Puzzler

An airplane flies in a straight line from airport A to airport B, then back again in a straight line from B to A. It travels with a constant engine speed and there is no wind. Will its travel time for the same round trip be greater, less, or the same if throughout both flights wind blows from A to B. Assume that the engine speed is always the same.

Solution: It takes longer with the wind

Example: A → B = 150 miles, v_{plane}= 50 mph, v_{wind} = 25 mph

No wind

A → B	:	150/50 = 3 hrs
B → A	:	150/50 = 3 hrs
Total	:	6 hrs

With wind @ 25 mph

A → B	:	150/75 = 2 hrs
B → A	:	150/25 = 6 hrs
Total	:	8 hrs

Introduction to Audio and Music Engineering

Lecture 9

- Acoustic resonances in tubes
- Waveform and timbre
- Acoustic resonances in 3-d
- Room modes
- Reverberation

Acoustic Modes

Acoustic waves obey the same wave equation as a string - just change the variables.

1-d

$$\frac{d^2 p(x,t)}{dt^2} = c^2 \frac{d^2 p(x,t)}{dx^2}$$

Boundary conditions:

open end → p = 0

closed end → p = maximum

Solutions of 1-d Acoustic wave equation

$$p(x,t) = \cos(n\omega_0 t)\left[\sin(n\pi\frac{x}{L}) \text{ or } \cos(n\pi\frac{x}{L})\right]$$

Oscillation in time

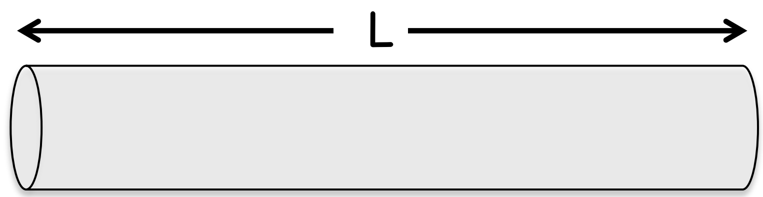
Both sine and cosine satisfy the wave equation.

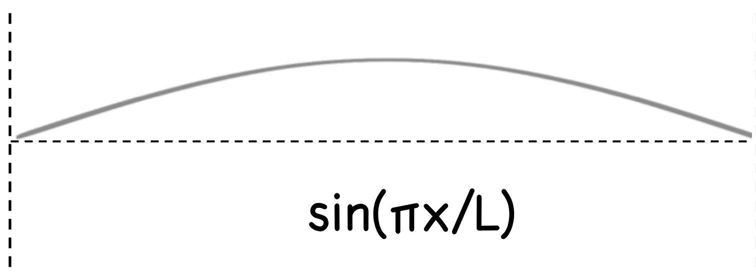
How do we know which solution to choose?

Choose the one that satisfies the boundary conditions.

