)}{(2)*(3)*(3)*(23)*(-7)} \begin{vmatrix} 3 & 5 & 1 \\ 0 & -7 & 1 \\ 0 & 0 & -72 \end{vmatrix} =$$

-24

$x_1 = \frac{\Delta_1}{\Delta} = -2$

$$\Delta_3 : \frac{(2)}{(2)*(3)*(3)} \begin{vmatrix} 3 & 2 & 5 \\ 0 & 5 & -7 \\ 0 & -1 & 23 \end{vmatrix} =$$

$$\frac{(2)}{(2)*(3)*(3)*(-5)} \begin{vmatrix} 3 & 2 & 5 \\ 0 & 5 & -7 \\ 0 & 0 & -108 \end{vmatrix} =$$

36

$x_2 = \frac{\Delta_2}{\Delta} = 3$

Answers: $x1 = 2, \quad x2 = -2, \quad x3 = 3$

25. **338**

Data:

$$\begin{cases} x_1 = 31 + 2x_2 = 31 + 4x_3 = 31 \\ 5x_1 = 29 + x_2 = 29 + 2x_3 = 29 \\ 3x_1 = 10 + x_2 = 10 - x_3 = 10 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 1 & 2 & 4 & 31 \\ 5 & 1 & 2 & 29 \\ 3 & -1 & 1 & 10 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 4 & 31 \\ 0 & 1 & 2 & 14 \\ 0 & 7 & 11 & 83 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 4 & 31 \\ 0 & 1 & 2 & 14 \\ 0 & 0 & 1 & 5 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 4 & 31 \\ 0 & 1 & 2 & 14 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

Answers: $x1 = 3, \quad x2 = 4, \quad x3 = 5$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 1 & 2 & 4 & 31 \\ 5 & 1 & 2 & 29 \\ 3 & -1 & 1 & 10 \end{vmatrix} =$$

$$\frac{(15)}{(15)*(3)*(5)} \begin{vmatrix} 1 & 2 & 4 \\ 0 & -27 & -54 \\ 0 & -35 & -55 \end{vmatrix} =$$

$$\frac{(15)*(405)}{(15)*(3)*(5)*(35)*(27)} \begin{vmatrix} 1 & 2 & 4 \\ 0 & -945 & -1890 \\ 0 & 0 & 1 \end{vmatrix} =$$

-27

$$\Delta_1 : \frac{(290)}{(290)*(310)*(899)} \begin{vmatrix} 31 & 2 & 4 \\ 0 & -270 & -540 \\ 0 & -1479 & -261 \end{vmatrix} =$$

$$\frac{(290)*(242730)}{(290)*(310)*(899)*(493)*(90)} \begin{vmatrix} 31 & 2 & 4 \\ 0 & -133110 & -266220 \\ 0 & 0 & 1 \end{vmatrix} =$$

-81

$x_0 = \frac{\Delta_0}{\Delta} = 3$

$$\Delta_2 : \frac{(15)}{(15)*(3)*(5)} \begin{vmatrix} 1 & 31 & 4 \\ 0 & -378 & -54 \\ 0 & -415 & -55 \end{vmatrix} =$$

$$\frac{(15)*(1620)}{(15)*(3)*(5)*(415)*(378)} \begin{vmatrix} 1 & 31 & 4 \\ 0 & -156870 & -22410 \\ 0 & 0 & 1 \end{vmatrix} =$$

-108

$x_1 = \frac{\Delta_1}{\Delta} = 4$

$$\Delta_3 : \frac{(15)}{(15)*(3)*(5)} \begin{vmatrix} 1 & 2 & 31 \\ 0 & -27 & -378 \\ 0 & -35 & -415 \end{vmatrix} =$$

$$\frac{(15)*(2025)}{(15)*(3)*(5)*(35)*(27)} \begin{vmatrix} 1 & 2 & 31 \\ 0 & -945 & -13230 \\ 0 & 0 & 1 \end{vmatrix} =$$

-135

$x_2 = \frac{\Delta_2}{\Delta} = 5$

Answers: $x1 = 3, \quad x2 = 4, \quad x3 = 5$

26. **339**

   Data:

$$\begin{cases} x_1 = 1 + x_2 = 1 + 2x_3 = 1 + 3x_4 = 1 \\ 3x_1 = -4 + x_2 = -4 - x_3 = -4 - 2x_4 = -4 \\ 2x_1 = -6 + 3x_2 = -6 + x_3 = -6 - x_4 = -6 \\ x_1 = -4 + 2x_2 = -4 + 3x_3 = -4 + x_4 = -4 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 1 & 1 & 2 & 3 & 1 \\ 3 & -1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 & -6 \\ 1 & 2 & 3 & -1 & -4 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 2 & 3 & 1 \\ 0 & 4 & 7 & 11 & 7 \\ 0 & 1 & -5 & -7 & -8 \\ 0 & 1 & 1 & -4 & -5 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 2 & 3 & 1 \\ 0 & 4 & 7 & 11 & 7 \\ 0 & 0 & 9 & 13 & 13 \\ 0 & 0 & 1 & 9 & 9 \end{bmatrix} \Rightarrow$$

14

$$\begin{bmatrix} 1 & 1 & 2 & 3 & 1 \\ 0 & 4 & 7 & 11 & 7 \\ 0 & 0 & 9 & 13 & 13 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 2 & 3 & 1 \\ 0 & 4 & 7 & 11 & 7 \\ 0 & 0 & 9 & 13 & 13 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Answers: $x1 = -1,\quad x2 = -1,\quad x3 = 0,\quad x4 = 1$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 1 & 1 & 2 & 3 & 1 \\ 3 & -1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 & -6 \\ 1 & 2 & 3 & -1 & -4 \end{vmatrix} =$$

$$\frac{(6)}{(6)*(2)*(3)*(6)} \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & -8 & -14 & -22 \\ 0 & 3 & -15 & -21 \\ 0 & 6 & 6 & -24 \end{vmatrix} =$$

$$\frac{(6)*(18)*(18)}{(6)*(2)*(3)*(6)*(3)*(-8)*(-4)} \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & -24 & -42 & -66 \\ 0 & 0 & 9 & 13 \\ 0 & 0 & 1 & 9 \end{vmatrix} =$$

$$\frac{(6)*(18)*(18)*(68)}{(6)*(2)*(3)*(6)*(3)*(-8)*(-4)*(9)} \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & -24 & -42 & -66 \\ 0 & 0 & 9 & 13 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

$$-153$$

$$\Delta_1 : \frac{(12)}{(12)*(-3)*(-2)*(-3)} \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & -9 & -21 & -30 \\ 0 & -18 & -22 & -34 \\ 0 & -18 & -33 & -33 \end{vmatrix} =$$

$$\frac{(12)*(2)*(9)}{(12)*(-3)*(-2)*(-3)*(2)} \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & -18 & -42 & -60 \\ 0 & 0 & 10 & 13 \\ 0 & 0 & 1 & 3 \end{vmatrix} =$$

$$\frac{(12)*(2)*(9)*(17)}{(12)*(-3)*(-2)*(-3)*(2)*(10)} \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & -18 & -42 & -60 \\ 0 & 0 & 10 & 13 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

$$153$$

$$x_0 = \frac{\Delta_0}{\Delta} = -1$$

$$\Delta_2 : \frac{(6)}{(6)*(2)*(3)*(6)} \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & -14 & -14 & -22 \\ 0 & -24 & -15 & -21 \\ 0 & -30 & 6 & -24 \end{vmatrix} =$$

$$\frac{(6)*(45)*(72)}{(6)*(2)*(3)*(6)*(60)*(35)*(28)} \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & -840 & -840 & -1320 \\ 0 & 0 & 7 & 13 \\ 0 & 0 & 14 & 9 \end{vmatrix} =$$

$$\frac{(6)*(45)*(72)*(2)}{(6)*(2)*(3)*(6)*(60)*(35)*(28)*(2)} \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & -840 & -840 & -1320 \\ 0 & 0 & 7 & 13 \\ 0 & 0 & 0 & -17 \end{vmatrix} =$$

153

$$x_1 = \frac{\Delta_1}{\Delta} = -1$$

$$\Delta_3 : \frac{(6)}{(6)*(2)*(3)*(6)} \begin{vmatrix} 1 & 1 & 1 & 3 \\ 0 & -8 & -14 & -22 \\ 0 & 3 & -24 & -21 \\ 0 & 6 & -30 & -24 \end{vmatrix} =$$

$$\frac{(6)*(234)*(162)}{(6)*(2)*(3)*(6)*(3)*(-8)*(-4)} \begin{vmatrix} 1 & 1 & 1 & 3 \\ 0 & -24 & -42 & -66 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} =$$

$$\frac{(6)*(234)*(162)}{(6)*(2)*(3)*(6)*(3)*(-8)*(-4)} \begin{vmatrix} 1 & 1 & 1 & 3 \\ 0 & -24 & -42 & -66 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{vmatrix} =$$

0

$$x_2 = \frac{\Delta_2}{\Delta} = 0$$

$$\Delta_4 : \frac{(6)}{(6)*(2)*(3)*(6)} \begin{vmatrix} 1 & 1 & 2 & 1 \\ 0 & -8 & -14 & -14 \\ 0 & 3 & -15 & -24 \\ 0 & 6 & 6 & -30 \end{vmatrix} =$$

$$\frac{(6)*(18)*(18)}{(6)*(2)*(3)*(6)*(3)*(-8)*(-4)} \begin{vmatrix} 1 & 1 & 2 & 1 \\ 0 & -24 & -42 & -42 \\ 0 & 0 & 9 & 13 \\ 0 & 0 & 1 & 9 \end{vmatrix} =$$

$$\frac{(6)*(18)*(18)*(68)}{(6)*(2)*(3)*(6)*(3)*(-8)*(-4)*(9)} \begin{vmatrix} 1 & 1 & 2 & 1 \\ 0 & -24 & -42 & -42 \\ 0 & 0 & 9 & 13 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

-153

$$x_3 = \frac{\Delta_3}{\Delta} = 1$$

Answers: $x1 = -1, \quad x2 = -1, \quad x3 = 0, \quad x4 = 1$

27. **340**

Data:

$$\begin{cases} x_1 = 6 + 2x_2 = 6 + 3x_3 = 6 + 2x_4 = 6 \\ 2x_1 = 8 + x_2 = 8 - 2x_3 = 8 - 3x_4 = 8 \\ 3x_1 = 4 + 2x_2 = 4 + x_3 = 4 - 2x_4 = 4 \\ 2x_1 = -8 + 3x_2 = -8 - 2x_3 = -8 + x_4 = -8 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 1 & 2 & 3 & -2 & 6 \\ 2 & -1 & -2 & -3 & 8 \\ 3 & 2 & -1 & 2 & 4 \\ 2 & -3 & 2 & 1 & -8 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & -2 & 6 \\ 0 & 5 & 8 & -1 & 4 \\ 0 & 2 & 5 & -4 & 7 \\ 0 & 7 & 4 & -5 & 20 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & -2 & 6 \\ 0 & 5 & 8 & -1 & 4 \\ 0 & 0 & 1 & -2 & 3 \\ 0 & 0 & 2 & 1 & -4 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & -2 & 6 \\ 0 & 5 & 8 & -1 & 4 \\ 0 & 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & -2 & 6 \\ 0 & 5 & 8 & -1 & 4 \\ 0 & 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

Answers: $x1 = 1, \quad x2 = 2, \quad x3 = -1, \quad x4 = -2$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 1 & 2 & 3 & -2 & 6 \\ 2 & -1 & -2 & -3 & 8 \\ 3 & 2 & -1 & 2 & 4 \\ 2 & -3 & 2 & 1 & -8 \end{vmatrix} =$$

$$\frac{(3)*(4)*(3)}{(6)*(3)*(2)*(3)} \begin{vmatrix} 6 & 12 & 18 & -12 \\ 0 & -5 & -8 & 1 \\ 0 & -2 & -5 & 4 \\ 0 & -7 & -4 & 5 \end{vmatrix} =$$

$$\frac{(3)*(4)*(3)*(14)*(63)*(36)}{(6)*(3)*(2)*(3)*(14)*(35)*(10)} \begin{vmatrix} 6 & 12 & 18 & -12 \\ 0 & -5 & -8 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 2 & 1 \end{vmatrix} =$$

$$\frac{(3)*(4)*(3)*(14)*(63)*(36)*(2)}{(6)*(3)*(2)*(3)*(14)*(35)*(10)*(2)*(-1)} \begin{vmatrix} 6 & 12 & 18 & -12 \\ 0 & -5 & -8 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & -5 \end{vmatrix} =$$

324

$$\Delta_1 : \frac{(2)}{(4)*(3)*(6)*(-3)} \begin{vmatrix} 24 & 8 & 12 & -8 \\ 0 & -11 & -18 & -1 \\ 0 & 2 & -9 & 10 \\ 0 & 1 & -18 & 5 \end{vmatrix} =$$

$$\frac{(2)}{(4)*(3)*(6)*(-3)*(2)*(-11)*(-22)} \begin{vmatrix} 24 & 8 & 12 & -8 \\ 0 & -22 & -36 & -2 \\ 0 & 0 & 135 & -108 \\ 0 & 0 & 432 & -108 \end{vmatrix} =$$

17

$$\frac{(2)*(1188)}{(4)*(3)*(6)*(-3)*(2)*(-11)*(-22)*(16)*(5)} \begin{vmatrix} 24 & 8 & 12 & -8 \\ 0 & -22 & -36 & -2 \\ 0 & 0 & 2160 & -1728 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

324

$x_0 = \frac{\Delta_0}{\Delta} = 1$

$$\Delta_2 : \frac{(3)*(4)*(3)}{(6)*(3)*(2)*(3)} \begin{vmatrix} 6 & 36 & 18 & -12 \\ 0 & -4 & -8 & 1 \\ 0 & -7 & -5 & 4 \\ 0 & -20 & -4 & 5 \end{vmatrix} =$$

$$\frac{(3)*(4)*(3)*(35)*(45)*(252)}{(6)*(3)*(2)*(3)*(35)*(20)*(7)} \begin{vmatrix} 6 & 36 & 18 & -12 \\ 0 & -4 & -8 & 1 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} =$$

$$\frac{(3)*(4)*(3)*(35)*(45)*(252)}{(6)*(3)*(2)*(3)*(35)*(20)*(7)*(4)} \begin{vmatrix} 6 & 36 & 18 & -12 \\ 0 & -4 & -8 & 1 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & -1 \end{vmatrix} =$$

648

$x_1 = \frac{\Delta_1}{\Delta} = 2$

$$\Delta_3 : \frac{(3)*(4)*(3)}{(6)*(3)*(2)*(3)} \begin{vmatrix} 6 & 12 & 36 & -12 \\ 0 & -5 & -4 & 1 \\ 0 & -2 & -7 & 4 \\ 0 & -7 & -20 & 5 \end{vmatrix} =$$

$$\frac{(3)*(4)*(3)*(14)*(63)*(36)}{(6)*(3)*(2)*(3)*(14)*(35)*(10)} \begin{vmatrix} 6 & 12 & 36 & -12 \\ 0 & -5 & -4 & 1 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -4 & 1 \end{vmatrix} =$$

$$\frac{(3)*(4)*(3)*(14)*(63)*(36)*(4)}{(6)*(3)*(2)*(3)*(14)*(35)*(10)*(4)*(3)} \begin{vmatrix} 6 & 12 & 36 & -12 \\ 0 & -5 & -4 & 1 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & -5 \end{vmatrix} =$$

