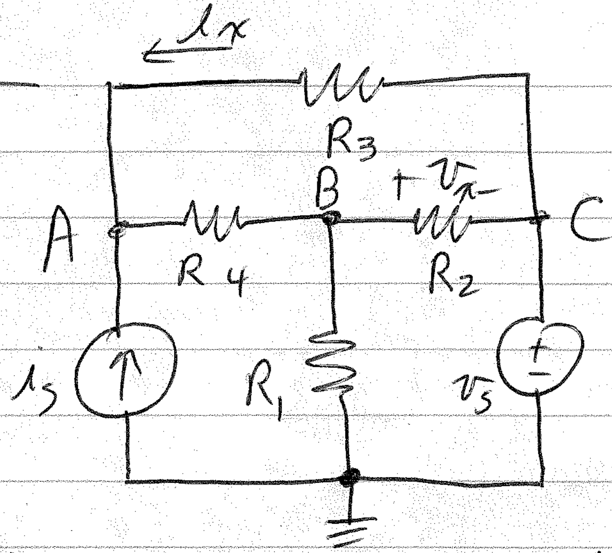
2014 – HW 3 Solutions

3-6



a) Choose ground and write N-V equations.

Answer: Choose "bottom" node as ground to eliminate 1 eqn. Label nodes as shown. $V_c = v_s$

(A) $$\frac{V_A - V_B}{R_4} - i_s + \frac{V_A - V_c}{R_3} = 0$$

$$\left(\frac{1}{R_3} + \frac{1}{R_4}\right) V_A - \frac{1}{R_4} V_B = i_s + \frac{v_s}{R_3}$$

(B) $$\frac{V_B - V_A}{R_4} + \frac{V_B}{R_1} + \frac{V_B - V_c}{R_2} = 0$$

$$-\frac{1}{R_4} V_A + \left(\frac{1}{R_4} + \frac{1}{R_1} + \frac{1}{R_2}\right) V_B = \frac{v_s}{R_2}$$

b.) Solve for v_x + i_x when $R_1 = R_2 = R_3 = R_4 = 10\,k\Omega$, ~~and~~ $v_s = 25V$, and $i_s = 1mA$.

Answer: First, write v_x + i_x in terms of V's:

$$v_x = V_B - V_C = \underline{V_B - v_s}$$

$$i_x = \frac{V_C - V_A}{R_3} = \underline{\frac{v_s - V_A}{R_3}}$$

So we need to solve for both V_A + V_B.

First let $R_1 = R_2 = R_3 = R_4 = \underline{R}$ and simplify:

A $\quad \frac{2}{R}V_A - \frac{1}{R}V_B = i_s + \frac{v_s}{R}$

$$2V_A - V_B = Ri_s + v_s$$

B $\quad -\frac{1}{R}V_A + \left(\frac{3}{R}\right)V_B = \frac{v_s}{R}$

$$-V_A + 3V_B = v_s$$

$$3V_B = v_s + V_A$$

$$V_B = \frac{1}{3}(v_s + V_A)$$

$$2V_A - \left(\frac{1}{3}\right)(v_s + V_A) = Ri_s + v_s$$

$$2V_A - \frac{1}{3}V_A = Ri_s + v_s + \frac{1}{3}v_s$$

$$\frac{5}{3}V_A = Ri_s + \frac{4}{3}v_s$$

$$V_A = \frac{3}{5}Ri_s + \frac{4}{5}v_s$$

Now substitute values:

$$V_A = \tfrac{3}{5}(10k\Omega)1mA + \tfrac{4}{5}(25V)$$

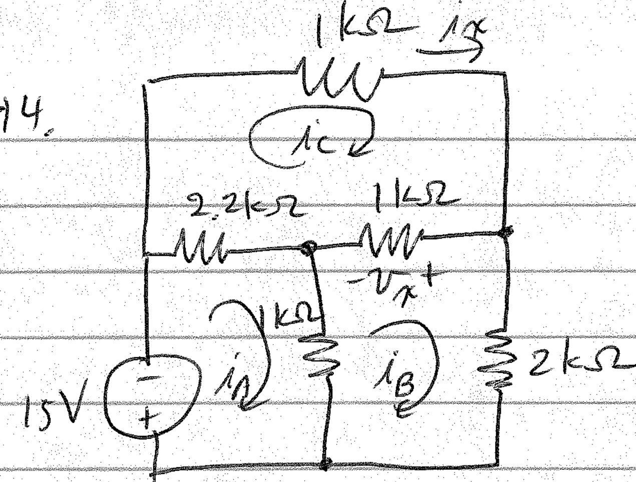
$$= 6V + 20V = \underline{\underline{26V}}$$

$$V_B = \tfrac{1}{3}(25V + 26V) = \tfrac{51}{3}V = \underline{\underline{17V}}$$

So $v_x = V_B - 25V = 17V - 25V = \underline{\underline{-8V}}$

$$i_x = \frac{25V - 26V}{10k\Omega} = \frac{-1V}{10k\Omega} = \underline{\underline{-0.1\,mA}}$$

