ECSE 403 Winter 2024
Lab assignment 1

Instructor: Prof. Caines
Lab TAs: Borna Sayedana, Jilan Samiuddin
Due date: January 19^{th}

August 21, 2024

1 Introduction

1.1 Lab Objectives

The main goal of this assignment is to (i) review some concepts from the linear systems and control course (ECSE 307), and (ii) learn some useful tools in Matlab which helps us in designing and implementing controllers.

1.2 Assignment

Students should answer all the questions and submit their answer in addition to Matlab codes to MyCourses. The submitted assignment should be in PDF format.

1.3 Model Description

The cart-pole system in the lab consists of two gears, a DC motor and sensors. The equation of motion of the DC motor is described by the following equations:

$$J_m\ddot{\theta} + (b + \frac{K_t K_e}{R_a})\dot{\theta} = \frac{K_t}{R_a}v_a$$

where θ is the shaft angle (in radians) of the motor and v_a is the voltage which is applied to the motor. The parameters of the system which are used in above equation are as following:

- $J_m = 0.01kgm^2$: the inertia of the rotor and the shaft.
- $b = 0.001Nmsec$: the viscous friction coefficient
- $K_e = 0.02Vsec$: the back emf constant
- $K_t = 0.02Nm/A$: the motor's torque constant.
- $R_a = 10\Omega$: the armature resistance

Note that using SI units $K_e = K_t$. Since in all the subsequent labs we use the equation of DC motor as a component of cart-pole system, in this lab we derive the transfer function of the DC motor, and look at its open-loop and closed-loop step responses.

2 Questions

2.1 Modeling and Programming Questions

1. Find the transfer function between the input voltage and the speed of the motor shaft, i.e.:

$$G(s) = \frac{W(s)}{V_a(s)} \quad \text{where,} \quad w = \dot{\theta}.$$

   (Take the Laplace transform of motor's equation and plug in the motors' coefficients).[5 marks]

2. Using the transfer function $G(s)$ and Matlab, plot the step response of $G(s)$ and find the steady-state response to the step input and the time constant of $G(s)$.[10 marks]
   Hint: Suppose you want to find the step response of the transfer function:

$$G(s) = \frac{s}{s^2 + 4s + 1}$$

   You can use following commands for defining a transfer function in Matlab:

```
s = tf('s')
G = s/(s^2 + 4*s+1)
```

   Or instead, you can just use polynomial coefficients.

```
G = tf([1],[1 4 1])
```

   After defining the transfer function $G(s)$, you can plot the step response using:

```
stepplot(G)
```

   You can get the information regarding the step response using:

```
stepinfo(G)
```

3. Write the definition of the rise time and the 2% settling time. Using the step response of $G(s)$, find the rise time and settling time of the system.[10 marks]

4. Using the final value theorem, theoretically compute the steady state speed of the motor to step response. Compare the theoretical value and the value you found in Question 2.[10 marks]

5. Find the transfer function between the shaft's angel and input voltage, i.e.:

$$F(s) = \frac{\Theta(s)}{V_a(s)}.$$

   Identify the order of the system with respect to new definition of input-output signals. [5 marks]

6. Consider the transfer function $G(s)$ in Question 1. Apply a unity feedback loop to the system and find the closed-loop transfer function. [10 marks]
   Hint: In Matlab, you can find the closed-loop transfer function of a unity feedback loop applied to the open-loop system $G(s)$, by using the following command:

```
G_cl = feedback(G,1).
```

7. Suppose a proportional controller is added to the system such that the open-loop transfer function is now $KG(s)$ instead of $G(s)$. (i) Plot the step response of the system for $K = \{0.1, 1, 10, 100\}$, in one figure. (ii) Describe the effect of proportional gain on behaviour of the closed-loop step response [15 marks].
   Hint: One way to plot different graphs on one figure in Matlab is as following:

```
figure(1);
stepplot(h1);
hold on;
stepplot(h2);
.
.
hold off;
```

8. Consider the transfer function $F(s)$ in Question 5. (i) Repeat steps of Question 7 for this system. (ii) Describe the effect of proportional gain on step response's behaviour [15 marks]. In this case you should explain the effect of proportional gain on the overshoot, rise-time, and 2% settling time.

