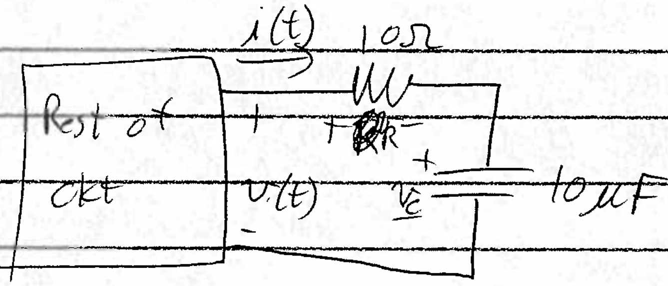
Everything we have done so far can be thought of as the limit as $\omega \to 0$ in what we are now going to do!

All of our analysis tools, Nodal Analysis, Mesh Analysis, Voltage + Current Division, Op Amps, ... can be done using Sinusoidal Steady State ~~and~~ and Phasor Notation.

Example: (8-5 of text)



Given that $i(t) = 4\cos(5000t)$ A, find $v(t)$

Solution: This is the Time Domain ckt. Convert to Phasor (or Frequency) Domain:

$\tilde{I} = 4\angle 0°$ A, $\omega = 5000$,

$\tilde{Z}_R = 10\Omega + j0$

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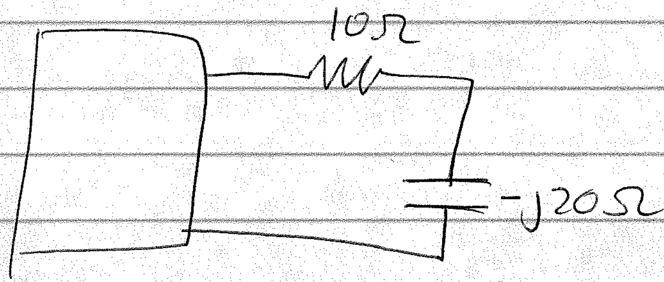
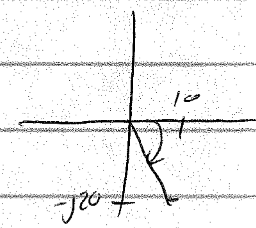
$$\tilde{Z}_C = \frac{1}{j\omega C} = \frac{1}{j(5000)(10\mu)} = -j\,\frac{1}{50\,m} = -j(0.02)(k)\,\Omega$$

$$\underline{= -j\,20\ \Omega}$$



$$\tilde{Z}_{Eq} = (10\,\Omega) + (-j20\,\Omega) = (10 - j20)\,\Omega$$



$$= 22.36\angle -63.43^\circ\ \Omega$$

$$\hat{V} = \tilde{Z}_{eq}\,\tilde{I} = (22.36\angle -63.43^\circ\,\Omega)(4\angle 0^\circ A)$$

$$\hat{V} = 89.44\angle -63.43^\circ\ V$$

$$\cancel{v(t) = 89.44 \cos(-63.43^\circ)}$$

$$v(t) = 89.44 \cos(5000t - 63.43^\circ)\ V$$

We can get $\tilde{v}_C$ or $\tilde{v}_R$ by voltage division

$$\tilde{V}_C = \frac{\tilde{Z}_C}{\tilde{Z}_R + \tilde{Z}_C}\tilde{V}$$

$$= \frac{-j20}{10-j20}\left(89.44\angle -63.43^\circ\ V\right)$$

$$= \frac{20\angle -90^\circ}{22.36\angle -63.43^\circ}\left(89.44\angle -63.43^\circ V\right)$$

$$= \left(0.894\angle -26.57\right)\left(89.44\angle -63.43^\circ V\right)$$

$$= \underline{\underline{80\angle -90^\circ\ V}} = \underline{\underline{-j\,80\ V}}$$

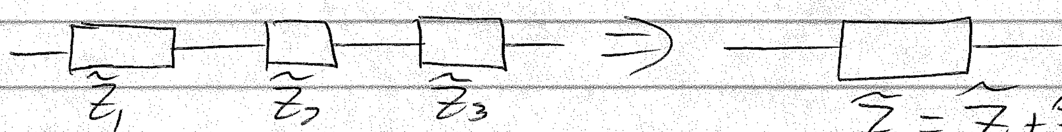
$$\tilde{V}_R = \frac{10\angle 0^\circ}{22.36\angle -63.43^\circ}\left(89.44\angle -63.43^\circ V\right)$$

$$= \frac{(10)(89.44)}{22.36}\angle\left(0^\circ \cancel{-63.43^\circ} - (\cancel{-63.43^\circ})\right)$$

$$\underline{\underline{\tilde{V}_R = 40\angle 0^\circ\ V = 40 + j0\ V}}$$

This gets us into Section 8-3, so you should read and do all exercises and examples through that.