flute

Pressure → 0 at open end





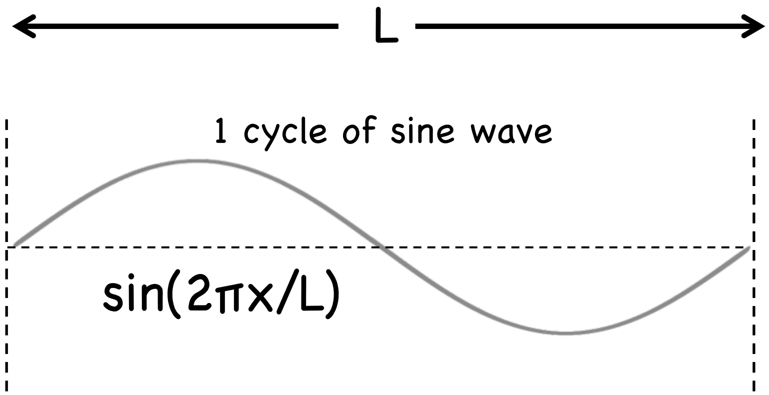
n = 1

One-half of a sine wave

$$f_1 = \frac{c}{2L}$$

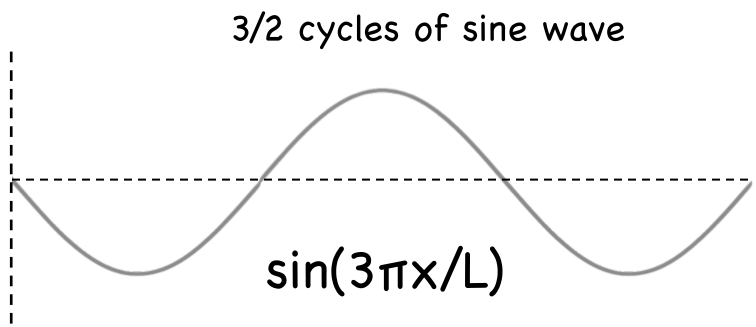
Same as fixed-fixed string!

Higher modes



n = 2

$$f_2 = 2\frac{c}{2L} = \frac{c}{L} = 2f_1$$



n = 3

$$f_3 = 3\frac{c}{2L} = \frac{3}{2}\frac{c}{L} = 3f_1$$

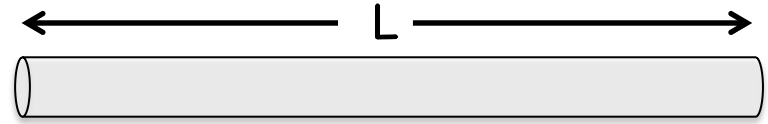
Modes of open-open tube are multiples of one-half of a sine wave.

Mode frequencies of open-open tube are the same as those for a fixed-fixed string.

