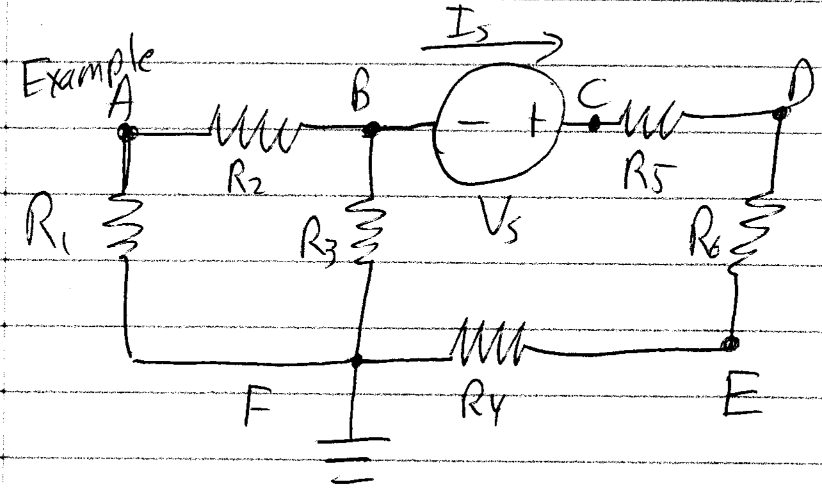
Back to Node Voltage Analysis

We saw how to set up the equations in Node Voltage Analysis on Monday, and we alluded to a special case that we should look at a little more today. It is the case of how to deal with a voltage source:

Example



6 Nodes, select F as Ground, or reference.

$V_F = 0$

(A) is easy: $\frac{V_A - V_B}{R_2} + \frac{V_A}{R_1} = 0$

(B) We ~~don't know how to~~ can start:

don't know how to write the current ~~through the voltage src.~~

$$\frac{V_B - V_A}{R_1} + \frac{V_B}{R_3} + I_s = 0$$

Call it I_s

C $-I_s + \frac{V_C - V_D}{R_5} = 0$

D $\frac{V_D - V_C}{R_5} + \frac{V_D - V_E}{R_6} = 0$

E $\frac{V_E - V_D}{R_6} + \frac{V_E}{R_4} = 0$

This gives me 5 equations, but now I have 6 unknowns, $V_A, V_B, V_C, V_D, V_E, + I_s$!

How ~~do~~ I deal with this?? I can combine

B + C: $\frac{V_B - V_A}{R_1} + \frac{V_B}{R_3} + \underbrace{\cancel{I_s} - \cancel{I_s}}_{\text{cancel, and I get}} + \frac{V_C - V_D}{R_5} = 0 + 0$

$$\frac{V_B - V_A}{R_1} + \frac{V_B}{R_3} + \frac{V_C - V_D}{R_5} = 0$$

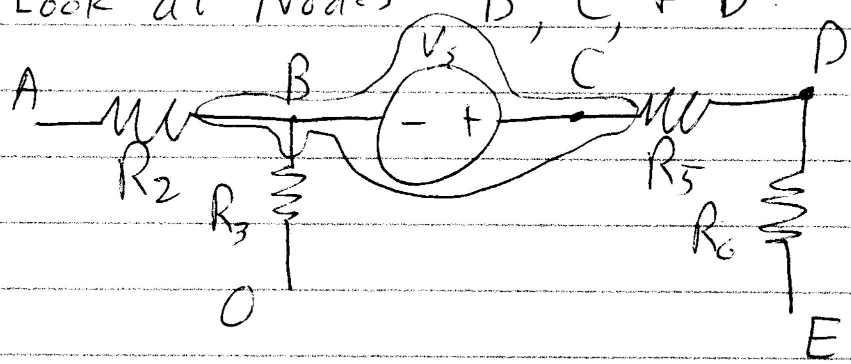
I now have eliminated I_s, but now I have only 4 equations: A, B+C, D, + E

4 Equations, 5 unknowns! I need one more equation. That comes from the "Linking Equation" or the nature of a Voltage Source:

$$\underline{\underline{V_C - V_B = V_s}}$$

Now I have 5 equations and 5 unknowns, so I can solve them.

