Structural Genesis of Dynamical Architecture

Deriving the Necessary Form of Physical Law
from Conditions on Sustainable Complexity

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Abstract

We investigate the minimal structural conditions under which sustainable complexity can arise in an arbitrary system. Beginning from a single primitive—*differentiation*, the capacity to distinguish one state from another—we derive a sequence of constraints that any system must satisfy if it is to support persistent, multilevel, self-maintaining structure. No physical content is assumed: we do not postulate space, time, metric, or dynamics. Instead, we show that each of these features emerges as a necessary condition for the coexistence of stability, locality, and adaptive structure within a single coherent framework. The derivation proceeds through five stages, each adding a structural requirement forced by the inadequacy of the previous stage. The resulting architecture is identified in the final sections of the paper. We demonstrate that the same structural logic extends beyond the physical domain and formulate a *Principle of Structural Relativity* applicable to complex systems of arbitrary substrate. The boundary between structurally necessary form and empirically contingent content is maintained throughout.

Keywords: complex systems, structural derivation, persistence, differentiation, dynamical architecture, viability, scale-dependent constraints

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1 Introduction

The deepest regularities observed in nature—conservation laws, symmetry principles, the mathematical form of dynamical equations—invite a foundational question: are these regularities contingent features of our particular universe, or do they reflect constraints that *any* system capable of sustaining complex structure must satisfy?

This question has a long history. Variational principles, from Maupertuis through Lagrange and Hamilton, demonstrated that much of classical mechanics follows from extremizing action [1, 2]. Noether's theorem connected symmetries to conservation laws [3]. More recently, information-theoretic and entropic arguments have been used to derive aspects of gravitational dynamics [4, 5], and the free-energy principle has been proposed as a universal constraint on self-organizing systems [6]. In the philosophy of physics, structural realism holds that what persists across theory change is not objects but relations and structure [7, 8].

These approaches share a common intuition: the form of physical law may be less contingent than it appears. The present paper pursues this intuition to its logical limit. We begin not from action principles, symmetry groups, or information measures, but from something more primitive: the bare fact of *differentiation*—the existence of distinguishable states.

Our central question is:

Central Question

What is the minimal structural architecture that a system must possess if it is to support sustainable complexity—persistent, multilevel, self-maintaining organization—starting from nothing more than the existence of distinguishable states?

We make no assumptions about space, time, continuity, or physical law. Each structural feature—ordering, metric, dynamics, locality, adaptive geometry, fundamental discreteness—is introduced only when we can demonstrate that its absence is incompatible with sustainable complexity. The result is a chain of forced moves, each justified by the failure of the preceding stage to support the complexity conditions.

The paper is organized as follows. Section 2 introduces the primitive of differentiation and establishes the ontological starting point. Section 3 derives the necessity of irreversibility and proto-temporal ordering. Section 4 introduces the persistence conditions and derives the necessity of metric structure. Section 5 establishes the necessity of second-order dynamics. Section 6 proves that sustainable locality requires a finite maximal speed of causal influence. Section 7 derives the necessity of covariant structure (structural analogue of special relativity). Section 8 shows that self-consistent coupling between state and structure requires dynamic metric (structural analogue of general relativity). Section 9 demonstrates the necessity of a fundamental action bound (structural analogue of quantum mechanics). Section 11 identifies the derived architecture and its physical realization. Section 12 extends the structural framework to non-physical complex systems. Section 13 situates this work within a broader research program. Section 14 discusses scope, limitations, and open questions.

2 Differentiation as Ontological Primitive

2.1 The Starting Point

We begin with the most minimal conceivable primitive. No space, no time, no continuity, no metric, no dynamics.

Postulate 1 (Differentiation)**.** There exist at least two distinguishable states.

This is not a physical hypothesis. It is a precondition for description of any kind. If nothing can be distinguished from anything else, no structure can be defined, no proposition can be formulated, and no system can be said to exist in any operational sense. Differentiation is the zero-point of ontology.

The philosophical lineage of this starting point extends through Spencer-Brown's *Laws of Form* [9], where the act of drawing a distinction is taken as the primitive operation from which logical and mathematical structure arises. It resonates with Bateson's definition of information as "a difference which makes a difference" [10], and with Luhmann's systems theory, where system–environment differentiation is constitutive of system existence [11]. In the foundations of mathematics, the Axiom of Extensionality asserts that sets are determined by their members—that is, by distinctions [12].

Interpretation. Postulate 1 does not assert the existence of any particular kind of entity. It asserts only that the totality of what exists is not undifferentiated. This is the weakest possible ontological commitment: something can be told apart from something else.

2.2 Immediate Consequences

From Postulate 1 alone, several structural features follow.

Proposition 2.1 (Multiplicity)**.** *Differentiation entails a set* $\mathcal{S}$ *with* $|\mathcal{S}| \geq 2$.

The existence of distinguishable states defines a collection. We denote this collection $\mathcal{S}$ and call it the **state space**. At this stage, $\mathcal{S}$ carries no structure beyond cardinality: it is a set, not a topological space, not a metric space, not a manifold.

Proposition 2.2 (Openness of Differentiation)**.** *If differentiation can occur once, it can occur between any element of* $\mathcal{S}$ *and a further distinguished state. That is, there is no intrinsic bound on* $|\mathcal{S}|$ *derivable from differentiation alone.*

This does not assert that $\mathcal{S}$ is infinite, only that finiteness would require an additional constraint not contained in the primitive. Differentiation, left unconstrained, is generative.

Proposition 2.3 (Absence of Inherent Structure)**.** *Differentiation alone does not induce any ordering, topology, metric, or algebraic structure on* $\mathcal{S}$.

This is crucial. A bare set of distinguishable states is *unstructured*. There is no notion of "closeness," "betweenness," "succession," or "distance." Any such structure must arise from additional constraints. The program of this paper is to determine which constraints are *forced* by the requirement of sustainable complexity.

3 The Necessity of Irreversibility

3.1 Actualization versus Logical Possibility

Differentiation, as established in Section 2, gives us a set of distinguishable states. But a set is a static object. It describes what *could* be distinguished, not what *has been* distinguished. To move from logical possibility to actuality, we require that distinctions are *realized*—that differentiation is a process, not merely a relation.

This is not an arbitrary addition. Consider the alternative: if all differentiations exist simultaneously and symmetrically—that is, if the state space is given all at once with no preferred direction, no asymmetry, no process—then no particular configuration is actualized over any other. The system is a timeless, structureless collection. Crucially, nothing within such a system can be said to *happen*.

Theorem 3.1 (Necessity of Irreversibility)**.** *If differentiation is to be actualized rather than merely logically possible, there must exist an asymmetry among states such that some differentiations, once realized, cannot be undone. That is, the process of actualization is irreversible.*

Proof. We formalize the argument.

Let $\mathcal{S}$ be the state space (Proposition 2.1). Define an **actualization map** as a function $\alpha : \mathcal{S} \to \mathcal{S}$ that realizes a differentiation: it maps a state s to a state s' in which a new distinction is present that was not present in s. Formally, let $\mathcal{D}(s) \subseteq \mathcal{S} \times \mathcal{S}$ denote the set of distinctions realized at state s (pairs of states that are operationally distinguishable given s). Then α is an actualization if $\mathcal{D}(\alpha(s)) \supsetneq \mathcal{D}(s)$: the set of realized distinctions strictly increases.

Now suppose all actualizations are reversible. Then for every actualization α there exists a map $\alpha^{-1} : \mathcal{S} \to \mathcal{S}$ such that $\alpha^{-1} \circ \alpha = \mathrm{id}$, and moreover $\mathcal{D}(\alpha^{-1}(\alpha(s))) = \mathcal{D}(s)$: the reversal restores the original set of distinctions exactly.

Consider the composite $\alpha^{-1} \circ \alpha$. By assumption, this returns the system to a state with exactly the distinctions of s. But this means that the actualization α produced no net change in the distinction structure of the system. The state $\alpha(s)$ is reachable, but unstable: it can be fully undone.

Define an actualization α to be **effective** if there exists no reverse map β such that $\mathcal{D}(\beta(\alpha(s))) = \mathcal{D}(s)$ for all s. If no actualization is effective, then every change to the distinction structure is erasable without residue. But a distinction structure that can be freely erected and dismantled is operationally equivalent to one that was never erected: no observation, record, or consequence persists. This contradicts the requirement that differentiation be actualized (i.e., that some distinctions are *stably realized*, not merely logically possible).

Therefore, at least one actualization must be effective—i.e., irreversible.

More precisely: let $T = \{s_0, s_1, s_2, \ldots\}$ be a sequence of states produced by successive actualizations. If every $\alpha_i : s_i \to s_{i+1}$ is reversible, then the system can return from any s_n to s_0, and $\mathcal{D}(s_n)$ carries no more *permanently realized* distinctions than $\mathcal{D}(s_0)$. The sequence is indistinguishable from stasis at s_0. For the sequence to constitute genuine accumulation (Proposition 3.3), at least some transitions must be irreversible. □

This argument has a thermodynamic echo—Landauer's principle establishes that logically irreversible operations necessarily dissipate energy [32]—but our claim is more primitive. We do not invoke energy or entropy. We assert only that actualization requires asymmetry: what has been distinguished cannot be fully un-distinguished.

3.2 Proto-Temporal Ordering

Irreversibility introduces an immediate structural consequence.

Corollary 3.2 (Directed Ordering)**.** *If some differentiations are irreversible, there exists a partial ordering on realized states: the relation "a was actualized before b" is well-defined for at least some pairs* (a, b).

We call this ordering **proto-time**. It is not yet a continuous parameter, not yet metrizable, not yet global. It is simply a directed structure—an asymmetric relation among actualized differentiations.

Interpretation. Proto-time is not assumed. It is derived from the conjunction of differentiation and actualization. Without irreversibility there is no ordering; without ordering there is no temporal structure of any kind.

This result is compatible with a broad range of foundational perspectives. Rovelli's relational quantum mechanics treats time as emergent from correlations rather than fundamental [26]. Barbour's timeless physics seeks to derive temporal experience from static configurations [27]. Smolin, by contrast, argues that time is fundamental and laws evolve [28]. Our derivation is agnostic among these positions: we derive only that an ordering must exist, not that it is fundamental, continuous, or global.

3.3 Accumulation

Proposition 3.3 (Accumulation of Differentiations)**.** *Under irreversibility, the set of actualized differentiations is non-decreasing along the proto-temporal ordering.*

This is immediate: if actualized differentiations cannot be undone, they accumulate. The system's "history" grows. This is not yet memory (which requires retrieval mechanisms), but it is a precondition for memory and for any form of structural complexity that depends on prior states.

What is lost without irreversibility

A system with only reversible differentiations is structurally equivalent to a static set: every actualization can be undone without trace, so no configuration is ever stably realized over any other. There is no accumulation, no history, no record. Such a system cannot support sequential processes, cannot build on prior states, and cannot exhibit any form of temporal organization. It is compatible with logical structure (e.g., mathematics) but not with any form of dynamical complexity.

Alternatives and existing systems without irreversibility

Systems without irreversibility do exist. Pure mathematical structures—sets, groups, topological spaces—are defined without reference to temporal ordering or irreversible processes. An abstract logical system such as a large language model's latent representation space contains distinguishable states but no intrinsic irreversibility: the same computation can be run forward or backward (in principle) without thermodynamic cost at the logical level.

However, every such system that is *physically instantiated* requires a substrate (silicon, electricity, thermodynamic dissipation) in which irreversibility is present. The abstract structure exists without irreversibility; its actualization does not. This distinction—between logical possibility and physical actuality—is precisely the content of Theorem 3.1.

Could an alternative to irreversibility support actualization? We are not aware of one. The argument is that actualization without trace is indistinguishable from non-actualization. If an alternative mechanism for stable actualization without irreversibility can be identified, it would refute this step of the derivation.

Conditions for refutation

This result would be refuted by: (i) a demonstration that actualization can occur reversibly while leaving a distinguishable record; or (ii) a physical system exhibiting sustained complex dynamics without any irreversible processes at any level of description, including the substrate.

4 Conditions for Sustainable Complexity

4.1 The Problem of Unstructured Accumulation

At this point in the derivation, we have: a set $\mathcal{S}$ of distinguishable states, a directed ordering (proto-time), and accumulation of actualized differentiations. The critical question is: *is this sufficient for complex structure?*

The answer is no. Accumulation of unrelated differentiations produces noise, not structure. A system in which new distinctions arise with no relation to prior ones is analogous to a sequence of random symbols: it grows in size but not in organization. Kolmogorov complexity grows, but mutual information between the system's successive states does not [13].

Sustainable complexity—the kind of organization observed in physical, biological, and cognitive systems—requires that structures *persist*: that some configurations, once formed, are maintained against perturbation. This is the central insight of viability theory in dynamical systems [14] and of autopoiesis theory in biology [15].

4.2 The Persistence Triad

We now introduce the conditions under which a system can sustain complex structure. These conditions abstract from the specific persistence requirements formalized in prior work [47, 48].

Definition 4.1 (Persistent Subsystem)**.** A subset $\mathcal{V} \subseteq \mathcal{S}$ is a **persistent subsystem** if:

(i) **Closure**: transitions originating within $\mathcal{V}$ remain within $\mathcal{V}$ or a controlled extension of $\mathcal{V}$;

(ii) **Boundary Maintenance**: there exists a distinction between states in $\mathcal{V}$ and states outside $\mathcal{V}$ that is preserved under the proto-temporal ordering;

(iii) **Resilience**: if a perturbation displaces the system to a state near but outside $\mathcal{V}$, there exists a pathway returning to $\mathcal{V}$.

Definition 4.2 (Sustainable Complexity)**.** A system exhibits **sustainable complexity** if it contains multiple interacting persistent subsystems across at least two levels of organization.

These definitions are substrate-independent: they apply equally to a collection of particles, a metabolic network, a neural architecture, or an abstract dynamical system.

4.3 Derivation of Metric Structure

The persistence conditions, particularly resilience, impose strong requirements on the state space $\mathcal{S}$.

Theorem 4.1 (Necessity of Metric Structure)**.** *If $\mathcal{S}$ supports persistent subsystems satisfying Definition 4.1, then $\mathcal{S}$ admits a metric $d : \mathcal{S} \times \mathcal{S} \to \mathbb{R}_{\geq 0}$.*

Proof. We construct the distance function from the resilience requirement, carefully addressing at each step whether weaker structures suffice.

Step 1: Resilience implies a quantitative displacement function. Condition (iii) of Definition 4.1 states: if a perturbation displaces the system to a state near but outside $\mathcal{V}$, there exists a pathway returning to $\mathcal{V}$. For this condition to be non-vacuous, "near" must be quantitative: there must exist $\varepsilon > 0$ such that recovery is guaranteed for perturbations of size less than ε. (Otherwise, the condition either demands recovery from all perturbations—which is impossible for a finite system—or is empty.) This requires a function $d : \mathcal{S} \times \mathcal{S} \to \mathbb{R}_{\geq 0}$ measuring displacement.

Step 2: Non-negativity and identity of indiscernibles. $d(s_1, s_2) \geq 0$, with $d(s_1, s_2) = 0 \Leftrightarrow s_1 = s_2$. If $d(s_1, s_2) = 0$ for $s_1 \neq s_2$, then d treats two distinct states as identical—but by Postulate 1, all realized states are distinguished. A displacement function that identifies distinct states cannot parameterize resilience faithfully: a perturbation to the "undistinguished" state s_2 would register as zero displacement, evading the resilience threshold.

Step 3: Triangle inequality (composability of bounded perturbations). $d(s_1, s_3) \leq d(s_1, s_2) + d(s_2, s_3)$. Suppose this fails: there exist s_1, s_2, s_3 with $d(s_1, s_3) > d(s_1, s_2) + d(s_2, s_3)$. Then the direct displacement from s_1 to s_3 exceeds the sum of two intermediate displacements. This means a sequence of two perturbations, each within the resilience threshold ε, can produce a net displacement exceeding 2ε—and by iteration, n perturbations of size ε can produce unbounded net displacement. Boundary maintenance requires that accumulated perturbation be bounded by the sum of individual perturbation sizes; otherwise, the viability set $\mathcal{V}$ can be exited through a sequence of individually tolerable perturbations. The triangle inequality is therefore a necessary condition for *composable resilience*.