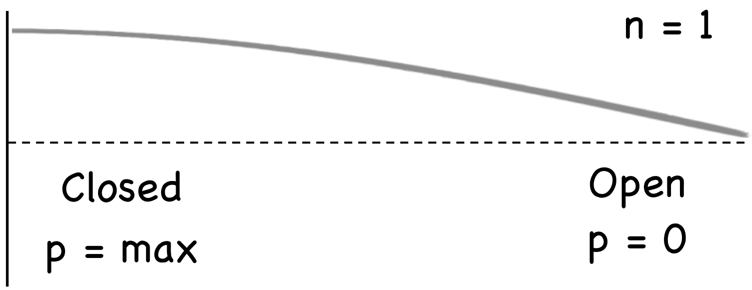
Closed-Open Boundary Condition

Clarinet

Closed



Open

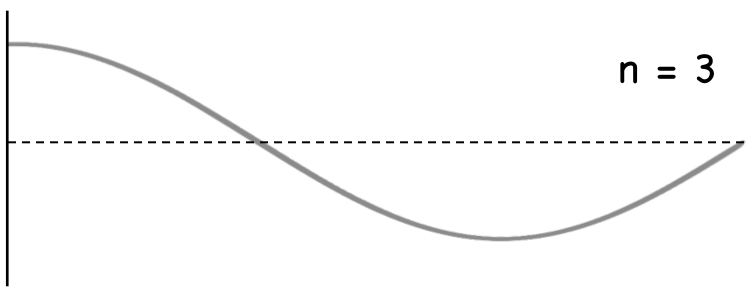


1/4 cycle of cosine

$$L = \frac{\lambda}{4}$$

$$\lambda_1 = 4L$$

$$f\lambda = c \quad \text{so} \quad f_1 = \frac{c}{\lambda_1} = \frac{c}{4L}$$

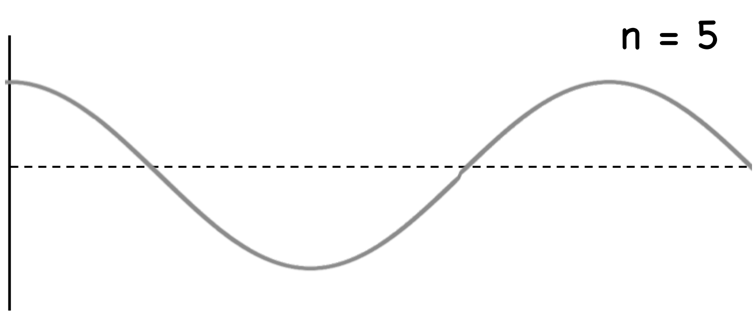


3/4 cycle of cosine

$$L = \frac{3}{4}\lambda$$

$$\lambda_3 = \frac{4L}{3}$$

$$f_3 = \frac{c}{\lambda_3} = 3\frac{c}{4L} = 3f_1$$



5/4 cycle of cosine

$$L = \frac{5}{4}\lambda$$

$$\lambda_5 = \frac{4L}{5}$$

$$f_5 = \frac{c}{\lambda_5} = 5\frac{c}{4L} = 5f_1$$

Summary

L

open-open

L ≈ 66 cm

$$f_1 = \frac{c}{2L}$$

C4

261.6 Hz

$$f_n = n\frac{c}{2L}$$

n = 1,2,3 ...

All harmonics

closed-open

L ≈ 60 cm

$$f_1 = \frac{c}{4L}$$

D3
"concert"

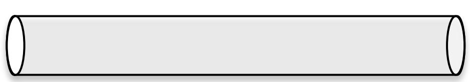
146.8 Hz

$$f_n = (2n-1)\frac{c}{4L}$$

n = 1,2,3 ...