Look at Nodes B, C, + D:



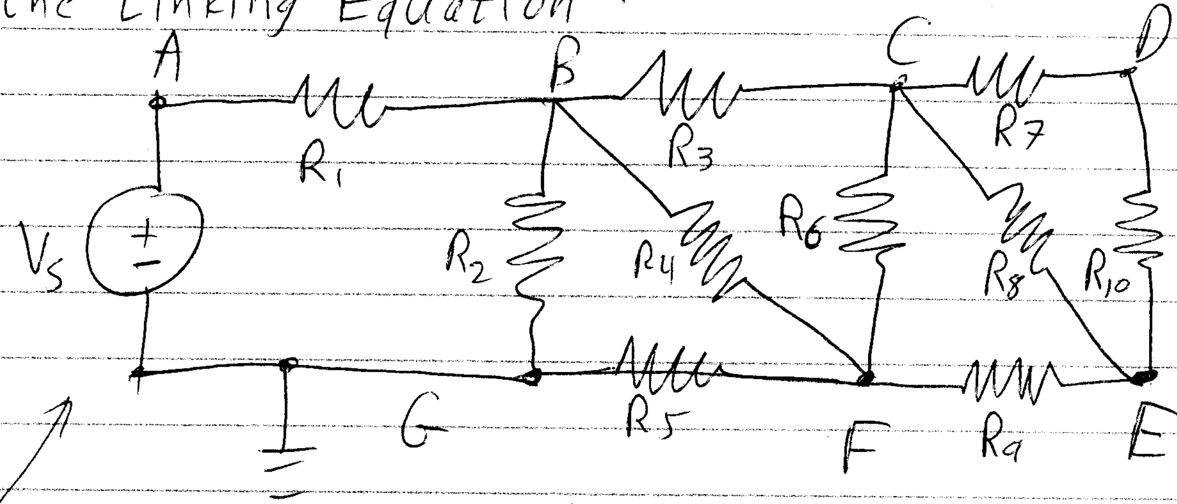
Since I had to combine B + C together, we can think of it as being a "Super Node" that includes the Voltage Source within it (draw).

Write the KCL equation for the Super Node:

$$\frac{V_B - V_A}{R_2} + \frac{V_B}{R_3} + \frac{V_C - V_D}{R_5} = 0 \qquad \text{as before}$$

and the "Linking Equation" $V_C - V_B = V_s$

We do the same thing when a Voltage Source is connected to the Reference Node, except that we do not write a KCL equation for the Reference Node, nor for a Supernode that includes the Reference node, just the Linking Equation:



~~a supernode here includes the reference node, so do not write KCL equation, only $V_A - 0 = V_s$~~

or $\underline{V_A = V_s}$ ⟶ Now there are 5 unknown V's

7 nodes, G is ref, so 6 unknown V's

(B) $$\frac{V_B - V_A}{R_1} + \frac{V_B}{R_2} + \frac{V_B - V_F}{R_4} + \frac{V_B - V_C}{R_3} = 0$$

$$-\frac{1}{R_1}V_A + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_3}\right)V_B - \frac{1}{R_3}V_C + 0\,V_D + 0\,V_E - \frac{1}{R_4}V_F = 0$$

Let's do the rest by inspection:

C $$0V_A - \frac{1}{R_3}V_B + \left(\frac{1}{R_3} + \frac{1}{R_6} + \frac{1}{R_8} + \frac{1}{R_7}\right)V_C - \frac{1}{R_7}V_D - \frac{1}{R_8}V_E - \frac{1}{R_6}V_F =$$