As I have already stated, or at least alluded to, we can combine impedances just like resistors:



$\Rightarrow$ $\tilde{Z}_{eq} = \tilde{Z}_1 + \tilde{Z}_2 + \tilde{Z}_3$

Ex:

time R C L $\longrightarrow$ cannot combine

freq: R $\quad \frac{1}{j\omega C}$ $\quad j\omega L \rightarrow \tilde{Z}_{eq} = R + \frac{1}{j\omega C} + j\omega L$

$$= R + j\left(-\frac{1}{\omega C} + \omega L\right)$$

$$= R + j\left(\frac{\omega^2 LC}{\omega C} - \frac{1}{\omega C}\right)$$

$$\tilde{Z}_{eq} = R + j\left(\frac{\omega^2 LC - 1}{\omega C}\right)$$

If we were told, $R = 1k\Omega$, $C = 1\mu F$, $L = 1mH$, $\omega = 1000$,

$$\text{then } \tilde{Z}_{eq} = 1k\Omega + j\left(\frac{(10^3)^2(1\mu F \cdot 1mH) - 1}{(10^3)(10^{-6})}\right)\Omega$$

$$= 1k\Omega + j\left(\frac{10^6(10^{-9}) - 1}{10^{-3}}\right)\Omega$$

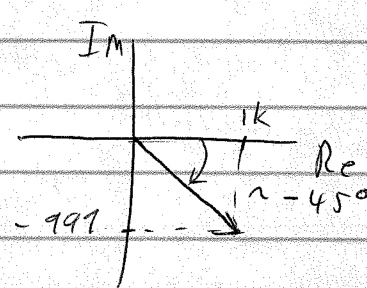
$$= 1k\Omega + j\left(\frac{10^{-3} - 1}{10^{-3}}\right)\Omega$$

Well, $10^{-3}-1=-0.999$, so

$$\tilde{Z}_{eq}=1k\Omega+j\left(\frac{-0.999}{10^{-3}}\right)\Omega$$

$$=1k\Omega-j\,999\,\Omega$$

$\uparrow$ $Im(\tilde{Z}_{eq})<0$, so capacitive



We could also have gone back to

$$\tilde{Z}_{eq}=\tilde{Z}_R+\tilde{Z}_c+\tilde{Z}_L$$

$$=1k\Omega+\left(-j\frac{1}{(10^3)(10^{-6})}\Omega\right)+j(10^3)(10^{-3})\,\Omega$$

Impedance of capacitor

$$=1k\Omega+(-j\,1k\Omega)+j(1)\,\Omega$$

impedance of inductor

$$=1k\Omega-j(1000-1)\Omega=1k\Omega-j\,999\,\Omega$$

$$|\tilde{Z}_c|>|\tilde{Z}_L| \Rightarrow \text{capacitive}$$

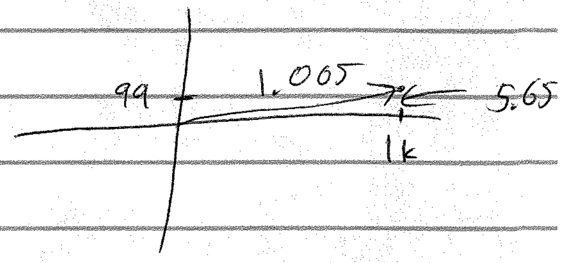
Try decreasing $|Z_c|$ by increasing C, + Increase Z_L by $\uparrow L$
Use $C=1000\,\mu F$ and $L=100\,mH$,

$$Z_{eq}=1k\Omega+j\left(-\frac{1}{(10^3)(10^{-3})}+(10^3)(10^{-1})\right)\Omega$$

$$=1k\Omega+j(-1+10^2)\,\Omega=1k\Omega+j\,99\,\Omega$$

This is now inductive, since $\mathrm{Im}(\tilde{Z}_{eq}) > 0$.

$$\tilde{Z}_{eq} = 1.005 \angle 5.65^\circ$$



Go back to the general expression:

$$\tilde{Z}_{eq} = R + j\left(\frac{\omega^2 LC - 1}{\omega C}\right) = R_{eq} + j\, X_{eq}$$

The circuit is on resonance at the frequency where the reactance goes to zero, that is, when $\frac{\omega_0^2 LC - 1}{\omega_0 C} = 0$

$$\text{or } \omega_0^2 LC - 1 = 0$$

$$\text{or } \omega_0^2 LC = 1$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Look familiar??

