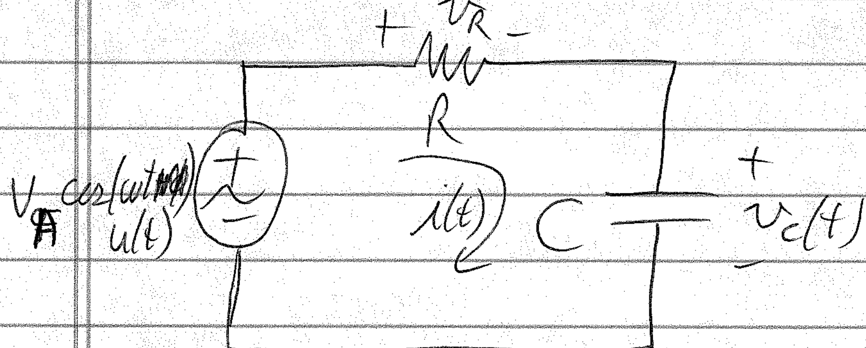
I will quickly touch on 1st order ckt responses to Sinusoidal Inputs, you should read the section 7.4 on this, and on the response to Exponential Inputs.

If a sinusoidal source is applied at $t=0$,




Find $v_c$.

~~We can write $\cos(\omega t \pm \phi) = a\cos(\omega t) + b\sin(\omega t)$~~

We are given (or can figure out) that $v_c(0) = V_0$

~~where $\tan\phi = \frac{b}{a}$~~

~~$\phi = \tan^{-1}\left(\frac{b}{a}\right)$~~

~~We need to find~~

~~Once again, $v_c(t)$ = Natural Response + Forced response~~

~~and the DE is $\frac{dv_c}{dt} + \frac{1}{RC} v_c = \frac{1}{RC} V_F(a\cos\omega t + b\sin\omega t)$~~

Once again, $v_c(t)$ = Natural Response + Forced Response

$$= Ke^{-t/RC} + V_F \cos(\omega t + \phi)$$

We can write: $V_F \cos(\omega t + \phi) = a\cos\omega t + b\sin\omega t$

where $\tan\phi = \frac{-b}{a}$ or $\phi = \tan^{-1}\left(\frac{-b}{a}\right)$

$+ V_F = \sqrt{a^2 + b^2}$

~~So $v_F$~~

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We will need to find $a + b$. ~~$+V_A$~~

Put $v_F$ into the DE:

$$RC\,\frac{d}{dt}(a\cos\omega t + b\sin\omega t) + (a\cos\omega t + b\sin\omega t) = V_A\cos\omega t$$

$$RC(-a\omega\sin\omega t + b\omega\cos\omega t) + (a\cos\omega t + b\sin\omega t) = V_A\cos\omega t$$

$$(b\omega RC + a)\cos\omega t + (-RC\omega a + b)\sin\omega t = V_A\cos\omega t\,(+\,0\sin\omega$$

Can only be true if coeff's of $\cos\omega t$ and coeff's of $\sin\omega t$ are equal:

$$\omega RCb + a = V_A \quad \text{and} \quad -\omega RCa + b = 0$$

$$b = \omega RCa$$

$$(\omega RC)(\omega RCa) + a = V_A$$

$$a(1 + (\omega RC)^2) = V_A$$

$$a = \frac{V_A}{(1 + (\omega RC)^2)} \longrightarrow b = \frac{\omega RC}{1 + (\omega RC)^2}V_A$$

So $v_c(t) = Ke^{-t/RC} + \frac{1}{1 + (\omega RC)^2}V_A\cos\omega t + \frac{\omega RC}{1 + (\omega RC)^2}V_A\sin\omega t$

I.C.: $v_c(0) = K + \frac{1}{1 + (\omega RC)^2}V_A(1) + 0 = V_0$

or $K = V_0 - \frac{1}{1 + (\omega RC)^2}V_A$

So:

$$v_c(t) = \left[V_0 - \frac{1}{1 + (\omega RC)^2}V_A\right]e^{-t/RC} + \frac{V_A}{1 + (\omega RC)^2}\left[\cos\omega t + \omega RC\sin\omega t\right]$$

$$v_c(t) = \underbrace{\left[V_0 - \frac{1}{1+(\omega RC)^2} V_A\right] e^{-t/RC}}_{\text{natural response}} + \underbrace{\frac{V_A}{1+(\omega RC)^2}\left[\sqrt{1+(\omega RC)^2}\cos(\omega t + \phi)\right]}_{\text{forced response.}}$$

where $\phi = \tan^{-1}\left(-\frac{\omega RC}{1}\right) = -\tan^{-1}(\omega RC)$

natural response ↑ decays to zero

forced response. ↑ continues forever,

1.) SAME FREQUENCY as input ($\omega$)

2.) Different Amplitude + Phase

3.) Proportional to input

$$v_c(t) = \left[V_0 - \frac{1}{1+(\omega RC)^2} V_A\right] e^{-t/RC} + \underbrace{\frac{V_A}{\sqrt{1+(\omega RC)^2}}\cos(\omega t + \phi)}_{\text{"sinusoidal steady state"}}, \quad \phi = \tan^{-1}(-\omega RC)$$

"sinusoidal steady state" $t \gg 0$, all natural responses have died out.

ALSO "AC response"

The step response we already spent time on is the ~~AC~~ "Zero Frequency Response": $\omega = 0$

$$v_c(t) = \left[V_0 - 1\,V_A\right] e^{-t/RC} + \frac{V_A}{1}(1)$$

$$= V_A + \left[V_0 - V_A\right] e^{-t/RC} \checkmark$$