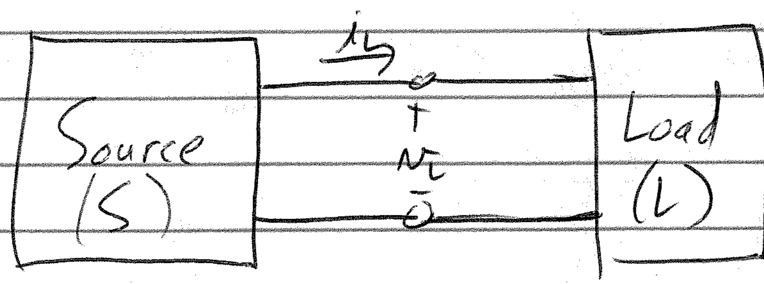
Thevenin + Norton Equivalents

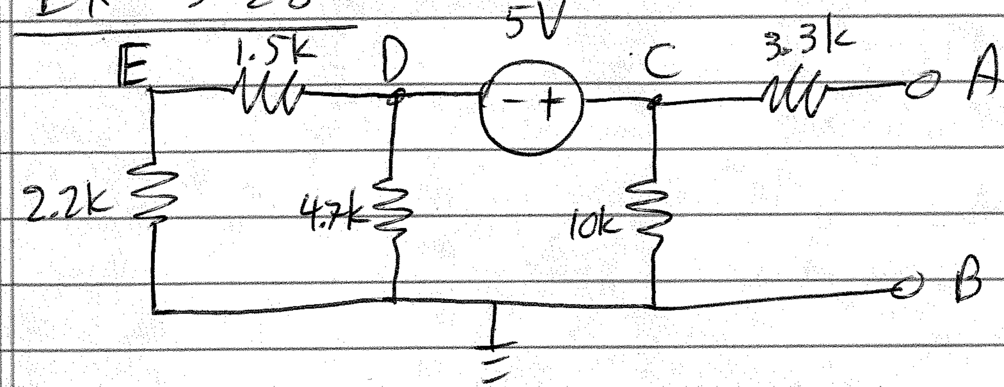
We have already talked about these as Equivalent Practical V + I srcs, you even had a question about them on the exam.

Thévenin proved for V src, and Norton for I src, that:

If a src ckt with 2 terminals is linear, then the "interface signals" $v + i$ do not change if src is replaced with its Thevenin or Norton equivalent.
($v_s + R_s$) ($i_s + R_s$)



Ex 3-28



Find Thev + Norton Equivalents between A + B.

Several possible methods

1) Source transformations?

$\underbrace{(2.2k + 1.5k) // 4.7k}_{\text{awkward}} + 5V = \text{Thev}(V_{src})$

Change to $I_{src} \Rightarrow$ $\underbrace{R_{eq} // 10k}_{\text{awkward}}$

Change to $V_{src} \Rightarrow$ $R''_{eq} + 3.3k = R_S$

Instead try

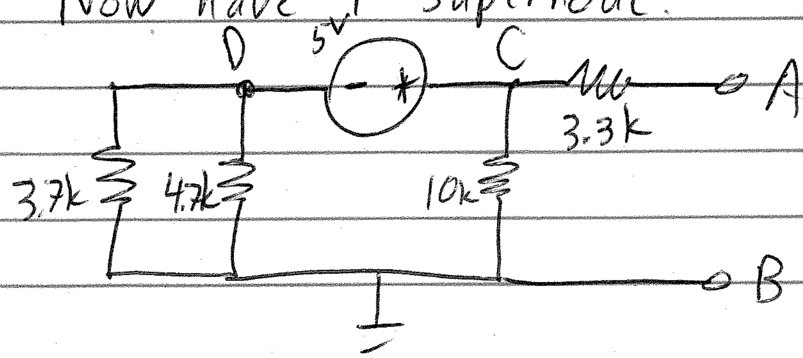
2.) Nodal analysis for $v_{oc} + i_{sc}$:

A-B open ckt to get v_{oc}

A-B short ckt to get i_{sc}

Combine $1.5k + 2.2k = 3.7k$ + eliminate node E.

