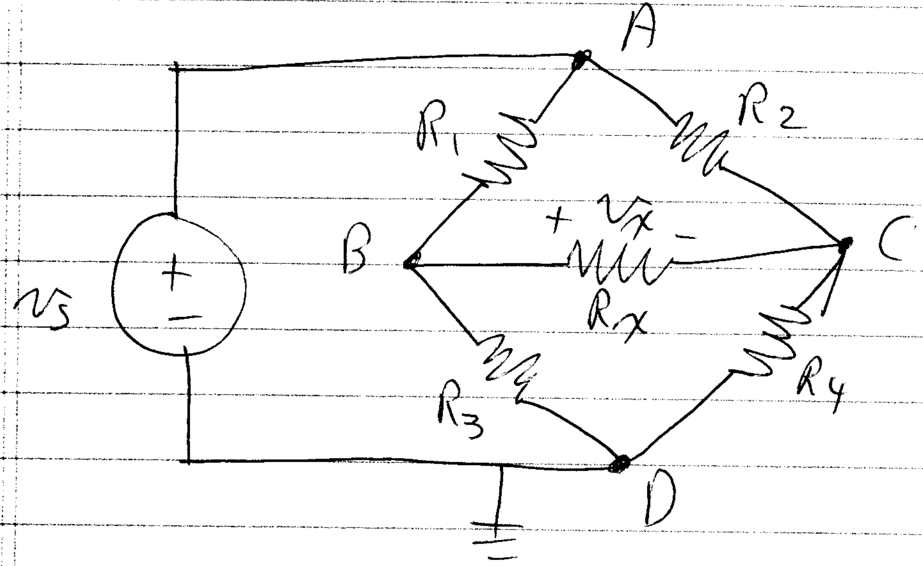
Let's look at another:

Prob 3-10 in text



D = reference

$v_A = v_S$ (supernode w/reference)

Only need $v_B + v_C$:

$$\frac{v_B - v_S}{R_1} + \frac{v_B - v_C}{R_x} + \frac{v_B}{R_3} = 0$$

B $$\left(\frac{1}{R_3} + \left\{\frac{1}{R_1} + \frac{1}{R_x}\right)v_B - \frac{1}{R_x}v_C = \frac{1}{R_1}v_S$$

$$\frac{v_C - v_S}{R_2} + \frac{v_C - v_B}{R_x} + \frac{v_C}{R_4} = 0$$

C $$-\frac{1}{R_x}v_B + \left(\frac{1}{R_2} + \frac{1}{R_x} + \frac{1}{R_4}\right)v_C = \frac{1}{R_2}v_S$$

Under what conditions will $v_x = 0$?

Write in terms of nodes: $v_x = v_B - v_C$

$$v_x = 0 = v_B - v_C \Rightarrow v_B = v_C$$

If $v_B = v_C$ then we have:

$$\left(\frac{1}{R_3}+\frac{1}{R_1}+\cancel{\frac{1}{R_x}}\right)v_B - \cancel{\frac{1}{R_x}v_B} = \frac{1}{R_1}v_S$$

$$\underline{\left(\frac{1}{R_3}+\frac{1}{R_1}\right)v_B = \frac{1}{R_1}v_S} = \left(\frac{R_1+R_3}{R_1R_3}\right)v_B$$

and $$\cancel{-\frac{1}{R_x}v_B} + \left(\frac{1}{R_2}+\cancel{\frac{1}{R_x}}+\frac{1}{R_4}\right)v_B = \frac{1}{R_2}v_S$$

$$\left(\frac{1}{R_2}+\frac{1}{R_4}\right)v_B = \frac{1}{R_2}v_S = \left(\frac{R_4+R_2}{R_2R_4}\right)v_B$$

Re-write these as:

$$v_S = R_1\left(\frac{R_1+R_3}{R_1R_3}\right)v_B = \frac{R_1+R_3}{R_3}v_B$$

and $$v_S = R_2\left(\frac{R_2+R_4}{R_2R_4}\right)v_B = \frac{R_2+R_4}{R_4}v_B$$

Equate these: $$\frac{R_1+R_3}{R_3}v_B = \frac{R_2+R_4}{R_4}v_B$$

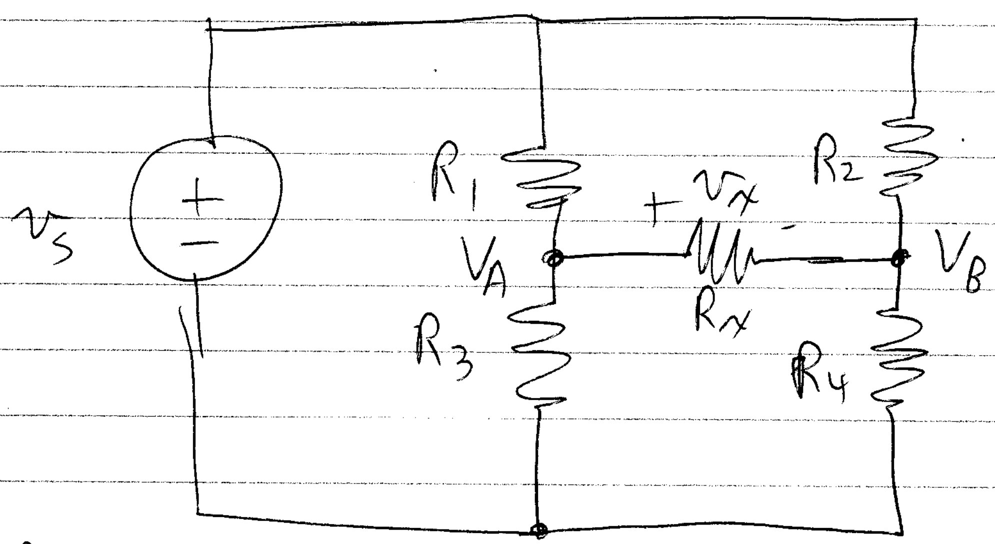
$$\frac{R_1+R_3}{R_3} = \frac{R_2+R_4}{R_4}$$

$$\frac{R_1}{R_3}+1 = \frac{R_2}{R_4}+1$$

$$\underline{\underline{\frac{R_1}{R_3} = \frac{R_2}{R_4}}}$$

This is often called a Wheatstone Bridge, and has taken on mystical stature. People immediately tense up when they hear ~~to~~ the name, but it really is very simple:

Another representation:



Do you agree this is the same?
If we remove R_x (erase) then we just have 2 voltage dividers, I can write

$$V_A = \frac{R_3}{R_1 + R_3} v_s \quad \text{and} \quad V_B = \frac{R_4}{R_2 + R_4} v_s$$

If these are to be equal, then

$$\cancel{V_A = \left(\frac{R_3}{R_1} + 1\right) v_s}$$

$$\frac{R_3}{R_1 + R_3} \cancel{v_s} = \frac{R_4}{R_2 + R_4} \cancel{v_s}$$

Take reciprocal: $\dfrac{R_1 + R_3}{R_3} = \dfrac{R_2 + R_4}{R_4}$

$$\frac{R_1}{R_3} \cancel{+1} = \frac{R_2}{R_4} \cancel{+1}$$

as before.

We can do this <u>because</u>, when $v_x = 0$, no current flows through R_x.