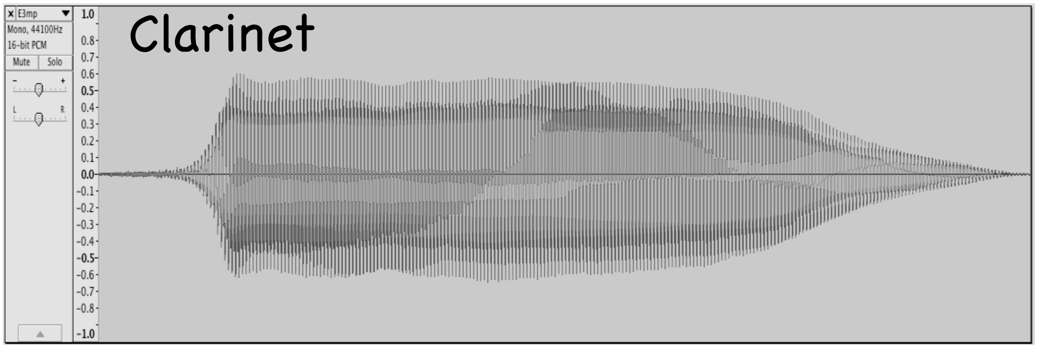
Only odd harmonics

closed-closed

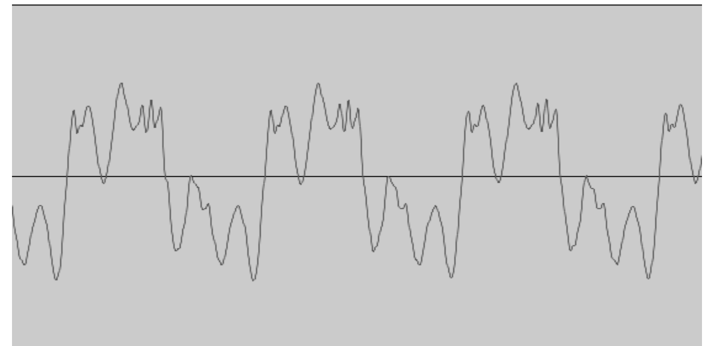
Closed  Closed

Waveform and timbre

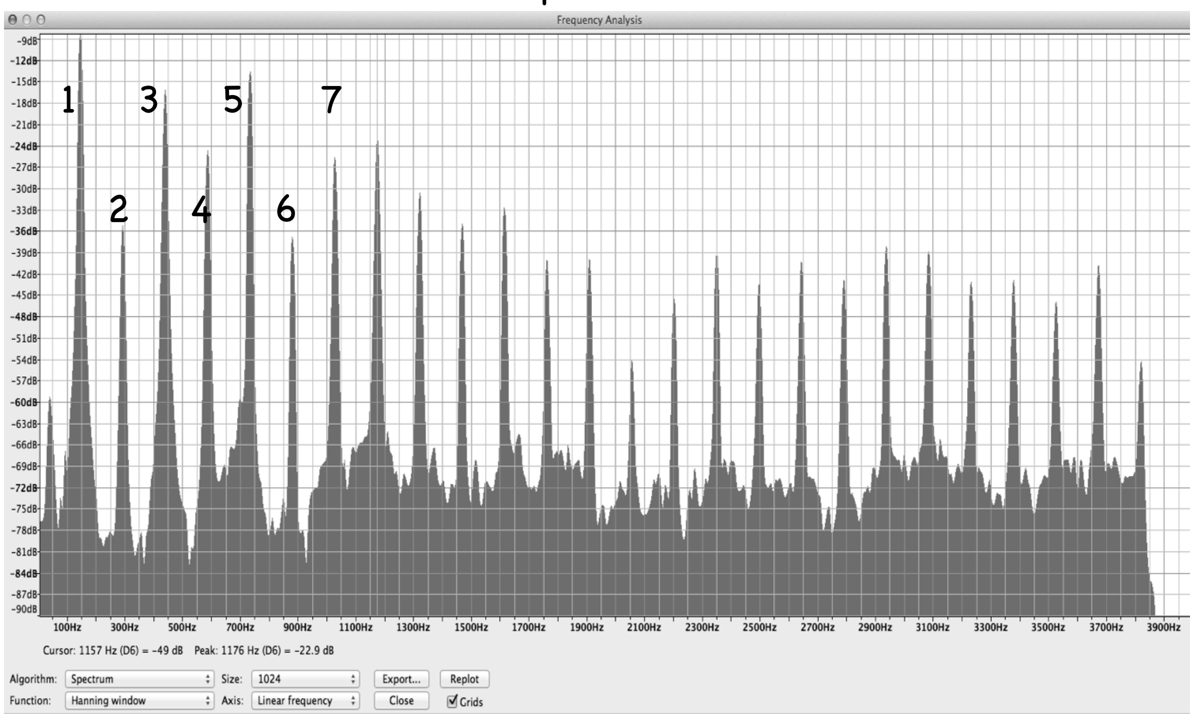
D3 "concert"



