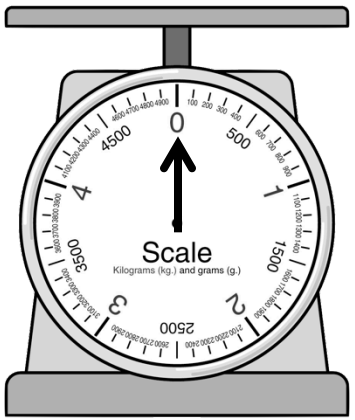
Puzzler

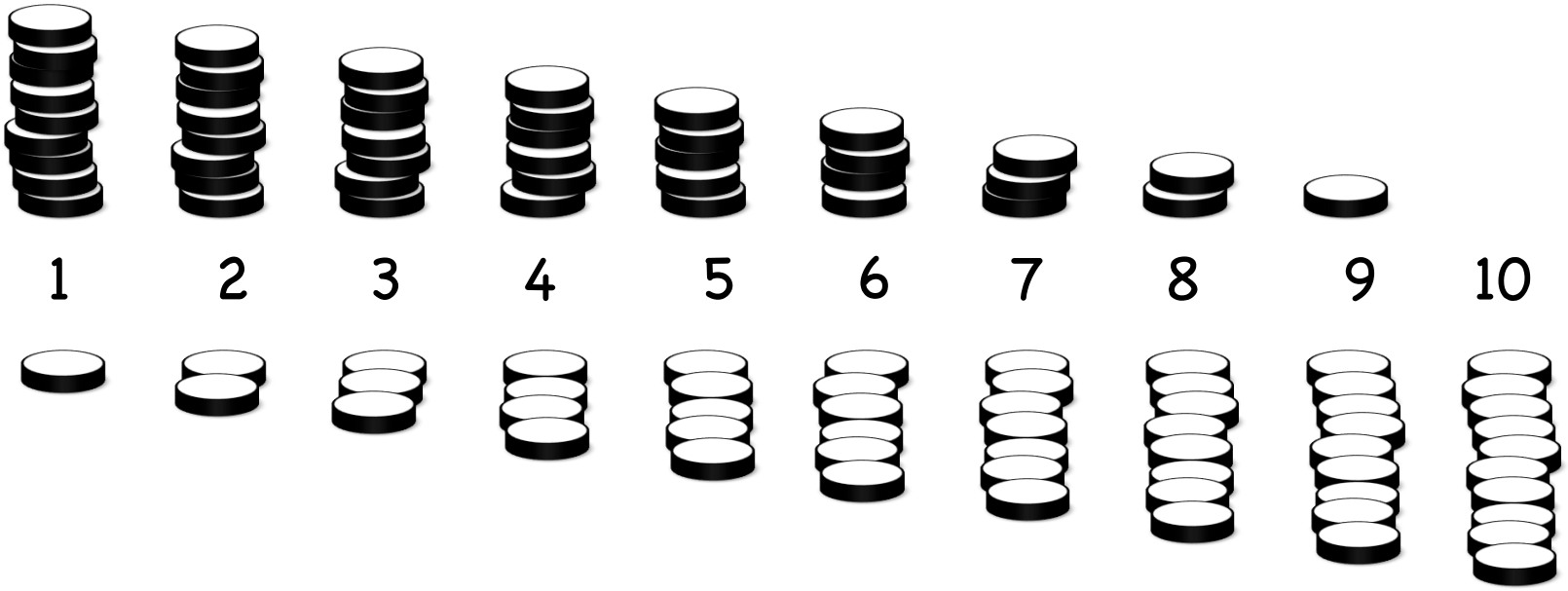
The Counterfeit Coins

You have 10 stacks of coins, each consisting of 10 half dollars. One entire stack is counterfeit, but you do not know which one. You do know the weight of a genuine half dollar and you are also told that each counterfeit coin weighs one gram more than it should. You may weigh the coins on a pointer scale. What is the smallest number of weighings necessary to determine which stack is counterfeit?



Solution: One!

Take one coin from the first stack, 2 from the second, 3 from the third and so forth. Weigh the entire group of coins. The number of grams in excess of 55x's the weight of one real coin tells you which stack is counterfeit.



