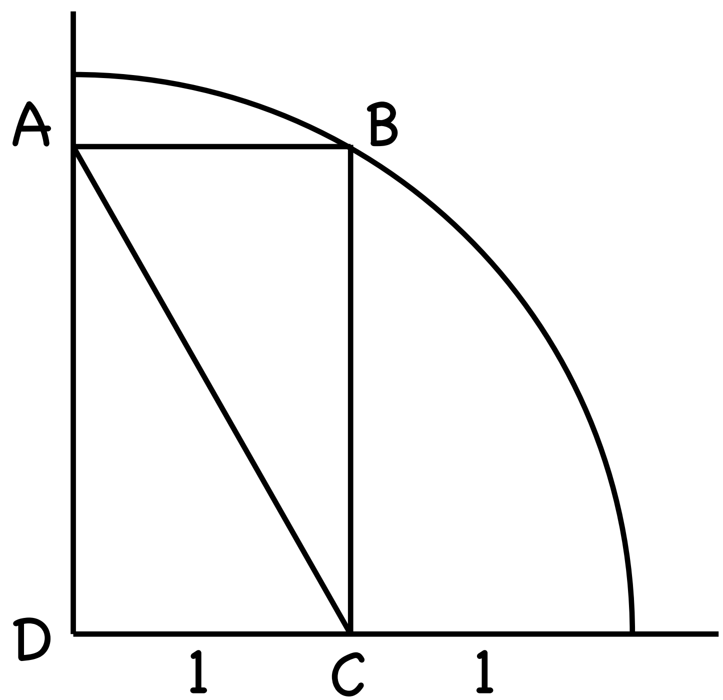
Puzzler

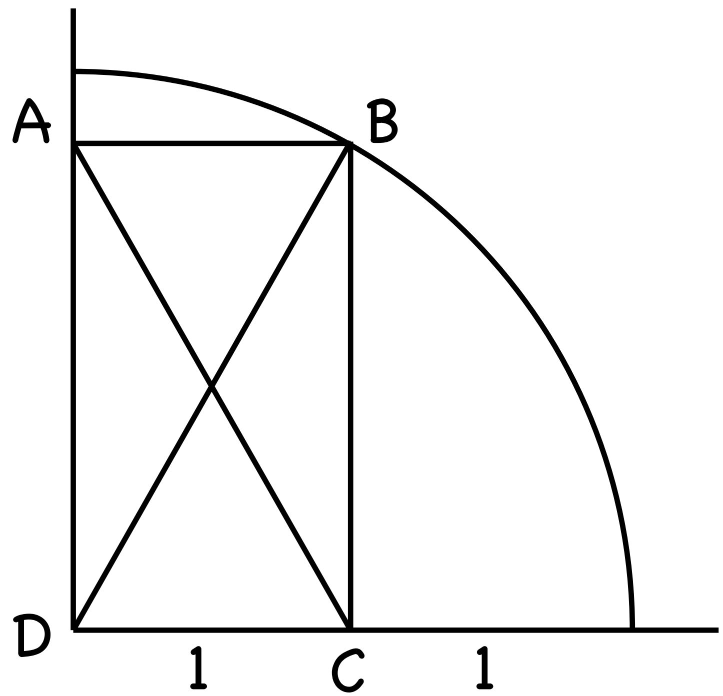
Guess the Diagonal

A rectangle is inscribed in the quadrant of a circle as shown. Given the distances indicated, can you accurately determine the length of the diagonal AC?



Solution: 2

Line AC is one diagonal of the rectangle. The other diagonal, BD, is also the radius of the circle, which is equal to 2. Since the two diagonals of the rectangle are equal, then AC = BD = 2 units!



