Linearity

Linear - has inputs that produce outputs.

Output is Proportional to Input

$$Out = (G) \, Input$$ (Homogeneity

$$f(x) = Gx$$ or Proportionality)

$$\cancel{Out(Sum) = K_1 In_1 + K_2 In_2 + K_3 In_3 + }$$

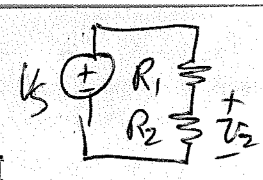
$$f(Kx) = K f(x)$$

Also Output(Sum of Inputs) = Sum of Outputs due to each Input

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

(Additivity or Superposition)

Proportionality only applies to currents and voltages, NOT TO POWER which is non-linear.

Examples in text: 

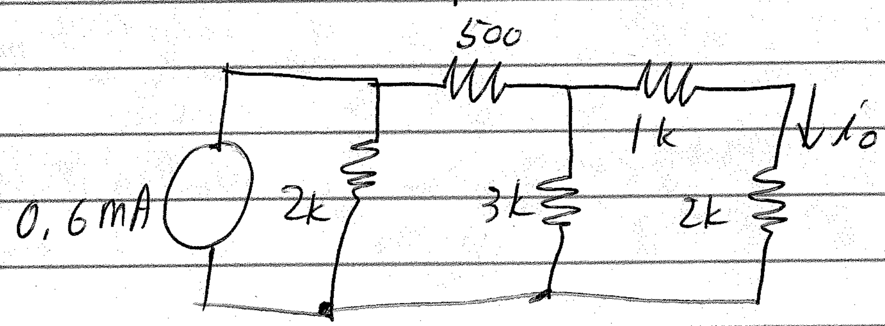
$v_2 = \frac{R_2}{R_1+R_2}(v_S)$ in text

$G = \frac{R_2}{R_1+R_2} (= K)$

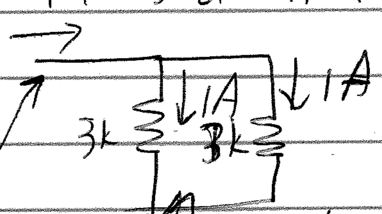
There are many ways of using these concepts in Circuit Analysis, one is the Unit Output Method, in which you ask "What input must be applied to get 1 (V or A or mV or mA, etc) for the output?"

Exercise 3-24:

Use Unit Output Method to find i_o in this:



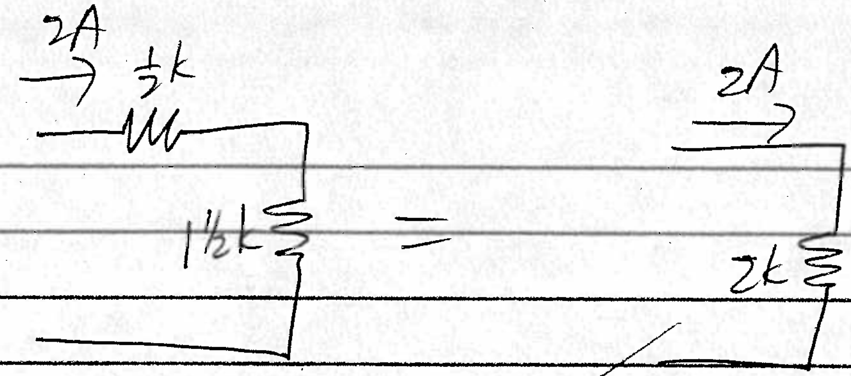
If 1A flows down in 2k, it also flows thru 1k,

