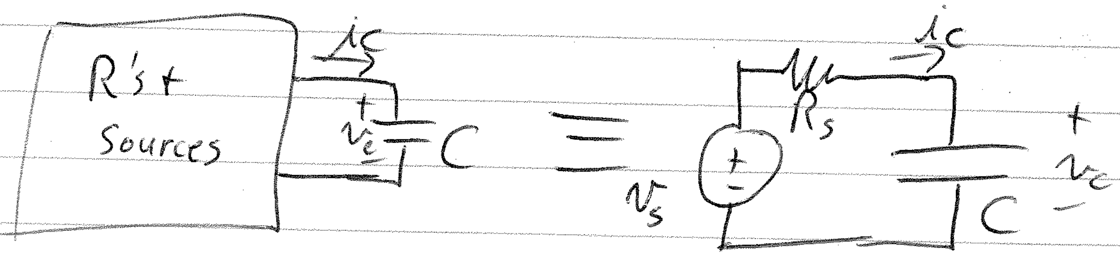
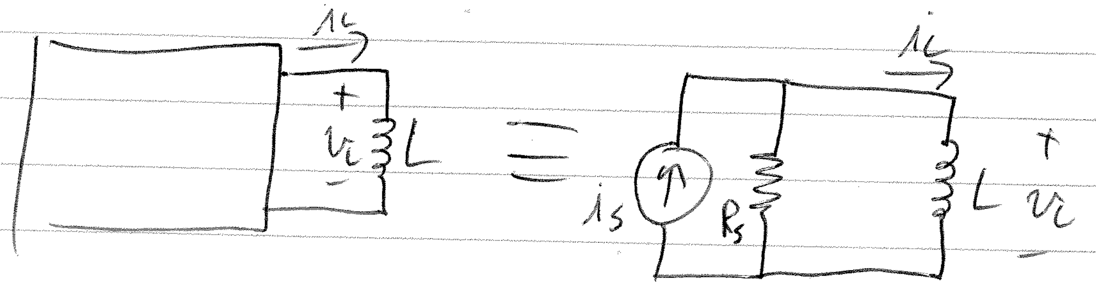
Read + Do examples + Exercises 6-13, 14, + 15.

We're going to move on to Ch. 7 - 1^{st} + 2^{nd} order ckts. Many things, even in the digital world today, still are governed or constrained by 1^{st} + 2^{nd} order phenomena, such as RC Time Constants, that limit our computer clock speed.

First Order Ckts contain one C or one L, and an equivalent resistance:





This is one reason we spent so much time learning to find Thev + Norton Equivalents

Look at RC:

KVL + Ω's Law: $R_s i_c + v_c - v_s = 0$

$$R_s\left(C\frac{dv_c}{dt}\right) + v_c = v_s$$

$$R_s C\frac{dv_c}{dt} + v_c = v_s$$

or $$\frac{dv_c}{dt} + \frac{1}{R_s C}v_c = \frac{1}{R_s C}v_s$$ in "standard form"

$$\frac{dv_c}{dt} + \frac{1}{\tau}v_c = \frac{1}{\tau}v_s$$

Inhomogeneous 1st order Linear DE w/ constant coeff's.

τ = Time Constant $= R_s C$

$v_c(t)$ is known as the "State Variable" because it determines the energy stored in the capacitor.

v_s is the input + v_c is the response.

Look at RL:

$$-i_s(t) + \frac{v_L(t)}{R_s} + i_L(t) = 0$$

$$\frac{1}{R_s}L\frac{di_L}{dt} + i_L(t) = +i_s(t)$$

$$\frac{di_L}{dt}(t) + \frac{R_s}{L}i_L(t) = \frac{R_s}{L}i_s(t)$$

$$\frac{di_L}{dt}(t) + \frac{1}{L/R_s}i_L(t) = \frac{1}{L/R_s}i_s(t)$$

$$\frac{di_L}{dt}(t) + \frac{1}{\tau}i_L(t) = \frac{1}{\tau}i_s(t)$$

$\tau = L/R_s$

Here i_L is the state variable.

i_L is the response to i_s as the input.

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To solve 1st order equations we need ~~to~~

~~have I known~~ know the response at at least one time, usually taken to be the Initial Value, or ~~I Con~~ Initial Condition, at t=0 (or whenever we "start" the ckt by switching something on, or off, or to a different value.)

First, note that the RHS of these equations can be thought of as:

$$\frac{1}{R_s C} v_s(t) + 0$$

$$\text{or} \quad \frac{R_s}{L} v_s(t) + 0$$

and we can think of the variables v_c + i_L as

$v_c(t) = v_{cf}(t) + v_{cn}(t)$, called the natural and forced responses.

and $i_L(t) = i_{Lf}(t) + i_{Ln}(t)$

→ insert 208a

The derivatives and constants distribute over the sums, so we can separate them as:

$$\frac{dv_{cf}}{dt} + \frac{1}{\tau} v_{cf} = \frac{1}{\tau} v_s \quad + \quad \frac{dv_{cn}}{dt} + \frac{1}{\tau} v_{cn} = 0$$

and $$\frac{di_{Lf}}{dt} + \frac{1}{\tau} i_{Lf} = \frac{1}{\tau} i_s \quad + \quad \frac{di_{Ln}}{dt} + \frac{1}{\tau} i_{Ln} = 0$$

called the Forced Response

Homogeneous, 1st Order DE's w/ Constant Coeff's.

Called the Zero-Input Response, or Natural Response

Solutions are of form

$v_{cn} = Ke^{st}$ and $i_{Ln} = Ke^{st}$

$\frac{dv_{cn}}{dt} = sKe^{st}$ $\qquad$ $\frac{di_{Ln}}{dt} = sKe^{st}$

Substitute:

$sKe^{st} + \frac{1}{\tau}Ke^{st} = 0$ and $sKe^{st} + \frac{1}{\tau}Ke^{st} = 0$

$(s + \frac{1}{\tau})Ke^{st} = 0 \quad \longleftrightarrow \quad (s + \frac{1}{\tau})Ke^{st} = 0$

3 possibilities $\qquad$ same!

$K = 0$ — trivial solution

$e^{st} = 0$ impossible except for $st = -\infty$

We can think of this like super position, the output due to both is the sum of the response due to one plus the response due to the other. Further, the nature of the response due to an input is the same as the nature of the input (ie: a constant input leads to a constant output.) This is often called a "particular response."

or $\left(s+\frac{1}{\tau}\right)=0$ The <u>characteristic equation</u>

$$\boxed{s=-\frac{1}{\tau}}$$

So $v_c = K e^{-t/\tau}$ or $i_L = K e^{-t/\tau}$

$\tau = R_s C$ $\qquad$ $\tau = L/R_s$

there is no forcing function v_s or $i_s = 0$ ~~then~~ and

If we ~~can~~ are given or can analyze the ckt to find $v_c(0)$ or $i_L(0)$ (IC's of the State Variables)

then we can write:

$$\underline{v_c(0) = Ke^{-0} = K} \quad \text{or} \quad \underline{i_L(0) = Ke^{-0} = K}$$

So now we know

$$v_c(t) = v_c(0)e^{-t/\tau} \quad \text{or} \quad i_L(t) = i_L(0)e^{-t/\tau}$$

↑ a constant usually called V_0 $\qquad$ ↑ called I_0

$$v_c(t) = V_0 e^{-t/\tau} \qquad i_L = I_0 e^{-t/\tau}$$