-324

$x_2 = \frac{\Delta_2}{\Delta} = -1$

$$\Delta_4 : \frac{(6)}{(6)*(3)*(2)*(3)} \begin{vmatrix} 1 & 2 & 3 & 6 \\ 0 & -15 & -24 & -12 \\ 0 & -8 & -20 & -28 \\ 0 & -21 & -12 & -60 \end{vmatrix} =$$

$$\frac{(6)}{(6)*(3)*(2)*(3)*(56)*(105)*(40)} \begin{vmatrix} 1 & 2 & 3 & 6 \\ 0 & -840 & -1344 & -672 \\ 0 & 0 & -756 & -2268 \\ 0 & 0 & 864 & -1728 \end{vmatrix} =$$

$$\frac{(6)*(30240)}{(6)*(3)*(2)*(3)*(56)*(105)*(40)*(8)*(-7)} \begin{vmatrix} 1 & 2 & 3 & 6 \\ 0 & -840 & -1344 & -672 \\ 0 & 0 & -6048 & -18144 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

-648

$x_3 = \frac{\Delta_3}{\Delta} = -2$

Answers: $x1 = 1, \quad x2 = 2, \quad x3 = -1, \quad x4 = -2$

28. **341**

Data:

$$\begin{cases} x_1 = 5 + 2x_2 = 5 + 3x_3 = 5 + 4x_4 = 5 \\ 2x_1 = 1 + x_2 = 1 + 2x_3 = 1 + 3x_4 = 1 \\ 3x_1 = 1 + 2x_2 = 1 + x_3 = 1 + 2x_4 = 1 \\ 4x_1 = -5 + 3x_2 = -5 + 2x_3 = -5 + x_4 = -5 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 2 & 3 & 1 \\ 3 & 2 & 1 & 2 & 1 \\ 4 & 3 & 2 & 1 & -5 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 3 & 4 & 5 & 9 \\ 0 & 2 & 4 & 5 & 7 \\ 0 & 1 & 2 & 3 & 5 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 3 & 4 & 5 & 9 \\ 0 & 0 & 4 & 5 & 3 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 3 & 4 & 5 & 9 \\ 0 & 0 & 4 & 5 & 3 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 3 & 4 & 5 & 9 \\ 0 & 0 & 4 & 5 & 3 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

Answers: $x1 = -2, \quad x2 = 2, \quad x3 = -3, \quad x4 = 3$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 2 & 3 & 1 \\ 3 & 2 & 1 & 2 & 1 \\ 4 & 3 & 2 & 1 & -5 \end{vmatrix} =$$

$$\frac{(12)}{(12)*(6)*(4)*(3)} \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -18 & -24 & -30 \\ 0 & -16 & -32 & -40 \\ 0 & -15 & -30 & -45 \end{vmatrix} =$$

$$\frac{(12)}{(12)*(6)*(4)*(3)*(40)*(45)*(48)} \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -720 & -960 & -1200 \\ 0 & 0 & -480 & -600 \\ 0 & 0 & -480 & -960 \end{vmatrix} =$$

$$\frac{(12)}{(12)*(6)*(4)*(3)*(40)*(45)*(48)} \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -720 & -960 & -1200 \\ 0 & 0 & -480 & -600 \\ 0 & 0 & 0 & -360 \end{vmatrix} =$$

$- 20$

$$\Delta_1 : \frac{(2)}{(5)*(5)*(-1)} \begin{vmatrix} 5 & 2 & 3 & 4 \\ 0 & 3 & 7 & 11 \\ 0 & 4 & 1 & 3 \\ 0 & -5 & -5 & -5 \end{vmatrix} =$$

$$\frac{(2)*(20)}{(5)*(5)*(-1)*(20)*(15)*(-12)} \begin{vmatrix} 5 & 2 & 3 & 4 \\ 0 & 3 & 7 & 11 \\ 0 & 0 & -125 & -175 \\ 0 & 0 & -80 & -160 \end{vmatrix} =$$

$$\frac{(2)*(20)}{(5)*(5)*(-1)*(20)*(15)*(-12)*(16)*(25)} \begin{vmatrix} 5 & 2 & 3 & 4 \\ 0 & 3 & 7 & 11 \\ 0 & 0 & -2000 & -2800 \\ 0 & 0 & 0 & -1200 \end{vmatrix} =$$

40

$x_0 = \frac{\Delta_0}{\Delta} = -2$

$$\Delta_2 : \frac{(12)}{(12)*(6)*(4)*(3)} \begin{vmatrix} 1 & 5 & 3 & 4 \\ 0 & -54 & -24 & -30 \\ 0 & -56 & -32 & -40 \\ 0 & -75 & -30 & -45 \end{vmatrix} =$$

$$\frac{(12)}{(12)*(6)*(4)*(3)*(700)*(675)*(504)} \begin{vmatrix} 1 & 5 & 3 & 4 \\ 0 & -37800 & -16800 & -21000 \\ 0 & 0 & -4800 & -6000 \\ 0 & 0 & 1680 & -1680 \end{vmatrix} =$$

$$\frac{(12)*(75600)}{(12)*(6)*(4)*(3)*(700)*(675)*(504)*(7)*(-20)} \begin{vmatrix} 1 & 5 & 3 & 4 \\ 0 & -37800 & -16800 & -21000 \\ 0 & 0 & -33600 & -42000 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

$- 40$

$x_1 = \frac{\Delta_1}{\Delta} = 2$

$$\Delta_3 : \frac{(12)}{(12)*(6)*(4)*(3)} \begin{vmatrix} 1 & 2 & 5 & 4 \\ 0 & -18 & -54 & -30 \\ 0 & -16 & -56 & -40 \\ 0 & -15 & -75 & -45 \end{vmatrix} =$$

$$\frac{(12)}{(12)*(6)*(4)*(3)*(40)*(45)*(48)} \begin{vmatrix} 1 & 2 & 5 & 4 \\ 0 & -720 & -2160 & -1200 \\ 0 & 0 & -360 & -600 \\ 0 & 0 & -1440 & -960 \end{vmatrix} =$$

$$\frac{(12)*(1440)}{(12)*(6)*(4)*(3)*(40)*(45)*(48)*(4)} \begin{vmatrix} 1 & 2 & 5 & 4 \\ 0 & -720 & -2160 & -1200 \\ 0 & 0 & -1440 & -2400 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

60

$x_2 = \frac{\Delta_2}{\Delta} = -3$

$$\Delta_4: \frac{(12)}{(12)*(6)*(4)*(3)} \begin{vmatrix} 1 & 2 & 3 & 5 \\ 0 & -18 & -24 & -54 \\ 0 & -16 & -32 & -56 \\ 0 & -15 & -30 & -75 \end{vmatrix} =$$

$$\frac{(12)}{(12)*(6)*(4)*(3)*(40)*(45)*(48)} \begin{vmatrix} 1 & 2 & 3 & 5 \\ 0 & -720 & -960 & -2160 \\ 0 & 0 & -480 & -360 \\ 0 & 0 & -480 & -1440 \end{vmatrix} =$$

$$\frac{(12)}{(12)*(6)*(4)*(3)*(40)*(45)*(48)} \begin{vmatrix} 1 & 2 & 3 & 5 \\ 0 & -720 & -960 & -2160 \\ 0 & 0 & -480 & -360 \\ 0 & 0 & 0 & -1080 \end{vmatrix} =$$

-60

$x_3 = \frac{\Delta_3}{\Delta} = 3$

Answers: $x1 = -2$, $x2 = 2$, $x3 = -3$, $x4 = 3$

29. **342**

Data:

$$\begin{cases} 0x_1 = -5 + x_2 = -5 + 3x_3 = -5 - 4x_4 = -5 \\ x_1 = -4 + 0x_2 = -4 + 2x_3 = -4 - 3x_4 = -4 \\ 3x_1 = 12 + 2x_2 = 12 + 0x_3 = 12 + 5x_4 = 12 \\ 4x_1 = 5 + 3x_2 = 5 + 5x_3 = 5 - 0x_4 = 5 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 0 & 1 & -3 & 4 & -5 \\ 1 & 0 & -2 & 3 & -4 \\ 3 & 2 & 0 & -5 & 12 \\ 4 & 3 & -5 & 0 & 5 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 0 & 1 & -3 & 4 & -5 \\ 1 & 0 & -2 & 3 & -4 \\ 0 & 1 & 3 & -7 & 12 \\ 0 & 1 & 1 & -4 & 7 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 0 & 1 & -3 & 4 & -5 \\ 1 & 0 & -2 & 3 & -4 \\ 0 & 0 & 6 & -11 & 17 \\ 0 & 0 & 1 & -2 & 3 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 0 & 1 & -3 & 4 & -5 \\ 1 & 0 & -2 & 3 & -4 \\ 0 & 0 & 6 & -11 & 17 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 0 & 1 & -3 & 4 & -5 \\ 1 & 0 & -2 & 3 & -4 \\ 0 & 0 & 6 & -11 & 17 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

Answers: $x1 = 1$, $x2 = 2$, $x3 = 1$, $x4 = -1$

Solution by Kramer method:

Answers: No solution

30. **343**

Data:

$$\begin{cases} 2x_1 = 4 + x_2 = 4 - 3x_3 = 4 + 2x_4 = 4 \\ 3x_1 = 6 + 3x_2 = 6 + 3x_3 = 6 + 2x_4 = 6 \\ 3x_1 = 6 + x_2 = 6 - x_3 = 6 - 2x_4 = 6 \\ 3x_1 = 6 + x_2 = 6 - 3x_3 = 6 + x_4 = 6 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 2 & -1 & 3 & 2 & 4 \\ 3 & 3 & 3 & 2 & 6 \\ 3 & -1 & -1 & 2 & 6 \\ 3 & -1 & 3 & -1 & 6 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & -1 & 3 & 2 & 4 \\ 0 & 9 & -3 & -2 & 0 \\ 0 & 1 & -11 & -2 & 0 \\ 0 & 1 & -3 & -8 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & -1 & 3 & 2 & 4 \\ 0 & 9 & -3 & -2 & 0 \\ 0 & 0 & 6 & 1 & 0 \\ 0 & 0 & 12 & 35 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & -1 & 3 & 2 & 4 \\ 0 & 9 & -3 & -2 & 0 \\ 0 & 0 & 6 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & -1 & 3 & 2 & 4 \\ 0 & 9 & -3 & -2 & 0 \\ 0 & 0 & 6 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Answers: $x1 = 2, \quad x2 = 0, \quad x3 = 0, \quad x4 = 0$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 2 & -1 & 3 & 2 & 4 \\ 3 & 3 & 3 & 2 & 6 \\ 3 & -1 & -1 & 2 & 6 \\ 3 & -1 & 3 & -1 & 6 \end{vmatrix} =$$

$$\frac{(3)}{(3)*(2)*(2)*(2)} \begin{vmatrix} 2 & -1 & 3 & 2 \\ 0 & 9 & -3 & -2 \\ 0 & 1 & -11 & -2 \\ 0 & 1 & -3 & -8 \end{vmatrix} =$$

$$\frac{(3)}{(3)*(2)*(2)*(2)*(9)*(9)} \begin{vmatrix} 2 & -1 & 3 & 2 \\ 0 & 9 & -3 & -2 \\ 0 & 0 & -96 & -16 \\ 0 & 0 & -24 & -70 \end{vmatrix} =$$

$$\frac{(3)}{(3)*(2)*(2)*(2)*(9)*(9)*(4)} \begin{vmatrix} 2 & -1 & 3 & 2 \\ 0 & 9 & -3 & -2 \\ 0 & 0 & -96 & -16 \\ 0 & 0 & 0 & -264 \end{vmatrix} =$$

176

$$\Delta_1 : \frac{(3)}{(3)*(2)*(2)*(2)} \begin{vmatrix} 4 & -1 & 3 & 2 \\ 0 & 9 & -3 & -2 \\ 0 & 1 & -11 & -2 \\ 0 & 1 & -3 & -8 \end{vmatrix} =$$

$$\frac{(3)}{(3)*(2)*(2)*(2)*(9)*(9)} \begin{vmatrix} 4 & -1 & 3 & 2 \\ 0 & 9 & -3 & -2 \\ 0 & 0 & -96 & -16 \\ 0 & 0 & -24 & -70 \end{vmatrix} =$$

$$\frac{(3)}{(3)*(2)*(2)*(2)*(9)*(9)*(4)} \begin{vmatrix} 4 & -1 & 3 & 2 \\ 0 & 9 & -3 & -2 \\ 0 & 0 & -96 & -16 \\ 0 & 0 & 0 & -264 \end{vmatrix} =$$

352

$$x_0 = \frac{\Delta_0}{\Delta} = 2$$

$$\Delta_2 : \frac{(3)}{(3)*(2)*(2)*(2)} \begin{vmatrix} 2 & 4 & 3 & 2 \\ 0 & 0 & -3 & -2 \\ 0 & 0 & -11 & -2 \\ 0 & 0 & -3 & -8 \end{vmatrix} =$$

$$\frac{(3)*(16)}{(3)*(2)*(2)*(2)*(11)*(3)*(11)} \begin{vmatrix} 2 & 4 & 3 & 2 \\ 0 & 0 & -33 & -22 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -66 \end{vmatrix} =$$

0

$$x_1 = \frac{\Delta_1}{\Delta} = 0$$

$$\Delta_3 : \frac{(3)}{(3)*(2)*(2)*(2)} \begin{vmatrix} 2 & -1 & 4 & 2 \\ 0 & 9 & 0 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 1 & 0 & -8 \end{vmatrix} =$$

$$\frac{(3)}{(3)*(2)*(2)*(2)*(9)*(9)} \begin{vmatrix} 2 & -1 & 4 & 2 \\ 0 & 9 & 0 & -2 \\ 0 & 0 & 0 & -16 \\ 0 & 0 & 0 & -70 \end{vmatrix} =$$

0

$$x_2 = \frac{\Delta_2}{\Delta} = 0$$

$$\Delta_4 : \frac{(3)}{(3)*(2)*(2)*(2)} \begin{vmatrix} 2 & -1 & 3 & 4 \\ 0 & 9 & -3 & 0 \\ 0 & 1 & -11 & 0 \\ 0 & 1 & -3 & 0 \end{vmatrix} =$$

$$\frac{(3)}{(3)*(2)*(2)*(2)*(9)*(9)} \begin{vmatrix} 2 & -1 & 3 & 4 \\ 0 & 9 & -3 & 0 \\ 0 & 0 & -96 & 0 \\ 0 & 0 & -24 & 0 \end{vmatrix} =$$

$$\frac{(3)}{(3)*(2)*(2)*(2)*(9)*(9)*(4)} \begin{vmatrix} 2 & -1 & 3 & 4 \\ 0 & 9 & -3 & 0 \\ 0 & 0 & -96 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} =$$

0

$x_3 = \frac{\Delta_3}{\Delta} = 0$

Answers: $x1 = 2, \quad x2 = 0, \quad x3 = 0, \quad x4 = 0$

31. **344**

Data:

$$\begin{cases} x_1 = 0 + x_2 = 0 + x_3 = 0 + x_4 = 0 \\ x_1 = 0 + 2x_2 = 0 + 3x_3 = 0 + 4x_4 = 0 \\ x_1 = 0 + 3x_2 = 0 + 6x_3 = 0 + 10x_4 = 0 \\ x_1 = 0 + 4x_2 = 0 + 10x_3 = 0 + 20x_4 = 0 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 4 & 0 \\ 1 & 3 & 6 & 10 & 0 \\ 1 & 4 & 10 & 20 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 2 & 5 & 9 & 0 \\ 0 & 3 & 9 & 19 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 3 & 10 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Answers: $x1 = 0, \quad x2 = 0, \quad x3 = 0, \quad x4 = 0$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 4 & 0 \\ 1 & 3 & 6 & 10 & 0 \\ 1 & 4 & 10 & 20 & 0 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 5 & 9 \\ 0 & 3 & 9 & 19 \end{vmatrix} =$$

$$\frac{(6)*(3)*(2)}{(6)*(3)*(2)} \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 3 & 10 \end{vmatrix} =$$

$$\frac{(6)*(3)*(2)*(3)}{(6)*(3)*(2)*(3)} \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

