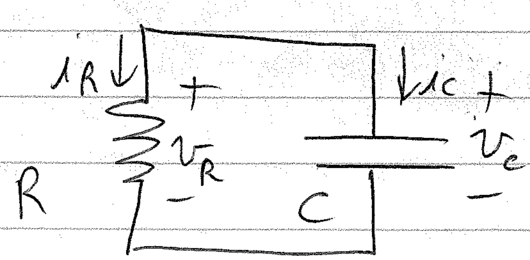
This is variously called the

Zero-Input Response (text)
Natural Response
Source-Free Response

We wrote all this down using the State Variables v_c + i_c, now I will tell you that the form (exponential decay) applies to every response in the ckt.

For example, if I asked about the current through the resistor in the RC ckt:



$$v_c(0^-) = V_0$$

$$v_c = \begin{cases} V_0 & t<0 \\ V_0 e^{-t/RC} & t \geq 0 \end{cases}$$

$$i_R = -i_c \quad \text{but} \quad i_c = C\frac{dv_c}{dt}$$

so $$\frac{dv_c}{dt} = -\frac{1}{RC}V_0 e^{-t/RC}$$

and $$i_R = -\left(-\frac{1}{RC}\right)V_0 e^{-t/RC} = \frac{C}{RC}V_0 e^{-t/RC} = \underline{\underline{\frac{1}{R}V_0 e^{-t/RC}}}$$

<u>OR</u> we could say:

This is a Zero-Input Response, so

$i_R(t) = I_0 e^{-t/RC}$ will be the form.

$I_0 = i_R(0^+) = \frac{v_R(0^+)}{R} = \frac{v_c(0^+)}{R}$ (R+C are in parallel)

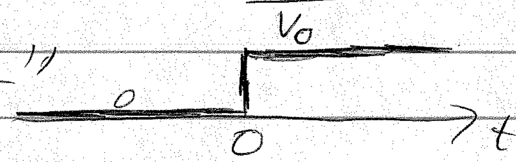
but $v_c(0^+) = v_c(0^-) = V_0$, so

$I_0 = \frac{V_0}{R}$ and $i(t) = \frac{V_0}{R} e^{-t/RC}$

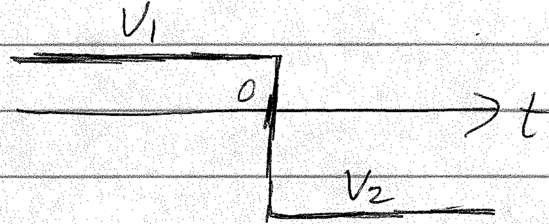
Do lots of examples and exercises.

The next step is to ask "What if the source is on for $t>0$, say as a constant?"

IE, a "step ~~resp~~ input"



or even



In this case we must include the "forcing function" for $t>0$, and the total response is the sum

$$v_T(t) = v_F(t) + v_N(t)$$

we know this is $v_N(t) = Ke^{-t/\tau}$ but we cannot evaluate K yet.

The forced response v_F is a solution to

$$\frac{dv_F}{dt} + \frac{1}{\tau}v_F = \frac{1}{\tau}V_A \quad \left(\text{or } \frac{1}{\tau}V_2\right) \text{ etc}$$

We are looking for a particular solution to this equation, and indeed taking $v_F = V_A$ is a solution, since $0 + \frac{1}{\tau}V_A = \frac{1}{\tau}V_A$ is true.

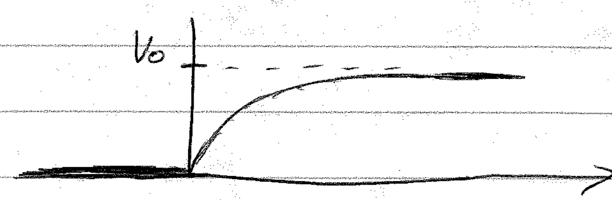
So $v_T(t) = V_0 + Ke^{-t/\tau}$ is the total solution.