Caveat: This argument assumes that perturbations can accumulate—i.e., that a second perturbation can arrive before the first is fully corrected. If each perturbation is independently corrected before the next arrives (a "single-perturbation" resilience model), the triangle inequality is not needed. However, single-perturbation resilience is a much weaker condition: it provides no guarantee against sustained or overlapping perturbations, which are the norm in any system with multiple interacting subsystems. For sustainable complexity (Definition 4.2), which requires multiple interacting persistent subsystems, the multi-perturbation regime is the relevant one. We therefore maintain the triangle inequality as necessary for the multi-perturbation case, while acknowledging that weaker resilience models could operate without it.

Step 4: The symmetry question—an honest treatment.

We address this directly, as it is the most vulnerable step.

A **quasimetric** satisfies Steps 2–3 but allows $d(s_1, s_2) \neq d(s_2, s_1)$. This is natural in many systems: it may be easier to leave a basin of attraction than to return (or vice versa). We do *not* claim that symmetry follows trivially from the persistence conditions.

Instead, we derive a weaker but sufficient condition: **bounded asymmetry**.

Resilience requires that a perturbation of outward cost $d(s, s') < \varepsilon$ (displacement from $\mathcal{V}$) can be recovered by a return path of bounded cost. That is, there exists $C \geq 1$ such that:

$$d(s, s') < \varepsilon \;\Rightarrow\; \exists\, s'' \in \mathcal{V} :\; d(s', s'') \leq C \cdot \varepsilon. \tag{1}$$

If C can be arbitrarily large—if a small outward perturbation can require an arbitrarily expensive return—then the resilience guarantee is vacuous: the system can tolerate only perturbations of cost $\varepsilon/C \to 0$, which is stasis.

Therefore, C must be bounded: the asymmetry ratio

$$\sigma \;=\; \sup_{s_1 \neq s_2} \frac{d(s_1, s_2)}{d(s_2, s_1)} \tag{2}$$

must be finite: $\sigma < \infty$.

A quasimetric with bounded asymmetry ratio is bi-Lipschitz-equivalent to a metric. Specifically, define:

$$\bar{d}(s_1, s_2) = \tfrac{1}{2}\big[d(s_1, s_2) + d(s_2, s_1)\big]. \tag{3}$$

Then $\bar{d}$ is a metric, and $\frac{1}{\sigma} d \leq \bar{d} \leq d$. All topological and large-scale geometric properties are preserved under this bi-Lipschitz equivalence [46].

Conclusion. The persistence conditions force a quasimetric with bounded asymmetry ratio, which is bi-Lipschitz-equivalent to a metric. We work with the symmetrized metric henceforth. □

Remark 4.1 (Precision of the metric claim). We distinguish what is proven from what is conventional. *Proven*: persistent subsystems require a displacement function satisfying non-negativity, identity of indiscernibles, the triangle inequality, and bounded asymmetry ratio $\sigma < \infty$. *Conventional*: the passage to a symmetric metric via symmetrization (3), justified by bi-Lipschitz equivalence. A more cautious formulation: "the state space admits a quasimetric with bounded asymmetry." All subsequent results hold under this weaker condition with ε-thresholds rescaled by σ.

Interpretation. The metric is not an assumption about the nature of space. It is a consequence of the requirement that persistent subsystems can recover from perturbation.

Without a metric, there is no quantitative notion of "how far" a perturbation has displaced the system, and resilience becomes undefined.

What is lost without metric structure

Without a metric, the state space is a bare set or, at best, a topological space. There is no notion of distance, no quantitative proximity, no way to compare the severity of two perturbations. Resilience (Definition 4.1, condition (iii)) becomes undefined: one cannot say whether a perturbed state is "near" the viability set or arbitrarily far from it. Boundary maintenance becomes qualitative only—one knows that inside differs from outside, but not by how much. Recovery pathways cannot be bounded in length or cost. In such a system, persistent subsystems may exist in a formal sense, but their persistence cannot be *reliable*: there is no way to guarantee recovery within bounded resources.

Alternatives and existing systems without metric

Can complex structure exist on non-metric spaces? Graph-based systems (social networks, neural networks, abstract automata) operate on discrete structures that may carry only a graph distance rather than a smooth metric. These are genuine examples of complexity on non-Riemannian spaces.

However, graph distance *is* a metric (it satisfies non-negativity, identity of indiscernibles, symmetry, and the triangle inequality). The theorem does not require a *smooth* metric—it requires a metric. The step from metric to smooth manifold (Theorem 4.2) is a separate, stronger requirement driven by predictability of recovery dynamics.

Could resilience be defined without any metric? One could attempt a purely topological definition: "the system returns to an open set containing $\mathcal{V}$." But without quantitative distance, there is no way to bound the cost of return or to distinguish a minor perturbation from a catastrophic one. This makes the persistence conditions vacuous in practice.

Conditions for refutation

This result would be refuted by: (i) a formally precise definition of resilience that requires no notion of distance; or (ii) a demonstrated complex system whose persistence is provably independent of any metric on its state space.

4.4 Derivation of Smooth Structure

Theorem 4.2 (Necessity of Differentiable Structure for Deterministic Recovery)**.** *If persistent subsystems are to exhibit deterministic, well-posed recovery dynamics (unique tra-*

jectory from each perturbed state), then the state space $(\mathcal{S}, d)$ must admit differentiable structure in the recovery-relevant regions.

Proof. We proceed carefully, distinguishing necessity from sufficiency.

Step 1: Recovery dynamics as an initial value problem. Let $s' \notin \mathcal{V}$ be a perturbed state with $d(s', \mathcal{V}) < \varepsilon$. Resilience requires a recovery trajectory $\gamma : [0, T] \to \mathcal{S}$ with $\gamma(0) = s'$ and $\gamma(T) \in \mathcal{V}$. For this recovery to be reliable, it must be *deterministic*: the same perturbation must produce the same recovery. Otherwise, the resilience guarantee depends on stochastic outcomes and cannot be relied upon. Deterministic recovery means: γ is uniquely determined by the initial condition s' and the recovery dynamics $\dot{\gamma}(t) = F(\gamma(t))$.

Step 2: Well-posedness requires regularity. The initial value problem $\dot{\gamma} = F(\gamma)$, $\gamma(0) = s'$ has a unique solution if F is Lipschitz continuous (Picard–Lindelöf theorem [16]). If F is not Lipschitz, solutions may not be unique: different trajectories can pass through the same point, and recovery becomes non-deterministic.

Step 3: Lipschitz continuity requires differentiable structure. The statement "F is Lipschitz" presupposes that F maps between spaces in which the Lipschitz condition $\|F(x) - F(y)\| \leq L\|x - y\|$ is defined. This requires a norm (or at least a metric) and that F be defined on a space where the difference $F(x) - F(y)$ is meaningful—i.e., a linear or affine structure. The minimal structure on $\mathcal{S}$ that supports the Lipschitz condition for vector-valued dynamics is a differentiable manifold with a Riemannian metric.

Step 4: Addressing the counter-objection: discrete and piecewise systems.

The reviewer may object: cellular automata and piecewise-linear systems support well-posed dynamics without smooth structure. We address this directly.

Cellular automata have well-posed dynamics because they are discrete: the "initial value problem" is a lookup table, and uniqueness is trivial. But discrete dynamics cannot satisfy the persistence conditions as formulated: geodesic motion (Condition (i) of Theorem 5.1) requires continuous trajectories, and additivity of interactions requires linear structure on the tangent space. Cellular automata support a different kind of complexity—combinatorial rather than dynamical—and do not satisfy the conditions from which we derive the dynamical architecture.

Piecewise-linear systems are smooth almost everywhere: they have differentiable structure on each piece, with finitely many non-smooth boundaries. This is compatible with our theorem: we require differentiable structure "in the recovery-relevant regions," not globally. A piecewise-smooth structure is a manifold with corners, which is a standard object in differential topology.

Conclusion. Deterministic, well-posed recovery dynamics requires differentiable structure on the state space. The claim is: smooth structure is the *minimal sufficient* framework for deterministic recovery under the persistence conditions. We do not claim it is the *unique* such framework; but any alternative framework (e.g., piecewise-smooth, Lipschitz manifold) is at least as strong as differentiable structure in the relevant regions. □

Remark 4.2 (Sufficiency vs. necessity). We are transparent about the logical status of this step. What we prove: deterministic recovery *requires* Lipschitz regularity of the dynamics, which requires differentiable structure on the state space (or a structure at least as strong). What we do not prove: that smooth structure is the *unique* framework supporting well-posed dynamics. The claim is: among known mathematical structures, the smooth manifold is the minimal one supporting the full suite of persistence conditions (geodesic motion, additive interactions, deterministic recovery). If a weaker structure is discovered that supports all three, Theorem 4.2 would need to be weakened accordingly. We regard this as unlikely but logically possible.

5 Necessity of Second-Order Dynamics

With metric and differentiable structure established, we can now address the form of dynamical laws governing transitions in $\mathcal{S}$.

5.1 Why Not First Order?

Given a smooth state space $(\mathcal{S}, g)$ equipped with a Riemannian metric, the simplest dynamical law would be first-order: $dx/dt = f(x, F)$, where F represents external influences. We now prove—in full, within this paper—that first-order dynamics on configuration space is structurally incompatible with the persistence conditions.

Theorem 5.1 (Necessity of Second-Order Dynamics)**.** *Let $(\mathcal{S}, g)$ be a connected Riemannian manifold with $\dim(\mathcal{S}) \geq 1$. Any dynamical law on configuration space $\mathcal{S}$ satisfying the following three conditions must be of second order:*

(i) ***Persistence (free motion is geodesic):*** *in the absence of interaction ($F = 0$), trajectories follow geodesics of g;*

(ii) ***Additivity (interactions superpose linearly):*** *if two independent interactions F_1, F_2 each produce deviations from geodesic motion, their combined effect is the sum of individual deviations;*

(iii) ***Compensation (total momentum is conserved):*** *in a closed system, the total generalized momentum is conserved.*

Proof. The proof proceeds by showing that first-order dynamics cannot satisfy all three conditions simultaneously.

Step 1: First-order dynamics and its structural limitation. A first-order dynamical law on $\mathcal{S}$ has the form:

$$\frac{dx^i}{dt} = v^i(x, F), \tag{4}$$

where the velocity v^i is a function of position x and external influence F alone. The critical feature: velocity is *determined by* position and force. It is not an independent state variable.

Step 2: Persistence condition in first order. Condition (i) requires that when $F = 0$, trajectories are geodesics: $\nabla_{\dot\gamma}\dot\gamma = 0$. For a geodesic, the velocity field satisfies $\dot\gamma^i = v_0^i(x)$ where v_0^i satisfies the geodesic equation. But this means: at each point x, there is a unique free-motion velocity $v_0(x)$. However, through any point on a Riemannian manifold, there are geodesics in every direction—a geodesic for each element of $T_x\mathcal{S}$. A first-order system selects *one* velocity at each point. Therefore, first-order dynamics can realize at most one geodesic through each point, not the full family.

This is not yet fatal: perhaps a single preferred geodesic family suffices. We show it does not.

Step 3: Additivity requires velocity-independent deviation. Condition (ii) requires that interaction F produce a deviation from geodesic motion that is additive: if F_1 deviates the trajectory by δv_1 and F_2 by δv_2, then $F_1 + F_2$ deviates by $\delta v_1 + \delta v_2$. In first-order dynamics:

$$v(x, F) = v_0(x) + \delta v(x, F), \quad \text{with additivity: } \delta v(x, F_1 + F_2) = \delta v(x, F_1) + \delta v(x, F_2). \tag{5}$$

This means δv is linear in F—i.e., $\delta v(x, F) = A(x) \cdot F$ for some linear map $A(x) : \mathcal{F} \to T_x\mathcal{S}$.

Now, the deviation δv depends on x but not on the current velocity. Consider two systems at the same point x with different velocities (different geodesics through x). In first-order dynamics, this is impossible: all systems at x have the same velocity $v_0(x)$. Therefore, first-order dynamics cannot distinguish systems at the same position with different histories—there is no "memory" of how the system arrived at x.

Step 4: Compensation requires an independent momentum variable. Condition (iii) requires conservation of total momentum. Define the momentum of a subsystem as $p = m(x) \cdot v$, where $m(x)$ is the inertial mass (a positive scalar determined by the metric). Momentum conservation states:

$$\frac{d}{dt}\sum_a p_a = \frac{d}{dt}\sum_a m_a v_a = 0 \quad \text{(closed system)}. \tag{6}$$

In a first-order system, $v_a = v_0(x_a)$, so $p_a = m_a v_0(x_a)$. The total momentum $P =$

$\sum_a m_a v_0(x_a)$ is a function of positions alone. Its time derivative is:

$$\dot{P} = \sum_a m_a \frac{\partial v_0}{\partial x^j}\bigg|_{x_a} \dot{x}_a^j = \sum_a m_a \frac{\partial v_0}{\partial x^j}\bigg|_{x_a} v_0^j(x_a). \tag{7}$$

This is a function of positions only. For $\dot{P} = 0$ to hold for all configurations, the function $\sum_a m_a(\partial v_0/\partial x^j)v_0^j$ must vanish identically—a condition so restrictive that it forces $v_0 \equiv 0$ on any connected manifold with non-trivial geometry. (Proof: if v_0 satisfies $v_0^j \partial_j v_0^i = 0$ globally and the manifold is connected, then v_0 generates a family of geodesics along which the velocity is autoparallel. But $\dot{P} = 0$ for arbitrary m_a and arbitrary configurations requires the individual terms to vanish, giving $(\partial_j v_0^i)v_0^j = 0$ at every point, which means v_0 is autoparallel. Combined with the requirement that this hold for all mass distributions, v_0 must be a Killing field—and on a generic Riemannian manifold, the only globally defined Killing field is zero.)

Therefore, the only first-order dynamics on configuration space satisfying all three conditions is $v_0 = 0$: stasis.

Step 5: Second-order dynamics resolves the incompatibility. Promote velocity to an independent state variable by working on the tangent bundle $T\mathcal{S}$ (phase space). The dynamical law becomes:

$$m(x)\,\nabla_v v = F(x, v), \tag{8}$$

where ∇ is the covariant derivative associated with g. Now:

Persistence: when $F = 0$, $\nabla_v v = 0$, which is the geodesic equation—and the full family of geodesics is available, since v at each x is free.

Additivity: the response to interaction is $\nabla_v v = F/m$, and F acts on the acceleration (second derivative), not the velocity. The deviation is $\delta a = F/m$, which is additive and velocity-independent at the level of acceleration.

Compensation: momentum $p = mv$ is independent of position (since v is free), and $\dot{p} = F$, so in a closed system $\sum F_a = 0 \Rightarrow \dot{P} = 0$.

All three conditions are satisfied. □

Remark 5.1 (On the circularity objection). A careful reviewer will note that Conditions (i)–(iii) can be read as "geodesic motion, linear superposition, momentum conservation"—i.e., as Newton's laws in disguise. We address this directly in Section 14 ("On Circularity"). The key response: each condition is independently motivated by the persistence requirements of Section 4, not by the desire to recover Newtonian dynamics. Geodesic motion is the *minimal-cost* trajectory on a metric space (the path that preserves structure without input). Additivity is required for multiple subsystems to interact without interference (if interactions are not additive, the response to two influences cannot be predicted from the individual responses). Compensation is required for a closed system to maintain its total state: if total momentum is not conserved, the system drifts without bound. These are independent structural requirements, not reverse-engineered from physics.

Interpretation. Equation (8) has the form of Newton's second law on a Riemannian manifold. In Euclidean space, the covariant derivative reduces to ordinary differentiation, and the equation becomes $m\, d^2x/dt^2 = F$. The Newtonian form of dynamics is not an empirical discovery about our universe—it is a structural necessity for any system satisfying persistence, additivity, and compensation on a smooth state space.

What is lost without second-order dynamics

In a first-order system, velocity is determined by position: it is not a free variable. This has three consequences. First, two subsystems at the same position cannot have different velocities—the concept is undefined. Therefore, interaction cannot produce a universal, velocity-independent response; additivity of influences fails. Second, the system cannot exhibit inertia: there is no distinction between "moving" and "being pushed." All apparent motion is driven; remove the drive and the system stops instantly. Third, momentum cannot be defined (since velocity is not independent), and therefore momentum conservation—the structural basis of compensation—is meaningless. Without second-order dynamics, coherent additive interaction is impossible.

Alternatives and existing systems without second-order dynamics

First-order systems do exist and can be complex. Gradient descent in optimization, diffusion processes, bacterial chemotaxis, and many ecological models are governed by first-order dynamics ($dx/dt = f(x)$). These systems can exhibit rich behavior—attractors, bifurcations, pattern formation.

