Puzzler

Three brothers shared 24 apples, each getting a number equal to his age 3 years ago.

The youngest one proposed a swap. "I will keep only half the apples I received and divide the rest between you two equally. But after I do this then you, my middle brother, shall keep half of your apples and divide the rest equally between our oldest brother and me, and following that then you, our oldest brother must do the same."

They agreed and the end result is that they each ended with 8 apples.

How old were the three brothers?

Solution: Work backwards

	Oldest	Middle	Youngest
Final	8	8	8
Oldest shares	16	4	4
Middle shares	14	8	2
Youngest shares	13	7	4
Ages (add 3)	16	10	7

Introduction to Audio and Music Engineering

Lecture 6

- Modes: space, time and frequency domains
- Fourier decomposition – Fourier coefficients
- Monitoring a string at a single point
- Signals & frequency content of a signal
- Harmonic content and tone quality (timbre)
- String plucking position and timbre
- Pickup position and timbre
- String vibration decay

Space – time – frequency

(keeping it all straight)

The string evolution as a function of space and time …

$$y(x,t) = \sum_n B_n \cos(n 2\pi f_0 t) \cdot \sin(n\pi \frac{x}{L})$$

time space

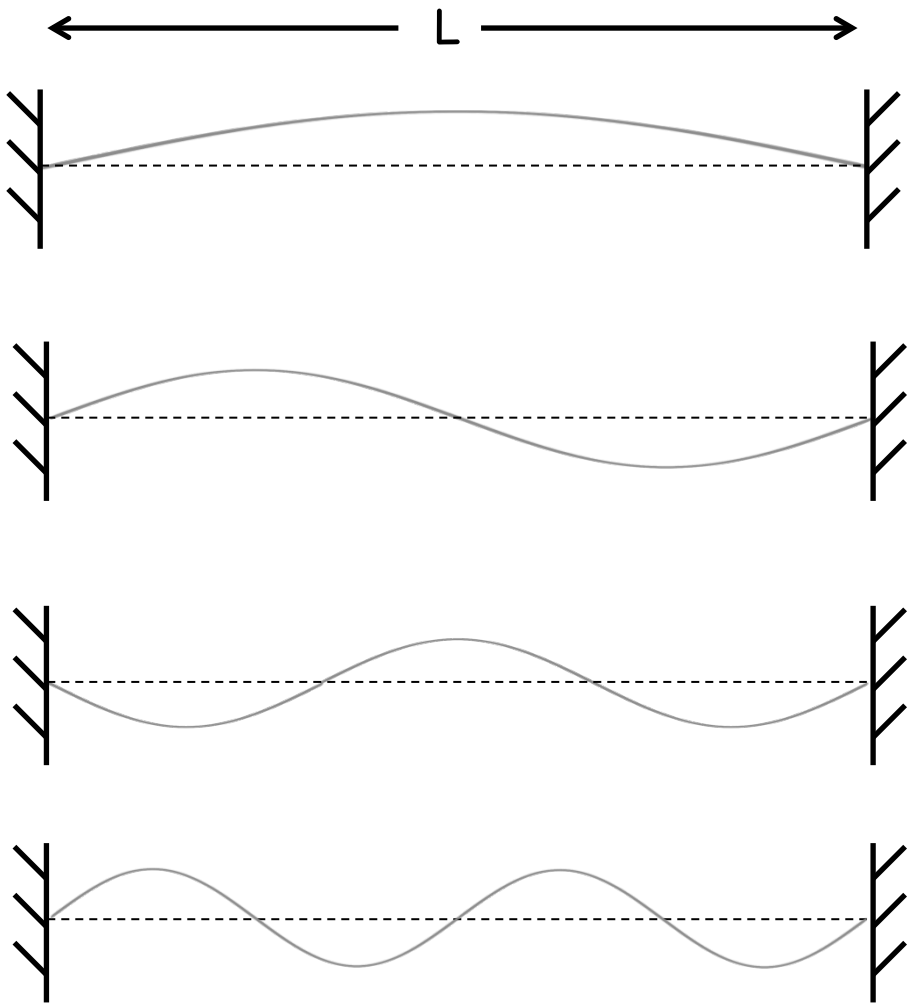
Amplitude of n'th mode

String motion is expressed as a sum of the string modes,

each one with its own <u>spatial</u> wavelength: $\lambda_n = \frac{2L}{n}$

and temporal frequency: $f_n = n f_0 = n \frac{c}{2L}$

A few normal modes



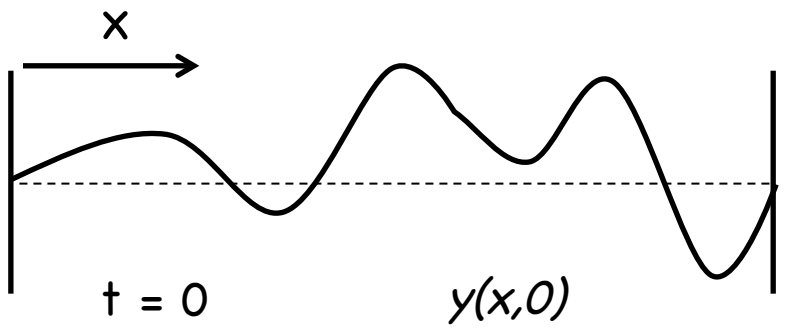
n = 1 $\lambda_1 = \frac{2L}{1}$ $f_1 = \frac{c}{2L}$

n = 2 $\lambda_2 = \frac{2L}{2}$ $f_1 = 2\frac{c}{2L}$

n = 3 $\lambda_3 = \frac{2L}{3}$ $f_1 = 3\frac{c}{2L}$

n = 4 $\lambda_4 = \frac{2L}{4}$ $f_1 = 4\frac{c}{2L}$

We can start with any shape



And then use the Fourier coefficient integral to find the amount of each mode present,

$$B_n = \frac{2}{L}\int_0^L y(x,0)\sin(\frac{n\pi x}{L})dx$$

This is just a measure of the similarity of each mode to the initial string shape.

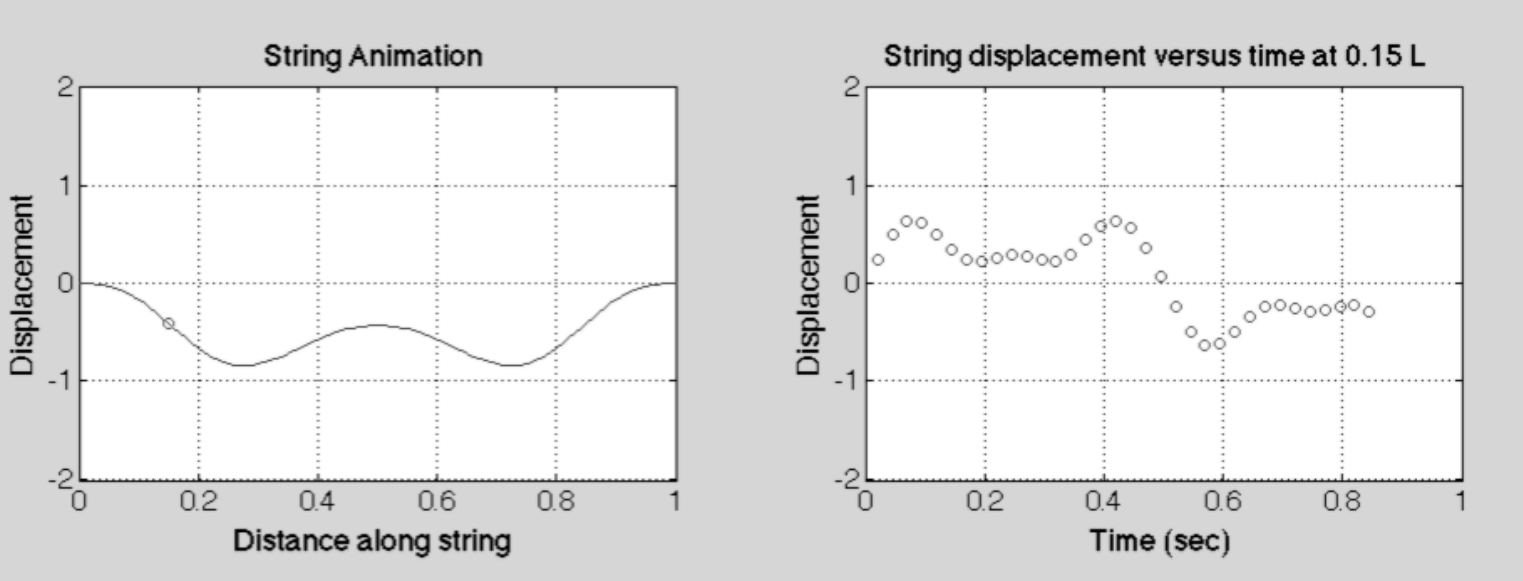
We then add up the modes with these "weights" and let it evolve as a function of space and time, each mode doing its own thing at its own frequency.