Now have 1 supernode:



$$\frac{V_D}{3.7k} + \frac{V_D}{4.7k} + \frac{V_C}{10k} + \frac{V_C - V_A}{3.3k} = 0$$

$$V_C - V_D = 5V$$

For open ckt, $V_A = V_C$, so last term $= 0$. $V_{oc} = V_C$
For short ckt, $V_A = 0$, so last term $= \frac{V_C}{3.3k}$; $I_{sc} = \frac{V_C}{3.3k}$

O.C.: $\left(\frac{1}{3.7k} + \frac{1}{4.7k}\right)V_D + \left(\frac{1}{10k}\right)V_C = 0$

$$V_D = V_C - 5V$$

$$\frac{4.7k + 3.7k}{(3.7k)(4.7k)}(V_C - 5V) + \frac{1}{10k}V_C = 0$$

$$\frac{0.483 \times 10^{-3}}{\Omega}V_C + \frac{0.1 \times 10^{-3}}{\Omega}V_C = \left(\frac{0.483 \times 10^{-3}}{\Omega}\right)5V$$

$$0.583 \times 10^{-3}\, V_C = 2.42\, mA$$

$$\underline{V_C = 4.15\, V = V_{oc}}$$

S.C. $0.483 \times 10^{-3} V_C - 2.42\, mA + \left(\frac{1}{10k} + \frac{1}{3.3k}\right)V_C = 0$

$$0.483 \times 10^{-3} V_C + 0.403 \times 10^{-3} V_C = 2.42\, mA$$

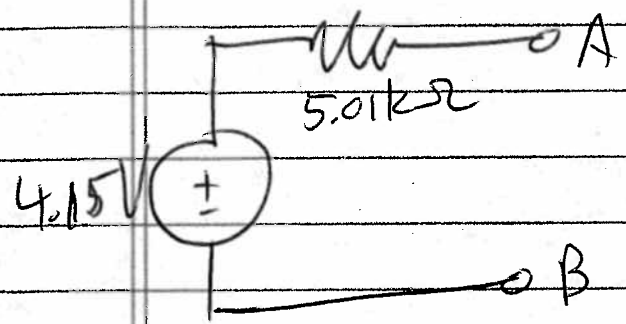
$$0.886 \times 10^{-3} V_C =$$

$$\underline{V_C = 2.73\, V}$$

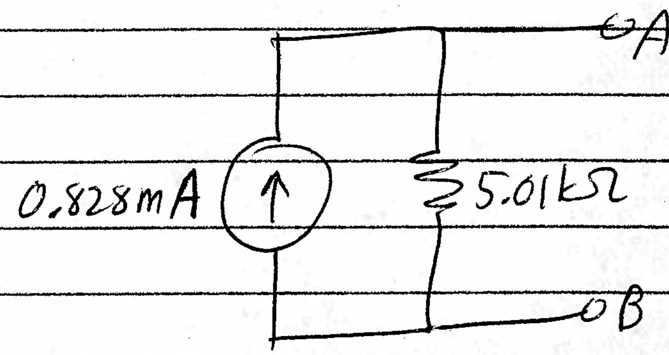
$$I_{sc} = \frac{V_C}{3.3k\Omega} = 0.828\, mA$$

$$R_S = \frac{V_{oc}}{I_{sc}} = \frac{4.15\, V}{0.828\, mA} = \underline{5.01\, k\Omega}$$

Thev. Eq.



