Cyclical Hierarchical Systems

A Modal Analysis of Self-Sufficient Structures

Boris Kriger

Abstract

This paper explores the possibility of cyclical hierarchical systems and the conceptual consequences of such structures. We provide mathematical conditions for their coherence using fixed-point theorems and topological closure properties (including homological characterizations), analyze candidate empirical manifestations with appropriate epistemic grading, and offer philosophical analysis of why traditional foundational questions may not apply to such structures. The central thesis is modal rather than assertoric: where the hierarchy of foundations is closed and atemporal, asking "what came first?" becomes a conceptual mismatch—akin to asking what is north of the North Pole. Applications extend from cosmology and mutual world-simulation to emerging architectures in artificial intelligence. Throughout, we maintain clear separation between mathematical results, philosophical interpretations, and speculative analogies.

Keywords: cyclical hierarchy, topological closure, fixed-point theory, grounding, mutual simulation, artificial intelligence, autopoiesis

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The author is grateful to Julian Barbour for generous personal correspondence that helped clarify the distinction between cosmological models and structural principles. His engagement sharpened the modal character of this theory: the principle concerns the coherent possibility of certain closed cyclical structures, not claims about the physical universe.

The author is grateful to Jonathan Schaffer for generous correspondence on grounding theory. It should be noted that the present paper does not stand in contradiction to Schaffer's work, nor does it seek to propose an alternative axiomatization of grounding. The cyclical hierarchies formalized here concern abstract dependency structures satisfying fixed-point conditions; they do not address the relationship between wholes and parts that is central to priority monism. Schaffer's observation that asymmetry is an axiom within grounding theory is fully compatible with our results, which operate in a different domain: the mathematical characterization of self-sufficient structures, without any claim that metaphysical grounding relations should or should not be asymmetric.

Dedication

This paper is dedicated to Julian Barbour, Nick Bostrom, and Stephen Hawking, whose ideas influenced my intellectual path over three decades and ultimately led to this work. Engaging with their thought made it possible to look back across twenty-five centuries of human wisdom without reverence for inherited certainties, yet with deep respect for the courage of those who first dared to think differently. Their influence helped shape a perspective in which the oldest questions are not merely answered, but re-examined in light of structure, limits, and possibility.

1 Introduction

The question of primary foundation has structured Western metaphysics from Aristotle's unmoved mover through Aquinas's cosmological arguments to contemporary debates about initial conditions and base reality. This paper explores the possibility that this assumption, while valid for open hierarchical structures, may represent a conceptual mismatch when applied to closed ones.

The contribution lies in four claims: (1) Mathematical conditions under which certain closed structures can exist can be precisely specified through fixed-point theory and topological constraints; (2) The concepts of source, foundation, and primacy, conflated in linear models, must be distinguished in cyclical ones; (3) Topological properties (compactness, connectedness, nontrivial homology) provide formal language for characterizing structural closure; (4) These distinctions may dissolve certain traditional questions by revealing them as conceptual mismatches.

The thesis is consistently modal: this analysis demonstrates that self-sufficient closed structures are coherently possible and that, for such structures, foundationalist questions may be conceptually misapplied. Whether any actual system instantiates this structure is a separate empirical question not addressed here.

2 Literature Review

2.1 Metaphysics of Grounding

Orthodox grounding theory [Schaffer, 2009, Rosen, 2010, Raven, 2015] holds grounding is irreflexive, asymmetric, and transitive. However, Bliss & Priest [2018] explore non-linear structures; Thompson [2016] argues for metaphysical interdependence; Barnes [2018] defends symmetric dependence. This paper specifies conditions under which mutual dependence is coherently possible without paradox.

2.2 Systems Theory and Autopoiesis

Maturana and Varela's autopoiesis [Maturana & Varela, 1980], Di Paolo [2005], Moreno & Mossio [2015], and Rosen's (M, R)-systems [Rosen, 2012] formalize organizational closure. Alcocer-Cuarón et al. [2014] show biological hierarchies form recurrent circuits. This paper extends this biological insight to ontological structures and provides explicit existence conditions.

