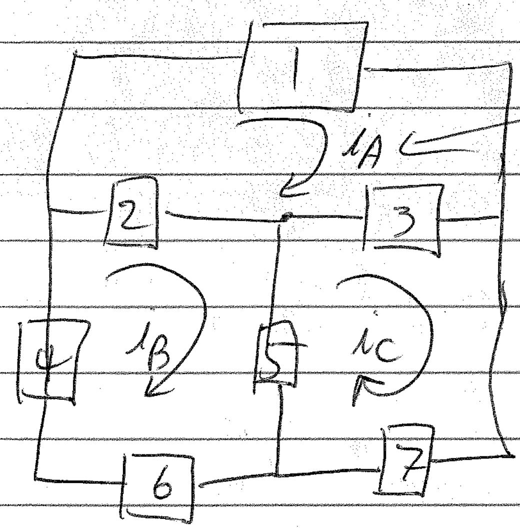
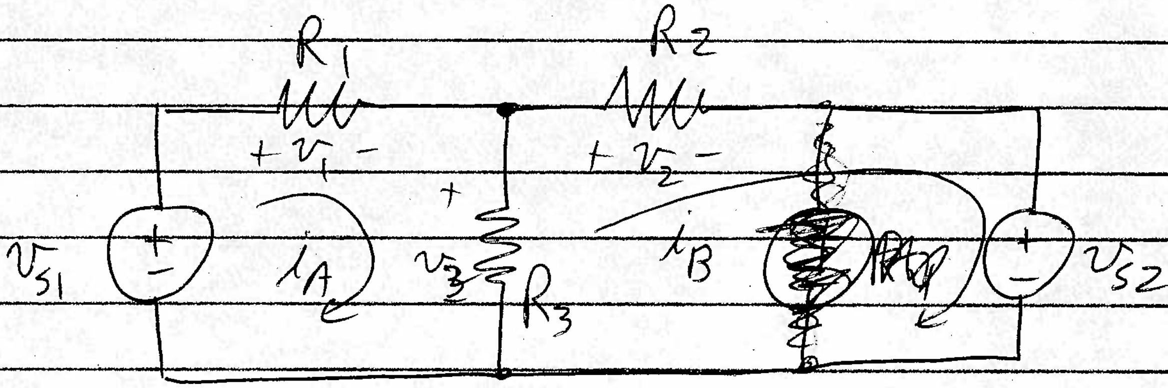
Mesh-Current Analysis

In a direct analogy to Node-Voltage Analysis, we can propose that Currents can be used to analyze circuits instead of Voltages. Any planar circuit can be drawn as meshes, loops of the ckt that do not contain other elements. Only 2-terminal elements can be allowed, and it can visualized as this:



"Mesh Currents," fictional, only 2 currents can flow thru a single element

We now use KVL around each mesh, and Ohm's Law, to write a set of M equations in M unknowns. Sometimes we can simplify these before we start. Let's take a look. Start with the example in the text:



1) We propose 2 currents, i_A and i_B flowing clockwise in each mesh.

2) There are no current sources, so both $i_A + i_B$ are unknown.

3) Write KVL (+ Apply Ohm's Law) around each ~~loop~~ Mesh. Book starts at bottom left, I tend to start at top left:

A $\quad +R_1 i_A + R_3(i_A - i_B) - v_{S1} = 0$

collect terms: $(R_1 + R_3)\, i_A - R_3 i_B = v_{S1}$

B $\quad R_2 i_B + v_{S2} + R_3(i_B - i_A) = 0$

$$-R_3 i_A + (R_2 + R_3) i_B = -v_{S2}$$

Matrix:
$$\begin{pmatrix} (R_1 + R_3) & -R_3 \\ -R_3 & (R_2 + R_3) \end{pmatrix} \begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} v_{S1} \\ -v_{S2} \end{pmatrix}$$

$$\Delta = (R_1 + R_3)(R_2 + R_3) - (-R_3)(-R_3)$$
$$= (R_1 R_2 + R_1 R_3 + R_2 R_3 \cancel{+ R_3^2}) \cancel{- R_3^2}$$

$$\Delta_A = \begin{vmatrix} v_{S1} & -R_3 \\ -v_{S2} & (R_2 + R_3) \end{vmatrix} = v_{S1}(R_2 + R_3) - (-v_{S2})(-R_3)$$
$$= v_{S1}(R_2 + R_3) - R_3 v_{S2}$$