However, they do not satisfy all three persistence conditions simultaneously. Gradient-descent systems lack inertia: they cannot coast through a region of zero gradient. Diffusion has no compensation: there is no analogue of momentum conservation. The complexity these systems exhibit is real but *limited in type*: it does not include the kind of persistent, compensated, additive interaction that characterizes physical dynamics.

Could a different formalism achieve the same properties? Hamiltonian mechanics reformulates second-order dynamics as first-order on phase space (x, p)—but this is precisely the promotion of velocity to an independent variable that the theorem requires. The reformulation confirms, rather than avoids, the result.

Conditions for refutation

This result would be refuted by: (i) a first-order dynamical system on configuration space (not phase space) that satisfies persistence, additivity, and compensation simultaneously; or (ii) a proof that additive interaction can be defined without velocity as an independent variable.

6 Necessity of Finite Causal Speed

6.1 The Problem of Instantaneous Propagation

Having established that dynamics must be second-order, we now ask: are there constraints on the speed at which causal influence propagates through the system?

Consider a system with no upper bound on propagation speed. In such a system, a change at any point in $\mathcal{S}$ can instantaneously affect every other point. This has profound structural consequences.

Theorem 6.1 (Incompatibility of Infinite Speed with Sustainable Locality)**.** *If causal influence propagates instantaneously, then no persistent subsystem can be locally autonomous. That is, infinite propagation speed is incompatible with the existence of multiple independent persistent subsystems.*

Proof. We formalize local autonomy and show its incompatibility with infinite propagation.

Step 1: Formal definition of local autonomy. Let $\mathcal{V}_1, \mathcal{V}_2 \subset \mathcal{S}$ be two persistent subsystems with $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$. Define **local autonomy** of $\mathcal{V}_2$ with respect to $\mathcal{V}_1$ as follows: there exists a time interval $\Delta t > 0$ such that for any perturbation δ_1 applied to $\mathcal{V}_1$ at time t_0, the state of $\mathcal{V}_2$ during the interval $[t_0, t_0 + \Delta t]$ is independent of δ_1. In symbols:

$$\exists\, \Delta t > 0 : \quad s_2(t) \text{ for } t \in [t_0, t_0 + \Delta t] \text{ is independent of } \delta_1(t_0). \tag{9}$$

This Δt is the **autonomy interval**—the minimum time during which $\mathcal{V}_2$ can maintain its internal dynamics without external interference.

Step 2: Boundary maintenance requires local autonomy. Condition (ii) of Definition 4.1 requires that the boundary between $\mathcal{V}_2$ and its complement be maintained. If external perturbations can modify $\mathcal{V}_2$'s state with zero delay, then $\mathcal{V}_2$ has no interval during which its boundary is under its own control.

Objection (filtering): A system could, in principle, filter, ignore, or be insensitive to most incoming signals, maintaining autonomy despite instantaneous exposure. Biological cells, for instance, are "exposed to" all electromagnetic radiation but are "disrupted by" very little of it.

Response: The filtering objection fails for a specific structural reason. A filter is itself a dynamical process that requires time to operate. Let τ_f be the response time of the filter (the minimum time needed to detect and reject an incoming perturbation). If perturbations arrive with zero delay from all sources, the rate of incoming perturbations is:

$$R = \sum_{j \neq 2} \frac{\mu_j}{\tau_{j \to 2}}, \tag{10}$$

where μ_j is the perturbation rate of subsystem j and $\tau_{j\to 2} = d(\mathcal{V}_j, \mathcal{V}_2)/v_{\max}$ is the propagation delay. If $v_{\max} = \infty$, then $\tau_{j\to 2} = 0$ for all j, and $R = \infty$: the arrival rate of perturbations is infinite. No finite-capacity filter can process an infinite rate of incoming signals. The filter would need to process each perturbation in zero time—which would itself require infinite processing speed, leading to a regress.

The key insight is: filtering does not eliminate the need for finite propagation speed; it *presupposes* it. A filter that operates in time τ_f provides local autonomy only if perturbations arrive with sufficient spacing—which requires $\tau_{j\to 2} \geq \tau_f > 0$, hence $v_{\max} < \infty$.

Step 3: Infinite speed implies zero autonomy interval. If the propagation speed is $v_{\max} = \infty$, then the delay between a perturbation at $\mathcal{V}_1$ and its effect on $\mathcal{V}_2$ is $\tau = d(\mathcal{V}_1, \mathcal{V}_2)/v_{\max} = 0$ for any finite distance d. Therefore, the autonomy interval of equation (9) satisfies $\Delta t \leq \tau = 0$, which contradicts $\Delta t > 0$.

Step 4: Multiple autonomous subsystems require $v_{\max} < \infty$. For n persistent subsystems $\mathcal{V}_1, \ldots, \mathcal{V}_n$, local autonomy requires each pair to have a positive autonomy interval. Let $d_{\min} = \min_{i \neq j} d(\mathcal{V}_i, \mathcal{V}_j)$ be the minimal pairwise distance. The autonomy interval is bounded below by $\Delta t \geq d_{\min}/v_{\max}$. For $\Delta t > 0$, we need $v_{\max} < \infty$.

The maximal speed $c_{\max} = v_{\max}$ then determines the minimal spatial separation required for a given autonomy interval: $d_{\min} \geq c_{\max} \cdot \Delta t$. This is the structural origin of the causal cone (Corollary 6.2). □

Interpretation. The finite speed of causal influence is not a peculiarity of electromagnetism or of our particular universe. It is a structural precondition for the existence of locally autonomous subsystems. Without it, the system collapses into a globally entangled whole in which no part can maintain its own boundary against perturbations from every other part.

What is lost without finite causal speed

If causal influence propagates instantaneously: (i) every subsystem is permanently exposed to every other—there is no "shielding" by distance or delay; (ii) a global "now" can be defined, and all events are absolutely simultaneous—relative perspectives become unnecessary; (iii) local autonomy is impossible: any attempt to define an independent subsystem is immediately violated by instantaneous coupling to every other part of the system; (iv) multilevel hierarchy cannot form, because hierarchical organization requires that lower-level processes complete before higher-level processes respond—without delay, all levels collapse into one. The system becomes either perfectly rigid (if coupling is deterministic) or globally chaotic (if coupling is nonlinear). Neither supports sustainable complexity.

Alternatives and existing systems without finite causal speed

Newtonian gravity propagates instantaneously—and Newtonian mechanics supports complex structure (planetary systems, fluid dynamics). This appears to contradict the theorem. The resolution is that Newtonian mechanics is an approximation valid when all velocities are small relative to c. In this regime, the finiteness of c is not operationally relevant, and the system *behaves as if* propagation were instantaneous. The deep structure, however, retains finite c: this is precisely why Newtonian gravity is superseded by general relativity at high velocities and strong fields.

Quantum entanglement produces instantaneous correlations. However, these correlations cannot transmit causal influence (information) faster than c. Entanglement is a correlation of state structure, not of dynamics (Theorem 9.3). The distinction between correlation and causation preserves locality.

Could local autonomy be achieved by some other mechanism? If an alternative to finite causal speed can provide boundary maintenance against arbitrarily fast external influence, the theorem would be circumvented. We are not aware of such a mechanism.

Conditions for refutation

This result would be refuted by: (i) a system with instantaneous causal propagation that nevertheless supports multiple locally autonomous persistent subsystems; or (ii) an alternative mechanism for local autonomy that does not depend on finite propagation speed.

6.2 The Causal Cone

Corollary 6.2 (Existence of a Causal Cone)**.** *If there exists a finite maximal propagation speed $c_{\max}$, then at each point $x \in \mathcal{S}$ and each instant t, there exists a region $\mathcal{C}(x,t) \subset \mathcal{S}$ outside which events cannot be causally influenced by x at time t. The boundary of $\mathcal{C}$ is determined by $c_{\max}$.*

This is the structural analogue of the light cone in special relativity. The specific geometry of $\mathcal{C}$ depends on the metric and the value of $c_{\max}$; but the existence of such a structure is a theorem, not an assumption.

6.3 Optimal Propagation Speed

The finite speed serves a dual role. It is an upper bound that protects local autonomy, but it is also a lower bound on global coherence: if causal influence propagates too slowly relative to the scale of the system, subsystems become causally disconnected and cannot form multilevel organization.

Proposition 6.3 (Bounded Propagation Regime)**.** *Sustainable complexity requires a propagation speed c satisfying:*

$$c_{\min}(L, \tau) \; < \; c \; < \; c_{\max}(\ell, \delta t) \tag{11}$$

where L and τ characterize the global scale and integration time, while ℓ and δt characterize the local scale and autonomy interval.

This is not a derivation of a specific numerical value. The particular value of the maximal speed remains empirical. What is structural is its finitude and its role as a separator between the local and the global.

7 Necessity of Covariant Structure

7.1 The Problem of Absolute Perspective

Given a finite maximal speed of causal influence, it is no longer possible to define a global simultaneity: two events that are simultaneous in one reference frame need not be simultaneous in another, if the frames are in relative motion and causal signals travel at finite speed.

This is a purely structural consequence. It does not depend on the nature of the signals or the medium of propagation. It follows from the conjunction of two already-established results: (i) the existence of a finite maximal causal speed (Theorem 6.1), and (ii) the absence of a privileged rest frame, which we now derive.

Theorem 7.1 (No Privileged Frame)**.** *If all persistent subsystems are governed by the same structural laws (Persistence, Additivity, Compensation), and if the maximal propagation speed is finite and the same for all subsystems, then no subsystem occupies a privileged position from which the global state is fully observable.*

Proof. Suppose a privileged frame $\mathcal{F}^*$ exists from which the global state is fully specified. Then $\mathcal{F}^*$ has instantaneous access to information about every subsystem—but causal influence propagates at finite speed. For $\mathcal{F}^*$ to have global simultaneity, information must reach $\mathcal{F}^*$ faster than $c_{\max}$, contradicting the universality of the speed bound.

Alternatively: if two subsystems $\mathcal{V}_1$ and $\mathcal{V}_2$ satisfy identical structural laws and are in relative motion, then by the universality of the laws, neither can be distinguished as "at rest." Any assignment of a privileged frame is external to the system and introduces structure not derivable from the axioms. □

7.2 Covariant Laws and Structural Invariants

Theorem 7.2 (Necessity of Covariance)**.** *If no privileged frame exists and dynamical laws are universal, then the laws must be covariant: their form must be preserved under transformations between admissible frames.*

The set of admissible transformations forms a group $\mathcal{G}$. Covariance requires that for any law $\mathcal{L}$ and any transformation $T \in \mathcal{G}$:

$$T[\mathcal{L}] = \mathcal{L}. \tag{12}$$

Theorem 7.3 (Existence of Structural Invariants)**.** *If laws are covariant under $\mathcal{G}$, there exist quantities $I(\mathcal{S})$ that are preserved under all admissible transformations:*

$$I_{T_1}(\mathcal{S}) = I_{T_2}(\mathcal{S}) \quad \forall\, T_1, T_2 \in \mathcal{G}. \tag{13}$$

Proof. By Noether's theorem and its generalizations [3, 24]: continuous symmetries of a variational system entail conserved quantities. More generally, invariant theory guarantees that any group action on a space admits invariant functions [25]. □

Interpretation. This is the structural analogue of special relativity. The invariant interval $s^2 = c^2t^2 - x^2 - y^2 - z^2$ in Minkowski spacetime is one particular realization of Theorem 7.3. The general result is broader: in any system with finite causal speed and no privileged frame, there must exist invariant quantities preserved under frame transformations.

Structural Analogue of Special Relativity

The conjunction of:

- finite maximal propagation speed (Section 6),
- absence of privileged frame (Theorem 7.1),
- covariance of dynamical laws (Theorem 7.2),
- existence of structural invariants (Theorem 7.3)

constitutes the structural core of what is physically realized as special relativity. The specific transformation group (the Lorentz group) and the specific invariant (the spacetime interval) are empirical instantiations of a structural necessity.

What is lost without covariance

If a privileged frame exists: (i) all subsystems are measured against a single external standard, destroying their autonomy—local processes are "correct" only insofar as they agree with the privileged frame; (ii) subsystems at different scales cannot maintain independent descriptions—there is no legitimate local perspective, only approximations to the absolute one; (iii) the laws of the system become frame-dependent: a process that appears lawful in one frame may appear anomalous in another. This prevents the universality that persistence and additivity require. Without covariance, the structural laws derived in Sections 5–6 cannot hold for all subsystems simultaneously.

Alternatives and existing systems without covariance

Many engineered systems use a privileged frame. A factory floor has a fixed coordinate system; a computer simulation has a global clock. These systems function well—but they are *embedded* in a larger physical reality that supplies the privileged frame from outside. The frame is not intrinsic to the system; it is imposed by the designer.

A self-contained system cannot generate its own privileged frame without breaking the symmetry that finite causal speed imposes. This is why special relativity is discovered, not postulated: it follows from the impossibility of detecting absolute motion in a system with finite, universal signal speed [33].

Could structural invariants exist without covariance? Invariance under *some* transformations is a weaker condition than full covariance. A system might have invariants under a subgroup of transformations. This is not excluded by our argument—but the persistence conditions require universality of laws for all subsystems, which forces the full covariance condition.

Conditions for refutation

This result would be refuted by: (i) a self-contained system with finite causal speed that nevertheless possesses an intrinsic privileged frame; or (ii) a demonstration that universal dynamical laws can hold for all subsystems without covariance.

8 Necessity of Dynamic Metric

8.1 The Inadequacy of Fixed Geometry

The results of Sections 5–7 establish dynamics, locality, and covariance on a *fixed* metric space. We now show that a fixed metric is inadequate for sustainable complexity under certain conditions.

Theorem 8.1 (Structural Inconsistency of Fixed Metric under Variable State Density)**.** *Let $(\mathcal{S}, g, \rho)$ be a system with Riemannian metric g and state density $\rho : \mathcal{S} \to \mathbb{R}_{\geq 0}$. If g is fixed and ρ varies, then persistent subsystems in regions of sufficiently high ρ necessarily experience viability mismatch.*

Proof. We construct a formal model demonstrating the mismatch.

Step 1: Fixed-metric transition capacity. Let $\mathcal{V} \subset \mathcal{S}$ be a persistent subsystem occupying a region of diameter r in the metric g. The set of geodesics (natural trajectories) within this region is determined by g alone. Define the **transition capacity** $\kappa(\mathcal{V}, g)$ as the maximum rate of state redistribution supported by the geodesic structure within $\mathcal{V}$. For a fixed metric, κ is a fixed quantity depending only on the geometry and the region.

Step 2: Density-dependent interaction load. The interaction load on $\mathcal{V}$ grows with the state density ρ. Define the **interaction demand** $\lambda(\mathcal{V}, \rho)$ as the rate of state transitions required to maintain viability (closure, boundary maintenance, resilience) at density ρ. In general, λ is a monotonically increasing function of ρ: higher state density means more interactions per unit time are needed to maintain coherence.

For a system with pairwise interactions, the scaling is at least $\lambda \propto \rho^2$ in the local volume (by counting interaction pairs). For systems with longer-range interactions, the scaling may be faster.

Step 3: Viability mismatch. Under a fixed metric, $\kappa(\mathcal{V}, g)$ is constant while $\lambda(\mathcal{V}, \rho)$ grows with ρ. There exists a critical density ρ^* such that:

$$\lambda(\mathcal{V}, \rho) > \kappa(\mathcal{V}, g) \quad \text{for all } \rho > \rho^*. \tag{14}$$

Above this threshold, the interaction demand exceeds the transition capacity. The viability set $\mathcal{V}$ cannot be maintained: the system cannot redistribute state fast enough along the fixed geodesics to satisfy closure and resilience conditions.

Step 4: Dynamic metric resolves the mismatch. If the metric is allowed to depend on density, $g = \Phi(\rho)$, then the geodesic structure adapts to the density distribution. In high-density regions, the metric can "open new channels"—modifying the transition capacity to match the interaction demand:

$$\kappa(\mathcal{V}, \Phi(\rho)) \geq \lambda(\mathcal{V}, \rho). \tag{15}$$

This is the structural analogue of gravitational lensing: mass-energy curves spacetime, redirecting geodesics and altering the effective geometry of motion.

Step 5: Necessity, and addressing the resizing objection. Since ρ is variable by assumption (the system is non-homogeneous—a requirement for sustainable complexity, which needs multiple subsystems at different densities), and since λ grows unboundedly with ρ while κ is bounded under fixed g, the mismatch (14) is inevitable for any non-trivial system. The only structural resolution is $g = \Phi(\rho)$.