3.4.



a) Formulate MC eqn's + put in matrix form.

(A) $2.2k\Omega(i_A - i_C) + 1k\Omega(i_A - i_B) + 15V = 0$

$3.2k\Omega\, i_A - 1k\Omega\, i_B - 2.2k\Omega\, i_C = -15V$

(B) $1k\Omega(i_B - i_C) + 2k\Omega\, i_B + 1k\Omega(i_B - i_A) = 0$

$-1k\Omega\, i_A + 4k\Omega\, i_B - 1k\Omega\, i_C = 0$

(C) $1k\Omega\, i_C + 1k\Omega(i_C - i_B) + 2.2k\Omega(i_C - i_A) = 0$

$-2.2k\Omega\, i_A - 1k\Omega\, i_B + (4.2k\Omega)\, i_C = 0$

Matrix form:

$$\begin{pmatrix} 3.2k\Omega & -1k\Omega & -2.2k\Omega \\ -1k\Omega & 4k\Omega & -1k\Omega \\ -2.2k\Omega & -1k\Omega & 4.2k\Omega \end{pmatrix} \begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix} = \begin{pmatrix} -15V \\ 0 \\ 0 \end{pmatrix}$$

b.) Solve by Cramer's Rule:

Δ = determinant of Resistance Matrix

$$= 3.2k\big((4k)(4.2k) - (-1k)(-1k)\big)$$
$$- (-1k)\big((-1k)(4.2k) - (-2.2k)(4k)\big)$$
$$+ (-2.2k)\big((-1k)(-1k) - (-2.2k)(4k)\big)$$

$$\Delta = 3.2k(16.8k^2 - 1k^2) + 1k(-4.2k^2 - 2.2k^2) - 2.2k(1k^2 + 8.8k^2)$$
$$= 3.2k(15.8k^2) + 1k(-6.4k^2) - 2.2k(9.8k^2)$$
$$= 50.56\,k^3 - 6.4k^3 - 21.56\,k^3$$
$$\underline{\Delta = 22.6\,k^3}$$

$$\Delta_A = (-15V)\big[(4k)(4.2k) - (-1k)(-1k)\big] \cancel{+0[\;]}$$
$$- (-1k)[0 - 0] + (-2.2k)[0 - 0]$$
$$= -15V[16.8k^2 - 1k^2] = -15V[15.8k^2]$$

$$\Delta_B = 3.2k[0 - 0] - (-15V)\big[(-1k)(4.2k) - (-2.2k)(-1k)\big] + (-2.2k)[0 - 0]$$

$$= 15V[-4.2k^2 - 2.2k^2] = 15V[-6.4k^2]$$

$$\Delta_C = 3.2k[0 - 0] - (-1k)[0 - 0] + (-15V)\big[(-1k)(-1k) - (-2.2k)(4k)\big]$$

$$= (-15V)[1k^2 + 8.8k^2] = -15V[9.8k^2]$$

So $$i_A = \frac{\Delta A}{\Delta} = \frac{-15V[15.8\,k^2\Omega^2]}{22.6\,k^3\Omega^3} = -\frac{15V}{k\Omega}\,\frac{15.8}{22.6} =$$

$$\underline{i_A = -10.5\ mA}$$

$$i_B = \frac{\Delta_B}{\Delta} = \frac{-15V(6.4k^2\Omega^2)}{22.6k^3\Omega^3}$$

$$\underline{i_B = -4.25\text{ mA}}$$

$$i_C = \frac{\Delta_C}{\Delta} = \frac{-15V(9.8k^2\Omega^2)}{22.6k^3\Omega^3}$$

$$\underline{i_C = -6.50\text{ mA}}$$

C) Use these to find v_x + i_x:

Answer: $v_x = 1k\Omega(i_C - i_B)$

$$= 1k\Omega(-6.50mA - (-4.25mA))$$

$$\underline{v_x =} -2.25V$$

$$\underline{i_x = i_C = -6.50\text{ mA}}$$

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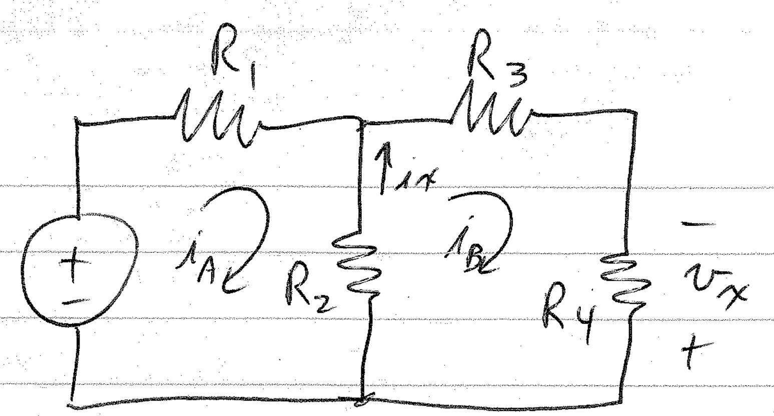


a) Formulate MC eqn's + write as matrix.

Answer: (A) $R_1 i_A + R_2(i_A - i_B) - V_s = 0$

$$\underline{(R_1+R_2)i_A - R_2 i_B = V_s}$$

(B) $R_3 i_B + R_4 i_B + R_2(i_B - i_A) = 0$

$$\underline{-R_2 i_A + (R_2+R_3+R_4) i_B = 0}$$

Matrix form:

$$\begin{pmatrix} (R_1+R_2) & -R_2 \\ -R_2 & (R_2+R_3+R_4) \end{pmatrix} \begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} V_s \\ 0 \end{pmatrix}$$

b.) Use these to find v_x + i_x.

Solve for i_A + i_B by Cramer's Rule:

$$\Delta = (R_1+R_2)(R_2+R_3+R_4) - (-R_2)(-R_2)$$
$$= (R_1+R_2)(R_2+R_3+R_4) - R_2^2$$
$$= R_1R_2 + R_1R_3 + R_1R_4 + \cancel{R_2^2} + R_2R_3 + R_2R_4 - \cancel{R_2^2}$$

$$\Delta_A = V_s(R_2+R_3+R_4) - \cancel{R}$$
$$= V_s(R_2+R_3+R_4)$$

$$\Delta_B = 0-(-R_2)(V_s)$$
$$= R_2 V_s$$

$$i_A = \frac{\Delta_A}{\Delta} = \frac{(R_2+R_3+R_4)}{R_1R_2+R_1R_3+R_1R_4+R_2R_3+R_2R_4} V_s$$

$$i_B = \frac{R_2}{R_1R_2+R_1R_3+R_1R_4+R_2R_3+R_2R_4} V_s$$

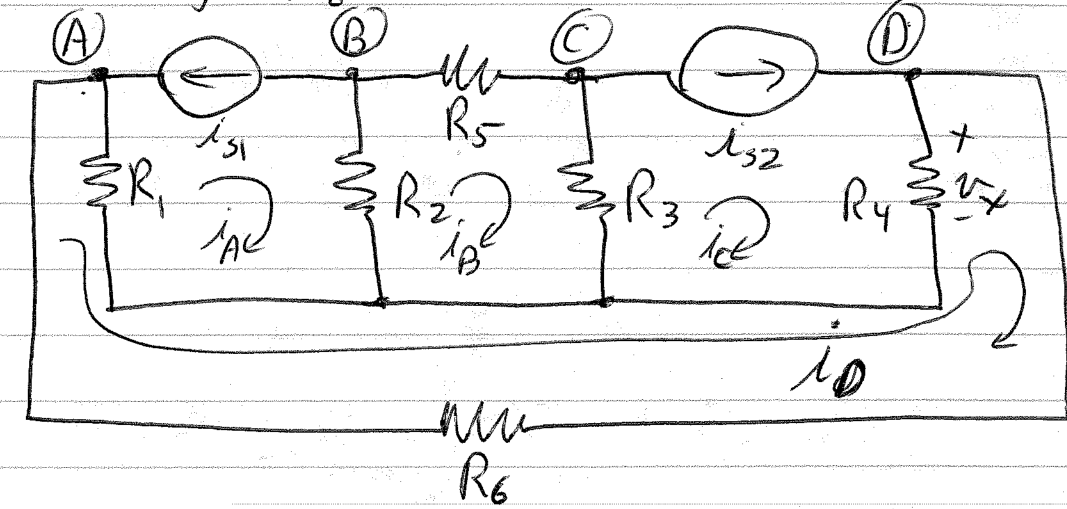
$$v_x = R_4(-i_B) = -\frac{R_4(R_2\,\cancel{})}{R_1R_2+R_1R_3+R_1R_4+R_2R_3+R_2R_4} V_s$$

