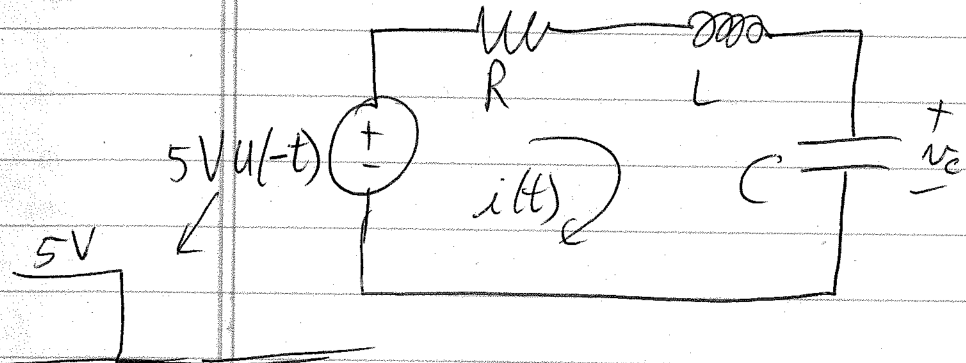
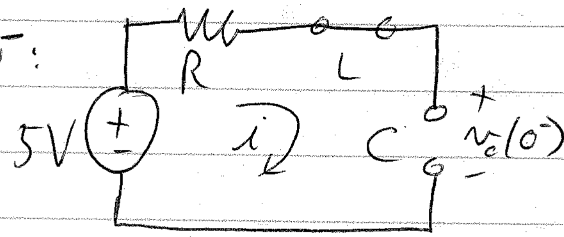
Example



Find $i(t)$ for $t > 0$.

$t \leq 0^-$:

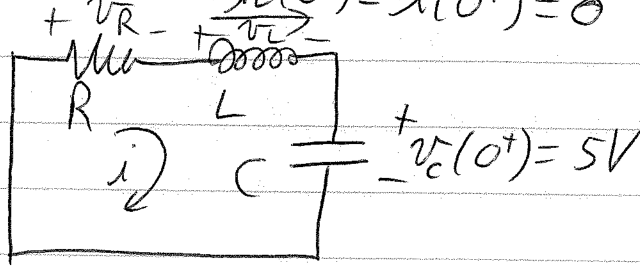


$i(0^-) = i_L(0^-) = 0$ (C is an open ckt.)

$v_c(0^-) = 5V$

$t = 0^+$:

$i_L(0^+) = i(0^+) = 0$



$v_R + v_L + v_c = 0$

$Ri + L\frac{di}{dt} + v_c = 0$

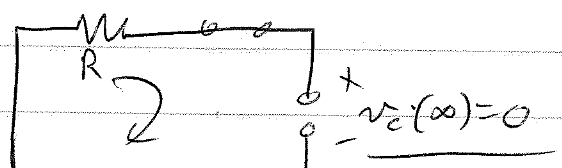
$\frac{di}{dt} = \frac{-1}{L}(v_c + Ri)$

$\frac{di}{dt}(0^+) = -\frac{1}{L}(v_c(0^+) + 0)$

$= -\frac{1}{L}(5V)$

$= -\frac{5V}{L}$

$t = \infty$:



$i(\infty) = 0$

Try different values of R — Leave these fixed $\underbrace{\hphantom{L = 1\,H,\ C = 0.25\,\mu F}}$ (234)

Values: $R = 8.5\,k\Omega$, $L = 1\,H$, $C = 0.25\,\mu F = \frac{1}{4}\mu F$

$$\alpha = \frac{R}{2L} = \frac{8.5\,k\Omega}{2(1\,H)} = 4{,}250\ s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1H)(\frac{1}{4}\mu F)}} = \frac{1}{\frac{1}{2}\,ms} = 2{,}000\ s^{-1}$$

$\alpha > \omega_0$, so this is <u>over damped</u>

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -4{,}250 \pm \sqrt{(4250)^2 - (2000)^2}$$
$$= -4{,}250 \pm 3{,}750$$

$$s_1 = -4{,}250 + 3750 = -500$$
$$s_2 = -4{,}250 - 3750 = -8{,}000$$

$$i(t) = Ae^{-500t} + Be^{-8000t}$$

$i(0) = 0 = A + B, \Rightarrow B = -A$, so

$$i(t) = A\left[e^{-500t} - e^{-8000t}\right]$$

$$\frac{di}{dt} = A\left[-500e^{-500t} + 8000e^{-8000t}\right]$$

$$\frac{di}{dt}(0^+) = A[-500 + 8000] = \underline{7500\,A}$$

~~[illegible] $v_R(0^+) + v_L(0^+) + v_C(0^+) = 0$~~ from $t = 0^+$:

