MATH 201 HOMEWORK 2 WRITTEN PROBLEMS

8. Prove that

$$\binom{n+m}{r} = \binom{n}{0} \binom{m}{r} + \binom{n}{1} \binom{m}{r-1} + \dots + \binom{n}{r} \binom{m}{0}$$

Hint: Consider a group of n men and m women. How many groups of size r are possible?

11. The following identity is known as Fermat's combinatorial identity:

$$\binom{n}{k} = \sum_{i=k}^{n} \binom{i-1}{k-1} \qquad n \ge k$$

Give a combinatorial argument (no computations are needed) to establish this identity. Hint: Consider the set of numbers 1 through n. How many subsets of size k have i as their highest numbered member?

13. Show that, for n > 0,

$$\sum_{i=0}^{n} (-1)^i \binom{n}{i} = 0$$

Hint: Use the binomial theorem.