$$i_x = i_B - i_A$$

$$i_x = \frac{R_2-(R_2+R_3+R_4)}{\Delta} V_s$$

$$i_x = \frac{-R_3-R_4}{\Delta} V_s$$

3-21 Rearrange R_6:

a) 

b) Mesh-Current Eqn's:

$$\underline{i_A = -i_{s1}}$$

$$\underline{i_C = i_{s2}}$$

(B) $R_5 i_B + R_3(i_B - i_C) + R_2(i_B - i_A) = 0$

$$\underline{(R_5 + R_3 + R_2) i_B = R_3 i_C + R_2 i_A = R_3 i_{s2} - R_2 i_{s1}}$$

(D) $R_1(i_D - i_A) + R_4(i_D - i_C) + R_6 i_D = 0$

$$\underline{(R_1 + R_4 + R_6) i_D = R_1 i_A + R_4 i_C = -R_1 i_{s1} + R_4 i_{s2}}$$

c) Solve for v_x when $R_1 = R_2 = R_3 = R_4 = 4k\Omega$, $R_5 = R_6 = 2k\Omega$, $i_{s1} = 80mA$, $i_{s2} = 40mA$.

$v_x = R_4(i_C - i_D)$ so we need to solve for i_D:

$$i_D = \frac{-R_1 i_{s1} + R_4 i_{s2}}{R_1 + R_4 + R_6}$$

$$= \frac{-(4k)(80mA) + (4k)(40mA)}{4k + \cancel{4mA}\,4k + 2k}$$

$$= \frac{-320V + 160V}{10k} = \frac{-160V}{10k\Omega} = -16mA$$

So $v_x = 4k\Omega(40mA + 16mA)$

$$= 4k\Omega(56mA) = 224V$$

(Answer in back of book gives 224V.
~~I reviewed equations parts a and b are same different. Simulated it in CircuitLab & got 128.84V.~~)

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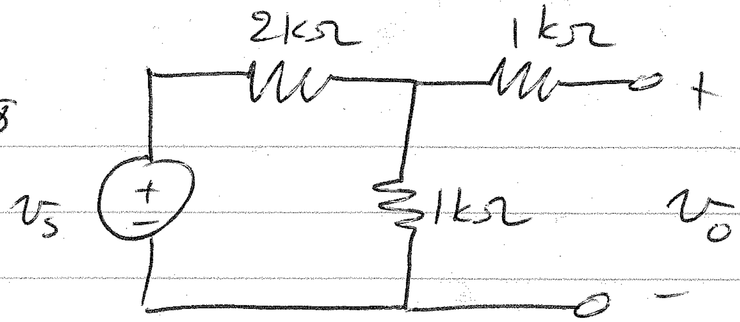

a) Find proportionality constant $K = v_o / v_s$.

Answer: No current thru 1 kΩ on top, so no voltage drop, so

$$v_o = \frac{1\,k\Omega}{1\,k\Omega + 2\,k\Omega} v_s \quad \text{by Voltage Div.}$$

$$v_o = \frac{1}{3} v_s$$

or $\underline{K = \frac{1}{3}}$

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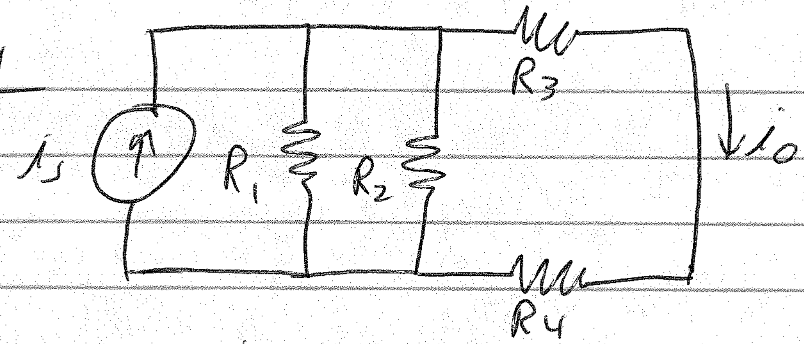
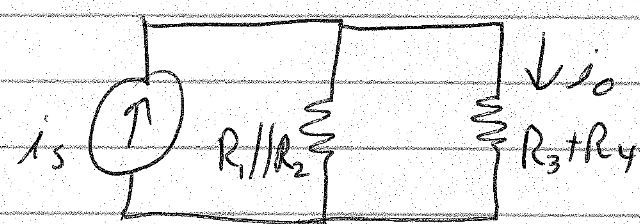


