(1) For each $k \in \mathbb{N}$, let E_k denote some event.
(a) Show that

$$\{ \text{ infinitely many } E_k \text{ occur } \} = \bigcap_{n=1}^{\infty} \left[\bigcup_{k=n}^{\infty} E_k \right].$$

This event is known as $\limsup_{k\to\infty} E_k$.
(b) Show that the indicator random variable of $\limsup_{k\to\infty} E_k$ is given by

$$\limsup_{k\to\infty} 1_{E_k}(\omega)$$

(c) We define $\liminf_{k\to\infty} E_k$ to be the event that all but finitely many E_k occur. Find and prove analogues of parts (a) and (b) for this event (use DeMorgan's laws).

(2) I repeatedly attempt the same task. My probability of success, on the k^{th} attempt is $k(k+2)/(k+1)^2$, independent of the outcomes of all previous attempts. Here $k \in \{1, 2, 3, \dots\}$. What is the probability that I never fail?

(3) (a) Starting at the origin on the line, I take a step of one unit to the left or to the right with probability 1/2. I do this repeatedly with independent steps. If I take $2n$ steps, what is the probability that I find myself back at the origin?
(b) You and I independently complete this same random walk. What is the probability that we end up in the same location?

(4) (a) Compute the probability that $X \sim \text{Geometric}(p)$ is even.
(b) Compute the probability that $X \sim \text{Poisson}(\lambda)$ is even.
(c) Compute the probability that $X \sim \text{Binomial}(n, p)$ is even.

(5) I carry out a sequence of independent Bernoulli(p) trials. Independently of me, you carry out a sequence of independent Bernoulli(q) trials. Here $p, q \in (0, 1)$. What is the probability we both have our first success after the same number of trials?

(6) Consider an infinite series of independent Bernoulli(p) trials, $0 < p < 1$. Given $r \in \mathbb{N}$, we define the random variable Y_r to be the number of trials completed when we achieve the r^{th} success. Find the PMF for Y_r.