A few cycles



Spectrum

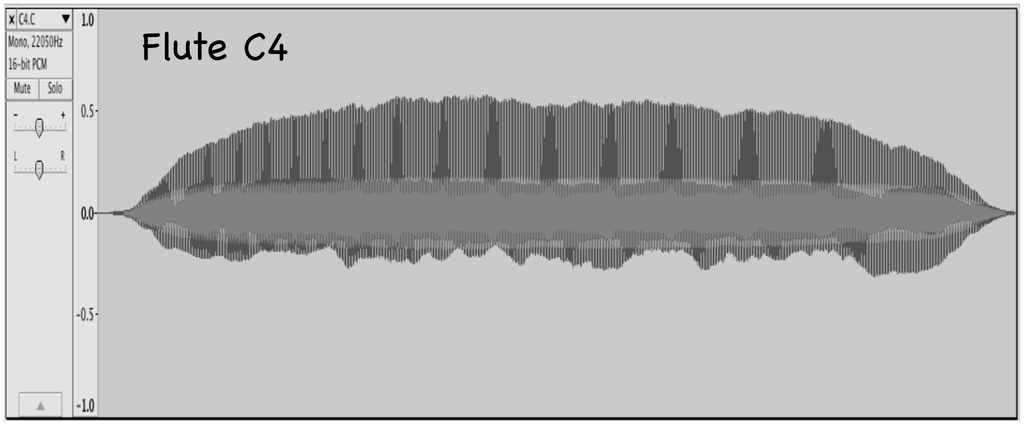


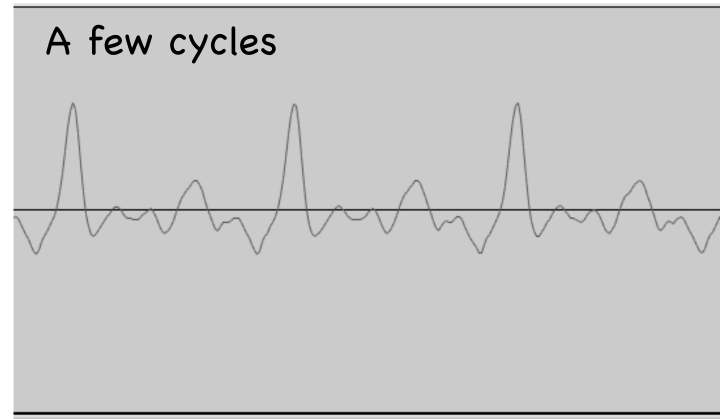
Note that the odd numbered harmonics have the greatest amplitudes.

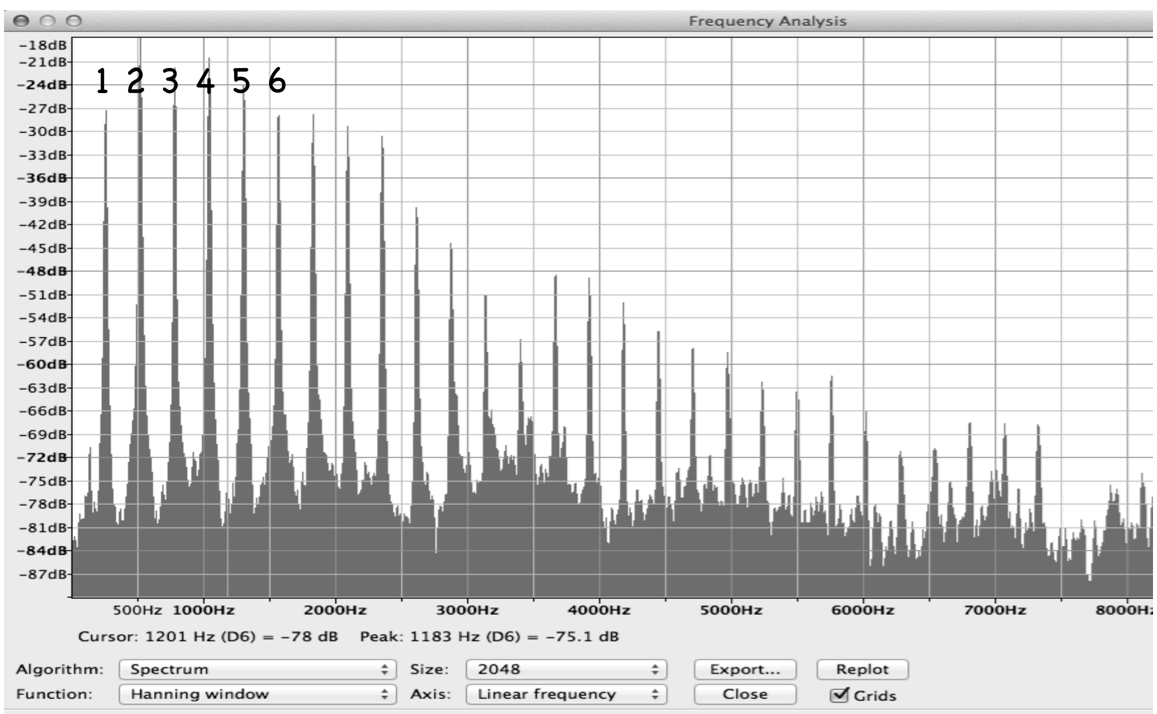
This is because the clarinet bore supports the odd numbered harmonics of the fundamental mode.