Find $K = \frac{i_o}{i_s}$ in this ckt.

Answer: i_o is current thru $R_3 + R_4$ in series, Combine $R_1 + R_2$ in parallel + get:



By Current Division,

$$i_o = \frac{R_1 || R_2}{R_1 || R_2 + (R_3 + R_4)} i_s$$

or

$$K = \frac{i_o}{i_s} = \frac{\frac{R_1 R_2}{R_1 + R_2}}{\frac{R_1 R_2}{R_1 + R_2} + (R_3 + R_4)}$$

$$K = \frac{\frac{R_1 R_2}{(R_1 + R_2)}}{\frac{R_1 R_2 + (R_1 + R_2)(R_3 + R_4)}{(R_1 + R_2)}} = \frac{R_1 R_2}{R_1 R_2 + (R_1 + R_2)(R_3 + R_4)}$$

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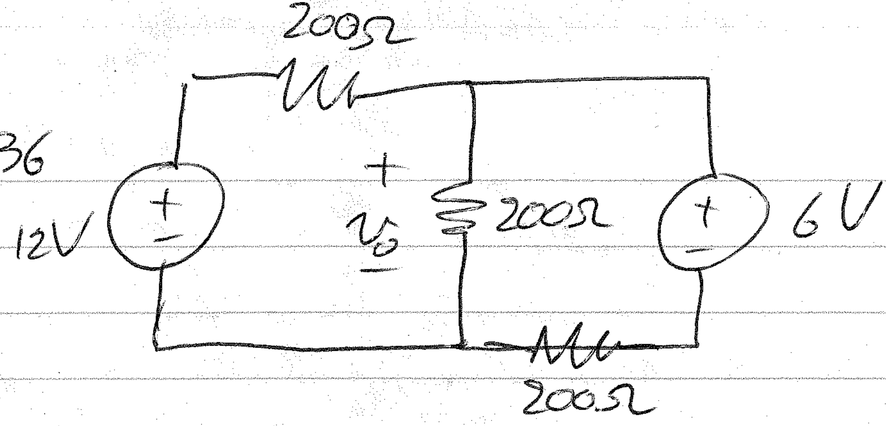
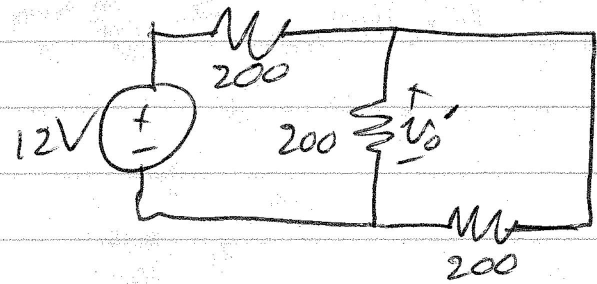


Use Superposition to find v_o.

Answer:

a) Turn off 6V src:

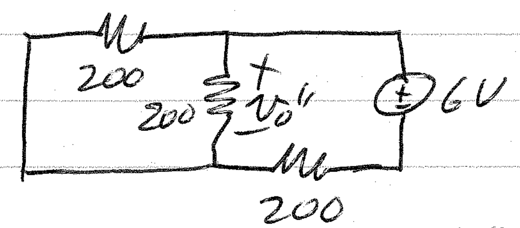


v_o' is Voltage across parallel R's, $R_{eq} = 100\Omega$

$$v_o' = \frac{100\Omega}{100\Omega + 200\Omega} 12V \quad \text{by Voltage Div.}$$

$$= \frac{1}{3} 12V = \underline{\underline{4V}}$$

Turn off 12V src:



Again: $v_o'' = \frac{100}{100+200} 6V = \frac{1}{3}(6V) = \underline{\underline{2V}}$

So $v_o = v_o' + v_o'' = 4V + 2V = \underline{\underline{6V}}$