Introduction to Audio and Music Engineering

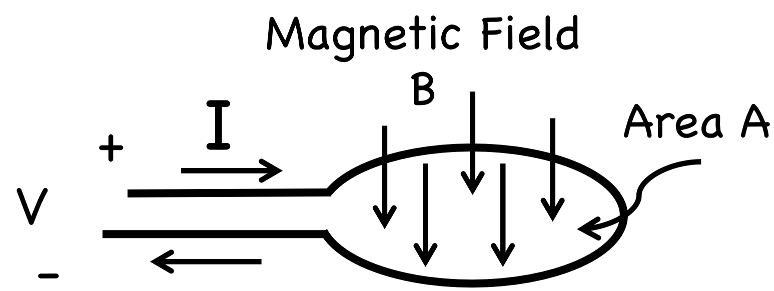
Lecture 15

Topics:

- Inductors
- Transients in RL circuits
- Low-pass and high-pass RL filters
- Phasors in AC circuit analysis
- Impedance
- Summary of simple RC and RL filters

Inductors

Magnetic Flux



$$\Phi_B = B \cdot A$$

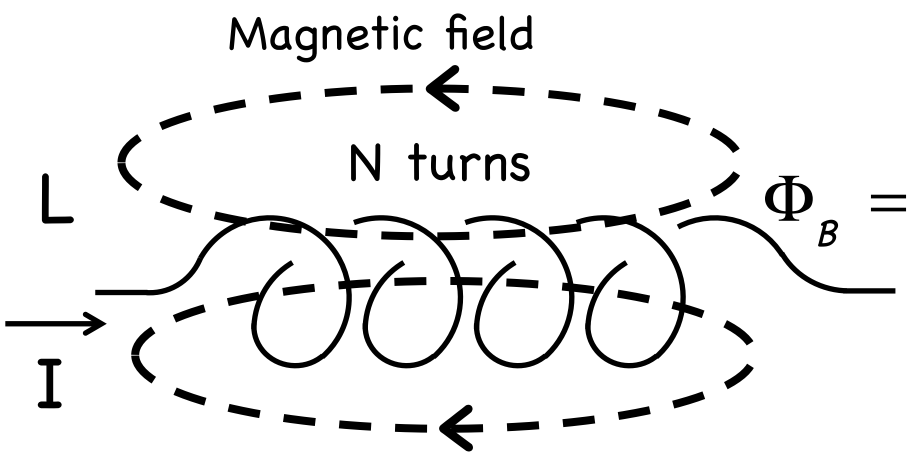
Faraday's Law

$$V = -N\frac{d\Phi_B}{dt}$$

N turns

Lenz's Law – A change in magnetic flux generates a voltage, which in turn drives a current that produces a magnetic field that opposes the change.