Evaluate K now:

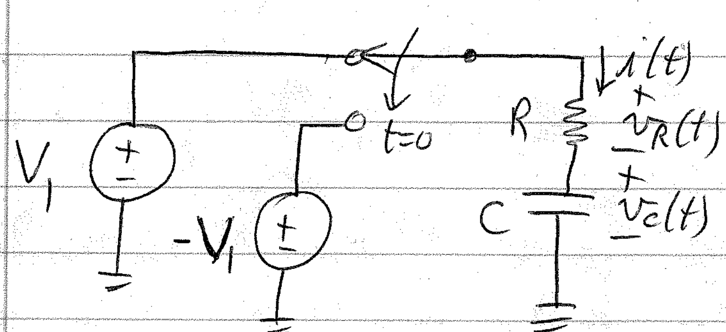
$$v_T(0^+) = V_0 + Ke^{-0} = 0$$

$$K = -V_0$$

So $v_T(t) = V_0 - V_0 e^{-t/\tau} = V_0(1 - e^{-t/\tau})$

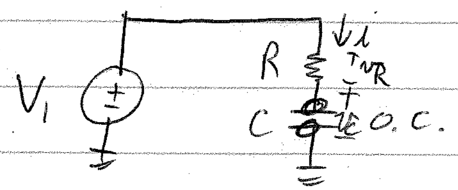


Example:



Find $v_C(t)$, $v_R(t)$, and $i(t)$

$t=0^-$: Find IC of state variable, others are interesting.

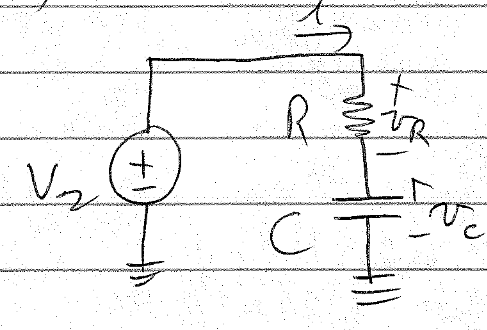


$v_C(0^-) = V_1$

$v_R(0^-) = 0$

$i(0^-) = 0$

2) $t = 0^+$:



$v_c(0^+) = v_c(0^-) = V_1$ (state var.)

KVL: $v_R + v_c - V_2 = 0$

$v_R + v_c = V_2$

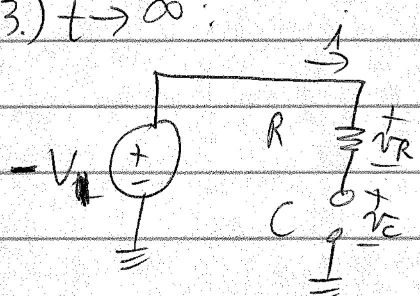
$\tau = RC$

or $v_R(0^+) + v_c(0^+) = V_2$

$\underline{v_R(0^+) = V_2 - v_c(0^+) = V_1 - V_1 = -2V_1}$ (not continuous)

$i = \frac{v_R}{R} \Rightarrow i(0^+) = \frac{v_R(0^+)}{R} = \frac{V_2 - V_1}{R} = -\frac{2V_1}{R}$ (not continuous)

3.) $t \to \infty$:



$v_c(\infty) = -V_1$ (not zero)

$v_R(\infty) = 0$

$i(\infty) = 0$

these are particular solutions for v_c, v_R, i

$\cancel{\text{So } v_c(t) = V_1}$

So, for $t > 0$:

$\cancel{v_c(t) = V_2 + V_1 e^{-t/\tau}}$

$v_c(t) = -V_1 + K e^{-t/\tau}$

$v_c(0^+) = V_1 = -V_1 + K$

$K = V_1 + V_1 = 2V_1$

so

$$v_c(t) = -V_1 + (2V_1)e^{-t/\tau}$$
$$= v_c(\infty) + (v_c(0^+) - v_c(\infty))\, e^{-t/\tau}$$

