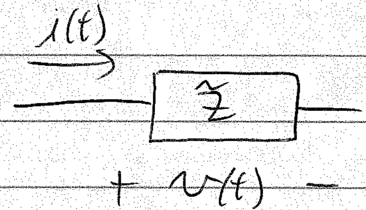
Now, let's ask about the <u>power dissipated</u> in an impedance. We must do the power calculation in the <u>Time</u> domain, because it is not linear:



$p(t) = v(t)\, i(t)$

Given $v(t) = V_0 \cos(\omega t)$

$\tilde{Z} = Z_0 \angle \theta$

Phasor Domain

$\tilde{V} = V_0 \angle 0^\circ$

$\tilde{Z} = Z_0 \angle \theta$

$\tilde{I} = \frac{\tilde{V}}{\tilde{Z}} = \frac{V_0 \angle 0^\circ}{Z_0 \angle \theta} = \frac{V_0}{Z_0} \angle -\theta$

$\longrightarrow \quad i(t) = \frac{V_0}{Z_0} \cos(\omega t - \theta)$

$$p(t) = V_0 \cos(\omega t) \left(\frac{V_0}{Z_0} \cos(\omega t - \theta) \right)$$

$$= \frac{V_0^2}{Z_0} \cos(\omega t) \cos(\omega t - \theta)$$

$$= \frac{V_0^2}{Z_0} \cos(\omega t) \left[\cos(\omega t) \cos(\theta) + \sin(\omega t) \sin(\theta) \right]$$

$$= \frac{V_0^2}{Z_0} \left[\cos^2(\omega t) \cos(\theta) + \cos(\omega t) \sin(\omega t) \sin(\theta) \right]$$

Recall:

$$\cos(2\omega t) = \cos(\omega t + \omega t)$$
$$= \cos(\omega t)\cos(\omega t) - \sin(\omega t)\sin(\omega t)$$
$$= \cos^2(\omega t) - \sin^2(\omega t)$$
$$= \cos^2(\omega t) - \left[1 - \cos^2(\omega t)\right] = 2\cos^2(\omega t) - 1$$

~~or $\cos^2(\omega t) = \cos(2\omega t) + \sin^2(\omega t)$~~

or $\cos^2(\omega t) = \frac{1}{2}\left(\cos(2\omega t) + 1\right)$

And $\sin(2\omega t) = \sin(\omega t + \omega t)$

$$= \sin(\omega t)\cos(\omega t) + \cos(\omega t)\sin(\omega t)$$
$$= 2\sin(\omega t)\cos(\omega t)$$

So $p(t) = \frac{V_0^2}{Z_0}\left[\frac{1}{2}\left(\cos(2\omega t) + 1\right)\cos(\theta) + \frac{1}{2}\sin(2\omega t)\sin(\theta)\right]$

$$= \frac{V_0^2}{Z_0}\left[\frac{1}{2}\cos\theta + \frac{1}{2}\left(\cos(2\omega t)\cos(\theta) + \sin(2\omega t)\sin(\theta)\right)\right]$$

$$= \frac{V_0^2}{Z_0}\left[\frac{1}{2}\cos(\theta) + \frac{1}{2}\left(\cos(2\omega t - \theta)\right)\right]$$

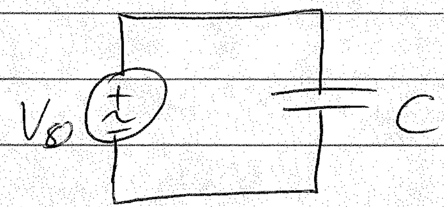
$$\underline{p(t) = \frac{1}{2}\frac{V_0^2}{Z_0}\left[\cos(\theta) + \cos(2\omega t - \theta)\right]}$$

First, assume $\theta = 0$, that is, $\tilde{Z}$ is a resistor: $Z_0 = R$

$$p_R(t) = \frac{1}{2}\frac{V_0^2}{R}\left[1 + \cos(2\omega t)\right]$$

$$p(t) = \frac{1}{2}\frac{V_0^2}{Z_0}\left[\cos(\theta) + \cos(2\omega t - \theta)\right]$$