2.3 Physics and Cosmology

Barbour [2020] proposes timeless cosmology. The Wheeler-DeWitt equation describes the universe without time parameter; Lehners [2023] reviews the no-boundary proposal. De Bianchi et al. [2025] introduce an "Atemporality Theorem." Topological quantum field theory [Atiyah, 1988, Witten, 1988] provides physical instantiation of topology-first modeling where structure precedes metric. Critical perspectives include Earman [2006], Kragh [2011], and Maudlin [2007], who defends time-asymmetric causation. It should be noted that this analysis makes no cosmological claims; correspondence with Barbour (2026) helped clarify that the principle concerns the logical structure of certain closed systems, not assertions about whether the physical universe has or lacks a first instant.

2.4 Category Theory and Topology

Lawvere's fixed-point theorem [Lawvere, 1969] shows when fixed points exist rather than lead to contradiction. Rutten [2000] develops universal coalgebra. Mac Lane & Moerdijk [2012] connect topology and logic through topos theory. Skowron [2023] applies topological philosophy. Fan et al. [2021] characterize cycle structure in networks via Betti numbers. Hatcher [2002] provides comprehensive treatment of algebraic topology including homology and cohomology. Munkres [2000] offers foundational treatment of topology including compactness and connectedness.

2.5 Simulation Theory and AI

Bostrom's simulation argument [Bostrom, 2003] assumes linear hierarchy. Chalmers [2022] examines implications. Wolpert [2025] enables cyclic mutual simulations. In AI, recursive self-improvement [Yampolskiy, 2015] and multi-agent alignment [Dafoe et al., 2021] suggest non-linear models. Wolfram [2002] raises computational irreducibility as potential obstacle to mutual recursion. Hossenfelder [2021] critiques simulation hypotheses as unfalsifiable—a concern this analysis addresses by maintaining modal rather than assertoric claims.

2.6 What Is Novel

This paper contributes: (a) tripartite distinction of source/foundation/primacy; (b) explicit mathematical and topological conditions including homological characterization with clear separation of formal model from philosophical interpretation; (c) systematic analysis of foundational questions as conceptual mismatches for certain structures; (d) applications maintaining consistent modal formulation with graded epistemic status.

3 Conceptual Analysis: The Foundational Mismatch

3.1 Category Errors in Philosophy

A category error [Ryle, 1949] occurs when a concept belonging to one logical type is misapplied to another. Ryle's paradigm: asking "where is the university?" after being shown buildings, libraries, and staff treats "university" as belonging to the same category as physical locations, when it is an institutional abstraction. The question is grammatically correct but logically malformed.

Similarly, Hume's is-ought distinction identifies a category error: deriving normative conclusions from purely descriptive premises treats "ought" as belonging to the same logical category as "is." The inference appears valid but crosses an unbridgeable categorical gap.

3.2 The Foundational Conceptual Mismatch

This analysis identifies a parallel conceptual mismatch: applying the question "what is the primary foundation?" to a closed structure treats "foundation" as a fixed position when, in such structures, it functions as a circulating role. The question presupposes that primacy is a fixed property when the structure's topology makes it a relative function.

Within a cyclical hierarchy, the question "what is the primary foundation?" loses its standard linear meaning. It becomes akin to asking for a "first point" on a circle—a misapplication of linear ordering concepts to a non-linear structure. This is not a logical error in the strict sense, but rather a conceptual mismatch: the question presupposes structural features (fixed endpoints, linear ordering) that the system lacks.

Consider: "What is north of the North Pole?" The question is grammatically well-formed but geographically inapplicable—it applies "north of" beyond its domain of applicability. Similarly, "what grounds a closed cyclical hierarchy?" may apply "grounds" beyond its proper domain. In open structures, the question is legitimate; in closed structures, it may be inapplicable—not unanswerable but potentially misframed.

3.3 Dissolution vs. Solution

Category errors are dissolved, not solved. One does not answer "where is the university?" by pointing somewhere new; one shows why the question was malformed. This analysis suggests that foundational questions for closed structures may be similarly dissolved by showing that such structures lack the categorical feature (fixed primacy) the question presupposes. This is not philosophical evasion but conceptual clarification—the question may never have been applicable.

4 Statement of the Principle

4.1 Formal Statement

The Principle of Cyclical Hierarchy of Systems:

> *There exist possible systems in which the hierarchy of levels and roles is closed and possesses no absolute beginning or apex. Elements perform distinct functions; yet the role of foundation circulates within the structure. That which is last in one relation acts as the condition of primacy in another. "First" and "last" designate structural roles, not positions in sequence. The question of primary origin may therefore be a conceptual mismatch when applied to such structures.*

Note: This is a modal possibility claim. The principle asserts that such structures are coherently possible and that foundational questions applied to them may be conceptually mismatched. Whether any actual system instantiates this structure is not claimed.

