I spent an extra day last Friday on design of OpAmp Interfaces, so we are slightly out of sync with the Syllabus (1 day) I will revise it to be better in step for upcoming lectures.

So, we are going to move on to learn about 2 new devices from an Engineering point of view, Capacitors and Inductors. Both of these only have significant effects in circuits in which the voltages and currents are changing with time, that is $\frac{dv}{dt} \neq 0$, $\frac{di}{dt} \neq 0$ for all times t.

So we must turn our attention to "signals", not just constant sources. Only a time-varying quantity can convey information.

There are a few basic types of signals that can be used to represent virtually every real signal out there:

In the text's words:

3 major types
- Step Waveform — "Off" or "On"
- Exponential Waveform – Exponential Decay or Growth
- Sinusoidal Waveform – Periodic Functions that change smoothly between limits, from $-\infty \le t \le \infty$.

I Add "DC" Waveforms – constants

- Composite Waveforms
- Partial Descriptors

Put ~~3~~ 4 main types together

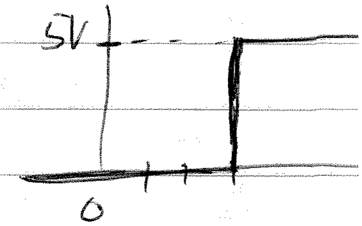
Let's go through them today!

Step: $u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$ called the "unit" step function.

or time shift it $u(t-t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t \geq t_0 \end{cases}$

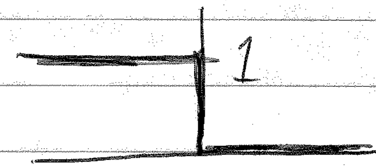
We can change the amplitude by multiplying by a constant:

$$5V\, u(t-3s) = \begin{cases} 0 & t < 3s \\ 5 & t \geq 3s \end{cases}$$




We reverse direction by ~~taking the~~ changing the sign of the argument:

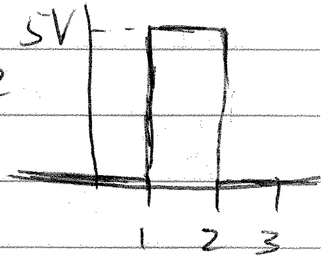
$$u(-t) = \begin{cases} 0 & t > 0 \\ 1 & t \leq 0 \end{cases}$$




We can combine steps to make "Gating Functions":

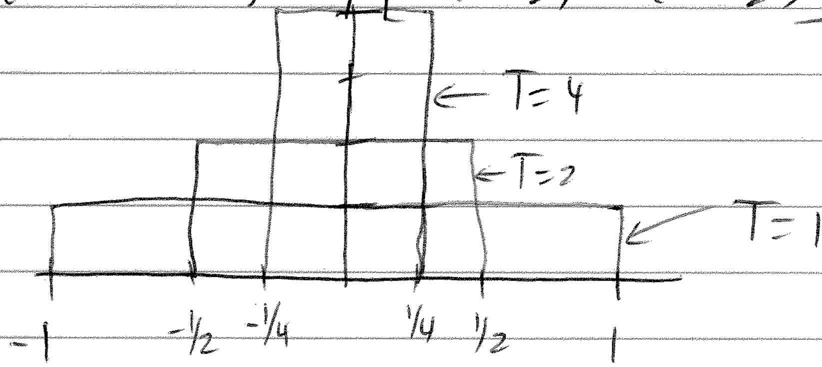
$$5V\, u(t-1) - 5V\, u(t-2) = \begin{cases} 0 & t < 1 \\ 5V & 1 \leq t < 2 \\ 0 & t \geq 2 \end{cases}$$

or $5V[u(t-1) - u(t-2)]$




Unit Impulse:

Let $v(t) = \frac{1}{T}\left[u(t+\frac{T}{2}) - u(t-\frac{T}{2})\right]$



For all of these, the area under them is $A = \int_{-T/2}^{T/2} v(t)\,dt = \frac{1}{T}\left(T/2 - (-T/2)\right) = \underline{1}$

If we take the limit that $T \to 0$, the width goes to zero but the area under it remains $\underline{1}$.

This is called the unit impulse, $\delta(t)$, also called the Dirac Delta, or Kronecker Delta.

$$\delta(t) = 0 \text{ for } t \neq 0, \quad \int_{-\infty}^{t} \delta(x)\,dx = u(t)$$

More on this later.