Norton Eq

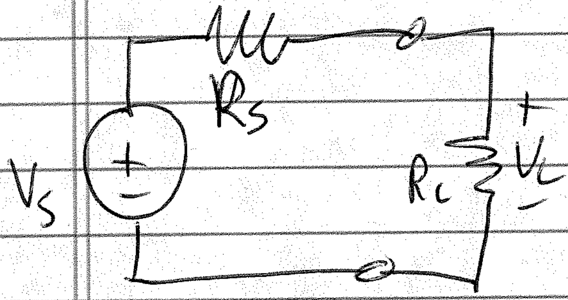


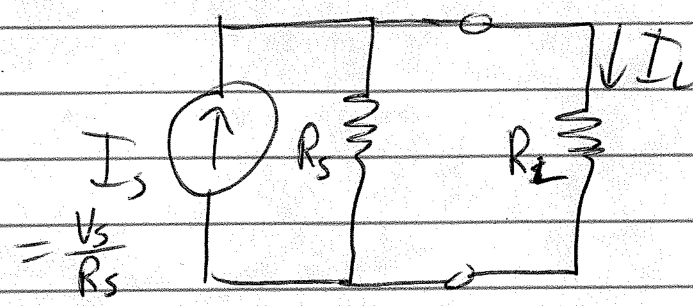
More exercises and examples in text, pages 116–125. Read them, understand them, do them.

I want to address 2 things a little out of order from the text, so I am swapping them.

I want to first look at questions of Power, then come back to applications to Nonlinear Loads.

Take our 2 types of Eq. Ckts with Loads:





Let $R_L = x R_S$, $x \in [0, \infty)$

This can express every situation:

$$\begin{cases} x = 0, & \text{short ckt} \\ 0 < x < 1, & R_L \text{ less than } R_S \\ x = 1, & R_L = R_S \\ 1 < x < \infty, & R_L \text{ greater than } R_S \\ x = \infty, & \text{open ckt.} \end{cases}$$

Division: $V_L = \frac{R_L}{R_S + R_L} V_S$
$= \frac{x R_S}{R_S + x R_S} V_S$
$= \frac{\cancel{R_S} \cdot x}{\cancel{R_S}(1+x)} V_S$

$I_L = \frac{R_S}{R_S + R_L} I_S$
$= \frac{R_S (1)}{R_S (1+x)} I_S$
$= \frac{1}{1+x} I_S$

Power $P_L = \frac{V_L^2}{R_L} = \frac{x^2}{(1+x)^2} V_S^2 \frac{1}{x R_S}$

$P_L = \frac{x}{(1+x)^2} \frac{V_S^2}{R_S}$

$P_L = I_L^2 R_L = \frac{1}{(1+x)^2} I_S x R_S$

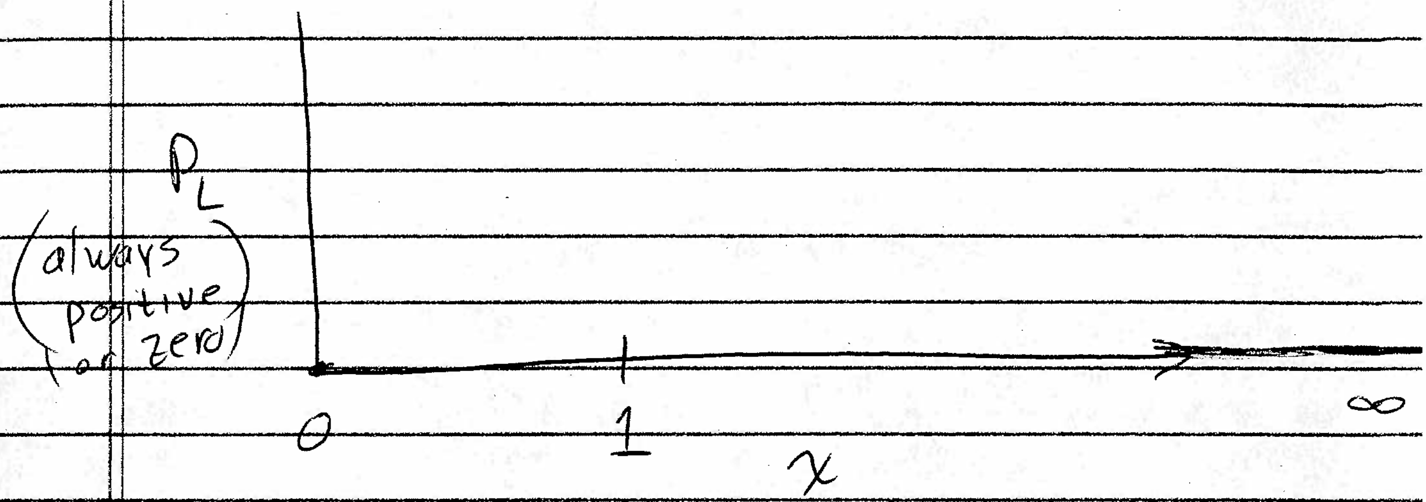
but $I_S = \frac{V_S}{R_S}$, so

$P_L = \frac{1}{(1+x)^2} \frac{V_S^2}{R_S^2} x R_S$

$P_L = \frac{x}{(1+x)^2} \frac{V_S^2}{R_S}$

Same!!