1

$\Delta_1 : \frac{(12)*(6)*(4)*(6)}{(12)*(6)*(4)*(3)} \begin{vmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 3 & 8 \end{vmatrix} =$

$\frac{(12)*(6)*(4)*(6)*(3)*(2)}{(12)*(6)*(4)*(3)*(3)} \begin{vmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$

0

$x_0 = \frac{\Delta_0}{\Delta} = 0$

$\Delta_2 : \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 5 & 9 \\ 0 & 0 & 9 & 19 \end{vmatrix} =$

$\frac{(45)*(27)*(55)}{(45)*(18)*(10)} \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$

0

$x_1 = \frac{\Delta_1}{\Delta} = 0$

$\Delta_3 : \begin{vmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 2 & 0 & 9 \\ 0 & 3 & 0 & 19 \end{vmatrix} =$

$\frac{(6)*(9)*(20)}{(6)*(3)*(2)} \begin{vmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$

0

$x_2 = \frac{\Delta_2}{\Delta} = 0$

$\Delta_4 : (3) \begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 5 & 0 \\ 0 & 1 & 3 & 0 \end{vmatrix} =$

$\frac{(3)*(2)*(2)}{(2)*(2)} \begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix} =$

$\frac{(3)*(2)*(2)}{(2)*(2)} \begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} =$

0

$x_3 = \frac{\Delta_3}{\Delta} = 0$

Answers: $x1 = 0, \quad x2 = 0, \quad x3 = 0, \quad x4 = 0$

32. **345**

Data:

$$\begin{cases} x_1 = 12 + 3x_2 = 12 + 5x_3 = 12 + 7x_4 = 12 \\ 3x_1 = 0 + 5x_2 = 0 + 7x_3 = 0 + x_4 = 0 \\ 5x_1 = 4 + 7x_2 = 4 + x_3 = 4 + 3x_4 = 4 \\ 7x_1 = 16 + x_2 = 16 + 3x_3 = 16 + 5x_4 = 16 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 1 & 3 & 5 & 7 & 12 \\ 3 & 5 & 7 & 1 & 0 \\ 5 & 7 & 1 & 3 & 4 \\ 7 & 1 & 3 & 5 & 16 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 3 & 5 & 7 & 12 \\ 0 & 1 & 2 & 5 & 9 \\ 0 & 1 & 3 & 4 & 7 \\ 0 & 5 & 8 & 11 & 17 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 3 & 5 & 7 & 12 \\ 0 & 1 & 2 & 5 & 9 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 7 & 14 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 3 & 5 & 7 & 12 \\ 0 & 1 & 2 & 5 & 9 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 3 & 5 & 7 & 12 \\ 0 & 1 & 2 & 5 & 9 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Answers: $x1 = 1, \quad x2 = -1, \quad x3 = 0, \quad x4 = 2$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 1 & 3 & 5 & 7 & 12 \\ 3 & 5 & 7 & 1 & 0 \\ 5 & 7 & 1 & 3 & 4 \\ 7 & 1 & 3 & 5 & 16 \end{vmatrix} =$$

$$\frac{(105)}{(105)*(35)*(21)*(15)} \begin{vmatrix} 1 & 3 & 5 & 7 \\ 0 & -140 & -280 & -700 \\ 0 & -168 & -504 & -672 \\ 0 & -300 & -480 & -660 \end{vmatrix} =$$

$$\frac{(105)*(4200)*(1680)}{(105)*(35)*(21)*(15)*(30)*(25)*(14)} \begin{vmatrix} 1 & 3 & 5 & 7 \\ 0 & -4200 & -8400 & -21000 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 7 \end{vmatrix} =$$

$$\frac{(105)*(4200)*(1680)}{(105)*(35)*(21)*(15)*(30)*(25)*(14)*(-1)} \begin{vmatrix} 1 & 3 & 5 & 7 \\ 0 & -4200 & -8400 & -21000 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -8 \end{vmatrix} =$$

2048

$$\Delta_1 : \frac{(4)*(8)}{(4)*(12)*(3)} \begin{vmatrix} 12 & 3 & 5 & 7 \\ 0 & 5 & 7 & 1 \\ 0 & 9 & -1 & 1 \\ 0 & -9 & -11 & -13 \end{vmatrix} =$$

$$\frac{(4)*(8)*(9)*(8)}{(4)*(12)*(3)*(9)*(5)*(-5)} \begin{vmatrix} 12 & 3 & 5 & 7 \\ 0 & 5 & 7 & 1 \\ 0 & 0 & -68 & -4 \\ 0 & 0 & -1 & 7 \end{vmatrix} =$$

$$\frac{(4)*(8)*(9)*(8)*(480)}{(4)*(12)*(3)*(9)*(5)*(-5)*(68)} \begin{vmatrix} 12 & 3 & 5 & 7 \\ 0 & 5 & 7 & 1 \\ 0 & 0 & -68 & -4 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

2048

$x_0 = \frac{\Delta_0}{\Delta} = 1$

$$\Delta_2 : \frac{(105)}{(105)*(35)*(21)*(15)} \begin{vmatrix} 1 & 12 & 5 & 7 \\ 0 & -1260 & -280 & -700 \\ 0 & -1176 & -504 & -672 \\ 0 & -1020 & -480 & -660 \end{vmatrix} =$$

$$\frac{(105)}{(105)*(35)*(21)*(15)*(238)*(255)*(294)} \begin{vmatrix} 1 & 12 & 5 & 7 \\ 0 & -299880 & -66640 & -166600 \\ 0 & 0 & -61880 & -4760 \\ 0 & 0 & -74480 & -27440 \end{vmatrix} =$$

$$\frac{(105)}{(105)*(35)*(21)*(15)*(238)*(255)*(294)*(266)*(221)} \begin{vmatrix} 1 & 12 & 5 & 7 \\ 0 & -299880 & -66640 & -166600 \\ 0 & 0 & -16460080 & -1266160 \\ 0 & 0 & 0 & -4798080 \end{vmatrix} =$$

-2048

$x_1 = \frac{\Delta_1}{\Delta} = -1$

$$\Delta_3 : \frac{(105)}{(105)*(35)*(21)*(15)} \begin{vmatrix} 1 & 3 & 12 & 7 \\ 0 & -140 & -1260 & -700 \\ 0 & -168 & -1176 & -672 \\ 0 & -300 & -1020 & -660 \end{vmatrix} =$$

$$\frac{(105)*(4200)*(11760)}{(105)*(35)*(21)*(15)*(30)*(25)*(14)} \begin{vmatrix} 1 & 3 & 12 & 7 \\ 0 & -4200 & -37800 & -21000 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 2 & 1 \end{vmatrix} =$$

$$\frac{(105)*(4200)*(11760)}{(105)*(35)*(21)*(15)*(30)*(25)*(14)} \begin{vmatrix} 1 & 3 & 12 & 7 \\ 0 & -4200 & -37800 & -21000 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{vmatrix} =$$

0

$x_2 = \frac{\Delta_2}{\Delta} = 0$

$$\Delta_4 : \frac{(105)}{(105)*(35)*(21)*(15)} \begin{vmatrix} 1 & 3 & 5 & 12 \\ 0 & -140 & -280 & -1260 \\ 0 & -168 & -504 & -1176 \\ 0 & -300 & -480 & -1020 \end{vmatrix} =$$

$$\frac{(105)*(4200)*(1680)}{(105)*(35)*(21)*(15)*(30)*(25)*(14)} \begin{vmatrix} 1 & 3 & 5 & 12 \\ 0 & -4200 & -8400 & -37800 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 14 \end{vmatrix} =$$

$$\frac{(105)*(4200)*(1680)}{(105)*(35)*(21)*(15)*(30)*(25)*(14)*(-1)} \begin{vmatrix} 1 & 3 & 5 & 12 \\ 0 & -4200 & -8400 & -37800 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & -16 \end{vmatrix} =$$

4096

$x_3 = \frac{\Delta_3}{\Delta} = 2$

Answers: $x1 = 1, \quad x2 = -1, \quad x3 = 0, \quad x4 = 2$

33. **346**

Data:

$$\begin{cases} x_1 = 0 + x_2 = 0 + x_3 = 0 + x_4 = 0 + x_5 = 0 \\ x_1 = 0 + x_2 = 0 - 2x_3 = 0 + 2x_4 = 0 - 3x_5 = 0 \\ x_1 = 0 + x_2 = 0 + 4x_3 = 0 + 4x_4 = 0 + 9x_5 = 0 \\ x_1 = 0 + x_2 = 0 - 8x_3 = 0 + 8x_4 = 0 - 27x_5 = 0 \\ x_1 = 0 + x_2 = 0 + 16x_3 = 0 + 16x_4 = 0 + 81x_5 = 0 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & -2 & 3 & 0 \\ 1 & 1 & 4 & 4 & 9 & 0 \\ 1 & -1 & 8 & -8 & 27 & 0 \\ 1 & 1 & 16 & 16 & 81 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 3 & -2 & 0 \\ 0 & 0 & 3 & 3 & 8 & 0 \\ 0 & 2 & -7 & 9 & -26 & 0 \\ 0 & 0 & 3 & 3 & 16 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 3 & -2 & 0 \\ 0 & 0 & 3 & 3 & 8 & 0 \\ 0 & 0 & 1 & -1 & 4 & 0 \\ 0 & 0 & 3 & 3 & 16 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 3 & -2 & 0 \\ 0 & 0 & 3 & 3 & 8 & 0 \\ 0 & 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 3 & -2 & 0 \\ 0 & 0 & 3 & 3 & 8 & 0 \\ 0 & 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 3 & -2 & 0 \\ 0 & 0 & 3 & 3 & 8 & 0 \\ 0 & 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Answers: $x1 = 0,$ $x2 = 0,$ $x3 = 0,$ $x4 = 0,$ $x5 = 0$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & -2 & 3 & 0 \\ 1 & 1 & 4 & 4 & 9 & 0 \\ 1 & -1 & 8 & -8 & 27 & 0 \\ 1 & 1 & 16 & 16 & 81 & 0 \end{vmatrix} =$$

$$(5) \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & -2 & 1 & -3 & 2 \\ 0 & 0 & 3 & 3 & 8 \\ 0 & -2 & 7 & -9 & 26 \\ 0 & 0 & 3 & 3 & 16 \end{vmatrix} =$$

$$(5)*(6) \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & -2 & 1 & -3 & 2 \\ 0 & 0 & 3 & 3 & 8 \\ 0 & 0 & 1 & -1 & 4 \\ 0 & 0 & 3 & 3 & 16 \end{vmatrix} =$$

$$\frac{(5)*(6)*(2)*(8)}{(3)} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & -2 & 1 & -3 & 2 \\ 0 & 0 & 3 & 3 & 8 \\ 0 & 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

$$\frac{(5)*(6)*(2)*(8)}{(3)} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & -2 & 1 & -3 & 2 \\ 0 & 0 & 3 & 3 & 8 \\ 0 & 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

2880

$$\Delta_1 : \frac{(5)}{(-1)*(-1)} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & -3 & 1 & -4 \\ 0 & 0 & 3 & 3 & 8 \\ 0 & 0 & -9 & 7 & -28 \\ 0 & 0 & 3 & 3 & 16 \end{vmatrix} =$$

$$\frac{(5)}{(-1)*(-1)*(3)*(-3)*(-3)} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & -9 & 3 & -12 \\ 0 & 0 & 0 & -12 & -12 \\ 0 & 0 & 0 & 4 & -16 \\ 0 & 0 & 0 & -12 & -36 \end{vmatrix} =$$

$$\frac{(5)*(60)}{(-1)*(-1)*(3)*(-3)*(-3)*(-3)} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & -9 & 3 & -12 \\ 0 & 0 & 0 & -12 & -12 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -24 \end{vmatrix} =$$

0

$x_0 = \frac{\Delta_0}{\Delta} = 0$

$$\Delta_2 : (5) \begin{vmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -3 & 2 \\ 0 & 0 & 3 & 3 & 8 \\ 0 & 0 & 7 & -9 & 26 \\ 0 & 0 & 3 & 3 & 16 \end{vmatrix} =$$

$$\frac{(5)*(21)*(14)*(36)*(14)}{(21)*(7)*(3)*(7)} \begin{vmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 6 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 6 & 5 \end{vmatrix} =$$

$$\frac{(5)*(21)*(14)*(36)*(14)*(5)*(4)}{(21)*(7)*(3)*(7)*(6)} \begin{vmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 6 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

0

$x_1 = \frac{\Delta_1}{\Delta} = 0$

$$\Delta_3 : (5) \begin{vmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & -2 & 0 & -3 & 2 \\ 0 & 0 & 0 & 3 & 8 \\ 0 & -2 & 0 & -9 & 26 \\ 0 & 0 & 0 & 3 & 16 \end{vmatrix} =$$

$$(5)*(6) \begin{vmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & -2 & 0 & -3 & 2 \\ 0 & 0 & 0 & 3 & 8 \\ 0 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 3 & 16 \end{vmatrix} =$$

$$\frac{(5)*(6)*(8)}{(-3)} \begin{vmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & -2 & 0 & -3 & 2 \\ 0 & 0 & 0 & 3 & 8 \\ 0 & 0 & 0 & 0 & -20 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

0

$x_2 = \frac{\Delta_2}{\Delta} = 0$

$$\Delta_4 : (5) \begin{vmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & -2 & 1 & 0 & 2 \\ 0 & 0 & 3 & 0 & 8 \\ 0 & -2 & 7 & 0 & 26 \\ 0 & 0 & 3 & 0 & 16 \end{vmatrix} =$$

$$(5)*(6) \begin{vmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & -2 & 1 & 0 & 2 \\ 0 & 0 & 3 & 0 & 8 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 3 & 0 & 16 \end{vmatrix} =$$

$$\frac{(5)*(6)*(4)*(8)}{(3)} \begin{vmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & -2 & 1 & 0 & 2 \\ 0 & 0 & 3 & 0 & 8 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

0

$x_3 = \frac{\Delta_3}{\Delta} = 0$

$$\Delta_5 : (3) * (15) \begin{vmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & -3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & -2 & 7 & -9 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{vmatrix} =$$

$$(3) * (15) \begin{vmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & -3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 6 & -6 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{vmatrix} =$$

$$\frac{(3)*(15)*(6)}{(6)*(6)} \begin{vmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & -3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -12 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix} =$$

$$\frac{(3)*(15)*(6)}{(6)*(6)} \begin{vmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & -3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -12 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix} =$$

