Iterative Modelling (Initial Model V.1)

Menorca Biodiversity Conservation Project

This initial iteration formulates the problem as a **Multi-Commodity Network Flow** model within a Mixed-Integer Programming (MIP) framework. The core strategy transforms the biological connectivity requirement into a **dynamic flow conservation problem**. In this approach:

1. **Dynamic Topology Discovery:** This model attempts to strictly construct the optimal graph topology from scratch during the optimization process.
2. **Flow Simulation:** Species are treated as "commodities" that must flow from existing population cores (source nodes) to newly restored habitats. A candidate site can only be selected if it receives a continuous "flow" of individuals from a neighbor, ensuring structural continuity without isolated islands.
3. **Big-M Formulation:** The coupling between the continuous flow variables (movement) and binary decision variables (corridor activation) is enforced via Big-M constraints.

Sets and Indices

- I: Set of grid cells (nodes).
- E: Set of edges connecting adjacent cells $(u, v) \in E$.
- S: Set of species $S = \{\text{atelerix, martes, eliomys, oryctolagus}\}$.

Decision Variables

- $x_{i,s} \in \{0, 1\}$: 1 if cell i is selected as habitat for species s.
- $y_{i,s} \in \{0, 1\}$: 1 if investment is made to restore cell i for species s.
- $z_{u,v} \in \{0, 1\}$: 1 if the ecological corridor between u and v is activated.
- $f_{u,v,s} \in \mathbb{Z}_{\geq 0}$: Flow of species s individuals through edge (u, v). Auxiliary variable for connectivity.
- $\text{stress}_i \in \{0, 1\}$: 1 if interspecific conflict exists in cell i.

Objective Function

Maximize the total biodiversity score, penalized by stress and costs:

$$\text{Maximize} \sum_{i \in I} \sum_{s \in S} (Q_{i,s} \cdot W_s \cdot A_i \cdot x_{i,s} + \Delta Q_{i,s} \cdot W_s \cdot A_i \cdot y_{i,s}) - \sum_{i \in I} P_{\text{stress}} \cdot \text{stress}_i - \sum_{(u,v) \in E} \epsilon \cdot z_{u,v} \tag{1}$$

Where $Q_{i,s}$ is habitat suitability, W_s is species weight, and A_i is cell area.

Constraints

1. Investment Logic: Restoration (y) is only possible if the cell is active (x). If the species is not native, investment is mandatory.

$$y_{i,s} \leq x_{i,s} \quad \forall i, s \tag{2}$$
$$x_{i,s} \leq y_{i,s} \quad \forall i, s \in S_{\text{non-native}} \tag{3}$$

2. Budget Constraint:

$$\sum_{i,s} (C^{\text{adapt}}_{i,s} \cdot y_{i,s}) + \sum_{(u,v) \in E} (C^{\text{corr}}_{u,v} \cdot z_{u,v}) \leq B_{\text{total}} \tag{4}$$

3. Biological Conflicts: Competitive exclusion between *Martes* and *Eliomys*, and stress calculation:

$$x_{i,\text{martes}} + x_{i,\text{eliomys}} \leq 1 \quad \forall i \tag{5}$$
$$\text{stress}_i \geq x_{i,\text{martes}} + x_{i,\text{oryctolagus}} - 1 \quad \forall i \tag{6}$$

4. Connectivity via Network Flow: To ensure spatial continuity, any active non-native cell must receive "flow" from a neighbor.

$$\sum_{j \in N(i)} f_{j,i,s} - \sum_{j \in N(i)} f_{i,j,s} \geq x_{i,s} \quad \forall i, s \notin \text{Native}_i \tag{7}$$

5. Flow-Linkage: Flow can only pass through an edge if the corridor is paid for ($z = 1$) and both cells are active for the species.

$$f_{u,v,s} = 0 \quad \text{if } z_{u,v} = 0 \tag{8}$$
$$f_{u,v,s} = 0 \quad \text{if } x_{u,s} = 0 \text{ or } x_{v,s} = 0 \tag{9}$$