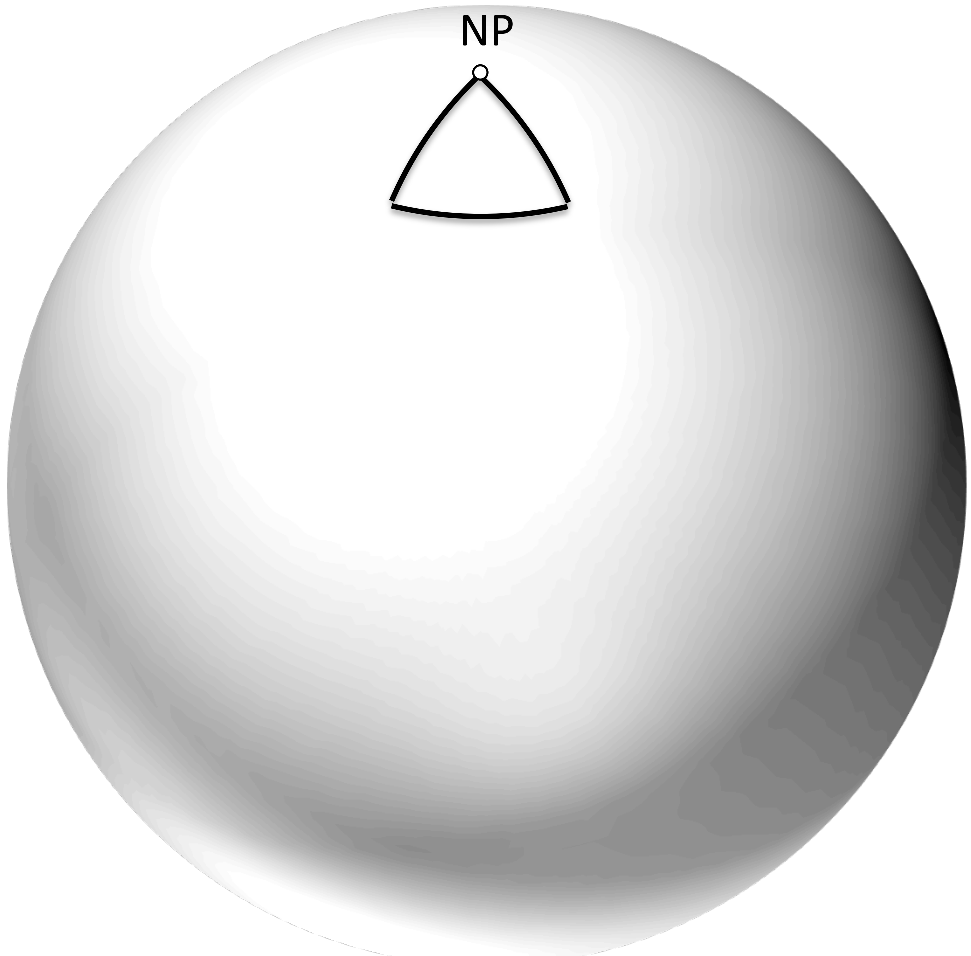
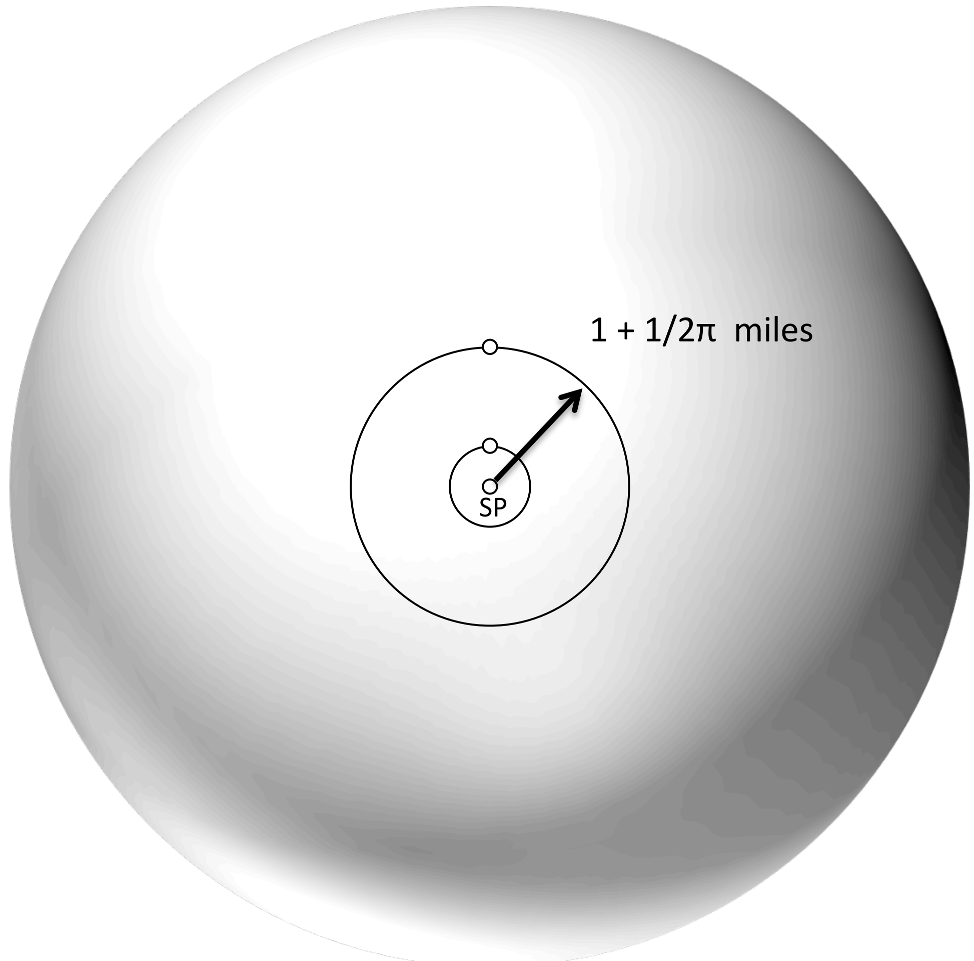
Puzzler