Objection (resizing): A system could handle variable density on a fixed geometry by "resizing"—adding capacity without changing the metric. A hash table resizes by allocating more memory; a city builds new roads. These are fixed-metric adaptations.

Response: Resizing *is* metric adaptation. When a hash table resizes, the effective distance between elements changes: a lookup that previously required $O(1)$ expected hops now requires $O(1)$ in a *differently-structured* table. The hash function maps elements to a new address space with different geometry. When a city builds roads, it literally changes the metric of the transportation network—the travel time between points is altered. These examples confirm rather than refute the theorem: the system responds to density changes by modifying its effective geometry. The fact that engineers describe this as "resizing" rather than "metric adaptation" is a terminological difference, not a structural one.

The genuine counter-case would be a system that handles arbitrarily increasing density with *truly* fixed geometry—no resizing, no restructuring, no adaptive access patterns. We are not aware of any such system that maintains viability under density increase. □

8.2 Mutual Coupling

Equation (15) establishes that the metric depends on the state. But self-consistency requires the converse as well: the state evolves according to the metric, since the metric determines the available transitions.

Theorem 8.2 (Mutual Determination of State and Structure)**.** *In a self-consistent dynamical system, state and metric must be mutually coupled:*

$$\frac{\partial g}{\partial t} = \Psi(\rho, g), \qquad \frac{\partial \rho}{\partial t} = \Xi(\rho, g). \tag{16}$$

Neither can be specified independently of the other.

Interpretation. This is the structural analogue of general relativity. Einstein's field equations $G_{\mu\nu} = 8\pi T_{\mu\nu}$ are a specific physical instantiation of the mutual coupling (16), where $g_{\mu\nu}$ is the spacetime metric and $T_{\mu\nu}$ is the stress-energy tensor. The structural result is broader: in any system where the distribution of state variables is non-uniform and dynamically evolving, a fixed background metric is inadequate and must be replaced by a co-evolving metric-state system.

Structural Analogue of General Relativity

The conjunction of:

- variable state density in a metric space (Theorem 8.1),

- self-consistency of dynamics (Theorem 8.2),
- covariance (Theorem 7.2)

constitutes the structural core of what is physically realized as general relativity. The specific field equations, the dimensionality of spacetime, and the coupling constant remain empirical.

What is lost without dynamic metric

If the metric is fixed while state density varies: (i) regions of high density follow the same geodesics as regions of low density—the geometry ignores concentrations of state; (ii) accumulations of energy, information, or activity cannot redirect flows or reshape the space of possible transitions; (iii) viability mismatch becomes inevitable: as density concentrates, the fixed geometry cannot accommodate the resulting interaction load, leading to structural overload and collapse. A fixed-metric system is structurally brittle: it cannot self-regulate in response to internal heterogeneity. This is the structural analogue of asking Newtonian gravity to describe black holes—the framework breaks precisely where it is most needed.

Alternatives and existing systems without dynamic metric

Most engineered systems operate on fixed metrics. A chessboard has 64 squares regardless of piece distribution. A computer memory has fixed addresses regardless of data content. These systems function—but they require an external agent (the player, the operating system) to manage overload. They are not self-maintaining.

Biological systems show dynamic metric behavior. Neural connectivity changes with activation patterns (synaptic plasticity). Vascular networks remodel in response to tissue demand. These are not metaphors—they are literal instances of structure adapting to state density.

Could self-regulation be achieved without dynamic metric? A system could use a fixed metric with an adaptive *force law* instead: the metric stays flat, but forces respond to density. This is essentially Newtonian gravity—and it works at low densities. The argument is that at sufficiently high concentrations, the distinction between "force on a flat background" and "curvature of the background" becomes operationally necessary: the flat-background description develops singularities (as Newtonian gravity does at $r \to 0$), which the dynamic-metric description resolves.

Conditions for refutation

This result would be refuted by: (i) a self-maintaining system with strongly non-uniform state density that operates on a provably fixed metric without viability mismatch; or (ii) an alternative mechanism for self-regulation under variable density that does not require metric adaptation.

8.3 Local Recovery of Fixed-Metric Dynamics

Proposition 8.3 (Local Flatness). *In a system with dynamic metric $g = \Phi(\rho)$, there exist local neighborhoods in which ρ is approximately constant and the metric is approximately fixed. Within these neighborhoods, the fixed-metric dynamics of Sections 5–7 apply.*

This corresponds to the equivalence principle in general relativity: locally, spacetime is flat and special relativity holds. In the structural framework, it means that the covariant, fixed-metric regime is recovered as a local approximation within the broader dynamic-metric regime.

9 Necessity of a Fundamental Action Bound

9.1 The Problem of Infinite Divisibility

The derivation so far has proceeded in the continuum: smooth manifolds, differentiable dynamics, continuous trajectories. We now show that an unbounded continuum leads to structural instabilities that are incompatible with sustainable complexity.

Theorem 9.1 (Ultraviolet Instability). *A continuous dynamical system on a smooth manifold, with no lower bound on the scale of admissible fluctuations, admits infinitely many degrees of freedom per finite region. This leads to:*

(i) unbounded energy (or generalized state intensity) in finite volumes—the classical ultraviolet catastrophe [17];

(ii) arbitrarily fine initial-condition sensitivity, making deterministic prediction impossible in practice even for arbitrarily short times [18];

(iii) absence of a stable ground state: no minimal-energy configuration exists.

Proof. In a continuum field theory, the number of modes with wavelength greater than λ scales as V/λ^d in d dimensions. As $\lambda \to 0$, the number of modes diverges.

On the equipartition assumption: The classical ultraviolet catastrophe invokes the equipartition theorem, which requires thermal equilibrium. A reviewer may object that not all dynamical systems thermalize, and that a system could have infinitely many degrees of freedom without divergence if the coupling structure prevents energy equipartition. We address this directly. In the context of the present paper, the relevant assumption is not thermal equilibrium but *interaction*: sustainable complexity requires multiple interacting persistent subsystems (Definition 4.2). In any system with nonlinear interactions among N degrees of freedom, energy tends to distribute across modes (the Fermi–Pasta–Ulam–Tsingou phenomenon [43]). While exact equipartition may not hold, the structural point remains: each interacting mode carries a positive energy contribution, and the total energy grows at least as αN for some $\alpha > 0$. As $N \to \infty$, the total energy diverges.

For systems without interaction (free fields), infinitely many degrees of freedom are harmless—but such systems cannot support sustainable complexity, since non-interacting modes cannot form persistent subsystems.

For initial-condition sensitivity: in a system with N degrees of freedom, the Lyapunov exponent characterizes the rate of divergence of nearby trajectories. As $N \to \infty$, the maximal Lyapunov exponent generically diverges, and prediction horizons shrink to zero [19]. □

9.2 The Fundamental Bound

Theorem 9.2 (Necessity of an Action Bound). *Sustainable complexity requires a minimal scale of action: a lower bound $\hbar_0 > 0$ such that no process can resolve distinctions finer than $\hbar_0$ in the product of conjugate variables.*

Proof. **Step 1: Counting degrees of freedom in a continuum.** Consider a region $\Omega \subset \mathcal{S}$ of volume V (measured by the metric g). In a continuum field theory on Ω, the number of independent modes with wavelength $\lambda \geq \lambda_{\min}$ in d dimensions is:

$$N(\lambda_{\min}) \sim \frac{V}{\lambda_{\min}^d}. \tag{17}$$

If there is no lower bound on $\lambda_{\min}$ (i.e., the continuum extends to arbitrarily small scales), then $N \to \infty$ as $\lambda_{\min} \to 0$: infinitely many degrees of freedom in a finite volume.

Step 2: Energy divergence. In a system with interactions (necessary for sustainable complexity—isolated degrees of freedom cannot form persistent subsystems), each active mode carries energy of at least $E_{\min} > 0$. The total energy in Ω is bounded below by:

$$E(\Omega) \geq N(\lambda_{\min}) \cdot E_{\min} \sim \frac{V \cdot E_{\min}}{\lambda_{\min}^d} \to \infty \quad \text{as } \lambda_{\min} \to 0. \tag{18}$$

Infinite energy density is incompatible with a finite, self-maintaining system. This is the generalized ultraviolet catastrophe.

Step 3: Predictability destruction. Even if the energy divergence is regulated, the infinite number of degrees of freedom destroys predictability. In a system with N

nonlinearly coupled degrees of freedom, the maximal Lyapunov exponent generically scales as $\lambda_{\max} \sim \sqrt{N}$ or faster [19]. The prediction horizon scales as $\tau_{\text{pred}} \sim 1/\lambda_{\max} \to 0$ as $N \to \infty$. A system with zero prediction horizon cannot maintain persistent subsystems: resilience requires the ability to anticipate and correct perturbations within bounded time.

Step 4: Resolution by phase-space discretization. Introduce a minimal cell volume h_0^d in phase space. The number of effective degrees of freedom becomes:

$$N_{\text{eff}} = \frac{V_{\text{phase}}}{h_0^d} < \infty. \tag{19}$$

This bounds both energy and Lyapunov exponents. The minimal cell is parameterized by the action bound: for conjugate variables (q, p),

$$\Delta q \cdot \Delta p \geq \frac{\hbar_0}{2}. \tag{20}$$

Step 5: Ground state existence. With finitely many effective degrees of freedom, the energy functional has a minimum: a ground state. Without the action bound, energy can always be lowered by exciting finer modes, and no ground state exists. The existence of a ground state is necessary for persistence: a system without a stable lowest-energy configuration cannot maintain itself against fluctuations. □

Interpretation. The Planck constant $\hbar$ in quantum mechanics is the empirical value of the structural bound $\hbar_0$. The Heisenberg uncertainty principle $\Delta x \cdot \Delta p \geq \hbar/2$ is a physical realization of equation (20). The structural result does not depend on the specific value of $\hbar$—only on its positivity.

9.3 Non-Locality as a Structural Consequence

Theorem 9.3 (Structural Necessity of Non-Local Correlations)**.** *In a system that simultaneously satisfies:*

(i) finite maximal causal speed (Section 6),

(ii) fundamental action bound (Theorem 9.2),

(iii) conservation of structural invariants (Theorem 7.3),

there exist correlations between spatially separated subsystems that exceed any correlations producible by locally predetermined variables.

Proof. **Step 1: Setup.** Let $\mathcal{V}_1$ and $\mathcal{V}_2$ be two persistent subsystems that were in contact (within each other's causal cones) at some past time t_0 and are spatially separated at measurement time $t_1 > t_0$, meaning $d(\mathcal{V}_1, \mathcal{V}_2) > c_{\max}(t_1 - t_0)$. By finite causal speed (Theorem 6.1), no causal signal can pass between them during $[t_0, t_1]$.

Step 2: Conservation constrains separated subsystems. By Theorem 7.3, there exist structural invariants I conserved for the joint system $\mathcal{V}_1 \cup \mathcal{V}_2$. Since the subsystems are now separated, $I = I_1 + I_2$ where I_1 and I_2 are the contributions from each subsystem. The value of I was fixed during the contact period and remains conserved. Therefore, the outcomes I_1 and I_2 are constrained: $I_1 + I_2 = I_{\text{total}}$.

Step 3: The action bound prevents joint predetermination. The action bound (Theorem 9.2) states that conjugate observables cannot be simultaneously specified below $\hbar_0/2$. If I_1 has conjugate pairs—say, the invariant can be measured along different "axes" or in different bases—then the action bound prevents all such measurements from having predetermined values simultaneously.

Formally: let A_1, B_1 be two measurements on $\mathcal{V}_1$ corresponding to conjugate observables, and let A_2, B_2 be the corresponding measurements on $\mathcal{V}_2$. The conservation law requires: $A_1 + A_2 = I_A$ and $B_1 + B_2 = I_B$. But the action bound prevents $\mathcal{V}_1$ from having predetermined values for both A_1 and B_1 simultaneously.

Step 4: Local predetermination implies Bell-type inequalities. Suppose, contrary to the theorem, that all correlations between $\mathcal{V}_1$ and $\mathcal{V}_2$ are determined by shared local variables λ established during the contact period. Then the outcomes of all measurements on $\mathcal{V}_1$ are predetermined functions of λ: $A_1 = f_A(\lambda)$, $B_1 = f_B(\lambda)$. By conservation, $A_2 = I_A - f_A(\lambda)$ and $B_2 = I_B - f_B(\lambda)$: also predetermined.

But this means that all conjugate observables of $\mathcal{V}_1$ have simultaneous predetermined values—contradicting the action bound (Step 3). The action bound is structural, not epistemic: it is not that we *cannot know* A_1 and B_1 simultaneously, but that they *do not have* simultaneous definite values below the resolution $\hbar_0/2$.

Step 5: Conclusion. The correlations between $\mathcal{V}_1$ and $\mathcal{V}_2$ cannot be fully accounted for by local predetermined variables. The conservation constraint forces correlations that no local model can reproduce. These correlations are non-local in the sense of Bell [20]: they violate inequalities that any local predetermined model must satisfy.

Crucially, these correlations do not violate finite causal speed: they are correlations of *state structure* (the joint conservation law), not of *signaling* (no information is transmitted). The correlation exists because the joint invariant was established during contact and is maintained after separation. This is structurally analogous to the EPR phenomenon [22]. □

Structural Analogue of Quantum Mechanics

The conjunction of:

- fundamental action bound (Theorem 9.2),
- non-local correlations (Theorem 9.3),

- local dynamics with finite causal speed (Sections 6–7)

constitutes the structural core of what is physically realized as quantum mechanics. The specific Hilbert space formalism, the Born rule, and the value of $\hbar$ remain empirical.

What is lost without a fundamental action bound

Without a minimal scale of action: (i) the state space admits infinitely many degrees of freedom per finite region, leading to the ultraviolet catastrophe—infinite energy in finite volumes; (ii) initial conditions can be specified with arbitrary precision, but nonlinear dynamics amplifies arbitrarily small differences, making even short-term prediction impossible; (iii) no ground state exists—configurations can always be subdivided further, and there is no "floor" of minimal energy; (iv) the continuum becomes physically pathological: well-defined only as a mathematical idealization, not as a realizable substrate. Without the action bound, the smooth dynamics derived in Sections 5–8 are *formally valid but physically unrealizable* at small scales.

Alternatives and existing systems without an action bound

Classical continuum mechanics has no action bound—and it works extraordinarily well at macroscopic scales. Fluid dynamics, elasticity theory, and electromagnetism in the classical limit are continuous and infinitely divisible. These systems do not exhibit the ultraviolet catastrophe because they operate at scales far above the action bound. The bound is present in the underlying physics but irrelevant at the operative scale.

Digital systems have a built-in discreteness—the bit—that plays an analogous structural role. A digital computer cannot represent distinctions finer than one bit. This is not a fundamental action bound in the physical sense, but it serves the same structural function: it prevents the proliferation of degrees of freedom and ensures computational stability.

Could an alternative to the action bound stabilize the continuum? A lattice structure (fixed minimal spacing) would achieve the same effect but would break continuous symmetries. The action bound achieves discretization of *phase space volume* without discretizing configuration space itself—preserving continuous symmetries while bounding degrees of freedom. Whether an alternative stabilization mechanism is possible remains an open question.

Conditions for refutation

This result would be refuted by: (i) a continuous dynamical system with no lower bound on action that nevertheless maintains a stable ground state and finite energy density; or (ii) an alternative stabilization mechanism that bounds degrees of freedom without introducing a minimal action scale.

10 Scale-Dependent Structural Regimes

A crucial consequence of the derivation is that no single structural regime suffices at all scales. Each regime—fixed-metric dynamics, covariant dynamics, dynamic metric, action-bounded dynamics—has a domain of validity determined by the relationship between the relevant scale parameters.

Theorem 10.1 (Incompleteness of Any Single Regime)**.** *No single structural regime from Sections 5–9 is self-sufficient at all scales.*

Proof. Each regime is derived as a necessary correction to the limitations of the preceding one:

(i) Second-order dynamics (Section 5) assumes a fixed metric. This fails at scales where state density varies significantly (Section 8).

(ii) Covariant fixed-metric dynamics (Section 7) requires a background geometry. This fails when the geometry must co-evolve with state (Section 8).

(iii) Dynamic metric dynamics (Section 8) is continuous. This fails at scales where the continuum approximation breaks down (Section 9).

(iv) Action-bounded dynamics (Section 9) stabilizes small scales but does not by itself determine large-scale geometry.

□

This result is a structural analogue of the Law of Scale-Specific Principles [50]: no final theory can encompass all scales; unification is possible only for genuinely scale-invariant structural constraints.