4.2 The Tripartite Distinction

Linear thinking conflates three concepts that diverge in closed structures:

Source (Quelle/explanatory derivation): That through which a system is explained or derived.

Foundation (Grund/ontological ground): That without which the system cannot exist.

Primacy (Priorität/structural role): Being the ground relative to other elements—a function, not a fixed position.

In linear causation, these collapse: the first cause is source, foundation, and primary. In cyclical hierarchy, they diverge: A may be the source of B while B is the foundation of A. Primacy circulates rather than inhering in any element.

5 Mathematical Framework

5.1 Fundamental Definitions

Definition 5.1 (Foundational Hierarchy). *A foundational hierarchy is a triple $(S, \leq, F)$ where: (i) S is a non-empty set (the state space); (ii) $\leq$ is a binary relation on S (the grounding relation); (iii) $F : S \to S$ is a function (the determination function) such that $x \leq y$ implies $F(y)$ is defined and determines x.*

Definition 5.2 (Well-Foundedness). *A foundational hierarchy $(S, \leq, F)$ is well-founded (or open) if $(S, \leq)$ contains no infinite descending chains: there is no sequence $(x_n)_{n\in\mathbb{N}}$ with $x_{n+1} < x_n$ for all n.*

Definition 5.3 (Cyclical Closure). *A foundational hierarchy is cyclically closed of order n if there exist elements $S_1, S_2, \ldots, S_n \in S$ and functions $F_1, F_2, \ldots, F_n : S \to S$ such that: (i) $S_i = F_i(S_{i+1})$ for $i = 1, \ldots, n-1$; (ii) $S_n = F_n(S_1)$; (iii) $S_i \leq S_{i+1}$ for all i (indices mod n).*

Proposition 5.1 (Fixed-Point Equivalence)**.** *A cyclically closed hierarchy of order n exists if and only if the composite operator $\Phi = F_1 \circ F_2 \circ \cdots \circ F_n$ has a fixed point.*

Proof. By substitution: $S_1 = F_1(S_2) = F_1(F_2(S_3)) = \cdots = F_1(F_2(\cdots F_n(S_1)\cdots)) = \Phi(S_1)$. Conversely, if $\Phi(S_1) = S_1$, define $S_{i+1} = (F_{i+1} \circ \cdots \circ F_n)(S_1)$ to recover the cycle. □

5.2 Existence Theorems

Proposition 5.4 reduces the existence of cyclically closed hierarchies to the existence of fixed points. The following classical theorems provide sufficient conditions.

Theorem 5.2 (Knaster-Tarski)**.** *Let $(L, \leq)$ be a complete lattice and let $\Phi : L \to L$ be order-preserving (monotonic). Then the set of fixed points of Φ is non-empty and forms a complete lattice under the induced order.*

Corollary 5.3. *If the state space S admits a complete lattice structure and each F_i is monotonic, then a cyclically closed hierarchy exists.*

Theorem 5.4 (Banach Contraction Principle)**.** *Let (X, d) be a complete metric space and let $\Phi : X \to X$ be a contraction, i.e., there exists $k \in [0, 1)$ such that $d(\Phi(x), \Phi(y)) \leq k \cdot d(x, y)$ for all $x, y \in X$. Then Φ has a unique fixed point $x^* \in X$, and for any $x_0 \in X$, the sequence $(\Phi^n(x_0))_n$ converges to x^*.*

Corollary 5.5. *If (S, d) is a complete metric space and $\Phi = F_1 \circ \cdots \circ F_n$ is a contraction, then a unique cyclically closed hierarchy exists.*

Theorem 5.6 (Brouwer)**.** *Let $K \subset \mathbb{R}^n$ be non-empty, compact, and convex. Let $\Phi : K \to K$ be continuous. Then Φ has at least one fixed point.*

Corollary 5.7. *If $S \subset \mathbb{R}^n$ is non-empty, compact, and convex, and each F_i is continuous with $\Phi(S) \subset S$, then a cyclically closed hierarchy exists.*

Remark 5.1 (Compactness)**.** *A topological space X is compact if every open cover of X has a finite subcover. In metric spaces, this is equivalent to sequential compactness (every sequence has a convergent subsequence). Compactness is strictly stronger than closedness; a closed subset of $\mathbb{R}^n$ need not be compact (e.g., $\mathbb{R}$ itself is closed but not compact).*

5.3 Topological Characterization

We characterize the essential cyclicity of hierarchical structures using algebraic topology. The key question is: when is the cyclic structure an intrinsic feature that cannot be removed by continuous deformation?