2.2 Laboratory Experiments

1. Follow the lab manual instructions and add the model of the cart-pole system in the Simulink. Apply a sinusoidal input wave with amplitude 1.5 and frequency $1Hz$ to the cart-pole system. Plot the position of the cart-pole system on the scope in the Simulink. [20 marks]

ECSE 403 Winter 2023
Lab assignment 2

Instructor: Prof. Caines
Lab TAs: Borna Sayedana, Jilan Samiuddin
Due date: January 26^{th}

August 21, 2024

1 Introduction

1.1 Lab Objectives

The main goal of this assignment is to (i) implement control blocks in Simulink, (ii) implement unity feedback controller for the position and velocity of the cart-pole system as a disturbance rejection and reference tracking controllers.

1.2 Assignment

Students should answer all the questions and submit their answer in addition to screen shot of block diagrams and wave signals to MyCourses. The submitted assignment should be in PDF format.

1.3 Model Description

The cart-pole system in the lab consists of two gears, a DC motor and sensors. The equation of motion of the DC motor is described by the following equations:

$$J_m\ddot{\theta} + (b + \frac{K_t K_e}{R_a})\dot{\theta} = \frac{K_t}{R_a} v_a$$

where θ is the shaft angle (in radians) of the motor and v_a is the voltage which is applied to the motor. The parameters of the system which are used in above equation are as following:

- $J_m = 0.01kgm^2$: the inertia of the rotor and the shaft.
- $b = 0.001Nmsec$: the viscous friction coefficient
- $K_e = 0.02Vsec$: the back emf constant
- $K_t = 0.02Nm/A$: the motor's torque constant.
- $R_a = 10\Omega$: the armature resistance

Note that using SI units $K_e = K_t$. Since in all the subsequent labs we use the equation of DC motor as a component of cart-pole system, in this lab we derive the transfer function of the DC motor, and look at its open-loop and closed-loop step responses.

2 Questions

2.1 Modeling Questions

1. Implement the transfer function you derived in previous lab $G(s) = \frac{W(s)}{V_a(s)}$ in Simulink. You can use blocks *Transfer Fcn* from *continuous* library, *scope* from *Sinks* library, and *Step*, from *Sources* library. [5 marks]

2. Using *Step*, find the rise-time and steady-state response of the system to a unit step function. Compare these values with the values you found in the previous lab. [5 marks].

2.2 Laboratory Experiments

1. Find the position gain which converts the output of the sensor to the position of the cart in cm using a ruler.[10 marks]

2. Let the position of the cart be the system's output $x(t)$, and the voltage be input $v_a(t)$, plot the step response of the cart-pole system. Explain qualitatively the behaviour of the step response.[10 marks]

3. Let the velocity of the cart be the system's output $\dot{x}(t)$, and the voltage be input $v_a(t)$, plot the step response of the cart-pole system. Explain qualitatively the behaviour of the step response.[10 marks]
   Hint: Note that in order to achieve the velocity of the cart you need to differentiate cart's position in the Simulink.

4. Explain why the observed velocity on the scope is noisier than the position.[10 marks]

   Hint: You can refer to lab manual to find a way to filter out the noise in the velocity signal.

