The Allure of Free Shipping: How to Choose the Best Policy for Online Retail

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Free shipping threshold policies – where shipping fees are waived for orders exceeding a minimum dollar threshold – are widely used in online retail, yet choosing the optimal threshold remains a complex challenge. This paper presents a practical, data-driven framework to (1) assess the profitability of a retailer's current free shipping threshold policy and (2) identify the optimal threshold. Our framework incorporates several consumer reactions to the chosen threshold: for some consumers a threshold could deter purchases altogether, while for others it could entice them to increase their order (order padding), all of which influences the likelihood and volume of returns. We estimate our framework using transaction-level sales and return data from a leading U.S. apparel retailer. By linking estimated behavioral effects to financial outcomes, our framework allows retailers to optimize their threshold policies. We find that either the retailer should always offer free shipping (i.e., a zero threshold) or, more generally (as with our partner retailer), an intermediate threshold is best – low (but not zero) thresholds are consistently sub-optimal. Our results offer actionable guidance for retailers to design more effective and financially sound free shipping threshold policies.

Key words: shipping fees; contingent free shipping; free shipping threshold; fulfillment policies; online retail

1. Introduction

Despite the ubiquitous prevalence of free shipping messaging, shipping a product is never truly free – someone always pays. For retailers, the decision of who bears this cost involves a strategic trade-off: passing shipping fees on to customers may introduce friction at checkout and deter purchases, while absorbing those costs themselves reduces margins. To navigate this tension, most retailers have adopted policies that waive shipping fees for orders exceeding a specified dollar value, known as the free shipping threshold.

Free shipping threshold policies are now widespread. Among the twenty largest global retailers, fifteen use some form of a threshold policy (Deloitte 2025), as do all of the ten largest global apparel retailers (Table 1). These policies combine strategic positioning and insights into consumer psychology. Customers are believed to be highly sensitive to shipping fees, as frequently highlighted in media coverage and consumer surveys: anecdotal reports suggest that 62% of shoppers would

Table 1 Free Shipping Thresholds for Top-10 Global Apparel Retailers by Total Revenue in 2023.

Retailer (Brand)	Shipping Policy	Total Revenue (billion USD)	Online Revenue (billion USD)	Country
Inditex, S.A. (Zara)	Free shipping above $70	39.0	10.3	Spain
Hennes & Mauritz (H&M)	Free shipping above $60	22.1	6.6	Sweden
Fast Retailing (Uniqlo)	Free shipping above $99	20.2	3.0	Japan
Gap	Free shipping above $50	14.9	6.0	U.S.
Lululemon	Free on all orders	9.1	4.3	U.S.
PVH (Calvin Klein)	Free shipping above $75	8.7	1.8	U.S.
Next	Free shipping above $90	6.8	3.7	U.K.
Ralph Lauren	Free on all orders	6.3	1.7	U.S.
Victoria's Secret & Co.	Free shipping above $100	5.8	2.2	U.S.
American Eagle	Free shipping above $75	5.0	2	U.S.

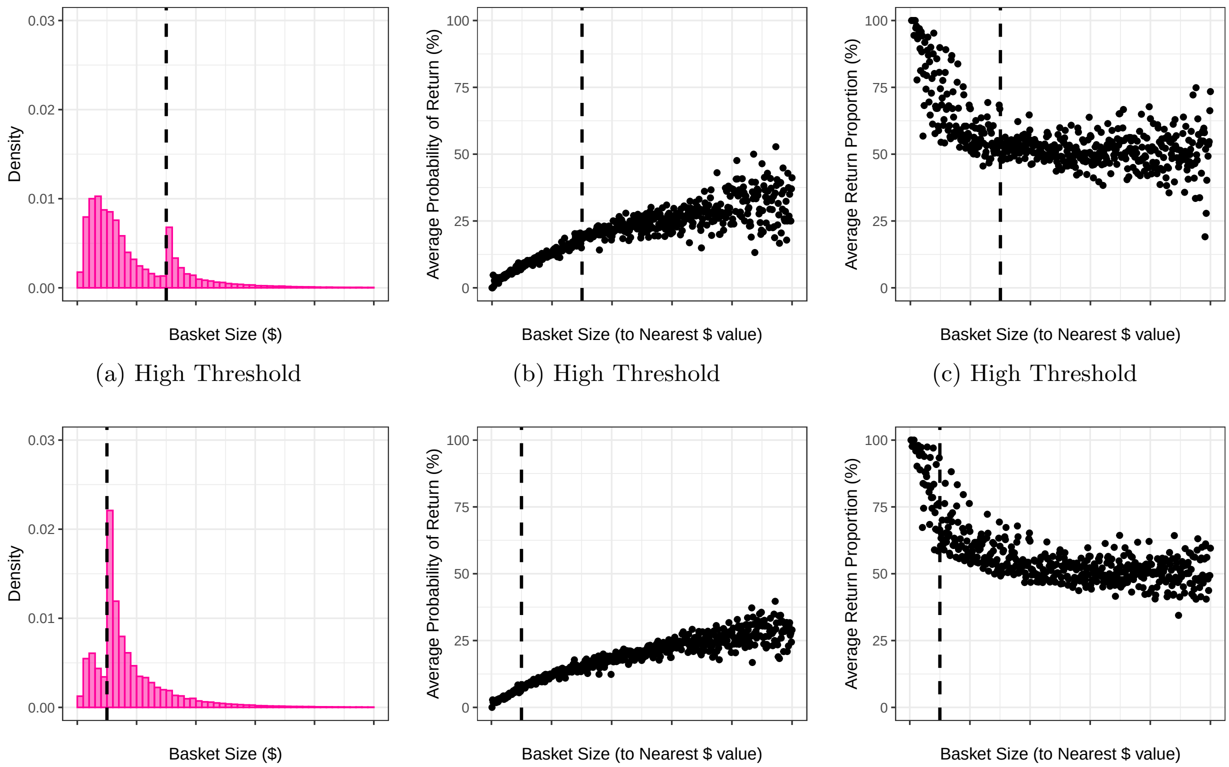
Source: Fast Retailing (2025).

stop purchasing from a retailer that does not offer a free shipping option, and 93% claim they would alter their shopping behavior to avoid paying shipping fees (Stevens and Banjo 2014, Young 2023). Despite their popularity, there is limited theoretical or empirical guidance on how to set free shipping thresholds effectively.

In this paper we develop a practical, data-driven framework to help retailers choose the best threshold policy, considering both the financial implications and customer behavior under that policy. Our framework accounts for three ways the threshold influences customer behavior. First, for customers who do not qualify for free shipping because their intended basket size falls below the threshold, shipping fees may act as a barrier to purchase, making them more likely to abandon their cart rather than pay a shipping fee. Second, customers with baskets just below the threshold may strategically add items to their order to qualify for free shipping, a behavior known as *order padding*. For example, if a retailer charges $10 for shipping and sets its free shipping threshold at $100, a customer who plans to spend $95 faces a total cost of $105. However, by adding a $6 item to reach a $101 basket, the customer would qualify for free shipping and effectively receive more while spending less. Third, customers who receive free shipping, particularly those who padded their orders to reach the threshold, may be more inclined to return part of their purchase because those items are primarily included to meet the threshold rather than due to genuine purchase intent.

Figure 1 illustrates these behaviors using transaction-level data from a leading U.S. apparel retailer. Panels (a) and (d) show the distribution of basket sizes under two different free shipping thresholds implemented by this retailer. In both cases, the histograms show a dip in the frequency of basket sizes just below the threshold and a sharp spike immediately above it, suggesting that customers engage in order padding to avoid shipping fees. The other panels in Figure 1 illustrate that larger baskets are more likely to involve a return, but a lower portion of the order is returned.

The customers' response to free shipping thresholds shapes critical retail outcomes, such as shipping revenue, order volume, basket size, and return-related costs. In turn, threshold policies have



(a) High Threshold (b) High Threshold (c) High Threshold

(d) Low Threshold (e) Low Threshold (f) Low Threshold

Figure 1 Our retail partner's basket size distribution (panels a and d), return probability (panels b and e), and return proportion conditional on return (panels c and f) from two free shipping thresholds.

major financial implications for retailers. Consider a retailer with a \$100 free-shipping threshold, a 40% gross margin, and fulfillment costs are \$10 each way. The shipping fee for the order, if charged, matches the retailer's cost, \$10, and the retailer offers free shipping on all returns. If a customer places a \$90 order, with a 25% chance of returning 25% of the order, the retailer makes a profit of \$31.25.[1] However, if the customer pads their order to \$110 to receive free shipping, the retailer's profit drops to \$28.75 even if the customer's return propensity behavior does not change – an 8% decline in profit despite a 22% increase in revenue.[2]

This example reflects the nuanced trade-offs involved when setting a free shipping threshold and underscores the importance of careful calibration. The free shipping threshold policy design requires balancing the desire to recover shipping costs with the need to attract and retain a substantial customer base (Lewis et al. 2006). A high threshold may discourage some customers from placing an order, but it can increase shipping revenue, since fewer orders will qualify for free shipping.

[1] Shipping revenue covers the shipping cost. A 40% margin on a \$90 order yields \$36, but the expected cost of returns (lost margin and shipping) is \$4.75, yielding a \$31.25 final profit.

[2] Now \$110 $\times$ 0.4 = \$44 is the base earnings, expected return costs are \$5.25, but now there is no shipping revenue to offset the order's shipping cost of \$10.

Moreover, a high threshold encourages customers to consolidate purchases. A low threshold, by contrast, removes frictions to place an order, encouraging small, frequent orders – at the expense of transferring the fulfillment costs from the customer to the retailer. This transfer can happen without necessarily boosting revenue, as customers no longer need to pad their orders to qualify for free shipping. Intermediate thresholds introduce additional complexities: customers with a basket value just below the threshold may pad their orders to avoid shipping fees, but may later adjust their return behavior, potentially returning items they were less committed to. Because these effects interact in subtle ways, the net effect of the free shipping threshold on a retailer's profitability is ambiguous *a priori*, and identifying the optimal free shipping threshold requires a unified approach that accounts for their combined impact on profitability, which our framework specifically provides.

We worked with an industry partner from the fashion apparel industry. Using their data, we evaluated the profitability of different threshold policies. Our analysis reveals that the profit-maximizing free shipping threshold is higher than both of the thresholds the retailer implemented during the study period. This downward bias reflects a broader industry trend in which retailers prioritize reducing customer friction, often at the expense of long-term profitability. For example, The Home Depot and Lowe's both set their thresholds at $45, while Walmart, Kroger, Target, Walgreens, and CVS cluster around $35. These patterns suggest that free shipping threshold policies are frequently used as competitive signals, with many firms setting thresholds by imitation rather than through careful analysis of their economic trade-offs. Nevertheless, it is useful to be able to quantify the impact of any chosen policy.

In addition to prescribing a retailer's profit-maximizing free shipping threshold, our model yields qualitative insights into customer behavior and threshold policy design. First, under these policies, customers exhibit strategic behavior both by padding orders to qualify for free shipping and by returning products; however, while the former has substantial implications when it comes to setting the optimal free shipping threshold, product returns only play a marginal role. Second, we find that setting the free shipping threshold below its optimal level is more harmful to profitability than setting it too high – a cautionary insight for retailers who may be tempted to lower their threshold simply to align with competitors. Third, our analysis uncovers a range of consistently suboptimal thresholds – values that are sufficiently high to discourage a significant share of customers from ordering, yet too low to encourage order padding or generate substantial shipping revenue. Finally, we show that the profit-maximizing threshold responds monotonically to several key operational parameters, such as fulfillment costs and gross margins, providing retailers with actionable guidance on how to adjust their free shipping threshold policies as business conditions evolve.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 describes the data and presents preliminary empirical evidence that motivates our modeling framework, which we introduce in Section 4 and estimate in Section 5. In light of the estimation results, Section 6 examines the behavioral and financial trade-offs associated with free shipping thresholds and discusses their implications for designing an optimal threshold policy. We explore the sensitivity of our results to the framework's key parameters in Section 7. Section 8 concludes.

2. Related Work

Before the widespread adoption of free shipping threshold policies in online retail, firms had long employed mechanisms to influence order size and frequency. Among the most common are quantity discounts, where customers receive a price reduction when purchasing larger quantities (Dolan 1987, Munson and Rosenblatt 1998).

Free shipping threshold policies – where shipping fees are waived on orders that exceed a minimum dollar amount – are conceptually related to quantity discounts in that they encourage and reward higher spending. However, the structure and consumer perception of the incentive differ meaningfully. Rather than a reduction in the marginal price, as is typical with traditional quantity discounts, threshold policies offer a one-time discount equal to the shipping fee, contingent on crossing a discrete purchase threshold. This design introduces distinct behavioral dynamics that set threshold policies apart from traditional quantity-based promotions.

Critically, free shipping thresholds represent a form of partitioned pricing, in which a surcharge (the shipping fee) is presented separately from the product price. Unlike quantity discounts, which take the form of direct price reductions, threshold policies frame the benefit as the conditional elimination of a fee – an approach that may heighten consumer sensitivity. Research on partitioned versus all-inclusive pricing (e.g., Thaler 1985; Morwitz et al. 1998; Bertini and Wathieu 2008; reviewed in Greenleaf et al. 2016) shows that consumers often respond more negatively to surcharges than to equivalent bundled prices. However, a key distinction in our context is that customers can avoid the fee by increasing their order value beyond the threshold, adding a strategic element absent in standard partitioned pricing scenarios.

Several studies have examined how different shipping fee structures affect consumer behavior. Lewis et al. (2006), using transaction data from a non-perishable goods retailer, find that lowering shipping fees increases order incidence and that threshold policies can raise average basket size by prompting customers to add items to reach the threshold – a phenomenon echoed in Hemmati et al. (2021a). Yet, both studies caution that these gains are often offset by lost shipping revenue, raising concerns about the profitability of threshold policies. A notable limitation in this work is the reliance on discrete basket-size categories, which restricts the ability to evaluate free shipping thresholds as continuous decision variables.

Complementing the empirical literature, a set of theoretical models has sought to characterize optimal threshold policies. These models treat the threshold as a strategic lever balancing fulfillment costs, demand, and margins. For instance, Leng and Parlar (2005) use a leader-follower game to model consumer response to thresholds, while Leng and Becerril-Arreola (2010) and Becerril-Arreola et al. (2013) explore joint pricing and free shipping threshold optimization in competitive and inventory-constrained settings. Gümüş et al. (2013) study whether to bundle or unbundle shipping fees using market data from electronics retailers. More recently, Li et al. (2023) incorporated consumer heterogeneity into threshold and pricing decisions in the grocery sector. While these models offer useful insights, they typically rest on strong functional assumptions that are difficult to validate empirically and often fail to reflect nuanced customer behavior observed in practice.

Building on this literature and new empirical evidence, we develop a data-driven framework that captures both the behavioral and financial consequences of free shipping threshold policies. In particular, our model addresses two key behavioral responses often overlooked in prior work: (1) adjustments to purchase behavior, including order incidence and basket size, and (2) changes in return behavior triggered by order padding. The latter is critical, as customers who pad their orders to qualify for free shipping may later return items they were less committed to buying (Hemmati et al. 2021b).