Definition 5.4 (Essential Cyclicity)**.** *Let X be a topological space encoding a hierarchical structure (e.g., a simplicial complex with vertices as states and edges as grounding relations). We say X is essentially cyclic if: (i) X is compact; (ii) X is connected; (iii) The first homology group $H_1(X; \mathbb{Z}) \neq 0$.*

Proposition 5.8. *If X is essentially cyclic, then there exists no continuous retraction $r : X \to \{x_0\}$ that preserves the cyclic structure. Equivalently, the cycle cannot be continuously contracted to a point.*

Proof. If such a retraction existed, it would induce a homomorphism $r_* : H_1(X; \mathbb{Z}) \to H_1(\{x_0\}; \mathbb{Z}) = 0$. But retractions induce surjections on homology, contradicting $H_1(X; \mathbb{Z}) \neq 0$. □

Example 5.1. *The circle S^1 has $H_1(S^1; \mathbb{Z}) \cong \mathbb{Z}$. Any encoding of a cyclical hierarchy as a circle-like structure inherits this non-trivial homology. The first Betti number $b_1 = \mathrm{rank}(H_1)$ counts the number of independent cycles; $b_1 > 0$ is a necessary condition for essential cyclicity.*

Remark 5.2 (Encoding Dependence). *The homological characterization depends on how the hierarchical structure is encoded as a topological space. The simplicial complex encoding (vertices as states, edges as grounding relations) is one natural choice, but not the only one. Different encodings may yield different homology groups. This is not a defect but a feature: the choice of encoding is part of the formal model, and different encodings may be appropriate for different applications. The key claim is conditional: if a structure admits an encoding with non-trivial H_1, then its cyclicity is essential in the precise sense of Proposition 5.13.*

5.4 Application: Self-Consistent Quantum States

We demonstrate the framework with a concrete example from quantum physics: self-consistent configurations in mutually entangled subsystems.

Setup. Consider a tripartite quantum system with subsystems A, B, C and Hilbert space $\mathcal{H} = \mathcal{H}^A \otimes \mathcal{H}^B \otimes \mathcal{H}^C$. The reduced density matrices satisfy mutual determination via partial traces:

$$\rho^A = \mathrm{Tr}_{BC}(\rho^{ABC}), \quad \rho^B = \mathrm{Tr}_{AC}(\rho^{ABC}), \quad \rho^C = \mathrm{Tr}_{AB}(\rho^{ABC})$$

For certain global states $|\Psi\rangle \in \mathcal{H}$, the reduced states form a cyclically determined structure: the properties of each subsystem are fixed by its correlations with the others, with no subsystem serving as an independent "foundation."

Proposition 5.9. *Let S be the space of valid density matrices on a finite-dimensional Hilbert space $\mathcal{H}$ (positive semidefinite, trace one). If $\Phi : S \to S$ is continuous, then a self-consistent configuration exists.*

Proof. The set S of density matrices on a finite-dimensional Hilbert space is non-empty, compact (closed and bounded in the operator norm), and convex. If Φ is continuous and maps S into S, then by Theorem 5.9 (Brouwer), Φ has a fixed point $\rho^* = \Phi(\rho^*)$. □

Remark 5.3. *This quantum example illustrates the framework's applicability to physics: entangled subsystems exist in self-consistent configurations without any subsystem being ontologically "primary." The question "which subsystem grounds the others?" has no distinguished answer—each is determined by its correlations with the rest.*

Remark 5.4 (Speculative Extension). *In some approaches to emergent spacetime [Van Raamsdonk, 2010, Ryu & Takayanagi, 2006], geometry arises from entanglement structure. Speculatively, if spacetime and quantum fields mutually determine each other, the resulting structure might exhibit cyclical closure. However, this remains a contested interpretive claim, not a demonstrated application of the framework. The connection between entanglement and geometry is an active research area with no consensus on its ontological implications.*

5.5 Philosophical Interpretation

The mathematical framework establishes that under appropriate conditions, mutually determining systems can exist without logical contradiction. The key insight is that what appears paradoxical from a temporal perspective ("how can A ground B if B grounds A?") may be coherent from a structural perspective, much as mutual gravitational attraction between bodies is coherent despite apparent circularity.

