Superposition –

1) "Turn off" all ~~source~~ independent ideal sources:

V src → short ckt ($V_s = 0$)

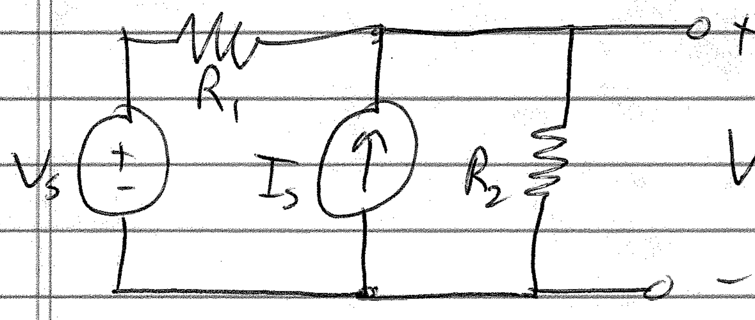
I src → open ckt ($I_s = 0$)

2) Turn 1 src "on" and find output due to that src alone, O_1. Turn src "off". Repeat for all N sources. ($O_{2,3,\ldots N}$)

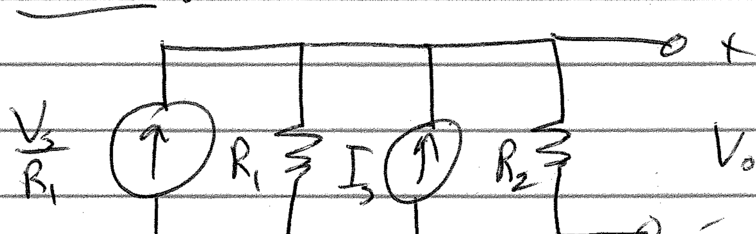
3) Output with all sources on is (algebraic) sum of ~~all~~ individual outputs:

$$O_T = \sum_{i=1}^{N} O_i$$

Text example:



Could do src transformation:



$$I_T = \frac{V_s}{R_1} + I_s = \frac{V_s + R_1 I_s}{R_1}$$

$$R_{eq} = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

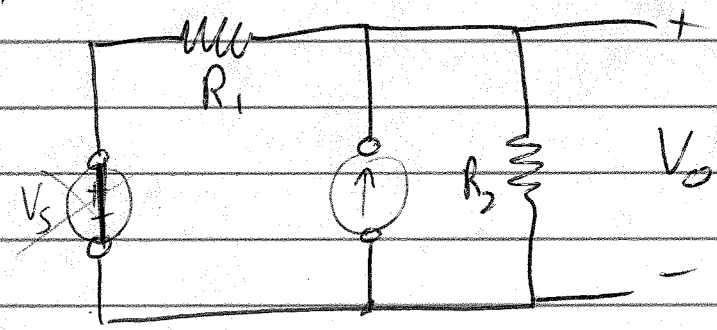
$$V_o = I_T R_{eq}$$

$$= \frac{V_s + R_1 I_s}{R_1} \cdot \frac{R_1 R_2}{(R_1 + R_2)}$$

$$\underline{V_o = \frac{R_2}{R_1+R_2} V_s + \frac{R_1 R_2}{R_1 + R_2} I_s}$$

Or, we could use syperposition!

1.)



2) a) Turn on V_s, leave $I_s = 0$:
(draw)
Voltage Division: $V_{o1} = \frac{R_2}{R_1+R_2} V_s$

