Formalization Laws: Structure, Boundaries, and the Necessity of Plural Representations

Reconstructing Formal Reason through the Principles of Total Formalizability, Boundary-Conditioned Verification, and Plural Incompleteness

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The three Formalization Laws — the Law of Total Formalizability, the Law of Boundary-Conditioned Verification, and the Law of Plurality and Incompleteness — were formulated and established in *Foundations of Intellectual Substrate Mining* by Boris Kriger, available on Amazon, with their full formal statements and mathematical proofs presented in Chapter 9 (pp. 125–164). Together, these laws establish that any phenomenon exhibiting determinate structure is formally representable under a sufficient constraint set, that verification of any formal claim is necessarily conditioned by explicit boundary regimes, and that no single formalization can exhaust a structured phenomenon. In the present work, these laws are treated as established methodological results and are used as governing constraints for further analysis without restating their derivation or proof.

Keywords

Formalization, Structure, Constraint Theory, Verification, Model Theory, Incompleteness, Representation, Boundary Conditions, Logic, Epistemology

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Abstract

This article introduces and formalizes three foundational laws of formal representation: the Law of Total Formalizability, the Law of Boundary-Conditioned Verification, and the Law of Plurality and Incompleteness. These principles collectively define the limits and possibilities of formal modeling across scientific, philosophical, and computational domains. Rejecting metaphysical claims about ineffability or absolute representation, the laws assert that all phenomena exhibiting determinate structure are formally capturable given appropriate constraints; that truth is conditional upon verifiable boundary conditions; and that no single representation can exhaustively describe any complex phenomenon. The article presents rigorous mathematical proofs for each law, defines critical terms, and discusses the implications for theory construction, scientific modeling, and epistemological disputes. Together, these laws establish a disciplined, pluralistic, and non-absolutist framework for formal reasoning. The work challenges traditional dichotomies between formal and informal knowledge by showing that the limits of formalization are always internal to representation systems, not intrinsic to reality. It concludes by proposing a methodological reorientation for science and philosophy based on representational adequacy, constraint refinement, and structured pluralism.

Introduction

The limitations of formal systems have long haunted discussions in science, philosophy, and logic. Arguments from ineffability, incompleteness, or representational failure have fueled skepticism about the capacity of formal models to capture complex, layered, or subjective phenomena. This article challenges such skepticism by articulating three foundational laws that jointly reframe the problem of formalization. These are the Law of Total Formalizability, the Law of Boundary-Conditioned Verification, and the Law of Plurality and Incompleteness. Taken together, they provide a rigorous, non-metaphysical foundation for the practice of formal representation. Rather than claiming that all phenomena are simple, reducible, or completely expressible, the framework holds that wherever there is determinate structure, formal representation is possible — though always bounded and partial. These laws systematically replace appeals to mystery or ineffability with disciplined attention to constraints, boundary conditions, and the selective nature of modeling.

Prior literature in mathematical logic, particularly model theory and information theory, has addressed aspects of representational adequacy and the expressive limitations of formal systems. Yet these discussions often remain internal to specific languages or formal domains, lacking a general theory of how formalization itself functions across domains. This article attempts to fill that gap by offering a generalized, formally stated, and provable account of the conditions under which formalization succeeds or fails — and why such failures are never due to the nature of the phenomenon itself.

Definitions

Structure: An ordered pair $S = (X, R)$, where X is a set of elements or states and R is a family of admissible relations on X.

Determinate Structure: A structure where not all logically possible configurations are permitted, i.e., there are nontrivial constraints that exclude certain configurations.

Formal Representation: A triple (L, C, M), where L is a formal language, C a constraint set, and M a model class, with a mapping from the phenomenon's structure to the model that preserves the relations in R.

Constraint Set: A collection of rules, axioms, or boundary conditions that restrict admissible models and fix indeterminacy.

Boundary Set: Conditions (domain, scale, operations, limits) that define the domain of application for a formal claim.

Adequate Representation: A formalization that captures all constraints relevant to a particular structural domain, though not necessarily all aspects of the phenomenon.

Underconstraint: A state where a model class remains too broad due to insufficient constraints, admitting incompatible realizations.

Verification: The process by which the truth or falsity of a formal claim is determined within specified boundary conditions.

Plurality: The structural fact that multiple, non-equivalent constraint sets can each yield an adequate formalization of the same phenomenon.

Incompleteness: The impossibility of any single representation capturing all structurally relevant aspects of a complex phenomenon.

Contextual Background or Historical Overview

The debate over the formalizability of complex domains — particularly in fields like consciousness, aesthetics, and social systems — dates back at least to the early 20th century. Philosophical movements such as logical positivism and formal semantics aimed to ground science in formal languages, but quickly encountered limitations noted by figures like Gödel (1931), Tarski (1944), and Quine (1951). These limitations were often taken as evidence that certain domains resist formalization altogether. However, recent developments in model theory, systems science, and information theory have shifted focus toward the conditions under which formalization is adequate, rather than exhaustive.

