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1. In nature, two periodic signals, a and b, will not sum to a periodic signal, c, if a period for signal c cannot be found to be a rational multiple of the periods of both signals a and b.

In other words, if there does not exsist a period which is a rational multiple of the periods of signals a and b, the sum of the two signals is not periodic.

With discrete signals, all periods are a rational multiple of the unit. This ensures that all signals can the be found to have a common multiple in the discrete time domain, no matter the size.

Continuous signals are different. Due to the "smooth" nature of the number system, two signals with periods that are different by an infinitesimally small amount will not have a common multiple. It can be see that if there were one, the value would be infinitesimally large. An example of this is the sinc(x) function, whose periodic components are made up of all signals with frequencies $0-\omega_0$ hz.

2. a) $y_1(t) + y_2(t) \stackrel{?}{=} f(x_1(t) + x_2(t))$

$\boxed{\text{Non-linear}}$

$$x_1(t)^2 + 1 + x_2(t)^2 + 1 = (x_1(t) + x_2(t))^2 + 1$$

$$x_1(t)^2 + x_2(t)^2 + 2 \neq x_1(t)^2 + 2x_1(t)x_2(t) + x_2(t)^2 + 1$$

b) $y_1(t) + y_2(t) \stackrel{?}{=} f(x_1(t) + x_2(t))$

$\boxed{\text{Non-linear}}$

$$\frac{1}{x_1(t)} + \frac{1}{x_2(t)} \neq \frac{1}{x_1(t) + x_2(t)}$$

c) $y_1(t) + y_2(t) \stackrel{?}{=} f(x_1(t) + x_2(t))$

$\boxed{\text{Linear}}$

$$e^{-t^2} \cdot x_1(t) + e^{-t^2} \cdot x_2(t) = e^{-t^2} \cdot (x_1(t) + x_2(t))$$

$$= e^{-t^2} \cdot x_1(t) + e^{-t^2} \cdot x_2(t) \checkmark$$

d) $y_1(t) + y_2(t) \stackrel{?}{=} f(x_1(t) + x_2(t))$ $\boxed{\text{Linear}}$

$x_1(t) \cdot \sin(w_c t + \theta) + x_2(t) \cdot \sin(w_c t + \theta) = (x_1(t) + x_2(t)) \cdot \sin(w_c t + \theta)$

$(x_1(t) + x_2(t)) \cdot \sin(w_c t + \theta) = (x_1(t) + x_2(t)) \cdot \sin(w_c t + \theta)$ ✓

e) $y_1(t) + y_2(t) \stackrel{?}{=} f(x_1(t) + x_2(t))$

$e^{2x_1(t)+1} + e^{2x_2(t)+1} = e^{2(x_1(t)+x_2(t))+1}$ $\boxed{\text{Non-Linear}}$

$e(e^{2x_1(t)} + e^{2x_2(t)}) = e^{2x_1(t)+2x_2(t)} \cdot e$

$\downarrow$

$\neq e(e^{2x_1(t)} \cdot e^{2x_2(t)})$

f) $y_1(t) + y_2(t) = f(x_1(t) + x_2(t))$

$x_1(t) + y_1(t-1) + y_1(t-2) + x_2(t) + y_2(t-1) + y_2(t-2) = x_1(t) + x_2(t) + y_1(t-1) +$
$y_2(t-1) + y_1(t-2) +$
$y_2(t-2)$

$\boxed{\text{Linear}}$

3. $P = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 \, dt$

$E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt$

a) Neither
b) Power
c) Power
d) Energy
e) Energy

4. a) $x(t) = \frac{1}{1+t^2} = \frac{1}{2} \cdot \left(\frac{2\alpha}{\alpha + t^2} \right)$; $\alpha = 1$

$X(\omega) = \frac{1}{2} \cdot e^{-|\omega|}$

b) $x(t) = rect(2t-3) + tri(2t+3)$

$X(\omega) = sinc(2\omega - 3) + sinc^2(2\omega + 3)$

c) $x(t) = t^2 sinc(t)$

d) $x(\omega) = \frac{1}{4\pi} \cdot \cos(\frac{1}{2} t)$

5 e) The plots indicate that, for this function, as resolution in the time domain increases, frequency content increases as well. On the graphs, an interesting result can be observed. It looks as though there is a resolution increase in the frequency domain as well. The catch, though, is that this visual effect is actually an increase in higher frequency content and the values of the frequencies present are also, in a way shifted.