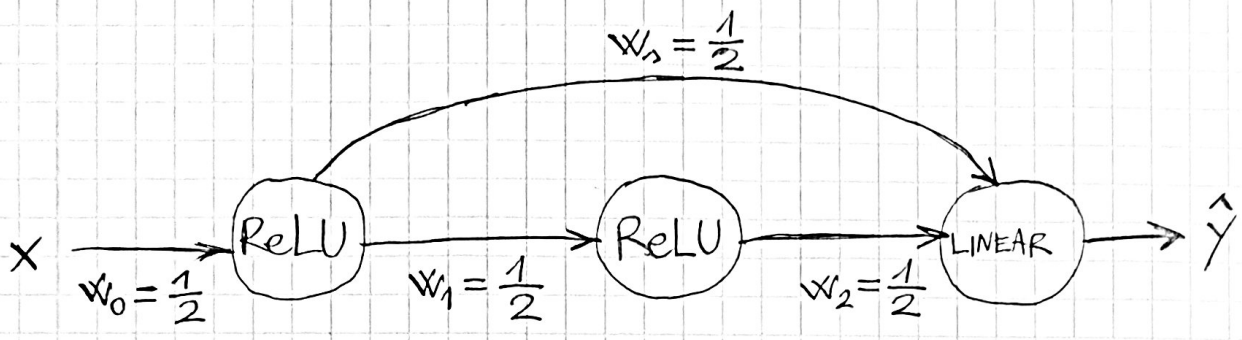
# FOUNDATIONS OF DEEP LEARNING
## - EXERCISE 2




### FORWARD PASS  $X=1$, $y=-3$

$$z_0 = w_0 X = \frac{1}{2}$$
$$h_0 = ReLU(z_0) = \frac{1}{2}$$

$$z_1 = w_1 h_0 = \frac{1}{4}$$
$$h_1 = ReLU(z_1) = \frac{1}{4}$$

$$z_2 = w_2 h_1 + w_3 h_0 = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$$
$$h_2 = z_2 = \frac{3}{8}$$

$$\hat{y} = \frac{3}{8}$$

### LOSS

$$L(\hat{y}, y) = |y - \hat{y}| = \begin{cases} y - \hat{y}, & y \geq \hat{y} \\ \hat{y} - y, & y < \hat{y} \end{cases} = \left| -3 - \frac{3}{8} \right| = \frac{27}{8}$$

### BACKWARD PASS

- WEIGHT $w_2$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_2} = \frac{\partial L}{\partial z_2} \cdot h_1 = \frac{1}{4}$$

$$\frac{\partial L}{\partial z_2} = \frac{\partial L}{\partial \hat{y}} = \begin{cases} -1, & y \geq \hat{y} \\ +1, & y < \hat{y} \end{cases} = 1$$

- WEIGHT $W_2$

$$\frac{\partial L}{\partial W_2} = \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial W_2} = \frac{\partial L}{\partial z_2} \cdot h_0 = \frac{1}{2}$$

- WEIGHT $W_1$

$$\frac{\partial L}{\partial W_1} = \frac{\partial L}{\partial z_1} \cdot \frac{\partial z_1}{\partial W_1} = \frac{\partial L}{\partial z_1} \cdot h_0 = \frac{1}{4}$$

$$\frac{\partial L}{\partial z_1} = \frac{\partial L}{\partial h_1} \cdot \frac{\partial h_1}{\partial z_1} = \frac{1}{2}$$

$$\frac{\partial L}{\partial h_1} = \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial h_1} = \frac{\partial L}{\partial z_2} \cdot W_2 = \frac{1}{2}$$

$$\frac{\partial h_1}{\partial z_1} = \left\{ \begin{matrix} 1, & z_1 \geq 0 \\ 0, & z_1 < 0 \end{matrix} \right\} = 1$$

- WEIGHT $W_0$

$$\frac{\partial L}{\partial W_0} = \frac{\partial L}{\partial z_0} \cdot \frac{\partial z_0}{\partial W_0} = \frac{\partial L}{\partial z_0} \cdot x = \frac{3}{4}$$

$$\frac{\partial L}{\partial z_0} = \frac{\partial L}{\partial h_0} \cdot \frac{\partial h_0}{\partial z_0} = \frac{3}{4}$$

$$\frac{\partial L}{\partial h_0} = \frac{\partial L}{\partial z_1} \cdot \frac{\partial z_1}{\partial h_0} + \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial h_0}$$

$$= \frac{\partial L}{\partial z_1} \cdot W_1 + \frac{\partial L}{\partial z_2} \cdot W_2$$

$$= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} = \frac{3}{4}$$

$$\frac{\partial h_0}{\partial z_0} = \left\{ \begin{matrix} 1, & z_0 \geq 0 \\ 0, & z_0 < 0 \end{matrix} \right\} = 1$$

## SKIP CONNECTION

Skip connection makes it easier for the gradient to flow backwards in order to decrease the effect of vanishing gradient.

## GRADIENT DESCENT STEP

$$W_2 \leftarrow W_2 - \frac{\partial L}{\partial W_2} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$W_3 \leftarrow W_3 - \frac{\partial L}{\partial W_3} = \frac{1}{2} - \frac{1}{2} = 0$$

$$W_1 \leftarrow W_1 - \frac{\partial L}{\partial W_1} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$W_0 \leftarrow W_0 - \frac{\partial L}{\partial W_0} = \frac{1}{2} - \frac{3}{4} = -\frac{1}{4}$$

## FORWARD PASS (AGAIN)

$$z_0 = W_0 X = -\frac{1}{4}$$

$$h_0 = \text{ReLU}(z_0) = 0$$

---

$$z_1 = W_1 h_0 = 0$$

$$h_1 = \text{ReLU}(z_1) = 0$$

---

$$z_2 = W_2 h_1 + W_3 h_0 = 0$$

$$h_2 = z_2 = 0$$

---

$$\hat{y} = 0$$

## LOSS (AGAIN)

$$L(\hat{y}, y) = |y - \hat{y}| = |-3 - 0| = 3$$