AND $v_R(t) = 0 + Ke^{-t/\tau}$

$v_R(0^+) = (-V_{\cancel{2}} - V_1) = K$

so $\underline{v_R(t) = (\cancel{V_2}-2V_1)e^{-t/\tau} = -2V_1 e^{-t/\tau}}$

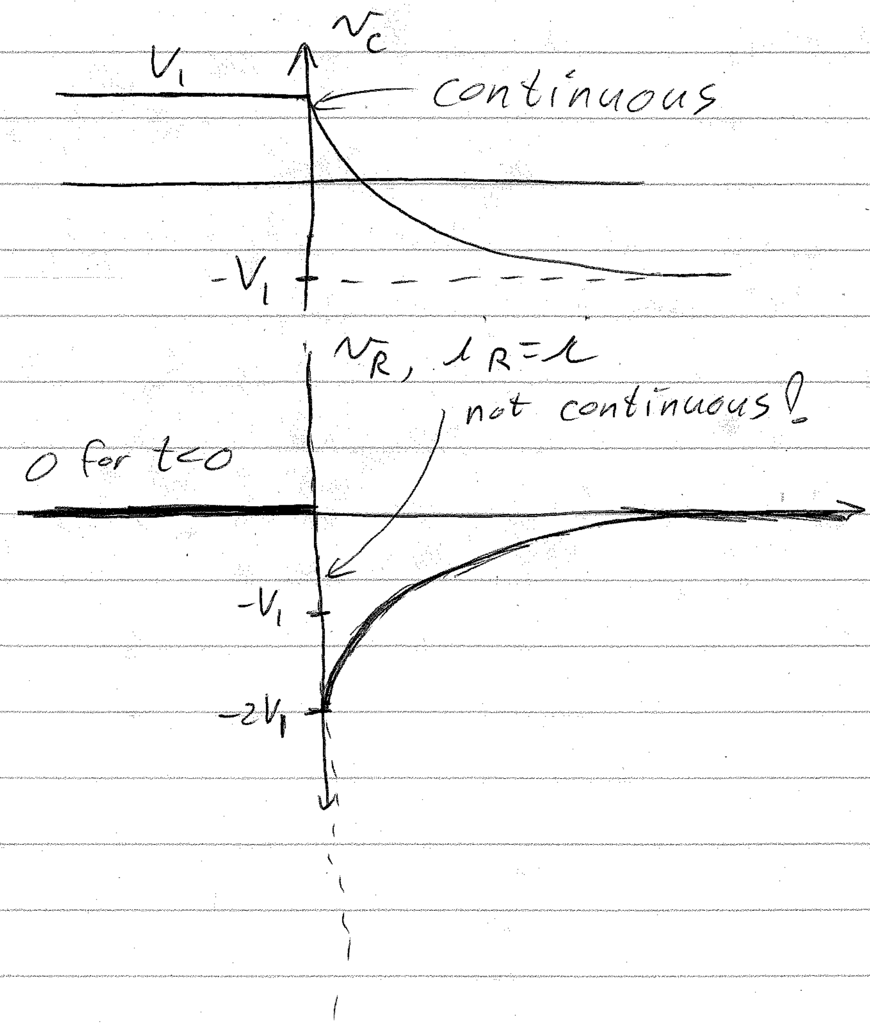
AND

$i(t) = \cancel{\frac{V_2-V_1}{R}}\, 0 + Ke^{-t/\tau}$

$i(0^+) = \frac{V_2 - V_1}{R} = K$

so $i(t) = \frac{V_2 - V_1}{R} e^{-t/\tau} = -\frac{2V_1}{R} e^{-t/\tau}$

Graph:

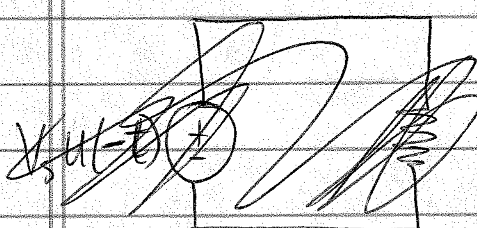


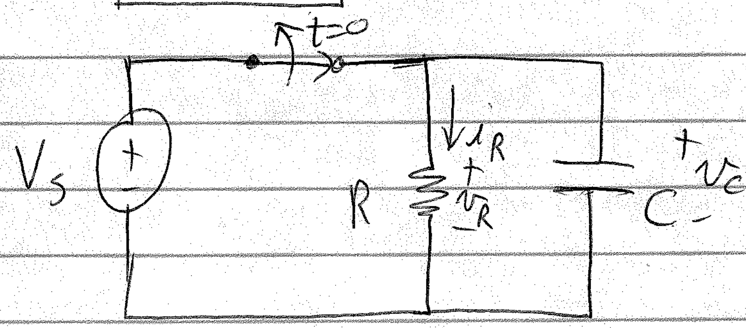
So, in general, the Total Response of Anything in a 1^{st} order ckt ~~is~~ to a step input is:

$$y(t)=\begin{cases} y(0^-) & t<0 \\ y(\infty)+\left(y(0^+)-y(\infty)\right)e^{-t/\tau} & t\geq 0 \end{cases}$$

First Order Step Response
(Zero Input is ~~one~~ a special case when $y(\infty)=0$)

Example





Find $i_R(t)$.

Solution: Note $v_R=v_c$ & $i_R=\frac{v_R}{R}=\frac{v_c}{R}$ (always)

$t=0^-$: $v_c(0^-)=V_s$

$v_R(0^-)=V_s$

$i_R(0^-)=\frac{V_s}{R}$

$t=0^+$: $v_c(0^+) = v_c(0^-) = V_s$ (<u>Must</u> be)

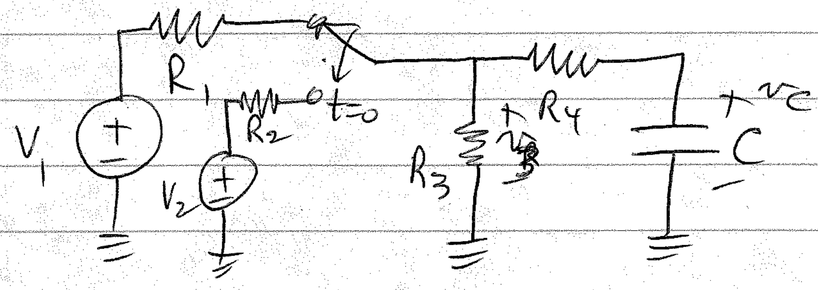
$$\left.\begin{array}{l} \text{so } v_R(0^+) = v_c(0^+) = V_s \\ \text{and } i_R(0^+) = \frac{V_s}{R} \end{array}\right\}$$

these are continuous because of circuit layout, not because they <u>have</u> to be.

$t=\infty$: $i_R(\infty) = 0$ (source-free)

$$\text{so } i_R(t) = \begin{cases} \frac{V_s}{R} & t<0 \\ \frac{V_s}{R} e^{-t/\tau} & t \geq 0 \end{cases}$$

Example:

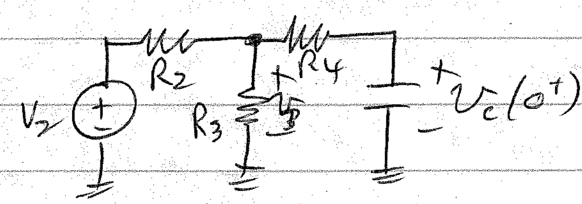


Find $v_3(t)$.

$t=0^-$: Ignore $V_2 + R_2$, C is an open ckt, ignore R_4.

$$\underline{v_C(0^-) = \frac{R_3}{R_1+R_3} V_1 = v_3(0^-)}$$

$t=0^+$: Ignore $V_1 + R_1$: $\underline{v_C(0^+) = v_C(0^-)}$



$$R_{eq} = \frac{R_2 R_3}{R_2+R_3} + R_4$$

$$\text{KCL: } \frac{v_3 - V_2}{R_2} + \frac{v_3 - v_C}{R_4} + \frac{v_3}{R_3} = 0$$

$$\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) v_3 = \frac{1}{R_2} V_2 + \frac{1}{R_4} v_C(0^+)$$