Product returns have attracted growing attention in operations research (e.g., Nageswaran et al. 2020; Patel et al. 2021), especially in light of their logistical and financial implications. While many studies explore optimal return policies (e.g., Su 2009; Akçay et al. 2013; Shang et al. 2017a,b), we hold the return policy fixed – our retail partner offers free returns – and focus instead on how the threshold shapes return behavior under this policy. Adjusting our analysis to incorporate different return policies is straightforward.

This study contributes to a broader stream of research that uses data-driven approaches to inform strategic decisions in retail operations. Applications span pricing (Caro and Gallien 2012, Ferreira et al. 2016), store location (Glaeser et al. 2019), staffing (Fisher et al. 2021), fulfillment (Acimovic and Graves 2015), and service design (Sousa et al. 2015, Buell et al. 2016). In this spirit, our work brings data-driven modeling to bear on a central and under-optimized lever in e-commerce: the free shipping threshold.

3. Empirical Setting and Data

We partner with a leading U.S. apparel retailer that provided a timestamped, transaction-level dataset covering all online sales and returns from January 2011 to December 2012. For each transaction, we observe the purchase amount and the amount returned, both in dollars, as well as what the free shipping threshold was at the time each order was placed. The dataset does not include any customer-level or demographic information.

A unique feature of this dataset is the presence of two distinct periods characterized by different free shipping thresholds. From January 1st to September 11th, 2011, the retailer maintained a "high" threshold of τ_1 dollars. On September 12th, 2011, the retailer lowered its threshold to $\tau_2 = \tau_1/3$ and maintained it through the end of the observation period.[3] This variation allows us to examine how changes in the free shipping threshold influence several business metrics of interest, namely, the number of online orders, average basket size, total sales, return probability, and the average proportion of an order that is returned. By investigating these effects, we uncover empirical patterns that inform our modeling choices as we develop the analytical framework in Section 4.

To assess how free shipping thresholds affect customer behavior and key retail outcomes, we construct a quasi-experimental dataset centered around a free shipping threshold policy change. Specifically, we focus on a 61-day window spanning 30 days before and after September 12 for both years. This window captures the retailer's decision to lower its free shipping threshold in 2011, while serving as a control period in 2012 when the policy remained unchanged. By comparing these intervals, while excluding promotional days when free shipping was temporarily offered on all orders (see Table EC.1), we isolate the causal effect of the threshold change on order volume, basket size, and product returns.

This setup lends itself to a difference-in-differences (DiD) approach, which we implement by defining the pre-intervention, high-threshold period in 2011 as the "treatment" condition and the post-change period as "control," a structure that inverts the conventional DiD formulation but retains its identification logic.

Figure 2 illustrates the impact of the free shipping threshold on the average basket size. The figure depicts the daily evolution of average basket size over the 61-day intervals around September 12 in both years, with horizontal lines indicating the 30-day averages before and after that date. In 2012, when the free shipping threshold remained constant, the basket size increased only slightly from \$88.81 to \$90.41, a difference that is statistically indistinguishable from zero. By contrast, 2011 shows a markedly different pattern: following a significant reduction in the free shipping threshold on September 12, the average basket size fell from \$94.49 in the pre-period to \$90.38 in the post-period, a statistically significant drop. Notably, the post-period average in 2011 aligns closely with the stable value observed in 2012 (\$90.38 vs. \$90.41), reinforcing the interpretation that the previous augmented basket size was driven by customers padding their orders to meet the higher free shipping threshold. This suggests that the high threshold prompted customers to pad their orders to qualify for free shipping, resulting in larger basket sizes. Figure EC.1 in the

[3] Due to confidentiality reasons, we cannot disclose the exact free shipping thresholds or provide additional descriptive statistics beyond those already presented in the paper. The high free shipping threshold, τ_1, was approximately twice the average basket size during the observation period.

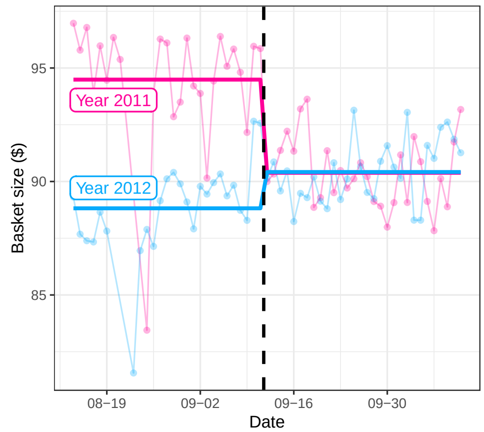


Figure 2 **Model-free evidence of the impact of free shipping threshold on the average basket size in the year it was not changed (blue, 2012) and in the year it was reduced (red, 2011).**

Online Appendix provides analogous evidence for the other business metrics of interest and can be interpreted in a similar manner.

We use the specification in Equation (1) to empirically estimate the impact of the free shipping threshold on the aforementioned metrics at the daily level. The dependent variable, Y_{dy}, represents a business metric on day d (between August 13 and October 12) in year $y \in \{2011, 2012\}$. The covariates include an indicator for whether the observation belongs to 2011, an indicator for whether the day falls before September 12, and their interaction, which captures the difference-in-differences effect. We also control for systematic temporal variations by including fixed effects for the day of the week and week of the year. This ensures that our estimates account for seasonal patterns and weekly fluctuations in customer behavior. The coefficient φ_3 provides the estimate of the effect of a *higher* free shipping threshold on the outcome variable.

$$Y_{dy} = \varphi_0 + \varphi_1 \text{year2011}_y + \varphi_2 \text{preSept12}_d + \varphi_3 \text{year2011}_y \times \text{preSept12}_d + \varphi_{\text{weekday}(dy)} + \varphi_{\text{week}(dy)} + \epsilon_{dy} \tag{1}$$

Table 2 presents the difference-in-differences (DiD) estimates and reveals key insights into how free shipping thresholds shape customer behavior. Column (1) shows that higher thresholds lead to fewer orders, supporting the idea that steeper requirements discourage customers from completing purchases. Column (2), however, indicates that when customers do place an order, their baskets tend to be larger, consistent with order padding. When these two effects – i.e., the reduced order frequency and increased basket size – are considered together, they largely cancel each other, leaving no significant change in total daily sales (column 3). Taken together, Columns (1) and (2) could also be interpreted as evidence of order consolidation, where customers delay purchases until they accumulate enough items to surpass the free shipping threshold, resulting in fewer but larger orders. While plausible in some contexts, this explanation is less likely in our setting. For our retail

Table 2 The effect of high free shipping threshold on several business metrics.

	Dependent variable:					
	# of orders	Avg. basket ($)	Total sales ($)	Pr(Return)	Avg. prop. returned (Returning orders)	Avg. prop. returned (All orders)
	Poisson	*OLS*	*OLS*	*OLS*	*OLS*	*OLS*
	(1)	(2)	(3)	(4)	(5)	(6)
year2011_y	-0.350^{***}	-0.003	$-156{,}450.700^{***}$	-0.0004	-0.005	-0.001
	(0.004)	(0.469)	(12,974.910)	(0.002)	(0.006)	(0.001)
preSept12_d	-0.123^{***}	0.558	$-47{,}883.010$	-0.010^{**}	0.011	-0.005^{*}
	(0.010)	(1.258)	(34,781.160)	(0.005)	(0.017)	(0.003)
$\text{year2011}_y \times \text{preSept12}_d$	-0.030^{***}	5.666^{***}	22,357.600	0.017^{***}	-0.030^{***}	0.006^{***}
	(0.006)	(0.686)	(18,979.830)	(0.003)	(0.009)	(0.002)
Day of the week FE	✓	✓	✓	✓	✓	✓
Week in the year FE	✓	✓	✓	✓	✓	✓
Observations	112	112	112	112	112	112
R^2		0.660	0.802	0.553	0.567	0.564
Adjusted R^2		0.594	0.764	0.466	0.483	0.480
Log Likelihood	$-3{,}191.721$					
Akaike Inf. Crit.	6,421.443					
Residual Std. Error (df = 93)		1.794	49,614.800	0.007	0.024	0.004
F Statistic (df = 18; 93)		10.029^{***}	20.950^{***}	6.382^{***}	6.759^{***}	6.688^{***}

Note: Observations at the daily level. $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$
We exclude 10 days when promotional discounts temporarily lowered free shipping thresholds.

partner, repeat purchases within the same fashion season, which typically lasts no more than three months, are rare, limiting customers' ability or incentive to postpone purchases for consolidation purposes.

From the analysis of return behavior, we observe the probability that a customer initiates a return increases when the free shipping threshold is high. However, when they do initiate a return, the proportion of the original order that is returned is smaller – consistent with customers padding their orders with smaller items and later returning them. The increase in return frequency outweighs the decrease in return proportion, resulting in a higher overall volume of returns under the high shipping threshold.

4. Model Description

This section presents a model that captures how the free shipping threshold can shape customer behavior in purchases and returns, and ultimately impact an online retailer's profitability.

4.1. The Online Retailer's Objective Function

The decision to set a particular free shipping threshold, denoted with τ, is a strategic one for retailers, as it directly affects both customer behavior and operational outcomes. By adjusting this threshold, the retailer influences (1) how often customers place orders, (2) how much they choose to spend per order, (3) the likelihood that an order falls below the threshold (and hence incurs a shipping fee), and (4) whether and to what extent customers return the products they previously purchased. These interactions collectively shape the firm's profitability.

To formalize this, we define four key functions, each depending on the free shipping threshold τ:

- $N(\tau)$ denotes the rate at which customers place orders, capturing the effect of the threshold on order incidence.

- $D(\tau)$ represents the average basket size, incorporating behavioral responses such as order padding.
- $S(\tau)$ corresponds to the average shipping revenue per order, depending on whether customers qualify for free shipping or not.
- $R(\tau)$ accounts for the average cost incurred from product returns, including return shipping and lost profit, as well as potential losses due to unsellable products.

Combining these components, the retailer's total profit can be written as:

$$\Pi(\tau) = N(\tau) \times \big(M\ D(\tau) + S(\tau) - K - R(\tau)\big) \tag{2}$$

where M is the gross margin rate and K is the fixed fulfillment cost per order. An exogenously-determined gross margin M is in line with the literature on category management (e.g., Anderson et al. 1992, Cachon and Kök 2007, Belavina et al. 2017) and is appropriate given retailers typically choose a free shipping threshold that applies across all of their products. Furthermore, in an omnichannel context – where retailers operate multiple channels, such as online and offline – it is typically not feasible to adjust the margin in one channel without affecting the others.

Equation (2) encapsulates the trade-off retailers face: a lower threshold may increase order volume but reduce shipping revenue and basket sizes, while a higher threshold might enhance per-order profitability – via shipping revenues or order padding – but deter some customers from ordering.

We now formalize how the threshold τ affects each component of Equation (2).

4.2. Orders

We take the number of online orders the retailer receives on a given day t to be Poisson distributed with rate $\lambda_t(\tau)$, where

$$\log\big(\lambda_t(\tau)\big) = \psi_0 + \psi_1 \log(\tau+1) + \psi_{\text{year}(t)} + \psi_{\text{weekday}(t)} + \psi_{\text{week}(t)} \tag{3}$$

Based on the order rate $\lambda_t(\tau)$, we define the (normalized) number of daily orders as the expected number of daily orders when the free shipping threshold is τ, divided by the expected number of daily orders when $\tau = 0$, i.e.:

$$N(\tau) = \frac{\lambda_t(\tau)}{\lambda_t(0)} = (\tau+1)^{\psi_1} \tag{4}$$

In this parametrization, $N(\tau)$ denotes the fraction of potential customers – those who would order if shipping were free – who still choose to order when the free shipping threshold is τ. Conversely, $1 - N(\tau)$ represents the share of potential customers who are deterred from ordering due to the threshold.

We use a logarithmic form in Equation (3) to ensure the sensitivity to the free shipping threshold decreases as the threshold becomes larger – raising the threshold from \$40 to \$50 has a stronger effect on customer purchase decisions than increasing it from \$490 to \$500 even though both changes represent the same nominal amount, \$10. Section EC.2 of the Online Appendix demonstrates that our main results remain robust under alternative specifications.

4.3. Basket Size

The relationship between the basket size function $D(\tau)$ and the free shipping threshold should account for *order padding*, a behavior by which customers who initially desire a basket size that falls below the threshold may consider adding a few extra items into their order, increasing their basket size to qualify for free shipping. Conversely, customers whose basket size exceeds the threshold have no incentive to increase their order. The likelihood that a customer pads their order should depend on how close their initial basket size is to the free shipping threshold, with customers just below the threshold being more inclined to pad their orders than customers further away from it.

We assume that a customer initially desires a basket size x, a realization from a random variable $X \sim F(\cdot)$ that represents the *base demand*. If the customer's initial basket exceeds the free shipping threshold, i.e., $x \geq \tau$, then they qualify for free shipping and place an order of size $d = x$. However, if x is below the free shipping threshold, the customer may choose to *pad* the order with probability $p(\tau, x) = \max\big(0, \alpha - \beta(\tau - x)\big)$, where $\alpha \in [0,1]$ and $\beta > 0$ are constants. If the customer pads, then they order $d = \tau + y$, where y is a realization from a random variable $Y \sim G(\cdot)$, independent from X. If the customer decides not to pad, they order $d = x$ and pay the shipping fee.

The random variable X represents the basket size distribution observed by the online retailer when free shipping is either universally granted ($\tau = 0$) or always charged ($\tau = \infty$). This formulation assumes that customer demand is primarily influenced by the retailer's prices, which remain fixed, despite the fact that the total purchase cost incurred by customers is lower when shipping is free for everyone compared to when shipping is always charged. In Section 7, we discuss the case where the retailer may choose to raise prices, and thereby increase its gross margin, to offset the loss in shipping revenue resulting from lowering its free shipping threshold.

The free shipping threshold τ divides the base demand distribution into two distinct regions. Orders with a sufficiently large initial basket size remain unaffected, whereas those with a base demand below the free shipping threshold may be padded with probability $p(\tau, x) = \max(0, \alpha - \beta(\tau - x))$. Here, α represents the base probability that a customer pads an order just below the threshold, and the padding probability $p(\tau, x)$ decays linearly at rate β as the gap between the initial basket size x and the threshold τ increases. If an order is padded, the final basket size d exceeds τ by a random amount Y. Since the retailer's assortment is limited, padding customers could end up spending an amount greater than τ, as exactly matching the free shipping threshold is not always feasible.