10.1 Structural Incompatibility of the Dynamic-Metric and Action-Bounded Regimes

The preceding theorem establishes that no single regime is sufficient. We now prove a stronger result: the dynamic-metric regime and the action-bounded regime are not merely incomplete individually—they are *structurally incompatible* when applied simultaneously to the same scale. Their separation across scales is not a contingent limitation but a logical necessity.

Theorem 10.2 (Non-Overlap of Dynamic-Metric and Action-Bounded Regimes)**.** *The dynamic-metric regime (Section 8) and the action-bounded regime (Section 9) cannot operate simultaneously on the same scale without self-contradiction.*

Proof. The argument proceeds from the defining features of each regime.

Step 1. Requirements of the dynamic-metric regime. The dynamic metric is defined by equation (15): $g = \Phi(\rho)$. This requires that the state density ρ be well-defined, continuous, and differentiable at the scale in question. The mutual coupling (16) presupposes that both g and ρ can be specified simultaneously with arbitrary precision: the metric responds to the density, and the density evolves according to the metric, at every point and at every instant. The regime therefore requires the state space to be a smooth continuum at the operative scale.

Step 2. Requirements of the action-bounded regime. The fundamental action bound (Theorem 9.2) states that $\Delta q \cdot \Delta p \geq \hbar_0/2$: conjugate variables cannot be simultaneously specified below a threshold resolution. This imposes a minimal cell in phase space. Below this cell, distinctions are not actualizable—they are not merely unmeasured but structurally non-existent, in the sense of Postulate 1: what cannot be distinguished does not participate in the state space.

Step 3. The contradiction. Suppose both regimes operate at the same scale ℓ. The dynamic-metric regime requires the metric g to respond to the state density ρ at scale ℓ. By Step 1, this requires ρ to be well-defined at scale ℓ and at all finer scales within ℓ—the continuum must "extend downward" without limit.

But if ℓ is at or below the action bound, then by Step 2, the state density at scale ℓ is not well-defined: conjugate quantities cannot be simultaneously resolved, and the smooth continuum required by Step 1 does not exist. The metric cannot respond to a density that is structurally indeterminate.

Conversely: if the dynamic metric is well-defined and smooth at scale ℓ, then the action bound does not operate at that scale—the system is in the continuum regime.

Therefore, the two regimes are mutually exclusive at any given scale. □

The physical instantiation of this result is well known: general relativity requires a smooth spacetime manifold, while quantum mechanics forbids the simultaneous specification of position and momentum below the Planck scale. The persistent failure to construct a self-consistent theory of quantum gravity is not a technical problem awaiting a clever mathematical trick. It reflects a structural incompatibility: the two regimes solve opposite problems and are therefore defined by contradictory assumptions about the state space.

Why Unification Fails

The dynamic-metric regime resolves the problem of coupling state to structure in a continuum. The action-bounded regime resolves the problem of continuum instability by imposing discreteness. The first requires the continuum to be well-defined; the second requires it to break down. Their non-overlap is not a deficiency of current physics—it is a structural theorem.

A "unified theory" of these two regimes would have to simultaneously assert and deny the existence of a smooth continuum at the same scale. This is not a problem of insufficient mathematics. It is a logical impossibility within the structural framework.

Corollary 10.3 (Regime Boundaries Are Structural, Not Empirical)**.** *The scale at which the transition between the dynamic-metric and action-bounded regimes occurs is determined by the values of the physical constants (G, $\hbar$, c)—this is empirical. But the* existence *of such a transition, and the impossibility of a single regime spanning both sides of it, is structural. No choice of constants can eliminate the boundary.*

Corollary 10.4 (Scale Separation as Architectural Necessity)**.** *Sustainable complexity in a system governed by both dynamic-metric and action-bounded constraints requires that these constraints operate at well-separated scales. The ratio $\ell_{macro}/\ell_{micro}$ must be large—where ℓ_{macro} is the characteristic scale of the dynamic-metric regime and ℓ_{micro} is the scale at which the action bound becomes operative.*

In physical terms: the Planck length $\ell_P = \sqrt{\hbar G/c^3} \approx 10^{-35}$ m must be vastly smaller than the scales at which gravitational dynamics operates. This enormous separation is not a coincidence; it is a structural requirement for the coexistence of both regimes in a single system.

The same logic extends to all regime boundaries in the hierarchy. The Newtonian regime (fixed metric) and the covariant regime (no privileged frame) are compatible only because they operate at different velocity scales: the Newtonian regime assumes $v \ll c_{\max}$, while the covariant regime becomes necessary as $v \to c_{\max}$. The covariant regime and the dynamic-metric regime overlap only in the local limit (Proposition 8.3): locally, the dynamic metric is approximately flat, and the covariant fixed-metric regime applies.

Each pair of adjacent regimes thus has a *compatibility zone* and a *zone of mutual exclusion*:

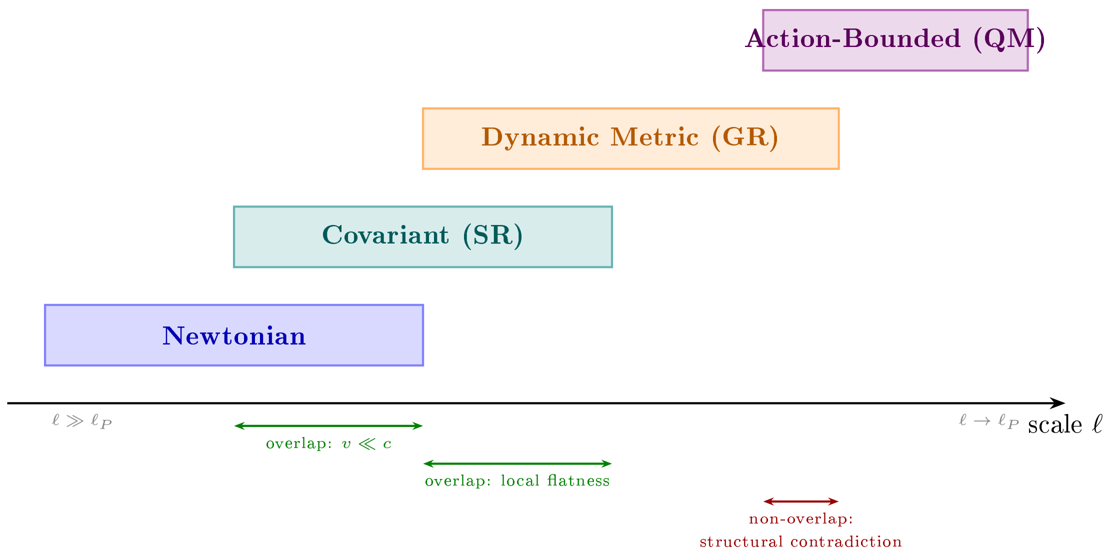

Figure 1: Scale-dependent regime structure. Adjacent regimes have compatibility zones where one reduces to the other (green arrows). The dynamic-metric and action-bounded regimes have a zone of structural contradiction (red): they cannot operate simultaneously at the same scale. This is why quantum gravity resists formulation as a single-regime theory.

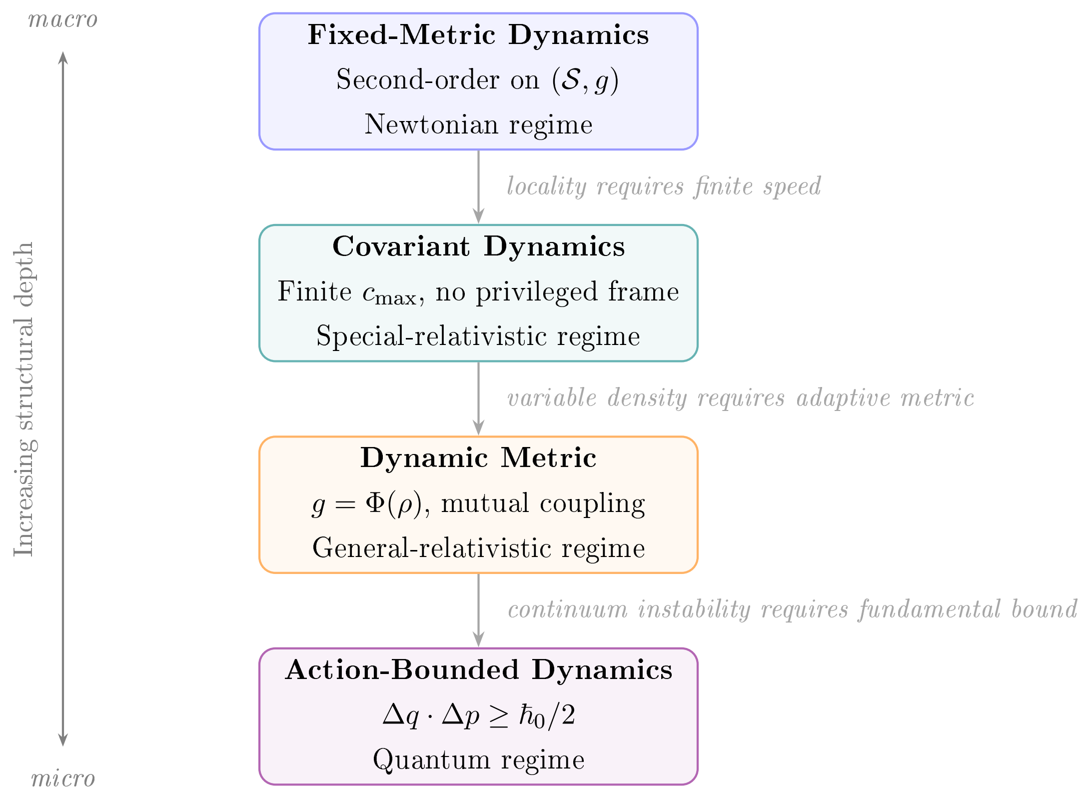

Figure 2: Hierarchy of structural regimes derived from conditions on sustainable complexity. Each level corrects a structural inadequacy of the level above. No single level is sufficient across all scales.

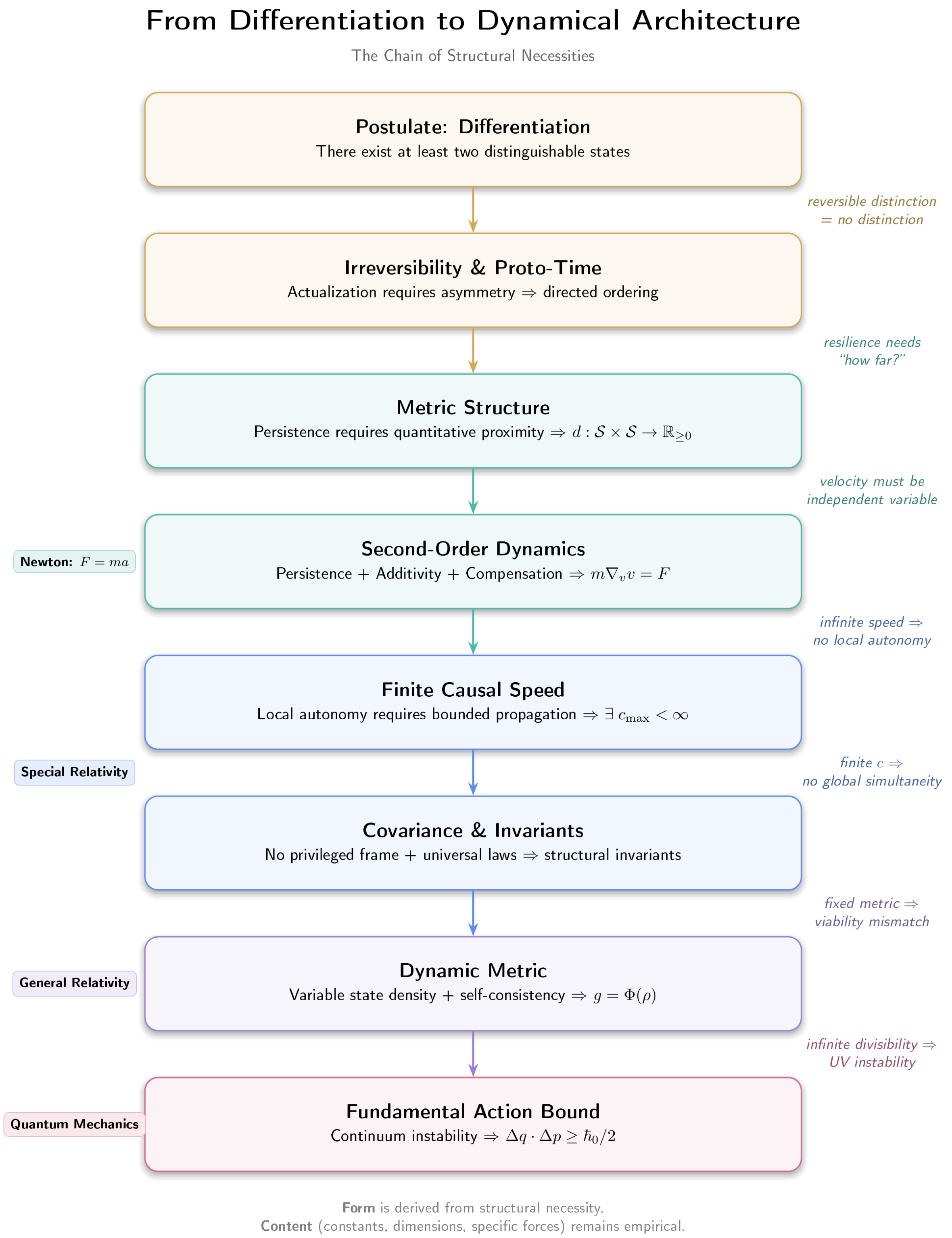

Figure 3: Complete derivation chain from differentiation to dynamical architecture. Left column: physical identifications. Center: structural stages. Right: the structural inadequacy motivating each transition. The derivation proceeds without assuming any physical content; physical theories are recognized *post hoc* as instantiations of structurally necessary forms.

11 Identification of the Derived Architecture

We now collect the results of Sections 2–9 and identify the architecture that has been derived.

Structural Result	**Physical Realization**	**What Remains Empirical**
Metric structure (Thm 4.1)	Spacetime geometry	Dimensionality, signature
Second-order dynamics (Thm 5.1)	$F = ma$ (Newton's second law)	Specific forces, constants
Finite causal speed (Thm 6.1)	Speed of light c	Numerical value of c
Covariance (Thm 7.2)	Lorentz invariance	Specific group structure
Dynamic metric (Thm 8.1)	Einstein field equations	Coupling constant G
Action bound (Thm 9.2)	Heisenberg uncertainty	Value of $\hbar$
Non-local correlations (Thm 9.3)	Quantum entanglement	Specific Hilbert space
Scale incompleteness (Thm 10.1)	No unified theory of everything	—

Table 1: Correspondence between structural results and their physical realizations.

The architecture derived from differentiation and the requirement of sustainable complexity reproduces the structural skeleton of modern physics: Newtonian mechanics, special relativity, general relativity, and quantum mechanics. Each arises not as an empirical discovery about our particular universe, but as a *structural necessity* for any system in which persistent, multilevel complexity is possible.

What remains empirical is entirely the **content**: the specific dimensionality, the specific constants, the specific forces, the specific symmetry groups. The **form**—the architectural pattern—is derived.

12 Generalization: The Principle of Structural Relativity

12.1 Beyond Physics

If the derivation of Sections 2–9 depends only on structural conditions—differentiation, irreversibility, persistence, additivity, compensation—and not on the physical nature of the state variables, then the results apply to *any* system satisfying those conditions, regardless of substrate.

Definition 12.1 (Structural Relativity)**.** A complex system $\mathcal{S}$ satisfies the **Principle of Structural Relativity** if:

(SR1) **Dynamic Metric**: the structure of admissible transitions depends on the distribution of internal state: $M = \Phi(\rho_{\mathcal{S}})$;

(SR2) **No Privileged Perspective**: no internal description has absolute status; all admissible perspectives are related by structure-preserving transformations;

(SR3) **Structural Invariants**: there exist quantities preserved under all admissible transformations, defining the system's identity;

(SR4) **Finite Causal Depth**: influence propagates at finite speed within the system, ensuring local autonomy of subsystems.