$$\cancel{R\,i(0^+) + L\frac{di}{dt}(0^+) + v_C(0^+)} = 0$$

$$\cancel{0 + L\frac{di}{dt}(0^+) + 5V = 0}$$

$$\underline{\frac{di}{dt}(0^+) = -\frac{5V}{L} = -5\,A/s}$$

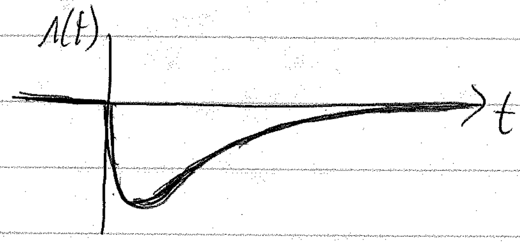
So $-5\ \frac{A}{s} = 7500\,s^{-1} A$

$$-\frac{5}{7500} = A$$

$$-\frac{2}{3}\,mA = A$$

$$i_\phi(t) = -\frac{2}{3}\,mA\left[e^{-500t} - e^{-8000t}\right]$$

this goes to zero fastest.



Now let $R = 4\,k\Omega$, $L + C$ remain.

$$\alpha = \frac{R}{2L} = \frac{4\,k\Omega}{2(1\,H)} = 2{,}000\ s^{-1}$$

$$\omega_0 = 2{,}000\ s^{-1} \text{ (as before)}$$

$\alpha = \omega_0 \Rightarrow$ critically damped.

$$i(t) = (C + Dt)e^{-\alpha t}$$

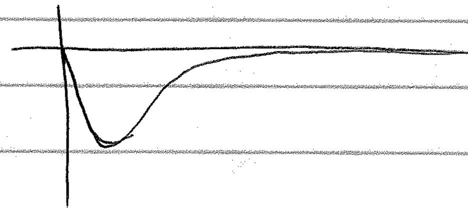
$$i(0^+) = C = 0, \text{ so}$$

$$i(t) = Dte^{-\alpha t}$$

$$\frac{di}{dt} = D\left[e^{-\alpha t} - \alpha t e^{-\alpha t}\right]$$

$$\frac{di}{dt}(0^+) = D[1-0] = D = -\frac{5}{L} = -5\ A/s.$$

So $i(t) = -5\, t e^{-2000t}$



looks like over-damped, just gets to 0 faster.

Let $R = 1\,k\Omega$:

$$\alpha = \frac{R}{2L} = \frac{1000}{2(1)} = 500\ s^{-1}$$

$$\omega_0 = 2{,}000\ s^{-1}$$

$\alpha < \omega_0 \Rightarrow$ underdamped

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{(2000)^2 - (500)^2} = 1{,}936\ s^{-1}$$

$$i(t) = e^{-\alpha t}\left[E\cos(\omega_d t) + F\sin(\omega_d t)\right]$$

$i(0^+) = E = 0$

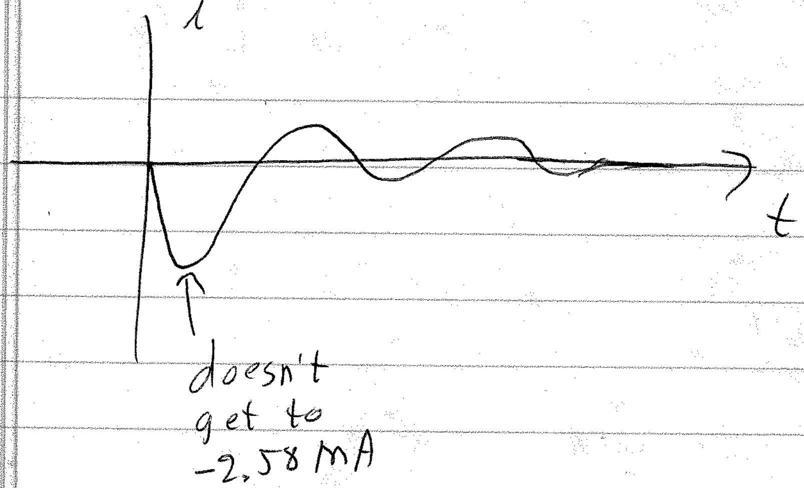
$$F = \frac{\frac{di}{dt}(0^+) + \alpha\, i(0^+)}{\omega_d} = \frac{-5\ A/s + \cancel{\alpha(0)}^{\,0}}{1936\ s^{-1}}$$

$F = -2.58\ mA$

So

$$i(t) = e^{-500t}\left[-2.58mA\ \sin(1{,}936t)\right]$$

$$= -2.58\ mA\ e^{-500t} \sin(1{,}936t)$$


i
t
doesn't
get to
-2.58 mA

From these we can get other ~~vol~~ parameters,

like $v_R(t) = Ri(t) \Rightarrow$ same shapes just a scale factor.

