1. **Jupyter notebooks**

   Most exercises will take place as jupyter notebooks: A document format to combine rich text with formulas, python code and plots.

   (a) Install jupyter with python 3 by following the instructions on `http://jupyter.org/install`. (0)

   (b) Learn how to start a notebook.. (0)

   (c) Work through the notebook `IntroductionToJupyterNotebooks.ipynb`. (0)

   (d) For course improvements, we would like your feedback about this question. At least tell us how much time you did invest, if you had major problems and if you think it's useful.

   Points for Question 1: 0

Note: The following questions are a repetition of linear algebra. Please also write down interim solutions. E.g. don't just use `numpy.linalg.norm(...)` to solve the question about vector norms.

2. **Eigenvalue Decomposition**

   Given the matrix

$$\mathbf{A} = \frac{1}{4}\begin{bmatrix} 7 & -\sqrt{3} \\ -\sqrt{3} & 5 \end{bmatrix}$$

   (a) Compute the eigenvalue decomposition $\mathbf{A} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^T$ with $\Lambda = \text{diag}(\lambda_1, \lambda_2)$. (2)

   (b) Show that the columns of $\mathbf{Q}$ are orthonormal, i.e. the columns are of unit length and orthogonal. (1)

   (c) Show that the matrix $\mathbf{Q}\mathbf{\Lambda}^{-1}\mathbf{Q}^T$ with $\mathbf{\Lambda}^{-1} = \text{diag}(\lambda_1^{-1}, \lambda_2^{-1})$ is the inverse of $\mathbf{A}$ (1)

   (d) For course improvements, we would like your feedback about this question. At least tell us how much time you did invest, if you had major problems and if you think it's useful.

   Points for Question 2: 4

3. **Matrix inversion**

   Given the matrix

$$\mathbf{A} = \begin{bmatrix} 3 & 4 \\ 6 & 13 \end{bmatrix}$$

   (a) Compute the LU decomposition of it, i.e. $\mathbf{A} = \mathbf{L} \cdot \mathbf{U}$, where $\mathbf{L}$ is a lowertriangular matrix and $\mathbf{U}$ is an uppertriangular matrix. Use this decomposition to solve $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$ with $\mathbf{b} = (1, 2)^T$ via forward and backward substitution. (2)

   (b) Use Gauss elimination to explicitly calculate the inverse of $\mathbf{A}$ and show that it yields the same solution $\mathbf{x} = \mathbf{A}^{-1} \cdot \mathbf{b}$ from part (a). (2)

   (c) For course improvements, we would like your feedback about this question. At least tell us how much time you did invest, if you had major problems and if you think it's useful.

   Points for Question 3: 4

4. **Vector Norms**

   The length of a vector is not a single number but can be defined in different ways. These vector norms share common properties but also have different characteristics.

   (a) Compute the $\|\cdot\|_1$, $\|\cdot\|_2$, $\|\cdot\|_8$ and $\|\cdot\|_\infty$ norms of the following vectors: (2)

$$\mathbf{x}_1 = \begin{bmatrix} 24 \\ 3 \\ 2 \\ 31 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 27 \\ 20 \\ 26 \\ 21 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 30 \\ 21 \\ 27 \\ 5 \end{bmatrix}, \quad \text{and} \quad \mathbf{x}_4 = \begin{bmatrix} 26 \\ 28 \\ 25 \\ 14 \end{bmatrix}.$$

   (b) Draw the set of points with $\|\mathbf{x}\|_i = 1$ for $i \in \{1, 2, 8, \infty\}$ and $\mathbf{x} \in \mathbb{R}^2$ (2)

(c) For course improvements, we would like your feedback about this question. At least tell us how much time you did invest, if you had major problems and if you think it's useful.

Points for Question 4: 4

5. **Special Orthorgonal Matrices**

A very important class of matrices are the special orthogonal group $SO(n)$. These matrices are characterized by a unit determinant and correspond to rotations around the origin.

(a) Given the matrix (2)

$$\mathbf{A} = \frac{1}{2}\begin{bmatrix} 7 & \sqrt{3} \\ \sqrt{3} & 5 \end{bmatrix}$$

compute $\det(\mathbf{A})$, $\mathrm{Tr}(\mathbf{A})$ and its eigenvalues.

(b) The matrix (1)

$$\mathbf{Q}(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

represents a rotation around an axis by the angle α. Compute $\mathbf{A}' = \mathbf{Q}\cdot\mathbf{A}\cdot\mathbf{Q}^T$, $\det(\mathbf{A}')$, $\mathrm{Tr}(\mathbf{A}')$ and its eigenvalues for $\alpha = \pi/12$.

(c) What would $\mathbf{A}'$ be for $\alpha = \pi/3$? (1)

(d) For course improvements, we would like your feedback about this question. At least tell us how much time you did invest, if you had major problems and if you think it's useful.

Points for Question 5: 4

You can achieve a total of **16 points** for this exercise. Additionally you can achieve **1 bonus point** for answering the feedback questions.

Please send a **single solution** as a **group of three** via ILIAS until **29.10.2018 12 pm**. You can hand in a jupyter notebook (**recommended**), a pdf or a scan (please write legibly).

Note: Jupyter notebooks will be executed **from top to bottom**. To **avoid point deduction** check your notebook by the following steps: 1. Use the python 3 kernel (`Kernel > Change kernel > Python 3`), 2. Run the full notebook (`Kernel > Restart & Run All`), 3. Save (`File > Save and Checkpoint`).