ECE 241 Signals: Homework 0

This homework is based on material that you should have covered in the pre-requisites for the course and its purpose is to provide you an opportunity to assess your preparedness and, if necessary, brush up on these pre-requisites.

1. RC **Circuit: Transient and Steady State Response** (*10 points*)

   Consider the RC circuit shown in Fig. 1. In this problem, we will consider the steady state and transient response of this circuit in a few different circumstances.

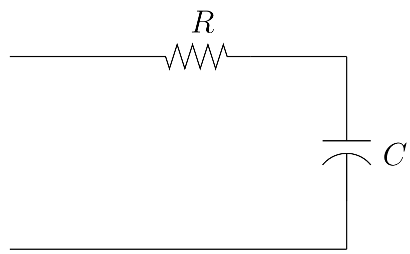

Figure 1: An RC circuit.

   (a) **Transient Response**

   Consider the RC circuit in the configuration shown in Fig. 2, where starting with the capacitor in a fully discharged state, at time $t = 0$, the switch is turned on connecting the circuit to a power supply with output voltage V_s.

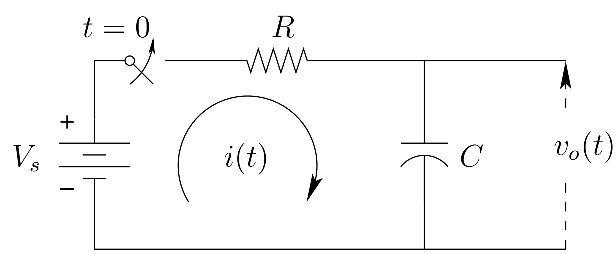

Figure 2: An RC circuit.

   i. Using Kirchhoff's voltage law and the $I - V$ characteristic for a capacitor, obtain a differential equation for the voltage $v_o(t)$ across the capacitor.

   ii. Solve the differential equation in the preceding part, to obtain an expression for the voltage $v_o(t)$ observed across the capacitor as a function of time.

   Hint: A linear differential equation in the standard form

$$\frac{dy}{dx} + P(x)y = Q(x). \tag{1}$$

   Equation 1 can be written as

$$\frac{d}{dx}\left(\mu y\right) = \mu Q, \tag{2}$$

where

$$\mu(x) = e^{\int P(x)dx} \tag{3}$$

is the integrating factor. We can see that the solution to (2) is

$$\mu y = \int \mu Q dx + c, \tag{4}$$

where c is the constant of integration.

(b) **Steady State Response** Now consider the RC circuit in the configuration shown in Fig. 3, where starting with the capacitor in a fully discharged state, at time $t = 0$, the switch is turned on connecting the circuit to a signal generator that generates the (voltage) signal

$$x(t) = \sin(2\pi f_c t) \tag{5}$$

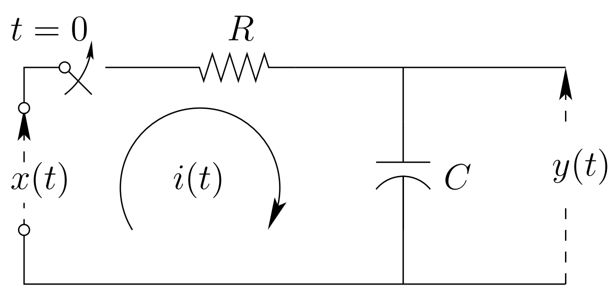

Figure 3: An RC circuit.

i. Using Kirchhoff's voltage law and the $I - V$ characteristic for a capacitor, obtain a differential equation for the voltage $y(t)$ across the capacitor (in terms of the input voltage $x(t)$ and the circuit parameters).

ii. For the case when the input signal is given by $x(t) = \sin(2\pi f_0 t + \theta)$, solve the differential equation obtained in the preceding part to obtain an expression for the output signal $y(t)$.
Hint: Use Euler's formula to obtain a representation for the signal $x(t)$ that allows you to simplify the solution of the simplification of the differential equation. It will *significantly simplify* your work.

iii. Simplify the answer you obtained in part 1(b)ii under the assumption that t is large in comparison to the time constant RC (specifically $t >> RC$) to obtain the steady state response $y_{ss}(t)$.

(c) Determine the steady state response of the circuit to the signal

$$x_2(t) = \sin(2\pi f_1 t) + \cos(2\pi f_2 t) \tag{6}$$

where f_1 and f_2 are two positive real numbers.
Hint: Use superposition to simplify your work. You do not have to solve differential equations again.

2. Problem 1.48 of text (*5 points*)

   Polar and cartesian representations of complex numbers

3. Problem 1.49 of text (*5 points*)

   Magnitude and phase for complex numbers

4. Problem 1.50 of text (*5 points*)

   Using Euler's formula

5. Problem 1.53 of text (*10 points*)

   Complex number relations

6. Problem 1.54 of text (*10 points*)

   Series summations

7. **Matlab Warm-up:**

   (a) Plot the function $\sin(2\pi t)$ over the domain $[0, 1]$. Clearly label the axes on your graph.

   (b) Now plot the function $f(t) = \sin(2\pi t) + 0.5 \sin^2(4\pi t)$ over the domain $[0, 1]$.

   (c) Program a bisection search function that determines a value of t such that $f(t) = \alpha$, where α is a real-value in the range of $f(t)$. Using your bisection search function, find all values of t such that $f(t) = 0.75$. To do this, you can use the plot of the function in Part 7b to identify, for each value of t such that $f(t) = 0.75$, a suitable pair of starting values for the bi-section search.

   Note: Your answers should be precise upto 5 decimal places. Describe, how you ensure this required level of precision in your bisection search.

   Important: Remember to label your plots clearly.