The existence of fixed points corresponds philosophically to the existence of stable, self-consistent configurations. The question "which came first?" presupposes a temporal ordering that the mathematical structure does not require. "Atemporal" means structural relations lack time parameter—states are co-given in the structure's topology.

6 Objections and Replies

Vicious Circle: *"This is circular reasoning."* Reply: Cyclical hierarchy involves structural mutual dependence among distinct elements, not justificatory circularity ("P because P"). The mediated circle through different elements is no more fallacious than mutual gravitational attraction.

Grounding Asymmetry: *"Grounding is asymmetric by definition."* Reply: This asymmetry is stipulated, not demonstrated. Barnes [2018] and Thompson [2016] defend symmetric dependence as coherent. Either grounding axioms require revision, or cyclical hierarchy concerns "mutual ontological dependence" distinct from technical grounding but doing equivalent metaphysical work.

"Merely Mathematical": *"Mathematical possibility doesn't entail metaphysical possibility."* Reply: Correct. Mathematical consistency removes one objection (logical impossibility) but doesn't establish metaphysical possibility. The positive case rests on: (a) no demonstrated impossibility beyond question-begging linear intuitions; (b) empirical instantiations (Section 7) providing defeasible evidence.

Explanatory Vacuity: *"Cyclical hierarchies explain nothing."* Reply: This presupposes linear explanation is uniquely legitimate. Holistic explanation—understanding parts through relations to whole—is common in mathematics, physics, and language. Linear explanation faces its own problem: infinite regress or brute first cause.

Causal Realism: *"Causation is time-asymmetric [Maudlin, 2007]."* Reply: This analysis claims structural determination, not efficient causation backwards in time. Whether to call this "causation" is terminological; the substantive claim is that mutual structural determination is coherent.

7 Examples and Candidate Instantiations

The following examples are graded by epistemic status. This grading distinguishes between strict formal instantiations, strong biological candidates, and weaker physical/speculative analogies. Throughout, the modal thesis is maintained: these examples suggest cyclical hierarchical structures may be instantiated, not that they definitively are.

7.1 Strict Instantiations (Formal Systems)

These examples instantiate cyclical hierarchy within formal systems. Extension to physical/metaphysical reality is a separate question.

Recursive function definitions: Fixed-point combinators (e.g., the Y combinator) enable recursive definitions without explicit self-reference. The existence of the recursive function is guaranteed by fixed-point theorems in domain theory [Scott, 1970].

Self-interpreting interpreters: A programming language that can interpret itself exhibits structural self-reference without vicious circularity.

Corecursive data structures: Infinite data structures defined by their unfolding (streams, infinite trees) instantiate structural closure without temporal origin.

7.2 Strong Biological Candidates

These biological systems exhibit structural mutual dependence. They illustrate structural closure, though their phylogenetic origin involved temporal processes.

Autopoiesis: In autopoietic systems [Maturana & Varela, 1980], membrane is produced by internal processes while those processes require membrane delimitation. This is genuine mutual dependence at systemic level. *Caveat:* Phylogenetically, the first cell had an origin (abiogenesis); cyclical hierarchy applies to ongoing structural organization, not to the absence of historical origin.

Metabolic Cycles: The Krebs cycle requires oxaloacetate to produce oxaloacetate. In steady state, $O = F(O)$—literally a fixed-point equation. The cycle exists when this equation has stable solution. *What this illustrates:* Structural closure (the cycle is self-maintaining). *What this does not prove:* Absence of temporal origin (the cycle had evolutionary origins).

7.3 Physical Analogies

These physical theories are compatible with cyclical hierarchical structures but do not prove their existence. Interpretations remain contested.

Wheeler-DeWitt equation: Describes quantum gravity without explicit time parameter. *Status:* Shows physics permits atemporal formulations; does not establish that the universe lacks temporal origin.

TQFT: Physical quantities depend only on topology, not metric. *Status:* Demonstrates topology-first modeling is physically viable; does not establish metaphysical priority of topology.