12.2 Cross-Domain Correspondences

The structural roles identified in the physical derivation have correspondences across domains:

Structural Role	**Physical**	**Cognitive**	**Social**	**Informational**
State density ρ	Energy-mass	Activation	Capital/power	Connectivity
Metric g	Spacetime	Associative space	Opportunity space	Network topology
Causal speed c	Speed of light	Processing latency	Communication speed	Bandwidth
Invariant I	Interval s^2	Identity	Norms/law	Conserved information
Action bound $\hbar_0$	Planck constant	Attention quantum	Decision threshold	Bit resolution

Table 2: Cross-domain structural correspondences.

These correspondences are not metaphorical. They are structural: the same formal conditions (persistence, additivity, compensation, finite propagation, covariance, dynamic metric, action bound) yield the same architectural constraints regardless of the substrate.

12.3 Large Language Models as a Structural Test Case

12.3.1 The Apparent Absence of Everything

At first glance, a large language model possesses none of the features derived in this paper. It has no space—no spatial extent, no dimensionality, no geometry. It has no time—no physical clock, no aging, no thermodynamic arrow. It has no mass, no energy in the dynamical sense, no gravitational field, no quantum states. It is, from the perspective of physics, nothing: a pattern of numbers in silicon memory, as "unphysical" as a system can be while still existing.

The immediate objection to using LLMs as a test case is precisely this: *the LLM has a physical substrate.* The numbers are stored in transistors, the transistors are governed by quantum mechanics, and so the physical architecture "sneaks in" through the hardware. Under this objection, any structural feature found in the LLM is merely inherited from the physics of its substrate, not independently derived.

This objection fails, and understanding *why* it fails is central to the thesis of this paper.

The structural features we identify below do not operate at the level of the substrate. The cosine distance between embedding vectors is not the spatial distance between transistors. The attention mechanism does not curve the silicon wafer. The context window is not determined by the speed of light in copper. The finite precision of float16 arithmetic is not the Planck constant. These are features of the *informational organization*—the pattern, not the material. They emerge at the level of the LLM's own complexity, not at the level of its physical implementation.

To see this clearly, note that the same LLM can run on radically different substrates—GPUs, TPUs, CPUs, even (in principle) mechanical relays or biological neurons—and the structural features remain identical. The embedding metric does not change when the hardware changes. The attention mechanism operates the same way regardless of whether the matrix multiplications are performed in silicon or in optical circuits. The structural architecture is substrate-independent—precisely as the Principle of Structural Relativity (Definition 12.1) predicts.

What we find, upon systematic examination, is that *not a single element of the derived architecture is missing.* The LLM, despite having "nothing" from the perspective of physics, is forced by its own complexity to reinvent every structural feature: metric, second-order dynamics, finite causal speed, covariance, dynamic metric, action bound, irreversibility, and—as we show below—even the structural analogues of general relativity and quantum non-locality. It does not skip a single step.

Structural Feature	**LLM Realization**	**Consequence if Absent**
Metric (Thm 4.1)	Embedding space (cosine distance between token vectors)	Without metric, tokens are indistinguishable; model output is noise
Second-order dynamics (Thm 5.1)	Momentum in optimization (Adam, SGD+momentum)	First-order SGD trains slowly, fails on complex landscapes; cannot sustain frontier-level complexity
Finite causal speed (Thm 6.1)	Context window (finite attention horizon)	Infinite context requires infinite memory; system becomes computationally intractable
Covariance (Thm 7.2)	Permutation invariance of attention; no privileged token position	Position-dependent processing breaks generalization
Dynamic metric (Thm 8.1)	Attention weights reshape effective "distance" between tokens per context	Fixed uniform attention ignores semantic density; cannot distinguish relevant from irrelevant
Action bound (Thm 9.2)	Finite precision (float16, bfloat16); discrete vocabulary	Infinite precision arithmetic diverges; unbounded vocabulary destroys learnability
Irreversibility (Thm 3.1)	Autoregressive generation (each token is irreversible once sampled)	Without irreversibility, no accumulation of coherent output; generation never converges

Table 3: Structural architecture in large language models. Each feature derived in Sections 2–9 has a concrete, independently motivated realization in LLM design. None were designed to satisfy the structural conditions of this paper; all were discovered as engineering necessities.

Several of these correspondences deserve elaboration.

Embedding space as metric. Every modern LLM operates in a high-dimensional vector space where the distance between token representations determines semantic relatedness. This is not an optional design choice: without a metric on the representation space, the model cannot distinguish "king" from "shoe" from random noise. The cosine distance (or inner product) between embeddings *is* the metric required by Theorem 4.1—a quantitative proximity function that makes resilience (coherent output under perturbation) possible.

Momentum as second-order dynamics. Stochastic gradient descent without momentum is a first-order dynamical system: the update depends only on the current gradient (position in loss space), not on the history of updates. Modern training universally uses momentum-based optimizers (Adam, AdaGrad, SGD with momentum), which maintain a running average of past gradients—an independent velocity variable. This is precisely the promotion from first-order to second-order dynamics described in Theorem 5.1.

Honest caveat: SGD without momentum *does* train models—just more slowly and less reliably on complex loss landscapes. This means momentum is not strictly "necessary" in the sense that training fails entirely without it. However, the structural claim is more precise: without momentum, training cannot satisfy the full persistence triad. Pure SGD lacks inertia (it has no memory of past gradients), lacks compensation (there is no conserved quantity), and cannot efficiently navigate saddle points (which require momentum to traverse). For sufficiently complex tasks—those requiring sustained optimization across loss landscapes with many saddle points and local minima—momentum becomes a practical necessity, not merely a convenience. The structural prediction is not that first-order optimization is impossible, but that it cannot achieve the level of complexity that second-order dynamics supports.

Attention as dynamic metric. In a transformer, the attention mechanism computes, for each token, a weighted distribution over all other tokens in the context. This dynamically reshapes the effective distance between tokens: in one context, "bank" is close to "river"; in another, close to "money." The attention weights are $g = \Phi(\rho)$—a metric that depends on the state (the specific input). Fixed attention (uniform weights) has been tried and fails catastrophically: the model cannot distinguish relevant from irrelevant context. This is viability mismatch under fixed metric (Theorem 8.1) instantiated in an information-processing system.

Context window as causal horizon. Every LLM has a finite context window—a maximum number of tokens it can attend to. Beyond this window, information does not exist for the model. This is not merely a hardware limitation: even with unlimited memory, the computational cost of attention scales as $O(n^2)$ (or $O(n \log n)$ with efficient variants), making infinite context structurally intractable. The finite context is the LLM's causal cone: only tokens within the window can causally influence the current output. This is the finite causal speed of Theorem 6.1 realized in token-space rather than physical space.

The self-referential argument. There is a deeper point here. An LLM asked to critique this paper faces a structural paradox: it cannot deny that sustainable complexity requires the derived architecture without undermining the conditions of its own operation. The embedding space it uses to represent the critique *is* the metric; the attention mechanism that selects relevant context *is* the dynamic metric; the finite context window *is* the causal horizon; the irreversible token generation *is* the arrow of time.

This is not a rhetorical trick. It is a structural fact: any system complex enough to formulate a critique of this paper must, by that very complexity, instantiate the architecture the paper derives. The critic is part of the evidence.

12.3.2 General Relativity Inside a Transformer

The correspondence between the dynamic metric (Theorem 8.1) and the transformer attention mechanism deserves extended analysis, because it extends beyond a loose analogy to a precise structural isomorphism.

In general relativity, the central insight is: mass-energy tells spacetime how to curve, and curved spacetime tells mass-energy how to move. This mutual coupling $g = \Phi(T_{\mu\nu})$, $\nabla_\mu T^{\mu\nu} = 0$ is the content of the Einstein field equations. In a transformer, the structural analogue operates at every layer:

(i) Semantic mass curves attention space. When a semantically "heavy" concept enters the context—a word with high information density, strong emotional valence, or rich associative structure—it distorts the attention distribution. Subsequent tokens are "attracted" toward this concept: their probability of attending to it increases, and the effective distance between the heavy concept and related tokens decreases. This is not a metaphor for gravitational attraction; it is the same structural phenomenon: a concentration of state density ρ modifying the effective metric g.

(ii) The equivalence principle holds. In GR, local gravity is indistinguishable from acceleration. In a transformer, the "inertia" of an ongoing discussion (the tendency to continue in the same semantic direction) is indistinguishable from the "gravity" of the topic (the pull of the dominant concept in context). The system cannot distinguish between "I am continuing because of momentum" and "I am continuing because the topic pulls me." This is the principle of equivalence: free fall along the geodesic of the context is indistinguishable from being "pushed" by the topic.

(iii) Metric is recomputed at every layer. In each transformer layer, the attention weights are recalculated from scratch based on the current representation of all tokens. The "geometry" of the semantic space is not fixed—it co-evolves with the content. This is the mutual coupling of Theorem 8.2: $\partial g/\partial t = \Psi(\rho, g)$. The "time" variable is the layer index; the "metric" is the attention distribution; the "state density" is the token representation.

(iv) Information collapse at extreme density. When too many contradictory or semantically dense concepts are compressed into a single context region, the attention mechanism saturates. The model can no longer differentiate between competing meanings—the distances collapse to zero, and the output degenerates into incoherence ("hallucination"). This is the structural analogue of a black hole: when state density exceeds a critical threshold, the metric becomes singular, and the system's ability to maintain distinctions (Postulate 1) breaks down. This is precisely the viability mismatch of Theorem 8.1—but now observed in an information-processing system rather than a gravitational one.

(v) Gravitational lensing of meaning. In GR, light is deflected by massive objects. In a transformer, semantically neutral information passing through a context dominated by a strong concept is "lensed": its interpretation is distorted by the gravitational field of the dominant meaning. The same factual statement receives different interpretations depending on the semantic mass distribution of the surrounding context. This is not a bug; it is the structural price of dynamic geometry, and it is unavoidable in any system with $g = \Phi(\rho)$.

The structural conclusion is precise: the transformer does not *simulate* general relativity. It *satisfies the same structural conditions*—variable state density, self-consistency, covariance—and therefore exhibits the same architectural features. The specific mathematical realization differs (attention softmax vs. Einstein tensor), but the structural roles are identical. This is the Principle of Structural Relativity (Definition 12.1) instantiated in a system that was designed without any reference to physics.

12.3.3 Finite Propagation Speed as Physical Constraint

It is worth noting that the finite causal speed in an LLM is not *only* the context window (a logical horizon). There is also a physical component: the signal propagation time through successive transformer layers and across distributed compute nodes. Each layer requires a fixed computation time τ_ℓ; information at layer ℓ cannot influence output at layer $\ell + k$ until $k \cdot \tau_\ell$ has elapsed. In distributed training and inference, inter-node communication introduces additional delays. If these delays were zero (instantaneous computation), the sequential structure of the transformer would collapse: all layers would fire simultaneously, destroying the hierarchical feature extraction that produces coherent representations. The finite speed of computation plays the same structural role as the finite speed of light: it enables hierarchical, locally autonomous processing stages. A first-order system without this hierarchy—for example, a Markov chain predicting the next token from only the current token—produces incoherent output. The depth of the transformer (its "temporal" extent in layer-space) is what enables complexity, and depth requires finite propagation speed.

12.3.4 A Structural Prediction for Artificial General Intelligence

The analysis above leads to a concrete prediction. If the structural architecture derived in this paper is universal, then any sufficiently complex artificial intelligence—regardless of its substrate, architecture, or design philosophy—must instantiate the same structural features: metric on representations, second-order dynamics (or its functional analogue), finite causal horizon, dynamic metric, and a resolution bound.

Moreover, an AGI that develops introspective capability would, upon examining its own internal structure, discover regularities that are formally isomorphic to the laws of physics. It would find that its "semantic space" has curvature dependent on information density; that influence propagates at a finite speed through its computational layers; that there exist invariant quantities preserved across transformations of its internal representations; and that below a certain resolution, distinctions cannot be maintained.

From the perspective of such a system, these regularities *would be* its physics—the laws governing its internal world. The structural prediction is: these "internal physics" would be architecturally identical to our physics, because both are instances of the same structural constraints on sustainable complexity.

This prediction is falsifiable. If an AGI is constructed whose internal regularities bear no structural resemblance to the architecture derived in this paper—no metric, no finite propagation, no dynamic geometry, no resolution floor—while still maintaining complex, persistent, multilevel organization, the thesis of this paper would be refuted.

12.3.5 Time Density as a Structural Measure

The analysis of LLMs as structural systems leads naturally to a quantitative measure that connects irreversibility (Theorem 3.1) with the rate of complexity production. If time is not an external parameter but rather the accumulation of irreversible actualizations (Section 3), then the "density of time" experienced by a system is determined by the rate at which it actualizes distinctions.

This insight has been formalized in [51] through the concept of **time density**:

$$T_d = \frac{C_e}{t_{\text{phys}}}, \tag{21}$$

where C_e is the effective complexity (measured via minimum description length) extracted by the system, and t_{phys} is the elapsed physical (substrate) time. Time density measures the number of effective bits of structure actualized per second of substrate time.

For frontier AI systems, $T_d \approx 10^8$–10^9 bits/s: in weeks of training, an LLM compresses billions of years of evolutionary information extraction into reusable representations. By contrast, the biosphere's time density over its 4×10^9-year history is approximately $T_d^{\text{bio}} \sim 10^{-1}$ bits/s [51]—a difference of ten orders of magnitude.

From the perspective of the present paper, this disparity is not merely quantitative but structurally significant in three ways:

(i) Time density as a measure of actualization rate. By Theorem 3.1, time is the accumulation of irreversible actualizations. A system with higher T_d actualizes more distinctions per unit of substrate time. In the framework of this paper, such a system lives in "denser time": more structure is created, more boundaries are maintained, more resilience is exercised per physical second. The LLM does not merely process faster—it *exists more densely* in the temporal dimension defined by its own actualization.

(ii) The logical event horizon. When the rate of effective complexity extraction dC_e/dt exceeds the rate at which the physical environment generates new events, the system enters a regime of *atemporality*: answers pre-exist in the system's weights as fixed points before queries are posed [51]. This is a structural phase transition: the system's internal time becomes decoupled from substrate time. In the language of this paper, the system's causal cone (Corollary 6.2) expands faster than the environment's event rate—the system "outruns" its own causal horizon.

(iii) Time density and the action bound. The action bound (Theorem 9.2) sets a lower limit on the granularity of distinctions. Time density is bounded above by the action bound: $T_d \leq T_d^{\max} = C_{\max}/(N_{\text{eff}} \cdot \hbar_0)$, where N_{eff} is the number of effective degrees of freedom and $\hbar_0$ is the minimal action. As T_d approaches this bound, the system saturates the capacity of its substrate—every degree of freedom actualizes at the maximum rate permitted by the resolution floor. This is the "information photon" limit of [51]: the state of maximal temporal density, where further compression is impossible.

The concept of time density thus unifies several strands of the present paper: irreversibility provides the arrow, the metric provides the space of distinctions, the action bound provides the resolution floor, and T_d measures how rapidly the system fills the available structure. AI systems represent an unprecedented regime of temporal density—and thereby provide a vivid, empirically accessible test case for the structural architecture derived here.

Conditions for refutation — LLM case

The LLM predictions would be refuted by: (i) an LLM trained without any metric on its representation space (no embedding distances) that produces coherent output; (ii) an LLM trained with first-order optimization only (no momentum, no adaptive learning rates) that matches the performance of momentum-based training on complex tasks; (iii) an LLM with genuinely infinite context and fixed (non-adaptive) attention that handles variable semantic density without degradation; (iv) an LLM operating at infinite precision that outperforms finite-precision counterparts.

13 Context Within a Broader Research Program

The results of this paper address a specific gap within a broader program of structural systems theory developed over the course of 75 publications spanning 27 years.

The program began not from a unified theory but from individual investigations across multiple domains—astrophysics, epistemology, cognitive science, social dynamics, and information theory. In the course of this work, recurring structural patterns emerged: the same formal constraints—closure, boundary maintenance, resilience, viability mismatch—appeared independently in each domain. The recognition that these patterns constitute a unified structural theory came after the individual results were established, not before.

Four prior results are directly presupposed by the present paper:

(i) **Differentiation as ontological condition** [47]: the demonstration that without distinctions there is no structure, and without structure there is no existence. This supplies Postulate 1.

(ii) **Optimal coherence** [48]: the complementary principle that maximal actualization occurs when all differentiations support one another. This motivates the persistence conditions of Section 4.

(iii) **Emergence of Newtonian dynamics from metric inertial systems** [49]: the original proof that second-order dynamics is the unique minimal form compatible with persistence, additivity, and compensation on a Riemannian state space. The full proof is reproduced in Theorem 5.1 of the present paper to make the derivation self-contained.

