Swept-Volume Geometry and Overlap Corrections in Protostellar Binary Accretion:

A Kinematic Upper Bound and Its Limitations

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January 2026

Abstract

We present a purely geometric–kinematic analysis of material encounter capacity in protostellar binary systems. Treating accretion as a swept-volume problem, we derive an upper-bound kinematic factor $\eta_{\rm kin} = \sqrt{1 + (v_{\rm orb}/v_{\rm bulk})^2}$, arising from the helical trajectories of orbiting components relative to bulk motion. For comparable orbital and bulk velocities, this yields $\eta_{\rm kin} \approx \sqrt{2}$. We emphasize that this result is not a physical accretion law but a geometric constraint on encounter capacity, which must be modulated by gravitational, hydrodynamic, and thermodynamic capture efficiencies. Using an area-conserving normalization and explicit overlap geometry, we show that for sufficiently close binaries swept-volume overlap eliminates any kinematic advantage, with a critical separation $a_{\rm crit} \simeq 0.66\, r$ for $\eta_{\rm kin} = \sqrt{2}$. We further explain why hydrodynamic simulations do not exhibit a clean $\sqrt{2}$ signature: reported enhancements are dominated by dynamical mechanisms rather than encounter geometry. The framework is intended as a diagnostic bookkeeping tool that bounds what kinematics alone can contribute before physical capture processes take over.

1 Motivation and Scope

Binary protostars frequently exhibit enhanced mass growth compared to isolated objects, both in simulations and observations (Bate, 2000; Offner & Krumholz, 2023). Most explanations emphasize gravitational focusing, disk-mediated inflows, streamers, and competitive accretion (Bonnell et al., 2001; Alves et al., 2019; Yen et al., 2019). The present work does not attempt to model these processes.

Instead, we isolate a logically prior question: how much material can a moving system *encounter* purely by virtue of its trajectory through space? The analysis is intentionally kinematic and geometric. It establishes an upper bound on encounter capacity, not a prediction of accreted mass.

While relative-velocity enhancements appear in some ballistic or Bondi–Hoyle treatments of binary accretion, we are not aware of a prior explicit derivation of this kinematic factor as a purely geometric upper bound on encounter capacity, bounded by overlap geometry, prior to invoking dynamical capture.

2 Kinematic Encounter Factor

Consider a system traversing a gaseous environment with bulk velocity $v_{\rm bulk}$ over a residence time t. A single object follows a straight path of length

$$L_{\rm single} = v_{\rm bulk} t. \tag{1}$$

In a binary system, each component additionally executes orbital motion with velocity $v_{\rm orb}$ about the center of mass, tracing a helical trajectory. The path length of each component is

$$L_{\rm comp} = t\sqrt{v_{\rm bulk}^2 + v_{\rm orb}^2}. \tag{2}$$

The ratio of path lengths defines a purely kinematic enhancement factor

$$\eta_{\rm kin} = \frac{L_{\rm comp}}{L_{\rm single}} = \sqrt{1 + \left(\frac{v_{\rm orb}}{v_{\rm bulk}}\right)^2}. \tag{3}$$

This result follows directly from vector addition of velocities and should be understood as dimensional analysis applied to trajectory geometry. For $v_{\rm orb} \sim v_{\rm bulk}$, one obtains $\eta_{\rm kin} \approx \sqrt{2}$.

Typical embedded binaries exhibit orbital velocities of order 1–10 $\rm km\,s^{-1}$, while turbulent motions in dense cores are typically 0.5–2 $\rm km\,s^{-1}$ (Tobin et al., 2016; Offner & Krumholz, 2023), placing many systems in the regime $v_{\rm orb}/v_{\rm bulk} \sim 0.5$–5. For tighter binaries ($a \lesssim 100$–300 au), orbital velocities can exceed bulk motions by factors of several, yielding $\eta_{\rm kin} \gtrsim 3$, though overlap effects then become increasingly important and suppress the net geometric contribution.

