NAME: _____ ID: _____

MTH 245 – FINAL EXAM

Instructions:

- Please remove any hats and headphones.
- Turn off all cell phones. Using your cell phone at all will be considered academic dishonesty and you will receive a zero on your exam. Don't risk it!
- You may use a 3×5 flashcard.
- A NON-graphing calculator is permitted.

There are two parts to this exam:

Section I is the Short Answer section, containing true/false, multiple choice, and/or fill-in-the-blank questions. You do not need to show work on this portion of test; just give your answer. Partial credit will not be earned on this part. Answers will be scored right or wrong. Follow all directions.

Section II is the Show Your Work section. You must show work on this part of the exam - partial credit may be earned on this part, but only if directions are followed and answers are justified by supporting work and explanations. Follow all directions in each problem.

Test-taking tips:

- Solve the problems in the order that is easiest for you.
- Skip problems that you find harder and come back to them later.
- You may prefer to work the longer problems at the end first and then continue with the shorter problems.
  - The final exam is scored out of 120 points.

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page 2	page 7
page 3	page 8
page 4	page 9
page 5	page 10

Total/120

Academic or Scholarly Dishonesty is prohibited and considered a serious violation of the Student Conduct Code. It is defined as an act of deception in which a Student seeks to claim credit for the work or effort of another person, or uses unauthorized materials or fabricated information in any academic work or research, either through the Student's own efforts or the efforts of another.

Good luck!

Do not open this exam until instructed to do so.

Section I: Multiple choice section.

1. (3 points) $\begin{cases} -4x + 3y &= \text{-1} \\ 12x - 9y &= 22 \end{cases}$

   If we solve the system of equations above using the method of elimination, we arrive at the statement:

   $$0 = 19.$$

   From this we know that:

   a) There is no solution to this system of equations.

   b) There is one unique solution at $(0, 19)$.

   c) There are infinitely many solutions and any real numbers will work for (x, y).

   d) There are infinitely many solutions of the form $\left(x, \frac{4}{3}x - \frac{11}{3}\right)$.

   e) The method of elimination is not a valid method for solving this type of problem.

2. (3 points) The probability that a part is defective is 0.38.

   The probability that a part was manufactured by machine A is 0.67.

   The probability that a part is defective and was produced by machine A is 0.21.

   Which expression will correctly calculate the probability that a part is defective or was manufactured by machine A?

   a) $\dfrac{0.38 + 0.67}{0.21}$

   b) $\dfrac{0.67 - 0.38}{0.21}$

   c) $0.38 + 0.67 - 0.21$

   d) $(0.38) \cdot (0.67) - 0.21$

   e) This situation cannot be modeled mathematically

3. (3 points) A normal random variable X has mean 140 and standard deviation 15. Find the probability that the random variable will have a value greater than 149. $P(X > 149)$.

   a) 0.1711

   b) 0.7257

   c) 0.1490

   d) 0.8289

   e) 0.2743

4. (4 points) A legislative body consists of 35 Blue Party members and 38 Yellow Party members. A measure is proposed which is supported by 13 Blues and 11 Yellows.

   If a person chosen at random from this legislative body is found to **NOT** support the measure, then what is the probability that the randomly chosen person belongs to the Blue Party?

   a) $\frac{24}{49}$

   b) $\frac{49}{73}$

   c) $\frac{11}{46}$

   d) $\frac{13}{46}$

   e) $\frac{22}{49}$

5. (3 points) Each lottery ticket has a probability of winning of 5.5%. Which of the following questions can be correctly answered with:

$$C(100, 6) \cdot (0.945)^{94} \cdot (0.055)^6?$$

   a) What is the probability that a person wins with the sixth ticket they buy?

   b) What is the probability that 94 out of 100 tickets purchased will be winners?

   c) What is the probability that a person wins on the 94^{th} ticket they buy?

   d) What is the probability that 6 out of 100 tickets purchased will be winners?

   e) None of the above questions can be correctly answered with $C(100, 6) \cdot (0.945)^{94} \cdot (0.055)^6$.

6. (3 points) What happens to the standard deviation and mean of a data set, if you subtract 4 from every number in the set? Choose the one best answer.

   a) The standard deviation decreases by 4 and the mean decreases by 4.

   b) The standard deviation stays the same and the mean decreases by 4.

   c) The standard deviation decreases by 4 and the mean stays the same.

   d) The standard deviation and the mean both stay the same as they were before.

   e) The question cannot be answered with the given information

7. (3 points) Suppose that you were solving a linear programming problem. After you graphed your feasible region, you found that it had corner points of $(0, 0), (5, 40), (20, 15)$, and $(25, 0)$. If your objective function is $z = 2x + 15y$, find the maximum value of z subject to the constraints.

   a) The maximum of z is achieved at $(0, 0)$.

   b) The maximum of z is achieved at $(5, 40)$.

   c) The maximum of z is achieved at $(20, 15)$.

   d) The maximum of z is achieved at $(35, 0)$.

   e) None of the above.