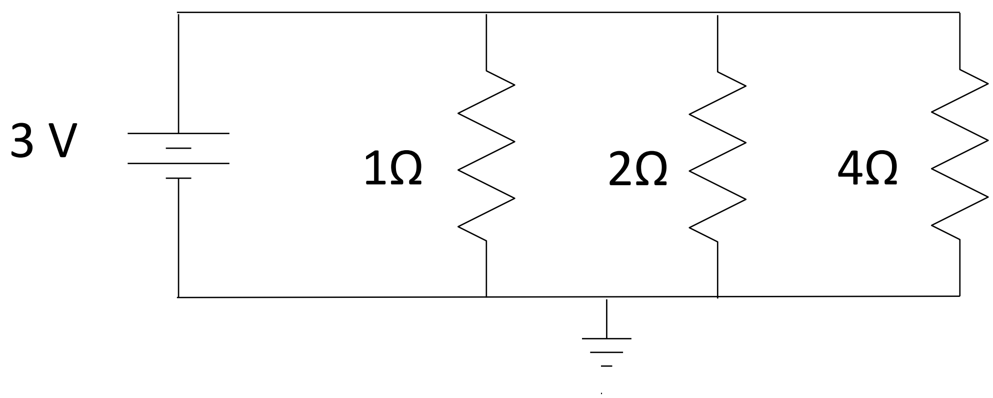
Introduction to Audio and Music Engineering

Lecture 13

Topics:

- Using KVL and KCL
- Energy and Power
- Impedance matching
- Introduction to Operational Amplifiers

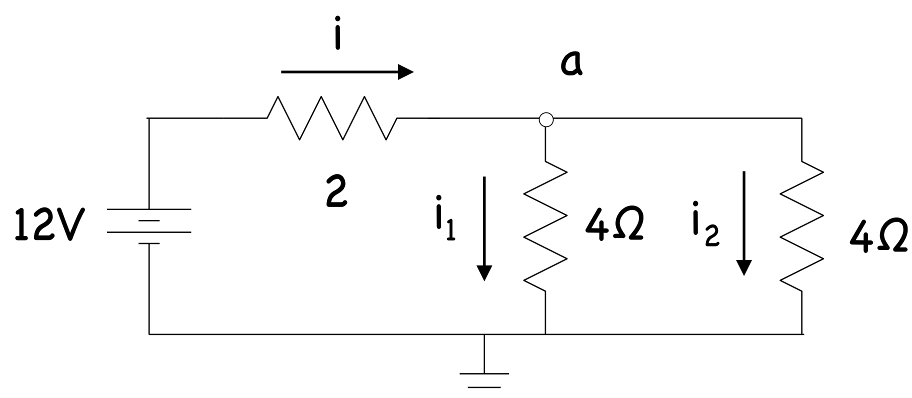
A larger current divider



Find the current in each resistor and the total current drawn from the battery.

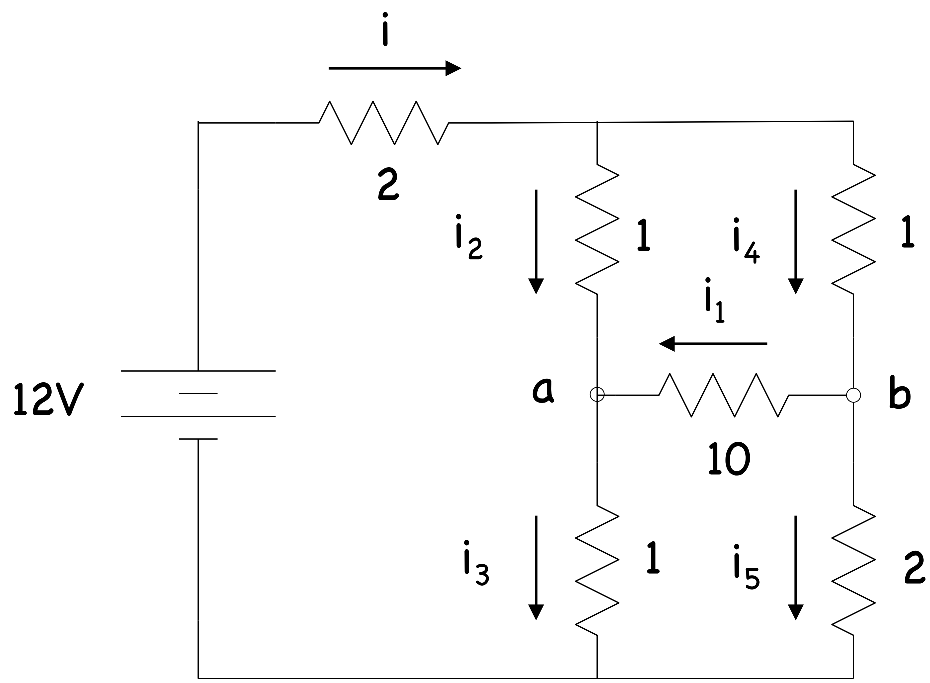
Combining KCL and KVL to solve more complicated circuits

