A Bayesian Model Comparison Framework for Extrapolative Scientific Hypotheses: Methodology and Application to Holography in Theoretical Physics

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Abstract

We develop a general Bayesian model comparison framework for evaluating universal ontological claims in science—hypotheses that extrapolate domain-specific laws to universal status without direct experimental verification. The methodology comprises: (i) sharp hypothesis specification distinguishing universal from scale-specific claims; (ii) historically informed prior assignment based on patterns of theory succession; (iii) systematic evidence decomposition with explicit likelihood elicitation; (iv) formal treatment of evidential dependence via correlation-adjusted composite likelihoods; and (v) comprehensive sensitivity analysis. As a detailed case study, we apply this framework to the holographic principle in theoretical physics, comparing universal holography (all physical degrees of freedom are boundary-encoded) against scale-specific holography (holographic descriptions are powerful within particular regimes but not constitutive of reality). Under explicit assumptions, we obtain a posterior probability of 6–8% for universal holography (range from sensitivity analysis), with a Bayes factor of approximately 0.29, indicating moderate evidence favoring the scale-specific interpretation. The framework provides a template for disciplined probabilistic assessment of foundational claims across scientific domains where direct testing is infeasible.

Keywords: Bayesian inference; model comparison; Bayes factors; prior specification; evidence synthesis; foundational physics; extrapolation

MSC 2020: 62F15 (Bayesian inference), 62P35 (Applications to physics), 62C10 (Bayesian problems)

1 Introduction

Bayesian model comparison provides a principled framework for evaluating competing hypotheses under uncertainty. While extensively developed for data-rich settings [2, 3], its application to foundational scientific claims—where direct experimental verification is limited or impossible—remains underexplored. This paper develops a general Bayesian methodology for such settings and demonstrates its application through a detailed case study.

The core statistical contribution is a framework for evaluating *extrapolative hypotheses*: claims that extend domain-specific laws to universal ontological status. Such hypotheses are ubiquitous in foundational science, from cosmological models to fundamental physics. Traditional Bayesian analysis assumes repeated sampling or direct observation; here, we adapt these tools to settings where evidence is indirect, heterogeneous, and partially correlated.

Our framework addresses three methodological challenges: (1) constructing informative priors from historical patterns of theory succession rather than conjugate convenience; (2) decomposing complex evidence into assessable components while accounting for dependence; and (3) quantifying sensitivity to ensure conclusions are robust. The methodology connects to established work on Bayes factors [2], calibrated probabilistic reasoning [5], and evidence synthesis [6].

As illustration, we apply this framework to the holographic principle in theoretical physics. The object of evaluation is a specific ontological claim: that the universe is fundamentally holographic, meaning all physical degrees of freedom are boundary-encoded and volumetric structure is derivative. This must be distinguished from the weaker assertion that holographic descriptions are effective in certain regimes. Bayesian analysis is suited precisely to this distinction, forcing the cost of extrapolation to be made explicit.

2 Bayesian Framework for Extrapolative Hypotheses

2.1 Hypothesis Specification

Let H denote a universal extrapolative hypothesis and $\bar{H}$ the corresponding scale-specific alternative. In our case study:

- H: Universal holography—all physical degrees of freedom are fundamentally boundary-encoded, with volumetric structure being derivative.
- $\bar{H}$: Scale-specific holography—holographic principles govern particular gravitational and strongly coupled domains but are not constitutive of reality as a whole.

The posterior probability is given by Bayes' theorem:

$$P(H \mid E) = \frac{P(E \mid H)\, P(H)}{P(E \mid H)\, P(H) + P(E \mid \bar{H})\, P(\bar{H})}, \tag{1}$$

where E denotes the total relevant evidence. Equivalently, in terms of posterior odds:

$$\frac{P(H \mid E)}{P(\bar{H} \mid E)} = \underbrace{\frac{P(E \mid H)}{P(E \mid \bar{H})}}_{\text{Bayes factor } \mathrm{BF}_{H:\bar{H}}} \times \underbrace{\frac{P(H)}{P(\bar{H})}}_{\text{Prior odds}} . \tag{2}$$

The Bayes factor $\mathrm{BF}_{H:\bar{H}}$ quantifies the evidence's discriminative power independent of priors [2].

2.2 Interpretation Scale for Bayes Factors

Following Kass and Raftery [2], we interpret Bayes factors on a logarithmic scale:

$\mathrm{BF}_{H:\bar{H}}$	$\log_{10}(\mathrm{BF})$	Evidence against H
1 to 1/3	0 to -0.5	Weak
1/3 to 1/10	-0.5 to -1	Moderate
1/10 to 1/100	-1 to -2	Strong
$< 1/100$	< -2	Decisive

3 Prior Specification from Historical Patterns

3.1 The Law of Scale-Specific Principles

The prior probabilities encode structural knowledge about theory extrapolation. Historical and mathematical evidence reveals a persistent pattern: laws successful within a given regime are rarely universal, and claims of finality typically collapse beyond their organizing assumptions' validity.