3 Normalization and What It Does (and Does Not) Mean

To compare binary and single architectures within a swept-volume picture, a normalization choice is unavoidable. Here we adopt an *area-conserving* normalization,

$$\pi R_0^2 = 2\pi r^2, \tag{4}$$

so that the total instantaneous encounter area is held fixed while isolating the effect of trajectory length.

This choice remains a modeling assumption rather than a physical prescription. It is adopted solely to prevent trivial size effects from dominating a kinematic comparison. Alternative scalings lead to different conclusions. For example, a Bondi-like scaling $r \propto M$ yields $\eta_{\rm kin} \approx \sqrt{2}/2 \approx 0.71$, corresponding to a net geometric deficit. This sensitivity underscores that the present analysis explores geometric scaling relationships rather than identifying a physically preferred capture law.

4 Overlap of Swept Volumes

The kinematic advantage implied by longer trajectories can be reduced or eliminated by overlap of swept regions. We approximate the time-averaged effective cross-section using the instantaneous union area of two circles, neglecting phase-dependent variations along the helical paths; this approximation is valid when the orbital period is short compared to the residence time through the medium.

For two components of radius r separated by distance a, the instantaneous union area is

$$A_{\rm union} = 2\pi r^2 - 2r^2 \cos^{-1}\left(\frac{a}{2r}\right) + \frac{a}{2}\sqrt{4r^2 - a^2}, \quad (a < 2r). \tag{5}$$

Including the kinematic factor, the effective geometric enhancement becomes

$$\eta_{\rm eff}(a) = \eta_{\rm kin}\,\frac{A_{\rm union}}{\pi R_0^2}. \tag{6}$$

Setting $\eta_{\rm eff} = 1$ with $\eta_{\rm kin} = \sqrt{2}$ and solving numerically for a/r yields a critical separation

$$a_{\rm crit} \simeq 0.66\, r, \tag{7}$$

or more precisely $a/r \approx 0.659$, below which overlap completely erases any kinematic advantage.

5 Why Simulations Do Not Show a Clean $\sqrt{2}$sqrt(2) Signature

Hydrodynamic simulations typically report accretion enhancements of factors 2–3 for binaries (Bate, 2000; Young et al., 2015). These enhancements are not inconsistent with the present bound. Rather, they are dominated by gravitational focusing, disk-mediated inflows, streamers, and episodic accretion bursts (Bonnell et al., 2001; Alves et al., 2019; Yen et al., 2019; Stamatellos et al., 2012).

The kinematic factor derived here is expected to be subdominant and often masked. Simulations do not isolate encounter geometry independently of capture efficiency, and there is no reason to expect a clean $\sqrt{2}$ signature to appear when dynamical processes vary by order unity or more.

6 Diagnostic Interpretation

The total accretion enhancement may be written schematically as

$$\frac{\dot{M}_{\rm bin}}{\dot{M}_{\rm single}} = \eta_{\rm eff}(a)\,\frac{\epsilon_{\rm bin}}{\epsilon_{\rm single}}, \tag{8}$$

where ϵ represents physical capture efficiency. If observed or simulated enhancements greatly exceed $\eta_{\rm eff}$, the excess must arise from dynamics rather than geometry. Conversely, $\eta_{\rm eff}$ provides an upper bound on what kinematics alone can supply.

7 Limitations

This work neglects gravitational binding, pressure support, magnetic fields, depletion of the ambient medium, and time-dependent feedback. Residence times and velocity ratios are treated parametrically. The analysis does not predict accretion rates and should not be interpreted as a physical model of protostellar growth.

8 Conclusions

Orbital motion increases encounter path length and therefore defines a kinematic upper bound on swept volume. When normalized to fixed encounter area and corrected for overlap, this contribution is modest and easily suppressed. The absence of a clean geometric signature in simulations is expected and indicates that physical capture processes dominate. The primary utility of the framework is diagnostic: it illustrates how orbital motion contributes to geometric encounter bounds before invoking dynamics.

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