0

$x_4 = \frac{\Delta_4}{\Delta} = 0$

Answers: $x1 = 0$, $x2 = 0$, $x3 = 0$, $x4 = 0$, $x5 = 0$

34. **347**

Data:

$$\begin{cases} x_1 = 0 + 2x_2 = 0 + 3x_3 = 0 + 4x_4 = 0 \\ x_1 = 0 + x_2 = 0 + 2x_3 = 0 + 3x_4 = 0 \\ x_1 = 0 + 5x_2 = 0 + x_3 = 0 + 2x_4 = 0 \\ x_1 = 0 + 5x_2 = 0 + 5x_3 = 0 + 2x_4 = 0 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 0 \\ 1 & 1 & 2 & 3 & 0 \\ 1 & 5 & 1 & 2 & 0 \\ 1 & 5 & 5 & 2 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 3 & -2 & -2 & 0 \\ 0 & 3 & 2 & -2 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 5 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Answers: $x1 = 0, \quad x2 = 0, \quad x3 = 0, \quad x4 = 0$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 1 & 2 & 3 & 4 & 0 \\ 1 & 1 & 2 & 3 & 0 \\ 1 & 5 & 1 & 2 & 0 \\ 1 & 5 & 5 & 2 & 0 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -1 & -1 \\ 0 & 3 & -2 & -2 \\ 0 & 3 & 2 & -2 \end{vmatrix} =$$

$$\frac{(5)}{(3)*(-1)*(-1)} \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -3 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 5 \end{vmatrix} =$$

$$\frac{(5)*(4)}{(3)*(-1)*(-1)} \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -3 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

$$- 20$$

$$\Delta_1 : \frac{(5)*(5)}{(5)*(10)*(2)*(2)} \begin{vmatrix} 0 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -13 & -16 \\ 0 & 0 & -5 & -16 \end{vmatrix} =$$

$$\frac{(5)*(5)*(65)*(78)}{(5)*(10)*(2)*(2)*(65)*(-5)*(-13)} \begin{vmatrix} 0 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -50 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

$$0$$

$$x_0 = \frac{\Delta_0}{\Delta} = 0$$

$$\Delta_2 : \begin{vmatrix} 1 & 0 & 3 & 4 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 2 & -2 \end{vmatrix} =$$

$$\frac{(4)}{(2)*(-1)} \begin{vmatrix} 1 & 0 & 3 & 4 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

$$0$$

$$x_1 = \frac{\Delta_1}{\Delta} = 0$$

$$\Delta_3 : \begin{vmatrix} 1 & 2 & 0 & 4 \\ 0 & -1 & 0 & -1 \\ 0 & 3 & 0 & -2 \\ 0 & 3 & 0 & -2 \end{vmatrix} =$$

$$\frac{(5)*(5)}{(3)*(-1)*(-1)} \begin{vmatrix} 1 & 2 & 0 & 4 \\ 0 & -3 & 0 & -3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

0

$x_2 = \frac{\Delta_2}{\Delta} = 0$

$$\Delta_4 : \begin{vmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 3 & -2 & 0 \\ 0 & 3 & 2 & 0 \end{vmatrix} =$$

$$\frac{(5)}{(3)*(-1)*(-1)} \begin{vmatrix} 1 & 2 & 3 & 0 \\ 0 & -3 & -3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix} =$$

$$\frac{(5)}{(3)*(-1)*(-1)} \begin{vmatrix} 1 & 2 & 3 & 0 \\ 0 & -3 & -3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} =$$

0

$x_3 = \frac{\Delta_3}{\Delta} = 0$

Answers: $x1 = 0, \quad x2 = 0, \quad x3 = 0, \quad x4 = 0$

35. **348**

Data:

$$\begin{cases} x_1 = 0 + x_2 = 0 + x_3 = 0 + x_4 = 0 + 0x_5 = 0 \\ 0x_1 = 0 + x_2 = 0 + x_3 = 0 + x_4 = 0 + x_5 = 0 \\ x_1 = 2 + 2x_2 = 2 + 3x_3 = 2 + 0x_4 = 2 + 0x_5 = 2 \\ 0x_1 = -2 + x_2 = -2 + 2x_3 = -2 + 3x_4 = -2 + 0x_5 = -2 \\ 0x_1 = 2 + 0x_2 = 2 + x_3 = 2 + 2x_4 = 2 + 3x_5 = 2 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 2 \\ 0 & 1 & 2 & 3 & 0 & -2 \\ 0 & 0 & 1 & 2 & 3 & 2 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 2 & 3 & 0 & -2 \\ 0 & 0 & 1 & 2 & 3 & 2 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 & 2 \\ 0 & 0 & 1 & 2 & -1 & -2 \\ 0 & 0 & 1 & 2 & 3 & 2 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Answers: $x1 = 1, \quad x2 = -1, \quad x3 = 1, \quad x4 = -1, \quad x5 = 1$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 2 \\ 0 & 1 & 2 & 3 & 0 & -2 \\ 0 & 0 & 1 & 2 & 3 & 2 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 2 & 3 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 2 & 3 \end{vmatrix} =$$

$$(4)*(4) \begin{vmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix} =$$

$$(4)*(4) \begin{vmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

16

$$\Delta_1 : \overline{(-1)} \begin{vmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 2 & 2 & 3 & 0 & 0 \\ 0 & -3 & -5 & -3 & 0 \\ 0 & -2 & -2 & 2 & 3 \end{vmatrix} =$$

34

$$\frac{(6)*(6)*(4)}{(-1)*(6)*(6)*(-2)*(-3)} \begin{vmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 2 & 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -12 & -9 \end{vmatrix} =$$

$$\frac{(6)*(6)*(4)}{(-1)*(6)*(6)*(-2)*(-3)} \begin{vmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 2 & 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -12 & -9 \end{vmatrix} =$$

$$\frac{(6)*(6)*(4)}{(-1)*(6)*(6)*(-2)*(-3)} \begin{vmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 2 & 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -12 & -9 \end{vmatrix} =$$

0

$x_0 = \frac{\Delta_0}{\Delta} = 0$

$$\Delta_2 : \begin{vmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 2 & 2 & -1 & 0 \\ 0 & -2 & 2 & 3 & 0 \\ 0 & 2 & 1 & 2 & 3 \end{vmatrix} =$$

$$\overline{(-1)} \begin{vmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 2 & 2 & -1 & 0 \\ 0 & 0 & -4 & -2 & 0 \\ 0 & 0 & -1 & 3 & 3 \end{vmatrix} =$$

$$\frac{(4)}{(-1)*(4)*(-1)*(-4)} \begin{vmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 2 & 2 & -1 & 0 \\ 0 & 0 & 0 & -2 & -4 \\ 0 & 0 & 0 & -16 & -16 \end{vmatrix} =$$

$$\frac{(4)*(16)}{(-1)*(4)*(-1)*(-4)*(8)} \begin{vmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 2 & 2 & -1 & 0 \\ 0 & 0 & 0 & -16 & -32 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

0

$x_1 = \frac{\Delta_1}{\Delta} = 0$

$$\Delta_3 : \begin{vmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & -2 & 3 & 0 \\ 0 & 0 & 2 & 2 & 3 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 2 & -2 & -1 \\ 0 & 0 & -2 & 2 & -1 \\ 0 & 0 & 2 & 2 & 3 \end{vmatrix} =$$

$$\frac{(2)*(4)}{(-1)} \begin{vmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 2 & -2 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix} =$$

0

$x_2 = \frac{\Delta_2}{\Delta} = 0$

$$\Delta_4 : \begin{vmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 2 & 0 \\ 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 1 & 2 & 3 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 2 & 3 \end{vmatrix} =$$

$$(4)\begin{vmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

-16

$x_3 = \frac{\Delta_3}{\Delta} = -1$

$$\Delta_5 : \begin{vmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & -1 & 2 \\ 0 & 1 & 2 & 3 & -2 \\ 0 & 0 & 1 & 2 & 2 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 1 & 2 & 2 \end{vmatrix} =$$

$$(4)\begin{vmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 4 & -4 \\ 0 & 0 & 0 & 1 & 0 \end{vmatrix} =$$

$$\frac{(4)*(4)}{(4)} \begin{vmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 4 & -4 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

16

$x_4 = \frac{\Delta_4}{\Delta} = 1$

Answers: $x1 = 0, \quad x2 = 0, \quad x3 = 0, \quad x4 = -1, \quad x5 = 1$

36. **349**

Data:

$$\begin{cases} x_1 = 0 + 4x_2 = 0 + 6x_3 = 0 + 4x_4 = 0 + x_5 = 0 \\ x_1 = 0 + x_2 = 0 + 4x_3 = 0 + 6x_4 = 0 + 4x_5 = 0 \\ 4x_1 = 0 + x_2 = 0 + x_3 = 0 + 4x_4 = 0 + 6x_5 = 0 \\ 6x_1 = 0 + 4x_2 = 0 + x_3 = 0 + x_4 = 0 + 4x_5 = 0 \\ 4x_1 = 0 + 6x_2 = 0 + 4x_3 = 0 + x_4 = 0 + x_5 = 0 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 1 & 4 & 6 & 4 & 1 & 0 \\ 1 & 1 & 4 & 6 & 4 & 0 \\ 4 & 1 & 1 & 4 & 6 & 0 \\ 6 & 4 & 1 & 1 & 4 & 0 \\ 4 & 6 & 4 & 1 & 1 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 3 & 2 & -2 & -3 & 0 \\ 0 & 15 & 23 & 12 & -2 & 0 \\ 0 & 20 & 35 & 23 & 2 & 0 \\ 0 & 10 & 20 & 15 & 3 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 3 & 2 & -2 & -3 & 0 \\ 0 & 0 & 13 & 22 & 13 & 0 \\ 0 & 0 & 65 & 109 & 66 & 0 \\ 0 & 0 & 40 & 65 & 39 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 3 & 2 & -2 & -3 & 0 \\ 0 & 0 & 13 & 22 & 13 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 35 & 13 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 3 & 2 & -2 & -3 & 0 \\ 0 & 0 & 13 & 22 & 13 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 3 & 2 & -2 & -3 & 0 \\ 0 & 0 & 13 & 22 & 13 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Answers: $x1 = 0, \quad x2 = 0, \quad x3 = 0, \quad x4 = 0, \quad x5 = 0$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 1 & 4 & 6 & 4 & 1 & 0 \\ 1 & 1 & 4 & 6 & 4 & 0 \\ 4 & 1 & 1 & 4 & 6 & 0 \\ 6 & 4 & 1 & 1 & 4 & 0 \\ 4 & 6 & 4 & 1 & 1 & 0 \end{vmatrix} =$$

$$\frac{(12)*(12)*(3)}{(12)*(12)*(3)*(2)*(3)} \begin{vmatrix} 1 & 4 & 6 & 4 & 1 \\ 0 & -3 & -2 & 2 & 3 \\ 0 & -15 & -23 & -12 & 2 \\ 0 & -40 & -70 & -46 & -4 \\ 0 & -30 & -60 & -45 & -9 \end{vmatrix} =$$

$$\frac{(12)*(12)*(3)*(40)}{(12)*(12)*(3)*(2)*(3)*(40)*(8)*(3)*(4)} \begin{vmatrix} 1 & 4 & 6 & 4 & 1 \\ 0 & -3 & -2 & 2 & 3 \\ 0 & 0 & -104 & -176 & -104 \\ 0 & 0 & -130 & -218 & -132 \\ 0 & 0 & -160 & -260 & -156 \end{vmatrix} =$$

$$\frac{(12)*(12)*(3)*(40)*(4)}{(12)*(12)*(3)*(2)*(3)*(40)*(8)*(3)*(4)*(20)*(16)*(13)} \begin{vmatrix} 1 & 4 & 6 & 4 & 1 \\ 0 & -3 & -2 & 2 & 3 \\ 0 & 0 & -2080 & -3520 & -2080 \\ 0 & 0 & 0 & 32 & -32 \\ 0 & 0 & 0 & 35 & 13 \end{vmatrix} =$$

$$\frac{(12)*(12)*(3)*(40)*(4)*(1536)}{(12)*(12)*(3)*(2)*(3)*(40)*(8)*(3)*(4)*(20)*(16)*(13)*(35)*(32)} \begin{vmatrix} 1 & 4 & 6 & 4 & 1 \\ 0 & -3 & -2 & 2 & 3 \\ 0 & 0 & -2080 & -3520 & -2080 \\ 0 & 0 & 0 & 1120 & -1120 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

16

$$\Delta_1 : \frac{(3)*(15)*(3)*(3)}{(3)*(12)*(12)*(3)*(2)} \begin{vmatrix} 0 & 4 & 6 & 4 & 1 \\ 0 & 0 & 2 & 4 & 3 \\ 0 & 0 & -2 & 12 & 23 \\ 0 & 0 & -5 & -3 & 3 \\ 0 & 0 & -10 & -10 & -1 \end{vmatrix} =$$

$$\frac{(3)*(15)*(3)*(3)*(5)}{(3)*(12)*(12)*(3)*(2)*(5)*(-5)*(-2)*(-1)} \begin{vmatrix} 0 & 4 & 6 & 4 & 1 \\ 0 & 0 & 2 & 4 & 3 \\ 0 & 0 & 0 & -80 & -130 \\ 0 & 0 & 0 & -14 & -21 \\ 0 & 0 & 0 & -10 & -14 \end{vmatrix} =$$

$$\frac{(3)*(15)*(3)*(3)*(5)*(70)*(126)}{(3)*(12)*(12)*(3)*(2)*(5)*(-5)*(-2)*(-1)*(7)*(40)*(56)} \begin{vmatrix} 0 & 4 & 6 & 4 & 1 \\ 0 & 0 & 2 & 4 & 3 \\ 0 & 0 & 0 & -560 & -910 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

0

$$x_0 = \frac{\Delta_0}{\Delta} = 0$$

$$\Delta_2 : \frac{(12)*(12)*(3)}{(12)*(12)*(3)*(2)*(3)} \begin{vmatrix} 1 & 0 & 6 & 4 & 1 \\ 0 & 0 & -2 & 2 & 3 \\ 0 & 0 & -23 & -12 & 2 \\ 0 & 0 & -70 & -46 & -4 \\ 0 & 0 & -60 & -45 & -9 \end{vmatrix} =$$

$$\frac{(12)*(12)*(3)*(4830)}{(12)*(12)*(3)*(2)*(3)*(4830)*(420)*(138)*(161)} \begin{vmatrix} 1 & 0 & 6 & 4 & 1 \\ 0 & 0 & -2 & 2 & 3 \\ 0 & 0 & 0 & -14700 & -13650 \\ 0 & 0 & 0 & -16008 & -15042 \\ 0 & 0 & 0 & -16905 & -15939 \end{vmatrix} =$$

$$\frac{(12)*(12)*(3)*(4830)}{(12)*(12)*(3)*(2)*(3)*(4830)*(420)*(138)*(161)*(1334)*(1225)*(1160)} \begin{vmatrix} 1 & 0 & 6 & 4 & 1 \\ 0 & 0 & -2 & 2 & 3 \\ 0 & 0 & 0 & -19609800 & -18209100 \\ 0 & 0 & 0 & 0 & -217350 \\ 0 & 0 & 0 & 0 & -280140 \end{vmatrix} =$$

0

$x_1 = \frac{\Delta_1}{\Delta} = 0$

$$\Delta_3 : \frac{(12)*(12)*(3)}{(12)*(12)*(3)*(2)*(3)} \begin{vmatrix} 1 & 4 & 0 & 4 & 1 \\ 0 & -3 & 0 & 2 & 3 \\ 0 & -15 & 0 & -12 & 2 \\ 0 & -40 & 0 & -46 & -4 \\ 0 & -30 & 0 & -45 & -9 \end{vmatrix} =$$

$$\frac{(12)*(12)*(3)*(40)}{(12)*(12)*(3)*(2)*(3)*(40)*(8)*(3)*(4)} \begin{vmatrix} 1 & 4 & 0 & 4 & 1 \\ 0 & -3 & 0 & 2 & 3 \\ 0 & 0 & 0 & -176 & -104 \\ 0 & 0 & 0 & -218 & -132 \\ 0 & 0 & 0 & -260 & -156 \end{vmatrix} =$$

$$\frac{(12)*(12)*(3)*(40)}{(12)*(12)*(3)*(2)*(3)*(40)*(8)*(3)*(4)*(7085)*(5720)*(4796)} \begin{vmatrix} 1 & 4 & 0 & 4 & 1 \\ 0 & -3 & 0 & 2 & 3 \\ 0 & 0 & 0 & -1246960 & -736840 \\ 0 & 0 & 0 & 0 & -18200 \\ 0 & 0 & 0 & 0 & -11336 \end{vmatrix} =$$