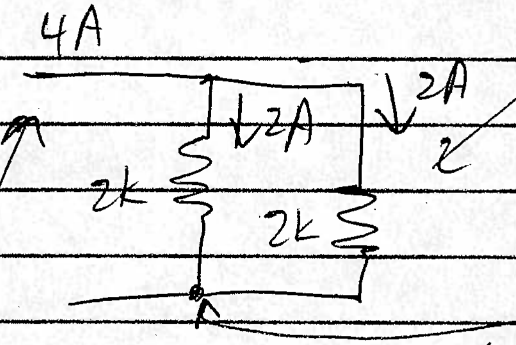


symmetry (or current div.) $\Rightarrow$ must be 1A in other 3k

So this must be 2A

Req = 3k // 3k = 1.5k



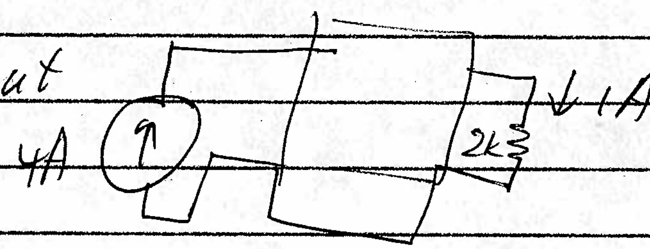


Again, symmetry $\Rightarrow$ 2A in other 2k

or a sum of 4A here

So, to get 1A at output we would

need 4A at input



So $1A = K(4A)$

$$K = \frac{1A\ (\text{output})}{4A\ (\text{input})} = \frac{1}{4}$$

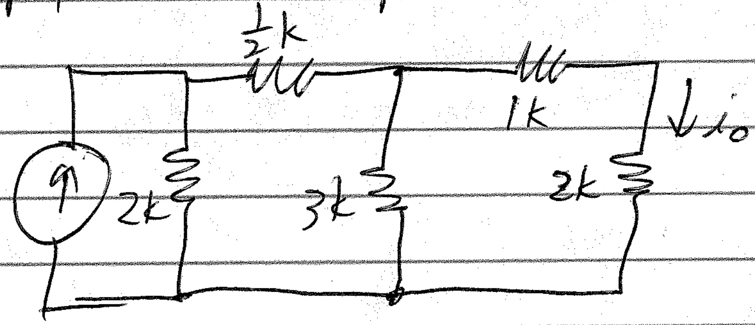
So $$i_o(\underset{\text{input}}{0.6\,mA}) = \frac{1}{4}(0.6\,mA) = 0.15\,mA$$

This is usually useful in highly symmetric and relatively simple ckts.

You can also use a "Unit Input Method", not in text, that goes "If a unit input is applied, what will the output be?" then

$$K = \frac{\text{output}}{1}$$

Apply to same problem:



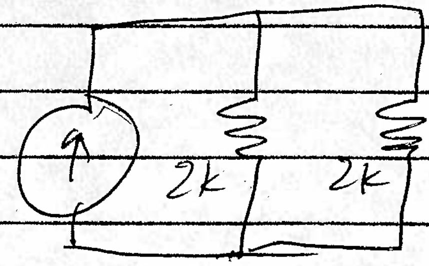
Go to far side + reduce:

$$1k + 2k = 3k$$

$$3k // 3k = 1.5k$$

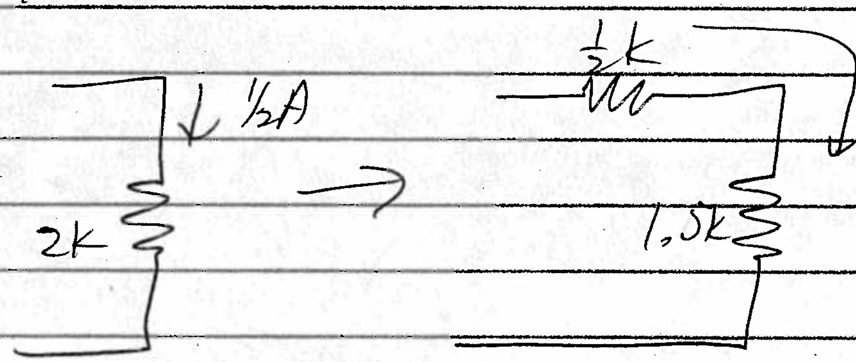
$$1.5k + 0.5k = 2k$$

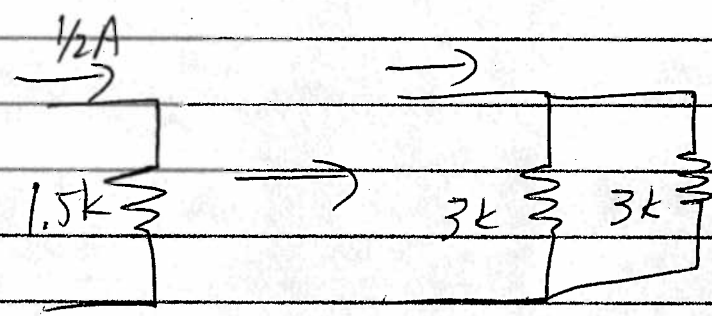
Now ask:



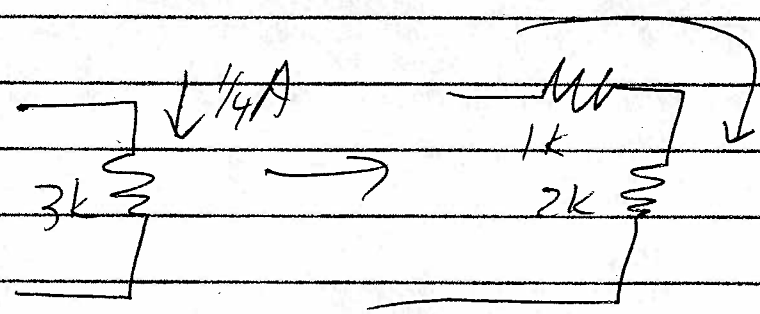
$\downarrow i ? = \frac{1}{2} A$ by symmetry

Back out





$\downarrow i = ?$ $\frac{1}{4} A$ by symmetry



$\frac{1}{4} A = i_o$

So 1A in produces $\frac{1}{4}$A out, $K = \frac{1/4 A}{1 A} = \frac{1}{4}$

So a 0.6 mA input produces $i_o = \frac{1}{4}(0.6 mA) = \underline{0.15 mA}$

Additivity (superposition)

$$y(x_1, x_2, x_3, \ldots) = K_1 x_1 + K_2 x_2 + \ldots$$

Output due to many inputs = Sum of outputs due to each source with all other sources turned off.