8. (3 points) Let $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$. Does matrix A has an inverse?

a) Yes, A^{-1} exits.

b) No, A^{-1} does not exist.

c) Not enough information is given to solve.

9. (3 points) Given the matrices A and B as shown below, identify which of the matrix operations are possible. CIRCLE ALL THAT APPLY.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 4 & 30 & 100 \\ 23 & 40 & 2 & 10 \\ 5 & 7 & 9 & 3 \end{bmatrix}$$

a) $A \cdot B$

b) $B \cdot A$

c) $B + A$

d) $A + B$

e) None of the above

10. (4 points) Solve the following system of linear equations:

$$\begin{cases} w - 2z = 3 \\ 4x + 12z = 4 \\ y = 5 \end{cases} .$$

Solve for (w, x, y, z), if there is a parameter let it be represented by the letter k.

a) $(5, -2, 5, 1)$

b) $(3 + 2k, 1 - 3k, 5, k)$ for k any real number

c) $(3 - 2k, 1 + 3k, 5, k)$ for k any real number

d) $(3 + 2k, 4 - 12k, 5, k)$ for k any real number

e) there is no solution

11. (3 points) Consider the following system of linear inequalities:

$$\begin{cases} 5x + y \leq 10 \\ 2x + 3y \leq 15 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Which of the following points is NOT in the feasible region?

a) $(0, 0)$

b) $(1, 2)$

c) $(2, 2)$

d) $(1, 3)$

e) ALL of them are in the feasible region

12. (4 points) Let $A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & -4 \\ -5 & 6 \end{bmatrix}$. What will be the entry in the second row, first column of the result of the matrix multiplication $B \cdot A$?

a) -4
b) 1
c) 2
d) 5
e) the indicated operation is not possible

13. (4 points) Find the mean of the data set $\{7, 30, 21, 26, 15, 85\}$

(a) 35.60
(b) 36.80
(c) 30.66
(d) 29.66
(e) 23.50
(f) None of the above

14. (4 points)

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & -6 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The augmented matrix above was obtained from a system of linear equations and is in reduced echelon form. How many solutions does the system of equations have?

a) exactly one solution
b) exactly two solutions
c) no solutions
d) infinitely many solutions
e) Cannot be determined from the given information

15. (3 points) A kindergarten class consists of 7 boys and 8 girls. The teacher randomly selects four children to feed the class pet this week and the teacher **always selects at least one girl.** If we define the random variable X as the number of boys selected, list all the possible values of X.

a) X = 0, 1, 2, 3, 4, 5, 6, 7
b) X = 0, 1, 2, 3, 4, 5, 6, 7, 8
c) X = 0, 1, 2, 3
d) X = 0, 1, 2, 3, 4
e) X = 1, 2, 3, 4
f) None of the above

16. (4 points) Perform the given row operation on the following matrix and write the resulting matrix

$$\begin{bmatrix} 1 & -1 & 3 & 3 \\ -2 & 3 & -11 & -4 \\ 1 & -2 & 8 & 6 \end{bmatrix} \xrightarrow{R_2+2R_1\to R_2}$$

a) $\begin{bmatrix} 1 & -1 & 3 & 3 \\ 0 & -1 & 5 & 2 \\ 1 & -2 & 8 & 6 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -1 & 3 & 3 \\ 0 & 1 & -5 & 2 \\ 1 & -2 & 8 & 6 \end{bmatrix}$

c) $\begin{bmatrix} 2 & -2 & 6 & 6 \\ -2 & 3 & -11 & -4 \\ 1 & -2 & 8 & 6 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 1 & -5 & 2 \\ -2 & 3 & -11 & -4 \\ 1 & -2 & 8 & 6 \end{bmatrix}$

e) None of the above.