0

$x_2 = \frac{\Delta_2}{\Delta} = 0$

$$\Delta_4 : \frac{(12)*(12)*(3)}{(12)*(12)*(3)*(2)*(3)} \begin{vmatrix} 1 & 4 & 6 & 0 & 1 \\ 0 & -3 & -2 & 0 & 3 \\ 0 & -15 & -23 & 0 & 2 \\ 0 & -40 & -70 & 0 & -4 \\ 0 & -30 & -60 & 0 & -9 \end{vmatrix} =$$

$$\frac{(12)*(12)*(3)*(40)}{(12)*(12)*(3)*(2)*(3)*(40)*(8)*(3)*(4)} \begin{vmatrix} 1 & 4 & 6 & 0 & 1 \\ 0 & -3 & -2 & 0 & 3 \\ 0 & 0 & -104 & 0 & -104 \\ 0 & 0 & -130 & 0 & -132 \\ 0 & 0 & -160 & 0 & -156 \end{vmatrix} =$$

$$\frac{(12)*(12)*(3)*(40)*(52)}{(12)*(12)*(3)*(2)*(3)*(40)*(8)*(3)*(4)*(20)*(16)*(13)} \begin{vmatrix} 1 & 4 & 6 & 0 & 1 \\ 0 & -3 & -2 & 0 & 3 \\ 0 & 0 & -2080 & 0 & -2080 \\ 0 & 0 & 0 & 0 & -32 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

0

$x_3 = \frac{\Delta_3}{\Delta} = 0$

$$\Delta_5 : \frac{(12)*(12)}{(12)*(12)*(3)*(2)*(3)} \begin{vmatrix} 1 & 4 & 6 & 4 & 0 \\ 0 & -3 & -2 & 2 & 0 \\ 0 & -45 & -69 & -36 & 0 \\ 0 & -40 & -70 & -46 & 0 \\ 0 & -30 & -60 & -45 & 0 \end{vmatrix} =$$

$$\frac{(12)*(12)*(120)}{(12)*(12)*(3)*(2)*(3)*(120)*(8)*(9)*(12)} \begin{vmatrix} 1 & 4 & 6 & 4 & 0 \\ 0 & -3 & -2 & 2 & 0 \\ 0 & 0 & -312 & -528 & 0 \\ 0 & 0 & -390 & -654 & 0 \\ 0 & 0 & -480 & -780 & 0 \end{vmatrix} =$$

$$\frac{(12)*(12)*(120)*(96)*(420)}{(12)*(12)*(3)*(2)*(3)*(120)*(8)*(9)*(12)*(20)*(16)*(13)} \begin{vmatrix} 1 & 4 & 6 & 4 & 0 \\ 0 & -3 & -2 & 2 & 0 \\ 0 & 0 & -6240 & -10560 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{vmatrix} =$$

$$\frac{(12)*(12)*(120)*(96)*(420)}{(12)*(12)*(3)*(2)*(3)*(120)*(8)*(9)*(12)*(20)*(16)*(13)} \begin{vmatrix} 1 & 4 & 6 & 4 & 0 \\ 0 & -3 & -2 & 2 & 0 \\ 0 & 0 & -6240 & -10560 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix} =$$

0

$x_4 = \frac{\Delta_4}{\Delta} = 0$

Answers: $x1 = 0, \quad x2 = 0, \quad x3 = 0, \quad x4 = 0, \quad x5 = 0$

37. **350**

Data:

$$\begin{cases} 2x_1 = 2 + x_2 = 2 + x_3 = 2 + x_4 = 2 + x_5 = 2 \\ x_1 = 0 + 2x_2 = 0 + x_3 = 0 + x_4 = 0 + x_5 = 0 \\ x_1 = 3 + x_2 = 3 + 3x_3 = 3 + x_4 = 3 + x_5 = 3 \\ x_1 = -2 + x_2 = -2 + x_3 = -2 + 4x_4 = -2 + x_5 = -2 \\ x_1 = 5 + x_2 = 5 + x_3 = 5 + x_4 = 5 + 5x_5 = 5 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 2 & 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 1 & 1 & 0 \\ 1 & 1 & 3 & 1 & 1 & 3 \\ 1 & 1 & 1 & 4 & 1 & -2 \\ 1 & 1 & 1 & 1 & 5 & 5 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & 1 & 1 & 1 & 1 & 2 \\ 0 & 3 & 1 & 1 & 1 & -2 \\ 0 & 1 & 5 & 1 & 1 & 4 \\ 0 & 1 & 1 & 7 & 1 & -6 \\ 0 & 1 & 1 & 1 & 9 & 8 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & 1 & 1 & 1 & 1 & 2 \\ 0 & 3 & 1 & 1 & 1 & -2 \\ 0 & 0 & 7 & 1 & 1 & 7 \\ 0 & 0 & 1 & 10 & 1 & -8 \\ 0 & 0 & 1 & 1 & 13 & 13 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & 1 & 1 & 1 & 1 & 2 \\ 0 & 3 & 1 & 1 & 1 & -2 \\ 0 & 0 & 7 & 1 & 1 & 7 \\ 0 & 0 & 0 & 23 & 2 & -21 \\ 0 & 0 & 0 & 1 & 15 & 14 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & 1 & 1 & 1 & 1 & 2 \\ 0 & 3 & 1 & 1 & 1 & -2 \\ 0 & 0 & 7 & 1 & 1 & 7 \\ 0 & 0 & 0 & 23 & 2 & -21 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & 1 & 1 & 1 & 1 & 2 \\ 0 & 3 & 1 & 1 & 1 & -2 \\ 0 & 0 & 7 & 1 & 1 & 7 \\ 0 & 0 & 0 & 23 & 2 & -21 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Answers: $x1 = 1, \quad x2 = -1, \quad x3 = 1, \quad x4 = -1, \quad x5 = 1$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 2 & 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 1 & 1 & 0 \\ 1 & 1 & 3 & 1 & 1 & 3 \\ 1 & 1 & 1 & 4 & 1 & -2 \\ 1 & 1 & 1 & 1 & 5 & 5 \end{vmatrix} =$$

$$\overline{(2)*(2)*(2)*(2)} \begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 0 & 3 & 1 & 1 & 1 \\ 0 & 1 & 5 & 1 & 1 \\ 0 & 1 & 1 & 7 & 1 \\ 0 & 1 & 1 & 1 & 9 \end{vmatrix} =$$

$$\frac{(2)*(2)*(2)}{(2)*(2)*(2)*(2)*(3)*(3)*(3)} \begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 0 & 3 & 1 & 1 & 1 \\ 0 & 0 & 7 & 1 & 1 \\ 0 & 0 & 1 & 10 & 1 \\ 0 & 0 & 1 & 1 & 13 \end{vmatrix} =$$

$$\frac{(2)*(2)*(2)*(3)*(6)}{(2)*(2)*(2)*(2)*(3)*(3)*(3)*(7)*(7)} \begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 0 & 3 & 1 & 1 & 1 \\ 0 & 0 & 7 & 1 & 1 \\ 0 & 0 & 0 & 23 & 2 \\ 0 & 0 & 0 & 1 & 15 \end{vmatrix} =$$

$$\frac{(2)*(2)*(2)*(3)*(6)*(343)}{(2)*(2)*(2)*(2)*(3)*(3)*(3)*(7)*(7)*(23)} \begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 0 & 3 & 1 & 1 & 1 \\ 0 & 0 & 7 & 1 & 1 \\ 0 & 0 & 0 & 23 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

98

$$\Delta_1 : \frac{(15)*(3)}{(15)*(10)*(-15)*(6)} \begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 & 1 \\ 0 & -5 & 15 & -5 & -5 \\ 0 & -30 & -30 & -75 & -30 \\ 0 & -3 & -3 & -3 & 5 \end{vmatrix} =$$

$$\frac{(15)*(3)*(15)*(15)*(15)}{(15)*(10)*(-15)*(6)*(15)*(-6)*(-1)*(-10)} \begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 & 1 \\ 0 & 0 & -7 & 1 & 1 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 15 & 15 & -65 \end{vmatrix} =$$

$$\frac{(15)*(3)*(15)*(15)*(15)*(15)*(40)}{(15)*(10)*(-15)*(6)*(15)*(-6)*(-1)*(-10)*(15)*(-105)*(-7)} \begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 & 1 \\ 0 & 0 & -7 & 1 & 1 \\ 0 & 0 & 0 & -435 & -120 \\ 0 & 0 & 0 & -3 & 11 \end{vmatrix} =$$

$$\frac{(15)*(3)*(15)*(15)*(15)*(15)*(40)*(1715)}{(15)*(10)*(-15)*(6)*(15)*(-6)*(-1)*(-10)*(15)*(-105)*(-7)*(145)} \begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 & 1 \\ 0 & 0 & -7 & 1 & 1 \\ 0 & 0 & 0 & -435 & -120 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

98

$x_0 = \frac{\Delta_0}{\Delta} = 1$

$$\Delta_2 : \frac{1}{(2)*(2)*(2)*(2)} \begin{vmatrix} 2 & 2 & 1 & 1 & 1 \\ 0 & -2 & 1 & 1 & 1 \\ 0 & 4 & 5 & 1 & 1 \\ 0 & -6 & 1 & 7 & 1 \\ 0 & 8 & 1 & 1 & 9 \end{vmatrix} =$$

$$\frac{(12)}{(2)*(2)*(2)*(2)*(12)*(-6)*(4)*(-3)} \begin{vmatrix} 2 & 2 & 1 & 1 & 1 \\ 0 & -2 & 1 & 1 & 1 \\ 0 & 0 & -42 & -18 & -18 \\ 0 & 0 & -8 & 16 & -8 \\ 0 & 0 & -15 & -15 & -39 \end{vmatrix} =$$

$$\frac{(12)}{(2)*(2)*(2)*(2)*(12)*(-6)*(4)*(-3)*(20)*(105)*(56)} \begin{vmatrix} 2 & 2 & 1 & 1 & 1 \\ 0 & -2 & 1 & 1 & 1 \\ 0 & 0 & -840 & -360 & -360 \\ 0 & 0 & 0 & 2040 & -480 \\ 0 & 0 & 0 & -480 & -1824 \end{vmatrix} =$$

$$\frac{(12)*(32928)}{(2)*(2)*(2)*(2)*(12)*(-6)*(4)*(-3)*(20)*(105)*(56)*(4)*(-17)} \begin{vmatrix} 2 & 2 & 1 & 1 & 1 \\ 0 & -2 & 1 & 1 & 1 \\ 0 & 0 & -840 & -360 & -360 \\ 0 & 0 & 0 & 8160 & -1920 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

-98

$x_1 = \frac{\Delta_1}{\Delta} = -1$

$$\Delta_3 : \frac{1}{(2)*(2)*(2)*(2)} \begin{vmatrix} 2 & 1 & 2 & 1 & 1 \\ 0 & 3 & -2 & 1 & 1 \\ 0 & 1 & 4 & 1 & 1 \\ 0 & 1 & -6 & 7 & 1 \\ 0 & 1 & 8 & 1 & 9 \end{vmatrix} =$$

$$\frac{(2)*(2)*(2)}{(2)*(2)*(2)*(2)*(3)*(3)*(3)} \begin{vmatrix} 2 & 1 & 2 & 1 & 1 \\ 0 & 3 & -2 & 1 & 1 \\ 0 & 0 & 7 & 1 & 1 \\ 0 & 0 & -8 & 10 & 1 \\ 0 & 0 & 13 & 1 & 13 \end{vmatrix} =$$

$$\frac{(2)*(2)*(2)*(104)*(48)}{(2)*(2)*(2)*(2)*(3)*(3)*(3)*(104)*(-91)*(56)} \begin{vmatrix} 2 & 1 & 2 & 1 & 1 \\ 0 & 3 & -2 & 1 & 1 \\ 0 & 0 & 7 & 1 & 1 \\ 0 & 0 & 0 & -1014 & -195 \\ 0 & 0 & 0 & -1 & 13 \end{vmatrix} =$$

$$\frac{(2)*(2)*(2)*(104)*(48)*(13377)}{(2)*(2)*(2)*(2)*(3)*(3)*(3)*(104)*(-91)*(56)*(1014)} \begin{vmatrix} 2 & 1 & 2 & 1 & 1 \\ 0 & 3 & -2 & 1 & 1 \\ 0 & 0 & 7 & 1 & 1 \\ 0 & 0 & 0 & -1014 & -195 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

98

$$x_2 = \frac{\Delta_2}{\Delta} = 1$$

$$\Delta_4 : \frac{1}{(2)*(2)*(2)*(2)} \begin{vmatrix} 2 & 1 & 1 & 2 & 1 \\ 0 & 3 & 1 & -2 & 1 \\ 0 & 1 & 5 & 4 & 1 \\ 0 & 1 & 1 & -6 & 1 \\ 0 & 1 & 1 & 8 & 9 \end{vmatrix} =$$

$$\frac{(2)*(2)*(2)}{(2)*(2)*(2)*(2)*(3)*(3)*(3)} \begin{vmatrix} 2 & 1 & 1 & 2 & 1 \\ 0 & 3 & 1 & -2 & 1 \\ 0 & 0 & 7 & 7 & 1 \\ 0 & 0 & 1 & -8 & 1 \\ 0 & 0 & 1 & 13 & 13 \end{vmatrix} =$$

$$\frac{(2)*(2)*(2)*(3)*(6)}{(2)*(2)*(2)*(2)*(3)*(3)*(3)*(7)*(7)} \begin{vmatrix} 2 & 1 & 1 & 2 & 1 \\ 0 & 3 & 1 & -2 & 1 \\ 0 & 0 & 7 & 7 & 1 \\ 0 & 0 & 0 & -21 & 2 \\ 0 & 0 & 0 & 14 & 15 \end{vmatrix} =$$

$$\frac{(2)*(2)*(2)*(3)*(6)*(2)}{(2)*(2)*(2)*(2)*(3)*(3)*(3)*(7)*(7)*(2)*(-3)} \begin{vmatrix} 2 & 1 & 1 & 2 & 1 \\ 0 & 3 & 1 & -2 & 1 \\ 0 & 0 & 7 & 7 & 1 \\ 0 & 0 & 0 & -21 & 2 \\ 0 & 0 & 0 & 0 & -49 \end{vmatrix} =$$

