In this part of our thesis, we would like to compare our setting to the setting presented by [Gazi, diploma thesis] (see Section [number of definitions Section]).

Definition 1. The language L is called *T-decomposable*, if there is a language L_{adv}, which is an effective advice for L.
Otherwise, we call L *T-undecomposable*.

Theorem 1. If there exists a non-trivial ASB-decomposition for language L [would like to write it in different way (using advisors, not decomposition), but did not find suitable definitions in Gazi's diploma thesis], then L is T-decomposable.

Proof. Easy to see, using an A-transducer computing the identity. □

However, the next theorem shows, that the reverse implication does not hold.

Theorem 2. There are infinitely many T-decomposable languages, thate are not ASB-decomposable.

Proof. Such languages are for example $L_x = \{u\$xv | u, v \in \{a, b\}^*\}$ for a fixed string $x \in \{a, b\}^*, |x| \geq 14$ and even.

We prove this claim in two steps. First, we need to show, that L_x is T-decomposable. It is easy to see, that a DFA accepting L_x needs at least $|x| + 1$ states, therefore $\mathscr{C}_{state}(L_x) = |x| + 1$.

However, we can use an advice to simplify the accepting automaton as follows: our A-transducer T will read the input word in the initial state with no output, until it finds the special marker \$. Then, using another three states, it encodes pairs of symbols (i. e. sequences aa, ab, ba, bb) into new letters c, d, e, f, respectively. If there is just one symbol in the end, T will read it and traverse into accepting state q_F with no further transitions, otherwise it will make an ϵ-transition into q_F. Note, that T uses just five states.

Now, the advise language $L_{x,adv} = \{x'v | x'$ is the aforementioned encoded form of x into symbols $c, d, e, f\}$. Clearly, $|x'| = \frac{|x|}{2}$ and $\mathscr{C}_{state}(L_{x,adv}) = \frac{|x|}{2} + 1$.

The decider D needs construct just an automaton for $\{a, b, \$\}^*$, since the advice gives full information about L_x. Alltogether, we used $5 + \frac{|x|}{2} + 1 + 1$ states, therefore for $|x| \geq 14$ is $L_{x,adv}$ with T an effective advice with regard to L_x.

Our next goal is to show, that L_x is not ASB-decomposable. [ToDo: directly or using necessary conditions from Gazi]□

Corollary 2.1. There are infinitely many T-decomposable languages.

Theorem 3. There are infinitely many T-undecomposable languages.

To prove this theorem, we show the following lemma.

Lemma 4. Each of the languages $L_n = \{w \in \{a,b\}^* | \#_a(w) \mod p \equiv 0, p$ is the n-th prime$\}$ is T-undecomposable.

Proof.

TODO: proof - probably using some alternation of pumping lemma, try to show, that the number of equivalence classes can't be reduced, since p is prime

□

As we have seen, the class of regular languages can be divided into three parts - both ASB-decomposable and T-decomposable languages; ASB-undecomposable, but T-decomposable; and T-undecomposable languages [I will try to write this in a better way]. In the next part of our thesis, we would like to investigate some properties of these classes.