From the basket size generation mechanism described above, the average basket size can be expressed as:

$$D(\tau) = \int_0^{\max\left(0,\tau-\frac{\alpha}{\beta}\right)} xf(x)dx + \int_{\max\left(0,\tau-\frac{\alpha}{\beta}\right)}^{\tau} \Big(p(\tau,x)(\tau+E[Y]) + \big(1-p(\tau,x)\big)x\Big) f(x)dx + \int_{\tau}^{\infty} xf(x)dx. \quad (5)$$

The first term in Equation (5) corresponds to orders in which the initial basket size is too small relative to the threshold τ for the customer to consider padding. The second term represents orders in which the initial basket size falls below the threshold, but close enough to it so that customers may consider padding. The last term represents orders with a base demand greater than the threshold that already qualify for free shipping, without the need of padding. It follows from Equation (5) that $D(\tau) \geq E[X]$ for all $\tau \geq 0$ and $D(0) = \lim_{\tau\to\infty} D(\tau) = E[X]$. These results follow because customers always order at least their base demand, and there is no incentive to pad orders when either all or none of the customers qualify for free shipping.

If the base demand distribution has a closed-form conditional moment expression (Kim 2010), the average basket size function $D(\tau)$ can be expressed in closed form as:

$$D(\tau) = \begin{cases} E[X] + F(\tau)\Big((\alpha-\beta\tau)\big(\tau+E[Y]-E[X|X\leq\tau]\big) + \beta\big(\tau+E[Y]\big)E[X|X\leq\tau] - \beta E[X^2|X\leq\tau]\Big) & \text{if } \tau-\frac{\alpha}{\beta}<0 \\ E[X] + \Big(F(\tau)-F\big(\tau-\frac{\alpha}{\beta}\big)\Big)\Big((\alpha-\beta\tau)\big(\tau+E[Y]-E\big[X\big|\tau-\frac{\alpha}{\beta}\leq X\leq\tau\big]\big) & \\ \qquad +\beta\big(\tau+E[Y]\big)E\big[X\big|\tau-\frac{\alpha}{\beta}\leq X\leq\tau\big] - \beta E\big[X^2\big|\tau-\frac{\alpha}{\beta}\leq X\leq\tau\big]\Big) & \text{if } \tau-\frac{\alpha}{\beta}\geq 0 \end{cases} \quad (6)$$

Proposition 1 establishes that, if X follows a log-concave distribution, the average basket size $D(\tau)$ is unimodal.[4] Consequently, there exists a unique free shipping threshold $\bar{\tau}$ that maximizes $D(\tau)$ over $\tau\in\mathbb{R}_+$, with the average basket size increasing for all $\tau<\bar{\tau}$ and decreasing for all $\tau>\bar{\tau}$.

PROPOSITION 1. *If the base demand X follows a log-concave distribution, then the average basket size $D(\tau)$ is unimodal.*

Proof: See the Online Appendix.

This result suggests that the free shipping threshold should be set strategically – high enough to incentivize additional spending, but not so high that it feels unattainable. If the threshold is too low, most customers qualify for free shipping without needing to pad their orders, diminishing its effectiveness. Conversely, if it is set too high, consumers may perceive it as unrealistic and disregard the incentive altogether. While Lewis et al. (2006) find that basket sizes increase with the free shipping threshold, our model accounts for its diminishing effectiveness in prompting order padding if it is set too high.

[4] Note that log-concavity of a probability distribution implies monotonically increasing hazard rates (An 1998). While log-concavity of the base demand distribution is a sufficient condition for the basket size function $D(\tau)$ to be unimodal, this is not a necessary condition. For instance, $D(\tau)$ is also unimodal if X follows a gamma distribution with decreasing hazard rate, which is not log-concave.

4.4. Shipping Revenue

As is common in e-commerce settings (Acimovic and Graves 2015), we assume the retailer incurs a fixed fulfillment cost of K dollars per order, covering expenses such as transportation and labor. The model can be extended to account for shipping costs that increase with the basket size, reflecting scenarios in which larger orders are more expensive to ship due to greater weight or volume, a higher likelihood of split shipments, or fulfillment from multiple locations. The online retailer charges customers a shipping fee that represents a fixed fraction $\kappa \in (0,1]$ of its fulfillment cost, collecting $\kappa \cdot K$ dollars for each order that does not qualify for free shipping:

$$S(\tau) = \kappa K \left[\int_0^{\max\left(0, \tau - \frac{\alpha}{\beta}\right)} f(x)dx + \int_{\max\left(0,\tau-\frac{\alpha}{\beta}\right)}^{\tau} \Big(1 - p(\tau, x)\Big) f(x)dx \right] \tag{7}$$

The terms in the square brackets represent the probability that an order does not qualify for free shipping – either because the initial basket size is too small relative to the free shipping threshold so that the customer does not consider padding, or because the customer chooses not to pad the order to receive free shipping. As in the case of $D(\tau)$, the shipping revenue function $S(\tau)$ can be expressed in closed form if the initial basket size distribution X has a closed-form conditional moment expression.

$$S(\tau) = \begin{cases} \kappa K F(\tau)\Big[1 - \alpha + \beta\Big(\tau - E[X|X \leq \tau]\Big)\Big] & \text{if } \tau - \frac{\alpha}{\beta} < 0 \\ \\ \kappa K\Big[F\Big(\tau - \frac{\alpha}{\beta}\Big) + \Big(F(\tau) - F\Big(\tau - \frac{\alpha}{\beta}\Big)\Big)\Big(1 - \alpha + \beta\Big(\tau - E\Big[X\Big|\tau - \frac{\alpha}{\beta} \leq X \leq \tau\Big]\Big)\Big)\Big] & \text{if } \tau - \frac{\alpha}{\beta} \geq 0 \end{cases} \tag{8}$$

4.5. Cost of Returns

We model product returns as the result from a two-step process. First, upon receiving an order, the customer evaluates the products and identifies any that may be unsatisfactory. Second, the customer decides whether to initiate a return for the unsatisfactory portion of the order. Because not all customers follow through with returns – some may decide that initiating a return is not worth the effort – the fraction of the order deemed unsatisfactory is only observed for those orders where a return is completed, but it remains unobserved for orders where no return occurs. To account for this process and correct for the selection bias that could potentially arise, we use the Heckman selection model (Heckman 1979) as the basis for modeling product returns.

Let d_i represent the original basket size of order i. Additionally, let the indicator variable IsPadded_i equal 1 if order i is padded, and 0 otherwise. Although IsPadded_i is not observed directly, it can be inferred from the data. We use r_i^* to denote the latent variable that represents the fraction of order i the customer dislikes, and r_i represents the fraction of the order they return. Note that $r_i = r_i^*$ only if the customer initiates a return for the unsatisfactory items, and $r_i = 0$ otherwise. We model the proportion of the order that the customer is dissatisfied with as:

$$r_i^* = \gamma_0 + \gamma_1 \cdot d_i + \gamma_2 \cdot (d_i - c) \cdot \mathbb{1}_{\{d_i \geq c\}} + \gamma_{pad} \cdot \text{IsPadded}_i + \epsilon_i \tag{9}$$

The constant $\gamma_0 \in [0,1]$ is the base proportion of the order the customer dislikes, while γ_1 and γ_2 allow the unsatisfactory proportion to depend on the basket size in a piecewise linear fashion, motivated by the evidence that the slope of the return proportion has a kink at basket size c, as shown in panels (c) and (f) of Figure 1. The constant γ_{pad} enables the fraction of an order that the customer dislikes to vary depending on whether the customer padded that order. We denote the error term with ϵ_i and assume it follows a normal distribution with mean zero and variance σ_ϵ^2. We refer to Equation (9) as the *outcome equation*.

Once the customer receives order i and decides what items they dislike, whether they initiate a return is governed by Equation (10), which we refer to as the *selection equation*. More precisely, we define an auxiliary latent variable z_i^* as:

$$z_i^* = \delta_0 + \delta_1 \cdot d_i + \delta_2 \cdot d_i^2 + \delta_{pad} \cdot \text{IsPadded}_i + \xi_i \tag{10}$$

and assume that a customer initiates a return if the perceived benefit of doing so, represented by z_i^*, exceeds a certain threshold, that is:

$$z_i = \begin{cases} 1 & \text{if } z_i^* \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Here, the indicator variable z_i equals 1 if the customer completes a return in relation to order i, and 0 otherwise. Assuming that the error term ξ_i in Equation (10) is drawn from a standard normal distribution, i.e., $\xi_i \sim N(0,1)$, yields the following return probability:

$$P(\text{Return}_i) = P(z_i = 1) = \Phi\big(\delta_0 + \delta_1 \cdot d_i + \delta_2 \cdot d_i^2 + \delta_{pad} \cdot \text{IsPadded}_i\big) \tag{12}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function (cdf) of the standard normal distribution. The constant $\delta_0 \in [0,1]$ captures the base rate of return, while δ_1 and δ_2 allow for the return probability to vary linearly and quadratically with the original basket size d_i – note that a quadratic model is a reasonable choice for the return probability model in light of panels (b) and (e) of Figure 1. The constant δ_{pad} accounts for the possibility that receiving free shipping through order padding could affect the probability of initiating a return.

To accommodate the possibility that unobserved factors affect both the proportion of the order a customer dislikes and the decision of whether they initiate a return or not, we let the errors in the selection and outcome equations, ξ_i and ϵ_i, be jointly normally-distributed with correlation ρ.

$$\begin{bmatrix} \xi_i \\ \epsilon_i \end{bmatrix} \sim \text{Normal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & \sigma_\epsilon^2 \end{bmatrix} \right) \tag{13}$$

Allowing for correlation between the errors in the outcome and selection equations accounts for the potential selection bias that could arise if unobserved factors influencing the selection decision also affect the outcome – i.e., if unobserved factors affect both the fraction of an order that the customer dislikes and the probability that they return it. For example, suppose some customers are pickier than others, and that picky customers (1) tend to find a larger fraction of their order undesirable, and (2) are more likely to initiate a return for the items they dislike. If the model did not incorporate this unobserved relationship between the proportion of the order the customer finds unsatisfactory and the probability they initiate a return, the estimates of the product return model would be biased.

If unobserved factors affect the selection and outcome decisions causing the errors ξ_i and ϵ_i to be correlated, then whether or not a customer initiates a return for a given order informs the fraction of the order they were unsatisfied with. Consequently, the unconditional mean of the outcome variable, $E(r_i^*) = \gamma_0 + \gamma_1 \cdot d_i + \gamma_2 \cdot (d_i - c) \cdot \mathbb{1}_{\{d_i \geq c\}} + \gamma_{pad} \cdot \text{IsPadded}_i$, differs from the mean of the outcome variable r_i^* conditional on r_i^* being observed, which only happens when the customer initiates a return. The latter, which we denote with $E(r_i^* \mid z_i = 1)$, is given by:

$$E(r_i^* \mid z_i = 1) = \gamma_0 + \gamma_1 \cdot d_i + \gamma_2 \cdot (d_i - c) \cdot \mathbb{1}_{\{d_i \geq c\}} + \gamma_{pad} \cdot \text{IsPadded}_i + \rho\sigma_\epsilon \frac{\phi(\delta_0 + \delta_1 \cdot d_i + \delta_2 \cdot d_i^2 + \delta_{pad} \cdot \text{IsPadded}_i)}{\Phi(\delta_0 + \delta_1 \cdot d_i + \delta_2 \cdot d_i^2 + \delta_{pad} \cdot \text{IsPadded}_i)} \tag{14}$$

The $\phi(\cdot)/\Phi(\cdot)$ term is often referred to as the Inverse Mills Ratio and corrects for the selection bias that results from the correlation in the selection and outcome decisions.

Online retailers face multiple costs associated with product returns. First, they incur a return shipping cost, which we assume as fixed and denote with K_r. Second, they forfeit the margin M per sales dollar that was originally earned when the item was sold. Third, it is possible that the returned item cannot be resold, which occurs with probability ω. In such cases, the retailer absorbs the cost of the item, which amounts to $1 - M$ per sales dollar. Therefore, the expected return cost for order i with basket size d_i, conditional on the customer initiating a return in relation to order i, is:

$$\begin{aligned} &\left(\gamma_0 + \gamma_1 \cdot d_i + \gamma_2 \cdot (d_i - c) \cdot 1_{\{d_i \geq c\}} + \gamma_{pad} \cdot \text{IsPadded}_i + \rho\sigma_\epsilon \frac{\phi(\delta_0 + \delta_1 \cdot d_i + \delta_2 \cdot d_i^2 + \delta_{pad} \cdot \text{IsPadded}_i)}{\Phi(\delta_0 + \delta_1 \cdot d_i + \delta_2 \cdot d_i^2 + \delta_{pad} \cdot \text{IsPadded}_i)}\right) \\ &\qquad \times \left(M + (1 - M)\omega\right) \cdot d_i + K_r \end{aligned} \tag{15}$$

By this logic, the expected cost of returns per order for a given free shipping threshold is:

$$\begin{aligned} R(\tau) = &\int_0^{\max(0, \tau - \frac{\alpha}{\beta})} \Big(\big(M + (1 - M)\omega\big) \cdot r_0^*(x) \cdot x + K_r\Big) \cdot \Phi\big(z_0^*(x)\big) f(x)dx \\ &+ \int_{\max(0, \tau - \frac{\alpha}{\beta})}^{\tau} \left[\int_0^\infty \Big(\big(M + (1 - M)\omega\big) \cdot r_1^*(\tau + v) \cdot (\tau + v) + K_r\Big) \cdot \Phi\big(z_1^*(\tau + v)\big) g(v)dv\right] p(\tau, x) f(x)dx \\ &+ \int_{\max(0, \tau - \frac{\alpha}{\beta})}^{\tau} \Big(\big(M + (1 - M)\omega\big) \cdot r_0^*(x) \cdot x + K_r\Big) \cdot \Phi\big(z_0^*(x)\big)\big(1 - p(\tau, x)\big) f(x)dx \\ &+ \int_\tau^\infty \Big(\big(M + (1 - M)\omega\big) \cdot r_0^*(x) \cdot x + K_r\Big) \cdot \Phi\big(z_0^*(x)\big) f(x)dx \end{aligned} \tag{16}$$

where $z_1^*(x)$ and $z_0^*(x)$ denote the expected value of the latent variable governing the return decision for an order of size x that was padded or not padded, respectively. Similarly, $r_1^*(x)$ and $r_0^*(x)$ represent the expected fraction of an order of size x that is returned, conditional on a return occurring, for padded and non-padded orders, respectively. That is,

$$\begin{aligned}
z_0^*(x) &= E(z^*|x, \text{IsPadded}=0) = \delta_0 + \delta_1 x + \delta_2 x^2 \\
z_1^*(x) &= E(z^*|x, \text{IsPadded}=1) = \delta_0 + \delta_1 x + \delta_2 x^2 + \delta_{pad} \\
r_0^*(x) &= E(r^*|x, z=1, \text{IsPadded}=0) = \gamma_0 + \gamma_1 x + \gamma_2 (x-c)\cdot \mathbb{1}_{\{x\geq c\}} + \rho\sigma_\epsilon \frac{\phi\big(z_0^*(x)\big)}{\Phi\big(z_0^*(x)\big)} \\
r_1^*(x) &= E(r^*|x, z=1, \text{IsPadded}=1) = \gamma_0 + \gamma_1 x + \gamma_2 (x-c)\cdot \mathbb{1}_{\{x\geq c\}} + \gamma_{pad} + \rho\sigma_\epsilon \frac{\phi\big(z_1^*(x)\big)}{\Phi\big(z_1^*(x)\big)}
\end{aligned} \tag{17}$$

The first term in Equation (16) captures the return cost for orders where demand is too low for order padding to take place. The second and third terms represent the product return costs for orders with an initial basket size below the free shipping threshold, depending on whether the customer chooses to pad the order or not, respectively. The fourth term accounts for the return cost of orders that qualify for free shipping due to a large initial demand. The function $R(\tau)$ can be expressed in closed form if the base distribution X has a closed-form conditional moment expression, but it is not necessarily unimodal, even if X is log-concave.