Is there any other point on the globe, besides the North Pole, from which you could walk a mile south, a mile east, and a mile north and find yourself back at the starting point?



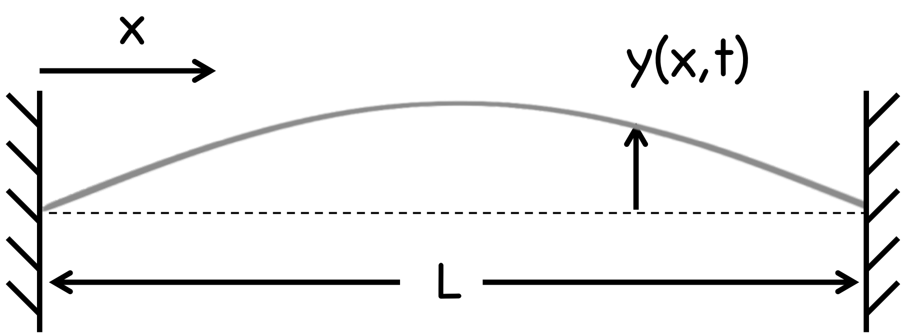

1 + 1/2π miles
SP

Introduction to Audio and Music Engineering

Lecture 3

- Strings: oscillations in time and space
- Modes of oscillation

Strings – Oscillations in time and space



T = tension

μ = mass/length

Speed of wave on string $\quad c \equiv \sqrt{\dfrac{T}{\mu}}$

$$\mu \frac{d^2 y(x,t)}{dt^2} = T \frac{d^2 y(x,t)}{dx^2}$$

mass x acceleration = tension x curvature

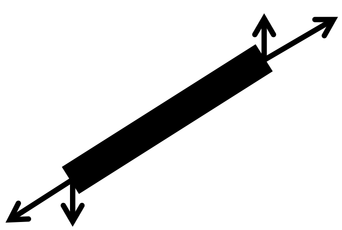
Solution is sinusoidal in time and space

$$y(x,t) = \sin(n\omega_0 t) \cdot \sin(n\pi \frac{x}{L})$$

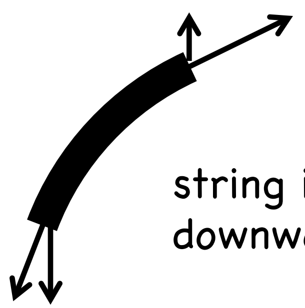
$y(0,t) = y(L,t) = 0$

$$\omega_0 = \pi \frac{c}{L} \qquad f_0 = \frac{c}{2L}$$

You can "prove" this by plugging solution into original equation



vertical forces cancel



string is pulled downward

Question

What is the speed of bending wave propagation for a string with mass density of 0.01 kg/m and a tension of 100 Nts?

a) 10,000 m/sec

b) 1,000 m/sec

c) $\boxed{\text{100 m/sec}}$

d) 10 m/sec

Typical guitar string mass per unit length and tension

640 mm length (25.2″)

