170A Killip Homework 1 Due Fri, Jan 12

(1) Let S be a set with $n \geq 0$ elements and let $0 \leq k \leq n$ be an integer.
(a) How many subsets does S have?
(b) In how many ways can make an (ordered) list of k elements from S.
(c) Show that the number of k-element subsets of S is given by the binomial coefficient

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

(d) Suppose $n = m_1 + m_2 + \cdots + m_\ell$ were all m_i are non-negative integers. Show that the number of ways of partitioning S into (distinguishable) subsets of sizes $m_1, m_2, \ldots, m_\ell$ is given by the multinomial coefficient

$$\binom{n}{m_1, m_2, \ldots, m_\ell} = \frac{n!}{m_1! m_2! \cdots m_\ell!}$$

(e) Use this interpretation to justify the multinomial expansion of

$$(x_1 + x_2 + \cdots + x_\ell)^n$$

(2) How many ways can we distribute n balls among k bags if
(a) the balls and bags are distinguishable (e.g. numbered).
(b) the bags are distinguishable; the balls are not.
(c) balls and bags are distinguishable, but the bags can contain at most one ball (necessarily, $k \geq n$).
(d) the bags are distinguishable, the balls are not, and the bags can contain at most one ball.
Hint: For part (b) imagine encoding configurations as strings of symbols by drawing o's for each ball and drawing | to separate bags.

(3) The principle of inclusion-exclusion is discussed in many places, including, pages 21 and 46 of the textbook. Illustrate the idea by using it to determine how many numbers between 1 and 100, inclusive, are multiples of 2 or 3 or 5 or 7.

(4) A number is to be chosen at random from the range $0 \leq x < 1$.
(a) Choose an appropriate sample space.
(b) We write x as a decimal (without recurring 9). Write down the event that the first or second digit (after the decimal point) is a three.

(5) A fair die is rolled and a fair coin is flipped.
(a) Write down a sample space for this experiment.
(b) Write down the event (as a set) that the coin lands heads.
(c) Name the events which have the smallest and largest probabilities.
(d) Write down the following event and compute its probability:

(the coin is heads and the die roll is odd, but not three) or (the die roll is three).