Series RLC:

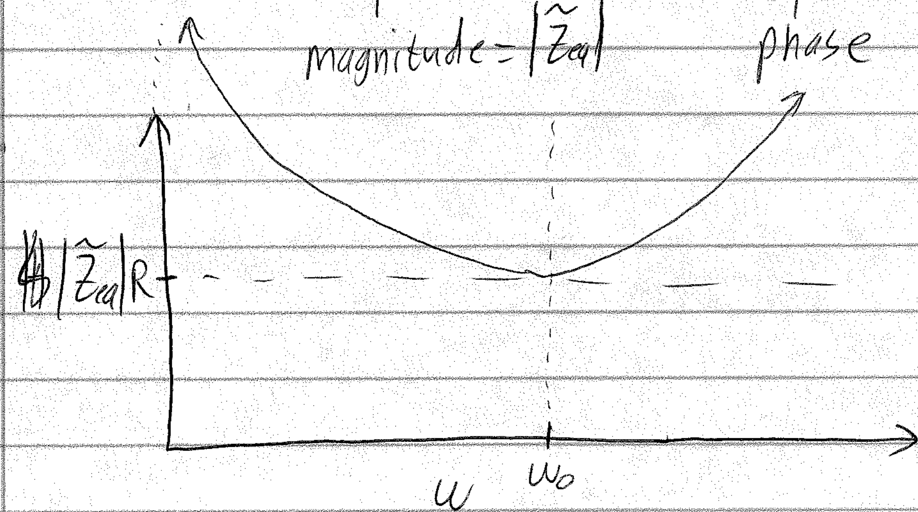
$$\tilde{Z}_{eq} = R + j\left(\frac{\omega^2 LC - 1}{\omega C}\right)$$

$$= \sqrt{R^2 + \left(\frac{\omega^2 LC - 1}{\omega C}\right)^2} \angle \tan^{-1}\left(\frac{\omega^2 LC - 1}{\omega C R}\right)$$

$$= \sqrt{\frac{(\omega RC)^2 + (\omega^2 LC - 1)^2}{(\omega C)^2}} \angle \tan^{-1}\left(\frac{\omega^2 LC - 1}{\omega RC}\right)$$

Get used to certain combinations, like $\omega RC + \omega^2 LC$.

$$\tilde{Z}_{eq} = \underbrace{\frac{\sqrt{(\omega RC)^2 + (\omega^2 LC - 1)^2}}{\omega C}}_{\text{magnitude} = |\tilde{Z}_{eq}|} \angle \underbrace{\tan^{-1}\left(\frac{\omega^2 LC - 1}{\omega RC}\right)}_{\text{phase}}$$

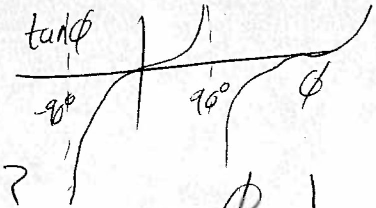


$$\lim_{\omega \to 0} |\tilde{Z}_{eq}| = \frac{\sqrt{0 + (0-1)^2}}{0} = \infty$$

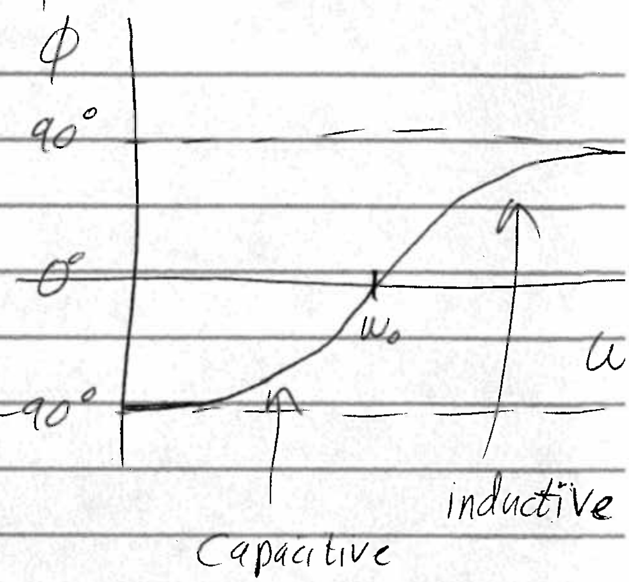
$$\lim_{\omega \to \infty} |\tilde{Z}_{eq}| = \frac{\sqrt{\infty^2 + (\infty^2)^2}}{\infty} = \frac{\infty^2}{\infty} = \infty$$

Can show that minimum occurs at $\omega_0 = \frac{1}{\sqrt{LC}}$

or $|\tilde{Z}_{eq}| = \frac{\sqrt{(\omega RC)^2}}{\omega C} = \frac{\omega RC}{\omega C} = R$



What about phase?



