Introduction to Audio and Music Engineering

Lecture 22

Topics:

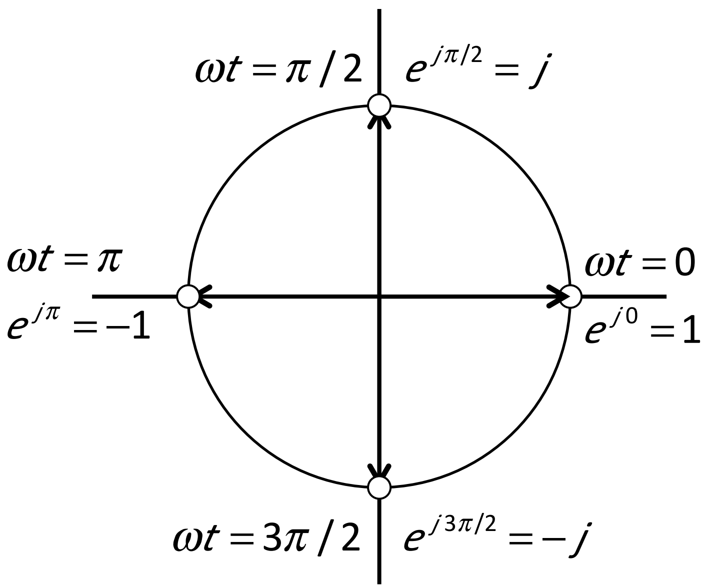
- A simple analog delay filter
- Inverse comb filter
- Filtering as interference
- Digital frequency
- The z-domain
- A simple digital filter

Manipulating digital sound

Delaying a phasor…

- We can decompose a complex sound into its sinusoidal components and consider delaying each one separately

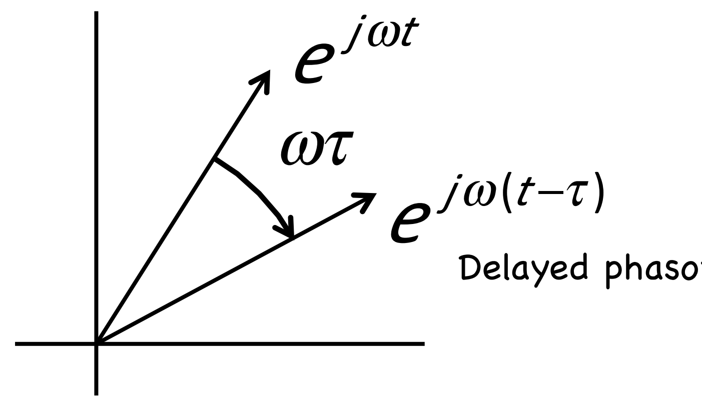
Unit amplitude phasor at frequency ω → $e^{j\omega t}$



We now delay the phasor by τ seconds.

