HOMEWORK 9

1. Find the invariant factors of the quotient group $\mathbb{Z}^3/N$, where N is generated by $(-4, 4, 2)$, $(16, -4, -8)$, $(12, 0, -6)$ and $(8, 4, 2)$.

2. Find the rational canonical form over $\mathbb{Q}$ of the matrix

$$\begin{pmatrix} -2 & 0 & 0 \\ -1 & -4 & -1 \\ 2 & 4 & 0 \end{pmatrix}$$

3. Find the rational canonical form over $\mathbb{Z}/2\mathbb{Z}$ of the matrix

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

4. Let $V \subset \mathbb{R}[x, y]$ be the subspace of all polynomials of the form $ax + by + c$, where $a, b, c \in \mathbb{R}$. Let $\mathcal{A}$ be a linear operator in V defined by

$$\mathcal{A}(ax + by + c) = a(x + 1) + b(y - 1) + c.$$

Find the elementary divisors and the canonical form of $\mathcal{A}$.

5. Find the Jordan canonical form over $\mathbb{C}$ of the matrix

$$\begin{pmatrix} 2i & 1 \\ 1 & 0 \end{pmatrix}$$

6. Prove that two 2×2 matrices over a field that are not scalar matrices are similar if and only if they have the same characteristic polynomials.

7. Prove that two 3×3 matrices are similar if and only if they have the same characteristic and the same minimal polynomials.

8. Show that the minimal polynomial of an $n \times n$-matrix A has the same irreducible divisors as the characteristic polynomial of A.

9. Let A be a nilpotent $n \times n$-matrix (that is $A^N = 0$ for some $N > 0$). Show that the invariant factors of A are powers of X. Prove that $A^n = 0$.

10. Prove that any $n \times n$-matrix A is similar to its transpose A^t.