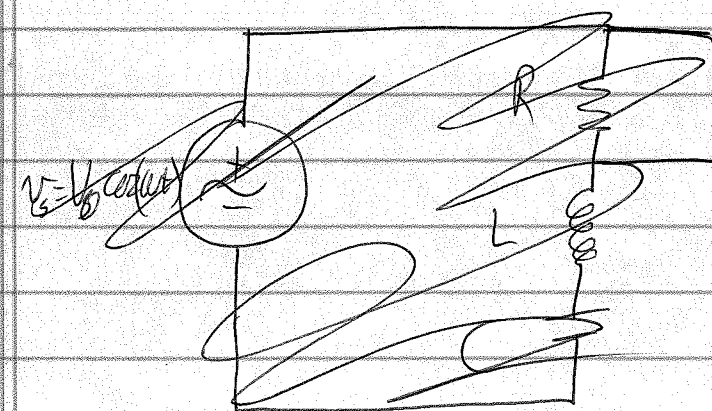
$$\phi = \tan^{-1}\left(\frac{\omega^2 LC - 1}{\omega RC}\right)$$

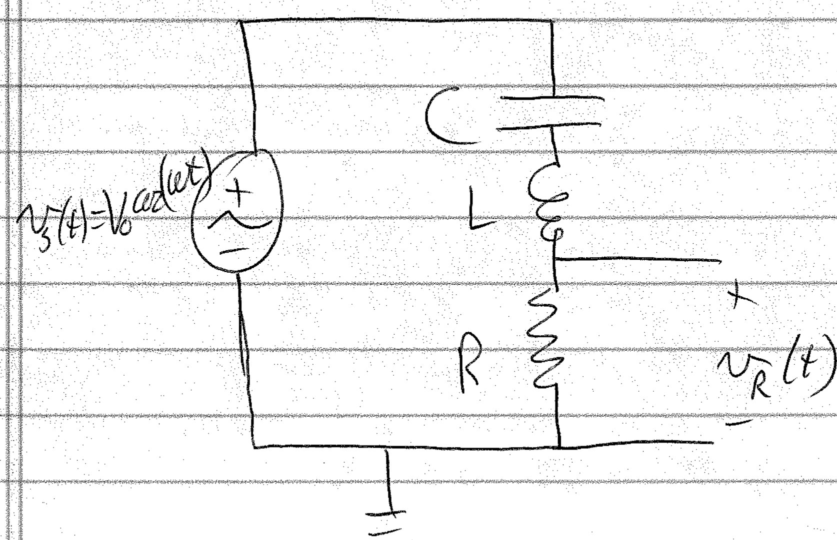
$$\lim_{\omega \to 0} \phi = \tan^{-1}\left(\frac{0^2 - 1}{0}\right)$$
$$= \tan^{-1}(-\infty) = -90^\circ$$

$$\lim_{\omega \to \infty} \phi = \tan^{-1}\left(\frac{\infty^2 - 1}{\infty}\right)$$
$$= \tan^{-1}(\infty) = 90^\circ$$

$$\omega = \omega_0 : \phi = \tan^{-1}\left(\frac{1 - 1}{\omega_0 RC}\right)$$
$$= \tan^{-1}(0) = 0^\circ$$

What if I asked you for the ~~current through this~~ voltage across the resistor, $v_R(t)$, if a voltage source $v_s(t) = V_0 \cos(\omega t)$ was applied to it:

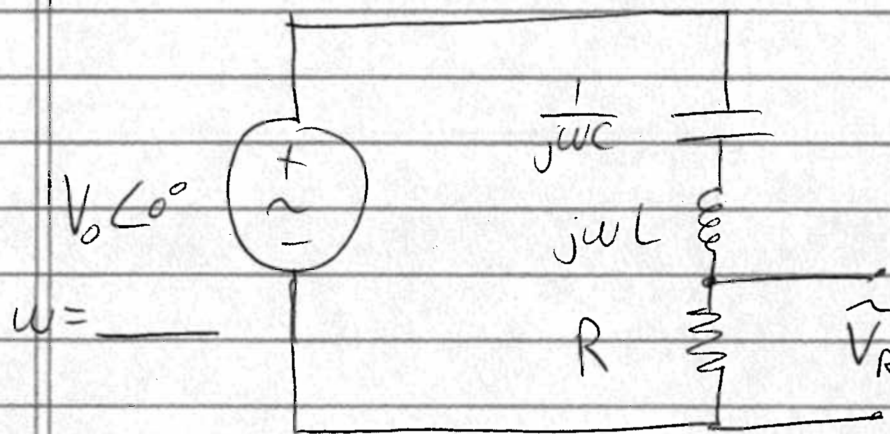




We could use $\tilde{Z}_{eq}$ from before, find $\tilde{I} = \frac{\tilde{V}_s}{\tilde{Z}_{eq}}$, then multiply by $\tilde{Z}_R = R$ to get $\tilde{V}_R = V_R \angle \phi_R$, then

$$v_R(t) = V_R \cos(\omega t + \phi_R)$$

Or, starting from scratch, we could convert to phasor (or frequency) domain:



Use Voltage Division:

$$\tilde{V}_R = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} V_0\angle 0^\circ$$

$$= \frac{R}{\frac{j\omega RC + (-\omega^2 LC) + 1}{j\omega C}} V_0\angle 0^\circ$$

$$= \frac{j\omega RC}{(1-\omega^2 LC) + j\omega RC} V_0\angle 0^\circ$$

$$= j\omega RC\left(\frac{1}{(1-\omega^2 LC) + j\omega RC}\right) V_0\angle 0^\circ$$

Recall: $\frac{1}{a+jb} = \frac{a-jb}{a^2+b^2}$

$$\tilde{V}_R = j\omega RC\left(\frac{(1-\omega^2LC)-j\omega RC}{(1-\omega^2LC)^2+(\omega RC)^2}\right) V_0\angle 0^\circ$$

~~multipl~~ move the j inside ( ), pull denom. out:

$$\tilde{V}_R = \frac{\omega RC}{(1-\omega^2LC)^2+(\omega RC)^2}\left(j(1-\omega^2LC)-(-1)\omega RC\right) V_0\angle 0^\circ$$

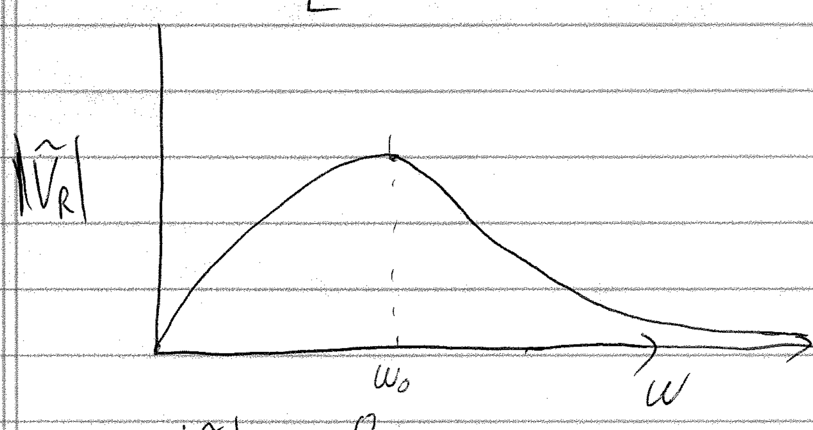
$$= \frac{\omega RC}{(1-\omega^2LC)^2+(\omega RC)^2}\left((\omega RC)+j(1-\omega^2LC)\right) V_0\angle 0^\circ$$

to multiply these we need both in polar form:

$$\tilde{V}_R = \frac{\omega RC}{(1-\omega^2LC)^2+(\omega RC)^2}\left[\sqrt{(\omega RC)^2+(1-\omega^2LC)^2}\angle \tan^{-1}\left(\frac{1-\omega^2LC}{\omega RC}\right)\right] V_0\angle 0^\circ$$

$$\frac{\sqrt{A}}{A} \quad + \quad \frac{\sqrt{A}}{A} = \frac{1}{\sqrt{A}}$$

$$\tilde{V}_R = \left[\frac{\omega RC}{\sqrt{(1-\omega^2LC)^2+(\omega RC)^2}}\angle \tan^{-1}\left(\frac{1-\omega^2LC}{\omega RC}\right)\right] V_0\angle 0^\circ$$



$\omega \to 0: |\tilde{V}_R| = \frac{0}{\sqrt{0+1}} = 0$ Peak at $\omega = \omega_0$

$\omega \to \infty: |\tilde{V}_R| = \frac{\infty}{\sqrt{(\infty^2)^2+\infty^2}} = \frac{\infty}{\infty^2+\infty} = 0$