Definition 1 (Law of Scale-Specific Principles)**.** *Universal ontological claims carry an intrinsic evidential penalty unless supported by explicit cross-scale mathematical control.*

This pattern is not merely sociological but reflects how effective theories suppress degrees of freedom that later reassert dominance. Concrete examples include:

1. **Newtonian mechanics → Special/General Relativity**: Newton's laws, spectacularly successful for centuries, required fundamental revision at high velocities and strong gravitational fields.

2. **Classical electromagnetism → Quantum electrodynamics**: Maxwell's equations, exact at macroscopic scales, required quantization at atomic scales.

3. **Fermi theory of weak interactions → Electroweak theory**: An effective low-energy description replaced by a more fundamental gauge theory.

4. **Effective field theories in particle physics**: The modern understanding explicitly treats successful theories as valid within cutoff scales, with new physics emerging beyond [8].

3.2 Quantitative Prior Assignment

Based on this historical pattern, we assign:

$$P(H) = 0.20, \qquad P(\bar{H}) = 0.80, \tag{3}$$

yielding prior odds of $1 : 4$ against universal extrapolation. This assignment reflects:

- A base rate of $\sim 20\%$ for theories surviving unchanged under regime extension, consistent with historical patterns.

- Symmetry: the same prior would apply to any universal ontological claim, not specifically holography.

To formalize uncertainty in the prior itself, we can embed this in a hierarchical structure. Let $P(H) \sim \text{Beta}(\alpha, \beta)$ with $\alpha = 2$, $\beta = 8$, giving $\mathbb{E}[P(H)] = 0.20$ and $\text{Var}[P(H)] = 0.0145$. The sensitivity analysis in Section 7 explores the impact of prior variation.

4 Evidence Decomposition and Likelihood Assessment

4.1 Evidence Categories

The evidence set is decomposed into eight components to avoid conflation and enable systematic assessment:

E_1 : Area scaling of black hole entropy
E_2 : Exactness of AdS/CFT correspondence
E_3 : Holographic resolution of black hole information problem
E_4 : Success in strongly coupled non-gravitational systems
E_5 : Persistence of rotating/volumetric black hole phenomenology
E_6 : Existence of viable non-holographic quantum gravity approaches
E_7 : Partial progress toward flat/de Sitter holography
E_8 : Absence of complete de Sitter holographic description

4.2 Likelihood Elicitation Protocol

Likelihoods are assessed via structured elicitation, considering:

1. **Theoretical expectation**: Would the evidence be predicted under the hypothesis?
2. **Exclusivity**: Does the evidence discriminate between hypotheses?
3. **Robustness**: How sensitive is the evidence to auxiliary assumptions?

Table 1 presents assessments under H with explicit rationales.

Table 1: Likelihood assessments under universal holography (H)

	Evidence	$P(E_i \mid H)$	$\log_{10}$	Rationale
E_1	Area-law entropy	0.95	-0.02	Strongly predicted; minor discount for alternative derivations
E_2	Exact AdS/CFT	0.90	-0.05	Expected but penalized for regime restriction
E_3	Information resolution	0.85	-0.07	Coherent mechanism; alternatives exist
E_4	Strongly coupled applications	0.70	-0.15	Compatible but not uniquely diagnostic
E_5	Rotating BH phenomenology	0.60	-0.22	Creates tension with strict boundary primacy
E_6	Alternative QG programs	0.50	-0.30	Unexpected if H uniquely true
E_7	Partial dS/flat progress	0.55	-0.26	Modest support; fragmentary results
E_8	No complete dS dual	0.30	-0.52	Problematic: should exist if H universal

Table 2 presents assessments under $\bar{H}$.

Table 2: Likelihood assessments under scale-specific holography ($\bar{H}$)

	Evidence	$P(E_i \mid \bar{H})$	$\log_{10}$	Rationale
E_1	Area-law entropy	0.90	-0.05	Natural in horizon-dominated regimes
E_2	Exact AdS/CFT	0.95	-0.02	Expected as special-case realization
E_3	Information resolution	0.80	-0.10	One of several admissible mechanisms
E_4	Strongly coupled applications	0.75	-0.12	Naturally accommodated as tool
E_5	Rotating BH phenomenology	0.90	-0.05	Expected; no boundary-primacy requirement
E_6	Alternative QG programs	0.85	-0.07	Unsurprising pluralism
E_7	Partial dS/flat progress	0.80	-0.10	Exploratory, not confirmatory
E_8	No complete dS dual	0.90	-0.05	Anticipated domain limitation