$$\frac{R_3 R_4 + R_2 R_4 + R_2 R_3}{R_2 R_3 R_4} v_3 =$$

$$v_3 = \frac{R_3 R_4}{R_2 R_3 + R_2 R_4 + R_3 R_4} V_2 + \frac{R_2 R_3}{R_2 R_3 + R_2 R_4 + R_3 R_4} \left(\frac{R_3}{R_1 + R_3}\right) V_1$$

$$v_3(0^+) = \frac{1}{\frac{R_2}{R_4} + \frac{R_3}{R_4} + 1} V_2 + \left(\frac{1}{1 + \frac{R_3}{R_4} + \frac{R_2}{R_3}}\right)\left(\frac{1}{1 + \frac{R_1}{R_3}}\right) V_1$$

$t \to \infty$: Ignore $V_1 + R_1$, C is an Open Ckt, ignore R_4.

$$v_3(\infty) = \frac{R_3}{R_2+R_3} V_2 = \frac{1}{1+\frac{R_2}{R_3}} V_2$$

So,
$$v_3(t) = \begin{cases} \frac{R_3}{R_1+R_3} V_1 = \frac{1}{1+\frac{R_1}{R_3}} V_1 & t<0 \\ \cancel{\frac{1}{1+\frac{R_2}{R_4}+\frac{R_3}{R_4}} V_2 + \left(\frac{1}{1+\frac{R_2}{R_3}+\frac{R_3}{R_4}}\right)\left(\frac{1}{1+\frac{R_1}{R_3}}\right) V_1} & \\ \frac{1}{1+\frac{R_2}{R_3}} V_2 + \left[\frac{1}{1+\frac{R_2+R_3}{R_4}} V_2 + \left(\frac{1}{1+\frac{R_2}{R_3}+\frac{R_3}{R_4}}\right)\left(\frac{1}{1+\frac{R_1}{R_3}}\right) V_1 - \frac{1}{1+\frac{R_2}{R_3}} V_2\right] e^{-t/\tau} & \end{cases}$$

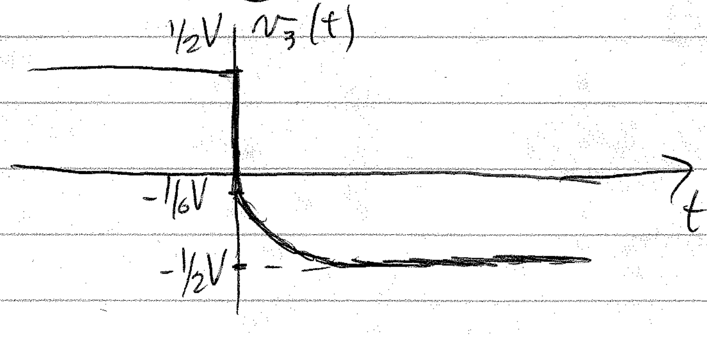
where $\tau = R_{eq} C = \left(R_4 + \frac{R_2 R_3}{R_2+R_3}\right) C$

Let all the R's be equal, so

$$v_3(t) = \begin{cases} \frac{1}{2}V_1 & t<0 \\ \frac{1}{2}V_2 + \left[\frac{1}{3}V_2 + \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)V_1 - \frac{1}{2}V_2\right]e^{-t/\tau} & t\geq 0 \\ = \frac{1}{2}V_2 + \left[\left(\frac{1}{3}-\frac{1}{2}\right)V_2 + \frac{1}{6}V_1\right]e^{-t/\tau} & \\ = \frac{1}{2}V_2 + \left[-\frac{1}{6}V_2 + \frac{1}{6}V_1\right]e^{-t/\tau} & \end{cases}$$

Pick $V_1 = 1V$, $V_2 = -1V$, so

$$v_3(t) = \begin{cases} \frac{1}{2}V & t<0 \\ -\frac{1}{2}V + \left[\frac{1}{6}V + \frac{1}{6}V\right]e^{-t/\tau} & t\geq 0 \\ = -\frac{1}{2}V + \frac{1}{3}Ve^{-t/\tau} & \end{cases}$$



$$\tau = \left(R + \frac{R^2}{2R}\right)C = \underline{\underline{\frac{3}{2}RC}}$$