Plot vs x:



limits: $R_L \to 0 \Rightarrow x \to 0$ (short ckt)

$$P_L \Rightarrow \frac{0}{(1+0)^2}\frac{V_s^2}{R_s} = 0$$

$R_L \to \infty \Rightarrow x \to \infty$ (open ckt)

$$P_L = \frac{\infty}{(1+\infty)^2}\frac{V_s^2}{R_s} = \frac{1}{\infty}\frac{V_s^2}{R_s} = 0$$

Now, we have something that is always positive or zero, starts at zero (at $x=0$) and ends at zero (at $x=\infty$).

Any infinities? (Any places where the denominator $=0$?) No. $\Rightarrow$ must be bounded.

So P_L must come up to a maximum and back to zero as $x \to 0 \to \infty$.

Where does maximum occur?

$$\frac{dP_L}{dx} = 0$$ (I will not expect you to ever derive this again, use the results!)

$$\frac{d}{dx}\left(\frac{x}{(1+x)^2}\frac{V_S^2}{R_S}\right) = \frac{V_S^2}{R_S}\left[\frac{(1+x)^2(1) - x2(1+x)(1)}{(1+x)^4}\right]$$

$$\cancel{= \frac{V_S^2}{R_S}\left[(1+x)(1-2x)\right.}$$

$$= \frac{V_S^2}{R_S}\left[\frac{(1+x)\{(1+x)-2x\}}{(1+x)^4}\right]$$

$$= \frac{V_S^2}{R_S}\left[\frac{1-x}{(1+x)^3}\right]$$

When does this $=0$? When numerator $=0$, or

When $1-x=0$

$$x=1$$

Maximum of P_L when $x=1$ or when $R_L = 1R_S$

$$R_L = R_S$$

Go back to sketch.

Value at max? $P_{L\,max} = \frac{1}{(1+1)^2}\frac{V_S^2}{R_S} = \frac{1}{4}\frac{V_S^2}{R_S}$

For Norton:

$$P_L = \frac{x}{(1+x)^2} R_s I_s^2$$

$$\cancel{\frac{d}{dx} = R_s I_s^2 \left(-2(1+x)(1+x)^{-3}\right)}$$

$$\cancel{= R_s I_s -2(1+x)}$$

$$\frac{dP_L}{dx} = R_s I_s^2 \left[\frac{(1)(1+x)^2 - 2(1+x)(1)}{(1+x)^4} \right]$$

$$\cancel{= R_s I_s^2 \left[(1+x)(1+x-2x)\right.}$$

$$= R_s I_s^2 \left[\frac{(1+x)(1+x-2)}{(1+x)^4} \right]$$

$$= R_s I_s^2 \left[\frac{x-1}{(1+x)^3} \right]$$

$= 0$ when $x = 1 \Rightarrow$ when $\underline{R_L = 1 R_s = R_s}$

Same sketch

Value at max? $P_L = \frac{1}{(1+1)^2} R_s I_s^2 = \frac{1}{4} R_s I_s^2$

let $I_s = \frac{V_s}{R_s}$, $P_L = \frac{1}{4} R_s \left(\frac{V_s}{R_s}\right)^2$

$$= \frac{1}{4} \frac{V_s^2}{R_s}$$

Same as for Thevenin !!

So, from point of view of load, the Voltage, Current, and Power dissipated are the same, so I cannot tell the difference between Thev. + Norton Equivalents.

Maximum Power Dissipated In Load

When $R_L = R_S$. !!!