Inductor



$$\Phi_B = L \cdot I$$

$$V = L\frac{dI}{dt}$$

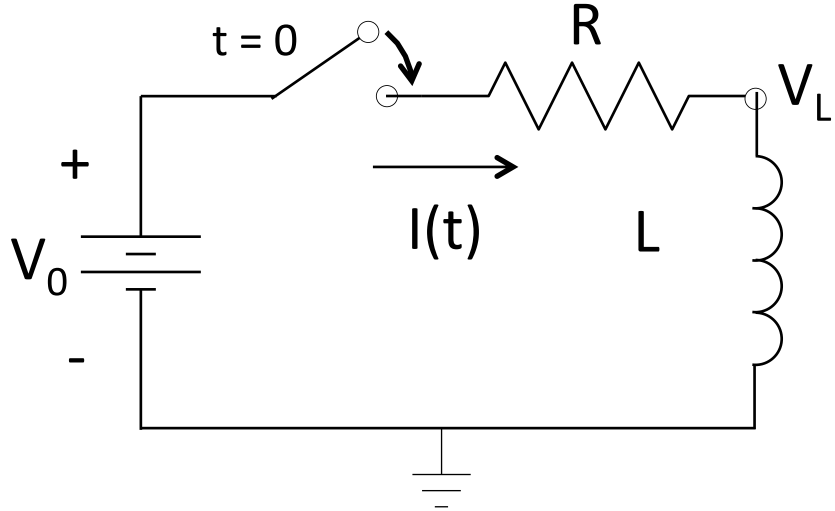
1 Volt = 1 Henry x 1 Amp/sec

$$L = \mu\frac{N^2 A}{l}$$

l is length of coil
A is area of a coil
μ = 4π x 10^{-7} Henry/m

Energy is stored by the magnetic field in the inductor.

Transient behavior of RL circuit



Kirchhoff's voltage law:

$$V_0 - V_R - V_L = 0 \rightarrow V_R + V_L = V_0$$

$$IR + L\frac{dI}{dt} = V_0$$

Initial Condition: I(t=0) = 0

Use solution method employed previously:

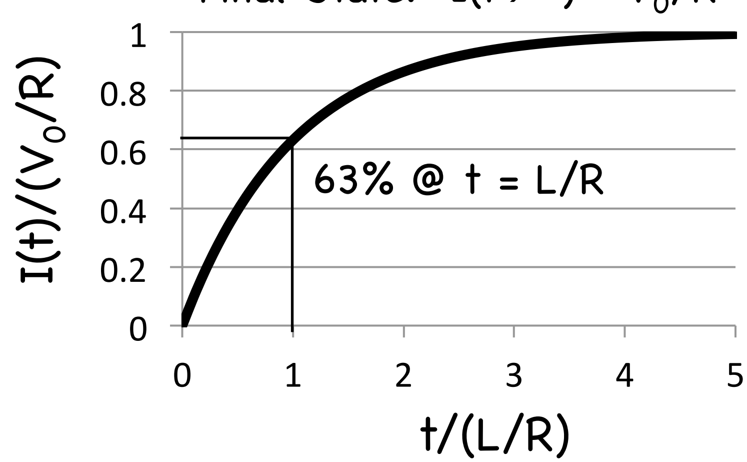
$$I(t) = I_p + I_g = \frac{V_0}{R} + Ae^{-\frac{R}{L}t}$$

$$I_p = \frac{V_0}{R} \qquad I_g(t) = Ae^{-\frac{R}{L}t}$$

$I(t=0) = 0 \rightarrow A = -V_0/R$

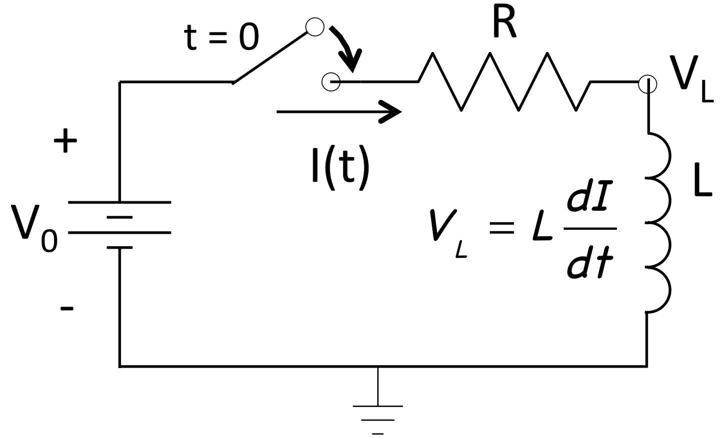
$$\boxed{I(t) = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right)}$$