De Bianchi et al. (2025) Atemporality Theorem: Derives atemporal structure from conservation laws. *Status:* Provides formal framework for atemporality in physics; interpretation and scope remain under discussion.

8 Applications

The following applications demonstrate the analytical power of cyclical hierarchical thinking. Throughout, the modal thesis is maintained: these are analyses of what would follow if certain structures obtained, not claims that they do obtain.

8.1 Mutual Simulation Without Base Reality

The simulation hypothesis [Bostrom, 2003] assumes linear hierarchy requiring base reality. Cyclical hierarchy suggests an alternative: W_1 determines rules of W_2, W_2 determines W_3, W_3 determines W_1. Temporally paradoxical; structurally coherent. The configuration has non-trivial H_1—cannot be contracted to linear chain.

Wolpert [2025] provides mathematical framework enabling such cyclic simulations where simulated universes can be computationally equivalent to simulators. No base reality is required because the configuration is topologically closed. Epistemic status: formal coherence demonstrated; actuality not claimed. The standard objection ("there must be a base") is revealed as presupposition, not logical requirement.

8.2 Mutual Cyclical AI

Standard AI narratives assume linear hierarchy: humans create AI, AI creates more advanced AI, with humans as ultimate origin. Cyclical hierarchy suggests an alternative model: cyclical mutual constitution where intelligences co-determine each other.

Formal Model: Consider AI systems A_1, A_2, A_3 where each system's parameters are refined by evaluation from others: $A_1 = R_1(A_2)$, $A_2 = R_2(A_3)$, $A_3 = R_3(A_1)$, where R_i are refinement operators. Let $\mathrm{P} = R_1 \circ R_2 \circ R_3$. Stable configuration exists iff P has fixed point.

Conditions for fixed-point existence: If we model the refinement space as a complete metric space (M, d) and each R_i is a contraction with constant $k_i < 1$, then by Banach's fixed-point theorem, P is also a contraction (with constant $k_1 k_2 k_3 < 1$), and a unique fixed point exists. *Whether actual AI systems satisfy these conditions is an empirical question* that depends on the specific architecture and training dynamics of the systems involved.

Computational Irreducibility Objection: Wolfram [2002] argues many computational processes are irreducible—no shortcut exists to determine their outcome. Could mutual AI recursion avoid stable fixed points, spiraling indefinitely? Reply: Computational irreducibility concerns prediction of specific states, not existence of fixed points. A chaotic system may be irreducible yet have attractors. For cyclical hierarchy, the question is whether stable configurations exist, not whether they're predictable. When refinement operators are contractive, convergence is guaranteed regardless of computational irreducibility.

Halting Problem Objection: Mutual recursion risks non-termination. Reply: Cyclical hierarchy concerns structural fixed points, not computational halting. A system can be structurally self-consistent without any temporal process "finishing." The fixed point is atemporal configuration, not endpoint of computation.

Multi-agent alignment research [Dafoe et al., 2021] and recursive self-improvement analysis [Yampolskiy, 2015] increasingly recognize non-linear dynamics in AI development. Cyclical hierarchy provides a conceptual framework: if AI systems achieved cyclical mutual constitution, foundational questions about "first intelligence" would become conceptual mismatches—not unanswerable but inapplicable.

Epistemic status: Speculative but formally grounded. Current AI systems are not cyclical hierarchies. Whether future systems could be is contingent on architectural developments. This framework provides tools for analysis if such developments occur, not prediction that they will.

9 Consequences and Limitations

9.1 What This Analysis Suggests (Modal Claims)

(1) Self-sufficient closed structures are mathematically and topologically coherent under specified conditions. (2) For such structures, the demand for primary foundation may be a conceptual mismatch—comparable to asking what is north of the North Pole. (3) The tripartite distinction (source/foundation/primacy) reveals hidden linear assumptions in traditional metaphysics. (4) Mutual simulation without base reality is formally possible. (5) If AI systems achieved cyclical mutual constitution, foundational questions about origins would become inapplicable.

9.2 What This Analysis Does Not Establish

(1) That reality is in fact a closed cyclical hierarchy. (2) That all systems are of this type—most are not. (3) That all foundational questions are conceptual mismatches—only those applied to closed structures. (4) Any specific cosmological, theological, or AI predictions. (5) The truth of any particular theological doctrine—only the structural coherence of certain theological models. (6) That any existing AI system instantiates cyclical hierarchy.