4.6. Optimality of Free Shipping Threshold Policies

The retailer's total profits $\Pi(\tau)$ may not be a well-behaved, unimodal function, mainly due to the decay introduced by $N(\tau)$ and the lack of structure in the return cost function $R(\tau)$. However, Proposition 2 establishes that there is some structure to the optimal, profit-maximizing free shipping threshold policy. In particular, when fulfillment costs are sufficiently high, offering free shipping on all orders ($\tau = 0$) cannot be optimal; conversely, when fulfillment costs are sufficiently low, charging shipping on all orders ($\tau = \infty$) cannot be optimal. For an intermediate range of fulfillment costs, an interior solution satisfying $0 < \tau^* < \infty$ exists.

Proposition 2. *Let the total profit function $\Pi(\tau)$ be defined as described previously and τ^* denote the profit-maximizing free shipping threshold, i.e., $\tau^* := \arg\max_{\tau\in\mathbb{R}_+} \Pi(\tau)$.*

There exist two thresholds $\underline{K}$ and $\bar{K}$ such that:

- *If $K > \underline{K}$, it is not optimal to offer free shipping on all orders, i.e., $\tau^* > 0$.*
- *If $K < \bar{K}$, it is not optimal to charge shipping fees on all orders, i.e., $\tau^* < \infty$.*
- *If K is such that $\underline{K} < K < \bar{K}$, then an interior solution is optimal, i.e., $0 < \tau^* < \infty$.*

Furthermore, $\underline{K} < \bar{K}$ guarantees the existence of a region of fulfillment costs K for which the optimal free shipping threshold τ^* is an interior solution.

Proof: See the Online Appendix.

Proposition 2 formalizes the following intuition. Free shipping thresholds act as an incentive device – i.e., a "carrot" –, encouraging consumers to add items to their basket to reach τ and qualify for free shipping. Although this padding behavior can enhance per-order profitability by amortizing the fulfillment cost over a larger basket, the profitability of this mechanism depends critically on the magnitude of the fulfillment cost. If fulfillment costs are sufficiently high, offering free shipping on all orders ($\tau = 0$) would require the retailer to absorb a substantial expense on every transaction, without benefiting from unplanned spending through order padding. In such cases, the retailer is better off by setting a positive threshold to recover part of these costs through shipping fees or larger basket sizes. Conversely, if fulfillment costs are sufficiently low, charging shipping fees on all orders ($\tau = \infty$) sacrifices the demand stimulation in exchange for a modest shipping charge, making a lower threshold more attractive. When fulfillment costs fall in an intermediate range, both extremes become suboptimal; in this case, an interior solution ($0 < \tau^* < \infty$) best balances the incremental gains from order padding with the partial recovery of fulfillment costs, thereby maximizing total profit.

5. Model Estimation and Validation

We now describe our approach to estimate the parameters of the order incidence, basket size, and product return models using transaction-level sales and return data from our retail partner. To ensure consistency and focus on typical customer behavior, we limit our estimation to orders with basket values below \$500, which represent 99.4% of the dataset. As in Section 3, we exclude orders placed during promotional periods when the retailer either temporarily lowered the free shipping threshold or offered universal free shipping.

5.1. Order Incidence Model

We estimate the order incidence model with Poisson regression. This analysis draws on the same 61-day windows described in Section 3, centered around September 12 in both 2011 – when the retailer reduced its free shipping threshold from τ_1 to $\tau_1/3$ – and 2012, which serves as a control period. To isolate the effect of the threshold change from underlying seasonal patterns, we include year, week, and day of the week fixed effects. As shown in Table 3, higher free shipping thresholds are associated with lower order frequency: a 10% increase in the threshold corresponds to an estimated 0.43% decline in daily order volume.

5.2. Basket Size Model

We estimate the basket size model by assuming the base demand X and the post-padding demand Y are gamma-distributed, i.e., $X \sim F(x; k_1, \theta_1)$ and $Y \sim G(y; k_2, \theta_2)$. Given a sequence $\{d_i, \tau_i\}$ of basket sizes and the corresponding free shipping thresholds observed in the data, we use maximum

Table 3 Estimates of the order incidence model.

	Dependent variable:
	Num. of orders
ψ_1	-0.043^{***}
	(0.005)
Day of the week FE	✓
Week in the year FE	✓
Year FEs	✓
Observations	112
Log Likelihood	−3,263.643
Akaike Inf. Crit.	6,563.287

*p<0.1; **p<0.05; ***p<0.01

likelihood to estimate the shape and scale parameters (k_1, θ_1, k_2, θ_2) of the demand distributions and the order padding parameters α and β. The log-likelihood function is:

$$\mathcal{LL}(\boldsymbol{\Theta}|\{\boldsymbol{d},\boldsymbol{\tau}\}) = \sum_i \left[f(d_i)\big(1-p(\tau_i,d_i)\big)\cdot \mathbb{1}_{\{d_i<\tau_i\}} + \left(f(d_i) + g(d_i-\tau_i)\int_{\max(0,\tau_i-\frac{\alpha}{\beta})}^{\tau_i} p(\tau_i,x)f(x)dx \right)\cdot \mathbb{1}_{\{d_i\geq\tau_i\}} \right] \quad (18)$$

The first term in the square brackets represents an order with a basket size below the free shipping threshold ($d_i < \tau_i$), which must originate from the base demand distribution X when no order padding occurs. The second term corresponds to an order exceeding the free shipping threshold ($d_i > \tau_i$), which may arise either from the base demand distribution alone or from a customer deliberately padding the order to qualify for free shipping.

Table 4 presents the maximum likelihood estimates of the basket size model using the data from the high threshold period.[5] We observe that the average basket size from the base demand distribution is given by $E[X] = k_1\theta_1 = \$88.30$. Additionally, customers who pad their orders spend an average of $E[Y] = k_2\theta_2 = \$12.34$ beyond the free shipping threshold. The probability that a customer pads their order when their intended basket size is right below the free shipping threshold is estimated to be 0.595. This likelihood declines at a rate of 0.0074 for each dollar their intended spend falls below the threshold. As a result, customers whose desired basket size is further than \$80.41 below the threshold (i.e., \$0.595/0.0074) do not consider padding their orders.

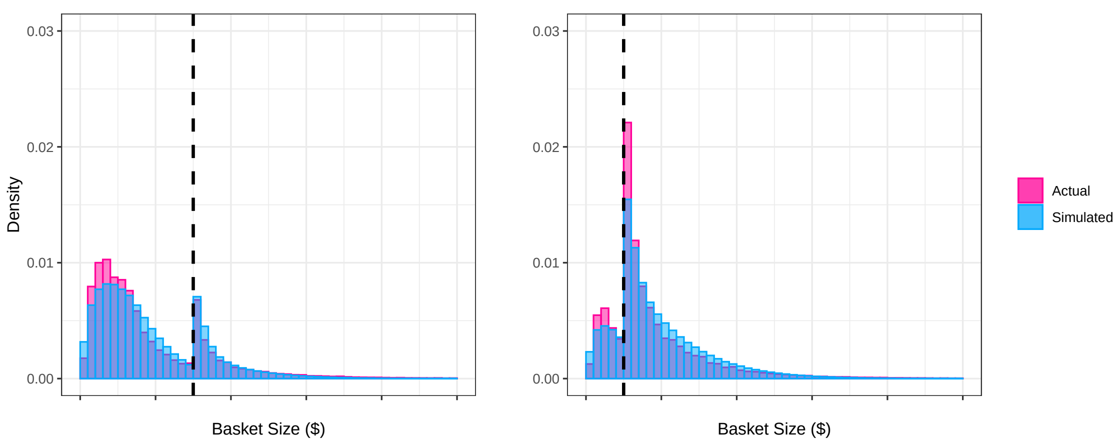
We validate the fit of the basket size model through simulation. Based on the basket size generation mechanism described in Section 4.3 and the estimates reported in Table 4, we simulate 1,000,000 orders for each one of two conditions: one with the high free shipping threshold of τ_1 and another with a low threshold of $\tau_2 = \tau_1/3$. The actual and simulated basket size distributions under both thresholds are illustrated in Figure 3. The 1-Wasserstein distances between the actual and simulated distributions are \$6.58 and \$7.87 during the high and low threshold periods, respectively

[5] Table EC.2 in the appendix presents the estimates from the low threshold period and the entire data sample.

Table 4 Estimates of basket size model.

	Dependent variable:
	Basket size
k_1	1.757***
	(0.003)
θ_1	50.257***
	(0.084)
k_2	1.279***
	(0.005)
θ_2	9.652***
	(0.013)
α	0.595***
	(0.002)
β	0.0074***
	(0.00004)
Observations	827,041
Log Likelihood	−4,456,886
Akaike Inf. Crit.	8,913,784

*p<0.1; **p<0.05; ***p<0.01



(a) High Threshold (b) Low Threshold

Figure 3 Model validation: Actual vs. simulated basket size distribution from (a) the high threshold period and (b) the low threshold period.

– representing 8.67% and 11.98% of the standard deviation of the data. This indicates a very good model fit.

Notably, while the actual data in the high threshold period (left panel of Figure 3) were used to estimate the parameters in Table 4, the results in the right panel serve as an out-of-sample validation, as the data from the low threshold period were not included in the estimation. In both cases, the basket size model effectively captures key empirical patterns observed in the data. In particular, it reproduces (1) a sharp increase in the distribution just above the free shipping threshold and (2) a noticeable decline below the threshold.

5.3. Product Returns Model

Finally, we estimate the parameters of the product return model using the Heckman Selection Model (Heckman 1979), which accounts for potential selection bias in product return behavior. The model comprises two components: a selection equation that models the likelihood of a return occurring, and an outcome equation that estimates the proportion of the order returned, conditional on a return taking place. To fit the model, we rely on a dataset that includes the basket sizes, return proportions, and corresponding free shipping thresholds for a sequence of orders, i.e., $\{d_i, r_i, \tau_i\}$.

The selection model estimates the likelihood of a customer initiating a return as a function of basket size and padding probability, using a probit regression.

$$P(r_i > 0) = \Phi\big(\delta_0 + \delta_1 \cdot d_i + \delta_2 \cdot d_i^2 + \delta_{pad} P[\text{Pad}|d_i, \tau_i]\big) \tag{19}$$

For each order i, we impute $P[\text{Pad}|d_i, \tau_i]$ based on estimates from the basket size model. Specifically, for any basket size exceeding the free shipping threshold ($d_i \geq \tau_i$), the probability that the order resulted from padding, rather than the customer genuinely desiring that exact basket size, is given by:

$$P[\text{Pad}|d_i, \tau_i] = \frac{P[\text{Pad}|\tau_i] g(d_i - \tau_i)}{f(d_i) + P[\text{Pad}|\tau_i] g(d_i - \tau_i)} \tag{20}$$

For orders with a basket size below the free shipping threshold ($d_i < \tau_i$), the probability of padding is zero.

We employ a piecewise linear specification to model the return proportion of order i (conditional on the order involving a return), incorporating a term that corrects for potential selection bias. This adjustment accounts for the possibility that unobserved factors influencing a customer's decision to return an order may be correlated with the proportion of the order that the customer dislikes and ultimately returns.

$$r_i = \gamma_0 + \gamma_1 d_i + \gamma_2 (d_i - c) \mathbb{1}_{\{d_i \geq c\}} + \gamma_{pad} P[\text{Pad}|d_i, \tau_i] + \gamma_{mills} \frac{\phi\big(\delta_0 + \delta_1 d_i + \delta_2 d_i^2 + \delta_{pad} P[\text{Pad}|d_i, \tau_i]\big)}{\Phi\big(\delta_0 + \delta_1 d_i + \delta_2 d_i^2 + \delta_{pad} P[\text{Pad}|d_i, \tau_i]\big)} + \epsilon_i, \tag{21}$$

By including the Inverse Mills Ratio term, $\phi(\cdot)/\Phi(\cdot)$, we ensure that the model estimates are not biased by the non-random nature of product return decisions.

We calibrate the location of the kink in the return proportion, c, by optimizing the fit of the product return model – specifically, by maximizing the adjusted R^2.

Table 5 displays the estimates for the product return model using the data from the high threshold period.[6] The findings indicate that when an order is padded, there is a statistically significant increase in the probability of return ($\delta_{pad} > 0$) but a statistically significant decrease in the proportion of the order returned ($\gamma_{pad} < 0$), with the latter declining by 5.2 percentage points.

[6] Table EC.3 in the appendix presents the estimates from the low threshold period and the entire data sample.

Table 5 Estimates of the product return model.

	Dependent variable:
	Return Proportion
Selection equation:	
δ_0	-1.739^{***}
	(0.005)
δ_1	0.007^{***}
	(0.00007)
δ_2	-0.00001^{***}
	(0.0000002)
δ_{pad}	0.080^{***}
	(0.010)
Outcome equation:	
γ_0	0.716^{***}
	(0.040)
γ_1	-0.003^{***}
	(0.0001)
γ_2	0.003^{***}
	(0.0001)
γ_{pad}	-0.052^{***}
	(0.005)
Observations	827,041
R^2	0.168
Adjusted R^2	0.168
Correlation, ρ	0.388
Inverse Mills Ratio	0.117^{***} (0.019)
c	104.94

*p<0.1; **p<0.05; ***p<0.01

The significant coefficient on the Inverse Mills Ratio suggests that selection bias arises from the selection process. This is because, as the positive correlation $\rho = 0.388$ between the selection and outcome errors indicates, customers who tend to find a larger share of their order unsatisfactory are more likely to initiate a return.

Figure 4 illustrates the model fit graphically. The top panels display the likelihood of initiating a return and the bottom panels depict the expected proportion of orders that customers find unsatisfactory, conditional on whether they initiate a return. As expected, and consistent with the results in Table 5, customers who initiate a return tend to dislike a larger fraction of their order. Notably, since only the high-threshold data are used to fit the product return models, the low-threshold panels in Figure 4 serve as out-of-sample validation.