Find all of the currents and then find the voltage at point a.



A more complicated circuit: the dc bridge

Find all of the currents and then find the voltages at points a and b.



Energy and Power

Power → Energy (Work) per unit time

Energy is measured in Joules.

(Force x distance) → kg m/sec^2 x m

Power is measured in Watts.

1 Watt = 1 Joule/1 sec

Units: Force x velocity → kg m/sec^2 x m/sec

Energy/time → kg m^2/sec^2 / sec

Same amount of work (energy).

3X Power

... same energy delivered in 1/3 of the time

Laser Bay 2 of the National Ignition Facility

192 laser beams deliver

500 trillion Watts !!

(1000 x US Power consumption)

But only for 3.6×10^{-9} secs

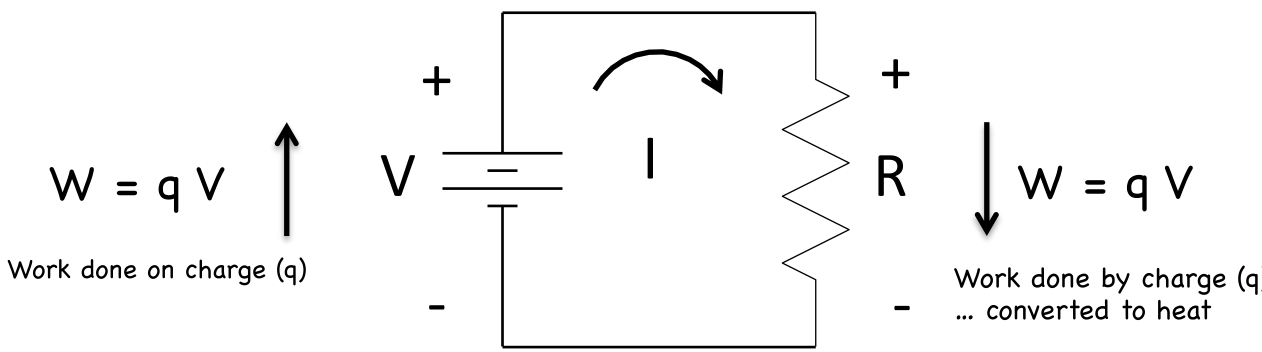
(500×10^{12}) W x (3.6×10^{-9}) sec

= 1.8×10^{6} Joules

= 1.8 Mega Joules (MJ)

This sounds like a lot of energy but this is only the amount of energy in about 50 ml of gasoline!

Power in an electrical circuit



Energy is delivered by the battery and absorbed by the resistor.

Power is Work (Energy) per unit time: Energy/time

$$P = qV/t = I \times V$$

Combine this expression with Ohm's Law to find power dissipated in a resistor, $V = IR$, $I = V/R$

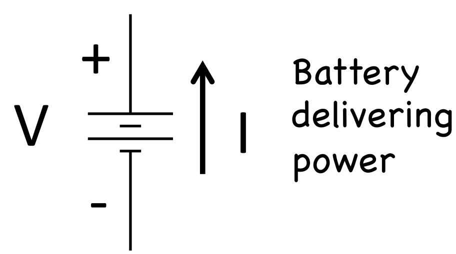
$$P = I \times V = V^2/R = I^2R$$

Note that it doesn't matter which direction the voltage is applied (or the current flows) through a resistor - the power dissipated is the same.