9.3 The Rehabilitated Causa Sui

Classical *causa sui* was rejected as paradoxical: self-causation before existence. Cyclical hierarchy rehabilitates it as structural and topological closure: a possible system whose existence conditions are internal, not through temporal self-causation (paradoxical) but through topologically complete structure (coherent). Self-sufficiency is a structural property, not a temporal achievement.

9.4 Relation to Grounding Theory

It should be emphasized that this paper does not propose an alternative axiomatization of grounding, nor does it stand in contradiction to standard grounding theory. The cyclical hierarchies formalized here are not about the relation between wholes and parts (as in priority monism), but about abstract dependency structures that satisfy certain closure conditions. In correspondence, Schaffer (2026) noted that asymmetry is an axiom within grounding theory; our results concern a different subject matter—mathematical structures satisfying fixed-point conditions—and make no claim about whether grounding relations in metaphysics should or should not be asymmetric.

10 Conclusion

This analysis has explored the possibility of closed hierarchical structures and their conceptual implications. The central finding is that such structures are coherently possible, and that for them, the demand for primary foundation may constitute a conceptual mismatch—not an unanswered question but a potentially inapplicable one, comparable to asking what is north of the North Pole.

The central result:

> ***Self-sufficient systems are coherently possible when the hierarchy of their foundations is closed and atemporal, structural functions satisfy fixed-point existence conditions, and the configuration space is topologically compact with non-trivial first homology. In such systems, a primary foundation is not absent but may be inadmissible: a conceptual mismatch arising from applying linear concepts to non-linear structure.***

The demand for a first principle, prevalent since Aristotle, may be a feature of open hierarchical structures mistaken for universal logical requirement. From cosmology to simulation theory to artificial intelligence, this analysis provides tools for recognizing when foundational questions may be misapplied—when we are asking for the first point on a circle.

The ouroboros, rightly understood—not as self-consumption but as self-generation, the serpent born from itself—captures the insight. There is no beginning because the structure is topologically complete. There is no end because closure is not termination but fulfillment. Whether reality, or AI, or consciousness instantiates this structure remains open. That they coherently could is what this paper has explored.

A Historical Development of Cyclical Structural Thinking

Note: This appendix presents a historical example of cyclical structural thinking for illustrative purposes only. It is not part of the scientific argument and does not claim to prove any theological doctrine true. The aim is limited: to show that cyclical hierarchical thinking has historical precedent in Western intellectual history, which may illuminate why such structures have proven resistant to linear formalization.

A.1 The Trinity as Historical Example

In orthodox Trinitarian theology, as developed through the ecumenical councils (Nicaea 325, Constantinople 381), Father, Son, and Spirit are characterized as: (a) not identical—each is distinct hypostasis; (b) not reducible to one another; (c) hierarchically related through origination (Father begets Son; Spirit proceeds); yet (d) none temporally prior—the hierarchy is eternal; (e) none existing without others—Father is not "Father" without Son.

This structure exhibits characteristics parallel to cyclical hierarchy: source, foundation, and primacy are distributed rather than localized. The Father is source of Son in one relation, but Son is foundation of Father's identity as Father. Spirit completes and closes the structure.

A.2 Why Alternative Formulations Were Rejected

Historical analysis reveals that alternative formulations were rejected precisely because they failed to preserve cyclical closure:

Modalism (Sabellianism) collapsed distinction, leaving no hierarchy to close. Subordinationism (Arianism) introduced linear hierarchy, destroying mutual determination—if the Son is ontologically lesser, the structure becomes a chain rather than a cycle. Tritheism destroyed closure entirely by positing three independent entities without mutual constitution.

Orthodox formulations preserved exactly the structural features that cyclical hierarchy identifies as characteristic of such systems.

A.3 Structural Reading of Traditional Formulas

The Gospel formula "the last shall be first" (Matthew 20:16), typically read as ethical teaching, admits a structural reading: "first" and "last" are circulating roles, not fixed positions. This represents cyclical hierarchical thinking *avant la lettre*—though the structural reading does not preclude ethical or eschatological interpretations.

This historical example demonstrates that cyclical hierarchical thinking is not a novel invention but has deep roots in Western intellectual history. The persistence of Trinitarian formulations despite centuries of attempted linearization suggests the intuitive appeal and structural coherence of cyclical models.

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