To assess the robustness of our return model, Table EC.4 in the appendix compares estimates under two different modeling assumptions: one that allows for correlation between the return incidence and return quantity decisions, and one that treats these decisions as independent. The correlated case corresponds to the Heckman Selection Model estimates reported in Table 5, while the uncorrelated case is constructed by separately estimating a probit model for the likelihood of a return and an OLS regression for the return quantity, using only the subset of orders that involved

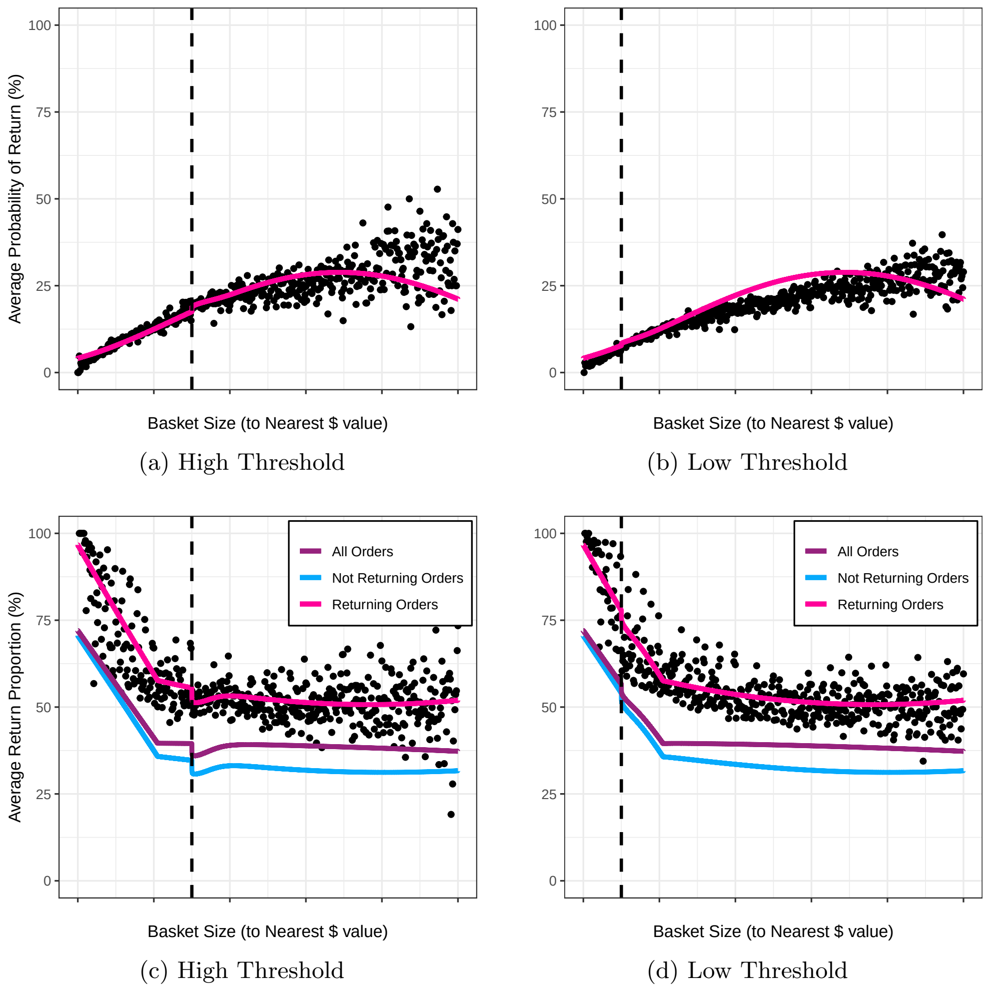


Figure 4 Model validation: Actual vs. fitted return probability (panels a and b) and return proportion conditional on return (panels c and d) from two free shipping thresholds.

a return. As expected, the estimates of the selection equation are identical across both approaches, but the outcome equation estimates diverge. Although using the more rigorous Heckman model does not materially alter our main findings – since return costs contribute only modestly to overall profitability and both approaches yield similar estimates of the return cost function $R(\tau)$ (see Figure EC.2) – accounting for selection bias may become more important in settings where return costs are more substantial or where the divergence between models is more pronounced.

6. Identifying the Optimal Free Shipping Threshold

The empirical results from the previous Section highlight that the selection of the free shipping threshold involves a balancing act shaped by competing behavioral and financial forces.

At one extreme, setting the threshold too low or offering free shipping on all orders drives a high volume of transactions, but often at the expense of profitability. Customers do not have an incentive to increase their spending, since free shipping is guaranteed regardless of basket size. This leads to a surge in small, low-margin orders, forcing the retailer to absorb substantial fulfillment costs. While fewer impulse purchases may result in lower return rates, the overall effect is often an erosion of profit due to the shipping expenses the retailer entirely covers.

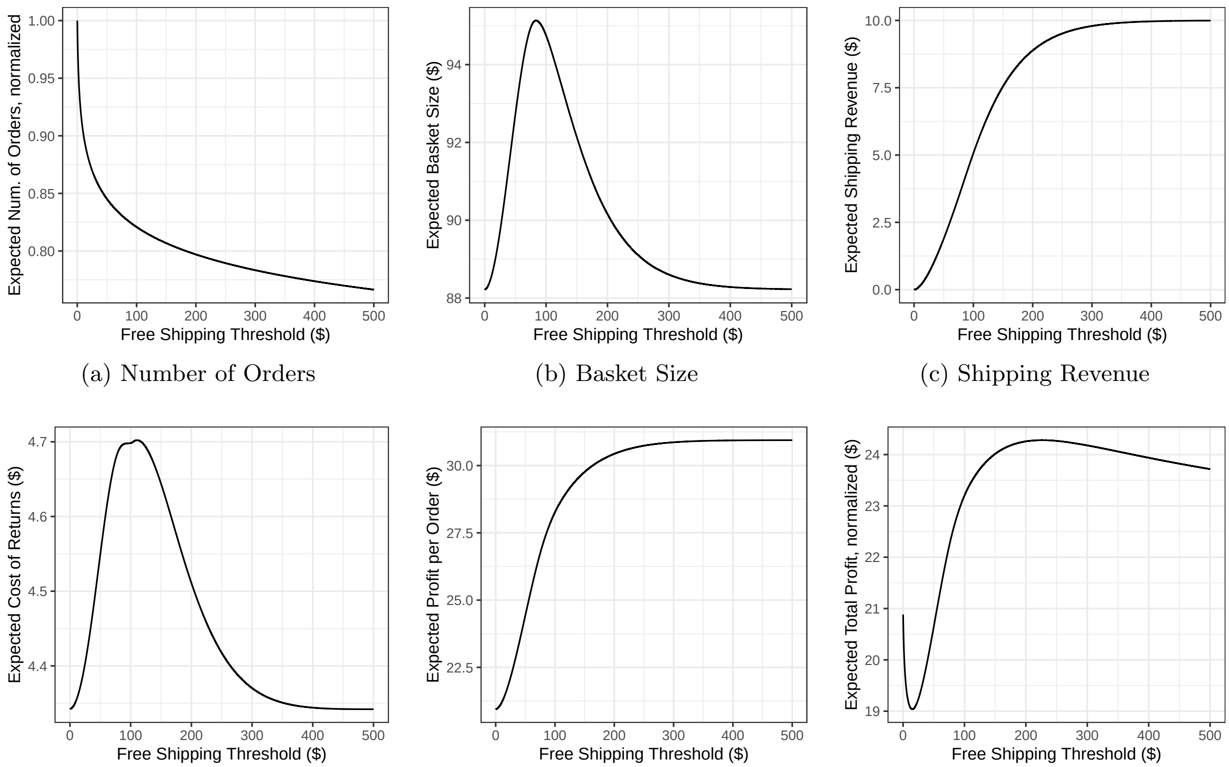
At the other extreme, setting the threshold too high effectively requires most customers to pay for shipping, which boosts shipping revenue but suppresses overall order volume. Moreover, without the incentive to qualify for free shipping, customers are less likely to add extra items to their carts, which reduces both average basket sizes and return rates at the potential expense of lost sales.

The optimal threshold likely lies somewhere between these two extremes, but finding the right balance is not straightforward. Our results show that intermediate thresholds encourage customers to pad their orders to qualify for free shipping, which can boost revenue in the short term. However, this behavior also increases return rates, as customers may add items they are less committed to keeping – which introduces additional logistical and financial costs for the retailer.

Our proposed framework, together with the parameters estimated in Section 5, provides a systematic and data-driven approach to identifying the profit-maximizing free shipping threshold and to quantify the financial impact of deviating from it. Figure 5 displays, for different values of the threshold $\tau \in [0, 500]$, the (normalized) expected number of orders, $N(\tau)$; the expected basket size, $D(\tau)$; the expected shipping revenue per order, $S(\tau)$; the expected cost of returns per order, $R(\tau)$; the expected profit per order, $M\ D(\tau) + S(\tau) - K - R(\tau)$; and the (normalized) total profits, as expressed in Equation (2). In these calculations, the fulfillment cost per order is $K = \$10$, and the retailer fully transfers the fulfillment costs to customers via the shipping fee, i.e., $\kappa = 1$. Furthermore, return shipping costs are $K_r = \$10$ and all returned items can be sold at full price in the future ($\omega = 0$). Lastly, the average gross margin rate for our retail partner is $M = 0.4$.

Figure 5 reveals several important patterns. As the free shipping threshold increases, the expected number of orders declines, reflecting the deterrent effect of higher thresholds on customer purchasing. When the threshold is set too low or too high, customers have little incentive to pad their orders, and the expected basket size tends to the mean of the base demand X, i.e., $E(X) = \$88.30$. In contrast, intermediate threshold levels create stronger incentives for padding: customers whose intended basket size falls just below the threshold often add items to qualify for free shipping. This behavior leads to an increase in the expected basket size, which peaks at \$95.05 when the threshold is $\tau = \$84.1$, a level above the initial spending intent of 57.7% of customers. This free shipping threshold leads 57.0% of customers – i.e., those who desire a basket size in the range $[\tau - \frac{\alpha}{\beta}, \tau] = [3.71, 84.12]$ – to consider padding their order.

Although order padding affects both the likelihood and value of product returns, the expected cost of returns $R(\tau)$ varies only modestly with τ, i.e., only \$0.36 between the lowest and highest



(a) Number of Orders (b) Basket Size (c) Shipping Revenue

(d) Cost of Returns (e) Profit per Order (f) Total Profit

Figure 5 Model predictions for the Number of Orders, Basket Size, Shipping Revenue, Cost of Returns, Profit per Order, and Total Profit, as a function of the free shipping threshold.

values. This points to a behavioral asymmetry: while order padding strongly influences the choice of the optimal free shipping threshold, product returns have a negligible impact on τ^*, as $R(\tau)$ remains virtually flat in τ. In other words, even though return costs are economically meaningful and relevant to the retailer's bottom line (it still incurs a return shipping cost on every order that involves a returned item), K_r has little impact on the choice of threshold in our setting – it shifts the profit function vertically rather than moving the optimal free shipping threshold.

Interestingly, the retailer's expected profit per order is maximized when all customers are charged for shipping – that is, when the free shipping threshold is $\tau = \infty$. While lowering the threshold encourages larger baskets through order padding, the associated gain in sales volume does not fully compensate for the loss in shipping revenue. This outcome is shaped by the retailer's financial structure (e.g. a gross margin of 40% and a shipping fee of \$10 for non-qualifying orders). Section 7 explores how changes in these parameters influence the optimal policy.

The total expected profit follows a non-monotonic relationship with the free shipping threshold. At low thresholds levels, it drops sharply, driven primarily by a steep decline in orders. As the threshold increases, profits rise rapidly, fueled by a combination of larger orders resulting from order padding and higher shipping revenues, since fewer orders qualify for free shipping. For our industry partner, the maximum profit (\$24.31) is attained at threshold of $\tau^* = \$224.49$. Beyond this point, profits begin to decline again, though at a slower rate, as the reduction in order volume becomes less pronounced.

The shape of the profit curve uncovers an important asymmetry: exceeding the optimal threshold is less costly than falling below it. A 50% deviation above the optimal free shipping threshold results in just a 0.8% profit loss, while falling short by the same amount leads to a 3.2% decline in profit. This asymmetry is particularly relevant for retailers facing external pressure to reduce thresholds for competitive reasons. This highlights the value of our framework as a practical decision support tool to help firms make informed trade-offs and avoid costly deviations from the optimal shipping policy.

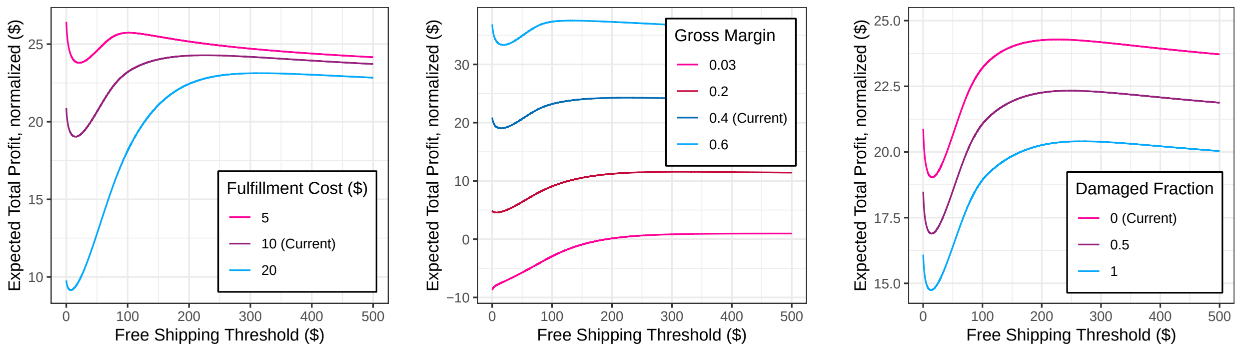
7. Sensitivity and Break-Even Analysis

In this section, we investigate how the optimal free shipping threshold policy adapts to changes in key operational parameters. Our analysis serves two primary objectives. First, we aim to understand how shifts in a retailer's internal cost structure or market environment – such as rising fulfillment expenses or changes in gross margin – should influence its free shipping threshold policy. Second, we explore how much these parameters would need to change to justify offering free shipping on all orders without compromising profitability. This allows us to identify when seemingly generous shipping policies may be financially sustainable – and when they are not.

7.1. Sensitivity of the Optimal Threshold to Key Operational Parameters

We begin by examining fulfillment costs. Each order incurs a fixed outbound fulfillment cost K, and orders that involve a return generate an additional return fulfillment cost K_r. These costs play a central role in shaping overall profitability. Figure 6, panel (a), illustrates how the total profit $\Pi(\tau)$ responds to changes in fulfillment costs, assuming $K = K_r$. While the retailer can offset outbound fulfillment costs by adjusting the free shipping threshold and passing some of the cost to customers, return-related fulfillment costs are fully borne by the retailer. As a result, increases in either component drive a noticeable and systematic decline in profitability.