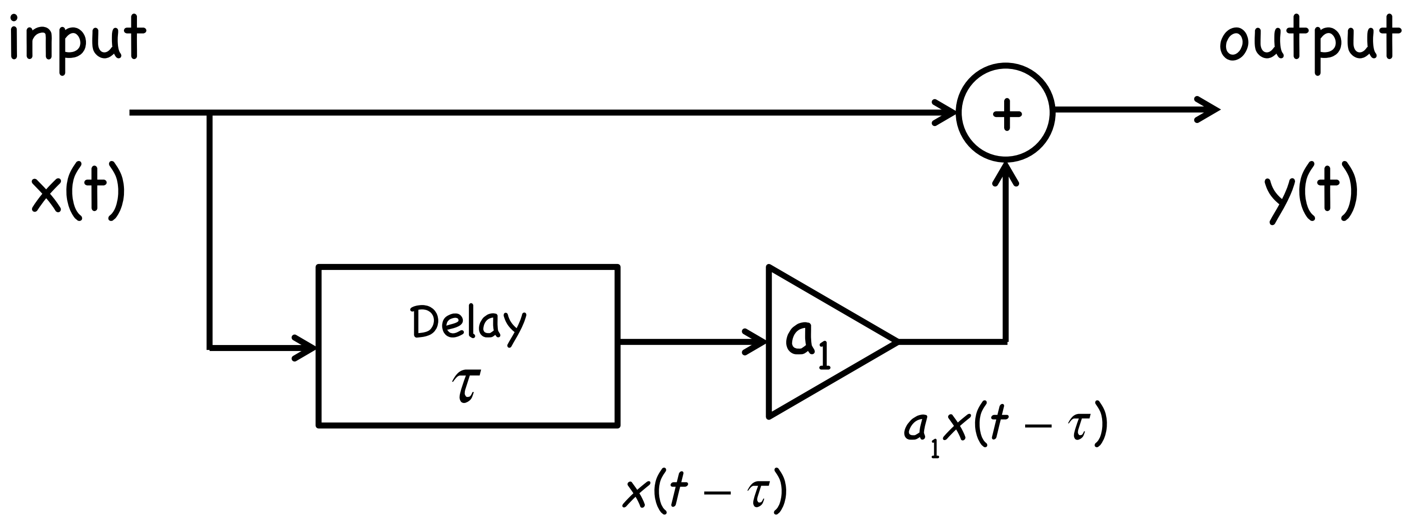
The delayed phasor is …

$$e^{j\omega(t-\tau)} = e^{-j\omega\tau} e^{j\omega t}$$



Delay is just a rotation of a phasor in the complex plane.

A first simple filter …



The filter function is $y(t) = x(t) + a_1 x(t-\tau)$

then for $x(t) = e^{j\omega t}$

$$y(t) = e^{j\omega t} + a_1 e^{j\omega(t-\tau)}$$

add a delayed version of original phasor

Factor the output expression: $y(t) = e^{j\omega t} + a_1 e^{j\omega(t-\tau)}$

$$y(t) = \underbrace{\left[1 + a_1 e^{-j\omega\tau}\right]}_{H(\omega)} e^{j\omega t}$$

Frequency response of the filter ...

note that it is complex and time independent.

$$H(\omega) = \left|H(\omega)\right| e^{j\theta(\omega)}$$

Phase response
(shifts the phase - rotation)

Magnitude response
(changes amplitude)

For the current filter ...

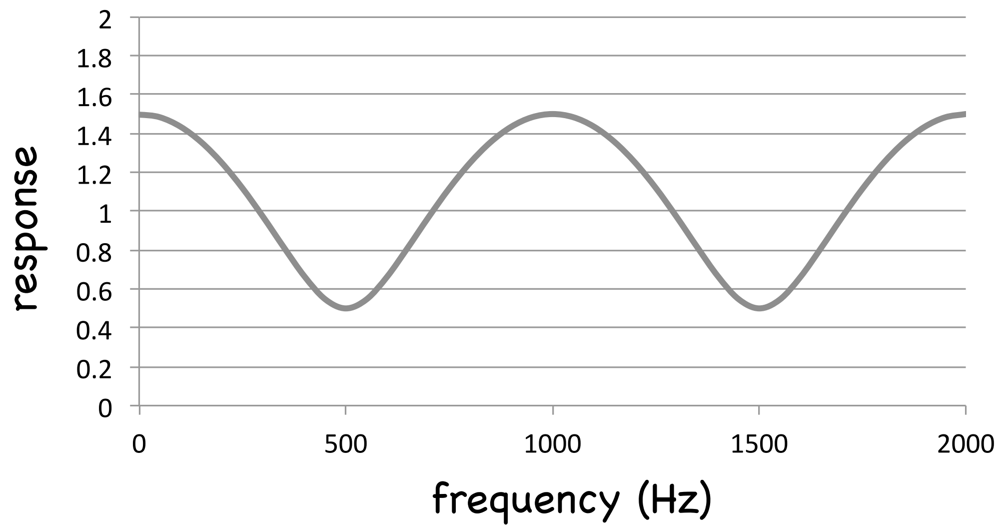
$$\left|H(\omega)\right| = \left|1 + a_1 e^{j\omega\tau}\right|$$

$$= \left[1 + a_1^2 + 2a_1 \cos\omega\tau\right]^{1/2}$$

Plot of the filter magnitude response … $\left|H(\omega)\right| = \left[1 + a_1^2 + 2a_1 \cos\omega\tau\right]^{1/2}$