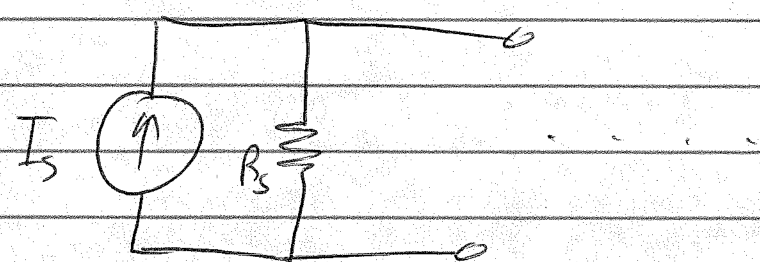
$$\Delta_B = \begin{vmatrix} (R_1 + R_3) & v_{S1} \\ -R_3 & -v_{S2} \end{vmatrix} = -(R_1 + R_3) v_{S2} - (-R_3)(v_{S1})$$
$$= R_3 v_{S1} - (R_1 + R_2) v_{S2}$$

$$i_A = \frac{\Delta_A}{\Delta} = \frac{v_{S1}(R_2 + R_3) - R_3 v_{S2}}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

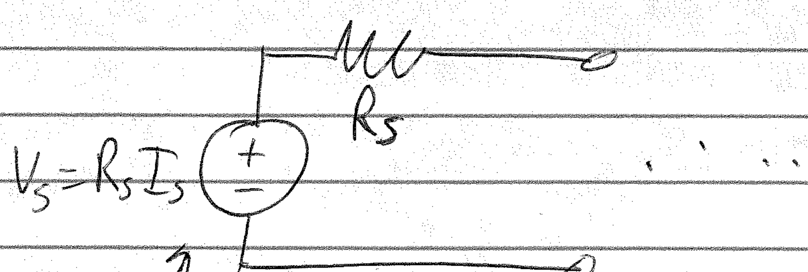
Read the section in the text, as well.

~~When~~ Recall that Voltage Sources required special handling in Node-Voltage Analysis. Well now Current Src's require some thought in Mesh-Current Analysis.

First, Practical Current Sources:



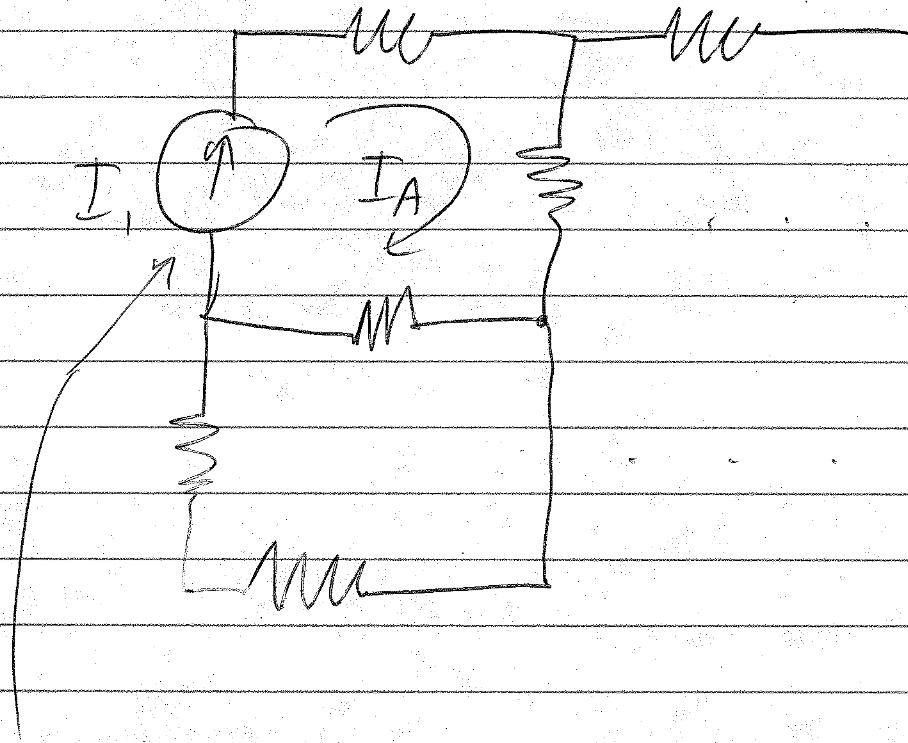
contains a mesh, so increases our Mesh Count by 1. Convert, if possible, to



Voltage sources are easily handled in M-C Analysis.

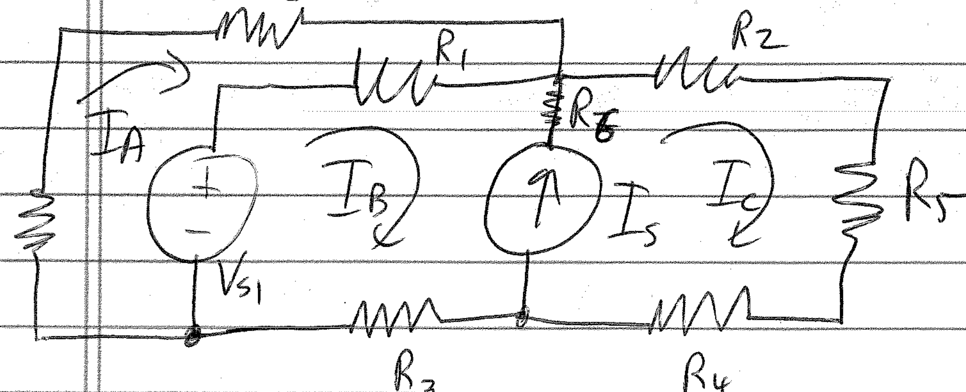
If you cannot convert a Current Source, then there are 2 possible cases:

1.) I source is on "outside" of a mesh:



Only 1 Mesh Current goes through Isrc, $I_A = I_1$, so reduces Mesh Count by 1.