stringmodes2.m demo

We can monitor the string at any selected point

and ask, what is the displacement of that point on the string as a function of time?

stringmodes2.m demo



We now have a function of time. We can then ask what frequencies are present. (This is what we hear!)

stringmodes3.m demo

Finding the frequency content of a "signal"

The string evolution at a chosen point, x_0, versus time …

$$y(x_0,t)=\sum_n B_n \cos(n2\pi f_0 t)\cdot\sin(n\pi\frac{x_0}{L})$$

Amplitude of n'th mode

Basis functions are now functions of time.

This is a fixed number.

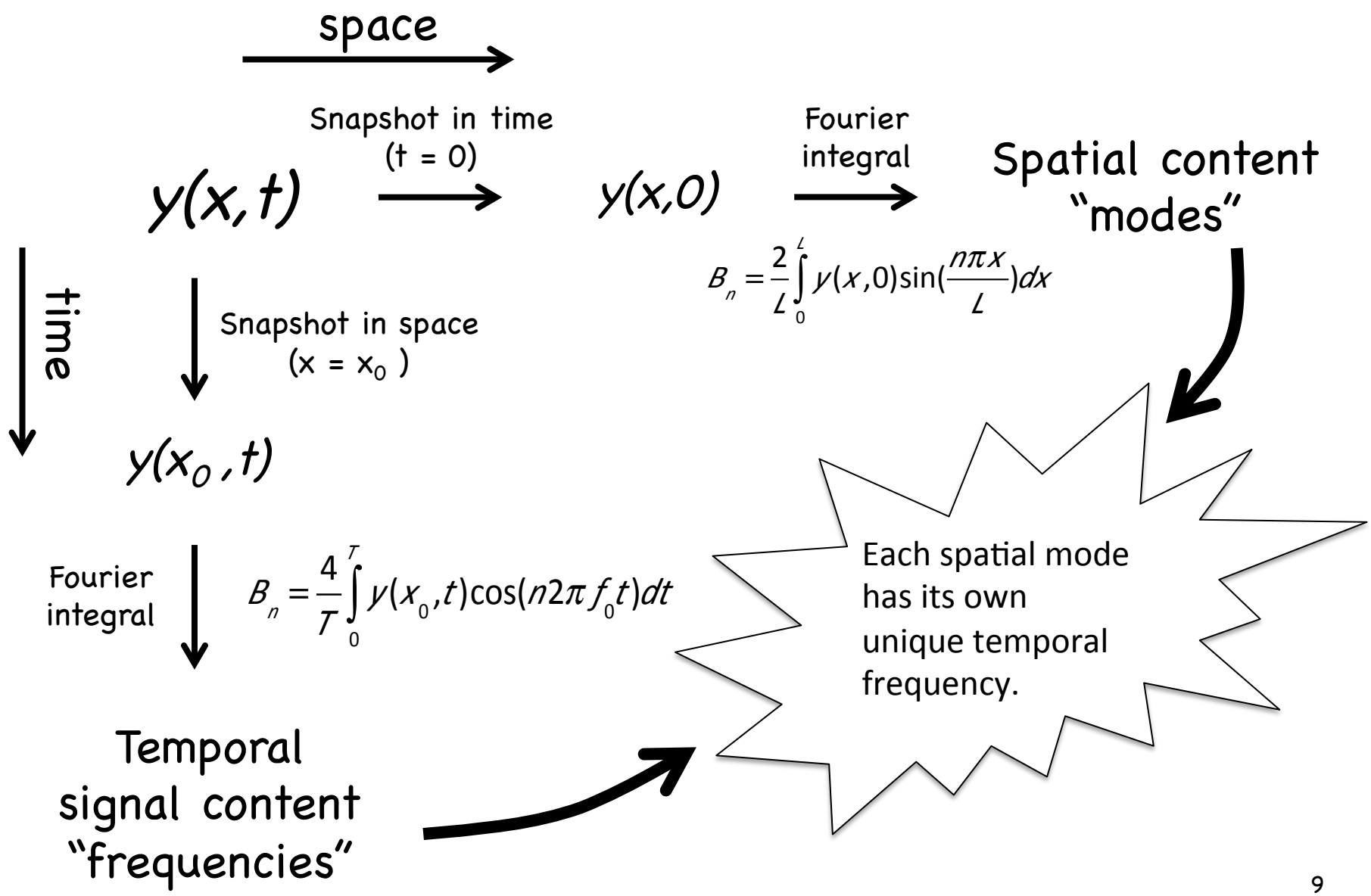
We can compute the Fourier series coefficients from this integral,