-98

$$x_3 = \frac{\Delta_3}{\Delta} = -1$$

$$\Delta_5 : \frac{1}{(2)*(2)*(2)*(2)} \begin{vmatrix} 2 & 1 & 1 & 1 & 2 \\ 0 & 3 & 1 & 1 & -2 \\ 0 & 1 & 5 & 1 & 4 \\ 0 & 1 & 1 & 7 & -6 \\ 0 & 1 & 1 & 1 & 8 \end{vmatrix} =$$

$$\frac{(2)*(2)}{(2)*(2)*(2)*(2)*(3)*(3)*(3)} \begin{vmatrix} 2 & 1 & 1 & 1 & 2 \\ 0 & 3 & 1 & 1 & -2 \\ 0 & 0 & 7 & 1 & 7 \\ 0 & 0 & 2 & 20 & -16 \\ 0 & 0 & 1 & 1 & 13 \end{vmatrix} =$$

$$\frac{(2)*(2)*(2)*(12)}{(2)*(2)*(2)*(2)*(3)*(3)*(3)*(2)*(7)*(14)} \begin{vmatrix} 2 & 1 & 1 & 1 & 2 \\ 0 & 3 & 1 & 1 & -2 \\ 0 & 0 & 7 & 1 & 7 \\ 0 & 0 & 0 & 138 & -126 \\ 0 & 0 & 0 & 1 & 14 \end{vmatrix} =$$

$$\frac{(2)*(2)*(2)*(12)*(2058)}{(2)*(2)*(2)*(2)*(3)*(3)*(3)*(2)*(7)*(14)*(138)} \begin{vmatrix} 2 & 1 & 1 & 1 & 2 \\ 0 & 3 & 1 & 1 & -2 \\ 0 & 0 & 7 & 1 & 7 \\ 0 & 0 & 0 & 138 & -126 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

98

$x_4 = \frac{\Delta_4}{\Delta} = 1$

Answers: $x1 = 1, \quad x2 = -1, \quad x3 = 1, \quad x4 = -1, \quad x5 = 1$

38. **351**

Data:

$$\begin{cases} x_1 = 13 + 2x_2 = 13 + 3x_3 = 13 + 4x_4 = 13 + 5x_5 = 13 \\ 2x_1 = 10 + x_2 = 10 + 2x_3 = 10 + 3x_4 = 10 + 4x_5 = 10 \\ 2x_1 = 11 + 2x_2 = 11 + x_3 = 11 + 2x_4 = 11 + 3x_5 = 11 \\ 2x_1 = 6 + 2x_2 = 6 + 2x_3 = 6 + x_4 = 6 + 2x_5 = 6 \\ 2x_1 = 3 + 2x_2 = 3 + 2x_3 = 3 + 2x_4 = 3 + x_5 = 3 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 13 \\ 2 & 1 & 2 & 3 & 4 & 10 \\ 2 & 2 & 1 & 2 & 3 & 11 \\ 2 & 2 & 2 & 1 & 2 & 6 \\ 2 & 2 & 2 & 2 & 1 & 3 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 13 \\ 0 & 3 & 4 & 5 & 6 & 16 \\ 0 & 2 & 5 & 6 & 7 & 15 \\ 0 & 2 & 4 & 7 & 8 & 20 \\ 0 & 2 & 4 & 6 & 9 & 23 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 13 \\ 0 & 3 & 4 & 5 & 6 & 16 \\ 0 & 0 & 7 & 8 & 9 & 13 \\ 0 & 0 & 4 & 11 & 12 & 28 \\ 0 & 0 & 4 & 8 & 15 & 37 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 13 \\ 0 & 3 & 4 & 5 & 6 & 16 \\ 0 & 0 & 7 & 8 & 9 & 13 \\ 0 & 0 & 0 & 15 & 16 & 48 \\ 0 & 0 & 0 & 8 & 23 & 69 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 13 \\ 0 & 3 & 4 & 5 & 6 & 16 \\ 0 & 0 & 7 & 8 & 9 & 13 \\ 0 & 0 & 0 & 15 & 16 & 48 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 13 \\ 0 & 3 & 4 & 5 & 6 & 16 \\ 0 & 0 & 7 & 8 & 9 & 13 \\ 0 & 0 & 0 & 15 & 16 & 48 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

Answers: $x1 = 0, \quad x2 = 2, \quad x3 = -2, \quad x4 = 0, \quad x5 = 3$

Solution by Kramer method:

$$\Delta: \begin{vmatrix} 1 & 2 & 3 & 4 & 5 & 13 \\ 2 & 1 & 2 & 3 & 4 & 10 \\ 2 & 2 & 1 & 2 & 3 & 11 \\ 2 & 2 & 2 & 1 & 2 & 6 \\ 2 & 2 & 2 & 2 & 1 & 3 \end{vmatrix} =$$

$$\frac{(2)}{(2)} \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -3 & -4 & -5 & -6 \\ 0 & -2 & -5 & -6 & -7 \\ 0 & -2 & -4 & -7 & -8 \\ 0 & -2 & -4 & -6 & -9 \end{vmatrix} =$$

$$\frac{(2)}{(2)*(2)*(3)*(3)*(3)} \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -6 & -8 & -10 & -12 \\ 0 & 0 & -7 & -8 & -9 \\ 0 & 0 & -4 & -11 & -12 \\ 0 & 0 & -4 & -8 & -15 \end{vmatrix} =$$

$$\frac{(2)}{(2)*(2)*(3)*(3)*(3)*(4)*(7)*(7)} \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -6 & -8 & -10 & -12 \\ 0 & 0 & -28 & -32 & -36 \\ 0 & 0 & 0 & -45 & -48 \\ 0 & 0 & 0 & -24 & -69 \end{vmatrix} =$$

$$\frac{(2)}{(2)*(2)*(3)*(3)*(3)*(4)*(7)*(7)*(8)*(15)} \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -6 & -8 & -10 & -12 \\ 0 & 0 & -28 & -32 & -36 \\ 0 & 0 & 0 & -360 & -384 \\ 0 & 0 & 0 & 0 & -651 \end{vmatrix} =$$

31

$$\Delta_1: \frac{(330)*(33)}{(330)*(429)*(390)*(715)*(1430)} \begin{vmatrix} 13 & 2 & 3 & 4 & 5 \\ 0 & -7 & -4 & -1 & 2 \\ 0 & 120 & -600 & -540 & -480 \\ 0 & 770 & 440 & -605 & -220 \\ 0 & 2200 & 1870 & 1540 & -220 \end{vmatrix} =$$

$$\frac{(330)*(33)*(6600)*(42900)*(42900)}{(330)*(429)*(390)*(715)*(1430)*(6600)*(-385)*(-60)*(-21)} \begin{vmatrix} 13 & 2 & 3 & 4 & 5 \\ 0 & -7 & -4 & -1 & 2 \\ 0 & 0 & 6 & 5 & 4 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -12870 & -25740 & -8580 \end{vmatrix} =$$

$$\frac{(330)*(33)*(6600)*(42900)*(42900)*(2145)*(15015)}{(330)*(429)*(390)*(715)*(1430)*(6600)*(-385)*(-60)*(-21)*(2145)*(-1)} \begin{vmatrix} 13 & 2 & 3 & 4 & 5 \\ 0 & -7 & -4 & -1 & 2 \\ 0 & 0 & 6 & 5 & 4 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{vmatrix} =$$

$$\frac{(330)*(33)*(6600)*(42900)*(42900)*(2145)*(15015)}{(330)*(429)*(390)*(715)*(1430)*(6600)*(-385)*(-60)*(-21)*(2145)*(-1)} \begin{vmatrix} 13 & 2 & 3 & 4 & 5 \\ 0 & -7 & -4 & -1 & 2 \\ 0 & 0 & 6 & 5 & 4 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix} =$$

0

$x_0 = \frac{\Delta_0}{\Delta} = 0$

$$\Delta_2 : \frac{(2)}{(2)} \begin{vmatrix} 1 & 13 & 3 & 4 & 5 \\ 0 & -16 & -4 & -5 & -6 \\ 0 & -15 & -5 & -6 & -7 \\ 0 & -20 & -4 & -7 & -8 \\ 0 & -23 & -4 & -6 & -9 \end{vmatrix} =$$

$$\frac{(2)}{(2)*(345)*(368)*(276)*(240)} \begin{vmatrix} 1 & 13 & 3 & 4 & 5 \\ 0 & -5520 & -1380 & -1725 & -2070 \\ 0 & 0 & -460 & -483 & -506 \\ 0 & 0 & 276 & -207 & -138 \\ 0 & 0 & 420 & 285 & -90 \end{vmatrix} =$$

$$\frac{(2)*(1932)*(276)}{(2)*(345)*(368)*(276)*(240)*(21)*(-35)*(-23)} \begin{vmatrix} 1 & 13 & 3 & 4 & 5 \\ 0 & -5520 & -1380 & -1725 & -2070 \\ 0 & 0 & -9660 & -10143 & -10626 \\ 0 & 0 & 0 & 9 & 8 \\ 0 & 0 & 0 & 13 & 46 \end{vmatrix} =$$

$$\frac{(2)*(1932)*(276)*(13)*(310)}{(2)*(345)*(368)*(276)*(240)*(21)*(-35)*(-23)*(13)*(9)} \begin{vmatrix} 1 & 13 & 3 & 4 & 5 \\ 0 & -5520 & -1380 & -1725 & -2070 \\ 0 & 0 & -9660 & -10143 & -10626 \\ 0 & 0 & 0 & 9 & 8 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

62

$$x_1 = \frac{\Delta_1}{\Delta} = 2$$

$$\Delta_3 : \frac{(2)}{(2)} \begin{vmatrix} 1 & 2 & 13 & 4 & 5 \\ 0 & -3 & -16 & -5 & -6 \\ 0 & -2 & -15 & -6 & -7 \\ 0 & -2 & -20 & -7 & -8 \\ 0 & -2 & -23 & -6 & -9 \end{vmatrix} =$$

$$\frac{(2)}{(2)*(2)*(3)*(3)*(3)} \begin{vmatrix} 1 & 2 & 13 & 4 & 5 \\ 0 & -6 & -32 & -10 & -12 \\ 0 & 0 & -13 & -8 & -9 \\ 0 & 0 & -28 & -11 & -12 \\ 0 & 0 & -37 & -8 & -15 \end{vmatrix} =$$

$$\frac{(2)*(111)*(168)}{(2)*(2)*(3)*(3)*(3)*(1036)*(481)*(364)} \begin{vmatrix} 1 & 2 & 13 & 4 & 5 \\ 0 & -6 & -32 & -10 & -12 \\ 0 & 0 & -13468 & -8288 & -9324 \\ 0 & 0 & 0 & 27 & 32 \\ 0 & 0 & 0 & 32 & 23 \end{vmatrix} =$$

$$\frac{(2)*(111)*(168)*(32)}{(2)*(2)*(3)*(3)*(3)*(1036)*(481)*(364)*(32)*(27)} \begin{vmatrix} 1 & 2 & 13 & 4 & 5 \\ 0 & -6 & -32 & -10 & -12 \\ 0 & 0 & -13468 & -8288 & -9324 \\ 0 & 0 & 0 & 27 & 32 \\ 0 & 0 & 0 & 0 & -403 \end{vmatrix} =$$

$- 62$

$$x_2 = \frac{\Delta_2}{\Delta} = -2$$

$$\Delta_4 : \frac{(2)}{(2)} \begin{vmatrix} 1 & 2 & 3 & 13 & 5 \\ 0 & -3 & -4 & -16 & -6 \\ 0 & -2 & -5 & -15 & -7 \\ 0 & -2 & -4 & -20 & -8 \\ 0 & -2 & -4 & -23 & -9 \end{vmatrix} =$$

$$\frac{(2)}{(2)*(2)*(3)*(3)*(3)} \begin{vmatrix} 1 & 2 & 3 & 13 & 5 \\ 0 & -6 & -8 & -32 & -12 \\ 0 & 0 & -7 & -13 & -9 \\ 0 & 0 & -4 & -28 & -12 \\ 0 & 0 & -4 & -37 & -15 \end{vmatrix} =$$

$$\frac{(2)}{(2)*(2)*(3)*(3)*(3)*(4)*(7)*(7)} \begin{vmatrix} 1 & 2 & 3 & 13 & 5 \\ 0 & -6 & -8 & -32 & -12 \\ 0 & 0 & -28 & -52 & -36 \\ 0 & 0 & 0 & -144 & -48 \\ 0 & 0 & 0 & -207 & -69 \end{vmatrix} =$$

$$\frac{(2)}{(2)*(2)*(3)*(3)*(3)*(4)*(7)*(7)*(23)*(16)} \begin{vmatrix} 1 & 2 & 3 & 13 & 5 \\ 0 & -6 & -8 & -32 & -12 \\ 0 & 0 & -28 & -52 & -36 \\ 0 & 0 & 0 & -3312 & -1104 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix} =$$

0

$x_3 = \frac{\Delta_3}{\Delta} = 0$

$$\Delta_5 : \frac{(2)}{(2)} \begin{vmatrix} 1 & 2 & 3 & 4 & 13 \\ 0 & -3 & -4 & -5 & -16 \\ 0 & -2 & -5 & -6 & -15 \\ 0 & -2 & -4 & -7 & -20 \\ 0 & -2 & -4 & -6 & -23 \end{vmatrix} =$$

$$\frac{(2)}{(2)*(2)*(3)*(3)*(3)} \begin{vmatrix} 1 & 2 & 3 & 4 & 13 \\ 0 & -6 & -8 & -10 & -32 \\ 0 & 0 & -7 & -8 & -13 \\ 0 & 0 & -4 & -11 & -28 \\ 0 & 0 & -4 & -8 & -37 \end{vmatrix} =$$

$$\frac{(2)}{(2)*(2)*(3)*(3)*(3)*(4)*(7)*(7)} \begin{vmatrix} 1 & 2 & 3 & 4 & 13 \\ 0 & -6 & -8 & -10 & -32 \\ 0 & 0 & -28 & -32 & -52 \\ 0 & 0 & 0 & -45 & -144 \\ 0 & 0 & 0 & -24 & -207 \end{vmatrix} =$$

$$\frac{(2)}{(2)*(2)*(3)*(3)*(3)*(4)*(7)*(7)*(8)*(15)} \begin{vmatrix} 1 & 2 & 3 & 4 & 13 \\ 0 & -6 & -8 & -10 & -32 \\ 0 & 0 & -28 & -32 & -52 \\ 0 & 0 & 0 & -360 & -1152 \\ 0 & 0 & 0 & 0 & -1953 \end{vmatrix} =$$