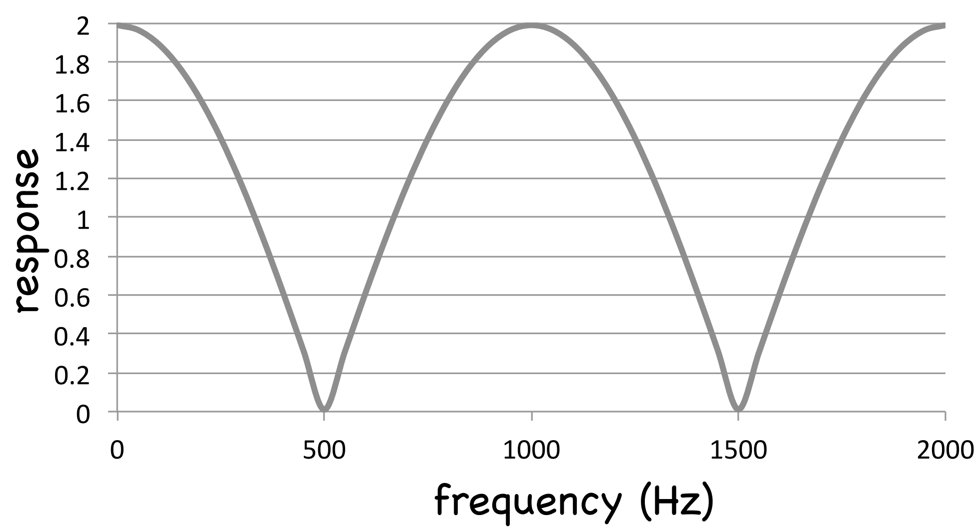
$\tau = 0.001\,\text{sec}$

$a_1 = 0.5$



$\tau = 0.001\,\text{sec}$

$a_1 = 0.99$

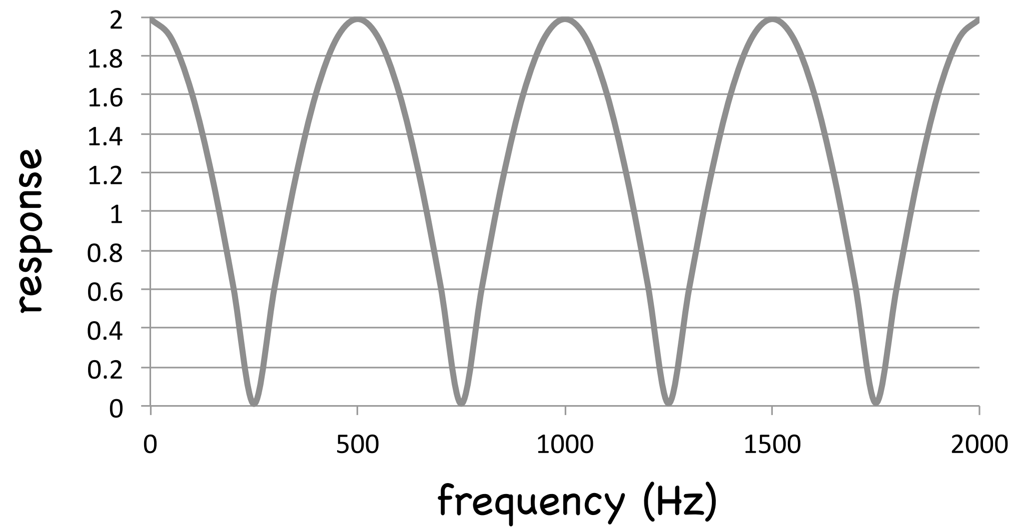


More examples ...