5. Estimate the coefficients of the step response $\dot{x}(t)/v_a(t)$ by setting the input to be a step function and measuring the time constant and steady state of the step response. [10 marks]

6. Implement a unity feedback controller for the position control. Plot response of the system $x(t)$ to a change in cart's position $\Delta x(t)$ and setting the reference signal to 0. Explain how this controller is a disturbance rejection controller.[10 marks]

7. Select (i) square wave and (ii) sine wave functions as reference signal $r(t)$. Plot response of the system $x(t)$ to the reference signal $r(t)$. Explain how this controller is a reference tracking controller.[10 marks]

8. Implement a unity feedback controller for the velocity control. Plot response of the system $\dot{x}(t)$ to a change in cart's position $\Delta x(t)$ and setting the reference signal to 0. Explain how this controller is a disturbance rejection controller.[10 marks]

9. Select (i) square wave and (ii) sine wave function as a reference signal $r(t)$. Plot response of the system $\dot{x}(t)$ to the reference signal $r(t)$. Explain how this controller is a reference tracking controller.[10 marks]

ECSE 403 Winter 2023
Lab assignment 3

Instructor: Prof. Caines
Lab TAs: Borna Sayedana, Jilan Samiuddin
Due date: February 2^{nd}

August 21, 2024

1 Introduction

1.1 Lab Objectives

The main goal of this assignment is to (i) test the linearity of the system, (ii) implement a frequency response system identification method

1.2 Assignment

Students should answer all the questions and submit their answer in addition to screen shot of block diagrams and wave signals to MyCourses. The submitted assignment should be in PDF format.

2 Experiments

1. Let the position of the system $x(t)$ be the output of the system. Choose two arbitrary input signals $v_1(t)$ and $v_2(t)$. (i) Plot the system response to these signals $H\{v_1(t)\}$ and $H\{v_2(t)\}$. (ii) Plot an arbitrary linear combination of the response signals $aH\{v_1(t)\} + bH\{v_2(t)\}$. (iii) Plot the response of the system to linear combination of the input signal i.e. $H\{av_1(t) + bv_2(t)\}$. (iv) By comparing two signals $H\{av_1(t) + bv_2(t)\}$ and $aH\{v_1(t)\} + bH\{v_2(t)\}$, verify the linearity of the system.[15 marks]
   Hint: (i) In order to export the data from Simulink environment you can use *To Works space* block. OR, (ii) Go to *Settings* of the scope, then under the tab *Logging*, check *Log data to workspace* and set the *Save format* to *Array*.

2. Suppose we observe a difference between the signals $H\{av_1(t) + bv_2(t)\}$ and $aH\{v_1(t)\} + bH\{v_2(t)\}$. Does that imply that the system is non-linear? Is it possible to argue that the cart-pole system is linear but this non-linearity effect exists because of a hidden linear feedback loop in the system? [10 marks]

3. Based on the theory of LTI systems, complex exponential signals e^{jwt} are eigenfunctions of LTI systems. This implies that LTI systems (in the steady-state response) only affect the gain and the phase of since waves (but not the type nor frequency of sine waves). (i) Explain the meaning of eigenfunction in the context of system theory. (ii) Measure amplitude and phase shift of the system for input of sine wave with amplitude 2 and

frequency of 1 Hz. Describe how the output signal is uniquely characterized with the amplitude and phase shift.[10 marks]

4. Plot response of the system to a square wave with amplitude 2 and frequency of 1 Hz. Is the square wave an eigenfunction for the LTI system?[10 marks]

5. Identify possible sources of non-linearity in the cart-pole system. [10 marks]

6. Let the position of the system $x(t)$ be the output of the system. Let the input signal be a sine wave with frequencies $w = [1, 5, 10, 50, 100]$ (rad/s), measure the gain of the system at these frequencies. Plot the magnitude bode diagram of the system using these measurements. [15 marks]
   Hint: Note that you have to use $w = log_{10}(input frequencies)$ as the x-axis and $20.log(gain)$ as y-axis to plot the magnitude Bode diagram from the data points.