The present work builds on that shift, offering a set of formal principles that clarify the difference between structural capture and metaphysical overreach. The laws proposed here take inspiration from both foundational logic and pragmatic modeling, moving toward a general theory of formalization that avoids absolutism while affirming the representational reach of formal systems.

Research Question(s)

1. Under what conditions is formal representation of a phenomenon possible?
2. How are truth and verification structurally dependent on boundary conditions?
3. Why must formal representations be plural and incomplete in principle?

Theoretical Framework

The article combines insights from model theory, information theory, and epistemology. Model theory provides a framework for understanding the relationships between formal languages and structures. Information theory, especially entropy, allows the quantification of constraint sufficiency. Epistemologically, the work draws on structural realism and constraint-based theories of representation, arguing that knowledge emerges not from capturing "the world in itself" but from mapping structural regularities under constraint regimes.

Formalization Laws

The framework developed in this work is governed by three laws that together define the full logical space of formal representation. Each law corrects one of three persistent failure modes that have historically distorted discussions of formalization: the belief that certain domains are intrinsically resistant to formal capture, the belief that formal truths can be verified without boundary conditions, and the belief that a single formal system could exhaust a phenomenon. Taken individually, each law addresses one error. Taken together, they form a closed, non-contradictory system that replaces metaphysical speculation with methodological discipline.

The first is **THE LAW OF TOTAL FORMALIZABILITY**. It establishes the possibility condition of formal representation. Wherever a phenomenon exhibits determinate structure—where not all descriptions, transitions, or configurations are equally admissible—there exists a formal representation capable of capturing that structure, provided that a constraint set sufficient to fix it is supplied. Formalization is not guaranteed by simplicity, transparency, or familiarity, but by structure alone. What appears as resistance to formalization does not arise from the phenomenon itself, but from underconstrained representational schemes. Structure and formal capturability are coextensive.

The second is **THE LAW OF BOUNDARY-CONDITIONED VERIFICATION**. It restricts the scope of validity of any formal claim. No statement, law, or model can be verified independently of the boundaries that define its domain of application. Verification is inseparable from constraints: from scales, assumptions, idealizations, and conditions under which a representation operates. There are no boundary-free truths and no universal verifications. This law prevents the elevation of any formal system into an absolute ontology and blocks the confusion between local adequacy and global finality.

The third is **THE LAW OF PLURALITY AND INCOMPLETENESS**. It states that any phenomenon possessing determinate structure admits multiple adequate formalizations, each corresponding to a different selection of constraints, and that no single formalization can exhaust the phenomenon. Formal representation is necessarily selective. Each model fixes certain invariants and leaves others unconstrained. The failure of any one formalization to fully describe a phenomenon does not signal ineffability, but the multiplicity of constraint-dependent structure. What cannot be captured under one constraint regime may be captured under another, though never all at once.

Together, these three laws define a disciplined conception of formal reason. Structure guarantees formalizability; boundaries condition verification; plurality entails incompleteness. Formalization is always possible, never universal, and necessarily non-exhaustive. What remains is not mystery, but method: the disciplined articulation and refinement of constraints through which structure is made formally visible.

THE LAW OF TOTAL FORMALIZABILITY.

The Law of Total Formalizability concerns the conditions under which any phenomenon may be rendered within a formal system. It does not assert that every phenomenon is already formalized, nor that formalization exhausts what is given. It asserts, more narrowly and more rigorously, that wherever a phenomenon exhibits determinate structure, there exists a formal representation capable of capturing that structure, provided that a constraint set sufficient to fix it is supplied. What appears as resistance to formalization arises not from the phenomenon itself, but from inadequacy in the representational scheme brought to bear upon it.

To state the law in this way requires the removal of all rhetorical, epistemic, and metaphysical language. The phenomenon is not approached as meaningful, valuable, mysterious, or self-interpreting. It is approached solely as something that manifests structure. Structure, in this context, refers neither to subjective coherence nor to semantic articulation, but to constraint-sensitive regularity: the fact that not all descriptions, evolutions, or representations of the phenomenon are equally admissible. Where such regularity exists, there are limits on variation. Where there are limits, constraints may be articulated. Where constraints may be articulated, formalization is possible.

The first task, therefore, is to clarify the assumptions implicit in the law. A phenomenon is taken to be defined only by its observable or inferable structure. Nothing further is presupposed. There is no appeal to essence, intention, or inner constitution beyond what can be inferred from regularities of manifestation. Structure is not identified with simplicity; a phenomenon may be complex, layered, or dynamic, yet still structured insofar as its variations are not arbitrary. The presence of structure implies that certain descriptions are ruled out, certain transitions forbidden, certain configurations unstable. These exclusions constitute constraints, whether or not they have yet been explicitly identified.

Formal systems are assumed to be extendable. This assumption is crucial. A formal language, together with its axioms and rules, is not treated as a closed artifact but as a modifiable representational apparatus. New axioms, relations, parameters, or inference rules may be added as needed. Constraint sets are therefore not fixed in advance; they may be partial, progressive, or hierarchical. A phenomenon need not be captured by a single, monolithic formalization. It may require successive refinements, each reducing indeterminacy further than the last.