Unit ramp: ~~$r(t) = t \quad t \geq 0$~~

$$r(t) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases}$$

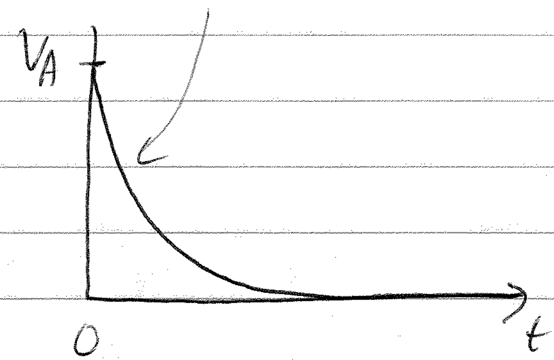
or $\quad r(t) = t\,u(t)$

or $\quad r(t) = \int_{-\infty}^{t} u(x)\,dx$

Read 5-2 + do the examples.

## Exponential Waveform

$$v(t) = \left[V_A e^{-t/T_c}\right] u(t)$$

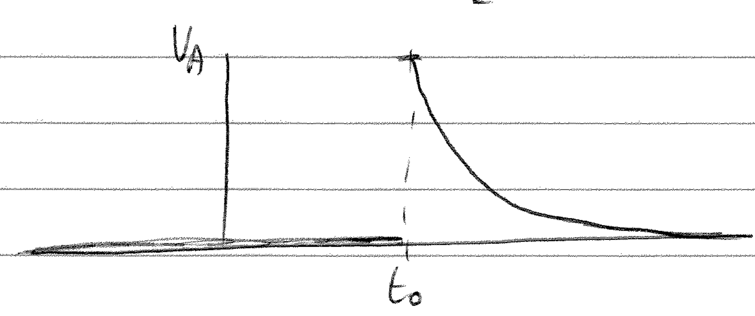



$T_c$ is called the "Time Constant". Mathematically speaking this function never reaches zero, but for Engineers the answer to "When does $v(t)$ equal zero?" is "5 $T_c$", because

$$e^{-5} = 0.00674 \quad \text{well less than } 1\%.$$

Time shift:

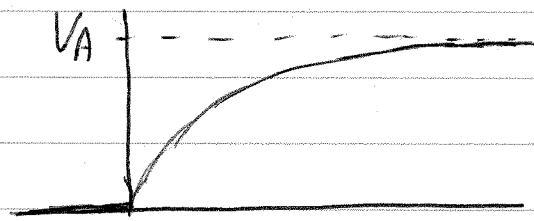
$$v(t-t_0) = \left[V_A e^{-(t-t_0)/T_c}\right] u(t-t_0)$$



This is like a thing "discharging," like water draining from a tank thru a hole in its side.

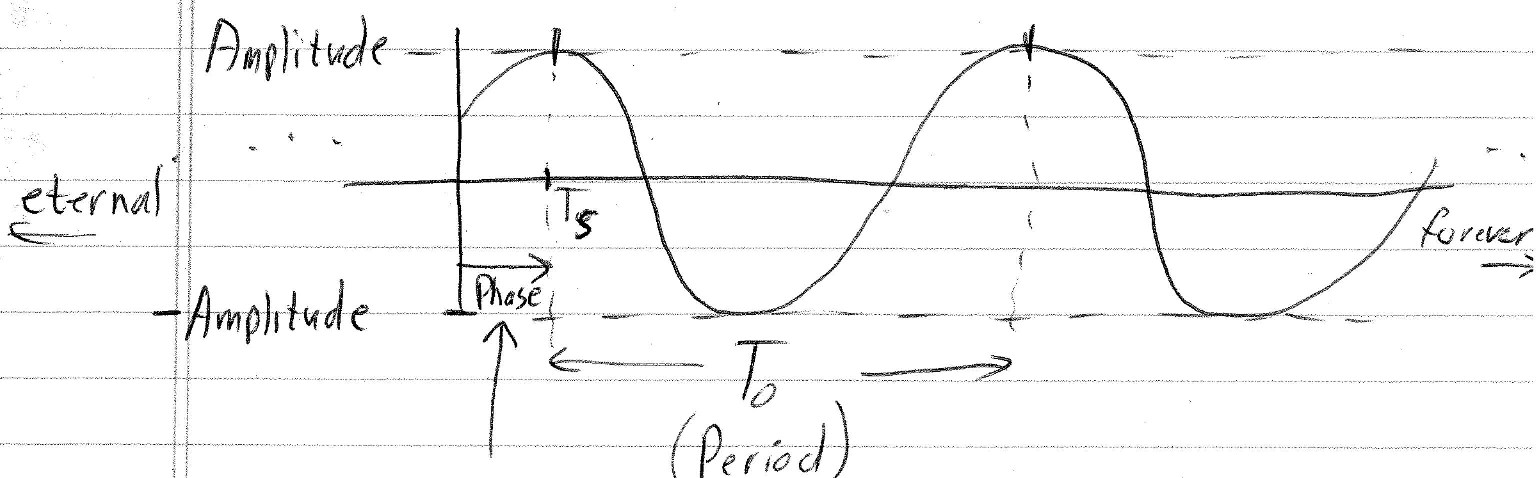
The "inverse" is "charging":