Final State: $I(t \to \infty) = V_0/R$



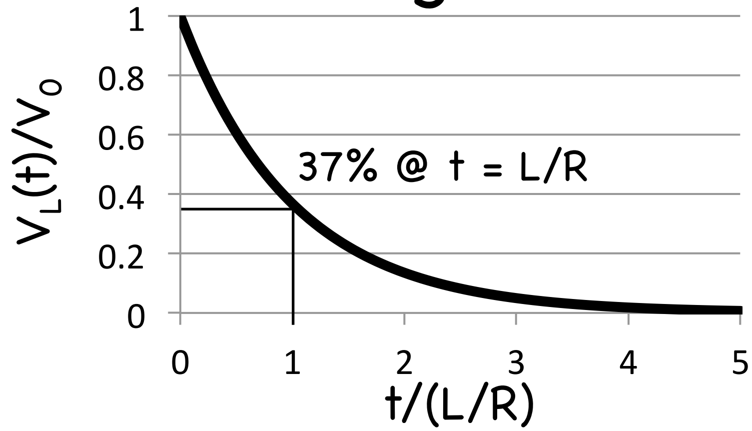
Two variations of the RL circuit …

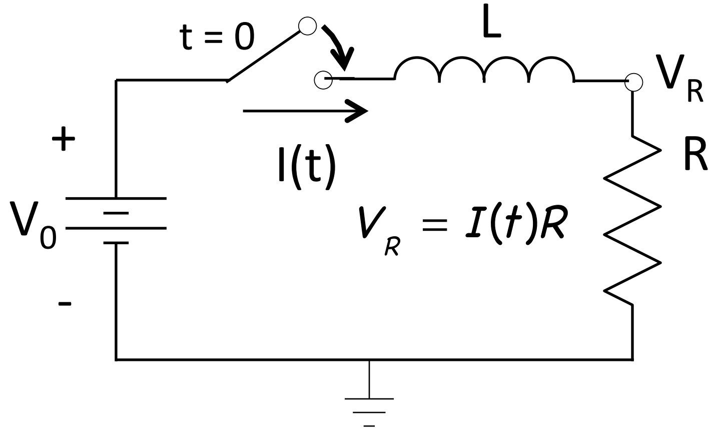
$$I(t) = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$



Open and close switch repeatedly.
Rapidly: $V_L \approx V_0$ Slowly: $V_L \approx 0$

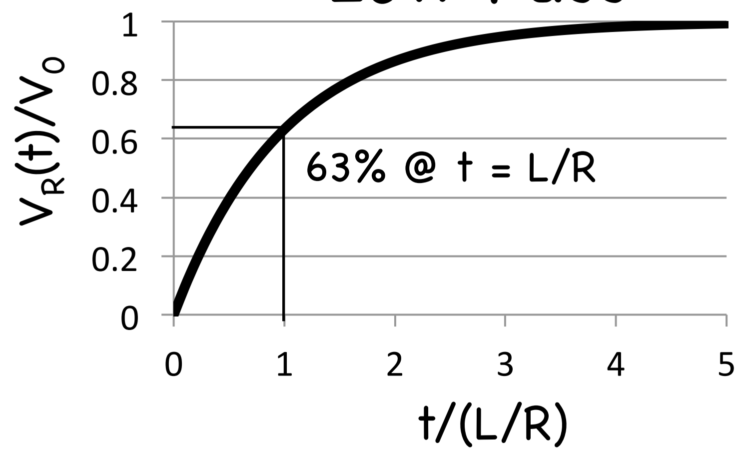
High Pass





Open and close switch repeatedly.
Rapidly: $V_R \approx 0$ Slowly: $V_R \approx V_0$

Low Pass

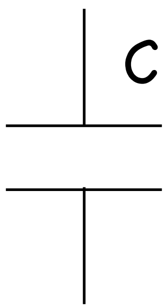


Using phasors in AC Circuit Analysis

Let... $I(t) = \tilde{I}(\omega)e^{j\omega t}$ $\quad V(t) = \tilde{V}(\omega)e^{j\omega t}$

Phasors: $\tilde{I}(\omega)$, $\tilde{V}(\omega)$ Phasors: time independent, complex