$$B_n=\frac{4}{T}\int_0^T y(x_0,t)\cos(n2\pi f_0 t)dt$$

The string motion <u>at one point on the string</u> can be represented as a Fourier series of functions of time as the basis functions.

Time → frequency transformation

Summary


space
Snapshot in time
(t = 0)
y(x, t)
y(x,0)
Fourier
integral
Spatial content
"modes"
$B_n = \frac{2}{L}\int_0^L y(x,0)\sin(\frac{n\pi x}{L})dx$
time
Snapshot in space
(x = x_0)
$y(x_0, t)$
Fourier
integral
$B_n = \frac{4}{T}\int_0^T y(x_0,t)\cos(n2\pi f_0 t)dt$
Each spatial mode
has its own
unique temporal
frequency.
Temporal
signal content
"frequencies"

Timbre (tone quality) and harmonic content

Go to the stringmodes2.m demo

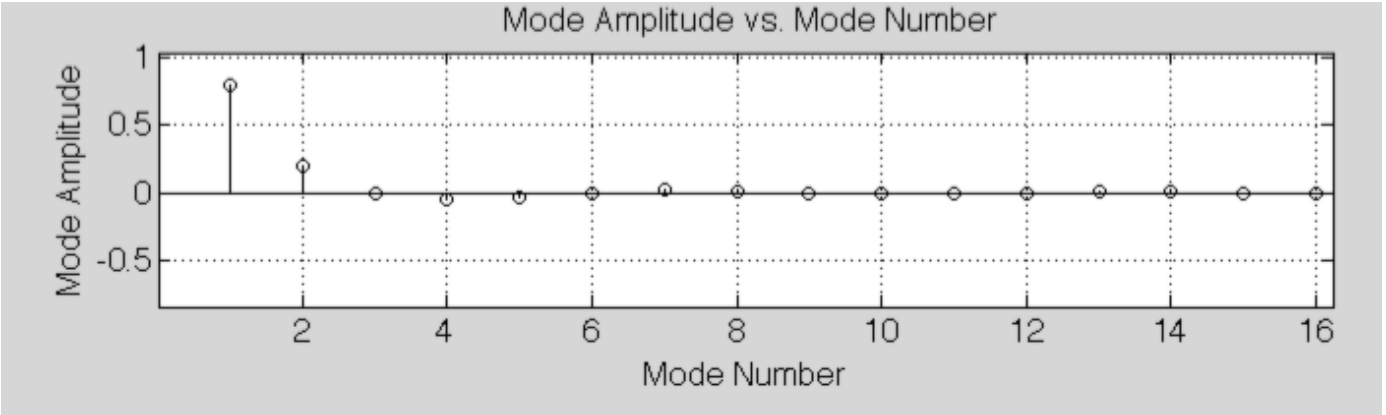
The greater the higher harmonic content the "brighter" the tone.