C4

Flute Timbre

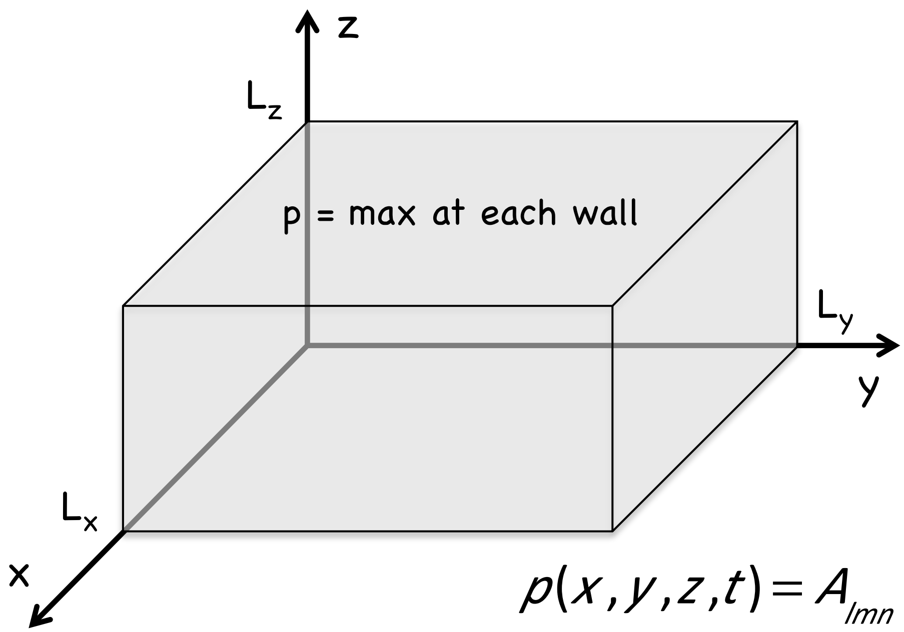






The Open-Open boundary condition of a flute supports all harmonics of the fundamental mode.

Acoustic resonances in higher dimensions



Acoustic wave equation in 3-d:

$$\frac{\partial^2 p}{\partial t^2} = c^2\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}\right)$$

Solutions:

$$p(x,y,z,t) = X(x)Y(y)Z(z)\cdot\cos(\omega t)$$

Separable into functions of x, y, z

$$p(x,y,z,t) = A_{lmn}\cos(lk_x x)\cdot\cos(mk_y y)\cdot\cos(nk_z z)\cdot\cos(\omega t)$$

$k_x = \pi/L_x$, $l = 0,1,2$ ...

$k_y = \pi/L_y$, $m = 0,1,2$ ...

$k_z = \pi/L_z$, $n = 0,1,2$ ...

$$\omega_{lmn} = c\sqrt{\left(\frac{l\pi}{L_x}\right)^2 + \left(\frac{m\pi}{L_y}\right)^2 + \left(\frac{n\pi}{L_z}\right)^2}$$

$$k^2 = k_x^2 + k_y^2 + k_z^2$$

for example: $\omega_{010} = c\dfrac{\pi}{L_y}$ $\quad \omega_{111} = c\sqrt{\left(\dfrac{\pi}{L_x}\right)^2 + \left(\dfrac{\pi}{L_y}\right)^2 + \left(\dfrac{\pi}{L_z}\right)^2}$

Keeping f, λ and ω, k all straight

$\omega = 2\pi f$ angular frequency (radians per second)