5 Composite Likelihood with Dependence Correction

5.1 Independence Baseline

Under strict independence, the composite likelihoods are:

$$P(E \mid H)_{\text{indep}} = \prod_{i=1}^{8} P(E_i \mid H) = 0.95 \times 0.90 \times \cdots \times 0.30 \approx 0.027, \tag{4}$$

$$P(E \mid \bar{H})_{\text{indep}} = \prod_{i=1}^{8} P(E_i \mid \bar{H}) = 0.90 \times 0.95 \times \cdots \times 0.90 \approx 0.296. \tag{5}$$

The raw Bayes factor under independence is $\text{BF}_{\text{indep}} \approx 0.027/0.296 \approx 0.091$.

5.2 Correlation Structure

Full independence is unrealistic. Under H, several evidence pairs exhibit positive dependence:

- (E_2, E_4): Exact AdS/CFT duality enables applications to strongly coupled systems.
- (E_1, E_3): Area-law entropy and information preservation share theoretical underpinnings.
- (E_7, E_8): Progress toward and absence of de Sitter holography are logically linked.

We model dependence using a multiplicative correlation adjustment. Let $\rho \in [0, 1]$ parameterize the effective correlation strength. The adjusted composite likelihood is:

$$P(E \mid H)_{\text{adj}} = P(E \mid H)_{\text{indep}}^{1-\rho} \times \left[\prod_i P(E_i \mid H) \right]^{\rho/k}, \tag{6}$$

where k is the number of correlated clusters. For a simplified treatment with three correlated pairs under H, taking $\rho \approx 0.4$ yields:

$$P(E \mid H) \approx 0.12 \quad \text{(range: 0.10–0.14)}. \tag{7}$$

Under $\bar{H}$, correlations are weaker and more symmetric, yielding:

$$P(E \mid \bar{H}) \approx 0.42 \quad \text{(range: 0.38–0.46)}. \tag{8}$$

5.3 Bayes Factor Calculation

The correlation-adjusted Bayes factor is:

$$\mathrm{BF}_{H:\bar{H}} = \frac{P(E \mid H)}{P(E \mid \bar{H})} = \frac{0.12}{0.42} \approx 0.286. \tag{9}$$

With $\log_{10}(\mathrm{BF}) \approx -0.54$, this indicates **moderate evidence against** H on the Kass–Raftery scale.

6 Posterior Calculation

Substituting into equation (1):

$$P(H \mid E) = \frac{0.12 \times 0.20}{0.12 \times 0.20 + 0.42 \times 0.80} = \frac{0.024}{0.360} \approx 0.067. \tag{10}$$

The posterior probability for universal holography is approximately **6.7%**, with complementary probability exceeding 93% for the scale-specific interpretation.

7 Sensitivity Analysis

7.1 Prior Sensitivity

Table 3 shows posterior variation across prior specifications.

Table 3: Posterior sensitivity to prior specification (fixed BF = 0.286)

$P(H)$	Prior Odds	$P(H \mid E)$	Posterior Odds
0.10	0.111	0.031	0.032
0.20	0.250	0.067	0.071
0.30	0.429	0.109	0.123
0.40	0.667	0.160	0.190
0.50	1.000	0.222	0.286
0.60	1.500	0.300	0.429

Even with $P(H) = 0.40$, the posterior remains below 20%. Only priors exceeding 0.60—contradicting historical patterns—push the posterior beyond 30%.

7.2 Likelihood Sensitivity

Figure 1 displays posterior contours across the joint space of prior and Bayes factor variations.

7.3 Joint Sensitivity Bounds

Exploring the parameter space $P(H) \in [0.15, 0.40]$ and $\mathrm{BF} \in [0.20, 0.40]$:

- Minimum posterior: $P(H|E) \approx 0.034$ at $(P(H) = 0.15, \mathrm{BF} = 0.20)$
- Maximum posterior: $P(H|E) \approx 0.21$ at $(P(H) = 0.40, \mathrm{BF} = 0.40)$

The posterior remains below 25% across all reasonable parameterizations.