Introduction to Audio and Music Engineering

Lecture 12

Topics:

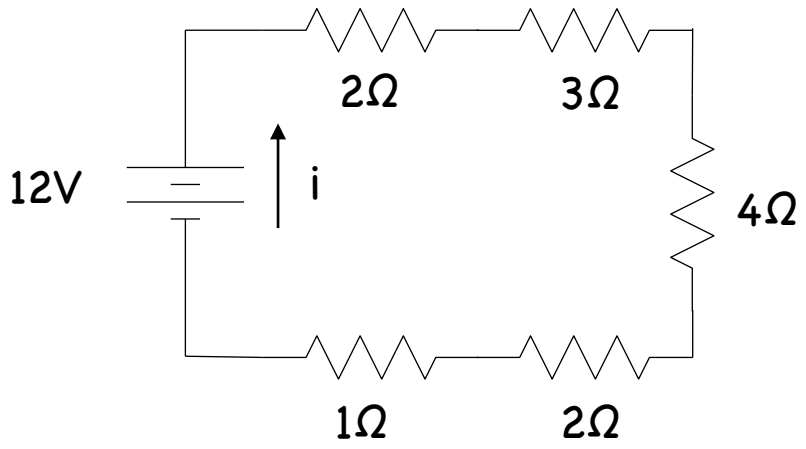
- Volume control circuit
- Kirchhoff's current law
- Voltage and current dividers
- Series and parallel resistors

Kirchhoff's Voltage Law

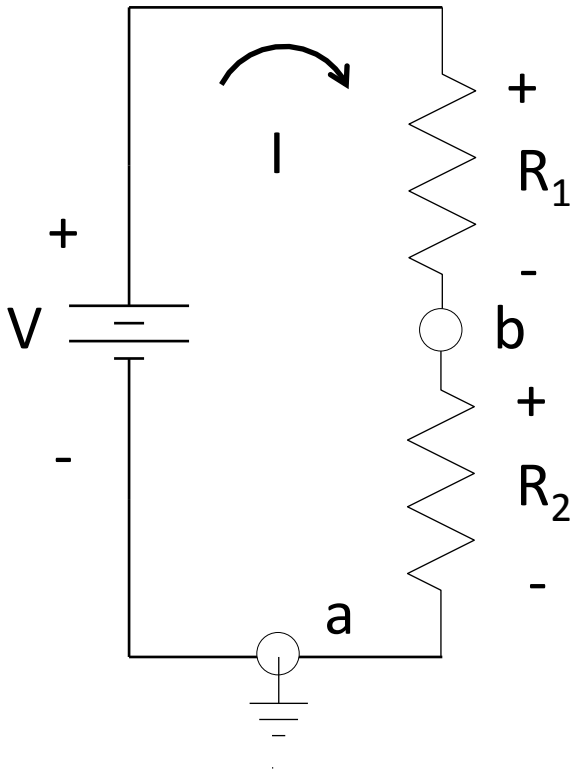
The sum of the voltages going around a closed path in a circuit is zero.

$$\sum_n V_n = 0$$

Find the voltage across each component in the circuit.



Voltage Divider



Find the current I

Find the voltage at b

KVL → $V - IR_1 - IR_2 = 0$

$$I = \frac{V}{R_1 + R_2}$$

Series resistors: $R_{series} = R_1 + R_2$

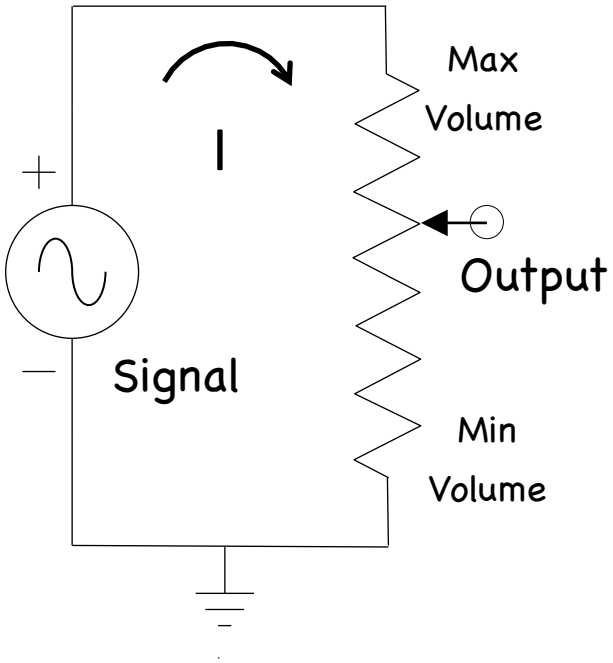
Voltage divider: $V_b = IR_2 = V\frac{R_2}{R_1 + R_2}$