93

$x_4 = \frac{\Delta_4}{\Delta} = 3$

Answers: $x1 = 0, \quad x2 = 2, \quad x3 = -2, \quad x4 = 0, \quad x5 = 3$

39. **352**

Data:

$$\begin{cases} x_1 = -1 + 2x_2 = -1 + 3x_3 = -1 - 4x_4 = -1 + x_5 = -1 \\ 2x_1 = 8 + x_2 = 8 - 3x_3 = 8 + 4x_4 = 8 - 2x_5 = 8 \\ 3x_1 = 3 + x_2 = 3 + x_3 = 3 - 2x_4 = 3 + x_5 = 3 \\ 4x_1 = -2 + 3x_2 = -2 + 4x_3 = -2 + 2x_4 = -2 + 2x_5 = -2 \\ x_1 = -3 + x_2 = -3 - x_3 = -3 - 2x_4 = -3 + 3x_5 = -3 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 1 & 2 & -3 & 4 & -1 & -1 \\ 2 & -1 & 3 & -4 & 2 & 8 \\ 3 & 1 & -1 & 2 & -1 & 3 \\ 4 & 3 & 4 & 2 & 2 & -2 \\ 1 & -1 & -1 & 2 & -3 & -3 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & -3 & 4 & -1 & -1 \\ 0 & 5 & -9 & 12 & -4 & -10 \\ 0 & 5 & -8 & 10 & -2 & -6 \\ 0 & 5 & -16 & 14 & -6 & -2 \\ 0 & 3 & -2 & 2 & 2 & 2 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & -3 & 4 & -1 & -1 \\ 0 & 5 & -9 & 12 & -4 & -10 \\ 0 & 0 & 1 & -2 & 2 & 4 \\ 0 & 0 & 7 & -2 & 2 & -8 \\ 0 & 0 & 17 & -26 & 22 & 40 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & -3 & 4 & -1 & -1 \\ 0 & 5 & -9 & 12 & -4 & -10 \\ 0 & 0 & 1 & -2 & 2 & 4 \\ 0 & 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 2 & -3 & -7 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & -3 & 4 & -1 & -1 \\ 0 & 5 & -9 & 12 & -4 & -10 \\ 0 & 0 & 1 & -2 & 2 & 4 \\ 0 & 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 2 & -3 & 4 & -1 & -1 \\ 0 & 5 & -9 & 12 & -4 & -10 \\ 0 & 0 & 1 & -2 & 2 & 4 \\ 0 & 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Answers: $x1 = 2, \quad x2 = 0, \quad x3 = -2, \quad x4 = -2, \quad x5 = 1$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 1 & 2 & -3 & 4 & -1 & -1 \\ 2 & -1 & 3 & -4 & 2 & 8 \\ 3 & 1 & -1 & 2 & -1 & 3 \\ 4 & 3 & 4 & 2 & 2 & -2 \\ 1 & -1 & -1 & 2 & -3 & -3 \end{vmatrix} =$$

$$\frac{(6)*(4)*(3)}{(12)*(6)*(4)*(3)*(12)} \begin{vmatrix} 12 & 24 & -36 & 48 & -12 \\ 0 & -5 & 9 & -12 & 4 \\ 0 & -5 & 8 & -10 & 2 \\ 0 & -5 & 16 & -14 & 6 \\ 0 & -36 & 24 & -24 & -24 \end{vmatrix} =$$

$$\frac{(6)*(4)*(3)*(36)*(36)}{(12)*(6)*(4)*(3)*(12)*(36)*(36)*(36)*(5)} \begin{vmatrix} 12 & 24 & -36 & 48 & -12 \\ 0 & -5 & 9 & -12 & 4 \\ 0 & 0 & -36 & 72 & -72 \\ 0 & 0 & 7 & -2 & 2 \\ 0 & 0 & -204 & 312 & -264 \end{vmatrix} =$$

$$\frac{(6)*(4)*(3)*(36)*(36)*(7344)*(1008)}{(12)*(6)*(4)*(3)*(12)*(36)*(36)*(36)*(5)*(119)*(-612)*(21)} \begin{vmatrix} 12 & 24 & -36 & 48 & -12 \\ 0 & -5 & 9 & -12 & 4 \\ 0 & 0 & -4284 & 8568 & -8568 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -2 & 3 \end{vmatrix} =$$

$$\frac{(6)*(4)*(3)*(36)*(36)*(7344)*(1008)*(2)}{(12)*(6)*(4)*(3)*(12)*(36)*(36)*(36)*(5)*(119)*(-612)*(21)*(2)} \begin{vmatrix} 12 & 24 & -36 & 48 & -12 \\ 0 & -5 & 9 & -12 & 4 \\ 0 & 0 & -4284 & 8568 & -8568 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

48

$$\Delta_1 : \frac{(3)*(8)*(12)}{(24)*(-3)*(-8)*(12)*(8)} \begin{vmatrix} -24 & 48 & -72 & 96 & -24 \\ 0 & -15 & 21 & -28 & 6 \\ 0 & -7 & 10 & -14 & 4 \\ 0 & -1 & 10 & -6 & 4 \\ 0 & -56 & 64 & -80 & 0 \end{vmatrix} =$$

$$\frac{(3)*(8)*(12)*(56)*(8)*(56)}{(24)*(-3)*(-8)*(12)*(8)*(56)*(120)*(840)*(15)} \begin{vmatrix} -24 & 48 & -72 & 96 & -24 \\ 0 & -15 & 21 & -28 & 6 \\ 0 & 0 & 3 & -14 & 18 \\ 0 & 0 & 129 & -62 & 54 \\ 0 & 0 & -216 & 368 & -336 \end{vmatrix} =$$

$$\frac{(3)*(8)*(12)*(56)*(8)*(56)*(3096)}{(24)*(-3)*(-8)*(12)*(8)*(56)*(120)*(840)*(15)*(3096)*(72)*(-43)} \begin{vmatrix} -24 & 48 & -72 & 96 & -24 \\ 0 & -15 & 21 & -28 & 6 \\ 0 & 0 & 3 & -14 & 18 \\ 0 & 0 & 0 & 38880 & -51840 \\ 0 & 0 & 0 & 27520 & -41280 \end{vmatrix} =$$

$$\frac{(3)*(8)*(12)*(56)*(8)*(56)*(3096)}{(24)*(-3)*(-8)*(12)*(8)*(56)*(120)*(840)*(15)*(3096)*(72)*(-43)*(172)*(243)} \begin{vmatrix} -24 & 48 & -72 & 96 & -24 \\ 0 & -15 & 21 & -28 & 6 \\ 0 & 0 & 3 & -14 & 18 \\ 0 & 0 & 0 & 6687360 & -8916480 \\ 0 & 0 & 0 & 0 & -1114560 \end{vmatrix} =$$

96

$$x_0 = \frac{\Delta_0}{\Delta} = 2$$

$$\Delta_2 : \frac{(6)*(8)*(6)}{(12)*(6)*(4)*(3)*(12)} \begin{vmatrix} 12 & -12 & -36 & 48 & -12 \\ 0 & 10 & 9 & -12 & 4 \\ 0 & 3 & 4 & -5 & 1 \\ 0 & 1 & 8 & -7 & 3 \\ 0 & -24 & 24 & -24 & -24 \end{vmatrix} =$$

$$\frac{(6)*(8)*(6)*(12)*(12)*(12)}{(12)*(6)*(4)*(3)*(12)*(12)*(40)*(120)*(-5)} \begin{vmatrix} 12 & -12 & -36 & 48 & -12 \\ 0 & 10 & 9 & -12 & 4 \\ 0 & 0 & 52 & -56 & -8 \\ 0 & 0 & 71 & -58 & 26 \\ 0 & 0 & -19 & 22 & 6 \end{vmatrix} =$$

$$\frac{(6)*(8)*(6)*(12)*(12)*(12)*(18240)}{(12)*(6)*(4)*(3)*(12)*(12)*(40)*(120)*(-5)*(1349)*(988)*(-3692)} \begin{vmatrix} 12 & -12 & -36 & 48 & -12 \\ 0 & 10 & 9 & -12 & 4 \\ 0 & 0 & 70148 & -75544 & -10792 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -5680 & -11360 \end{vmatrix} =$$

$$\frac{(6)*(8)*(6)*(12)*(12)*(12)*(18240)*(5680)}{(12)*(6)*(4)*(3)*(12)*(12)*(40)*(120)*(-5)*(1349)*(988)*(-3692)*(5680)*(-1)} \begin{vmatrix} 12 & -12 & -36 & 48 & -12 \\ 0 & 10 & 9 & -12 & 4 \\ 0 & 0 & 70148 & -75544 & -10792 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix} =$$

0

$$x_1 = \frac{\Delta_1}{\Delta} = 0$$

$$\Delta_3 : \frac{(6)*(4)*(3)}{(12)*(6)*(4)*(3)*(12)} \begin{vmatrix} 12 & 24 & -12 & 48 & -12 \\ 0 & -5 & 10 & -12 & 4 \\ 0 & -5 & 6 & -10 & 2 \\ 0 & -5 & 2 & -14 & 6 \\ 0 & -36 & -24 & -24 & -24 \end{vmatrix} =$$

$$\frac{(6)*(4)*(3)*(36)*(72)}{(12)*(6)*(4)*(3)*(12)*(36)*(36)*(36)*(5)} \begin{vmatrix} 12 & 24 & -12 & 48 & -12 \\ 0 & -5 & 10 & -12 & 4 \\ 0 & 0 & -144 & 72 & -72 \\ 0 & 0 & -4 & -1 & 1 \\ 0 & 0 & -480 & 312 & -264 \end{vmatrix} =$$

$$\frac{(6)*(4)*(3)*(36)*(72)*(1080)}{(12)*(6)*(4)*(3)*(12)*(36)*(36)*(36)*(5)*(10)*(360)*(3)} \begin{vmatrix} 12 & 24 & -12 & 48 & -12 \\ 0 & -5 & 10 & -12 & 4 \\ 0 & 0 & -1440 & 720 & -720 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 216 & -72 \end{vmatrix} =$$

$$\frac{(6)*(4)*(3)*(36)*(72)*(1080)*(216)}{(12)*(6)*(4)*(3)*(12)*(36)*(36)*(36)*(5)*(10)*(360)*(3)*(216)*(-1)} \begin{vmatrix} 12 & 24 & -12 & 48 & -12 \\ 0 & -5 & 10 & -12 & 4 \\ 0 & 0 & -1440 & 720 & -720 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -144 \end{vmatrix} =$$

$$-96$$

$$x_2 = \frac{\Delta_2}{\Delta} = -2$$

$$\Delta_4 : \frac{(6)*(4)*(3)}{(12)*(6)*(4)*(3)*(12)} \begin{vmatrix} 12 & 24 & -36 & -12 & -12 \\ 0 & -5 & 9 & 10 & 4 \\ 0 & -5 & 8 & 6 & 2 \\ 0 & -5 & 16 & 2 & 6 \\ 0 & -36 & 24 & -24 & -24 \end{vmatrix} =$$

$$\frac{(6)*(4)*(3)*(36)*(36)}{(12)*(6)*(4)*(3)*(12)*(36)*(36)*(36)*(5)} \begin{vmatrix} 12 & 24 & -36 & -12 & -12 \\ 0 & -5 & 9 & 10 & 4 \\ 0 & 0 & -36 & -144 & -72 \\ 0 & 0 & 7 & -8 & 2 \\ 0 & 0 & -204 & -480 & -264 \end{vmatrix} =$$

$$\frac{(6)*(4)*(3)*(36)*(36)*(7344)*(1008)}{(12)*(6)*(4)*(3)*(12)*(36)*(36)*(36)*(5)*(119)*(-612)*(21)} \begin{vmatrix} 12 & 24 & -36 & -12 & -12 \\ 0 & -5 & 9 & 10 & 4 \\ 0 & 0 & -4284 & -17136 & -8568 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 7 & 3 \end{vmatrix} =$$

$$\frac{(6)*(4)*(3)*(36)*(36)*(7344)*(1008)*(7)*(2)}{(12)*(6)*(4)*(3)*(12)*(36)*(36)*(36)*(5)*(119)*(-612)*(21)*(7)*(3)} \begin{vmatrix} 12 & 24 & -36 & -12 & -12 \\ 0 & -5 & 9 & 10 & 4 \\ 0 & 0 & -4284 & -17136 & -8568 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

$$- 96$$

$$x_3 = \frac{\Delta_3}{\Delta} = -2$$

$$\Delta_5 : \frac{(6)*(4)*(3)}{(12)*(6)*(4)*(3)*(12)} \begin{vmatrix} 12 & 24 & -36 & 48 & -12 \\ 0 & -5 & 9 & -12 & 10 \\ 0 & -5 & 8 & -10 & 6 \\ 0 & -5 & 16 & -14 & 2 \\ 0 & -36 & 24 & -24 & -24 \end{vmatrix} =$$

$$\frac{(6)*(4)*(3)*(36)}{(12)*(6)*(4)*(3)*(12)*(36)*(36)*(36)*(5)} \begin{vmatrix} 12 & 24 & -36 & 48 & -12 \\ 0 & -5 & 9 & -12 & 10 \\ 0 & 0 & -36 & 72 & -144 \\ 0 & 0 & 252 & -72 & -288 \\ 0 & 0 & -204 & 312 & -480 \end{vmatrix} =$$

$$\frac{(6)*(4)*(3)*(36)*(7344)*(1008)}{(12)*(6)*(4)*(3)*(12)*(36)*(36)*(36)*(5)*(119)*(-17)*(21)} \begin{vmatrix} 12 & 24 & -36 & 48 & -12 \\ 0 & -5 & 9 & -12 & 10 \\ 0 & 0 & -4284 & 8568 & -17136 \\ 0 & 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & -2 & 7 \end{vmatrix} =$$

$$\frac{(6)*(4)*(3)*(36)*(7344)*(1008)*(2)}{(12)*(6)*(4)*(3)*(12)*(36)*(36)*(36)*(5)*(119)*(-17)*(21)*(2)} \begin{vmatrix} 12 & 24 & -36 & 48 & -12 \\ 0 & -5 & 9 & -12 & 10 \\ 0 & 0 & -4284 & 8568 & -17136 \\ 0 & 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} =$$

$$48$$

$$x_4 = \frac{\Delta_4}{\Delta} = 1$$

Answers: $x1 = 2, \quad x2 = 0, \quad x3 = -2, \quad x4 = -2, \quad x5 = 1$

40. **353**
    Data:

$$\begin{cases} 2x_1 = 0 + 3x_2 = 0 - 4x_3 = 0 + 3x_4 = 0 \\ 3x_1 = 0 + x_2 = 0 - 11x_3 = 0 + 13x_4 = 0 \\ 4x_1 = 0 + 5x_2 = 0 + 7x_3 = 0 - 2x_4 = 0 \\ 13x_1 = 0 + 25x_2 = 0 - x_3 = 0 + 11x_4 = 0 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 2 & -3 & 4 & -3 & 0 \\ 3 & -1 & 11 & -13 & 0 \\ 4 & 5 & -7 & -2 & 0 \\ 13 & -25 & 1 & 11 & 0 \end{bmatrix} \Rightarrow$$

Answers: Infinity of solutions

Solution by Kramer method:

Answers: No solution

41. **Test1**
    Data:

$$\begin{cases} 3x_1 = 5 + 4x_2 = 5 - 2x_3 = 5 \\ 2x_1 = 9 + x_2 = 9 + 4x_3 = 9 \\ 5x_1 = 3 + 2x_2 = 3 - x_3 = 3 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 3 & -4 & 2 & 5 \\ 2 & 1 & 4 & 9 \\ 5 & -2 & -1 & 3 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 3 & -4 & 2 & 5 \\ 0 & 11 & 8 & 17 \\ 0 & 14 & -13 & -16 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 3 & -4 & 2 & 5 \\ 0 & 11 & 8 & 17 \\ 0 & 0 & 85 & 138 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 3 & -4 & 2 & 5 \\ 0 & 11 & 8 & 17 \\ 0 & 0 & 85 & 138 \end{bmatrix}$$