Let's look at 2 more things before we do more with this statement:

Look at power dissipated in R_s and power supplied by V_s or I_s for $R_L = xR_s$:

Thev.

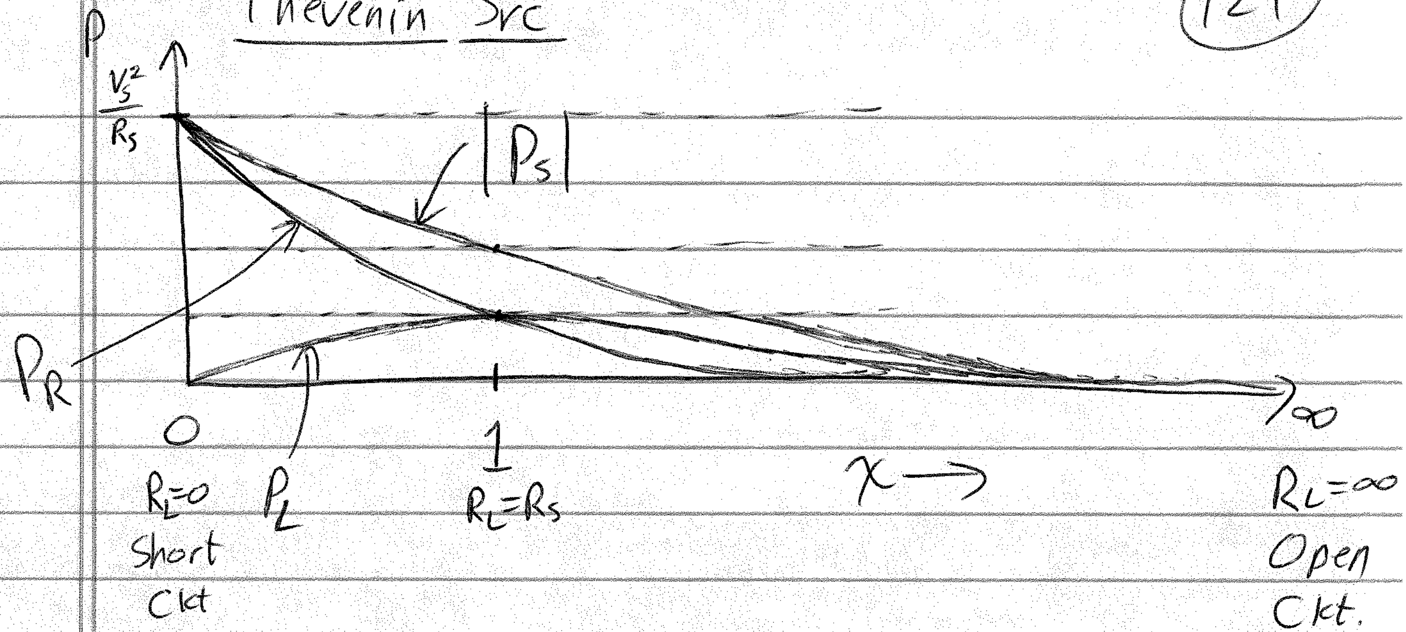
$$P_R = \frac{\left(\frac{R_s}{R_s + xR_s}\right)^2 V_s^2}{R_s} = \frac{R_s}{(R_s + xR_s)^2} V_s^2 = \frac{1}{(1+x)^2} \frac{V_s^2}{R_s}$$

Graph: $P_R(0) = \frac{V_s^2}{R_s}$, $P_R(1) = \frac{1}{4}\frac{V_s^2}{R_s}$, $P_R(\infty) = 0$

$$P_s = -IV_s = -\left(\frac{V_s}{R_s + xR_s}\right) V_s = -\frac{1}{(1+x)} \frac{V_s^2}{R_s}$$

Graph: $P_s(0) = -\frac{V_s^2}{R_s}$, $P_s(1) = -\frac{1}{2}\frac{V_s^2}{R_s}$, $P_s(\infty) = -0$

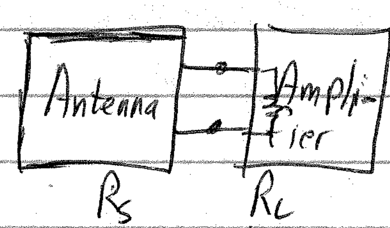
Thevenin Src



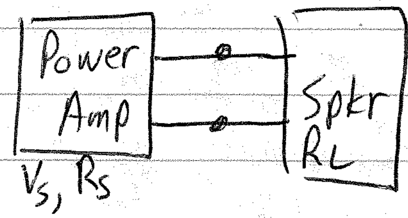
So, P_s supplies the most power, and all of that power is dissipated inside the box, when it is shorted out (why batteries get hot when shorted out.)