Special Cases:

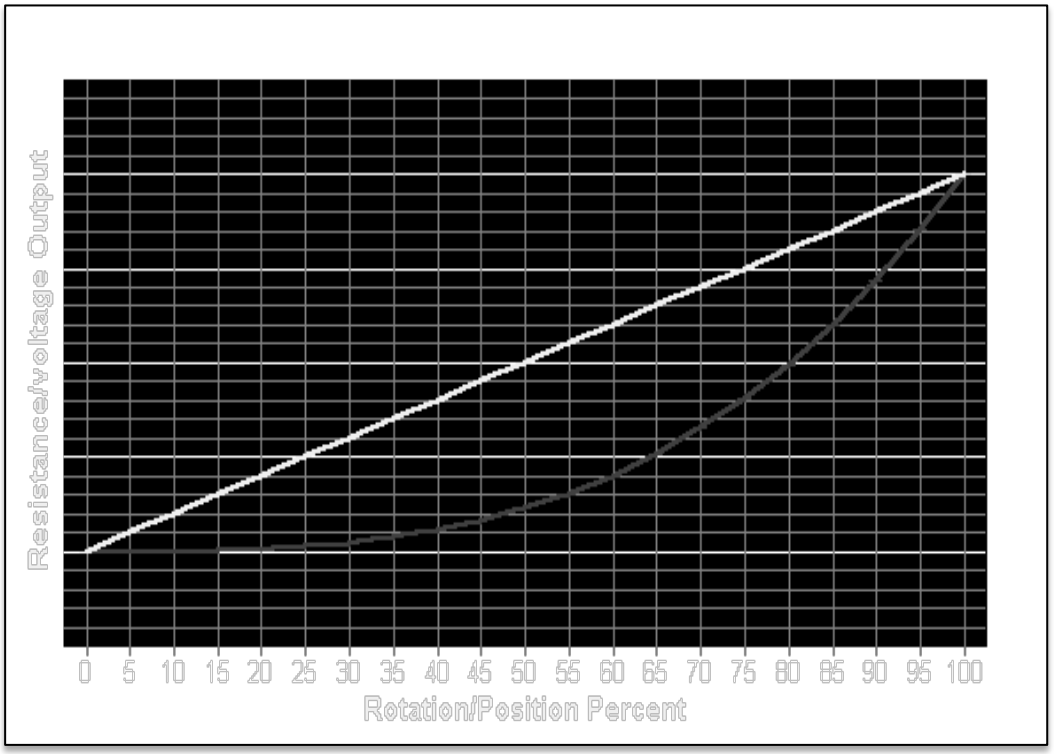
$R_1 = R_2$; $V_b = \frac{1}{2} V$

$R_2 >> R_1$; $V_b \rightarrow V$

Volume Control Circuit



Logarithmic vs. linear potentiometer



Why logarithmic pots are used for volume control …

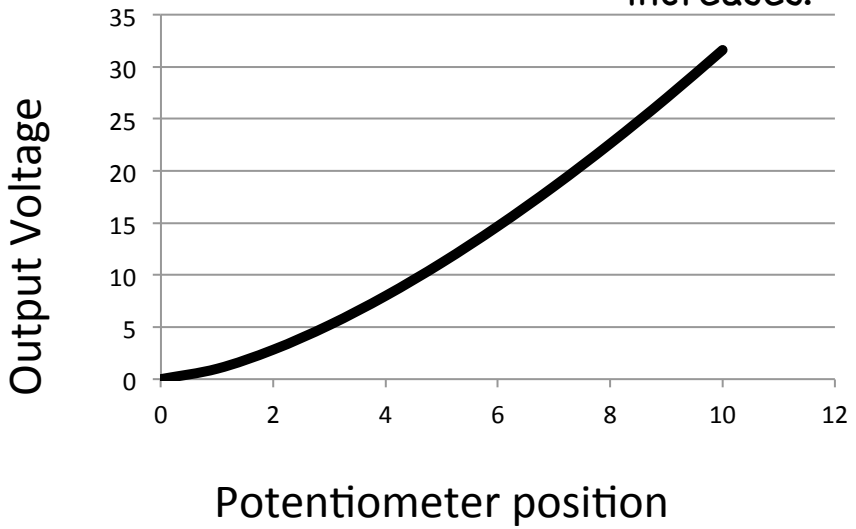
Human perception of loudness is logarithmic.