Answers: $x1 = \frac{91}{85}, \quad x2 = \frac{31}{85}, \quad x3 = \frac{138}{85}$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 3 & -4 & 2 & 5 \\ 2 & 1 & 4 & 9 \\ 5 & -2 & -1 & 3 \end{vmatrix} =$$

$$\frac{(10)*(5)}{(10)*(15)*(6)} \begin{vmatrix} 3 & -4 & 2 \\ 0 & 11 & 8 \\ 0 & 28 & -26 \end{vmatrix} =$$

$$\frac{(10)*(5)*(28)}{(10)*(15)*(6)*(28)*(11)} \begin{vmatrix} 3 & -4 & 2 \\ 0 & 11 & 8 \\ 0 & 0 & -510 \end{vmatrix} =$$

$$-85$$

$$\Delta_1 : \frac{(9)}{(9)*(5)*(15)} \begin{vmatrix} 5 & -4 & 2 \\ 0 & 41 & 2 \\ 0 & 6 & -33 \end{vmatrix} =$$

$$\frac{(9)*(6)}{(9)*(5)*(15)*(6)*(41)} \begin{vmatrix} 5 & -4 & 2 \\ 0 & 41 & 2 \\ 0 & 0 & -1365 \end{vmatrix} =$$

$$-91$$

$x_0 = \frac{\Delta_0}{\Delta} = \frac{91}{85}$

$$\Delta_2 : \frac{(10)*(5)}{(10)*(15)*(6)} \begin{vmatrix} 3 & 5 & 2 \\ 0 & 17 & 8 \\ 0 & -32 & -26 \end{vmatrix} =$$

$$\frac{(10)*(5)*(32)*(186)}{(10)*(15)*(6)*(32)*(-17)} \begin{vmatrix} 3 & 5 & 2 \\ 0 & 17 & 8 \\ 0 & 0 & 1 \end{vmatrix} =$$

$$-31$$

$x_1 = \frac{\Delta_1}{\Delta} = \frac{31}{85}$

$$\Delta_3 : \frac{(10)*(5)}{(10)*(15)*(6)} \begin{vmatrix} 3 & -4 & 5 \\ 0 & 11 & 17 \\ 0 & 28 & -32 \end{vmatrix} =$$

$$\frac{(10)*(5)*(28)}{(10)*(15)*(6)*(28)*(11)} \begin{vmatrix} 3 & -4 & 5 \\ 0 & 11 & 17 \\ 0 & 0 & -828 \end{vmatrix} =$$

$$-138$$

$$x_2 = \frac{\Delta_2}{\Delta} = \frac{138}{85}$$

Answers: $x1 = \frac{91}{85}, \quad x2 = \frac{31}{85}, \quad x3 = \frac{138}{85}$

42. **Test2**

Data:

$$\begin{cases} 3x_1 = 5 + x_2 = 5 - 0x_3 = 5 \\ 2x_1 = 0 - x_2 = 0 + x_3 = 0 \\ 2x_1 = 15 + x_2 = 15 - 4x_3 = 15 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 3 & -1 & 0 & 5 \\ 2 & -1 & -1 & 0 \\ 2 & -1 & 4 & 15 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 3 & -1 & 0 & 5 \\ 0 & 1 & 3 & 10 \\ 0 & 1 & -12 & -35 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 3 & -1 & 0 & 5 \\ 0 & 1 & 3 & 10 \\ 0 & 0 & 1 & 3 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 3 & -1 & 0 & 5 \\ 0 & 1 & 3 & 10 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

Answers: $x1 = 2, \quad x2 = 1, \quad x3 = 3$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 3 & -1 & 0 & 5 \\ -2 & 1 & 1 & 0 \\ 2 & -1 & 4 & 15 \end{vmatrix} =$$

$$\frac{1}{(2)*(-3)*(3)} \begin{vmatrix} 6 & -2 & 0 \\ 0 & -1 & -3 \\ 0 & -1 & 12 \end{vmatrix} =$$

$$\frac{(15)}{(2)*(-3)*(3)} \begin{vmatrix} 6 & -2 & 0 \\ 0 & -1 & -3 \\ 0 & 0 & 1 \end{vmatrix} =$$

$$5$$

$$\Delta_1 : \frac{(2)}{(3)} \begin{vmatrix} 15 & -3 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{vmatrix} =$$

$$\frac{(2)}{(3)} \begin{vmatrix} 15 & -3 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} =$$

$$10$$

$$x_0 = \frac{\Delta_0}{\Delta} = 2$$

$$\Delta_2 : \frac{(2)}{(2)*(-3)*(3)} \begin{vmatrix} 3 & 5 & 0 \\ 0 & -10 & -3 \\ 0 & 35 & 12 \end{vmatrix} =$$

$$\frac{(2)}{(2)*(-3)*(3)*(7)*(-2)} \begin{vmatrix} 3 & 5 & 0 \\ 0 & -70 & -21 \\ 0 & 0 & -3 \end{vmatrix} =$$

5

$$x_1 = \frac{\Delta_1}{\Delta} = 1$$

$$\Delta_3 : \frac{(2)}{(2)*(-3)*(3)} \begin{vmatrix} 3 & -1 & 5 \\ 0 & -1 & -10 \\ 0 & -1 & 35 \end{vmatrix} =$$

$$\frac{(2)*(45)}{(2)*(-3)*(3)} \begin{vmatrix} 3 & -1 & 5 \\ 0 & -1 & -10 \\ 0 & 0 & 1 \end{vmatrix} =$$

15

$$x_2 = \frac{\Delta_2}{\Delta} = 3$$

Answers: $x1 = 2, \quad x2 = 1, \quad x3 = 3$

43. **Test3**
Data:

$$\begin{cases} 2x_1 = 9 + x_2 = 9 - 3x_3 = 9 \\ 3x_1 = -4 + 5x_2 = -4 - x_3 = -4 \\ 4x_1 = 5 + 7x_2 = 5 - x_3 = 5 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 2 & -1 & 3 & 9 \\ 3 & -5 & 1 & -4 \\ 4 & -7 & 1 & 5 \end{bmatrix} \Rightarrow$$

Answers: No solutions

Solution by Kramer method:

Answers: No solution

44. **Test4**
Data:

$$\begin{cases} 2x_1 = 4 + 2x_2 = 4 + x_3 = 4 - x_4 = 4 \\ 4x_1 = 6 + 3x_2 = 6 + x_3 = 6 - 2x_4 = 6 \\ 8x_1 = 12 + 5x_2 = 12 + 3x_3 = 12 - 4x_4 = 12 \\ 3x_1 = 6 + 3x_2 = 6 + 2x_3 = 6 - 2x_4 = 6 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 2 & 2 & -1 & 1 & 4 \\ 4 & 3 & -1 & 2 & 6 \\ 8 & 5 & -3 & 4 & 12 \\ 3 & 3 & -2 & 2 & 6 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & 2 & -1 & 1 & 4 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 3 & -1 & 0 & 4 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & 2 & -1 & 1 & 4 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & 2 & -1 & 1 & 4 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & 2 & -1 & 1 & 4 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

Answers: $x1 = 1, \quad x2 = 1, \quad x3 = -1, \quad x4 = -1$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 2 & 2 & -1 & 1 & 4 \\ 4 & 3 & -1 & 2 & 6 \\ 8 & 5 & -3 & 4 & 12 \\ 3 & 3 & -2 & 2 & 6 \end{vmatrix} =$$

$$\frac{(12)*(6)*(3)*(4)}{(12)*(6)*(3)*(8)} \begin{vmatrix} 2 & 2 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{vmatrix} =$$

$$\frac{(12)*(6)*(3)*(4)*(3)}{(12)*(6)*(3)*(8)*(3)} \begin{vmatrix} 2 & 2 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & -1 & 1 \end{vmatrix} =$$

$$\frac{(12)*(6)*(3)*(4)*(3)*(2)}{(12)*(6)*(3)*(8)*(3)*(2)} \begin{vmatrix} 2 & 2 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

2

$$\Delta_1 : \frac{(3)}{(3)*(2)*(2)} \begin{vmatrix} 4 & 2 & -1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{vmatrix} =$$

$$\frac{(3)}{(3)*(2)*(2)} \begin{vmatrix} 4 & 2 & -1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{vmatrix} =$$

$$\frac{(3)}{(3)*(2)*(2)*(-1)} \begin{vmatrix} 4 & 2 & -1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -2 \end{vmatrix} =$$

0

$x_0 = \frac{\Delta_0}{\Delta} = 0$

$\Delta_2 : \frac{(12)*(6)*(3)*(4)}{(12)*(6)*(3)*(8)} \begin{vmatrix} 2 & 4 & -1 & 1 \\ 0 & -2 & 1 & 0 \\ 0 & -4 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{vmatrix} =$

$\frac{(12)*(6)*(3)*(4)*(2)}{(12)*(6)*(3)*(8)*(2)} \begin{vmatrix} 2 & 4 & -1 & 1 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{vmatrix} =$

$\frac{(12)*(6)*(3)*(4)*(2)}{(12)*(6)*(3)*(8)*(2)} \begin{vmatrix} 2 & 4 & -1 & 1 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$

2

$x_1 = \frac{\Delta_1}{\Delta} = 1$

$\Delta_3 : \frac{(12)*(4)}{(12)*(6)*(3)*(8)} \begin{vmatrix} 2 & 2 & 4 & 1 \\ 0 & -6 & -12 & 0 \\ 0 & -9 & -12 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$

$\frac{(12)*(4)*(12)}{(12)*(6)*(3)*(8)*(3)*(2)} \begin{vmatrix} 2 & 2 & 4 & 1 \\ 0 & -18 & -36 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$

$\frac{(12)*(4)*(12)}{(12)*(6)*(3)*(8)*(3)*(2)} \begin{vmatrix} 2 & 2 & 4 & 1 \\ 0 & -18 & -36 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$

-2

$x_2 = \frac{\Delta_2}{\Delta} = -1$

$\Delta_4 : \frac{(12)}{(12)*(6)*(3)*(8)} \begin{vmatrix} 2 & 2 & -1 & 4 \\ 0 & -6 & 6 & -12 \\ 0 & -9 & 3 & -12 \\ 0 & 0 & -4 & 0 \end{vmatrix} =$

$\frac{(12)*(12)}{(12)*(6)*(3)*(8)*(3)*(2)} \begin{vmatrix} 2 & 2 & -1 & 4 \\ 0 & -18 & 18 & -36 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -4 & 0 \end{vmatrix} =$

$\frac{(12)*(12)*(4)}{(12)*(6)*(3)*(8)*(3)*(2)*(4)} \begin{vmatrix} 2 & 2 & -1 & 4 \\ 0 & -18 & 18 & -36 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -4 \end{vmatrix} =$

-2

$x_3 = \frac{\Delta_3}{\Delta} = -1$

Answers: $x1 = 0$, $x2 = 1$, $x3 = -1$, $x4 = -1$

45. **Test5**
   Data:

$$\begin{cases} 2x_1 = 0 + 3x_2 = 0 + 4x_3 = 0 - x_4 = 0 \\ x_1 = -1 + 0x_2 = -1 + 0x_3 = -1 + x_4 = -1 \\ x_1 = 1 - x_2 = 1 - 2x_3 = 1 + x_4 = 1 \\ 2x_1 = -2 + x_2 = -2 - x_3 = -2 - 2x_4 = -2 \end{cases}$$

Solution by Gauss method:

$$\begin{bmatrix} 2 & 3 & -4 & 1 & 0 \\ 1 & 0 & 0 & 1 & -1 \\ 1 & 1 & -2 & 1 & -1 \\ 2 & -1 & -1 & 2 & -2 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & 3 & -4 & 1 & 0 \\ 0 & 3 & -4 & -1 & 2 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 4 & -3 & -1 & 2 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & 3 & -4 & 1 & 0 \\ 0 & 3 & -4 & -1 & 2 \\ 0 & 0 & 2 & -1 & 2 \\ 0 & 0 & 7 & 1 & -2 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & 3 & -4 & 1 & 0 \\ 0 & 3 & -4 & -1 & 2 \\ 0 & 0 & 2 & -1 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 2 & 3 & -4 & 1 & 0 \\ 0 & 3 & -4 & -1 & 2 \\ 0 & 0 & 2 & -1 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

Answers: $x1 = 1, \quad x2 = 0, \quad x3 = 0, \quad x4 = -2$

Solution by Kramer method:

$$\Delta : \begin{vmatrix} 2 & 3 & -4 & 1 & 0 \\ 1 & 0 & 0 & 1 & -1 \\ -1 & -1 & 2 & -1 & 1 \\ 2 & -1 & -1 & 2 & -2 \end{vmatrix} =$$

$$\overline{(2)*(-2)} \begin{vmatrix} 2 & 3 & -4 & 1 \\ 0 & -3 & 4 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & -4 & 3 & 1 \end{vmatrix} =$$

$$\frac{(4)*(8)}{(2)*(-2)*(4)*(12)*(3)} \begin{vmatrix} 2 & 3 & -4 & 1 \\ 0 & -3 & 4 & 1 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & -7 & -1 \end{vmatrix} =$$

$$\frac{(4)*(8)*(7)}{(2)*(-2)*(4)*(12)*(3)*(7)*(2)} \begin{vmatrix} 2 & 3 & -4 & 1 \\ 0 & -3 & 4 & 1 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -9 \end{vmatrix} =$$

3

$$\Delta_1 : \frac{(2)}{(2)*(-2)} \begin{vmatrix} 0 & 3 & -4 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 2 & -4 & 0 \\ 0 & -1 & -1 & 0 \end{vmatrix} =$$

$$\frac{(2)*(2)}{(2)*(-2)*(2)*(3)*(-6)} \begin{vmatrix} 0 & 3 & -4 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -4 & -2 \\ 0 & 0 & 14 & -2 \end{vmatrix} =$$

$$\frac{(2)*(2)*(18)}{(2)*(-2)*(2)*(3)*(-6)*(7)*(-2)} \begin{vmatrix} 0 & 3 & -4 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -28 & -14 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

0

$x_0 = \frac{\Delta_0}{\Delta} = 0$

$$\Delta_2 : \frac{1}{(2)*(-2)} \begin{vmatrix} 2 & 0 & -4 & 1 \\ 0 & -2 & 4 & 1 \\ 0 & -2 & 0 & 1 \\ 0 & -2 & 3 & 1 \end{vmatrix} =$$

$$\frac{1}{(2)*(-2)} \begin{vmatrix} 2 & 0 & -4 & 1 \\ 0 & -2 & 4 & 1 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & -1 & 0 \end{vmatrix} =$$

$$\frac{1}{(2)*(-2)*(4)} \begin{vmatrix} 2 & 0 & -4 & 1 \\ 0 & -2 & 4 & 1 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} =$$

0

$x_1 = \frac{\Delta_1}{\Delta} = 0$

$$\Delta_3 : \frac{1}{(2)*(-2)} \begin{vmatrix} 2 & 3 & 0 & 1 \\ 0 & -3 & -2 & 1 \\ 0 & -1 & -2 & 1 \\ 0 & -4 & -2 & 1 \end{vmatrix} =$$

$$\frac{(4)*(8)}{(2)*(-2)*(4)*(12)*(3)} \begin{vmatrix} 2 & 3 & 0 & 1 \\ 0 & -3 & -2 & 1 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 2 & -1 \end{vmatrix} =$$

$$\frac{(4)*(8)}{(2)*(-2)*(4)*(12)*(3)*(-1)} \begin{vmatrix} 2 & 3 & 0 & 1 \\ 0 & -3 & -2 & 1 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{vmatrix} =$$

0

$x_2 = \frac{\Delta_2}{\Delta} = 0$

$$\Delta_4 : \frac{1}{(2)*(-2)} \begin{vmatrix} 2 & 3 & -4 & 0 \\ 0 & -3 & 4 & -2 \\ 0 & -1 & 0 & -2 \\ 0 & -4 & 3 & -2 \end{vmatrix} =$$

$$\frac{1}{(2)*(-2)*(4)*(12)*(3)} \begin{vmatrix} 2 & 3 & -4 & 0 \\ 0 & -12 & 16 & -8 \\ 0 & 0 & -16 & -16 \\ 0 & 0 & -7 & 2 \end{vmatrix} =$$

$$\frac{(144)}{(2)*(-2)*(4)*(12)*(3)*(7)*(16)} \begin{vmatrix} 2 & 3 & -4 & 0 \\ 0 & -12 & 16 & -8 \\ 0 & 0 & -112 & -112 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

-6

$x_3 = \frac{\Delta_3}{\Delta} = -2$

Answers: $x1 = 0, \quad x2 = 0, \quad x3 = 0, \quad x4 = -2$