$$\left|H(\omega)\right| = \left[1 + a_1^2 + 2a_1 \cos \omega\tau\right]^{1/2}$$

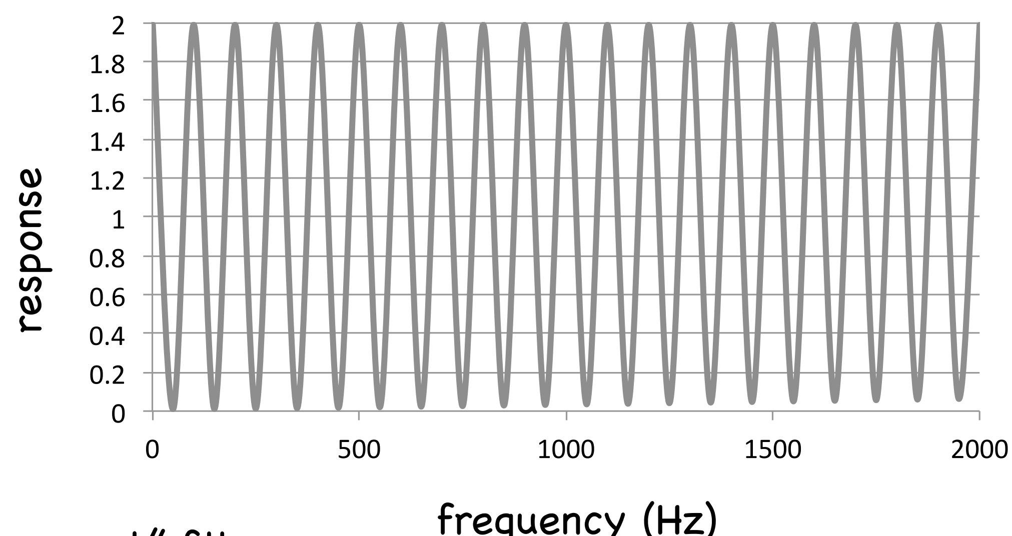
$\tau = 0.002 \sec$

$a_1 = 0.99$



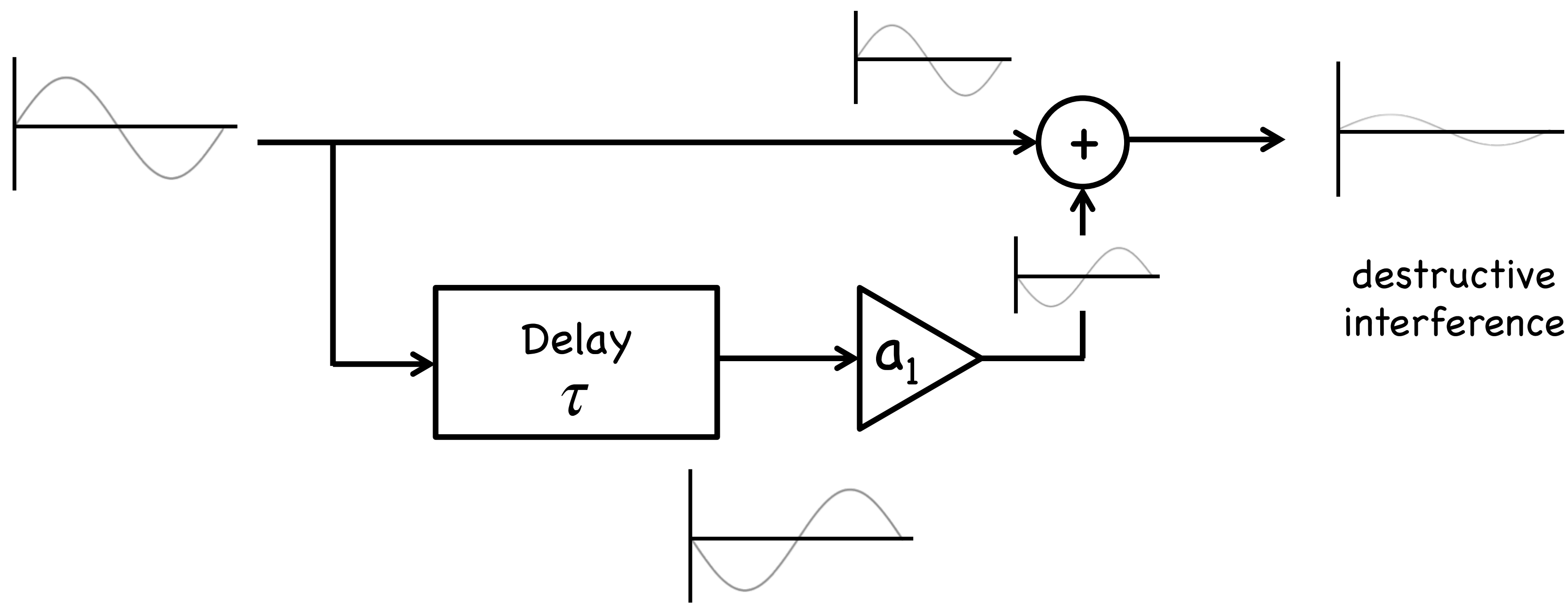
$\tau = 0.01 \sec$

$a_1 = 0.99$



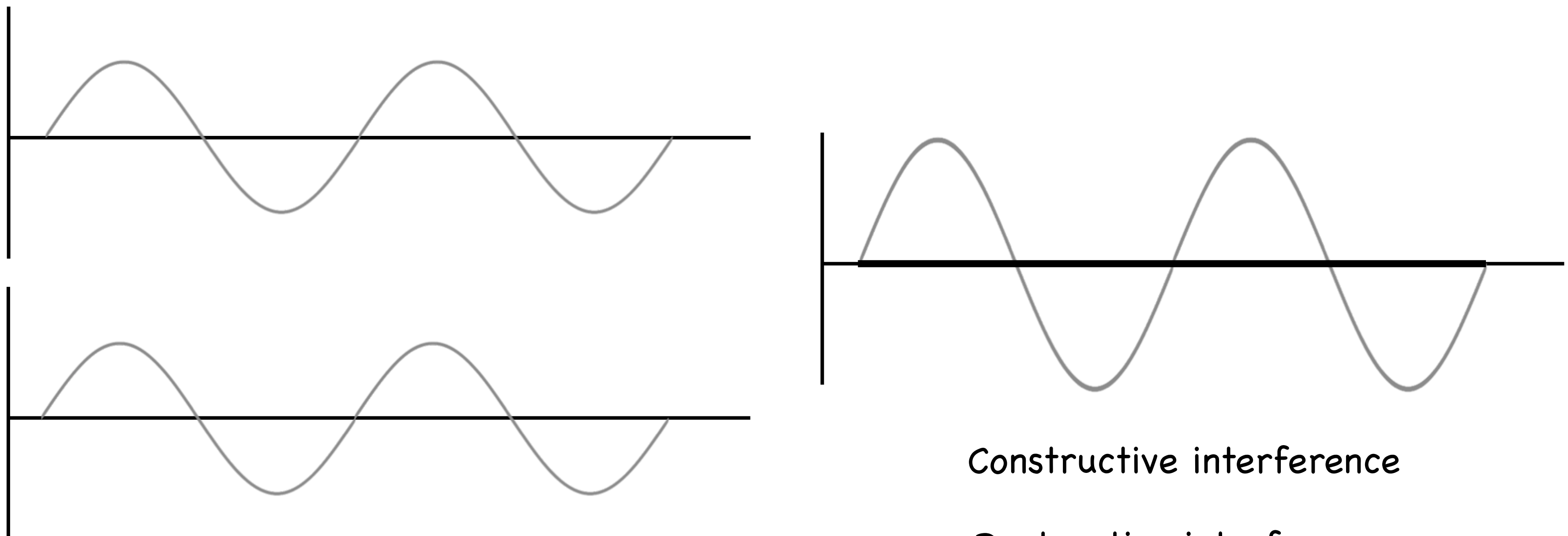
This is called an “inverse comb” filter.

What's happening?



It's simply constructive and destructive interference of the delayed phasor with the input phasor.

What's happening?

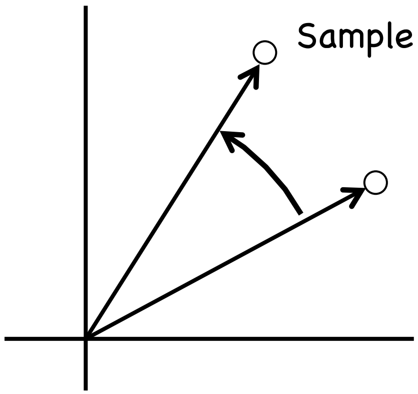


Constructive interference

Destructive interference

When the delay goes from 0 to one complete period of the phasor the interference goes from constructive to destructive and then back to constructive.