D $$0V_A + 0V_B - \frac{1}{R_7}V_C + \left(\frac{1}{R_7} + \frac{1}{R_{10}}\right)V_D - \frac{1}{R_{10}}V_E + 0V_F = 0$$

E $$0V_A + 0V_B - \frac{1}{R_8}V_C - \frac{1}{R_{10}}V_D + \left(\frac{1}{R_8} + \frac{1}{R_{10}} + \frac{1}{R_9}\right)V_E - \frac{1}{R_9}V_F = 0$$

F $$0V_A - \frac{1}{R_4}V_B - \frac{1}{R_6}V_C + 0V_D - \frac{1}{R_9}V_E + \left(\frac{1}{R_5} + \frac{1}{R_4} + \frac{1}{R_6} + \frac{1}{R_9}\right)V_F =$$

Remember, $V_A = V_s$ (a known quantity, presumably.)

So we really only need to solve for V_{B-F}

Re-arrange:

B $$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_3}\right)V_B - \frac{1}{R_3}V_C + 0V_D + 0V_E - \frac{1}{R_4}V_F = \frac{1}{R_1}V_S$$

C $$-\frac{1}{R_3}V_B + \left(\frac{1}{R_3} + \frac{1}{R_6} + \frac{1}{R_8} + \frac{1}{R_7}\right)V_C - \frac{1}{R_7}V_D - \frac{1}{R_8}V_E - \frac{1}{R_6}V_F = 0$$

D $$0V_B - \frac{1}{R_7}V_C + \left(\frac{1}{R_7} + \frac{1}{R_{10}}\right)V_D - \frac{1}{R_{10}}V_E + 0V_F = 0$$

E $$0V_B - \frac{1}{R_8}V_C - \frac{1}{R_{10}}V_D + \left(\frac{1}{R_8} + \frac{1}{R_{10}} + \frac{1}{R_9}\right)V_E - \frac{1}{R_9}V_F = 0$$

F $$-\frac{1}{R_4}V_B - \frac{1}{R_6}V_C + 0V_D - \frac{1}{R_9}V_E + \left(\frac{1}{R_5} + \frac{1}{R_4} + \frac{1}{R_6} + \frac{1}{R_9}\right)V_F = 0$$

Write as a matrix:

$$
\begin{array}{c}
B \\ C \\ D \\ E \\ F
\end{array}
\underbrace{\begin{pmatrix}
\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_3}\right) & -\frac{1}{R_3} & 0 & 0 & -\frac{1}{R_4} \\
-\frac{1}{R_3} & \left(\frac{1}{R_3}+\frac{1}{R_6}+\frac{1}{R_8}+\frac{1}{R_7}\right) & -\frac{1}{R_7} & -\frac{1}{R_8} & -\frac{1}{R_6} \\
0 & -\frac{1}{R_7} & \left(\frac{1}{R_7}+\frac{1}{R_{10}}\right) & -\frac{1}{R_{10}} & 0 \\
0 & -\frac{1}{R_8} & -\frac{1}{R_{10}} & \left(\frac{1}{R_8}+\frac{1}{R_{10}}+\frac{1}{R_9}\right) & -\frac{1}{R_9} \\
-\frac{1}{R_4} & -\frac{1}{R_6} & 0 & -\frac{1}{R_9} & \left(\frac{1}{R_5}+\frac{1}{R_4}+\frac{1}{R_6}+\frac{1}{R_9}\right)
\end{pmatrix}}_{\text{Conductance Matrix}}
\underbrace{\begin{pmatrix} V_B \\ V_C \\ V_D \\ V_E \\ V_F \end{pmatrix}}_{\text{Node Voltage Vector}}
=
\underbrace{\begin{pmatrix} \frac{V}{R} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{\text{Source Vector}}
$$

Notice: Diagonal Symmetry

+ on diagonal
− off diagonal.

This is bigger than I would ever have you do on a test. HW maybe, but not on an exam.

How to solve?