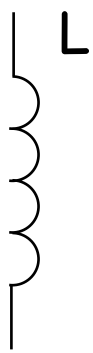
Notation: drop ω, $\tilde{I}(\omega) \to \tilde{I}$, $\tilde{V}(\omega) \to \tilde{V}$



$$I = C\frac{dV}{dt} \qquad \tilde{I}e^{j\omega t} = C\frac{d}{dt}\left(\tilde{V}e^{j\omega t}\right) = C\tilde{V}j\omega e^{j\omega t}$$

$$\tilde{I} = j\omega C\tilde{V} \qquad \tilde{V} = \frac{1}{j\omega C}\tilde{I}$$

Impedance: $\boxed{\dfrac{1}{j\omega C}}$

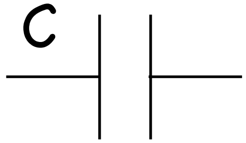
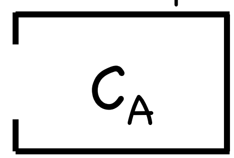
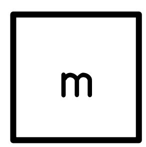
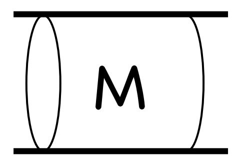


$$V = L\frac{dI}{dt} \qquad \tilde{V}e^{j\omega t} = L\frac{d}{dt}\left(\tilde{I}e^{j\omega t}\right) = L\tilde{I}j\omega e^{j\omega t}$$

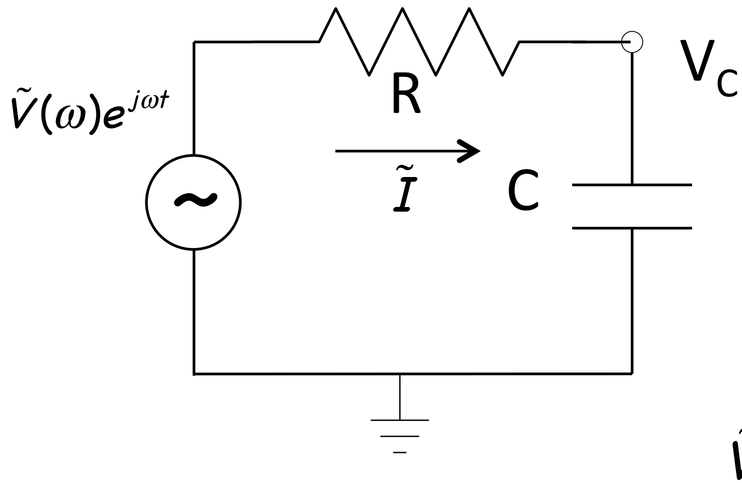
$$\tilde{V} = j\omega L\tilde{I}$$

Impedance: $\boxed{j\omega L}$

Generalized Impedance

	Electrical V = voltage I = current	Mechanical F = force v = velocity	Acoustic P = pressure u = flow velocity
Dissipation	R  $\tilde{V} = R\tilde{I}$	R_m  $\tilde{F} = R_m\tilde{v}$	R_A Acoustic Baffle  $\tilde{P} = R_A\tilde{u}$
Spring (potential)	C  $\tilde{V} = \frac{1}{j\omega C}\tilde{I}$	k  $\tilde{F} = \frac{k}{j\omega}\tilde{v}$	Acoustic “spring”  $\tilde{P} = \frac{C_A}{j\omega}\tilde{u}$
Inertia (kinetic)	L  $\tilde{V} = j\omega L\tilde{I}$	 $\tilde{F} = j\omega m\tilde{v}$	Mass of Air  $\tilde{P} = j\omega M\tilde{u}$

Circuit analysis example



Find V_C

KVL: $$\tilde{V} - \tilde{I}R - \frac{1}{j\omega C}\tilde{I} = 0$$

$$\tilde{I} = \frac{\tilde{V}}{R + 1/j\omega C}$$

$$\tilde{V}_c = \frac{1}{j\omega C}\tilde{I} = \frac{1/j\omega C}{R + 1/j\omega C}\tilde{V} = \frac{1}{j\omega CR + 1}\tilde{V}$$