Digital frequency

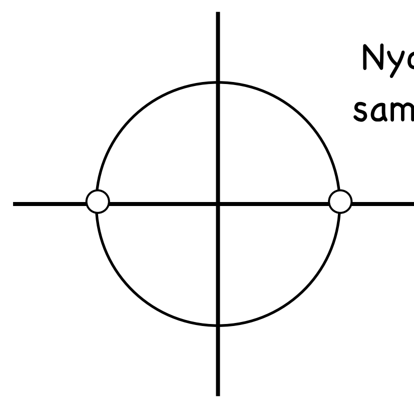


The convention is that the phasor rotates counterclockwise as time advances

Digital frequency of signal = phase advance of the signal in one sample time.

$$\omega = \frac{f}{R/2}\pi$$

$$0 \le \omega \le \pi$$ Nyquist frequency



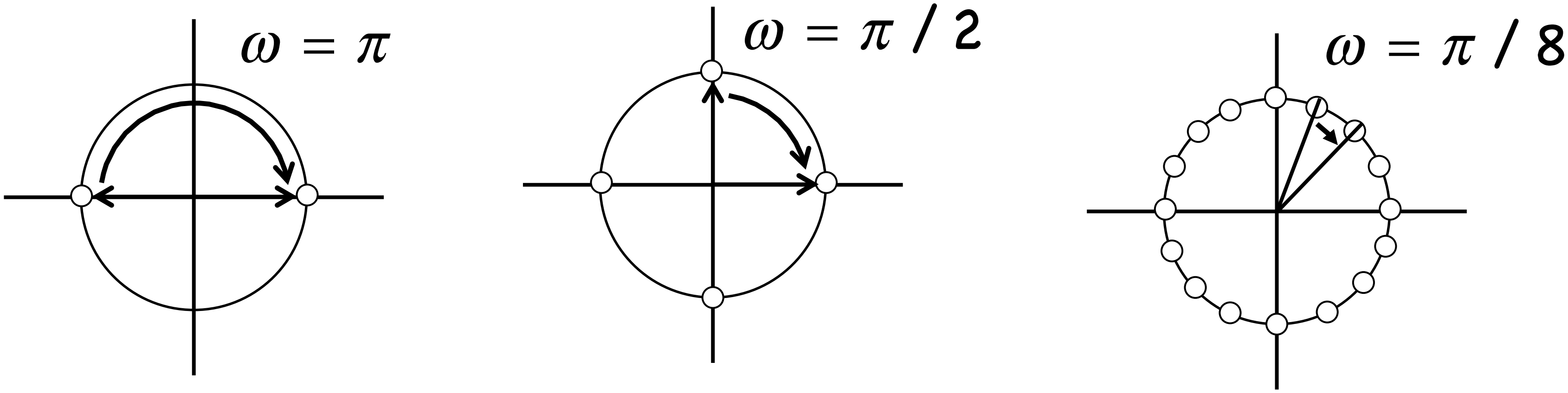
In the digital domain delays are always an integer number of samples.

So a one sample delay of a phasor at frequency ω is like multiplying by $e^{-j\omega}$