To demonstrate this add in successive harmonics (1, 1/2, 1/3, …)

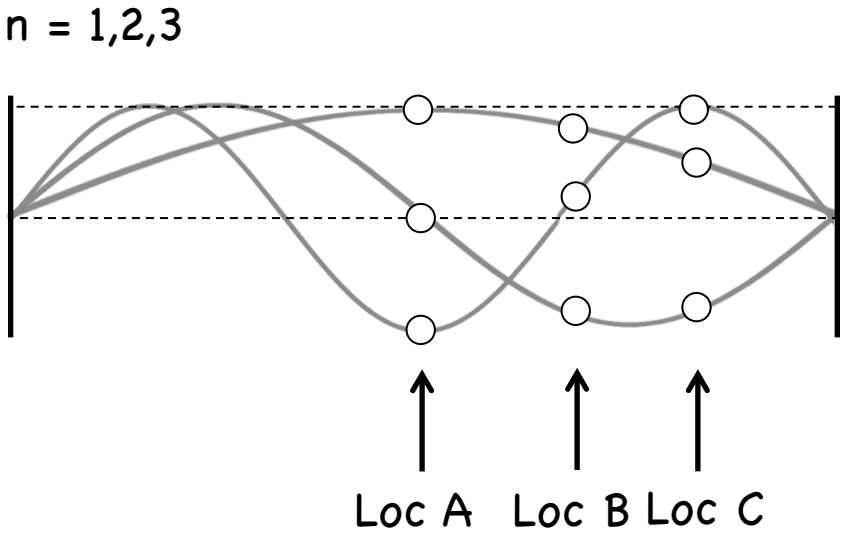
Go to the pluckmodes.m demo …

The closer the plucking point is to the bridge the more high frequency content in the Fourier series and the "brighter" the tone quality.

Interpret the mode amplitude versus node number graph as the spectrum of the sound.



Monitoring the string at different locations along its length



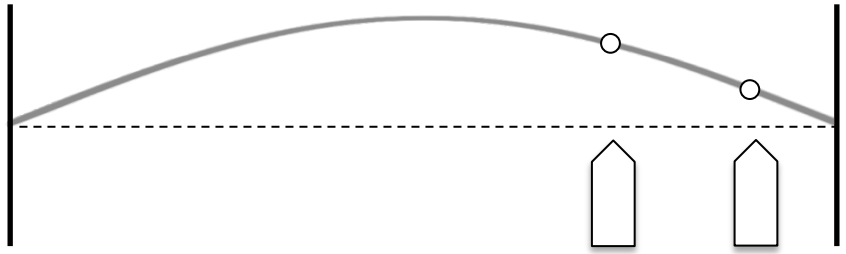
Consider only three modes - each with the same amplitude.

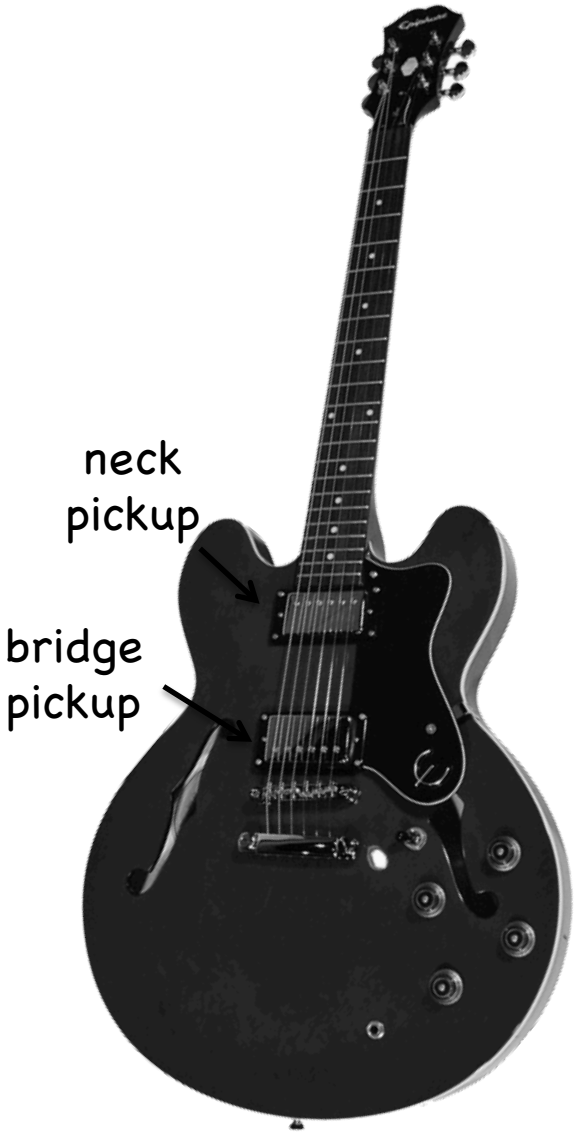
What is the maximum amplitude of each mode when measured at different locations along the string?