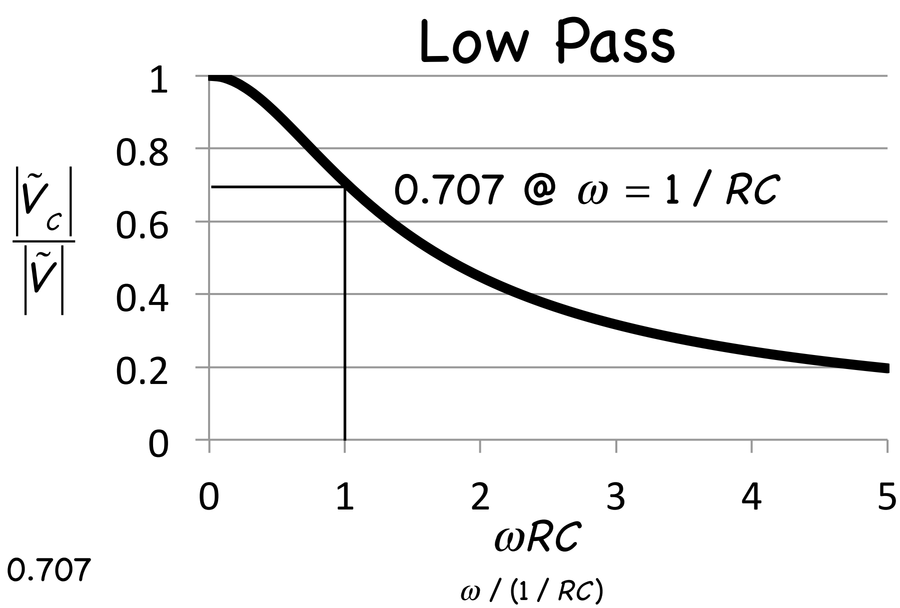
Compute magnitude of V_C

$$\frac{|\tilde{V}_c|}{|\tilde{V}|} = \left|\frac{1}{1 + j\omega CR}\right|$$

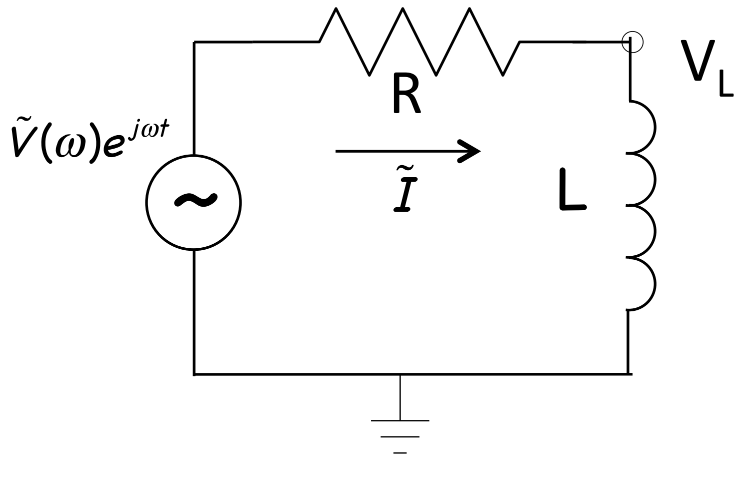
$$\frac{|\tilde{V}_c|}{|\tilde{V}|} = \left[\frac{1}{1 + j\omega CR} \cdot \frac{1}{1 - j\omega CR}\right]^{1/2}$$

$$\frac{|\tilde{V}_c|}{|\tilde{V}|} = \left[\frac{1}{1 + \omega^2 R^2 C^2}\right]^{1/2}$$

when $\omega = \frac{1}{RC} \rightarrow \left[\frac{1}{1+1}\right]^{1/2} = 0.707$