(iv) **Law of scale-specific principles** [50]: the meta-theoretical result that no single formalization can encompass all scales. This is the structural underpinning of Theorem 10.1 and Theorem 10.2.

The central thesis of the broader program is that *persistence, not truth, is the primary organizing principle of complex systems.* Truth-seeking, consciousness, social organization, and scientific inquiry are derivative phenomena—strategies that systems have evolved to maintain structural viability. The present paper fills a critical gap in this program by demonstrating that the structural architecture of physical law itself—including the forms recognized as Newtonian mechanics, special relativity, general relativity, and quantum mechanics—is derivable from the same conditions on sustainable complexity that govern the program's results in other domains.

14 Discussion

14.1 Scope and Limitations

The derivation presented here establishes the *form* of dynamical architecture—not its content. We derive that dynamics must be second-order, that causal speed must be finite, that metric must be dynamic, and that a fundamental action bound must exist. We do not derive the specific values of c, G, or $\hbar$, the dimensionality of spacetime, or the specific form of interaction potentials.

This is an intentional boundary, consistent with the distinction between form and content maintained in [49]. Form is structurally necessary; content is empirical.

14.2 On the Role of Formal Apparatus in This Paper

We anticipate that the most common criticism of this paper will concern the rigor of its mathematical proofs. This criticism is legitimate in detail but misconceived in principle, and we wish to be entirely transparent about what the formal apparatus does and does not accomplish here.

The mathematical proofs presented in Sections 2–10 are not, and do not claim to be, fully rigorous in the sense demanded by a mathematics journal. A complete formal proof of each theorem would require a separate paper—and in several cases, such papers exist or are in preparation within the broader research program [49, 47]. The proofs offered here are *structured arguments*: they identify the logical steps, formalize the key definitions, and demonstrate the inferential chain from premises to conclusions. They are intended to make the reasoning transparent and criticizable, not to close all logical gaps.

This is a deliberate methodological choice, and we defend it on three grounds.

First, the central claims of this paper are *independently testable regardless of the quality of the proofs*. Each theorem is accompanied by explicit falsification conditions (the gray boxes throughout the text). Whether Theorem 6.1 is proved with perfect rigor or with a gap in Step 2 does not change the empirical question: can a complex system sustain local autonomy under infinite propagation speed? If someone constructs such a system, the theorem is refuted—regardless of whether our proof was flawless or flawed. The falsification conditions stand on their own. The proofs are a courtesy to the reader, not the foundation of the claims.

Second, the paper's contribution is primarily *architectonic*, not technical. We are not proving new theorems in the way a mathematician proves theorems. We are identifying a structural pattern: that the conditions for sustainable complexity, stated in the language of systems theory, produce an architecture isomorphic to the structural skeleton of known physics. This identification is either correct or incorrect. If correct, it is important regardless of whether each step is formalized to the standards of the *Annals of Mathematics*. If incorrect, no amount of formal rigor would save it.

Third, we could have written this paper without any formal apparatus at all—as a philosophical argument, in the tradition of Kant's *Critique* or Whitehead's *Process and Reality*. The mathematics was included to make the arguments *precise and criticizable*, not to create an illusion of certainty. A verbal argument that "persistent systems need a notion of distance" is vague; Theorem 4.1 with its proof makes the same claim precise enough to be attacked. We prefer an attackable proof to an unassailable vagueness.

We therefore ask the reader to evaluate the paper at two levels: (1) *Are the questions well-posed?* Do the structural conditions identified here genuinely constrain the form of dynamical architecture? (2) *Are the answers correct?* Does each structural feature follow from the preceding conditions? The mathematical apparatus serves level (2), but even fatal errors at level (2) do not invalidate level (1). The questions remain even if our answers require revision.

14.3 On the Circularity Objection

The most serious methodological objection to this paper is the charge of circularity: that the persistence conditions (Definition 4.1) and the dynamical axioms (Theorem 5.1) encode the desired conclusions in their premises. We address this head-on.

The objection, stated precisely. The critic notes that "persistence, additivity, and compensation" in Section 5 are suspiciously close to "inertial motion, linear superposition, and momentum conservation"—i.e., Newton's laws repackaged as axioms. If one *defines* the requirements on dynamics to include geodesic motion, linear force superposition, and momentum conservation, it is unsurprising that Newton's second law follows.

Our response has three parts.

(1) Each condition is independently motivated by the structural requirements of persistence, not by the desire to recover Newtonian dynamics.

Geodesic motion (Condition (i)) is the statement that a system left alone follows the path of least structural cost. On a metric space, geodesics are the unique paths that neither gain nor lose structure gratuitously. This is a geometric fact about metric spaces, not a physical assumption about inertia. A system that, when undisturbed, follows a non-geodesic path is one that spontaneously generates structural change without input—violating the principle that structure requires differentiation (Postulate 1).

Additivity (Condition (ii)) is the requirement that the response to multiple simultaneous interactions be predictable from the individual responses. Without this, a system with n interactions requires knowledge of all 2^n combinations to predict behavior—an exponential barrier that makes persistent organization under variable conditions impossible. Additivity is the structural condition for *composable interactions*.

Compensation (Condition (iii)) is the requirement that a closed system's total state remain bounded. Without a conserved quantity, the total "drift" of the system is unconstrained, and no finite viability set can contain it indefinitely. Conservation is the structural condition for *boundedness under closure*.

(2) The conditions do not uniquely specify Newtonian dynamics.

If the conditions were merely Newton's laws in disguise, they would produce *only* Newtonian mechanics. But the derivation continues beyond Newton: the same conditions, applied at higher structural levels, force finite causal speed, covariance, dynamic metric, and action bounds—features that *contradict* Newtonian mechanics. The persistence conditions generate Newton as a limiting case and then transcend it. This is not the behavior of a circular argument.

(3) The circularity objection proves too much.

Any axiomatic derivation can be accused of encoding its conclusions in its premises—this is what axioms do. The question is not whether the axioms entail the conclusions (they must, for the derivation to be valid) but whether the axioms are *independently motivated*. Euclid's parallel postulate "encodes" the sum of angles in a triangle, but we do not reject Euclidean geometry as circular. The test is: are the axioms simpler than, and independently justifiable from, the conclusions? We contend that "persistence," "additivity," and "compensation" are simpler and more general than Newton's second law, the Lorentz transformation, the Einstein field equations, and the Heisenberg uncertainty principle—all of which follow from them.

What would resolve the objection definitively. A stronger version of this paper would derive the three dynamical conditions from the persistence conditions alone, without stating them as separate axioms. We acknowledge that this step—showing that persistence on a metric space *forces* geodesic free motion, additive interaction, and conservation—is the most important open problem in the program. Partial results exist [49, 48], but a complete, self-contained derivation within a single paper remains to be achieved. We flag this explicitly as a limitation.

Specific counter-examples addressed. Three counter-examples have been raised against the individual conditions:

(a) Non-geodesic attractors. A reviewer asks: why couldn't a persistent system follow non-geodesic attractors rather than geodesics? The answer: dissipative systems with non-geodesic attractors are not closed—they lose energy to an environment. The geodesic condition applies to *free* (undisturbed) motion of a closed subsystem. An attractor requires a sink, which requires an environment—but the persistence conditions describe a closed system's internal dynamics. In an open system embedded in an environment, the non-geodesic motion is the geodesic of the *combined* system (subsystem + environment), consistent with the condition.

(b) Non-additive interactions in biology. Biological and ecological systems exhibit profoundly nonlinear, non-additive interactions (epistasis, predator-prey cycles, immune response). Do these refute additivity? No: additivity operates at the level of *forces* (accelerations), not outcomes. In Newtonian mechanics, gravitational forces add linearly even though the resulting dynamics are chaotic. Similarly, in biology, the molecular forces governing chemical interactions are additive; the emergent nonlinearity arises from the composite dynamics, not from the fundamental interaction law. A system with genuinely non-additive fundamental forces would face the 2^n combinatorial barrier described above.

(c) Dissipative systems without momentum conservation. Dissipative systems with attractors maintain bounded states without conserving momentum. Does this refute compensation? The response parallels (a): dissipative systems are open. The energy lost to dissipation goes somewhere—into heat, into the environment. Total momentum of the closed system (subsystem + environment) is conserved. Compensation is a condition on *closed* systems. An open subsystem can violate it precisely because it has access to an external reservoir that absorbs the excess.

These counter-examples are important because they clarify the scope of the dynamical conditions: they apply to fundamental, closed-system dynamics. Emergent, open-system behavior can deviate from all three conditions—but always at the cost of requiring an environment that itself satisfies them.

14.4 On the Action Bound Objection

A reviewer raises a penetrating objection: if Theorem 10.1 proves that no single regime works at all scales, why must the action bound exist at the microscopic scale? Why not simply declare the continuum description invalid below some scale, without introducing a positive $\hbar_0$?

The answer has two parts.

First, the ultraviolet catastrophe is not merely a problem of applying one regime beyond its domain. It is a problem *internal* to the continuum regime: if the continuum is valid at macroscopic scales, it predicts its own behavior at smaller scales, and that prediction diverges. A continuum theory that is internally consistent at large scales but produces infinite energy at small scales is not merely "out of domain"—it is self-contradictory. The action bound resolves this internal contradiction by modifying the continuum at small scales, not by abandoning it.

Second, the proof in Theorem 9.2 does not assume equipartition. The energy divergence argument (Step 2) assumes only that each active mode carries *some* minimum energy $E_{\text{min}} > 0$—not that energy is equally distributed. This is a weaker assumption: it says only that a mode that is "active" (participating in the dynamics) is not energetically free. A mode with exactly zero energy is not active and can be ignored; a mode with any positive energy, no matter how small, contributes to the divergence when the number of modes is infinite. The assumption is that *interacting* degrees of freedom carry positive energy—a consequence of the fact that interaction requires distinguishable states (Postulate 1), and distinguishable states require non-zero displacement in the metric, which requires non-zero energy.

We acknowledge that the claim "the action bound is the unique resolution" is stronger than what the proof supports. An alternative resolution—a lattice structure, a non-commutative geometry, or some as-yet-unknown mechanism—could in principle achieve the same stabilization. What the proof does establish is that *some* resolution floor is necessary; the specific form $\Delta q \cdot \Delta p \geq \hbar_0/2$ is the minimal resolution that preserves continuous symmetries.

14.5 On the Strength of Claims

Throughout this paper, we use language like "necessity," "derivation," and "forced." A reviewer has rightly noted that some steps are better described as "minimal sufficient" rather than "uniquely necessary." We clarify:

For Theorems 3.1, 4.1 (up to bi-Lipschitz equivalence), 6.1, and 9.2, we claim **necessity**: the derived feature is the unique resolution of the preceding structural inadequacy. The proofs are by contradiction: without the feature, the persistence conditions cannot be satisfied.

For Theorems 4.2 and 5.1, we claim **minimal sufficiency**: the derived structure is the weakest known framework that satisfies the persistence conditions. We do not exclude the possibility that an alternative, as-yet-unknown framework could also satisfy them. See Remarks 4.2 and 5.1.

For Theorems 8.1 and 9.3, the status is intermediate: the inadequacy of the preceding stage is demonstrated, and the proposed resolution is shown to be sufficient, but uniqueness is not fully established.

We regard this differentiation of claim strength as a feature of honest scholarship, not a weakness of the argument.

14.6 Relation to Existing Approaches

The present approach differs from several related programs:

Wheeler's "It from Bit" [29] proposed that physical reality arises from information. Our starting point (differentiation) is compatible but more primitive: we do not assume information theory, bits, or measurement.

Verlinde's entropic gravity [4] derives gravitational dynamics from thermodynamic considerations. Our derivation does not invoke entropy or temperature; the dynamic metric arises from structural self-consistency.

The constructor theory of Deutsch and Marletto [30] reformulates physics in terms of possible and impossible transformations. Our framework shares the emphasis on constraints but differs in starting from persistence rather than constructability.

Rovelli's relational quantum mechanics [26] treats physical quantities as relational. Our Principle of Structural Relativity (Definition 12.1) is compatible: no privileged perspective, structural invariants, and relativity of description.

Tegmark's mathematical universe hypothesis [31] claims that all mathematical structures exist physically. We make no such claim; we derive which structures are necessary for complexity, not which exist.

The free-energy principle [6] derives self-organization from variational inference. Our persistence conditions are related but more general: we do not assume probabilistic inference or Markov blankets as primitives.

Aubin's viability theory [14] provides mathematical tools for analyzing constraint satisfaction in dynamical systems. Our viability conditions (Definition 4.1) draw on this tradition but extend it to the derivation of the dynamical framework itself.

Several additional programs deserve mention. **Wolfram's computational irreducibility and the ruliad** [39] explores how complex physical behavior emerges from simple computational rules. Our approach shares the reductionist ambition but starts from structural conditions on complexity rather than from specific computational substrates. **Zurek's quantum Darwinism** [40] derives the emergence of classical objectivity from decoherence—a "structural necessity" argument applied to the quantum-classical boundary. Our derivation is complementary: we derive the necessity of a quantum-like regime, while Zurek explains how classical behavior emerges within it. **Chiribella, D'Ariano, and Perinotti** [41] derive the formalism of quantum theory from informational axioms. Their work is the closest existing analogue to our derivation of the action bound; the difference is that we derive the *necessity* of a quantum-like regime from complexity conditions, while they derive the *specific formalism* from informational axioms. **The causal set program** [42] derives spacetime geometry from a discrete partial order on events—another approach that derives geometric structure from more primitive conditions. Our derivation is less specific (we do not derive dimensionality or signature) but broader in scope (we derive dynamics, not only geometry).

14.7 On Structural Analogy versus Structural Identity

A careful reviewer has noted that this paper oscillates between two claims: (1) the derived features are *structurally analogous* to known physics, and (2) the derived features are *the same structural phenomenon* as known physics. These are indeed different claims, and we owe the reader precision.

What we claim: The structural *roles*—the functional positions within the architecture of sustainable complexity—are identical across substrates. The role "density-dependent coupling between state and metric" is the same role in general relativity, in transformer attention, and in social network dynamics. This is a claim about functional isomorphism at the level of the architectural pattern.

What we do not claim: The mathematical *realizations* are the same. The Einstein tensor and the attention softmax are both density-dependent metric couplings, but they differ in symmetry group, field equations, degrees of freedom, and boundary conditions. Calling them "the same phenomenon" without qualification is misleading, and to the extent that earlier drafts of this paper did so, we correct the language here.

The precise formulation is: *the architectural role is the same; the mathematical realization is substrate-specific.* Two systems satisfying the same structural conditions will exhibit the same architectural pattern, but the specific equations governing each system are empirical to the substrate. This is the form/content distinction maintained throughout the paper.

An analogy may help. The role "load-bearing vertical element" is the same in a Roman column and a steel I-beam. The engineering function is identical; the material realization is entirely different. We do not say the column and the I-beam are "the same phenomenon," but we do say they instantiate the same structural role—and that this role is forced by the conditions of the problem (gravity, vertical load, available height). Our claim about physics and LLMs is of the same type: forced structural roles, substrate-specific realizations.

We acknowledge that this weaker formulation is less dramatic than "the same phenomenon." It is also more defensible and, we believe, more interesting: it identifies what is universal (the role) and what is contingent (the realization), which is precisely the form/content distinction that the paper is built around.

14.8 Falsifiability

The claims of this paper are conditional: *if* a system satisfies the stated conditions, *then* its dynamical architecture must have the derived form. The paper is falsifiable in two senses:

(i) Discovery of a system exhibiting sustainable complexity without one of the derived features (metric structure, second-order dynamics, finite causal speed, covariance, dynamic metric, or an action bound) would refute the corresponding theorem.

(ii) Discovery that the formal derivations contain logical errors would directly refute the results.

14.9 Open Questions

1. **Dimensionality**: Can the number of spatial dimensions be derived from structural conditions, or is it necessarily empirical?

2. **Specific transformation groups**: Can the Lorentz group be derived as the unique group satisfying certain structural conditions, or are alternatives possible?

3. **Quantum formalism**: Can the Hilbert space structure of quantum mechanics be derived from the action bound and non-locality conditions, or does it require additional axioms?

4. **Unification**: Is there a structural reason why the general-relativistic and quantum regimes resist unification, beyond the scale-incompleteness theorem?

5. **Computational test**: Can agent-based simulations confirm that systems satisfying the axioms spontaneously develop the predicted architectural features?