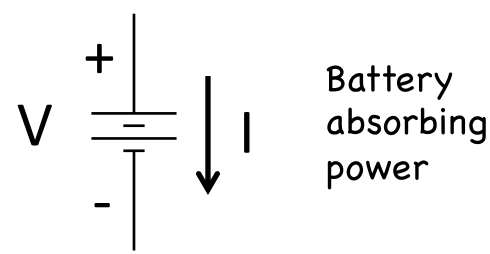
$$P = V^2/R = I^2R$$

Sign Convention: (To be consistent we really should make P(res) negative.)

+P → delivering power; -P → absorbing power



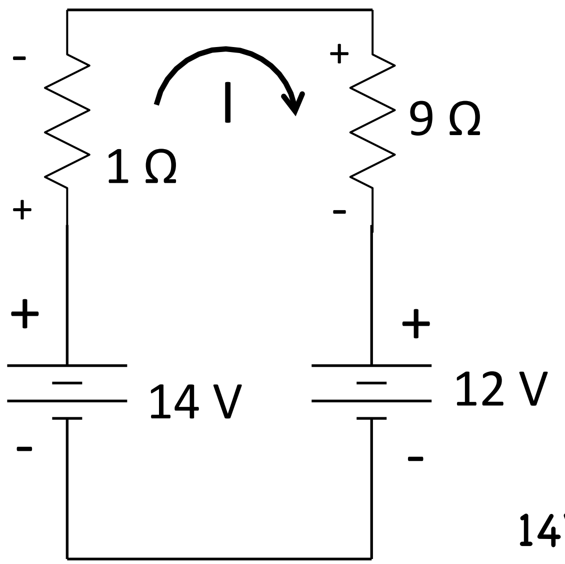
I, V in same direction
I x V is positive



I, V opposed
I x V is negative

Example: Battery Charging circuit

Find the Power absorbed or delivered by each element in the circuit.



KVL: $14 - 1\,I - 9\,I - 12 = 0$

$2 - 10\,I = 0 \rightarrow I = 0.2\text{ A}$

14V battery: $\overset{+}{\underset{-}{V}} \uparrow I$ $14 \times 0.2 = 2.8$ Watts (delivered)

12V battery: $\overset{+}{\underset{-}{V}} \downarrow I$ $12 \times (-0.2) = -\ 2.4$ Watts (absorbed)

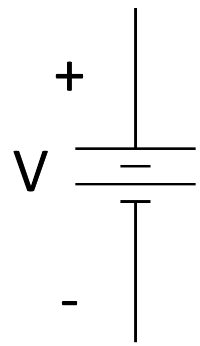
1Ω resistor: $\overset{-}{\underset{+}{V}} \uparrow I$ $I^2 \times 1 = -0.04$ Watts (absorbed)

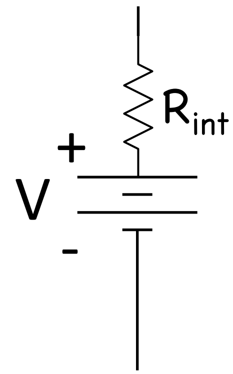
9Ω resistor: $\overset{+}{\underset{-}{V}} \downarrow I$ $I^2 \times 9 = -0.36$ Watts (absorbed)

Total = 0

Real Voltage and Current Sources

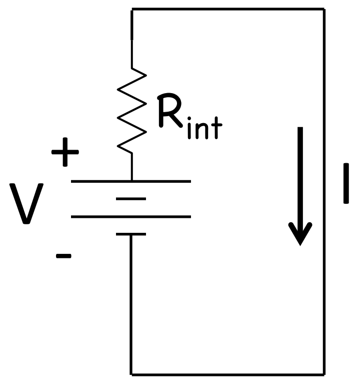
Ideal voltage source: Voltage remains fixed no matter how much current is drawn.