If we have a capacitor only,

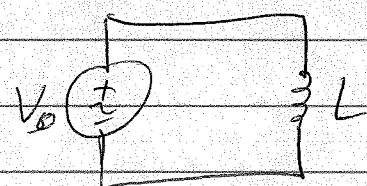


then $Z_c = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90°$, $\theta = -90°$ and $\cos(\theta) = 0$.

$$p_C(t) = \frac{1}{2}\frac{V_0^2}{Z_0}\left[\cos(2\omega t + 90°)\right]$$

$$p_c(t) = \frac{1}{2}\,\omega C V_0^2\left[\cos(2\omega t + 90°)\right]$$

If an inductor:



$Z_L = j\omega L = \omega L\angle 90°$

$\cos(90°) = 0$

$$p_L(t) = \frac{1}{2}\frac{V_0^2}{\omega L}\left[\cos(2\omega t - 90°)\right]$$

The instantaneous power is interesting, but what matters more is the Time Average Power,

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \text{Average Power}, \text{ where } T = \frac{1}{f} = 1 \text{ period of } \cos(\omega t)$$

$$\cancel{P_R = \frac{1}{T}\int_0^T \frac{1}{2}V_R I_R \cos(2\omega t)\,dt = 0}$$

Look at P for each component:

$$P_R = \frac{1}{T}\int_0^T \frac{1}{2}V_R I_R\left(1 + \cos(2\omega t)\right)dt$$

$$= \frac{1}{T}\left(\frac{1}{2}V_R I_R\right)\left[\int_0^T dt + \int_0^T \cos(2\omega t)\,dt\right]$$

$$P_R = \frac{1}{T}\left(\frac{1}{2}V_R I_R\right)\left[T + 0\right]$$

$$= \frac{1}{2}V_R I_R = \frac{1}{2}\frac{V_0^2}{R} = \frac{1}{2}RI_R^2$$

<u>Note</u>: ~~If I~~ Do not forget the $\frac{1}{2}$!!

Note: If I distribute the 2 into $V_R + I_R$,

and call them V_{eff} or $V_{RMS} = \frac{V_R}{\sqrt{2}}$

and I_{eff} or $I_{RMS} = \frac{I_R}{\sqrt{2}}$

then $\underline{P_R = V_{RMS} I_{RMS} = \frac{V_{RMS}^2}{R} = I_{RMS}^2 R}$ just like DC.

RMS is a way to get a positive number:
(Root Mean Square)

$A\cos(\omega t)$ Avg $= 0$!

$A^2\cos^2(\omega t)$ Avg $= \frac{1}{2}A^2$

$\sqrt{A^2\cos^2(\omega t)}$ Root MS $= \frac{1}{\sqrt{2}}A$

Others: $P_c = \frac{1}{T}\int_0^T \frac{1}{2}\omega C V_c^2 \cos(2\omega t - 90°)\,dt$

$$= \frac{1}{T}\frac{1}{2}\frac{2\pi}{T} C V_c^2 \underbrace{\int_0^T \cos(2\omega t - 90°)\,dt}_{=0}$$

$$\underline{\underline{P_c = 0}}$$

And. $P_L = \frac{1}{T}\frac{1}{2\omega L} V_L^2 \underbrace{\int_0^T \cos(2\omega t + 90°)\,dt}_{=0}$

$$\underline{\underline{P_L = 0}}$$

Inductors and Capacitors <u>store</u> some energy during $\frac{1}{2}$ the cycle, then <u>return</u> it to the circuit during the other half of the cycle, but do not dissipate any energy.

For a general impedance $\underset{= Z_0 \angle \theta}{\tilde{Z}}$, let $\frac{V_0}{Z_0} = I_0$, then

$$p(t) = \frac{1}{2} V_0 I_0 \left[\cos(\theta) + \cos(2\omega t - \theta)\right]$$

and $P = \frac{1}{2} V_0 I_0 \left[\frac{1}{T}\int_0^T \cos(\theta)\,dt + \frac{1}{T}\int_0^T \cos(2\omega t - \theta)\,dt\right]$

$$= \frac{1}{2} V_0 I_0 \left[\cos(\theta) + 0\right]$$

$$= \frac{1}{2} V_0 I_0 \cos(\theta) = V_{RMS} I_{RMS} \cos\theta$$