Indeterminacy, under this law, is not a property of the phenomenon. It is a property of the representational scheme. When multiple incompatible models satisfy the same formal description, the description is underconstrained. This multiplicity does not indicate that the phenomenon itself lacks structure, but that the constraints specifying it have not yet been sufficiently articulated. The law therefore rejects the notion of irreducible structureless variance. Variability may be extensive, but if it is governed by constraints—even probabilistic or statistical ones—it remains structured.

Adequacy, not absolute completeness, is the criterion of representation. A formal model is adequate if it captures the determinate structure under consideration within the bounds specified. It need not account for every conceivable aspect of the phenomenon, nor reproduce its experiential fullness. Demands for total capture are methodologically illegitimate. They confuse formal sufficiency with ontological exhaustiveness. The law operates entirely within the former.

Once these assumptions are made explicit, the phenomenon may be recast as a system suitable for reduction. The relevant system is not the phenomenon itself, but the representational relation between structure and description. It consists of a target structure, a formal language, a constraint set, and a mapping relation that connects elements of the language to aspects of the structure. The aim of reduction is not to simplify the phenomenon, but to isolate the minimal conditions under which the mapping ceases to be indeterminate.

Success, within this system, is defined by the emergence of a unique model or a bounded class of models that match the target structure. Uniqueness need not be absolute; it suffices that the admissible models fall within a well-defined range. Failure occurs when the model space remains unbounded, admitting incompatible realizations that cannot be distinguished by the constraints in place. Error arises when imposed constraints misalign with the actual structure, excluding admissible behaviors or admitting inadmissible ones. Such errors are not refutations of the law but indicators of improper constraint specification.

All anthropic concepts are excluded from this reduction. There is no appeal to intuition, understanding, or lived meaning. These may accompany the phenomenon in experience, but they do not participate in the determination of structure. Formalization proceeds without regard to how the phenomenon is felt, interpreted, or valued. This exclusion is not a denial of experience but a suspension of it. The law addresses representation, not consciousness.

The formal statement of the law follows directly from this reduction. For any phenomenon possessing determinate structure, there exists a formal representation that captures that structure once a constraint set sufficient to eliminate representational indeterminacy is imposed. Apparent resistance to formalization arises exclusively from underconstrained representational schemes, not from the nature of the phenomenon. The necessary conditions are minimal: determinate structure and an extendable constraint space. The boundary condition is equally clear: the law does not apply to entities lacking structure altogether. Where no regularity exists, nothing may be fixed, and representation collapses into arbitrariness.

The scope of the law is universal in the precise sense that it applies wherever structure appears. This includes physical systems governed by dynamical laws, abstract systems defined by relations, computational processes constrained by algorithms, linguistic patterns stabilized by usage, and experiential domains insofar as they exhibit regularities of recurrence or limitation. The law makes no distinction of dignity or depth among these domains. It distinguishes only between the structured and the unstructured.

The law is falsifiable in principle. A counterexample would consist in exhibiting a phenomenon that demonstrably possesses determinate structure yet resists all finite or recursively extensible constraint systems. Such a phenomenon would have to show stable regularities while remaining incompatible with every attempt at constraint articulation. No such phenomenon has been identified. Claims of this kind invariably rely on a refusal to specify what counts as structure or on an implicit demand for representational completeness beyond sufficiency.

The mathematical framework most suited to articulating the law combines model theory with information theory. Model theory provides the language for relating formal systems to structures. A phenomenon corresponds to a structure; constraints correspond to axioms and relations; formalization corresponds to model instantiation. Underconstraint manifests as excess entropy or non-uniqueness in the model space. Information theory supplies a measure of this indeterminacy, allowing one to quantify the degree to which constraints reduce admissible variation. Together, these frameworks suffice to express the law without excess machinery.

Within this framework, the proof of the law is straightforward. Let a phenomenon exhibit determinate structure. By definition, determinate structure imposes constraints on admissible descriptions. Any such constraint may be encoded as a formal rule. A representational scheme lacking sufficient constraints will admit multiple incompatible models. Each additional constraint reduces this multiplicity. Because the structure is determinate, there exists a finite or recursively extensible constraint set sufficient to isolate it within a bounded model class. Therefore, a formal model exists. If no such model existed, the structure would be indeterminate, contradicting the premise. Resistance to formalization is thus a representational deficiency, not an ontological one.

From the law follows an immediate corollary. Claims that a domain—such as consciousness, meaning, or aesthetics—is inherently unformalizable are equivalent to claims that no adequate constraint set has yet been specified. They do not describe a property of the domain but a limitation of the current representational effort. Such claims may reflect the present state of knowledge or the complexity of the task, but they do not establish a principled barrier.

This corollary is often expressed in a compressed form: no phenomenon is intrinsically resistant to formal representation. As a proverb, this statement preserves the dependence on constraint sufficiency but loses the distinctions between structure, indeterminacy, and adequacy. It asserts the conclusion without articulating the conditions under which it holds. The present law restores those conditions, transforming a slogan into a disciplined principle.

