Iterative Modelling (Final Model V.3)

Menorca Biodiversity Conservation Project

This final iteration adopts a **hybrid approach combining Graph Theory and Constraint Programming** to solve the conservation planning problem. Specifically, it implements a **Column Generation-inspired strategy** where viable connectivity paths are pre-computed rather than dynamically discovered during optimization.

The strategy consists of two phases:

1. **Graph Pre-processing (Dijkstra + Pruning):** Instead of allowing the solver to explore an exponential number of potential paths (Network Flow approach), we model the landscape as a weighted graph where edge weights represent real geodetic distances (Haversine metric) adjusted by friction costs. For every potential habitat site, the optimal path to the nearest existing population core is pre-calculated using Dijkstra's algorithm. To ensure computational tractability, paths exceeding a cost feasibility threshold are pruned from the decision space.
2. **Combinatorial Optimization (CP-SAT):** The problem is then formulated as a Path Selection Model where selecting a habitat site implicitly enforces the activation of its pre-calculated connection path. This transforms the complex connectivity constraints into linear implications, allowing the solver to focus on balancing multi-objective trade-offs such as biodiversity gain, budget allocation, and biological equity.

This methodology effectively decouples the pathfinding complexity from the combinatorial selection problem, ensuring both geographic accuracy and the ability to prove global optimality within reasonable time limits.

Sets and Indices

- I: Set of candidate sites (grid cells).
- S: Set of target species $S = \{\text{atelerix, martes, eliomys, oryctolagus}\}$.
- E: Set of all potential corridor edges connecting adjacent sites (u, v).
- $P_{i,s} \subseteq E$: Set of edges representing the shortest path from the nearest existing population of species s to candidate site i. This parameter is pre-computed to enforce connectivity efficiently.

Parameters

- A_i: Area of site i (km^2).
- $Q_{i,s}$: Habitat suitability index for species s in site i.
- $\Delta Q_{i,s}$: Gain in habitat quality if restoration is applied.
- $C_{i,s}^{\text{adapt}}$: Cost of land acquisition or adaptation for species s at site i.

- C_e^{corr}: Cost of activating corridor edge e, calculated based on geodesic distance and terrain friction.
- B_{total}: Total available budget.
- $\text{MinPct}_s, \text{MaxPct}_s$: Minimum and maximum percentage of total conserved area allocated to species s (biological equity targets).

Decision Variables

- $x_{i,s} \in \{0, 1\}$: 1 if site i is selected as habitat for species s.
- $y_{i,s} \in \{0, 1\}$: 1 if investment is made to restore site i for species s.
- $z_e \in \{0, 1\}$: 1 if corridor edge e is activated.
- $\text{stress}_i \in \{0, 1\}$: 1 if interspecific conflict exists in site i.

Objective Function

The objective maximizes the total weighted ecological value of the network while minimizing interspecific stress and redundant corridor costs:

$$\text{Maximize} \sum_{i \in I} \sum_{s \in S} W_s A_i \left(Q_{i,s} x_{i,s} + \Delta Q_{i,s} y_{i,s} \right) - \sum_{i \in I} P_{\text{stress}} \cdot \text{stress}_i - \sum_{e \in E} \epsilon \cdot z_e \tag{1}$$

Constraints

1. Investment Logic

Restoration (y) is only possible if the site is selected (x). For non-native sites, restoration investment is mandatory for habitation.

$$y_{i,s} \leq x_{i,s} \quad \forall i, s \tag{2}$$
$$x_{i,s} \leq y_{i,s} \quad \forall i, s \in S_{\text{non-native}} \tag{3}$$

2. Connectivity via Path Implication

This constraint enforces structural connectivity without flow variables. If a site i is selected for species s, the entire path $P_{i,s}$ connecting it to a core population must be activated.

$$x_{i,s} \leq z_e \quad \forall i, s, \forall e \in P_{i,s} \tag{4}$$

3. Feasibility Cutoff (Pruning)

Sites whose connection cost exceeds a feasibility threshold (e.g., due to extreme distance) are deemed unreachable.

$$x_{i,s} = 0 \quad \text{if } P_{i,s} = \emptyset \tag{5}$$

4. Budget Constraint

The sum of adaptation costs and corridor infrastructure costs must not exceed the budget.

$$\sum_{i \in I} \sum_{s \in S} C_{i,s}^{\text{adapt}} y_{i,s} + \sum_{e \in E} C_e^{\text{corr}} z_e \leq B_{\text{total}} \tag{6}$$

5. Biological Equity

To prevent a single species from dominating the solution, the total area conserved for each species s is bounded relative to the total network size.

$$\mathrm{MinPct}_s \cdot \mathcal{A}_{\text{total}} \leq \sum_{i \in I} A_i x_{i,s} \leq \mathrm{MaxPct}_s \cdot \mathcal{A}_{\text{total}} \tag{7}$$

Where $\mathcal{A}_{\text{total}} = \sum_{k \in S} \sum_{j \in I} A_j x_{j,k}$.

6. Interspecific Conflict

Competitive exclusion between incompatible species (e.g., *Martes* and *Eliomys*) is enforced to prevent co-occurrence in the same cell.

$$x_{i,\text{martes}} + x_{i,\text{eliomys}} \leq 1 \quad \forall i \tag{8}$$