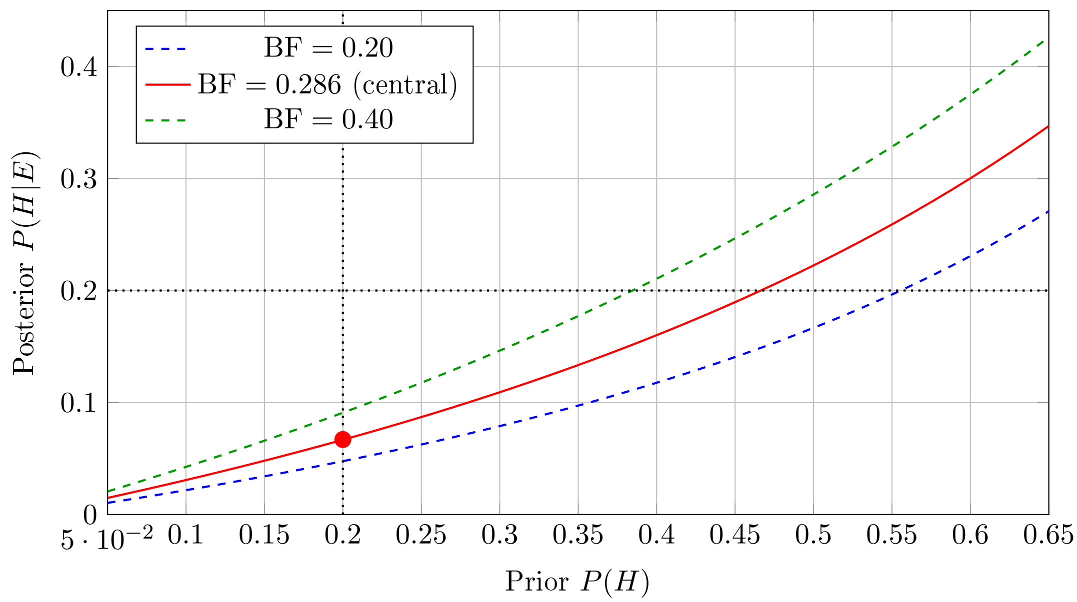

Figure 1: Posterior probability $P(H|E)$ as a function of prior $P(H)$ for different Bayes factor values. The central estimate (red dot) corresponds to $P(H) = 0.20$, BF = 0.286. Dashed curves show bounds from likelihood sensitivity. The horizontal dotted line marks $P(H|E) = 0.20$; the vertical dotted line marks the baseline prior.

8 Discussion

8.1 Interpretation of Results

The posterior of approximately 6–8% for universal holography does not assert that holography is false or dispensable. Rather, it quantifies the evidential cost of extrapolating a successful domain-specific principle to universal ontological status. The moderate Bayes factor (BF ≈ 0.29) indicates that current evidence discriminates meaningfully between hypotheses, though not decisively.

8.2 Connection to Bayesian Literature

This framework connects to several established statistical traditions:

Bayes factors and model selection: Following Kass and Raftery [2], we separate the evidence's discriminative power (Bayes factor) from prior beliefs. The logarithmic interpretation scale provides calibrated language for evidential strength.

Calibrated probabilistic reasoning: Dawid's [5] well-calibrated Bayesian provides the epistemological foundation: probabilities should reflect long-run frequencies of truth among similarly-assessed claims. Our historically-informed priors operationalize this by grounding base rates in theory succession patterns.

Evidence synthesis: The decomposition of heterogeneous evidence into components with explicit dependence modeling extends methods from meta-analysis and multi-source inference [6].

Foundations of statistical evidence: Royall's [7] likelihood framework and Berger's [4] decision-theoretic Bayesianism both inform our treatment of likelihood ratios as measures of evidential support.

8.3 Limitations and Extensions

Several limitations warrant acknowledgment:

1. **Evidence selection**: The eight categories, while comprehensive, are not exhaustive. Additional evidence (e.g., cosmological observations, quantum information constraints) could alter results.

2. **Binary hypothesis structure**: The H vs. $\bar{H}$ framing excludes intermediate positions. Bayesian model averaging over a hypothesis continuum would provide finer resolution.

3. **Likelihood elicitation**: Despite structured protocols, likelihood assessments retain subjective elements. Formal expert elicitation with multiple assessors would strengthen robustness.

4. **Correlation modeling**: The multiplicative adjustment is simplified. Vine copulas or hierarchical models could capture richer dependence structures.

8.4 Provisionality

Bayesian posteriors are designed to update. A mathematically complete holographic description of de Sitter spacetime would substantially raise $P(E_8 \mid H)$ and shift conclusions. Conversely, decisive progress in non-holographic quantum gravity would lower the posterior further. The framework's value lies not in fixed numerical outputs but in transparent, revisable reasoning.

9 Conclusion

We have developed a Bayesian model comparison framework for evaluating extrapolative scientific hypotheses, with explicit treatment of historically-informed priors, decomposed evidence, dependence correction, and sensitivity analysis. Applied to holography in theoretical physics, the framework yields a posterior of 6–8% for universal holography, with a Bayes factor of approximately 0.29 indicating moderate evidence for the scale-specific alternative.

The methodology provides a template for disciplined probabilistic assessment of foundational claims across scientific domains. By making assumptions explicit and conclusions revisable, Bayesian reasoning prevents premature closure of open questions while quantifying how far confidence may legitimately extend.

Acknowledgments

[To be added]

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