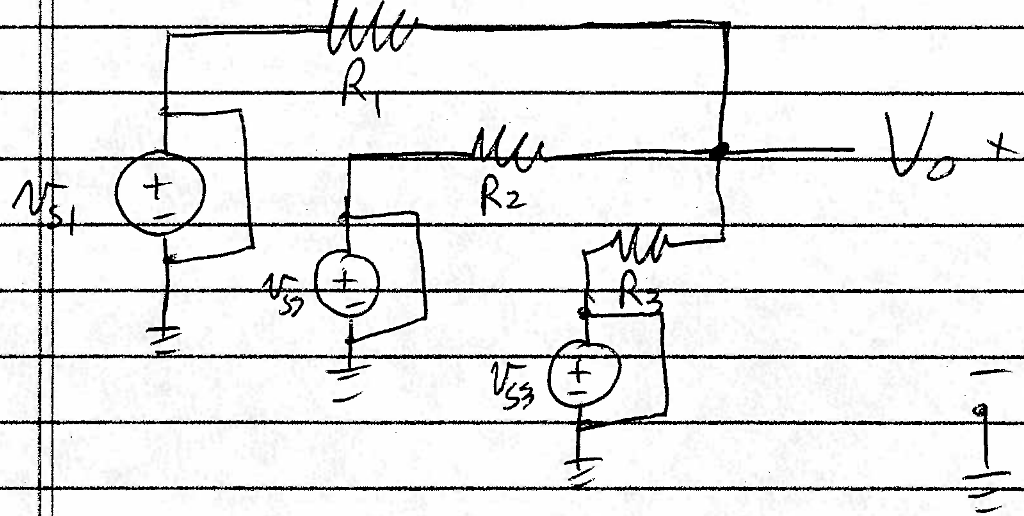
b) V_s off, I_s on: I_s into parallel R's:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}, \quad V_{o2} = \frac{R_1 R_2}{R_1+R_2} I_s$$

3) Total:

$$\underline{V_o = V_{o1} + V_{o2} = \frac{R_2}{R_1+R_2} V_s + \frac{R_1 R_2}{R_1+R_2} I_s}$$

Expand Example in Text:



Superposition:

1.) All off: . . .

2.) a.) v_{S_1} on:

$$V_{O_1} = \frac{R_2 // R_3}{R_1 + R_2 // R_3} v_{S_1}$$

$$= \frac{\frac{R_2 R_3}{R_2 + R_3}}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} v_{S_1}$$

$$= \frac{\frac{R_2 R_3}{R_2 + R_3}}{\frac{R_1(R_2+R_3) + R_2 R_3}{R_2 + R_3}} v_{S_1}$$

$$V_{O_1} = \frac{R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} v_{S_1}$$

turn v_{S_1} off:

b.) v_{S_2} on: $V_{O2} = \frac{R_1 // R_3}{R_2 + R_1 // R_3} v_{S2}$

$$= \frac{R_1 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} v_{S2}$$

turn v_{S_2} off

c) turn v_{s3} on: $V_{o3} = \dfrac{R_1 // R_2}{R_3 + R_1 // R_2} v_{s3}$

$$V_{o3} = \frac{R_1 R_2}{R_1R_2 + R_1R_3 + R_2R_3} v_{s3}$$

3.) Turn all on:

$$V_o = V_{o1} + V_{o2} + V_{o3}$$

$$= \frac{R_2R_3 v_{s1} + R_1R_3 v_{s2} + R_1R_2 v_{s3}}{R_1R_2 + R_1R_3 + R_2R_3}$$

If all R's are equal: $R_1 = R_2 = R_3 = R$, then

$$V_o = \frac{R^2 v_{s1} + R^2 v_{s2} + R^2 v_{s3}}{R^2 + R^2 + R^2} = \frac{R^2(v_{s1} + v_{s2} + v_{s3})}{3R^2}$$

$$V_o = \frac{1}{3}(v_{s1} + v_{s2} + v_{s3}) \quad \text{(Average)}$$

or $$= \frac{1}{3}v_{s1} + \frac{1}{3}v_{s2} + \frac{1}{3}v_{s3} \quad \text{(Weighted Sum)}$$

If R's not equal:

$$V_o = \frac{R_2R_3}{\underbrace{R_1R_2 + R_1R_3 + R_2R_3}_{R^{2\prime}}} v_{s1} + \frac{R_1R_3}{R^{2\prime}} v_{s2} + \frac{R_1R_2}{R^{2\prime}} v_{s3}$$

$$V_o = \frac{1}{\frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2R_3}} v_{s1} + \cdots$$

$$V_o = \frac{1}{1 + \frac{R_1}{R_3} + \frac{R_1}{R_2}} v_{s1} + \cdots$$