7. Let the velocity of the system $\dot{x}(t)$ be the output of the system. Let the input signal be a sine wave with frequencies $w = [1, 5, 10, 50, 100]$ (rad/s), measure the gain of the system at these frequencies. Plot the magnitude Bode diagram of the system using these measurements. [15 marks]

8. Use the estimated coefficient of the step response $\dot{x}(t)/v(t)$ that you found in Question 5 of Lab 2.(i) Plot the bode diagram of the estimated system using Matlab. (ii) Compare this Bode diagram with the Bode diagram you found in Question 6. [15 marks]

ECSE 403 Winter 2023
Lab assignment 4

Instructor: Prof. Caines
Lab TAs: Borna Sayedana, Jilan Samiuddin
Due date: February 9^{th}

August 21, 2024

1 Introduction

1.1 Lab Objectives

The main goal of this assignment is to (i) derive the theoretical transfer function of cart-pole system, (ii) design the proportional controllers using root locus diagram, and (iii) implement a proportional controller for the cart-pole system.

1.2 Assignment

Students should answer all the questions and submit their answer in addition to screen shot of block diagrams and wave signals to MyCourses. The submitted assignment should be in PDF format.

1.3 Model description

A simplified electrical model of the DC motor of the cart-pole system in the lab is shown in Figure 1.

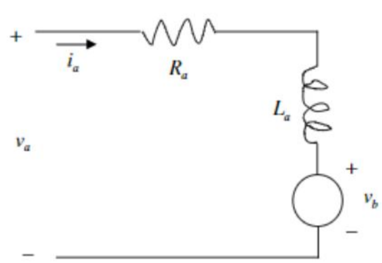

Figure 1: Simplified model of the DC motor

By applying KVL and substituting back electromotive force, we have :

$$v_a = R_a i_a + L_a \frac{d}{dt} i_a + v_b, \qquad v_b = K_e \omega_m \tag{1}$$

where v_a is the armature voltage, i_a is the armature current, R_a is armature resistance, L_a is armature inductance, and ω_m is the angular velocity of the motor shaft. For the motor torque, we have:

$$\tau_m = K_m i_a \tag{2}$$

The model of the cart is shown in Figure 2. By applying Newton's second law of motion, to the cart we get:

$$F_c - F_f = m_c \ddot{x}_c \tag{3}$$

where F_c is the force applied on the cart by the motor, F_f is the friction force, and x_c is the linear position of the cart. Moreover, $x_c = r_g \theta_g$ where r_g is the radius of the pinion and θ_g is the angle of the pinion. let the angular

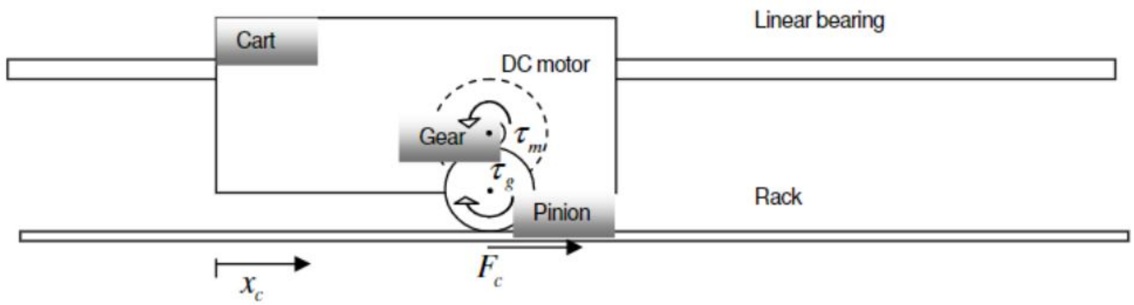

Figure 2: model of the cart

velocity of the pinion be $\omega_g = d\theta_g/dt$. The angular velocity and torque of the pinion and the motor are related as following:

$$\omega_g = \frac{1}{K_g}\omega_m, \qquad \tau_m = \frac{1}{K_g}\tau_g \tag{4}$$

where K_g is the gear ratio. We can write the force applied to the cart as:

$$F_c = \frac{1}{r_g}\tau_g \tag{5}$$

Following constants are from lab manual.