Where do you want to operate?

It depends:



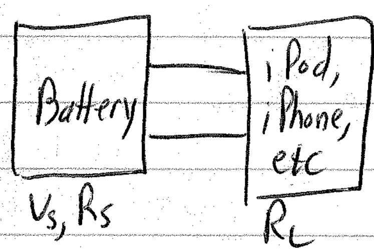
Probably want max power tranfer, so want $R_L = R_s$



Want max power, so want $R_L = R_S$

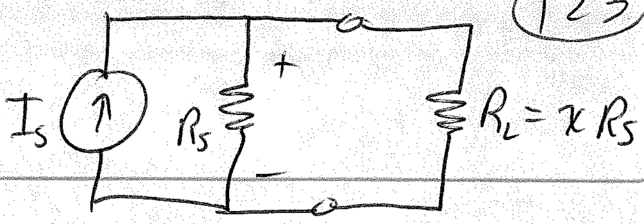
If $R_L \neq R_s$, no damage to speakers, less power is delivered to either, $\Rightarrow$ not as loud.

But if $R_L < R_s$ the power supplied (P_s) and dissipated internally (P_R) go up.



Don't want max power, as that would give minimum battery life. Want minimum power delivered, for max. battery life. $R_L \to \infty$

Now, Norton Equiv.



$$P_L = \frac{x}{(1+x)^2} R_s I_s^2$$

$x=0$: $P_L = 0$, $x = 1$: $P_L = \frac{1}{4} R_s I_s^2$; $x \to \infty$: $P_L = 0$

$$P_R = R_s \left[\frac{x R_s}{R_s + x R_s} I_s\right]^2 = R_s \left[\frac{x}{1+x} I_s\right]^2 = \frac{x^2}{(1+x)^2} R_s I_s^2$$

$x=0$: $P_R = 0$

$x=1$: $P_R = \frac{1}{4} R_s I_s^2$

$x=\infty$: $P_R = R_s I_s^2$

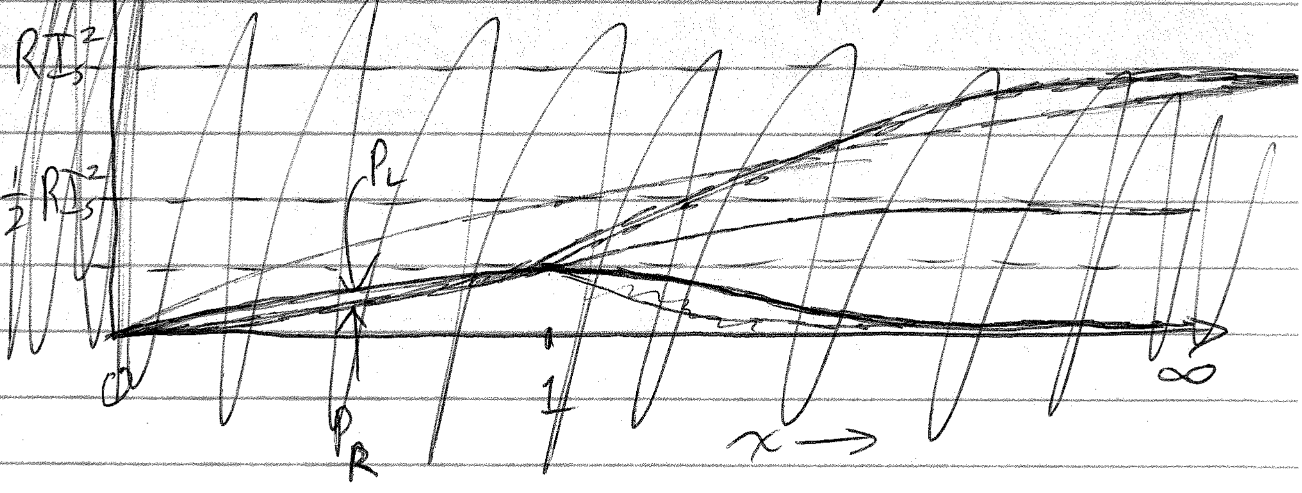
$$P_L = -I_s\left(\frac{(R_s)(xR_s)}{R_s + xR_s} I_s\right) = -I_s^2\left(\frac{x}{1+x}\right)R_s = -\frac{x}{1+x} R_s I_s^2$$