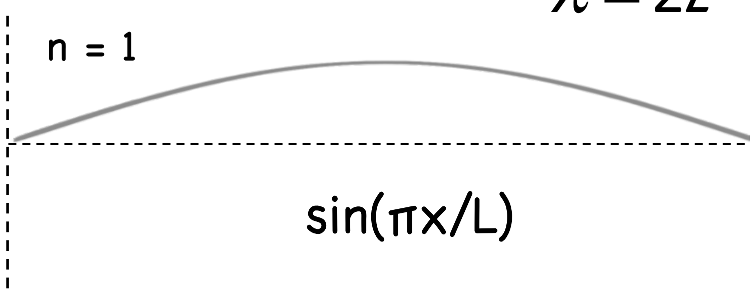
$k = \dfrac{2\pi}{\lambda}$ wavenumber (radians/meter)

$f = \dfrac{2\pi}{\omega}$ frequency (cycles/sec)

$\lambda = \dfrac{2\pi}{k}$ wavelength (meters/wave)

$$\lambda f = c \quad \rightarrow \quad \frac{2\pi}{k}\frac{\omega}{2\pi} = \frac{\omega}{k} = c$$

$$\lambda = 2L$$



remember!

$$\boxed{\lambda f = c} \rightarrow f = \frac{c}{2L}$$

$$\lambda = \frac{2\pi}{k} = 2L \rightarrow L = \frac{\pi}{k} \rightarrow k = \frac{\pi}{L}$$

$$\omega = ck = c\frac{\pi}{L}$$

Resonances in rooms

$$f_{lmn} = \frac{\omega_{lmn}}{2\pi} = \frac{c}{2\pi}\sqrt{\left(\frac{l\pi}{L_x}\right)^2 + \left(\frac{m\pi}{L_y}\right)^2 + \left(\frac{n\pi}{L_z}\right)^2} = \frac{c}{2}\sqrt{\left(\frac{l}{L_x}\right)^2 + \left(\frac{m}{L_y}\right)^2 + \left(\frac{n}{L_z}\right)^2}$$

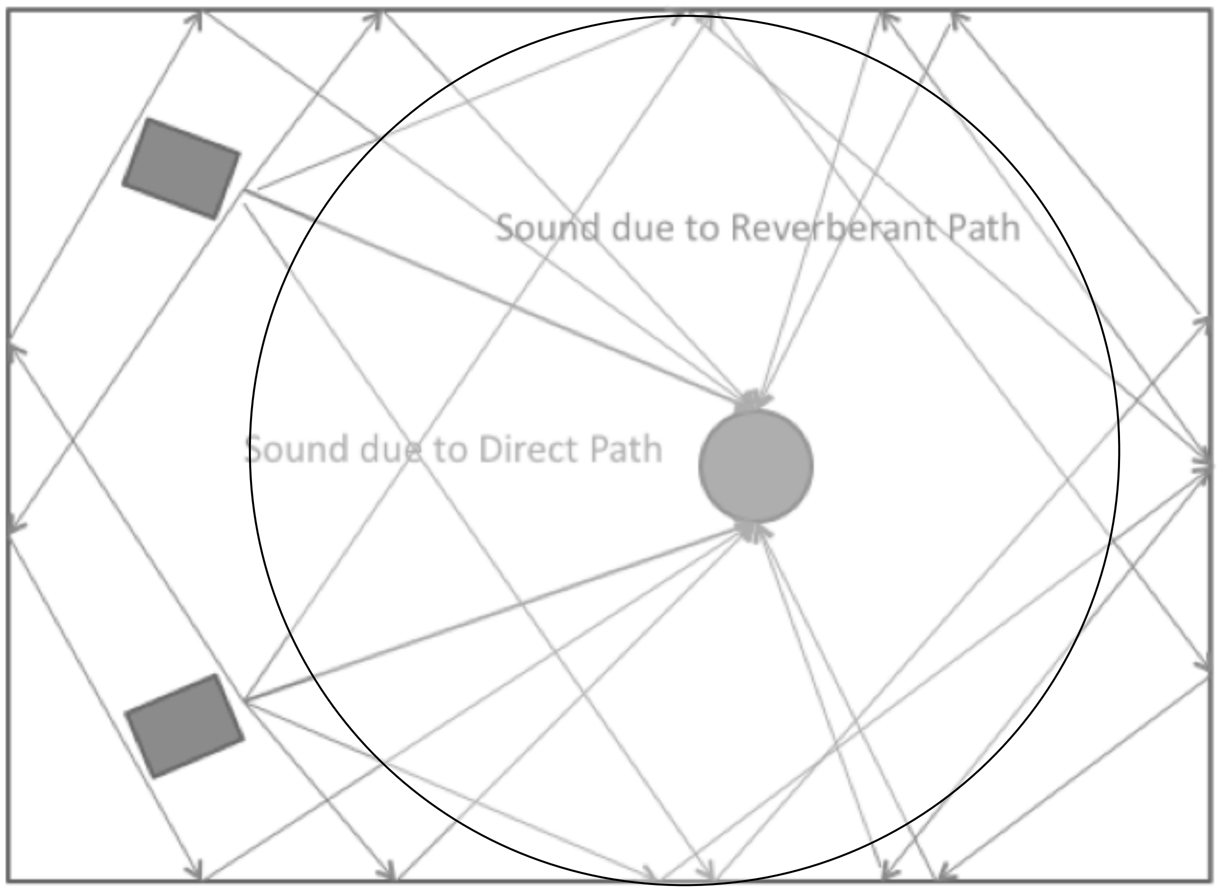
Room Dimensions: 3 meters high
4 meters wide
5 meters long