- $K_m = K_e = 0.0077 N.m/amp$: Motor and back EMF constants
- $R_a = 2.6 ohms$: Armature resistance
- $L_a = 180 \mu Henry$: Armature inductance
- $K_g = 3.7/1$: Internal gear ratio
- $r_g = 0.0064m$: Motor gear radius
- $m_c = 0.526kg$: Cart mass

2 Questions

2.1 Modeling and Programming Questions

1. Let the friction F_f in Equation (3) be negligible. (i) Using Equations (2),(4),(5) and substituting back in Equation (3), find the ratio $i_a/\ddot{x}_c$ in terms of constants of the cart system. (ii) Substitute the expression for i_a into Equation (1) to find the ODE describing v_a in terms of x and its derivatives.[25 marks]

2. Take the Laplace transform of the ODE you found in Question 1 to find the transfer function. what is the order of the system.[10 marks]

$$F(s) = \frac{X_c(s)}{V_a(s)}$$

3. Using the fact that $L_a \ll R_a$, (i) find a second order approximation of the system $G(s)$. (ii) Plot the step response of accurate model and approximate model and compare the two responses. [10 marks]

4. For the rest of the lab, for simplicity, use the **Approximate** transfer function $G(s)$. Let $v_c(t) = \dot{x}(t)$ denote the velocity of the cart. (i) Find the transfer function with the velocity as the output.

$$H(s) = \frac{V_c(s)}{V_a(s)}$$

   (ii) Plot and compare the step response of $H(s)$ and step response of transfer function that you found experimentally in Question 5 of Lab 2.[10 marks]

5. Consider the transfer function $G(s)$. (i) Explain what root locus diagram depicts. (ii) Plot the root locus of $G(s)$ in Matlab. Is there any value of the gain for which the closed loop system of the cart pole system becomes unstable? [10 marks]

6. Consider the transfer function $H(s)$. (i) Plot the root locus of $H(s)$ in Matlab. Is there any gain for which the closed loop system of the cart pole system becomes unstable? [10 marks]

2.2 Experiments

1. Use the root locus diagram in Question 5, find the proportional gain K, which results in the fastest rise time with no overshoot. Implement a proportional controller in Simulink for the position of the cart system with proportional gain K. [10 marks]

2. Choosing the gains $K \in \{0.1, 0.2, 0.5, 1, 2\}$ describe the effect of increasing the proportional gain on the step response of the cart-pole system. Describe how this behaviour matches with the root locus diagram. [15 marks]
   Hint: You can use a square wave with low frequency instead of step input.

ECSE 403 Winter 2024
Lab assignment 5

Instructor: Prof. Caines
Lab TAs: Borna Sayedana, Jilan Samiuddin
Due date: February 16^{th}

August 21, 2024

1 Introduction

1.1 Lab Objectives

The main goal of this assignment is to (i) design proportional controllers using Matlab PID tuner, (ii) design proportional-derivative controllers using Matlab PID tuner, (iii) implement proportional controller for the position control of the cart, and (iv) implement proportional-derivative controller for the position control of the cart system.

1.2 Assignment

Students should answer all the questions and submit their answer in addition to screen shot of block diagrams and wave signals to MyCourses. The submitted assignment should be in PDF format.

2 Questions

2.1 Proportional controller

1. (i) Use the transfer function $G(s)$ that you found in Lab 4, Question 2.1.3. Implement a PID controller using the blocks : *PID controller*, *Transfer Function* $= G(s)$ and a unity feedback structure. (ii) Set the controller type to the proportional controller (**P**) and use the PID tuner to find the proportional gain P which decreases the rise time by 50% compare to the unity feedback loop. (iii) What is the effect of the decrease in the rise time on settling time and the percentage of the overshoot? (iv) Plot the response of the system to a square wave with frequency $0.25Hz$. [15 marks]

