From now on you can use these general results, which can be obtained in several ways:

Given a complex number

$$\hat{A} = a + jb = \sqrt{a^2+b^2} \angle \tan^{-1}\left(\frac{b}{a}\right)$$

complex conjuga

$$\frac{1}{\hat{A}} = \frac{1}{a+jb} = \frac{a-jb}{(a^2+b^2)} = \frac{\hat{A}^*}{|A|^2} = \frac{1}{\sqrt{a^2+b^2}\angle \tan^{-1}\left(\frac{b}{a}\right)} = \frac{1\angle 0°}{\sqrt{}\angle \tan^{-}}$$

$$= \frac{\sqrt{a^2+b^2}\angle \tan^{-1}\left(\frac{-b}{a}\right)}{(a^2+b^2)}$$

$$= \frac{1}{\sqrt{a^2+b^2}} \angle -\tan^{-1}\left(\frac{b}{a}\right) \qquad = \frac{1}{\sqrt{a^2+b^2}} \angle -\tan^{-1}\left(\frac{b}{a}\right)$$

So, in the future, if you have

$$\frac{\hat{A}}{\tilde{B}} = \hat{A}\left(\frac{1}{\tilde{B}}\right) = \hat{A}\left(\frac{\tilde{B}^*}{|B|^2}\right)$$

$$= \frac{\hat{A}\tilde{B}^*}{|B|^2}$$

So,

$$\frac{3+4j}{4+3j} = \frac{(3+4j)(4-3j)}{(4^2+3^2)} = \frac{(3)(4)+(4)(3)+j(16-9)}{25}$$

$$= \frac{24+j7}{25} = \frac{\sqrt{24^2+7^2}\angle \tan^{-1}\left(\frac{7}{24}\right)}{25} = 1 \angle \tan^{-1}\left(\frac{7}{24}\right)$$

$$= 1 \angle 16.26°$$

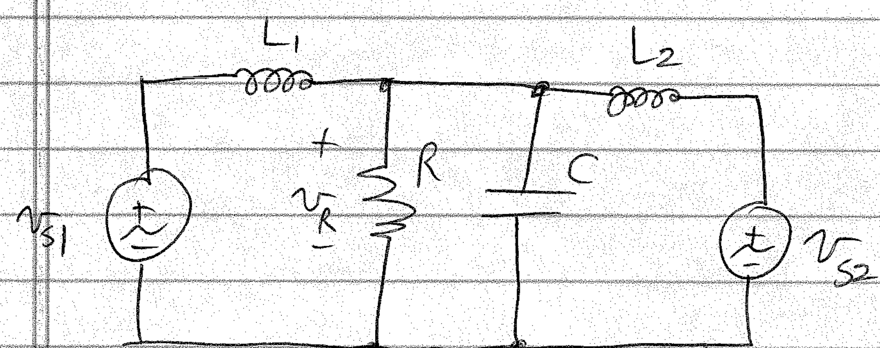
Superposition in Phasor Domain

Superposition always holds in time domain. ~~all the~~

" holds in phasor domain at <u>single frequency</u>.

If sources have different frequencies, analyse at each frequency, convert to time domain, <u>then</u> add up.

Example 8-15



Find $v_R(t)$ for $R = 20\,\Omega$, $L_1 = 2\,mH$, $L_2 = 6\,mH$, $C = 20\,\mu F$,

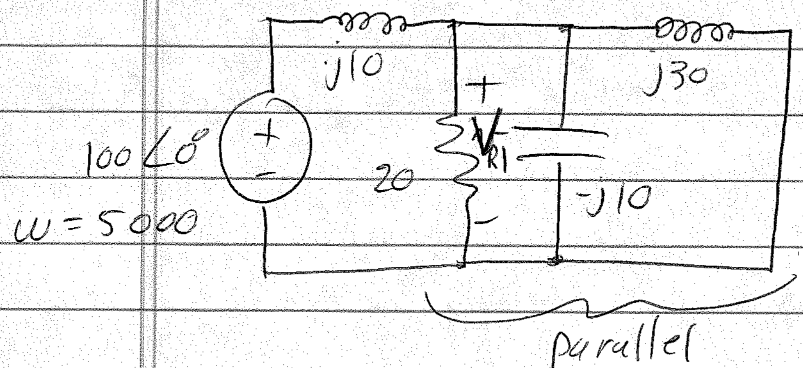
$v_{S_1} = 100\cos(5000t)\,V$ & $v_{S_2} = 120\cos(5000t + 30^\circ)\,V$.