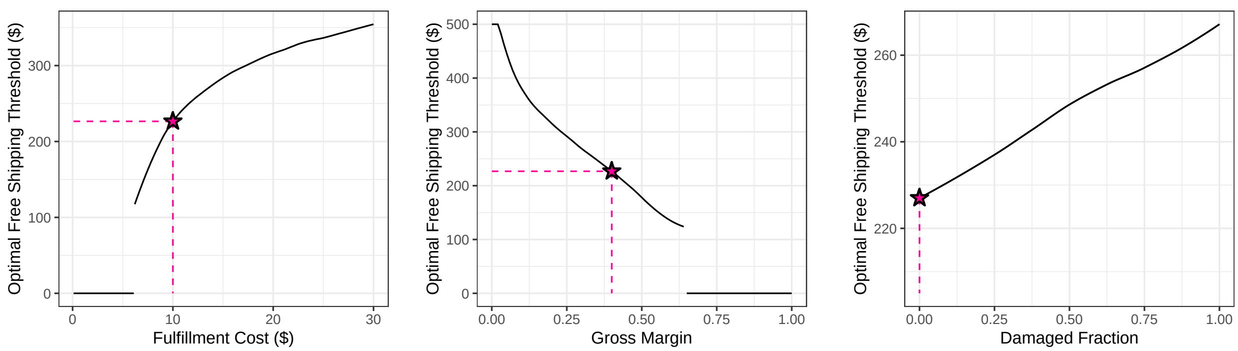
The retailer's decision to set a free shipping threshold can be viewed as determining the minimum basket size at which it becomes economically viable to absorb fulfillment costs. As formalized in Proposition 2, when fulfillment costs are low, the retailer is willing to waive shipping fees at relatively low order values, making a lower threshold optimal. Conversely, as fulfillment costs rise, covering those expenses becomes less attractive, prompting the retailer to raise the threshold to shift more of the burden onto customers. This relationship – where the optimal free shipping threshold increases with fulfillment costs – is illustrated in panel (d) of Figure 6.



(a) Expected total profit, as a function of fulfillment cost K

(b) Expected total profit, as a function of gross margin M

(c) Expected total profit, as a function of the damaged fraction ω



(d) Optimal free shipping threshold, as a function of fulfillment cost K

(e) Optimal free shipping threshold, as a function of gross margin M

(f) Optimal free shipping threshold, as a function of the damaged fraction ω

Figure 6 **Sensitivity of the retailer's profits and the optimal free shipping threshold with respect to fulfillment cost, gross margin, and the fraction of returns that are damaged.**

Interestingly, the optimal free shipping threshold is discontinuous in the fulfillment costs, indicating the existence of an interval of thresholds that are not optimal for any values of $K = K_r$. For example, in panel (d) of Figure 6, it is never optimal to choose a free shipping threshold in the range \$1 to about \$104, i.e., the optimal free shipping is either to always offer free shipping ($\tau^* = 0$) or to have some threshold that is at least \$104. Inside this "no-go" interval, the loss in order volume that results from not offering universal free shipping is not sufficiently offset by gains in basket sizes or shipping revenues, making these thresholds consistently suboptimal. Graphically, this is reflected in the "S-shape" of the profit function $\Pi(\tau)$, where the global maximum abruptly "jumps" from the boundary point at $\tau = 0$ to an interior local maximum as fulfillment costs increase.

Another critical parameter in our framework is the gross margin M, which determines the retailer's earnings from each dollar of sales that is not subsequently returned – and thus has a direct and substantial impact on overall profitability. Panel (b) of Figure 6 illustrates how the profit function $\Pi(\tau)$ varies as the gross margin increases from 3% (typical of the grocery sector) to 60% (common in the luxury jewelry industry).

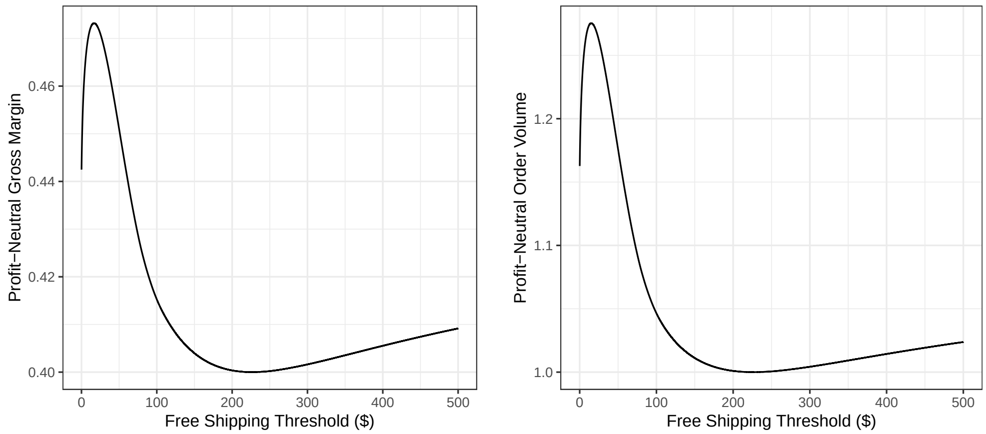
Due to its impact on the retailer's bottom line, the gross margin also influences the optimal, profit-maximizing free shipping threshold τ^*. In fact, the sufficient conditions for the existence of an interior solution presented in Proposition 2 can be expressed as conditions on the gross margin M. When the gross margin is low, individual orders yield limited profit, making it unappealing for the retailer to forgo shipping revenue in an attempt to boost the volume of low-margin orders. Conversely, if the gross margin is high, each sale is more profitable, and the retailer will be more willing to absorb part of the fulfillment cost to encourage customers to place high-margin orders. Consistent with this intuition, the optimal free shipping threshold decreases with the gross margin, as illustrated in panel (e) of Figure 6. Notably, as in the case of the fulfillment costs, there exists a range of near-zero thresholds that are suboptimal for any value of the gross margin.

Similar to the gross margin, the fraction of returned products that cannot be resold at full price directly influences the retailer's per-order profitability and, by extension, its overall financial performance. Throughout our analysis, we have assumed that any product returned by a customer can be resold to another customer at full price, implying that the fraction of returned items that are irreparably damaged is $\omega = 0$. However, this assumption may not always hold, as returned goods are sometimes damaged or otherwise rendered unsellable. For example, it is believed that Amazon.com recovers only a small fraction – between 5% and 10% – of a returned product's value.

If returned goods cannot be resold at full price, returns become more costly for the retailer, reducing the profitability of each order and lowering overall profit, as illustrated in panel (c) of Figure 6. As in the case of a low gross margin, the less profitable each individual order becomes, the less inclined the retailer is to subsidize shipping. Consequently, the optimal free shipping threshold increases with the fraction of returns that cannot be resold at full price, ω. Nevertheless, the optimal threshold remains relatively stable, ranging between \$224 and \$269 across all $\omega \in [0, 1]$, as shown in panel (f) of Figure 6.

7.2. Conditions for Profit-Neutral Universal Free Shipping

Our analysis takes the gross margin M as constant and exogenously determined. However, a retailer may be able to raise prices – and thereby increase the gross margin – to offset the loss of shipping revenue that results from lowering the free shipping threshold. This pricing adjustment is motivated by behavioral research suggesting that consumers often prefer bundled pricing, where shipping is perceived as "free," rather than partitioned pricing where the shipping fee is itemized separately. This preference is supported by Thaler (1985), who argue that explicitly itemizing secondary costs, such as shipping, can draw undue attention to them and reduce the likelihood of purchase. Consistent with this view, Smith and Brynjolfsson (2001) find that consumers are nearly twice as sensitive to shipping fees as they are to the price of the product.



(a) Profit-neutral gross margin rate, $\hat{M}(\tau)$ (b) Profit-neutral order volume

Figure 7 Profit-neutral gross margin rate and order volume.

Let $\hat{M}(\tau)$ denote the *profit-neutral gross margin*, that is, the gross margin rate with a free shipping threshold τ that yields the same total profit as $\tau^* = \$224.49$ at the base gross margin of 40%. In other words, $\hat{M}(\tau)$ represents the gross margin that would justify offering free shipping at a threshold τ different from τ^* without sacrificing overall profitability. As illustrated in panel (a) of Figure 7, a reduction in the free shipping threshold from $\tau^* = \$224.49$ to $\tau = \$0$ would require our retail partner to raise its gross margin from 40% to 44.2% so that its overall profitability would not be affected. This is equivalent to a 12.5% price increase.

The final "what-if" scenario we explore measures the extent to which the retailer's customer base would need to expand in order for a free shipping threshold τ to deliver the same total profit as the optimal threshold $\tau^* = \$224.49$ under a normalized customer base of size 1. To quantify this, we define the *profit-neutral order volume*, which is illustrated in panel (b) of Figure 7. Our results indicate that offering free shipping on all orders becomes profit-neutral only if, by doing so, the customer base increases by at least 16%. Such an outcome may be feasible if there exists a substantial segment of consumers, often referred to as *shipping charge skeptics*, that would only consider purchasing if shipping is completely free (Schindler et al. 2005).

8. Conclusion

Free shipping threshold policies have become a defining feature of retail. Their widespread adoption underscores their strategic significance, but identifying the optimal threshold remains a complex challenge. The threshold not only affects whether customers complete a purchase, but also how much they spend and whether they later return items. In light of these interdependent trade-offs, our paper introduces a practical, data-driven framework that quantifies the full financial impact of threshold policies and supports retailers in making informed, profit-maximizing threshold decisions.

Motivated by our initial empirical findings, we develop an analytical model that captures how the threshold influences four key dimensions of the retailer's business: the number of orders it receives, the value of each order, the probability that a customer initiates a return, and the quantity returned when they do. We model order incidence with a Poisson regression, return behavior with a Heckman selection model, and basket sizes using a custom formulation that combines two gamma random variables – for the base and post-padding demands – with a threshold-dependent order padding probability. We estimate these models using a proprietary dataset from the online channel of a leading U.S. apparel retailer, which allows us to compute the retailer's counterfactual profits across a wide range of free shipping thresholds. This makes it possible to identify the profit-maximizing threshold and to quantify the potential profit loss the retailer would incur if it deviated from the optimum in response, for example, to competitive pressure.

In addition to prescribing the optimal free shipping threshold, our framework yields broader insights into how threshold policies influence customer behavior and retailer profitability. First, the free shipping threshold serves as a strategic lever to stimulate unplanned spending: when set high enough to encourage order padding but not so high as to feel unattainable, it can meaningfully boost basket sizes. Second, by integrating this behavioral effect with operational considerations, we compute an optimal threshold that adjusts predictably to business conditions: it increases when fulfillment becomes more costly or margins shrink. Third, when the optimal policy lies between offering free shipping to all and charging everyone, erring on the side of a higher threshold is less harmful to profits than setting it too low. Finally, we identify a band of consistently suboptimal thresholds that both deter purchases and fail to generate sufficient order padding or shipping revenue, offering a clear warning against poorly calibrated thresholds.

Managers seeking to operationalize these results can start by estimating four empirical curves from their own data: order incidence, average basket value, expected shipping revenue, and expected return cost. These curves jointly determine the profit function, so computing the optimal threshold is straightforward once they are in hand. Our results offer two practical guardrails when choosing τ^*. On the one hand, retailers should avoid the "no-go" band of near-zero thresholds that may seem attractive from a marketing perspective but erode profit by giving away shipping without meaningfully increasing basket sizes. On the other hand, if external constraints require deviating from the optimal threshold τ^*, it is preferable to lean slightly toward the high side: being too generous (setting τ too low) is usually more detrimental than being too strict (setting τ too high), because the foregone shipping revenue is rarely offset by incremental demand. If branding or competitive positioning dictates a threshold below τ^*, managers can analyze profit-neutral trade-offs to make the policy whole: quantify the required lift in margin, price, or traffic needed to offset the deviation, and revisit these calibrations regularly as shipping costs, margins, and product return environments evolve.

While the framework presented in this paper captures a rich set of behavioral and operational dynamics, several aspects are beyond its scope and offer promising avenues for future research. Because most customers in our dataset make only one purchase per fashion season, we interpret the observed decline in order incidence as the result of discouraged purchases rather than order consolidation. In settings where customers place repeated orders, however, a high free shipping threshold could motivate them to consolidate orders, placing fewer, larger orders in an attempt to qualify for free shipping or to reduce the number of orders that incur shipping fees. Furthermore, in a richer setting with more varied data, one could examine how purchase and return decisions respond not only to the free shipping threshold but also to the specific shipping fee charged, and derive the optimal shipping policy as a joint decision over both the threshold and the fee. Finally, two emerging retail practices offer fertile ground for future exploration: some retailers have begun to charge fees for product returns, introducing a new layer of cost that may alter customer behavior and the optimal design of shipping policies, while others now offer subscription-based plans (e.g., Amazon Prime or Walmart+) that decouple shipping benefits from individual transactions, effectively transforming shipping fees into a flat-rate commitment (Fang et al. 2021, Wagner et al. 2021, Guo and Liu 2023). Exploring these directions can broaden the applicability of our proposed framework and deepen the understanding of how different shipping policies affect customer behavior and profitability.

This paper offers a practical, data-driven framework to design and evaluate free shipping threshold policies. By integrating behavioral and operational dynamics, it equips retailers with tools to balance the competing forces of order incidence, basket size, shipping revenue, and return costs – informing profit-maximizing decisions, even as market conditions evolve. We bridge academic insight with managerial action, showing that free shipping can be more than just a marketing perk, and transforming the threshold from a blanket giveaway into a disciplined, data-driven lever for profitability and long-term competitive advantage.

Acknowledgments

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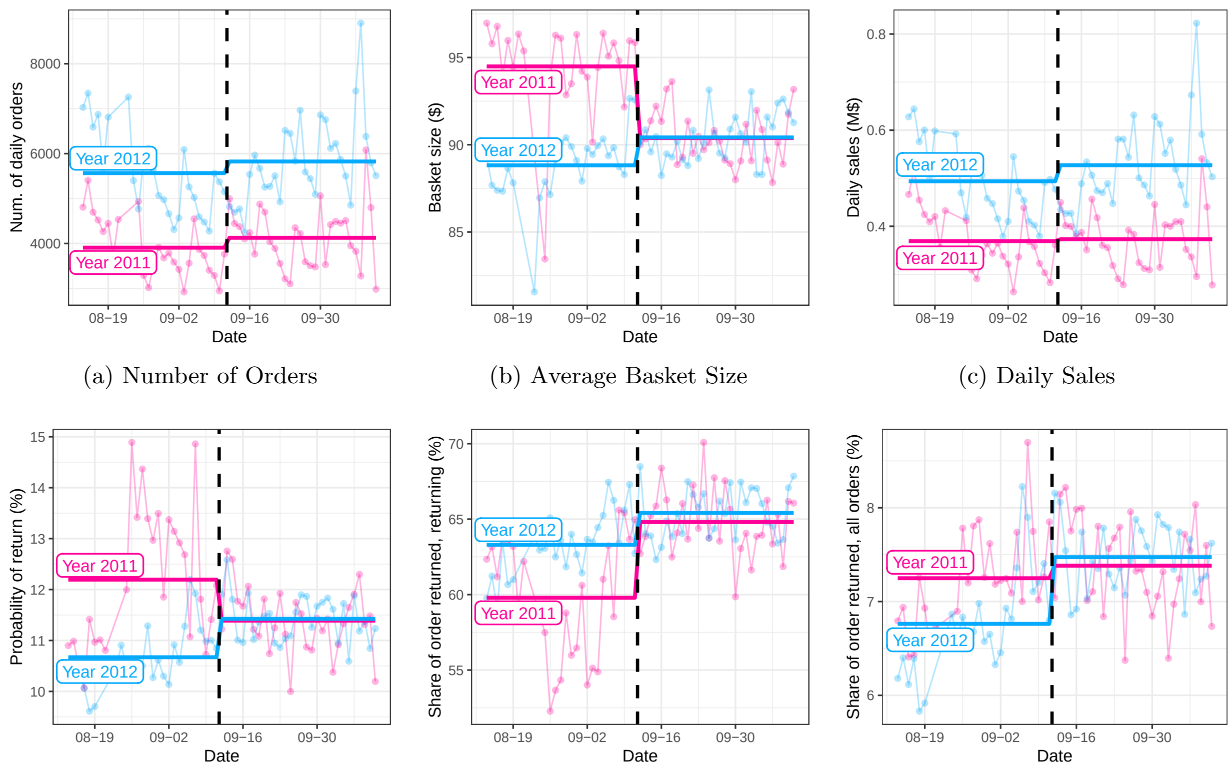
E-companion to *Measuring the Financial Impact of Free Shipping Thresholds in Online Retail*

EC.1. Additional Tables and Figures

Table EC.1 Free Shipping Threshold Periods.