To double perceived loudness the sound pressure level must increase by a factor of about 2.8, which is about (9 decibels).

$$dB(Volts) = 20\log\left(\frac{V}{V_{ref}}\right) \Rightarrow V = V_{ref}\,10^{\frac{dBV}{20}} \qquad 10^{\frac{9}{20}} \approx 2.8$$

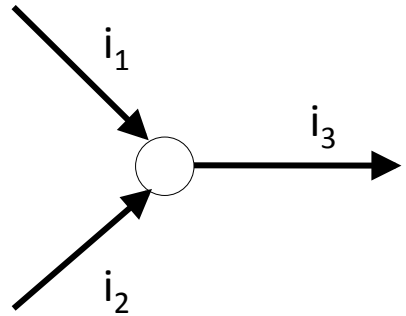
Poten. Position	Voltage Increase
1	1 (ref)
2	2.8
4	2.8 x 2.8 = 7.8
8	2.8 x 7.8 = 22

Voltage must increase faster as the voltage increases.



Kirchhoff's Current Law

The sum of all currents entering or leaving a node of a circuit is zero.



Pick a consistent convention:

current flowing into node is positive

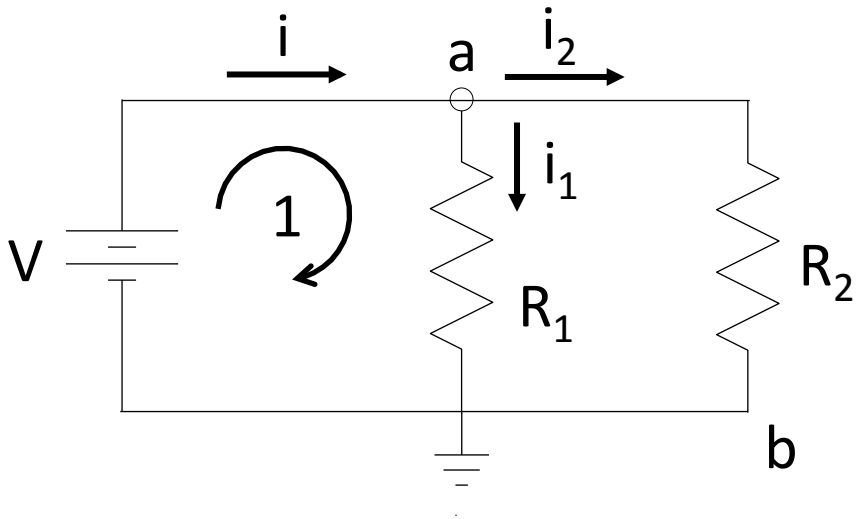
current flowing out of node is negative

$$i_1 + i_2 - i_3 = 0$$

Charge is neither destroyed or created in a circuit.

What goes in must come out.

Current divider



Find i, i_1, i_2

KVL Loop 1:

(1) $V - i_1R_1 = 0 \rightarrow i_1 = V/R_1$

KVL Outer Loop:

(2) $V - i_2R_2 = 0 \rightarrow i_2 = V/R_2$

KCL @ node a:

(3) $i - i_1 - i_2 = 0$

$i - V/R_1 - V/R_2 = 0$

$i = V(1/R_1 + 1/R_2) = V/R_{//}$

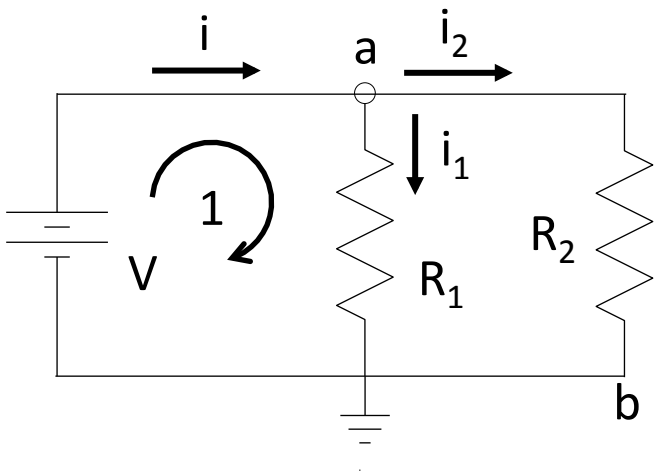
$1/R_{//} = 1/R_1 + 1/R_2$

$1/R_{//} = (R_2 + R_1)/R_1R_2$

Parallel Combination R_1, R_2

$$R_{//} = \frac{R_1 R_2}{R_1 + R_2}$$

Current divider cont.