2. (i) Implement the previous proportional controller for the cart system. (ii) Where should the saturation block be implemented in the loop to prevent the controller to send high voltage inputs to the DC motor? (iii) Explain how we can incorporate the constraints on the cart's input voltage in the PID tuning? [15 marks]

3. (i) Repeat the steps of Question 1 to find the gain P which makes the percentage of the overshoot under 5%. (ii) What is the effect of the decrease in the percentage of the overshoot on settling time and the rise time of the closed loop system? [10 marks]

4. (i) Implement the previous proportional controller for the cart system. (ii) Plot the response of the system to a square wave with frequency $0.25Hz$, (iii) Tune the coefficients of the P controller manually to meet the design specification in Question 3. [10 marks]

2.1.1 Proportional-Derivative Controller

5. (i) Repeat the steps of Question 1. Set the controller type to the proportional-derivative controller (**PD**) and use the PID tuner to find the coefficients P, N, D which decreases the percentage of the overshoot to under 5% without any change in the rise time. (ii) Make sure that the designed controller's output is within the range of the saturation block. [10 marks]

6. (i) Implement the previous proportional controller for the cart system. (ii) Plot the response of the system to a square wave with frequency $0.25Hz$, (iii) Tune the coefficients of the PD controller manually to make sure that the percentage of the overshoot is under 5%. [10 marks]

7. (i) Repeat the steps of Question 5. Find the coefficients P, N, D which makes the rise-time of the closed-loop system $0.1s$ and the percentage of the overshoot $< 1\%$. (ii) Make sure that the designed controller's output is within the range of the saturation block. [10 marks]

8. (i) Implement the previous proportional controller for the cart system. (ii) Plot the response of the system to a square wave with frequency $0.25Hz$, (iii) Tune the coefficients of the PD controller manually to meet the design specification in Question 7. [10 marks]

ECSE 403 Winter 2024
Lab assignment 6

Instructor: Prof. Caines
Lab TAs: Borna Sayedana, Jilan Samiuddin
Due date: February 23^{rd}

August 21, 2024

1 Introduction

1.1 Lab Objectives

The main goal of this assignment is to (i) design Proportional-Derivative-Integral (PID) controller for the position control of the cart system using PID tuner (ii) implement PID controller for the position control of the cart system, (iii) design PID controller for the velocity control of the cart system using PID tuner (iv) implement PID controller for the velocity control of the cart system.

1.2 Assignment

Students should answer all the questions and submit their answer in addition to screen shot of block diagrams and wave signals to MyCourses. The submitted assignment should be in PDF format.

2 Questions

2.1 Proportional controller

1. (i) Use the transfer function $G(s)$ that you found in Lab 4, Question 2.1.3. Implement a PID controller using the blocks : *PID controller*, *Transfer Function* $= G(s)$ and a unity feedback structure. (ii) Set the controller type to the **PID** controller and use the PID tuner to find parameters P, N, D, I which makes the percentage of the overshoot under 10% and the rise time under $0.1s$. [10 marks]

2. (i) Implement the previous PID controller for the position control of the cart system. (ii) Plot the response of the system to a square wave with frequency $0.25Hz$. (iii) Find the rise time and percentage of the overshoot of the closed loop cart system to the step response . [10 marks]

3. (i) Use the transfer function for the velocity of cart system $H(s)$ that you found in Lab 4, Question 2.1.4. (i) Repeat the steps of Question 1 and set the controller type to **P**. (ii) Find the proportional gain P which makes the rise time under $0.05s$. (iii) What is the effect of increasing the proportional gain on the rise-time and steady state-error? [10 marks]

4. (i) Add the noise cancelling filter for the velocity signal to your model. (ii) Repeat the steps of Question 1 and set the controller type to **P**. Find the proportional gain P which makes the rise time under $0.05s$ and the percentage of the overshoot under 5%. (iii) What is the effect of increasing the proportional gain on the rise-time, over-shoot and steady-state error? [10 marks]

5. (i) Explain the effect of adding the filter on the closed loop step response by comparing the results of Questions 3 and 4. (ii) Using the pole-zero plots, explain why the effect of the filter can be neglected in the open loop step response. (iii) Using the root-locus diagram, explain why the effect of velocity filter becomes non-negligible in the closed loop dynamics. [15 marks]
   Hint : You can plot the pole-zero plot in Matlab using the command *pzplot*.

