The text website has an Appendix A on it that describes some methods, Cramer's Rule and Linear Algebra. I have put the Web Appendix A on the Black board site under Course Materials.

Cramer's Rule:

Given an array equation:

$$G V = S$$

$$\begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}$$

then $V_i = \frac{\Delta_i}{\Delta}$

where $\Delta = |G|$ and $\Delta_i = \left| \begin{matrix} G \text{ with} \\ S \text{ in place} \\ \text{of the } i^{th} \\ \text{column} \end{matrix} \right|$

$$\Delta = \begin{vmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{vmatrix} = (G_{11}G_{22} - G_{12}G_{21})$$

$$\Delta_1 = \begin{vmatrix} S_1 & G_{12} \\ S_2 & G_{22} \end{vmatrix} = S_1 G_{22} - S_2 G_{12}$$

and $\Delta_2 = \begin{vmatrix} G_{11} & S_1 \\ G_{21} & S_2 \end{vmatrix} = (G_{11}S_2 - G_{21}S_1)$

Then $V_1 = \frac{\Delta_1}{\Delta} = \frac{S_1 G_{22} - S_2 G_{12}}{G_{11}G_{22} - G_{12}G_{21}}$

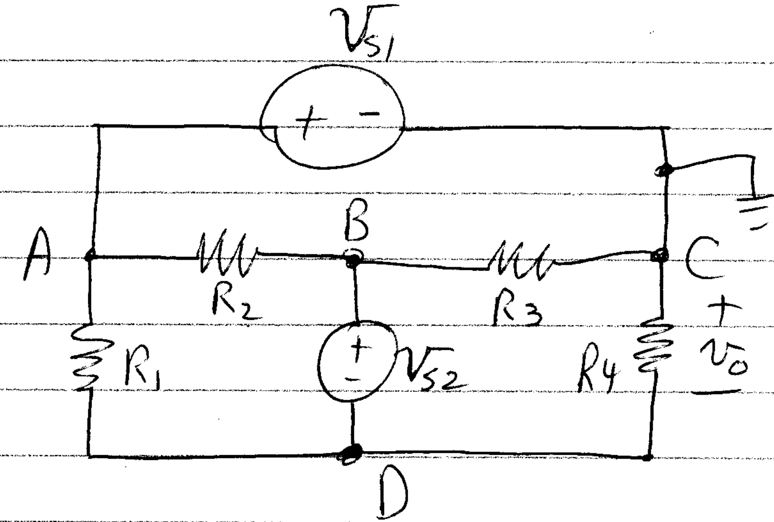
$V_2 = \frac{\Delta_2}{\Delta} = \ldots$

The larger an array becomes, the more complicated the determinant is. I'm not going to go further now. Many calculators, and Excel, Matlab, Wolfram Alpha, etc., and CircuitLab will do them for you.

Page 91 of text has a pretty good summary of Node-Voltage Analysis, look it over, good idea to develop your own, prime candidate for an exam.

Let's do another example, I'm going to do Ex 3-7 from text, but with a twist:

Ckt:



With $R_1 = R_4 = 2k\Omega$ and $R_2 = R_3 = 4k\Omega$

In text, they selected D as reference, as I usually would, and got $v_0 = v_C = \frac{v_{S2}}{3} - \frac{v_{S1}}{2}$

To show that any choice works, I'm going to select C as the reference (mark) and solve:

① Now- $v_o = v_C - v_D = 0 - v_D = -v_D \longleftarrow$ D.O

② Count- 4 Node Voltages $v_A = v_{S1}$ $v_C = 0$

$v_B = ?$ $v_D = ?$

2 unknowns, need 2 equations

③ V source supernodes:

① v_{S1} includes reference, so

$v_A = v_{S1}$

② v_{S2} includes B + D, so

$$\underline{v_B - v_D = v_{S2}}$$

(B+D) $\frac{v_B - v_A}{R_2} + \frac{v_D - v_A}{R_1} + \frac{v_D}{R_4} + \frac{v_B}{R_3} = 0$

$$\left(-\frac{1}{R_2} - \frac{1}{R_1}\right)v_A + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)v_B + \left(\frac{1}{R_1} + \frac{1}{R_4}\right)v_D = 0$$

Insert values: $\left(-\frac{1}{4k} - \frac{1}{2k}\right)v_A + \left(\frac{1}{4k} + \frac{1}{4k}\right)v_B + \left(\frac{1}{2k} + \frac{1}{2k}\right)v_D = 0$

X 4k : $(-1-2)v_A + (1+1)v_B + (2+2)v_D = 0$

$$\underline{-3v_A + 2v_B + 4v_D = 0}$$

Use $\underline{v_A = v_{S1}}$ + $\underline{v_B = v_D + v_{S2}}$

$$-3v_{S1} + 2(v_D + v_{S2}) + 4v_D = 0$$

$$6v_D = 3v_{S1} - 2v_{S2}$$

$$v_D = \frac{1}{2}v_{S1} - \frac{1}{3}v_{S2}$$

So $\underline{v_o = -v_D = \frac{1}{3}v_{S2} - \frac{1}{2}v_{S1}}$

Could I write as matrix?
Put in $v_A = v_{S1}$ to start:

$\cancel{-3 \quad 2 \quad 4 \quad v_A \quad v_B}$

$$2v_B + 4v_D = 3v_{S1}$$

$$v_B - v_D = v_{S2}$$

Becomes: $\begin{pmatrix} 2 & 4 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} v_B \\ v_D \end{pmatrix} = \begin{pmatrix} 3v_{S1} \\ v_{S2} \end{pmatrix}$

$$v_D = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 2 & 3v_{S1} \\ 1 & v_{S2} \end{vmatrix}}{\begin{vmatrix} 2 & 4 \\ 1 & -1 \end{vmatrix}} = \frac{2v_{S2} - 3v_{S1}}{((2)(-1) - (1)(4))}$$

$$= \frac{2v_{S2} - 3v_{S1}}{-2-4} = \frac{2v_{S2} - 3v_{S1}}{-6}$$

$$= -\frac{1}{3}v_{S2} + \frac{1}{2}v_{S1}$$