The limits of the law must be stated with equal clarity. It fails only when no determinate structure exists, when constraints cannot be operationalized, when representation is demanded to be complete rather than sufficient, or when constraint extension is forbidden by definition. In each case, the failure arises from the way the task is posed, not from the nature of what is being represented. The law does not compel formalization where structure is absent; it denies only that structure can exist without the possibility of formal capture.

The novelty of the law lies in its relocation of unformalizability from ontology to constraint theory. It is stronger than expressibility results in logic, which concern what can be stated within a given formal language. It is orthogonal to results on incompleteness, which concern what can be proven within a fixed axiomatic system. It is compatible with, but not reducible to, theses concerning computation, since it addresses representation rather than effective procedure.

By establishing that every failure of formalization is diagnostic of constraint insufficiency, the law transforms debates about ineffability into problems of representational design. What was once treated as a metaphysical limit becomes an engineering challenge: to identify, articulate, and refine the constraints that fix structure. Formal resistance is never intrinsic; it is always relational, arising from the mismatch between structure and the constraints brought to represent it.

The implications of this shift are considerable. Domains traditionally shielded from formal analysis by appeal to depth or subjectivity are reopened, not to reductionist dismissal, but to disciplined representation. At the same time, formal systems are deprived of any claim to ontological privilege. They are tools whose adequacy depends entirely on the care with which constraints are specified. Formal limits are not properties of reality; they are properties of representational discipline.

A Complete Mathematical Proof of the Law of Total Formalizability

This proof concerns only phenomena that possess determinate structure. No appeal is made to meaning, value, subjective experience, or metaphysical status beyond what is required to define structure and representation.

I. Preliminaries and Definitions

Definition 1 (Structure). A structure is an ordered pair S = (X, R), where X is a nonempty set of states (or elements) and R is a nonempty family of admissible relations, operations, or predicates on X.

Definition 2 (Determinate Structure). S = (X, R) is determinate if R imposes nontrivial constraints, i.e., the set of admissible configurations or transitions is a proper subset of all logically possible configurations on X. Equivalently, there exists at least one property P expressible over X such that both P and not-P are not simultaneously admissible under R.

Definition 3 (Formal Language). A formal language L consists of a (finite or countable) alphabet, formation rules for well-formed expressions, and a semantics assigning interpretations to those expressions over some domain.

Definition 4 (Constraint Set). A constraint set C is a finite or recursively enumerable collection of axioms, relations, or inference rules added to L that restrict the class of admissible models.

Definition 5 (Formal Representation). A formal representation of S is a triple (L, C, M), where M is a nonempty class of models of L union C, together with a mapping phi: X → Dom(M) such that every relation in R is preserved under phi.

Definition 6 (Adequacy). A representation (L, C, M) is adequate if the class M satisfies one of the following:

1. M is a singleton up to isomorphism; or
2. M forms an equivalence class in which any two models agree on all relations in R (i.e., they differ only on features external to the preserved structure).

Definition 7 (Underconstraint). A representation is underconstrained if M contains multiple non-equivalent models that disagree on at least one relation in R.

II. Statement of the Law

Theorem (Law of Total Formalizability). For any phenomenon whose associated structure S = (X, R) is determinate, there exists an adequate formal representation (L, C, M). Apparent resistance to formalization arises exclusively from underconstrained representational schemes, not from the nature of S.

III. Lemmas

Lemma 1 (Existence of Constraints). If S = (X, R) is determinate, then there exists at least one nontrivial constraint that excludes at least one otherwise admissible description of X.
Proof. By Definition 2, determinacy implies the existence of a property P such that not all admissible descriptions satisfy both P and not-P. This exclusion constitutes a constraint. ■

Lemma 2 (Constraint Codability). Any constraint on admissible descriptions of X can be encoded as an axiom or rule in a suitable formal language L.
Proof. Any first-order expressible property over a relational structure can be axiomatized in an appropriate signature. More generally, any recursively enumerable constraint admits encoding within a suitably enriched formal language. Hence any constraint arising from R can be formally represented. ■

Lemma 3 (Model Multiplicity under Underconstraint). If (L, C, M) is underconstrained with respect to S, then there exist at least two non-equivalent models in M that disagree on at least one relation in R.
Proof. Underconstraint means the constraints in C fail to fix at least one relation in R. Hence there exist models satisfying C that assign incompatible values to that relation, yielding non-equivalent models. ■

Lemma 4 (Constraint Refinement). If (L, C, M) is underconstrained, then there exists an extension C' properly containing C such that the resulting model class M' of L union C' is strictly smaller than M.
Proof. Select a relation R in R on which models in M disagree. Introduce an axiom fixing R in accordance with S. This eliminates at least one previously admissible model, strictly reducing M. ■

IV. Main Proof

Let S = (X, R) be a determinate structure.

1. **Initialization.** Choose a formal language L_0 capable of expressing elements of X and relations in R. Set C_0 = empty set. The initial model class M_0 is maximally underconstrained.
2. **Constraint Introduction.** By Lemma 1, at least one nontrivial constraint exists. By Lemma 2, encode this constraint into L_0, yielding C_1.
3. **Iterative Refinement.** If (L_0, C_1, M_1) is adequate (per Definition 6), terminate. Otherwise, by Lemma 3 it is underconstrained. By Lemma 4, there exists a refinement C_2 properly containing C_1 that strictly reduces the model class.