Magnitude of maximum string displacement versus A,B,C

Location	**n = 1**	**n = 2**	**n = 3**
A	1	0	1
B	0.8	0.8	0.2
C	0.5	0.8	1

Electric guitar pickup placement



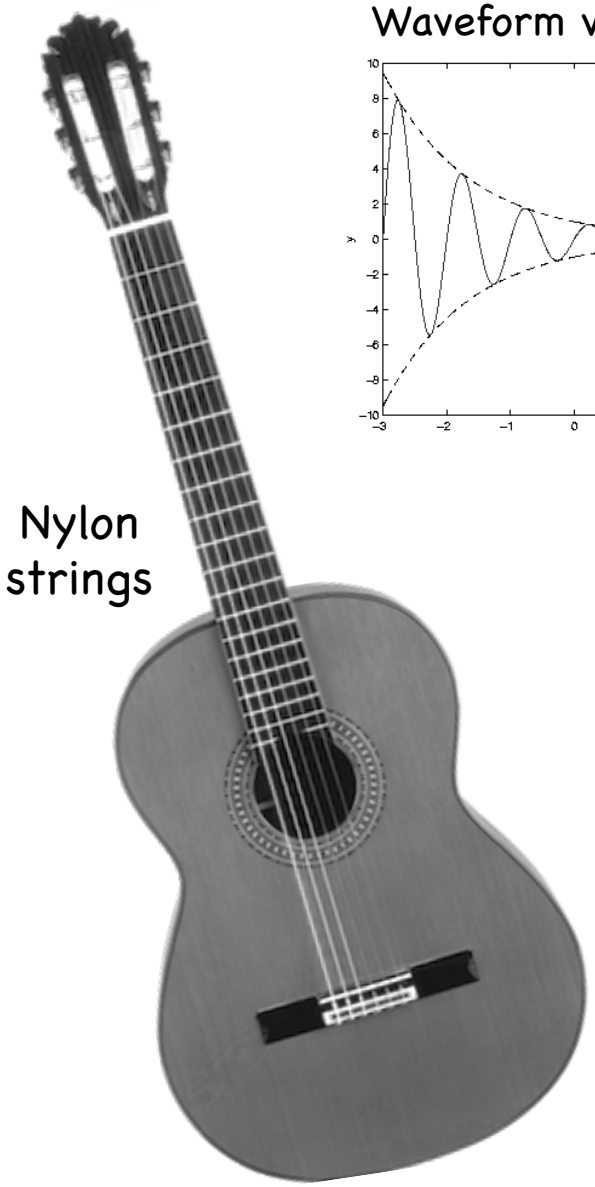


Amplitudes of lower modes (small n) generally are smaller near the bridge pickup. There are more "highs" and the sound is "brighter".

Amplitudes of lower modes (small n) generally are larger near the neck pickup. There are more "lows" and the sound is "darker".

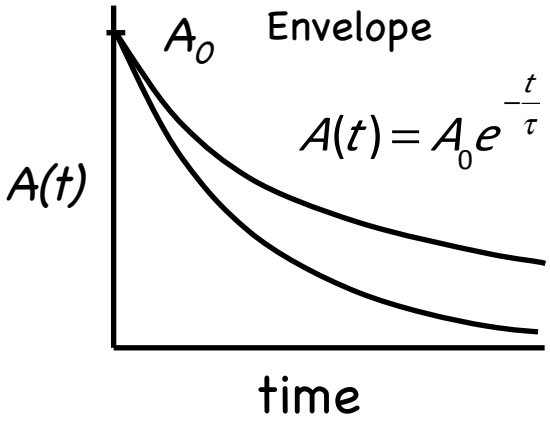
demonstrate pluckmodes.m

Decay of guitar string vibrations



Nylon strings

Waveform versus time



$$\tau \propto \frac{1}{f^d}$$

Higher frequencies decay faster

d ≈ 1 for nylon
d ≈ ½ for steel

demonstrate pluckmodes.m & newpluck.m

Steel strings