17. (4 points) Write a system of linear equations to represent the following situation: The OSU Math club sold two kinds of pies, pecan pie and apple pie, to raise money for their club activities. They sold each pecan pie for \$10 and each apple pie for \$9. They sold 120 pies for a total revenue of \$563. How many pies of each type did they sell? Let x represent the number pecan pies and let y represent the number of apples pies. You do not need to solve the system.

a) $\begin{cases} x + y = 563 \\ 10x + 9y = 120 \end{cases}$

b) $\begin{cases} x + y = 120 \\ 10x + 9y = 563 \end{cases}$

c) $\begin{cases} 10x + y = 563 \\ x + 9y = 120 \end{cases}$

d) $\begin{cases} x + y = 563 \\ x + y = 120 \end{cases}$

e) $\begin{cases} x + y = 120 \\ 9x + 10y = 563 \end{cases}$

18. (4 points) Given the system of equations in $AX = B$ form:

$$\begin{bmatrix} 4 & -2 & 3 \\ 8 & -3 & 5 \\ 7 & -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}.$$

Also given that $A^{-1} = \begin{bmatrix} -2 & 2 & -1 \\ 3 & -5 & 4 \\ 5 & -6 & 4 \end{bmatrix}$. Then the solution is:

a) $\begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$

b) $\begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix}$

c) $\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$

e) None of the above

Section II: Show Your Work.

The following questions are open ended. Please justify each of your answers

19. (5 points) With a certain scratch–off lottery ticket a player either win \$100 or won nothing. The probability that a person wins \$100 on a single ticket is 0.369. If Peter buys 9 tickets, what is the probability of winning \$100 each on 5 of the tickets?

20. (6 points) Suppose that a test for hepatitis has 95% chance of being positive in the presence of the condition; and 90% chances of a negative test in the absence of the condition. A person is selected at random from a large population, of which 0.05% of the people have hepatitis, and given the test. What is the probability of having the condition, given a positive test result?

21. (4 points) The lifetime of a certain brand of tires is normally distributed with mean $\mu = 30,000$ miles and standard deviation $\sigma = 5000$. The company has decided to issue a warranty for the tires but does not want to replace more than 2% of the tires that it sells. At what mileage should the warranty expires?

22. (4 points) The tables below gives the probability distribution for the possible returns from two different investments. Compute the mean and the variance for each investment. **Show all your calculations**.

Investment A		Investment B	
Return ($ millions)	Probability	Return ($ millions)	Probability
-10	1/5	0	.3
20	3/5	10	.4
25	1/5	30	.3

$\mu_A =$ _____, $\mu_B =$ _____

$\sigma_A^2 =$ _____, $\sigma_B^2 =$ _____

23. (2 points) Consider again the previous question. Which investment has the highest expected return? Which investment is less risky? **Justify your answer.**

Use the following information to solve the next four questions

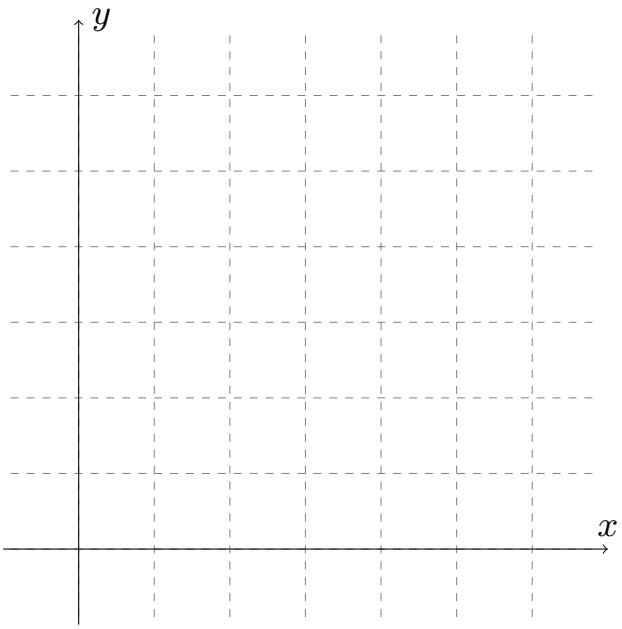
A nutritionist, working for NASA, must meet certain nutritional requirements for astronauts and yet keep the weight of the food at a minimum. He is considering a combination of two foods, which are packaged in tubes. Each tube of **food A** contains 5 units of protein, 2 units of carbohydrates, and 2 units of fat and weights 3 pounds. Each tube of **food B** contains 4 units of protein, 8 units of carbohydrates, and 1 unit of fat and weights 2 pounds. The requirement calls for 60 units of protein, 40 units of carbohydrates, and 18 units of fat.

24. (4 points) Fill the following table

	Food A	Food B
Protein		
Carbohydrates		
Fat		
Weight		

25. (10 points) Let x be the number of tubes of food A and y the number of tubes of food B. Fill the following using the constraints in the previous question. Write test points that you used to fill out the last column and graph the feasible region for the problem

Inequality	Line to graph	x-intcpt	y-intcpt



26. (7 points) Write the objective function. Fill out the following table (the first column is for the corners (vertices) of the feasible region. Evaluate the objective function on each vertex for the second column).