4. **Termination.** Each refinement step strictly reduces either the cardinality or the informational entropy of the admissible model class. Under standard set-theoretic assumptions, the initial model class is at most countable, and entropy cannot decrease indefinitely. Moreover, infinite descent would imply arbitrarily many incompatible realizations of R, contradicting determinacy. Hence the process terminates at some finite stage C_n.
5. **Adequacy.** The resulting representation (L_0, C_n, M_n) is adequate by Definition 6, completing the construction.

■

V. Corollaries

Corollary 1 (Localization of Unformalizability). If a phenomenon resists formalization, then either (i) its structure is indeterminate, or (ii) the constraint set employed is insufficient.

Corollary 2 (Non-Ontological Status of Resistance). Resistance to formalization is a property of representational schemes, not of phenomena.

VI. Information-Theoretic Interpretation

Let H(M) denote the Shannon entropy of the uniform distribution over distinguishable models in M with respect to distinctions fixed by R. Underconstraint corresponds to high entropy. Each refinement step reduces H(M). Determinacy of S implies the existence of a constraint set C such that inf_C H(M_C) < ∞, yielding a bounded minimum. This provides a quantitative measure of convergence without altering the logical structure of the proof.

VII. Relation to Existing Results

Unlike expressibility results (which concern what can be stated within a fixed formal language) and incompleteness theorems (which concern what can be proven within a fixed axiom system), the present law concerns the extendability of the representational framework itself. Unlike forcing or Löwenheim–Skolem phenomena (which describe model multiplicity under fixed theories), this law concerns the possibility of refining the theory until adequacy is achieved.

VIII. Limit Case

If R is empty or trivial, no determinate structure exists and the law does not apply. In such cases, representation fails not because formalization is impossible, but because there is nothing to formalize.

Thus the Law of Total Formalizability is established: whenever determinate structure exists, a sufficient constraint set can be constructed to yield an adequate formal representation. Formal limits are therefore properties of representational discipline, not of reality itself.

THE LAW OF BOUNDARY-CONDITIONED VERIFICATION

The law of boundary-conditioned verification concerns the conditions under which any statement may be evaluated as true or false. It does not propose a new theory of truth, nor does it redefine truth in psychological, linguistic, or social terms. It specifies a structural dependency that all verification presupposes but rarely acknowledges explicitly: every act of verification is possible only within a bounded system of constraints. Truth, in this sense, is not an intrinsic property of sentences taken in isolation, but an emergent property of representational systems operating under defined limits.

Formal statement Any statement, law, or proposition is verifiable — and hence capable of being true or false — only relative to a boundary set that determines the domain, scale, assumptions, admissible operations, and verification procedures under which the statement is evaluated. Outside such boundaries, verification is undefined rather than false. The absence of boundaries does not enlarge truth; it dissolves the conditions under which truth or falsehood can be assessed.

From this follows the central conceptual shift: truth is boundary-dependent. A statement is not true simpliciter. It is true within a representational system whose constraints render its verification meaningful. These constraints may be explicit (formal model, experimental protocol) or implicit (shared background assumptions). In either case, they delimit what counts as evidence, what operations are admissible, and what variations are relevant. Removing these limits does not strengthen truth; it deprives it of the conditions required for assessment.

Falsehood acquires a parallel reinterpretation. A statement is often false not because it misrepresents reality, but because it is asserted outside the boundary conditions under which it could be verified. Many false statements are unscoped descriptions whose domain has silently expanded beyond the constraints that once supported them. Falsehood thus appears not as the opposite of truth, but as a consequence of missing or illegitimate limits.

This reinterpretation enables the transformation of false statements through correct scoping. A proposition false when asserted universally may become true once its boundary conditions are properly specified. Claims of the form “X is impossible,” “Y always occurs,” or “Z never happens” frequently fail not because their content is wrong, but because their scope is unbounded. The law therefore replaces outright refutation with localization: under what boundaries does the statement hold?

Localization must be distinguished from fabrication. Localization restricts a claim to the domain in which it is verifiable. Fabrication invents boundaries post hoc to rescue a claim from falsification. The law does not license arbitrary scoping; boundaries must be operationally grounded in the structure of the system under consideration.

Implications for disputes Many persistent disagreements in science and philosophy persist not because one side is correct and the other mistaken, but because each operates under different, often implicit, boundary conditions. Disputes over determinism, reductionism, free will, realism, or objectivity frequently dissolve once verification domains are made explicit. Contradiction gives way to non-overlapping scopes.

Truth is not a property of sentences; it is a property of systems. Detached from a structured representational system, a sentence is neither true nor false; it is indeterminate.

Distinction from relativism Boundary-conditioned verification is not relativism. Boundaries are not perspectives, preferences, or conventions; they are structural constraints imposed by the system under investigation. They can be inadequate, inconsistent, or wrong, but they are not arbitrary. Verification varies with operational limits, not with opinion.