Another circuit analysis example



Find V_L

KVL: $\tilde{V} - \tilde{I}R - j\omega L\tilde{I} = 0$

$$\tilde{I} = \frac{\tilde{V}}{R + j\omega L}$$

$$\tilde{V}_L = j\omega L\tilde{I} = \frac{j\omega L}{R + j\omega L}\tilde{V} = \frac{j\omega\, L/R}{1 + j\omega\, L/R}\tilde{V}$$

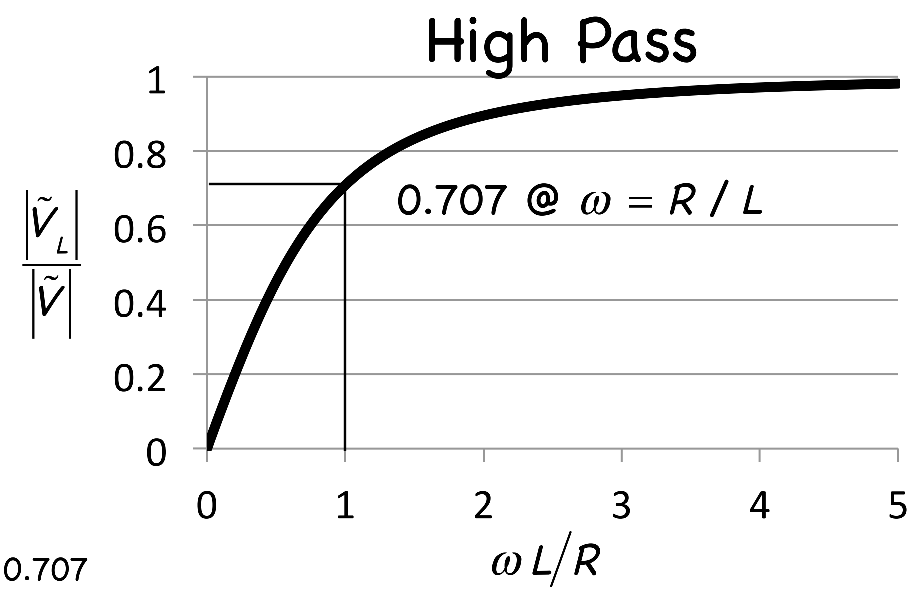
Compute magnitude of V_L

$$\frac{|\tilde{V}_L|}{|\tilde{V}|} = \left|\frac{j\omega\, L/R}{1 + j\omega\, L/R}\right|$$

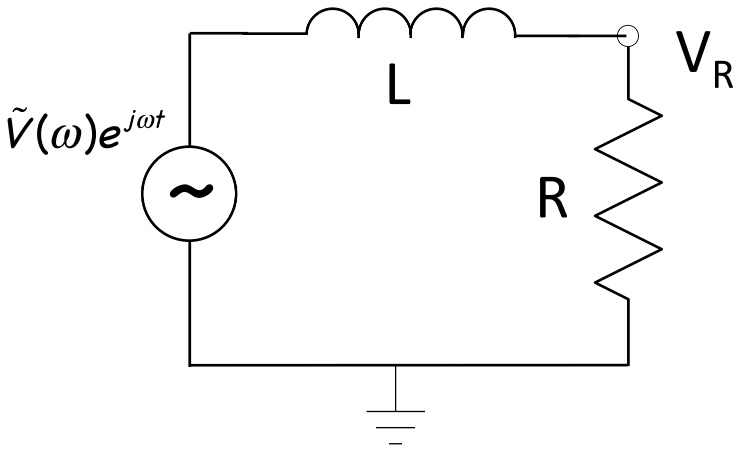
$$\frac{|\tilde{V}_L|}{|\tilde{V}|} = \left[\frac{j\omega\, L/R}{1 + j\omega\, L/R} \cdot \frac{-j\omega\, L/R}{1 - j\omega\, L/R}\right]^{1/2}$$