Start Date	*End Date*	*Free Shipping Threshold*		
		High	Low	Zero
January 1, 2011	March 6, 2011	✓		
March 7, 2011	March 31, 2011		✓	
April 1, 2011	June 12, 2011	✓		
June 13, 2011	June 15, 2011		✓	
June 16, 2011	June 29, 2011	✓		
June 30, 2011	July 4, 2011		✓	
July 5, 2011	August 21, 2011	✓		
August 22, 2011	August 24, 2011			✓
August 25, 2011	September 11, 2011	✓		
September 12, 2011	December 8, 2011		✓	
December 9, 2011	December 12, 2011			✓
December 13, 2011	March 25, 2012		✓	
March 26, 2012	March 28, 2012			✓
March 29, 2012	April 24, 2012		✓	
April 25, 2012	April 25, 2012			✓
April 26, 2012	June 10, 2012		✓	
June 11, 2012	June 13, 2012			✓
June 14, 2012	August 19, 2012		✓	
August 20, 2012	August 22, 2012			✓
August 23, 2012	December 31, 2012		✓	



(a) Number of Orders

(b) Average Basket Size

(c) Daily Sales

(d) Probability of Return

(e) Return Proportion, returning orders

(f) Return Proportion, all orders

Figure EC.1 Model-free evidence of the impact of free shipping threshold change on some performance metrics.

Table EC.2 Estimates of basket size model, using different data samples. The first column presents the same model as in Table 4 in the main text.

	Dependent variable:		
	Basket size		
k_1	1.757***	1.614***	1.771***
	(0.003)	(0.002)	(0.001)
θ_1	50.257***	50.576***	47.578***
	(0.084)	(0.058)	(0.042)
k_2	1.279***	1.980***	2.019***
	(0.005)	(0.006)	(0.006)
θ_2	9.652***	4.747***	4.408***
	(0.013)	(0.022)	(0.021)
α	0.595***	0.478***	0.580***
	(0.002)	(0.0006)	(0.001)
β	0.0074***	0	0.0067***
	(0.00004)		(0.00003)
Data Sample	Jan 2011 - Sept 2011	Sept 2011 - Dec 2012	Jan 2011 - Dec 2012
Free Shipping Threshold	High	Low	High and Low
Observations	827,041	2,624,712	3,451,753
Log Likelihood	−4,456,886	−13,679,776	−18,151,031
Akaike Inf. Crit.	8,913,784	27,359,563	36,302,074

Note: In the estimates from the low threshold period, $\beta = 0$ means that a customer that desires an order under the free shipping threshold τ pads their order with probability $\alpha = 0.478$, for any values of her initial demand. *p<0.1; **p<0.05; ***p<0.01

Table EC.3 Estimates of the product return model, using different data samples. The first column presents the same model as in Table 5 in the main text.

	Dependent variable:		
	Return Proportion		
Selection equation:			
δ_0	-1.739^{***}	-1.803^{***}	-1.781^{***}
	(0.005)	(0.003)	(0.003)
δ_1	0.007^{***}	0.007^{***}	0.007^{***}
	(0.00007)	(0.00004)	(0.00004)
δ_2	-0.00001^{***}	-0.00001^{***}	-0.00001^{***}
	(0.0000002)	(0.0000002)	(0.0000001)
δ_{pad}	0.080^{***}	0.118^{***}	0.101^{***}
	(0.010)	(0.005)	(0.004)
Outcome equation:			
γ_0	0.716^{***}	0.565^{***}	0.575^{***}
	(0.040)	(0.026)	(0.022)
γ_1	-0.003^{***}	-0.003^{***}	-0.003^{***}
	(0.0001)	(0.0001)	(0.0001)
γ_2	0.003^{***}	0.003^{***}	0.003^{***}
	(0.0001)	(0.0001)	(0.00005)
γ_{pad}	-0.052^{***}	-0.147^{***}	-0.110^{***}
	(0.005)	(0.003)	(0.002)
Data Sample	Jan 2011 - Sept 2011	Sept 2011 - Dec 2012	Jan 2011 - Dec 2012
Free Shipping Threshold	High	Low	High and Low
Observations	827,041	2,624,712	3,451,753
R^2	0.168	0.107	0.123
Adjusted R^2	0.168	0.107	0.123
Correlation, ρ	0.388	0.588	0.551
Inverse Mills Ratio	0.117^{***} (0.019)	0.199^{***} (0.012)	0.181^{***} (0.010)
c	104.94	107.18	104.75

Note: The location of the kink in the return proportion, c, is calibrated by optimizing the model fit.

*p<0.1; **p<0.05; ***p<0.01

Table EC.4 Estimates of the product return model, allowing for correlation between the return incidence and quantity decisions (Heckman Selection Model) vs. not allowing for such correlation (Probit model for return incidence and OLS for return proportion). The Heckman selection model in the first column corresponds to the model in Table 5 in the main text.

	Dependent variable:		
	Return Proportion	Return Decision	Return Proportion
	Heckman	*Probit*	*OLS*
Selection equation:			
δ_0	-1.739^{***}	-1.739^{***}	
	(0.005)	(0.005)	
δ_1	0.007^{***}	0.007^{***}	
	(0.00007)	(0.00007)	
δ_2	-0.00001^{***}	-0.00001^{***}	
	(0.0000002)	(0.0000002)	
δ_{pad}	0.080^{***}	0.080^{***}	
	(0.010)	(0.010)	
Outcome equation:			
γ_0	0.716^{***}		0.965^{***}
	(0.040)		(0.003)
γ_1	-0.003^{***}		-0.004^{***}
	(0.0001)		(0.00004)
γ_2	0.003^{***}		0.004^{***}
	(0.0001)		(0.0001)
γ_{pad}	-0.052^{***}		-0.062^{***}
	(0.005)		(0.042)
Data Sample	Jan 2011 - Sept 2011	Jan 2011 - Sept 2011	Jan 2011 - Sept 2011
Free Shipping Threshold	High	High	High
Observations	827,041	827,041	97,214
R^2	0.168		0.168
Adjusted R^2	0.168		0.168
Correlation, ρ	0.388		
Inverse Mills Ratio	0.117^{***} (0.019)		
c	104.94		107.17
Log Likelihood		$-283,957$	
Akaike Inf. Crit.		567,923	
Residual Std. Error (df = 97210)			0.283
F Statistic (df = 3; 97210)			$6,547.7^{***}$

Notes: The location of the kink in the return proportion models, c, is calibrated by optimizing the model fit. The OLS version of the return proportion model uses the subsample of orders that involved a return. *p<0.1; **p<0.05; ***p<0.01

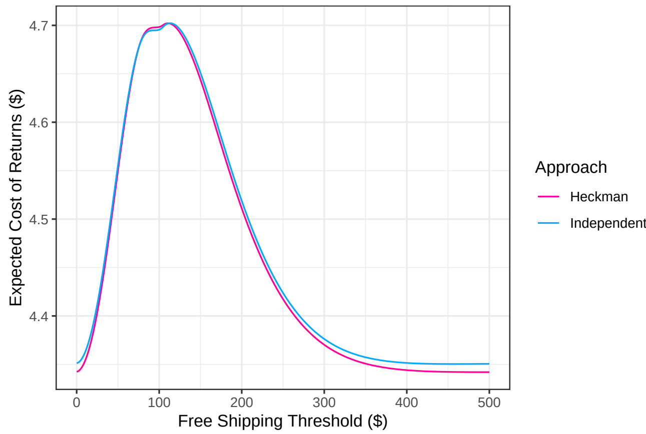


Figure EC.2 Expected cost of returns $R(\tau)$, estimated using the Heckman Selection Model which accounts for correlation between the return incidence and quantity, vs. assuming return incidence and quantity are independent.

EC.2. Alternative Specification for the Order Incidence Model

The data contains transactions from periods with two different free shipping thresholds, τ_1 and $\tau_2 = \tau_1/3$. Because only two thresholds are observed, assumptions are needed to characterize the functional form that relates the daily rate of orders that the retailer receives and the free shipping threshold. While we argue that the specification in Equation (3) is reasonable, in this section we check an alternative specification to test the robustness of our findings.

In this alternative specification, we let the Poisson rate of orders on day t, λ_t, be given by:

$$\log\big(\lambda_t(\tau)\big) = \psi'_0 + \psi'_1\tau + \psi'_{\text{year}(t)} + \psi'_{\text{weekday}(t)} + \psi'_{\text{week}(t)} \tag{EC.1}$$

and we define the $N(\tau)$ as the share of potential customers (i.e., those who would purchase if shipping were free) who continue to order when the free shipping threshold is τ, i.e.,

$$N(\tau) = \frac{\lambda_t(\tau)}{\lambda_t(0)} = \exp(\psi'_1\tau) \tag{EC.2}$$

We fit this model using Poisson regression and present the results in Table EC.5. As in the case of the original specification, the negative and significant coefficient suggests that a high free shipping threshold can deter customers from placing their orders.

Figure EC.3 presents the expected order volume $N(\tau)$ and the expected total profit $\Pi(\tau)$ under the alternative specification. The decline in order volume is less pronounced than in the main specification, what causes the profit-maximizing free shipping threshold to decrease from the original \$224.49 to \$171.20. Panel (b) supports the finding that setting the threshold above the optimum leads to smaller profit losses than setting it below.

Figure EC.4 presents, under this alternative specification for the order incidence model, the sensitivity of the profit and the optimal free shipping threshold with respect to the gross margin M, the fulfillment costs $K = K_r$, and the fraction of returned products that are damaged and cannot be sold at full price ω. As in Figure 6 in the main body of the paper, we observe that the optimal free shipping threshold is monotonic in these key operational parameters.

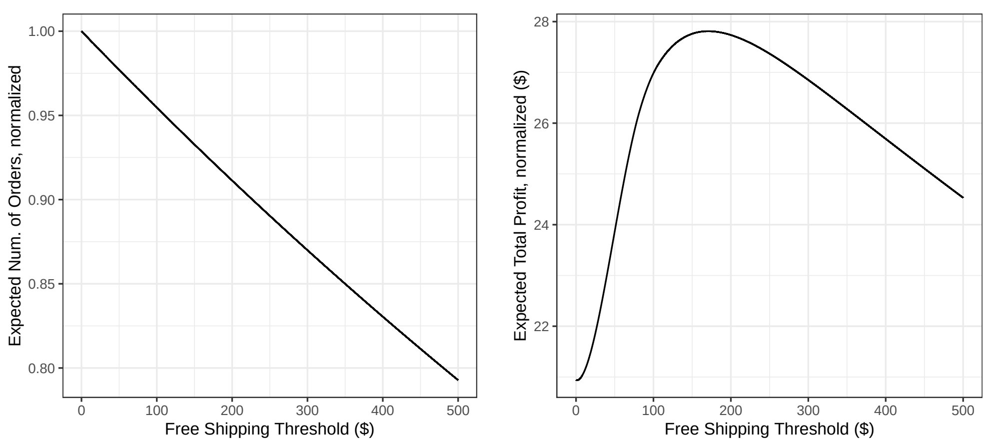
Interestingly, under the alternative specification of the order incidence model, low thresholds remain suboptimal, albeit for a different reason. Because order volume is less sensitive to the retailer's free shipping threshold, the framework tends to favor higher thresholds, since these thresholds impact order incidence less adversely than in the case of the original specification. As a result, offering free shipping on all (or most) orders is not optimal, even when fulfillment costs are minimal or gross margins are high – precisely the conditions under which a retailer would typically be most willing to absorb shipping costs to stimulate demand.

These findings reinforce the takeaways presented in the main body of the paper, highlight the value of our framework, and underscore the risks associated with poorly calibrated free shipping thresholds.

Table EC.5 Estimates of the order incidence model with alternative specification.

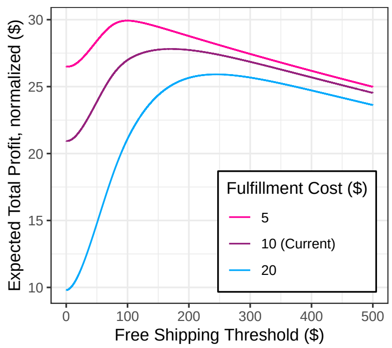
	Dependent variable:
	Num. of orders
ψ'_1	-0.00046^{***}
	(0.0001)
Day of the week FE	✓
Week in the year FE	✓
Year FEs	✓
Observations	112
Log Likelihood	−3,263.643
Akaike Inf. Crit.	6,563.287

*p<0.1; **p<0.05; ***p<0.01

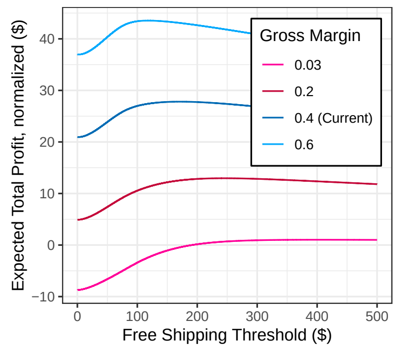


(a) Number of Orders (b) Total Profit

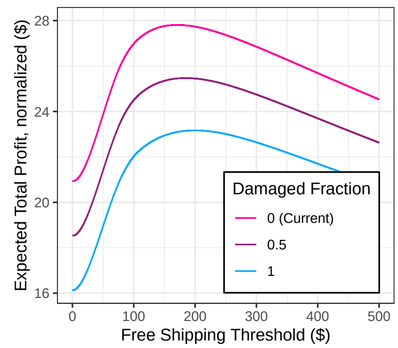
Figure EC.3 Model predictions for the Number of Orders and Total Profit as a function of the free shipping threshold, under the alternative specification for the order incidence model.