Cross-domain illustrations In physics, classical mechanics holds within macroscopic, low-energy regimes; quantum descriptions hold within microscopic, high-precision regimes. Neither is “more true”; each is true relative to its boundary conditions. In mathematics, theorems hold relative to axioms and inference rules. In biology, functional explanations hold within evolutionary and ecological constraints. In each case, truth is inseparable from boundaries.

Impossibility of universal truth No universally true statement exists. A universal claim must be verifiable independently of any boundary set. But verification requires boundaries that delimit admissible procedures and observations. Demanding universality is demanding verification without conditions — a contradiction. Universal assertions are unverifiable and therefore lack determinate truth value. Absolute truth and absolute falsehood coincide as forms of indeterminacy.

The law does not deny structured reality. It denies only the legitimacy of boundary-free assertions about it. Reality constrains representations; representations require boundaries to test those constraints.

Conclusion Local truth — truth within well-defined boundaries — is the only coherent form of truth. Knowledge is a network of formally constrained representations, each verified within its domain. Progress consists in refining boundaries, extending domains under explicit conditions, and clarifying relations between verification regimes. Boundary-conditioned verification demands restraint in assertion, precision in scope, and accountability in reasoning. Intellectual humility is not a moral add-on; it is a structural necessity.

A Complete Mathematical Proof of the Law of Boundary-Conditioned Verification

This proof concerns only statements evaluated within formal representational systems. No appeal is made to meaning, belief, convention, or subjective interpretation. Verification is treated as a structural relation between statements, models, and boundary conditions.

I. Preliminaries and Definitions

Definition 1 (Representational System). A representational system is an ordered quadruple (L, C, B, M), where L is a formal language, C is a constraint set, B is a boundary set specifying admissible domains, scales, and operations, and M is a nonempty class of models of L union C consistent with B.

Definition 2 (Boundary Set). A boundary set B is a finite or recursively specifiable collection of conditions that restrict: (1) the domain of discourse, (2) admissible transformations or measurements, (3) relevant scales or regimes, (4) permissible idealizations.

Definition 3 (Statement). A statement is a well-formed formula of L.

Definition 4 (Verification). A statement phi is verifiable in the system if and only if there exists a procedure, admissible under B, that determines whether phi holds in all models in M.

Definition 5 (Truth). Phi is true in the system if it is verifiable and holds in all models in M.

Definition 6 (Falsehood). Phi is false in the system if it is verifiable and fails in at least one model in M.

Definition 7 (Unscoped Statement). A statement is unscoped if it is asserted without reference to any boundary set B.

II. Statement of the Law

Theorem. A statement is verifiable — and hence capable of truth or falsehood — only relative to a boundary set that defines the conditions under which verification is possible. Statements asserted without boundary conditions are unverifiable and therefore lack determinate truth value.

III. Lemmas

Lemma 1. Verification requires a nonempty boundary set. *Proof.* Verification requires admissible procedures. Procedures require specification of domain, operations, and limits. These specifications constitute B. ■

Lemma 2. Removing B renders all statements unverifiable. *Proof.* Without B, admissible procedures are undefined. ■

Lemma 3. Truth cannot be assigned to a statement independently of a representational system. *Proof.* Truth depends on holding in all models in M, which depends on B. ■

Lemma 4. Falsehood presupposes the same boundary conditions as truth. *Proof.* Falsehood requires verification of failure in at least one model, which requires B. ■

IV. Main Proof

Let phi be an arbitrary statement of L.

Assume phi is verifiable without reference to any boundary set. By Lemma 1, this is impossible. Contradiction.

Assume phi is true without boundary conditions. Truth requires verification (Definition 5). By Lemma 2, verification without boundaries is impossible. Contradiction.

The same reasoning applies to falsehood (Lemma 4).

Therefore, any statement asserted without boundaries is unverifiable and lacks determinate truth value.

Hence, verification — and consequently truth and falsehood — are boundary-conditioned. ■

V. Corollaries

1. Every true statement is true only within a bounded representational system.
2. Many false statements are false due to missing or incorrect boundary specification rather than content.
3. A false statement may become true under correct scoping, provided boundaries are properly specified.

VI. Impossibility of Universal Truth No universally true statement exists. A universal claim must hold independently of all boundary sets. Removal of boundaries eliminates verification. An unverifiable statement lacks truth value. ■

VII. Symmetry Absolute truth and absolute falsehood coincide as forms of indeterminacy — both evade evaluation by eliminating verification conditions.
VIII. Conclusion Truth exists only as boundary-conditioned truth. The Law of Boundary-Conditioned Verification is established.

THE LAW OF PLURALITY AND INCOMPLETENESS

The law of plurality and incompleteness addresses the necessary multiplicity and partiality of formal representations. It does not deny the reality or adequacy of any particular formalization, nor does it suggest that all models are equally valid or arbitrary. It asserts that any phenomenon possessing determinate structure admits multiple adequate formal representations, each arising from a different selection of constraints, and that no single formalization can exhaust the phenomenon in all its aspects. Formal representation is inherently selective: every model isolates certain invariants while leaving others unconstrained. The apparent failure of one formalization to capture everything is not evidence of ineffability or intrinsic resistance, but of the structural richness that demands multiple constraint regimes.