Vertex	Objective function=

27. (3 points) Are you minimizing or maximizing the objective function? How many tubes of each food should be supplied to the astronauts? Justify your assertions.

28. (5 points) A bin contains 85 black balls and 15 red balls. A contestant pays \$25 to enter to the game and randomly draws a ball from the bin. If the ball is red, the contestant receives \$100. If the ball is black, the contestant receives nothing. Find the expected value for this game.

Outcome x_i	Probability p_i

29. (6 points) A box contains 10 parts of which 4 are defective. 3 parts are chosen at random from the box. What is the expected number of defective parts?

30. (2 points) Consider the Bernoulli (binomial) experiment 2500 trials with a probability of success of 0.2 for each trial. Use the normal distribution to estimate the probability of at most 2467 (inclusive) successes.

APPENDICES

APPENDIX A
Areas Under the Standard Normal Curve

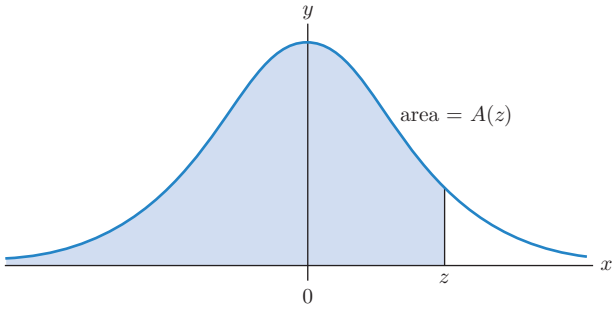


TABLE 1 Areas under the standard normal curve

z	$A(z)$	z	$A(z)$	z	$A(z)$	z	$A(z)$	z	$A(z)$
−3.50	.0002	−2.00	.0228	−.50	.3085	1.00	.8413	2.45	.9929
−3.45	.0003	−1.95	.0256	−.45	.3264	1.05	.8531	2.50	.9938
−3.40	.0003	−1.90	.0287	−.40	.3446	1.10	.8643	2.55	.9946
−3.35	.0004	−1.85	.0322	−.35	.3632	1.15	.8749	2.60	.9953
−3.30	.0005	−1.80	.0359	−.30	.3821	1.20	.8849	2.65	.9960
−3.25	.0006	−1.75	.0401	−.25	.4013	1.25	.8944	2.70	.9965
−3.20	.0007	−1.70	.0446	−.20	.4207	1.2813	.9000	2.75	.9970
−3.15	.0008	−1.65	.0495	−.15	.4404	1.30	.9032	2.80	.9974
−3.10	.0010	−1.60	.0548	−.10	.4602	1.35	.9115	2.85	.9978
−3.05	.0011	−1.55	.0606	−.05	.4801	1.40	.9192	2.90	.9981
−3.00	.0013	−1.50	.0668	.00	.5000	1.45	.9265	2.95	.9984
−2.95	.0016	−1.45	.0735	.05	.5199	1.50	.9332	3.00	.9987
−2.90	.0019	−1.40	.0808	.10	.5398	1.55	.9394	3.05	.9989
−2.85	.0022	−1.35	.0885	.15	.5596	1.60	.9452	3.10	.9990
−2.80	.0026	−1.30	.0968	.20	.5793	1.65	.9505	3.15	.9992
−2.75	.0030	−1.25	.1056	.25	.5987	1.70	.9554	3.20	.9993
−2.70	.0035	−1.20	.1151	.30	.6179	1.75	.9599	3.25	.9994
−2.65	.0040	−1.15	.1251	.35	.6368	1.80	.9641	3.30	.9995
−2.60	.0047	−1.10	.1357	.40	.6554	1.85	.9678	3.35	.9996
−2.55	.0054	−1.05	.1469	.45	.6736	1.90	.9713	3.40	.9997
−2.50	.0062	−1.00	.1587	.50	.6915	1.95	.9744	3.45	.9997
−2.45	.0071	−.95	.1711	.55	.7088	2.00	.9772	3.50	.9998
−2.40	.0082	−.90	.1841	.60	.7257	2.05	.9798		
−2.35	.0094	−.85	.1977	.65	.7422	2.10	.9821		
−2.30	.0107	−.80	.2119	.70	.7580	2.15	.9842		
−2.25	.0122	−.75	.2266	.75	.7734	2.20	.9861		
−2.20	.0139	−.70	.2420	.80	.7881	2.25	.9878		
−2.15	.0158	−.65	.2578	.85	.8023	2.30	.9893		
−2.10	.0179	−.60	.2743	.90	.8159	2.35	.9906		
−2.05	.0202	−.55	.2912	.95	.8289	2.40	.9918		