$x=0$: $P_L = 0$

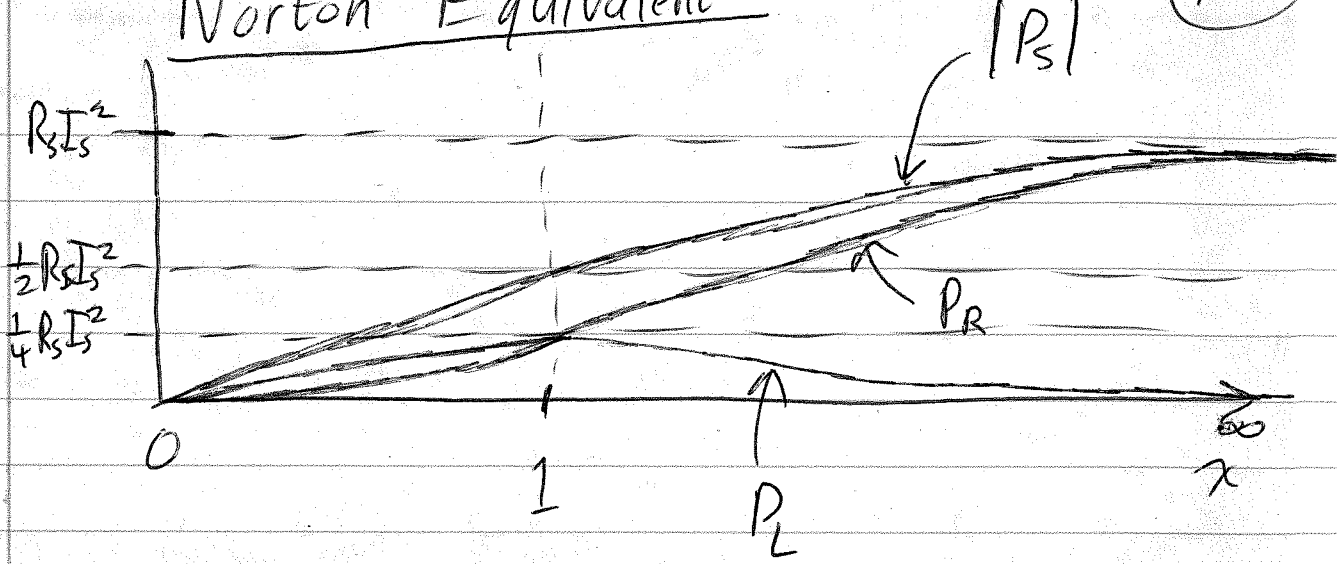
$x=1$: $P_L = -\frac{1}{2} R_s I_s^2$

$x=\infty$: $P_L = -R_s I_s^2$

see next page



Norton Equivalent



Now, if the "Black Box" really behaves like a Current Source, it dissipates its maximum ~~ener~~ power internally When It Is Open Circuited not short cktd.!

Very few things really behave in this way. Laboratory devices that are unprotected current sources have big warning labels on them, "Do not disconnect load with power on" or "Do not operate without load connected."

So, most devices (batteries, power supplies, etc.) really behave like ~~Vo~~ Practical Voltage Sources, or Thevenin Equivalents.

Sometimes using an equivalent Practical Current Source as a Model can make analysis easier.