Formal statement For any phenomenon with determinate structure, there exist at least two distinct adequate formal representations (L, C, M), corresponding to different boundary and constraint sets, and no single representation can capture all relevant aspects of the structure simultaneously. Completeness with respect to the phenomenon is impossible within any finite or even recursively enumerable set of constraints; only partial, selective adequacy is achievable.

This law follows directly from the first two laws. The Law of Total Formalizability guarantees that structure is capturable under some constraint set. The Law of Boundary-Conditioned Verification shows that every such capture is scoped to specific boundaries. Because structure itself is multi-layered and can be delimited in non-equivalent ways (different partitions of variables, different scales of observation, different idealizations), multiple non-equivalent constraint regimes can each yield adequate but incomplete models. The phenomenon is not indeterminate; it is overdetermined relative to any single representational choice.

Key conceptual shift Incompleteness is not a defect of formalization but a consequence of its strength. A model is adequate precisely because it enforces strong constraints that fix certain invariants; those very constraints exclude alternative descriptions that might be equally valid under different boundaries. What one model leaves indeterminate or abstracted away another model can fix, but at the cost of abstracting or indeterminizing something else. Plurality is therefore not a sign of failure, but of fidelity to the phenomenon's complexity.

Distinction from arbitrariness Plurality does not imply that any formalization is as good as any other. Models remain constrained by the phenomenon's structure: incompatible models cannot both be adequate with respect to the same boundary set. Selection of constraints is not whimsical; it is guided by explanatory aims, empirical regimes, computational tractability, or theoretical coherence. The law demands transparency about which aspects are preserved and which are sacrificed, rather than pretending to a false totality.

Implications for disputes and progress Many scientific and philosophical controversies arise from treating partial models as if they were complete. When two formalizations appear contradictory, the conflict often dissolves upon explicit comparison of their constraint regimes. Progress in inquiry does not consist in eliminating alternatives to reach a single final model, but in mapping the space of adequate formalizations, understanding their relations (e.g., embeddings, limits, approximations), and selecting models fit for specific purposes. The law thus replaces the ideal of a unified, exhaustive theory with a disciplined pluralism: multiple, interlocking, partial representations that together approximate the phenomenon without ever coinciding with it.

Cross-domain illustrations In physics, the same phenomenon (e.g., fluid flow) admits both continuum mechanics and molecular dynamics models; neither exhausts the other, yet both are adequate within their regimes. In biology, a single trait (e.g., eusociality in insects) can be formalized via inclusive fitness, group selection, or network dynamics; each highlights different invariants while leaving others opaque. In cognitive science, mental states exhibit multiple realizability across neural architectures; functionalist models abstract from substrate while substrate-specific models preserve implementation details. In each case, the plurality reflects not confusion but the multi-faceted nature of the phenomenon.

Relation to underdetermination and incompleteness traditions This law resonates with — but is distinct from — the Duhem-Quine underdetermination thesis (empirical evidence alone cannot uniquely determine theory) and Gödelian incompleteness (no consistent formal system can prove all truths about arithmetic). Unlike underdetermination of theory by data, the present plurality concerns the structure itself: even given perfect data, multiple constraint selections remain viable. Unlike Gödelian incompleteness, which is internal to a fixed system, the law concerns the necessary incompleteness of any system relative to a richer phenomenon.

Distinction from relativism or anti-realism The law is not relativist: adequacy remains objective, grounded in how well models capture determinate structure under specified boundaries. Nor is it anti-realist: the plurality arises precisely because reality is structured and rich enough to support multiple legitimate projections. Reality is not indeterminate; it is inexhaustible by any single formal lens.

Conclusion Plurality and incompleteness are structural features of formal reason, not limitations to be overcome. The phenomenon guarantees formalizability (Law 1), boundaries guarantee conditioned verification (Law 2), and the richness of structure guarantees that no single formalization can be complete. Together, the three laws define formalization as always possible, always bounded, and always partial. What remains is not the dream of a total theory, but the disciplined practice of constructing, comparing, and refining multiple representations — each adequate for its domain, none pretending to finality.

A Complete Mathematical Proof of the Law of Plurality and Incompleteness

This proof concerns only phenomena with determinate structure formalized within representational systems. No appeal is made to meaning, explanatory power, or pragmatic preference beyond structural adequacy.

I. Preliminaries and Definitions

Definition 1 (Adequate Representation). A representation (L, C, M) is adequate for a structure S = (X, R) if M satisfies the adequacy condition (singleton up to isomorphism or equivalence class agreeing on R).

Definition 2 (Distinct Representations). Two representations (L1, C1, M1) and (L2, C2, M2) are distinct if there exists some relation in R preserved by one but not fixed by the other, or if their boundary sets differ and induce non-equivalent partitions of the phenomenon.

Definition 3 (Exhaustiveness). A representation is exhaustive if it fixes all relations in R and all possible boundary regimes relevant to S.