2.) I source is common to 2 Meshes:



Now, we write that the Source Current is Equal to the Sum of the Mesh Currents:

$$I_S = I_C - I_B$$

M.C. in same direction as I_S

M.C. in opposite direction as I_S.

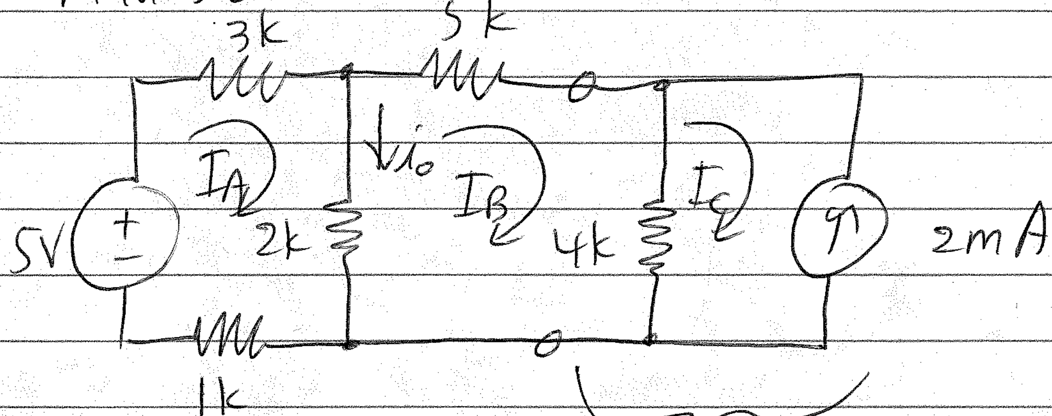
And now we ~~write~~ combine B+C into a "Super Mesh":

$$R_1(I_B - I_A) + R_2 I_C + R_5 I_C + R_4 I_C + R_3 I_B - \cancel{A} + V_{S1} = 0$$

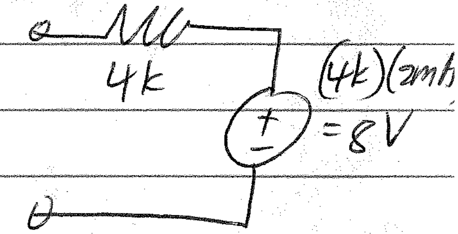
Note— R_7 does not appear and ~~is~~ does not affect the mesh currents.

Text Example 3-9:

Find i_o:



in text they ~~redu~~ conver to V. src:



We will assume we did not see that possibility and continue.

Propose I_A, I_B, I_C (draw).

Express Desired Output in terms of I's:

$$\underline{i_o = I_A - I_B}$$

Find I src's: There is 1, 2mA, on outside:

$$\underline{\underline{I_c = -2mA}}$$

Write KVL:

$$3kI_A + 2k(I_A - I_B) + 1kI_A - 5V = 0$$

$$(3k + 2k + 1k)I_A \quad - 2kI_B = 5V$$

$$\underline{6kI_A - 2kI_B = 5V}$$

$$5kI_B + 4k(I_B - I_C) + 2k(I_B - I_A) = 0$$

$$-2kI_A + (5k + 4k + 2k)I_B - 4k(-2mA) = 0$$

$$\underline{-2kI_A + 11kI_B = -8V}$$

$$\Delta = \begin{vmatrix} 6k & -2k \\ -2k & 11k \end{vmatrix} = (6k)(11k) - (-2k)(-2k)$$

$$= 66M - 4M = 62\,k^2$$

$$\Delta_A = \begin{vmatrix} 5V & -2k \\ -8V & 11k \end{vmatrix} = (11k)(5V) - (-8V)(-2k)$$

$$= 55\,kV - 16\,kV = 39\,kV$$

$$I_A = \frac{39\,kV}{62\,k^2} = \frac{39}{62}\frac{V}{k\Omega} = \underline{0.629 \; mA}$$

$$\Delta_B = \begin{vmatrix} 6k & 5V \\ -2k & -8V \end{vmatrix} = (6k)(-8V) - (-2k)(5V)$$

$$= -48\,kV + 10\,kV = -38\,kV$$

$$I_B = \frac{-38\,k\Omega V}{62\,(k\Omega)^2} = \underline{-0.613 \; mA}$$

So $i_o = I_A - I_B = 0.629\ mA - (-0.613\ mA)$
$= 1.242\ mA$