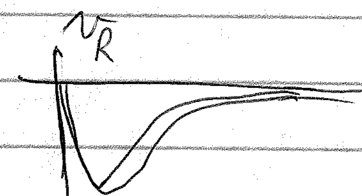
$$v_L(t) = L\frac{di}{dt}(t)$$

$$v_C(t) = \frac{1}{C}\int_0^t i(x)\,dx + v_C(0^+)$$

or use KVL

$$\underline{v_C(t) = -v_L(t) - v_R(t)}$$

Case A:

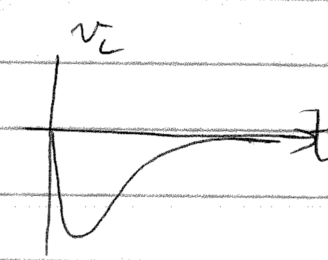


$$v_R(t) = R\,i(t)$$
$$= R\left(-\tfrac{2}{3}mA\right)\left[e^{-500t} - e^{-8000t}\right] = -5.667V\left[e^{-500t} - e^{-8000t}\right]$$

$$v_L(t) = L\left(-\tfrac{2}{3}mA\right)\left[-500e^{-500t} + 8000e^{-8000t}\right]$$

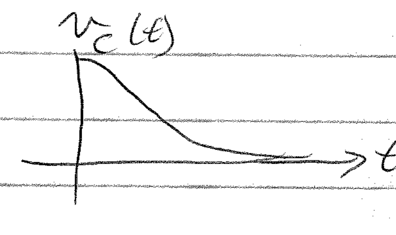
$$= -\tfrac{2}{3}mV\left[-500e^{-500t} + 8000e^{-8000t}\right]$$

$$= +0.333Ve^{-500t} - 5.333Ve^{-8000t}$$

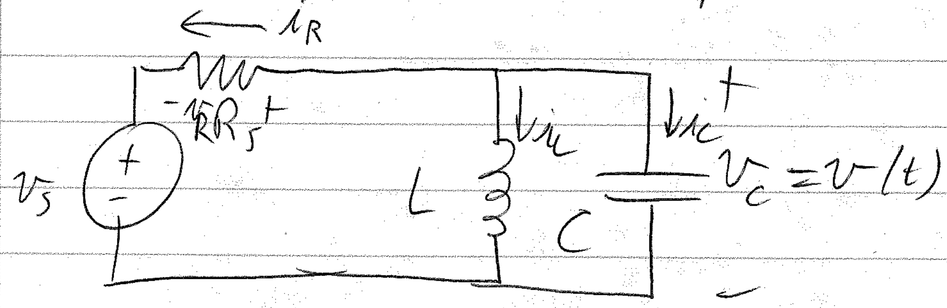


$$v_C(t) = -0.333Ve^{-500t} + 5.333Ve^{-8000t} + 5.667Ve^{-500t} - 5.667e^{-8000t}$$

$$= 5.333Ve^{-500t} - 0.333Ve^{-8000t}$$



What about L + C in parallel?



KCL: $i_R + i_L + i_C = 0$

$$\frac{v - v_s}{R} + \frac{1}{L}\int_0^t v(x)\,dx + i_L(0) + C\frac{dv}{dt} = 0$$

~~To get zero input~~

Take $\frac{d}{dt}$: $C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v(t) = \frac{1}{R}\frac{dv_s}{dt}$

Characteristic eqn: $Cs^2 + \frac{1}{R}s + \frac{1}{L} = 0$

or $s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$

$$s_{1,2} = \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - 4(1)\left(\frac{1}{LC}\right)}}{2(1)}$$

$$= -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{LC}\right)}$$

Now let $\alpha_{par} = \frac{1}{2RC}$ + $\omega_0 = \frac{1}{\sqrt{LC}}$ (as before)

$$s_{1,2} = -\alpha_{par} \pm \sqrt{\alpha_{par}^2 - \omega_0^2}$$

So, parallel LC gives same results except

$$\alpha_{par} = \frac{1}{2RC} = \frac{1}{2\tau_c}$$

For series we had

$$\alpha_{ser} = \frac{R}{2L} = \frac{1}{2(L/R)} = \frac{1}{2\tau_L}$$

I remember these as follows:

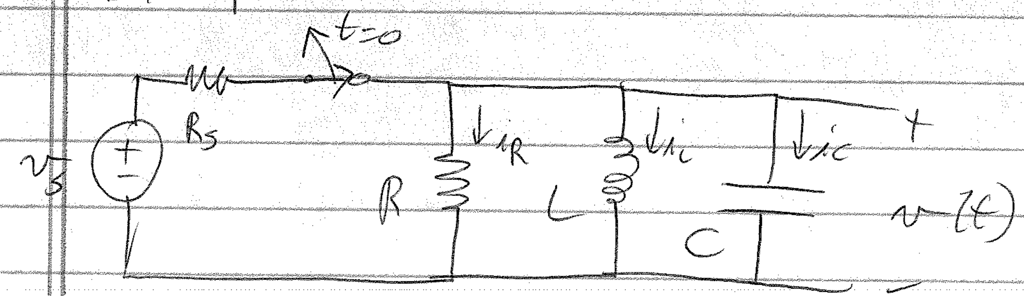
Series LC has a common Current. Current is the state variable for an Inductor,

so $\alpha_{ser} = \frac{1}{2\tau_L}$

Parallel LC has a common Voltage. Voltage is the state variable for a Capacitor,

so $\alpha_{par} = \frac{1}{2\tau_C}$

Example:



Find $v(t)$.