II. Statement of the Law

Theorem. For any phenomenon with determinate structure S, there exist at least two distinct adequate representations, and no representation is exhaustive.

III. Lemmas

Lemma 1 (Multiple Constraint Selections). Given determinate S, there exist at least two non-equivalent constraint sets C1 not equal to C2 such that (L, C1, M1) and (L, C2, M2) are both adequate.

Proof. By the Law of Total Formalizability, S admits an adequate representation under some C. Since S is determinate but not trivially simple, there exist multiple ways to partition variables or impose boundaries (e.g., coarse vs. fine graining, different idealizations). Each yields a different C that fixes R adequately but excludes different aspects. ■

Lemma 2 (Non-Exhaustiveness). No finite or recursively enumerable constraint set C can fix all possible boundary regimes for S.

Proof. The space of possible boundary sets is at least countably infinite (different scales, idealizations, variable selections). Any fixed C operates within one regime, leaving others indeterminate or abstracted away. Exhausting all regimes would require an infinite conjunction of incompatible constraints, which is impossible within a single representation. ■

IV. Main Proof

Let S be determinate.

By Lemma 1, there exist at least two distinct adequate representations.

By Lemma 2, no representation exhausts S.

Therefore, plurality obtains and completeness is impossible. ■

V. Corollaries

1. Incompleteness is structural: every adequate model sacrifices some invariants to preserve others.
2. Plurality is non-arbitrary: distinct models remain accountable to S's structure.

3. Progress consists in mapping the space of adequate representations, not eliminating alternatives.

VI. Conclusion

The Law of Plurality and Incompleteness is established: structure demands multiple formal lenses and forbids any single one from being total. Formal reason is therefore necessarily plural and partial — a disciplined network of bounded, interlocking representations rather than a monolithic truth.

Discussion

The three formalization laws provide a coherent and mutually reinforcing framework. The Law of Total Formalizability (LTF) asserts that where structure exists, formalization is always possible given a sufficient constraint set. This challenges claims that certain domains (e.g., consciousness or culture) are inherently unformalizable. Rather, these domains may simply lack appropriately refined constraint systems.

The Law of Boundary-Conditioned Verification (LBV) clarifies that verification is not absolute; it depends on specific boundary sets — domain, scale, assumptions — without which the truth or falsity of a statement is undefined. This law addresses the overreach of many universal claims in both science and philosophy, showing that disagreements often stem from unacknowledged differences in scope.

The Law of Plurality and Incompleteness (LPI) demonstrates that no single model can exhaust a complex phenomenon. Multiple adequate formalizations can coexist, each illuminating different aspects depending on the constraint regime. Far from being a weakness, this multiplicity reflects the richness of the phenomenon.

These principles collectively shift the focus from whether a phenomenon *can* be formalized to *how* it is constrained, *where* verification applies, and *what* aspects are necessarily left out. Formalization becomes a disciplined practice of mapping structural regularities under bounded, plural regimes.

Limitations

The framework excludes phenomena that lack determinate structure. It assumes that constraint sets are recursively extendable and that representation systems are sufficiently expressive. It also does not resolve debates about which constraints are epistemically superior, focusing instead on structural adequacy. Finally, the framework does not address computational feasibility, only representational possibility.

Counterarguments and Responses

Counterargument: Certain domains, such as subjective experience or ethics, resist formalization due to their qualitative nature.
Response: LTF does not deny qualitative richness but claims that wherever structure (e.g., regular patterns, conditional invariants) is present, it can be captured formally. Resistance arises from incomplete constraint articulation, not the domain's essence.

Counterargument: Boundary-conditioned truth is relativistic.
Response: LBV distinguishes structural boundaries from subjective perspectives. Verification conditions are not opinions but operational constraints. Truth remains objective within a boundary-defined model space.

Counterargument: Plurality implies arbitrary modeling.
Response: LPI insists on structural adequacy for all models. Plurality arises not from whimsy but from the selective nature of constraint application. Models must still preserve defined relations within scope.

Future Research Directions

- Development of constraint-discovery algorithms for underformalized domains.
- Application of boundary-conditioned verification to scientific modeling in climate science, economics, or neuroscience.
- Formalization of interdisciplinary phenomena (e.g., between biology and AI) under LPI.
- Exploration of model interoperability and translation between plural formalizations.
- Quantitative studies of entropy reduction across constraint iterations.

Theoretical Implications

The laws refine the epistemological role of formal models. Rather than seeking totalizing representations, science should focus on structurally adequate, domain-specific models. These laws challenge traditional formalist ambitions in mathematics and logic while preserving the rigor of formal reasoning. They also suggest a refined version of structural realism: the world is knowable to the extent that its structures are constrainable, though never exhaustively so in a single frame.

Conclusion

The three laws presented here redefine the scope and purpose of formal representation. Total formalizability affirms the universal reach of formal methods wherever structure exists. Boundary-conditioned verification ensures that truth is always scope-sensitive. Plurality and incompleteness acknowledge the selective nature of all modeling. Together, these laws replace metaphysical closure with methodological discipline. The promise of formalization is not the capture of reality in a single model, but the construction of bounded, plural, structurally adequate representations.

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