(1) $V - i_1R_1 = 0 \rightarrow i_1 = V/R_1$

(2) $V - i_2R_2 = 0 \rightarrow i_2 = V/R_2$

(3) $i - i_1 - i_2 = 0$

Find total current i : $i = V/R_{//}$ where $R_{//} = R_1R_2/(R_2 + R_1)$

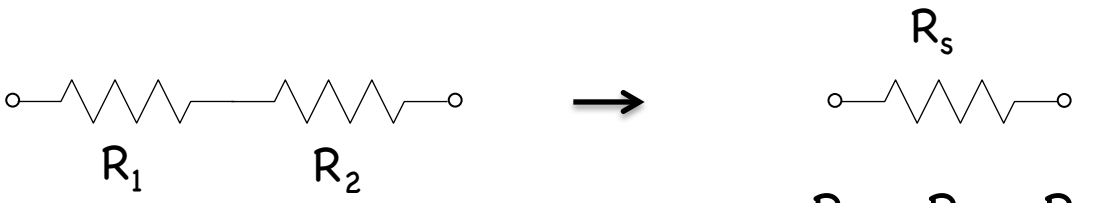
Then find i_1 : $i_1 = V/R_1 = iR_{//}/R_1 = iR_2/(R_2 + R_1)$

and i_2 is : $i_2 = iR_1/(R_2 + R_1)$

Note that the greater current passes through the smaller resistor.

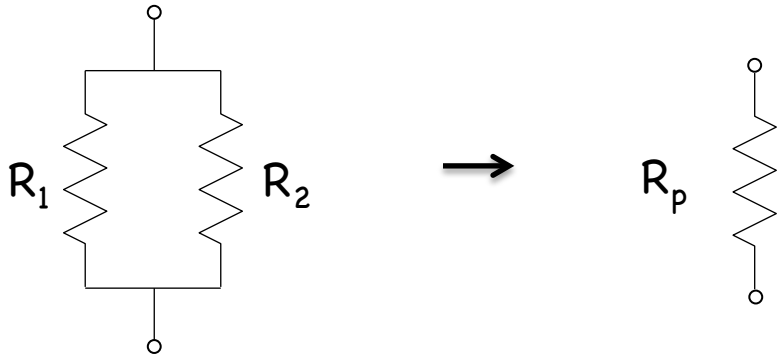
Series and parallel resistors

Series



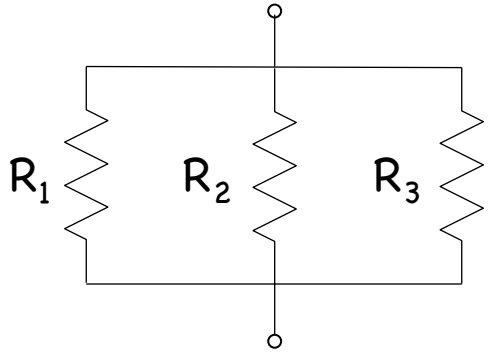
$R_s = R_1 + R_2$

Parallel



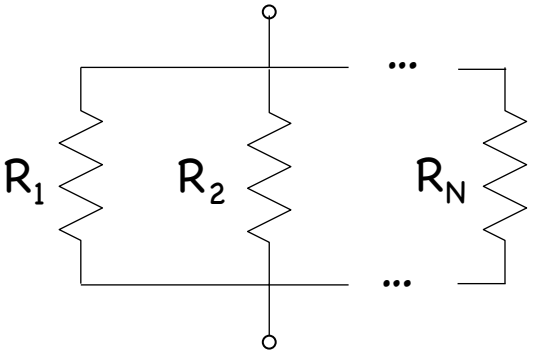
$R_p = R_1R_2/(R_2 + R_1)$

Multiple parallel resistors



R_p = ?

What is R_p when $R_1 = R_2 = R_3 = 3$?



$$R_p = \prod_{i=1}^{N} R_i \Bigg/ \sum_{k=1}^{N} \frac{\prod_{i=1}^{N} R_i}{R_k}$$

N = 4 $$R_p = \frac{R_1 R_2 R_3 R_4}{R_1 R_2 R_3 + R_1 R_2 R_4 + R_1 R_3 R_4 + R_2 R_3 R_4}$$

Let's say that you need a resistor of 1 Ohm with a rating of 5 Watts but that you only have an assortment of ¼ Watt resistors. How could you construct the required resistor?