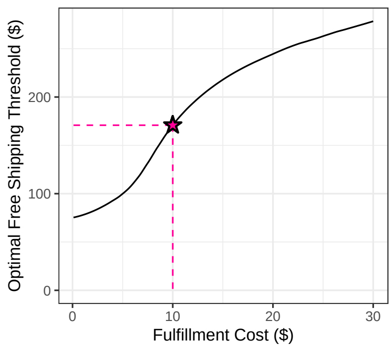
(a) Expected total profit, as a function of fulfillment cost K



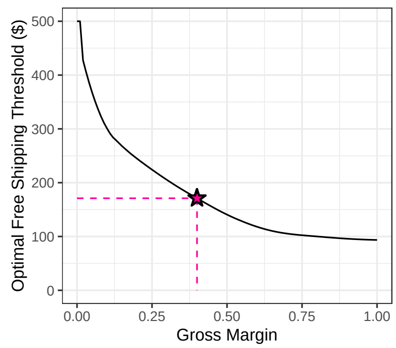
(b) Expected total profit, as a function of gross margin M



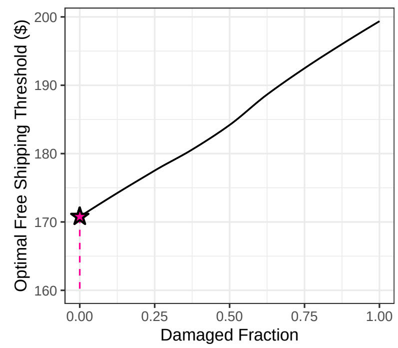
(c) Expected total profit, as a function of the damaged fraction ω



(d) Optimal free shipping threshold, as a function of fulfillment cost K



(e) Optimal free shipping threshold, as a function of gross margin M



(f) Optimal free shipping threshold, as a function of the damaged fraction ω

Figure EC.4 Sensitivity of the retailer's profits and the optimal free shipping threshold with respect to fulfillment cost, gross margin, and the fraction of returns that are damaged, under the alternative specification for the order incidence model.

EC.3. Mathematical Proofs

EC.3.1. Proof of Proposition 1.

Let X follow a log-concave distribution, and $f(x)$ and $F(x)$ represent its pdf and cdf, respectively. By definition, $\alpha \in [0,1]$, $\beta > 0$ and $E(Y) > 0$.

The average basket size $D(\tau)$ is given by Equation (6). Taking the derivative with respect to τ we obtain:

$$D'(\tau) = \begin{cases} \alpha f(\tau)E[Y] + F(\tau)\Big(\alpha - \beta\big(E[Y] + 2(\tau - E[X|X \leq \tau])\big)\Big) & \text{if } \tau - \frac{\alpha}{\beta} < 0 \\ \alpha f(\tau)E[Y] + \Big(F(\tau) - F(\tau - \frac{\alpha}{\beta})\Big)\Big(\alpha - \beta\big(E[Y] + 2(\tau - E[X|\tau - \frac{\alpha}{\beta} \leq X \leq \tau])\big)\Big) & \text{if } \tau - \frac{\alpha}{\beta} \geq 0 \end{cases} \tag{EC.3}$$

For $D(\tau)$ to be unimodal, $D'(\tau)$ must be positive for τ near zero, negative for large τ, and cross zero exactly once.

Part 1: $D'(\tau) > 0$ for τ near zero, and $D'(\tau) < 0$ for large τ: From the shape of $D(\tau)$, we can show that $D(0) = E(X)$, $\lim_{\tau\to\infty} D(\tau) = E(X)$, and $D(\tau) > E(X)$ for all $\tau > 0$. Since $D(\tau)$ is continuous, we can combine these results to conclude that $D(\tau)$ must (1) be increasing near $\tau = 0$ and (2) converge to $E(X)$ from above as $\tau \to \infty$. This shows that $D'(\tau) > 0$ at τ near zero and $D'(\tau) < 0$ for a sufficiently large τ.

Part 2: $D'(\tau)$ crosses 0 exactly once: Because $D'(\tau) > 0$ for a sufficiently small τ and $D'(\tau) < 0$ for a large τ, by the Intermediate Value Theorem, $D'(\tau)$ must cross 0 at least once in the positive domain.

We now show that $D'(\tau) = 0$ at most once. Let $\bar{\tau}$ denote the smallest τ such that $D(\tau) = 0$ and suppose that $\bar{\tau} \geq \frac{\alpha}{\beta}$. The first-order condition must hold at $\bar{\tau}$, which implies:

$$\alpha - \beta E[Y] = 2\beta\left(\bar{\tau} - E\left[X\middle|\bar{\tau} - \frac{\alpha}{\beta} \leq X \leq \bar{\tau}\right]\right) - \alpha E[Y]\frac{f(\bar{\tau})}{F(\bar{\tau}) - F(\bar{\tau} - \frac{\alpha}{\beta})} \tag{EC.4}$$

The left-hand side $\alpha - \beta E[Y]$ is a constant, and the right-hand side is a function of τ.

From Corollary 1 in An (1998), we know that the right tail of a log-concave density is at most exponential. As a result, if $\lambda > 0$ denotes the rate of the exponential distribution that bounds the log-concave distribution, we have:

$$\tau - E\left[X\middle|\tau - \frac{\alpha}{\beta} \leq X \leq \tau\right] \geq \frac{\alpha}{\beta}\left(\frac{\exp\left(\frac{\lambda\alpha}{\beta}\right)}{\exp\left(\frac{\lambda\alpha}{\beta}\right) - 1}\right) - \frac{1}{\lambda} \tag{EC.5}$$

and

$$\frac{f(\tau)}{F(\tau) - F(\tau - \frac{\alpha}{\beta})} \leq \frac{\lambda}{\exp\left(\frac{\lambda\alpha}{\beta}\right) - 1} \tag{EC.6}$$

The former is always greater than $\frac{\alpha}{2\beta}$, and the latter is bounded between 0 and $\frac{\beta}{\alpha}$. Thus, $D'(\tau) < 0$ for all $\tau > \bar{\tau}$, so it follows that the first-order condition $D'(\tau) = 0$ is met exactly once.

Similarly, suppose that $\bar{\tau} < \frac{\alpha}{\beta}$. By the first order condition, we have:

$$\alpha - \beta E[Y] = 2\beta\big(\bar{\tau} - E[X|X \leq \bar{\tau}]\big) - \alpha E[Y]\frac{f(\bar{\tau})}{F(\bar{\tau})} \tag{EC.7}$$

In this case, $\tau - E[X|X \leq \tau]$ is increasing in τ and $\frac{f(\tau)}{F(\tau)}$ is decreasing in τ – the latter follows because the log-concavity of $f(x)$ implies the log-concavity of $F(x)$, as shown in Proposition 1 in An (1998). Hence, $D'(\tau) < 0$ for all $\tau > \bar{\tau}$ and $D'(\tau) = 0$ is satisfied exactly once.

Therefore, because $D'(\tau)$ is positive for τ near zero, negative for a large τ, and crosses zero exactly once, we conclude that the average basket size function $D(\tau)$ is unimodal if the base demand X follows a log-concave distribution. □

EC.3.2. Proof of Proposition 2.

To establish this result, we derive conditions for the total profits function $\Pi(\tau)$ to be (1) increasing at $\tau = 0$ and (2) decreasing as $\tau \to \infty$. If $\Pi'(0) > 0$, then there will exist an $\epsilon > 0$ such that $\Pi(\epsilon) > \Pi(0)$, so $\tau^* = \arg\max_{\tau \in \mathbb{R}_+} \Pi(\tau) \neq 0$, that is, it is not optimal for the retailer to offer free shipping on all orders. Similarly, if $\Pi'(\tau) < 0$ as $\tau \to \infty$, then $\tau^* \neq \infty$, that is, charging shipping fees on all orders is not optimal.

We let X denote the base demand distribution, and $f(x)$ and $F(x)$ be its pdf and cdf, respectively. By definition, $\alpha \in [0,1]$, $\beta > 0$, $\kappa \in (0,1]$, and $E(X)$, $E(Y) > 0$. By economic principle, $\psi_1 < 0$.

The total profit function is:

$$\Pi(\tau) = N(\tau) \times \big(M\ D(\tau) + S(\tau) - K - R(\tau)\big) \tag{EC.8}$$

and its derivative with respect to τ is:

$$\Pi'(\tau) = N'(\tau) \times \big(M\ D(\tau) + S(\tau) - K - R(\tau)\big) + N(\tau) \times \big(M\ D'(\tau) + S'(\tau) - R'(\tau)\big) \tag{EC.9}$$

The functions $N(\tau)$, $D(\tau)$, $S(\tau)$, and $R(\tau)$ are defined by Equations (4), (6), (8) and (16) in the paper, respectively. Differentiating $N(\tau)$ and $S(\tau)$ with respect to τ we obtain:

$$N'(\tau) = \psi_1(1+\tau)^{\psi_1 - 1} \tag{EC.10}$$

and

$$S'(\tau) = \begin{cases} \kappa K\Big((1-\alpha)f(\tau) + \beta F(\tau)\Big) & \text{if } \tau - \frac{\alpha}{\beta} < 0 \\ \\ \kappa K\Big((1-\alpha)f(\tau) + \beta\big(F(\tau) - F(\tau - \frac{\alpha}{\beta})\big)\Big) & \text{if } \tau - \frac{\alpha}{\beta} \geq 0 \end{cases} \tag{EC.11}$$

The derivative of $D(\tau)$ with respect to τ, $D'(\tau)$, is expressed in Equation (EC.3). The derivative of $R(\tau)$ with respect to τ admits a closed-form expression that is algebraically cumbersome. Accordingly, we only present its evaluations at key points, such as $\tau = 0$ – see Equation (EC.12).

Part 1: Conditions for $\Pi'(0)>0$**:** At $\tau=0$, we know that $N(0)=1$, $N'(0)=\psi_1$, $D(0)=E(X)$, and $S(0)=0$. Furthermore, because $F(0)=0$, it follows that $D'(0)=\alpha f(0)E[Y]$ and $S'(0)=(1-\alpha)f(0)\kappa K$. Lastly, $R(0)$ can be computed using Equation (16), and $R'(0)$ is given by:

$$R'(0)=\alpha f(0)\left[\int_0^\infty \left(M+(1-M)\omega\right) r_1^*(v)\, v\, \Phi\big(z_1^*(v)\big)\, g(v)\, dv+\int_0^\infty K_r\, \Big(\Phi\big(z_1^*(v)\big)-\Phi\big(\delta_0\big)\Big)\, g(v)\, dv\right] \tag{EC.12}$$

Consequently, the derivative of the the profit function at $\tau=0$ is:

$$\Pi'(0)=\psi_1\big(M\ E(X)-K-R(0)\big)+\alpha f(0)M\ E(Y)+(1-\alpha)f(0)\kappa K-R'(0) \tag{EC.13}$$

It is not optimal for the retailer to offer free shipping on all orders if $\Pi'(0)>0$ or, equivalently, if the fulfillment cost K is such that:

$$K>\frac{-\psi_1\big(ME(X)-R(0)\big)-\alpha f(0)ME(Y)+R'(0)}{(1-\alpha)f(0)\kappa-\psi_1}=\underline{K} \tag{EC.14}$$

Part 2: Conditions for $\Pi'(\tau)<0$ **as** $\tau\to\infty$**:** It can be shown that $N(\tau)$, $N'(\tau)$, $D'(\tau)$, $S'(\tau)$ and $R'(\tau)$ all tend to 0 as $\tau\to\infty$. Similarly, because a high τ gives customers no incentive to pad their orders (just like $\tau=0$), then $\lim_{\tau\to\infty}R(\tau)=R(0)$ These results, combined with the fact that $MD(\tau)+S(\tau)-K-R(0)$ is finite for all $\tau\in[0,\infty)$, ensures that both $\Pi(\tau)$ and $\Pi'(\tau)$ tend to 0 as $\tau\to\infty$.

Because $\lim_{\tau\to\infty}\Pi(\tau)=0$, for a large enough τ, the total profit $\Pi(\tau)$ must be decreasing if $\Pi(\tau)>0$ and increasing if $\Pi(\tau)<0$. Hence, since $N(\tau)>0$ for all $\tau\in\mathbb{R}_+$, the condition for $\Pi(\tau)$ to be decreasing as $\tau\to\infty$ is that $\lim_{\tau\to\infty}MD(\tau)+S(\tau)-K-R(\tau)=ME(X)-(1-\kappa)K-R(0)>0$, or the fulfillment cost must satisfy $K<\frac{ME(X)-R(0)}{1-\kappa}=\bar{K}$.

Part 3: Show that it is possible that $\Pi(\tau)$ **is increasing at** $\tau=0$ **and decreasing as** $\tau\to\infty$**, and hence has an interior maximizer:** This happens if $\underline{K}<\bar{K}$, i.e.,

$$\frac{-\psi_1\big(ME(X)-R(0)\big)-\alpha f(0)ME(Y)+R'(0)}{(1-\alpha)f(0)\kappa-\psi_1}<\frac{ME(X)-R(0)}{1-\kappa} \tag{EC.15}$$

which can be rewritten as

$$\psi_1\kappa\big(ME(X)-R(0)\big)<(1-\alpha)f(0)\kappa\big(ME(X)-R(0)\big)+\alpha f(0)(1-\kappa)ME(Y)-(1-\kappa)R'(0) \tag{EC.16}$$

Because $\psi_1<0$, the left-hand side is always negative under the mild assumption that the retailer turns a profit from selling merchandise (disregarding fulfillment costs and shipping revenues), i.e., $ME(X)-R(0)>0$. The right-hand side is non-negative, because $f(0)\geq 0$, and because $R'(0)$ can be interpreted as a non-negative constant $\alpha f(0)$ times the (hypothetical) average cost of returns for the padded fraction of an order if the firm sets $\tau=0$. We denote the latter, which represents the term in the square brackets in Equation (EC.12), as $R_P(0)$. Hence, we can rewrite $R'(0) = \alpha f(0) R_P(0)$. Similarly, we can denote the average cost of returns for the base demand fraction of the order if the firm sets $\tau = 0$ as $R_B(0) = R(0)$.

It follows that the RHS in Equation (EC.16) can be rewritten as

$$(1-\alpha) f(0) \kappa \big(ME(X) - R_B(0)\big) + \alpha f(0)(1-\kappa)\big(ME(Y) - R_P(0)\big) \tag{EC.17}$$

which is non-negative under the mild assumption that firm makes a non-negative profit from selling merchandise for both the base and padded fractions of an average order. The former, i.e., $ME(X) - R_B(0) \geq 0$, is required for the retailer's business to be sustainable. The latter, $ME(Y) - R_P(0) \geq 0$, is necessary so that it makes sense for a retailer to deploy a free shipping threshold policy as a means to improve its bottom line. For these reasons, these conditions will be always satisfied for any retailer doing business sustainably and that has a sensible incentive to consider a free shipping threshold policy. This completes the proof that $\underline{K} < \bar{K}$, and hence, there exists an interval of fulfillment costs K such that $\Pi(\tau)$ has an interior maximizer $\tau^* \in (0, \infty)$. $\square$