C = 340 m/sec

Lx	4	m
Ly	5	m
Lz	3	m

l	m	n	f(l,m,n)	
1	0	0	42.50	Hertz
0	1	0	34.00	
0	0	1	56.67	
1	1	0	54.43	
0	1	1	66.08	
1	0	1	70.83	
1	1	1	78.57	
2	0	0	85.00	
0	2	0	68.00	
0	0	2	113.33	
2	1	0	91.55	
2	0	1	102.16	
2	1	1	107.67	
2	2	2	157.14	

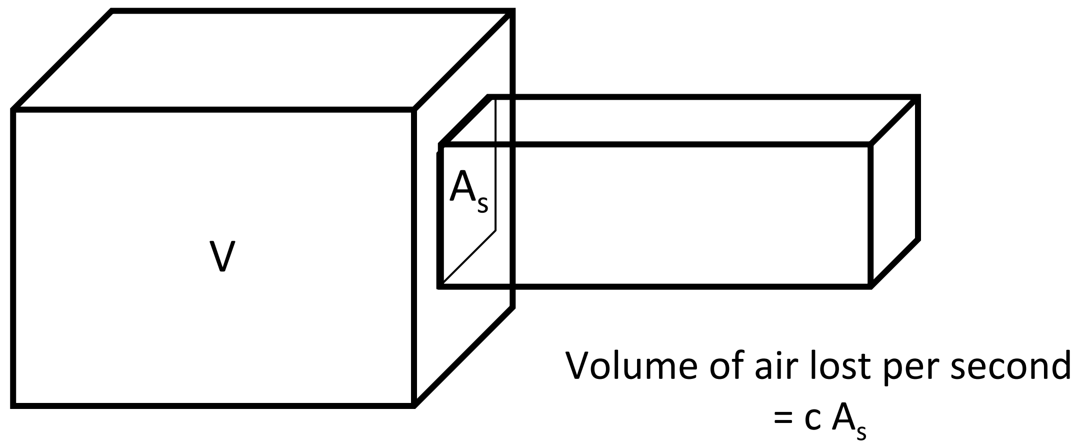
Direct versus reverberant sound



Radius of reverberation

Sabine-Franklin-Jaeger Theory of Room Acoustics

$$T_{60} = 6\log_{10}\left(\frac{4V}{cA_s}\right)$$



- T_{60} = time required for sound to decay 60 dB
- V = Volume of room
- A_s = equivalent area of an open window resulting from all sound absorption in room
- T_{60} is essentially the time it takes to empty all of the air in the room out of the window 4x