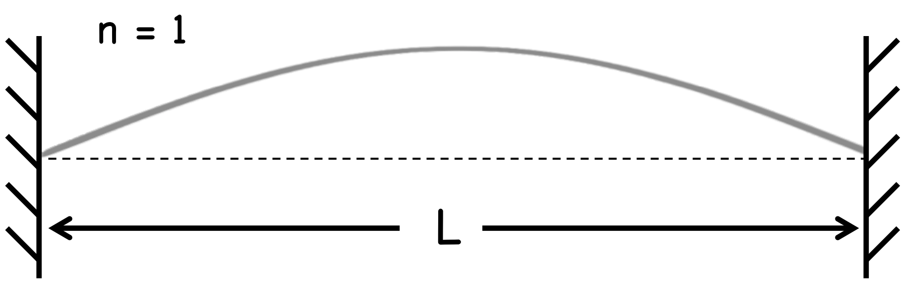
String	Diameter (inches)	Frequency (Hz)	Mass/Length kg/m	Tension kg (lbs)	Wave Speed (m/sec)
E (1st)	0.010	329.63	0.401×10^{-3}	7.28 (16.0)	421.8
B (2nd)	0.013	246.94	0.708×10^{-3}	7.22 (15.9)	316.1
G (3rd)	0.017	196.00	1.140×10^{-3}	7.32 (16.1)	290.3
D (4th)	0.026 (0.014 core)	146.82	2.333×10^{-3}	8.41 (18.5)	188.0
A (5th)	0.036 (0.015 core)	110.00	4.466×10^{-3}	9.03 (19.9)	140.8
E (6th)	0.046 (0.016 core)	82.41	6.790×10^{-3}	7.71 (17.0)	105.5
			Total	103.4 lbs	

round wound string

A typical grand piano has 230 strings and nearly 30 tons of tension in total!

Modes of oscillation

Fundamental "mode" of oscillation



$\lambda = 2L$ Spatial wavelength

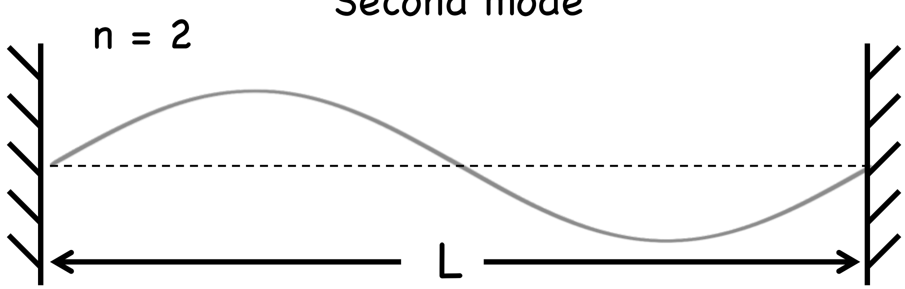
Round trip distance = 2L

Round trip time T = 2L/c

$$f_0 \equiv \frac{1}{T} = \frac{c}{2L} \qquad \omega_0 \equiv 2\pi f = \pi \frac{c}{L}$$

From previous slide

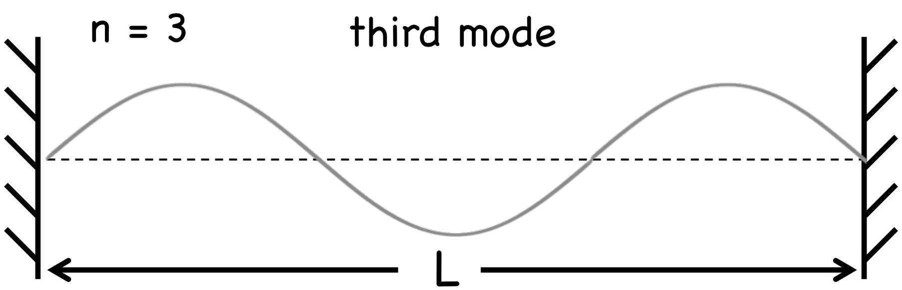
Second mode



$\lambda = L$ Spatial wavelength

$$\lambda = \frac{2L}{2} = L \qquad f_2 = 2\frac{c}{2L} = 2f_0$$

Higher modes



$$\lambda = \frac{2}{3}L$$ Spatial wavelength

$$\lambda = \frac{2L}{3} \qquad f_3 = 3\frac{c}{2L} = 3f_0$$

Spatial wavelength and frequency of the n'th mode

$$\lambda_n = \frac{2L}{n} \qquad f_n = n\frac{c}{2L} = nf_0$$

Question

What is frequency of the fundamental mode of a string of length 1 meter with a mass density of 0.01 kg/m and a tension of 100 Nts?

a) $\boxed{\text{50 Hz}}$

b) 100 Hz

c) 200 Hz

d) 500 Hz