A real voltage source has a small internal resistance. (AA battery ~ 0.1 Ω)

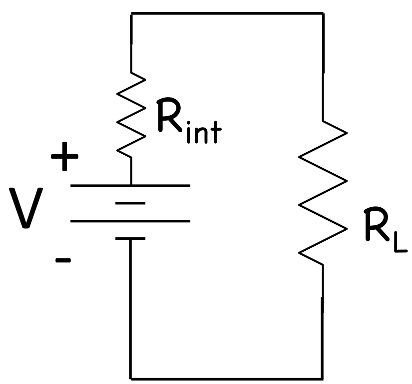
This limits the maximum current available from the source.



If we short circuit the source the current flow is limited.

(AA battery: $V = 1.5V$, $R_{int} = 0.1\ \Omega$)

$$I = V/R_{int} = 1.5/0.1 = 15 \text{ Amps}$$

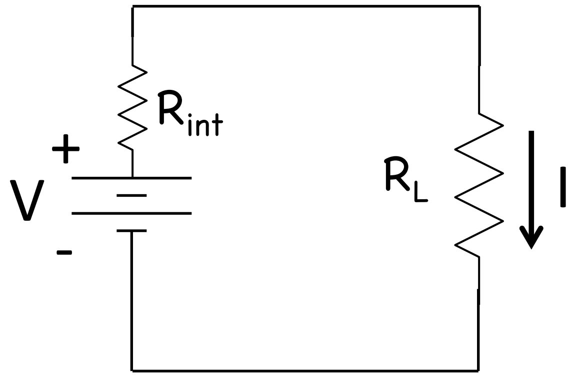


R_L and R_{int} form a voltage divider

Voltage developed across R_L:

$$V_L = V[R_L/(R_L + R_{int})]$$

Impedance Matching



What is the value of R_L that gives maximum power transfer from source to load?

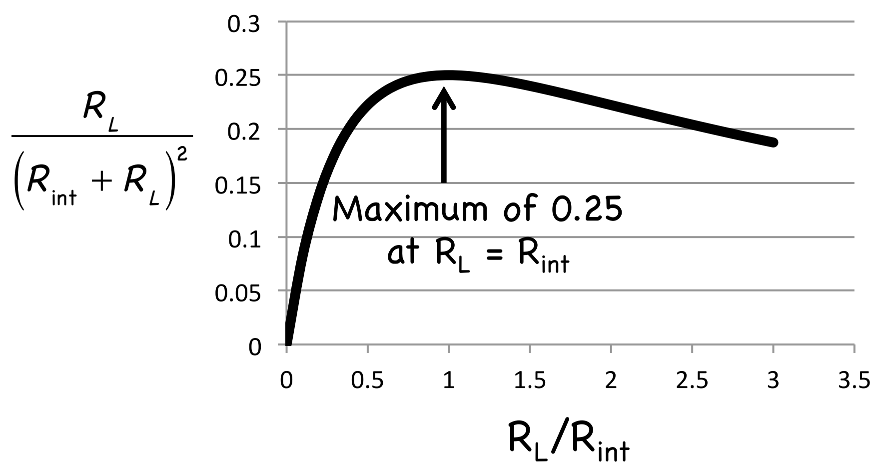
$P = I \times V_L$

$I = V/(R_{int} + R_L)$

$V_L = I\ R_L = V\ R_L/(R_{int} + R_L)$

so … $$P = V^2 \frac{R_L}{\left(R_{int} + R_L\right)^2}$$

Find the value of R_L that makes this expression a maximum.



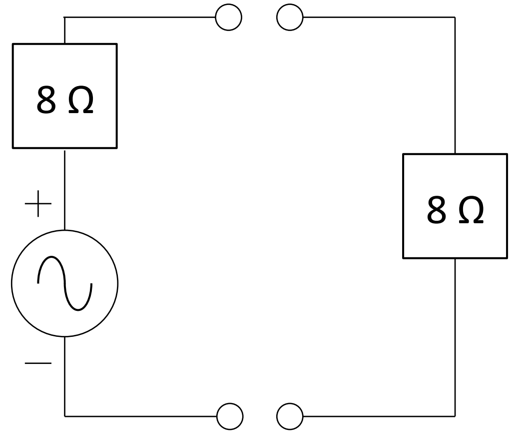
We can use calculus to find the maximum.