this is a clockwise rotation by an angle equal to the digital frequency, ω

Examples of digital delay

1 unit delay $\rightarrow e^{-j\omega}$



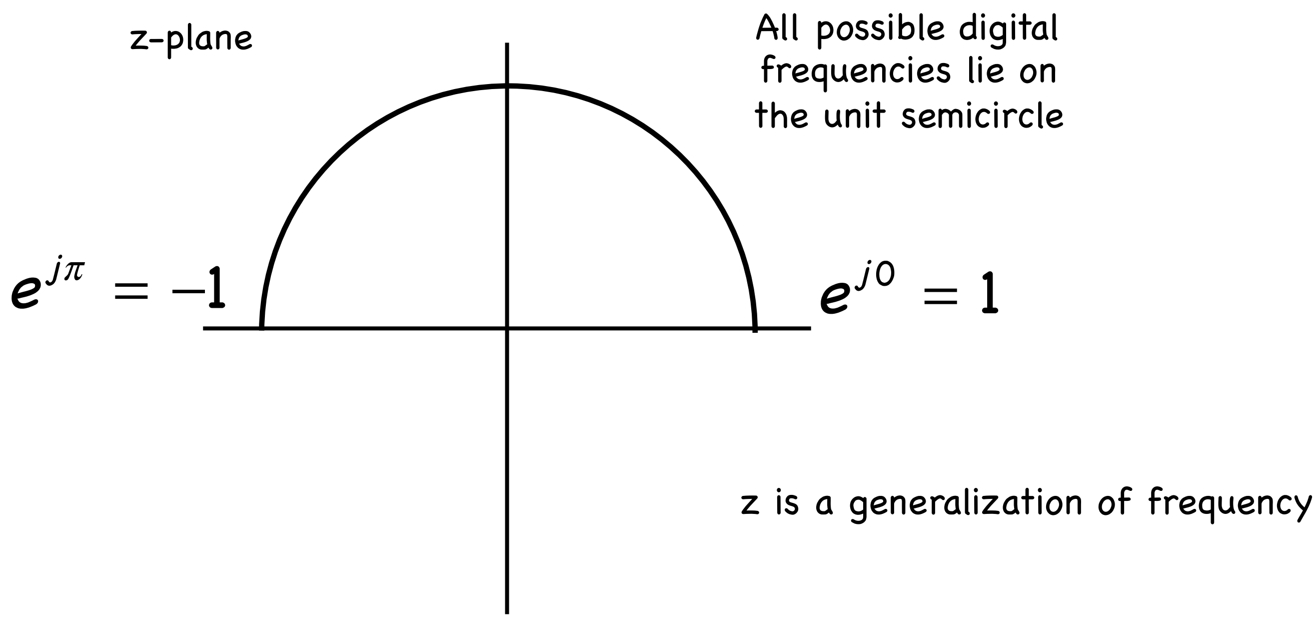
We tire of writing $e^{j\omega}$ all the time so we just use z.

$$e^{j\omega} \rightarrow z \qquad e^{-j\omega} \rightarrow z^{-1}$$

z^{-1} is the unit delay "operator"

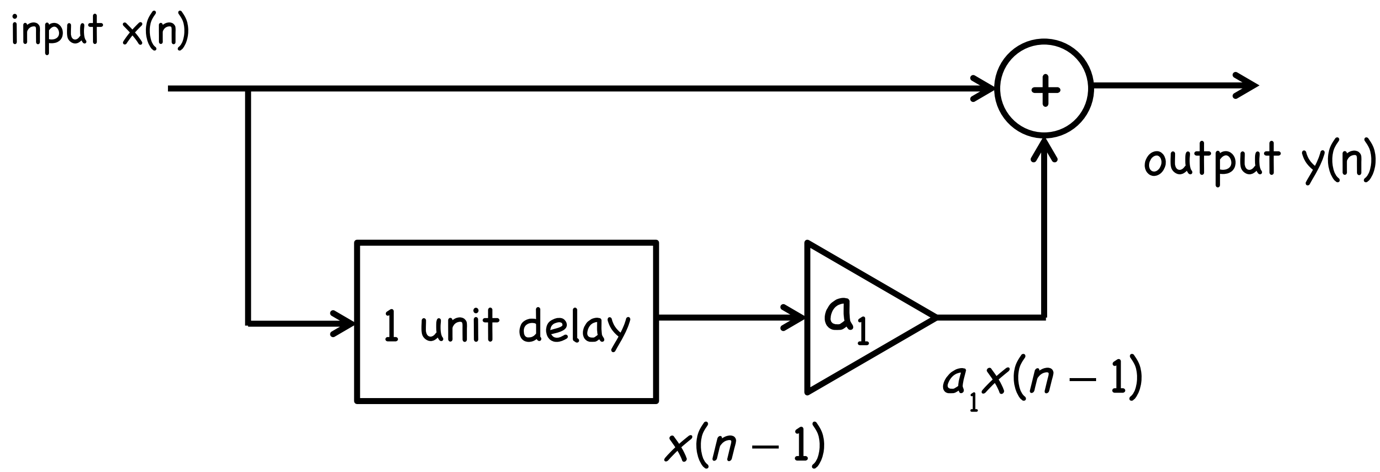
This isn't the entire story … there is an entire z-plane of complex frequencies.

The z-domain



... but there is much more to the z-plane as we shall soon see ...

Back to our first simple filter ... but now in the digital domain



The digital filter function is: $y(n) = x(n) + a_1 x(n-1)$

Transform to the z domain: $Y = X + a_1 z^{-1} X$

$$Y = \left(1 + a_1 z^{-1}\right) X \qquad Y = H(z)X \qquad H(z) = 1 + a_1 z^{-1}$$

H(z) tells us everything about the filter!