(278)

Text solution:

$$j\omega L_1 = j(5000)(2mH) = j10\,\Omega$$
$$j\omega L_2 = j(5000)(6mH) = j30\,\Omega$$
$$\frac{1}{j\omega C} = -j\frac{1}{(5000)(20\mu F)} = -j\frac{1}{100m} = -j10\,\Omega$$

Turn off src 2:



$$\tilde{Z}_{eq} = \frac{1}{\frac{1}{20}+\frac{1}{-j10}+\frac{1}{j30}} = \cancel{\frac{1}{\frac{1}{20}+\frac{1}{j30}+\frac{1}{j10}}}$$

Common denom? j60

$$= \frac{1}{\frac{j3-6+2}{j60}} = \frac{j60}{-4+j3}\cdot\frac{-4-j3}{-4-j3}$$

$$= \frac{180-j240}{4^2+3^2} = \frac{180-j240}{16+9} = \frac{180-j240}{25} = \underline{7.2-j9.6}$$

$$\underline{\tilde{Z}_{eq} = 12\angle -53.13}$$

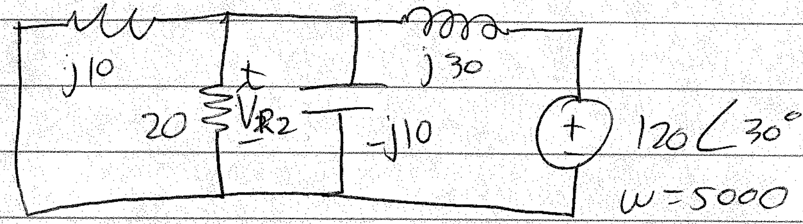
By Voltage Division: $\tilde{V}_{R1} = \frac{\tilde{Z}_{eq}}{j10+\tilde{Z}_{eq}}100\angle 0^\circ$

$$= \frac{7.2-j9.6}{7.2+j0.4}100\angle 0^\circ$$

$$= \frac{12\angle -53.13^\circ}{7.211\angle 3.18^\circ}100\angle 0^\circ = 166\angle -56.31^\circ$$

$\tilde{V}_{R_1} = 92.08 - j138 \text{ V}$

Turn on S_2 + S_1 off



parallel

$$\tilde{Z}_{eq} = \frac{1}{\frac{1}{20} + \frac{1}{j10} + \frac{1}{-j10}} = \frac{1}{\frac{1}{20} - j\frac{1}{10} + j\frac{1}{10}} = 20\Omega - j0$$

Voltage Division: $\tilde{V}_{R2} = \frac{20\Omega}{20\Omega + j30\Omega} V_s = \frac{20\angle 0°}{36.06\angle 56.31°} 120\angle 30°$

$$= (0.555\angle -56.31°)(120\angle 30°)$$

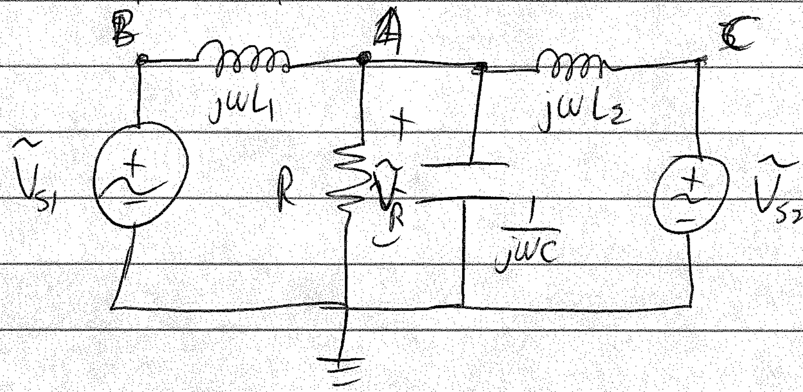
$$\tilde{V}_{R2} = 66.56\angle -26.31 = 59.66 - j29.50$$

$$\tilde{V}_R = \tilde{V}_{R_1} + \tilde{V}_{R_2} = \cancel{20}(92.08 - j138) + (59.66 - j29.50) \text{ V}$$

$$= 151.74 - j167.5 = 226\angle -47.8°$$

Alternate:

Write Nodal Analysis Eqns in Phasor domain w/ ω as variable:



$$\frac{\hat{V}_A-\hat{V}_{S_1}}{j\omega L_1}+\frac{\hat{V}_A}{R}+\frac{\hat{V}_A}{1/j\omega C}+\frac{\hat{V}_A-\tilde{V}_{S2}}{j\omega L_2}=0$$

$$\left(\frac{1}{j\omega L_1}+\frac{1}{R}+j\omega C+\frac{1}{j\omega L_2}\right)\tilde{V}_A=\frac{1}{j\omega L_1}\tilde{V}_{S1}+\frac{1}{j\omega L_2}\tilde{V}_{S2}$$

common denom $j\omega L_1 L_2 R$

$$\left(\frac{RL_2+j\omega L_1L_2+j\omega C(j\omega L_1L_2R)+L_1R}{j\omega L_1L_2R}\right)\tilde{V}_A=\frac{1}{j\omega}\left(\frac{1}{L_1}\tilde{V}_{S_1}+\frac{1}{L_2}\tilde{V}_{S2}\right)$$

$$\cancel{R(L_1+L_2)+j\omega L_1L_2}$$

$$\frac{R(L_1+L_2)-\omega^2L_1L_2RC+j\omega L_1L_2}{\cancel{j\omega} L_1L_2R}\tilde{V}_A=\frac{1}{\cancel{j\omega}}\left(\frac{1}{L_1}\tilde{V}_{S_1}+\frac{1}{L_2}\tilde{V}_{S2}\right)$$

$$\cancel{\frac{1}{\omega L_1L_2R}\left(-j\left[R(L_1+L_2)-\omega^2L_1L_2RC\right]+\omega L_1L_2\right)\tilde{V}_A=\frac{1}{j\omega}\left(\frac{1}{L_1}\tilde{V}_{S_1}+\frac{1}{L_2}\tilde{V}_{S2}\right)}$$

$$\frac{1}{L_1L_2R}\left[R(L_1+L_2)-\omega^2L_1L_2RC+j\omega L_1L_2\right]\tilde{V}_A=\left(\frac{1}{L_1}\tilde{V}_{S_1}+\frac{1}{L_2}\tilde{V}_{S2}\right)$$

$$\tilde{V}_A = \frac{L_1 L_2 R}{\left[R(L_1+L_2) - \omega^2 L_1 L_2 RC + j\omega L_1 L_2\right]} \left[\frac{1}{L_1}\tilde{V}_{S_1} + \frac{1}{L_2}\tilde{V}_{S_2}\right]$$

$$= \frac{L_1 L_2 R}{\sqrt{\left[R(L_1+L_2) - \omega^2 L_1 L_2 RC\right]^2 + (\omega L_1 L_2)^2}} \angle -\tan^{-1}\left(\frac{\omega L_1 L_2}{R(L_1+L_2) - \omega^2 L_1 L_2 RC}\right) \left[\frac{1}{L_1}\tilde{V}_{S_1} + \frac{1}{L_2}\tilde{V}_{S_2}\right]$$

$$= \tilde{K}\left[\frac{1}{L_1}\tilde{V}_{S_1} + \frac{1}{L_2}\tilde{V}_{S_2}\right]$$

Check values:

$$|\tilde{K}| = \frac{(2mH)(6mH)(20\Omega)}{\sqrt{\left[20(2mH+6mH) - (5000)^2(2mH)(6mH)(20)(20\mu F)\right]^2 + ((5000)(2mH)(6mH))^2}}$$

$$= \frac{240\times10^{-6}}{\sqrt{\left[160\times10^{-3} - 120\times10^{-3}\right]^2 + (60\times10^{-3})^2}} = \frac{240\times10^{-6}}{\sqrt{(40\times10^{-3})^2 + (60\times10^{-3})^2}}$$

$$= \frac{240\times10^{-6}}{72.11\times10^{-3}} = 3.328\times10^{-3}$$

$$\tilde{V}_A = 3.328\times10^{-3} \angle -\tan^{-1}\left(\frac{60\times10^{-3}}{40\times10^{-3}}\right) \left[\frac{1}{L_1}\tilde{V}_{S_1} + \frac{1}{L_2}\tilde{V}_{S_2}\right]$$

$$= 3.328\times10^{-3} \angle -56.31^\circ \left[\frac{1}{2mH} 100\angle 0^\circ + \frac{1}{6mH} 120\angle 30^\circ\right]$$

$$\left[50{,}000\angle 0^\circ + 20{,}000\angle 30^\circ\right]$$

$$\left[50{,}000 + j0 + 17{,}320 + j10{,}000\right]$$

$$\left[67{,}320 + j10{,}000\right]$$

$$\left[68{,}059\angle 8.45^\circ\right]$$

$$\tilde{V}_A = 226\angle -47.9^\circ \checkmark$$

Look back at

$$\tilde{V}_A = \frac{L_1 L_2 R}{\left[R(L_1+L_2) - \omega^2 L_1 L_2 RC + j\omega L_1 L_2\right]} \left[\frac{1}{L_1}\tilde{V}_{S1} + \frac{1}{L_2}\tilde{V}_{S2}\right]$$

If V_{S1} + V_{S2} had been at different frequencies, we could have evaluated this at each frequency (With V_s at the other frequency set to zero), converted each to the time domain, <u>then</u> added them up.