set $\frac{d}{dR_L}\left[\frac{R_L}{\left(R_{int} + R_L\right)^2}\right] = 0$ and solve for R_L

sol'n, $R_L = R_{int}$ $\qquad P_{max} = \frac{1}{4}\frac{V^2}{R_{int}}$

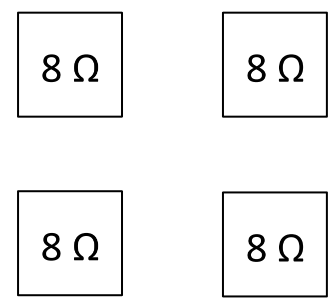
Impedance "matched" for maximum power transfer.

Impedance matching: Amplifier & Loudspeaker



Amp Loudspeaker

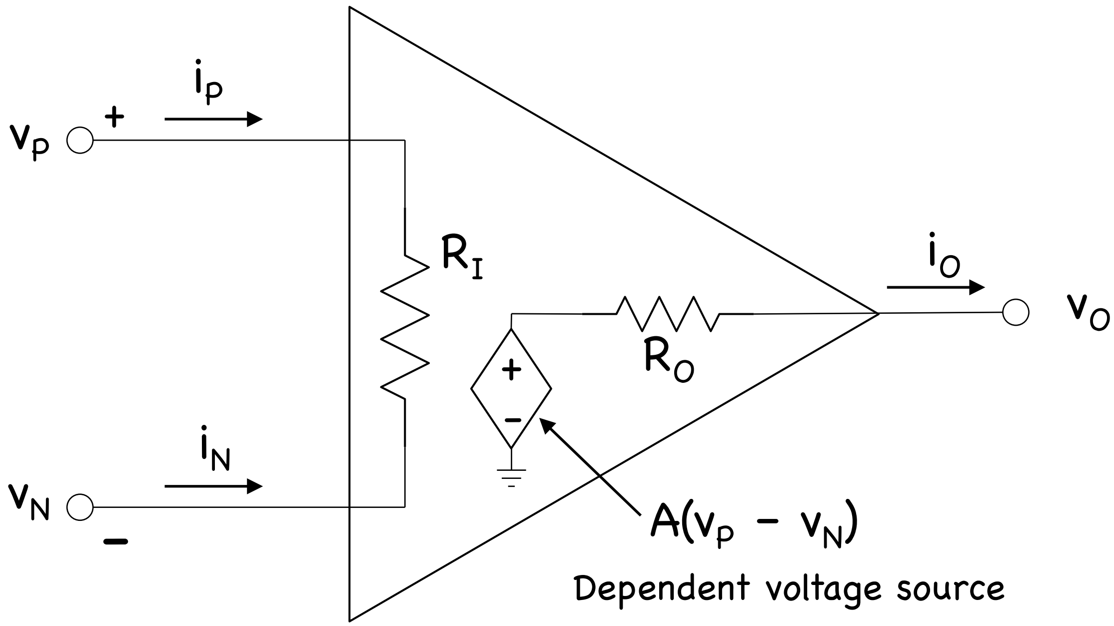
How to connect 4 speakers
(and maintain 8 Ohms)



Maximum power transfer when loudspeaker and amplifier have the same impedance

Operational Amplifier

Op-amp – dependent voltage source model



Typical Values

$10^6 < R_I < 10^{12}\ \Omega$

$10 < R_O < 100\ \Omega$

$10^5 < A < 10^8$

Output voltage is limited to power supply voltage and A is large → $v_P \approx v_N$