$$v(t) = \left[V_A\left(1 - e^{-t/T_c}\right)\right] u(t)$$




## Sinusoidal Waveform

cosine + sine are both called "Sinusoidal"




$$v(t) = V_A \cos\left(\frac{2\pi}{T_0}t - \phi\right) = V_A \cos\left(\frac{2\pi}{T_0}(t - T_s)\right)$$

$$= V_A \cos\left(\frac{2\pi}{T_0}t - 2\pi\frac{T_s}{T_0}\right)$$

Remember that $2\pi$ (radians) $= 360°$, so in degrees we write:

$$v(t) = V_A \cos\left(\frac{360°}{T_0} t + \phi\right)$$

and $\phi = -\frac{360°}{T_0} T_s = -360° \underbrace{\frac{T_s}{T_0}}_{\text{shift as fraction of period}}$

## Alternative Form:

$$v(t) = a \cos\left(\frac{2\pi}{T_0} t\right) + b \sin\left(\frac{2\pi}{T_0} t\right)$$

in which case $V_A = \sqrt{a^2 + b^2}$

and $\phi = \tan^{-1}\left(\frac{-b}{a}\right)$

## Frequency:

Cyclic Frequency $f_0 = \frac{1}{T_0}$ (in $\frac{\text{cycles}}{\text{sec}}$ or Hz)

Angular Frequency $\omega_0 = \frac{2\pi}{T_0}$ (in $\frac{\text{radians}}{\text{sec}}$

$= 2\pi f_0$

There are 3 parameters that describe this sinusoid: Amplitude

Frequency

Phase (w.r.t. some "zero" of time)

Properties:

Periodic: $v(t+nT_0) = v(t)$ where $\overbrace{n = -\infty, \ldots -3, -2, -1, 1, 2, 3, \ldots \infty}^{n \text{ is an integer}}$

Additive: Adding 2 or more sinusoids with the same frequency yields a sinusoid of the same frequency but a different amplitude and phase.

Derivatives and Integrals wrt $t$ yield sinusoids of the same frequency.

Read 5-4!!!

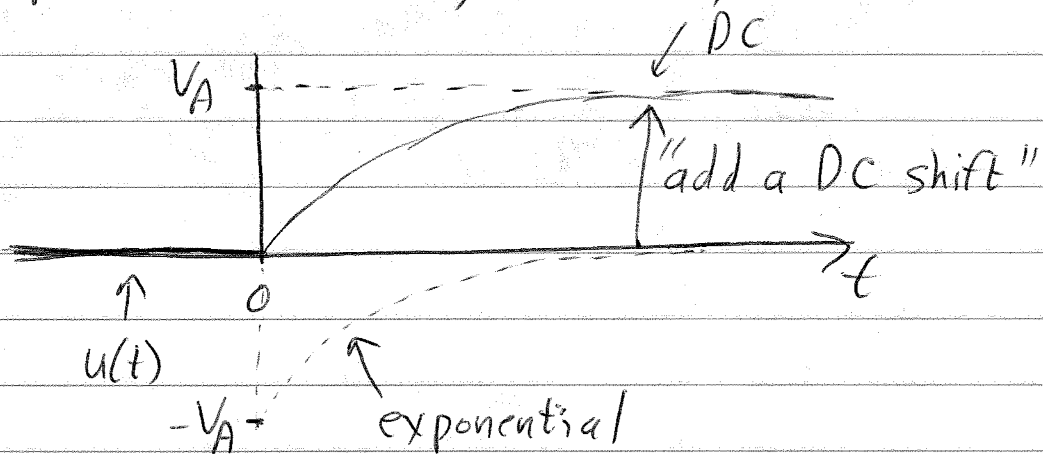
"DC" Waveforms – Constants

Composite Waveforms

I already alluded to one:

$$V_A\left[1 - e^{-t/\tau_c}\right]u(t)$$

puts together DC, exponential, and step to get:




Read Section 5-5!!

We will do more with these as we go.