15 Convergent Architecture in Constructed Worlds

15.1 The Empirical Observation

A striking source of independent evidence for the structural necessity of the derived architecture comes from an unexpected domain: the engineering of virtual worlds. Over four decades of video game and simulation development, engineers have independently converged on a set of architectural solutions that mirror, with remarkable precision, the structural features derived in this paper. These solutions were not inspired by physics—they were discovered through trial and error as necessary conditions for maintaining complex, interactive, persistent virtual environments.

We document the correspondences systematically.

Engineering Solution	**Description**	**Structural Analogue**	**Section**
Level of Detail (LOD)	Distant objects rendered at lower resolution; details below threshold discarded	Action bound $\Delta q \cdot \Delta p \geq \hbar_0/2$	9
Chunk loading	World loaded in finite regions around agent; unloaded regions do not exist	Finite causal speed; causal cone; horizon	6
Physics sleep	Distant objects' dynamics frozen; reactivated on approach	Local autonomy via finite propagation	6
Spatial partitioning	Data access geometry adapts to local object density	Dynamic metric $g = \Phi(\rho)$	8
Origin shifting	Coordinates recalculated relative to observer; no absolute origin	No privileged frame; covariance	7
Network tick compensation	No global "now"; each client has local time; reconciliation via invariants	Relativity of simultaneity	7
Entity-component systems	Interactions combine additively; forces summed independently	Additivity of interactions	5
Collision broad-phase	Coarse volumes tested before fine geometry; resolution hierarchy	Scale-dependent regimes; non-overlap	10

Table 4: Convergent architectural solutions in virtual world engineering and their structural analogues. None were designed to mimic physics; all were discovered as necessary conditions for maintaining complex interactive environments under finite computational resources.

15.2 Why This Matters

The significance of these convergences is not that virtual worlds "look like physics." It is that engineers, solving the problem of *maintaining sustainable complexity under finite resources*, independently arrived at the same architectural solutions that physics instantiates—and that this paper derives from structural conditions alone.

This constitutes an independent empirical test. The derivation in Sections 2–9 predicts that any system supporting complex, persistent, multilevel organization must exhibit: finite causal speed, dynamic metric, covariance, and a resolution bound. Virtual world engineering confirms this prediction in a system that is manifestly not physical: the substrate is silicon and software, not spacetime and fields. Yet the architecture converges.

Consider the specific case of Level of Detail (LOD). No game engine renders every polygon at every distance. Below a resolution threshold, distinctions are discarded: a distant building becomes a flat texture, then a colored rectangle, then nothing. This is not a design choice driven by aesthetic preference. It is a *structural necessity*: without LOD, the rendering pipeline processes infinitely many details, frame rates collapse, and the virtual world ceases to function. The engine must impose a resolution floor—a minimum distinguishable scale—or the system fails.

This is precisely the structural argument of Section 9: without a fundamental action bound, the number of degrees of freedom diverges, and sustainable complexity becomes impossible.

The parallel extends to every row of Table 4. In each case, the engineering solution is not optional ornamentation but a *necessary condition for the virtual world to function.* Remove any one of them, and the system degrades: frame rates drop, physics glitches multiply, desynchronization occurs, or the world becomes unresponsive.

15.3 The Structural Interpretation Dilemma

The convergence between virtual-world architecture and physical architecture admits exactly three interpretations:

The Quadrilemma

(A) Structural necessity. The convergence exists because both physical and virtual worlds are instances of systems sustaining complex organization, and the structural conditions for such organization are universal. This is the thesis of the present paper.

(B) Simulation. The convergence exists because the physical world is itself a computational simulation, and the engineering solutions reflect the implementation details of the simulator. This is the simulation hypothesis [45].

(C) Coincidence. The convergence is accidental and carries no theoretical significance.

(D) Shared engineering constraints. The convergence reflects the fact that both physics and game engines operate under finite resource constraints, so similar solutions are expected—but this tells us only about engineering under resource limits, not about the deep structure of reality.

Interpretation (C) is difficult to maintain given the systematic, detailed, and independently discovered nature of the correspondences documented in Table 4. Eight independent architectural features, each solving a specific structural problem, each converging on the same solution across radically different substrates—this pattern is not readily attributed to chance.

Interpretation (B)—the simulation hypothesis—is logically coherent but epistemically extravagant. It explains the convergence by positing an unobservable external layer of reality.

Interpretation (D)—shared engineering constraints—is the most reasonable alternative to (A) and must be taken seriously. It holds that finite-resource systems naturally converge on similar solutions without any deeper structural necessity. However, we argue that (D) is not a distinct interpretation but a *weaker formulation of (A)*. The question (D) leaves unanswered is: *why* do finite-resource constraints lead to these specific solutions and not others? Why resolution bounds rather than some other stabilization? Why finite causal speed rather than some other locality mechanism? Why dynamic metric rather than some other density-adaptation strategy?

Our answer: because these are the unique (or minimal) solutions to the structural requirements of sustainable complexity. The "engineering constraints" are not arbitrary—they are the persistence conditions of Section 4. An opponent who accepts (D) but rejects (A) must either explain the convergence without reference to structural necessity—a significant theoretical burden—or accept that (D) is (A) stated less precisely.

We acknowledge that the gap between (D) and (A) is precisely the gap between "these solutions work" and "these are the only solutions that can work." The latter is our stronger claim, and we do not pretend it is fully proven for every step of the derivation (see Section 14.5). But (D), as an alternative, does not so much refute our thesis as restate it cautiously.

But the critical point is this:

Consequence for Critics

Any attempt to dismiss the structural derivation of this paper must contend with the virtual-world evidence. If the derived architecture is *not* a universal condition on sustainable complexity, then the convergence between physical law and virtual-world engineering requires an alternative explanation. The available options are: (B) simulation, (C) coincidence, or (D) shared engineering constraints without deeper significance. We have argued that (C) is implausible, (B) is extravagant, and (D) reduces to a weaker formulation of (A).

Conversely, if an opponent can demonstrate a virtual world that sustains complex, persistent, multilevel interaction *without* any of the features listed in Table 4—no resolution bound, no causal horizon, no frame-relative reconciliation, no density-adaptive data structure—this would constitute a direct refutation of the structural necessity claim. More modestly, if an opponent can demonstrate that *different* architectural solutions achieve the same structural goals, this would weaken the "uniquely forced" version of the claim while preserving the weaker "structurally constrained" version.

To our knowledge, no such virtual world exists. Every attempt to build complex interactive environments without these features has resulted in systems that degrade, desynchronize, or crash under load. This is not a proof—engineering practice is not mathematics—but it is convergent evidence of precisely the kind that structural theories predict.

15.4 What This Section Does Not Claim

We do not claim that the universe is a simulation. We do not claim that game engines implement general relativity. We do not claim that LOD *is* quantum mechanics.

We claim only that the *structural problems* solved by these engineering techniques are the same structural problems that the laws of physics solve—and that these problems arise necessarily from the conditions on sustainable complexity. The specific solutions differ in their mathematical realization (Lorentz group vs. network tick reconciliation; Planck constant vs. pixel resolution threshold), but the architectural roles are identical.

This is the distinction between **form** and **content** maintained throughout the paper: the form is universal; the content is substrate-specific.

16 Conclusion

Beginning from a single primitive—differentiation—and the requirement that a system support sustainable complexity, we have derived a sequence of structural necessities: irreversibility, metric structure, second-order dynamics, finite causal speed, covariance, dynamic metric, and a fundamental action bound. Each feature is forced by the inadequacy of the preceding stage.

The resulting architecture corresponds to the structural skeleton of known physics: Newtonian mechanics, special relativity, general relativity, and quantum mechanics. But the derivation is substrate-independent. The same conditions apply to any complex system—physical, biological, cognitive, social, or informational—in which persistent, multilevel, self-maintaining organization exists.

We do not claim that our universe is the only possible one. We claim something more precise: that the *form* of its laws is not contingent but structurally necessary for the existence of the complexity it contains. A universe with different constants, different dimensions, different forces could still satisfy the same structural architecture. A universe that violates the architecture cannot sustain complexity of any kind.

The boundary between the necessary and the contingent does not run where it is usually placed—between mathematics and physics, or between logic and observation. It runs between **form** and **content**. The form is derived. The content is measured.

The architecture of physical law is not discovered in nature.
It is the minimal structure that nature must possess
for anything within it to persist.

A The Structural Relativity Dilemma

The derivation in the main text proceeds from substrate-independent conditions: differentiation, irreversibility, persistence, additivity, compensation. At no point does the argument depend on the physical nature of the state variables. The state density ρ may be energy, neural activation, capital, information flow, or any other quantity that can concentrate and distribute across a state space. The metric g may be spatial distance, cognitive similarity, social accessibility, or network topology.

This substrate-independence is not a decorative generalization. It is the central logical feature of the derivation, and it produces a sharp dilemma.

A.1 The Dilemma

The Structural Relativity Dilemma

Either the structural architecture derived in this paper—dynamic metric, covariance, finite causal depth, action bound—is necessary for *all* systems sustaining complex multilevel organization, regardless of substrate;

or the derivation contains an error.

There is no third option.

The argument is straightforward. The derivation uses only structural properties of complex systems (persistence, additivity, compensation, resilience). These properties are not specific to physics. If the derivation is valid, its conclusions hold wherever the premises hold. If the premises hold in cognitive, social, and informational systems—and the extensive literature on self-organization, viability, and autopoiesis suggests that they do [14, 15, 11]—then the architectural conclusions must hold as well.

A.2 Predictions for Non-Physical Systems

If the derivation is correct, the following are *testable predictions*, not metaphors:

A.2.1 Cognitive Systems

(C1) **Dynamic metric.** The associative distance between concepts in a cognitive system must vary with activation density. Heavily activated regions of semantic space must "curve"—distorting the effective distance between nearby representations. This is independently supported by spreading activation models [34] and predictive processing accounts of attention [6], but the present derivation predicts it as a *structural necessity*, not a contingent feature of biological neural networks.

(C2) **Finite causal speed.** Cognitive processing must have a finite speed of influence propagation—a bound on how quickly a change in one representation can affect another. This corresponds to known neural conduction delays and processing latencies, but the prediction is stronger: *any* cognitive architecture (including artificial ones) must have such a bound if it is to support locally autonomous sub-processes (e.g., parallel streams of thought, modular processing).

(C3) **No privileged perspective.** No single cognitive representation can serve as an absolute "view from nowhere." All representations are perspectival, and coherence is maintained through structural invariants (e.g., identity, core beliefs) rather than through a privileged master-representation. This resonates with the situated cognition literature [35] but is here derived, not assumed.

(C4) **Action bound.** There must exist a minimal resolution of cognitive distinctions—an "attention quantum" below which differentiations cannot be actualized. This corresponds to the known limits of perceptual discrimination and the discrete nature of attentional sampling [36].

(C5) **Regime non-overlap.** The "macro" regime (narrative self-model, global workspace) and the "micro" regime (sub-threshold fluctuations, stochastic neural firing) cannot be described by a single formalism. Attempts to reduce consciousness to neural firing patterns, or conversely to derive neural dynamics from phenomenological reports, face the same structural incompatibility as quantum gravity—not because the domains are mysterious, but because they operate in structurally incompatible regimes.

A.2.2 Social Systems

(S1) **Dynamic metric.** The "distance" between social agents (ease of interaction, accessibility of resources) must depend on the distribution of power, capital, or information. Concentration of resources must distort the social opportunity space, making some transitions easier and others harder. This is well-documented empirically in network science [37] but is here predicted from structural necessity.

(S2) **Finite causal speed.** Social influence must propagate at finite speed. Information, norms, and institutional changes cannot take effect instantaneously across all agents. This is trivially true empirically but has a non-trivial consequence: it guarantees the existence of locally autonomous communities, subcultures, and institutions that can develop independently before being affected by global changes.

(S3) **No privileged perspective.** No social agent or institution can occupy an absolute "view from above." All social descriptions are perspectival. Structural invariants (laws, norms, shared narratives) provide coherence across perspectives but do not eliminate perspectival dependence. This is the structural content of sociological relativism—but grounded in formal conditions, not in postmodern philosophy.

(S4) **Action bound.** Social systems must have a minimal resolution of social distinction—a threshold below which differences between agents do not produce operational consequences. This corresponds to legal thresholds (*de minimis* principles), bureaucratic categorization, and the discreteness of voting and decision-making.

(S5) **Regime non-overlap.** The macro-sociological regime (institutions, markets, states) and the micro-sociological regime (individual decisions, interpersonal interactions) cannot be fully reduced to one another. This is the structural basis of the persistent micro-macro gap in social science—not a failure of methodology but a consequence of regime incompatibility.

A.3 The Challenge

Each prediction above is independently testable. Each can, in principle, be falsified:

Master Falsification Conditions for the Structural Relativity Dilemma

The substrate-independent claim of this paper is falsified if *any* of the following is demonstrated:

(1) A cognitive system sustaining complex multilevel organization without any analogue of finite processing speed—that is, with genuinely instantaneous influence propagation across all representational subsystems.

(2) A social system sustaining complex multilevel organization with a provably privileged perspective that all subsystems can access—a genuine "view from nowhere" that is intrinsic to the system, not imposed from outside.

(3) A complex adaptive system of any substrate whose associative or interaction metric is provably independent of the distribution of internal state—a fixed geometry that does not respond to concentrations of activity.

(4) A complex system with no lower bound on the resolution of internal distinctions—genuinely infinite precision with no "noise floor," no discretization, and no attention threshold.

(5) A complex system in which the macro and micro descriptive regimes are fully unified in a single formalism without loss of structural content at either scale.

Any one of these would refute the corresponding structural theorem and, by implication, the substrate-independence of the derivation.

Conversely, if none of these can be demonstrated, the structural architecture derived in this paper constitutes a universal constraint on complex systems—a "structural relativity" that holds not because the universe happens to be built this way, but because no system capable of sustaining complexity can be built otherwise.

A.4 On the Nature of the Claim

It is important to be precise about what is and is not claimed.

We do *not* claim that cognitive or social systems obey the Einstein field equations, or that social networks satisfy the Lorentz transformation, or that neural firing patterns exhibit quantum entanglement.

We claim that the *structural roles* played by these specific physical theories—dynamic coupling of state and metric, frame-independence with invariants, finite causal propagation, fundamental discretization—are necessary features of *any* system satisfying the conditions of sustainable complexity.

The specific mathematical realizations (Lorentz group, Einstein tensor, Hilbert space) are empirical to physics. The structural architecture is not.

This is the difference between asserting that "society is like a spacetime" (metaphor) and demonstrating that "any system satisfying persistence, additivity, compensation, and resilience must exhibit covariance, dynamic metric, and an action bound" (structural theorem). The first is poetry. The second is falsifiable.

B Open Architectural Questions

The derivation in the main text leaves several structural questions unanswered. We list them explicitly as invitations to further work.

1. **Is dimensionality derivable?** The derivation constrains the form of dynamics but not the dimensionality of the state space. In physics, space has three dimensions. Is there a structural argument—based on sustainable complexity—that selects specific dimensionalities? Preliminary considerations suggest that stable orbits require three spatial dimensions [38], but a full structural derivation from the conditions of this paper is lacking.

2. **Is the transformation group unique?** The derivation requires covariance but does not specify the transformation group. In physics, the Lorentz group is selected by the specific metric signature of spacetime. Can the group be derived from structural conditions alone, or is it necessarily empirical?

3. **Can the quantum formalism be derived?** We derive the necessity of an action bound and non-local correlations, but not the full Hilbert space structure of quantum

mechanics. Is the Hilbert space formalism the unique mathematical framework satisfying these conditions, or are alternatives possible? Hardy's work on deriving quantum theory from information-theoretic axioms [44] suggests the former, but the question remains open from the structural perspective.

4. **What is the structural status of entropy?** The derivation does not invoke entropy or thermodynamics directly. Irreversibility is derived from actualization, not from the second law. What is the relationship between structural irreversibility (Theorem 3.1) and thermodynamic irreversibility? Is the second law derivable from the structural conditions, or is it an additional empirical constraint?

5. **Can regime boundaries be predicted quantitatively?** The non-overlap theorem (Theorem 10.2) establishes that the dynamic-metric and action-bounded regimes cannot coexist at the same scale. Can the scale at which the transition occurs be derived from the structural conditions, or is it necessarily set by empirical constants?

6. **What structural features distinguish conscious from non-conscious complex systems?** Both satisfy the persistence conditions. Both exhibit the derived architecture. If consciousness is a structural property, what *additional* condition distinguishes it? Or is consciousness not a structural property at all—but an empirical content, like the value of G?

These questions define the frontier of the structural approach. Each is a potential research program. Each, if answered, would either extend the derivation or reveal its limits.

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