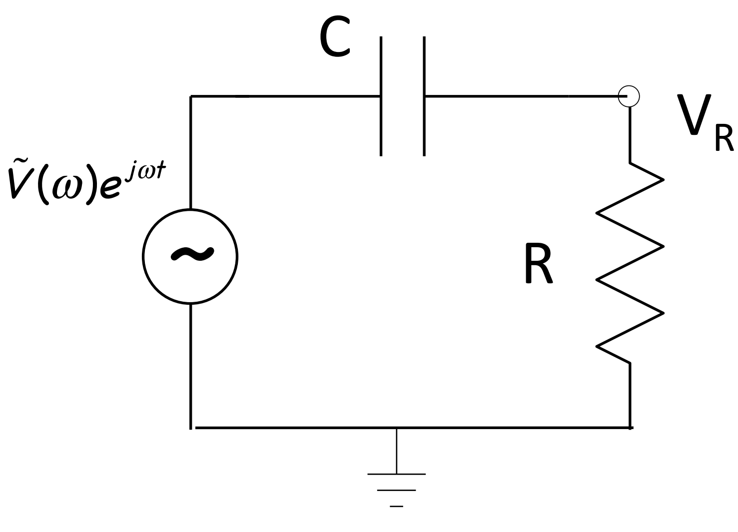
$$\frac{|\tilde{V}_L|}{|\tilde{V}|} = \left[\frac{\omega^2\, L^2/R^2}{1 + \omega^2\, L^2/R^2}\right]^{1/2}$$

when $\omega = \frac{R}{L} \rightarrow \left[\frac{1}{1+1}\right]^{1/2} = 0.707$

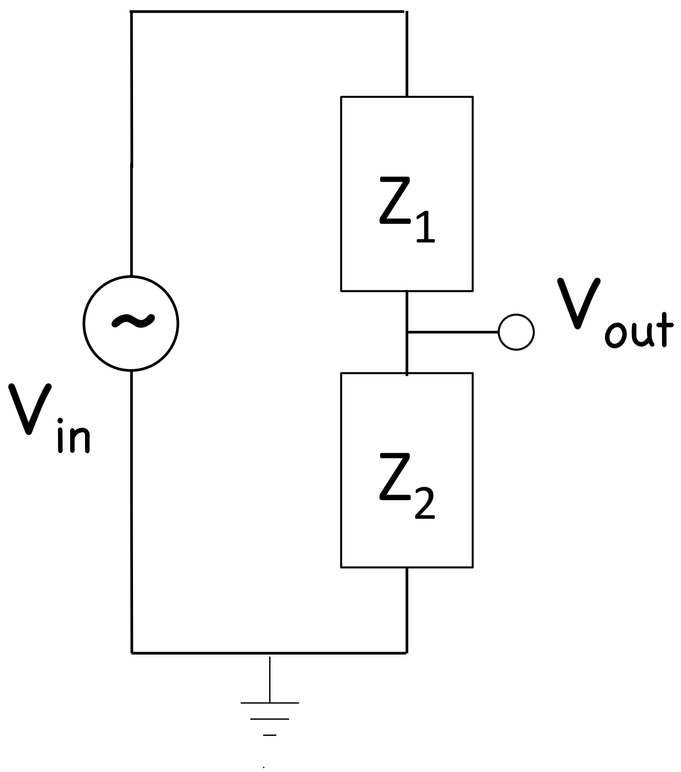


Find the transfer functions …





Filters are simply frequency dependent voltage dividers ...



$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{|Z_2|}{|Z_1 + Z_2|}$$

Z	**Z**	**Filter type**	**Cutoff Freq**
R	$\frac{1}{j\omega C}$	Low	$\frac{1}{2\pi}\frac{1}{RC}$
R	$j\omega L$	High	$\frac{1}{2\pi}\frac{R}{L}$
$\frac{1}{j\omega C}$	R	High	$\frac{1}{2\pi}\frac{1}{RC}$
$j\omega L$	R	Low	$\frac{1}{2\pi}\frac{R}{L}$