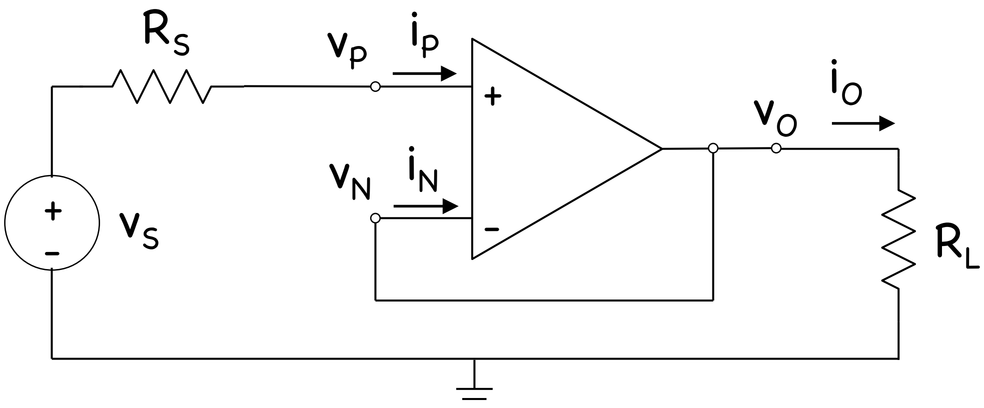
Ideal Op-Amp Model: $A \to \infty$ → $v_P = v_N$ & $i_P = i_N = 0$

The ideal model is adequate for analyzing many op-amp circuits.

Simple Op Amp Circuits

(Analyzed using the idealized op amp model)

Voltage Follower (buffer)



Feedback forces: $v_N = v_O$

$i_P = 0$, no voltage drop across R_S

then, $v_P = v_S$

ideal op amp model → $v_P = v_N$

therefore $v_O = v_S$

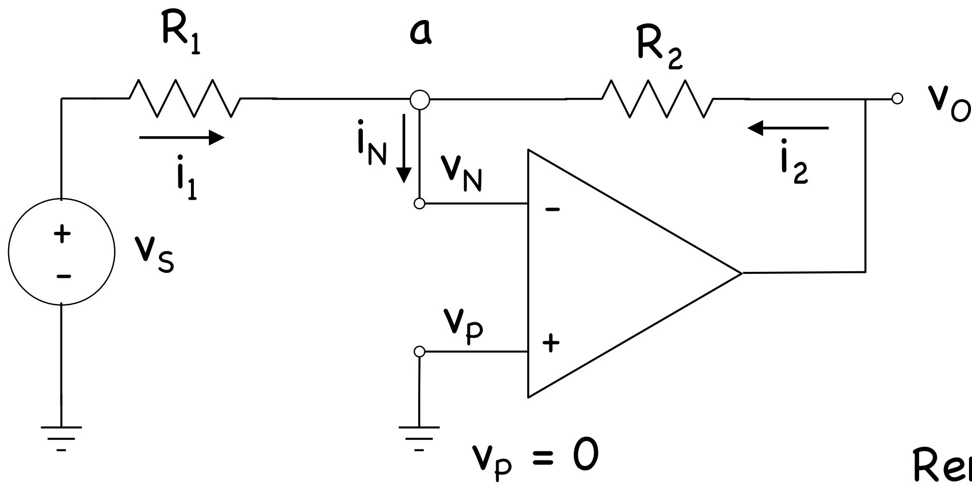
The output voltage simply "follows" the input voltage.

However there is no current drawn from the source and the output voltage is independent of the source resistance. (The source is buffered.)

Why can't you plug headphones directly into an electric guitar ... and hear anything?

You need a buffer amplifier.

Inverting Amplifier



Apply KCL at node a:

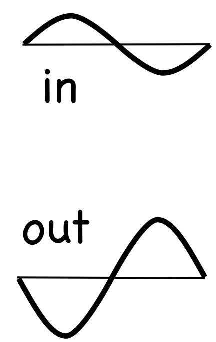
$$i_1 + i_2 - i_N = 0$$

$$\rightarrow \quad \frac{v_S - v_N}{R_1} + \frac{v_O - v_N}{R_2} - i_N = 0$$

Remember that … $v_P = v_N$ & $i_P = i_N = 0$

We made $v_P = 0$, and $v_P = v_N$ so $v_N = 0$

$$\frac{v_S}{R_1} + \frac{v_O}{R_2} = 0 \qquad \text{so ...} \quad v_O = -\frac{R_2}{R_1} v_S$$



Control the gain by adjusting R_2/R_1 .