6. (i) Implement the proportional controller in Question 4 for the velocity control of cart system. (ii) Plot the response of the system to a square wave with frequency $0.25Hz$. (iii) Find the rise time, overshoot and the steady state error of the cart system. [15 marks]

7. (i) For the velocity control of the cart system (with the noise cancelling filter), set the controller type to **PID** and find the parameters P, I, N, D which makes the rise time under $0.1s$, percentage of the overshoot under 1% and the steady-state error $= 0$. [15 marks]

8. (i) Implement the PID controller in Question 7 for the cart system. (ii) Plot the response of the system to a square wave with frequency $0.25Hz$. (iii) Find the rise time, overshoot and the steady state error of the closed loop cart system for the step response. [15 marks]

ECSE 403 Winter 2024
Lab assignment 7

Instructor: Prof. Caines
Lab TAs: Borna Sayedana, Jilan Samiuddin
Due date: March 1^{st}

August 21, 2024

1 Introduction

1.1 Lab Objectives

The main goal of this assignment is to (i) design lead and lag controller for the position control of the cart system , (ii) implement lead and lag controller for the position control of the cart system.

1.2 Assignment

Students should answer all the questions and submit their answer in addition to screen shot of block diagrams and wave signals to MyCourses. The submitted assignment should be in PDF format.

2 Questions

2.1 Lead controller

1. (i) Write the definition of stability margins (i.e. gain margin and phase margin). (ii) Find the gain margin and phase margin for the transfer function $G(s)$ that you found in Lab 4, Question 2.1.3. (Include the gain so that the output of the system is in cm) [10 marks]
   Hint: You can use the command *margin(G(s))* in Matlab.

2. Design a Lead controller for the transfer function $G(s)$ that you found in Lab 4, Question 2.1.3. to increase the open-loop phase margin to 60°.[20 marks]
   Hint: You can follow subsequent steps to design a lead controller.

   (a) Suppose the Lead controller is of the form:

   $$C_{lead}(s) = K_{lead}\frac{\alpha.Ts+1}{Ts+1}$$

   (b) Set $K_{lead} = 1$, since there is no requirement on steady-state error.

   (c) Find the phase margin of $G(s)$.

(d) Calculate the required phase $\phi_{\max}$ such that the open loop has 60° phase margin plus a margin of 15°.

(e) Find the coefficient α, using following formula:

$$\phi_{max} = \sin^{-1}\left(\frac{\alpha - 1}{\alpha + 1}\right)$$

(f) Find the desired frequency w_m, for which the gain is $-20\log(K_{lead}\sqrt{\alpha})dB$ on the Bode diagram of $G(s)$.

(g) Find the parameter T using the following equation:

$$T = \frac{1}{w_m\sqrt{\alpha}}$$

(h) Plugin the parameters α and T in the formula to find the lead controller:

$$C_{lead}(s) = K_{lead}\frac{\alpha.Ts + 1}{Ts + 1}$$

(i) Plot the Bode diagram of $C_{lead}(s)G(s)$, and verify the phase margin of the compensated system.

3. Plot the Bode diagram of $G(s)$ and $C_{lead}(s)G(s)$ and explain the effect of lead controller on the gain crossover frequency and phase margin. [5 marks]

4. Plot the unity feedback closed loop step responses of $G(s)$ and $C_{lead}(s)G(s)$. Explain the effect of lead controller on the step response. [10 marks]

5. Implement the lead controller for the position control of the cart system. Compare the performance of compensated and uncompensated controllers by measuring the rise time and the overshoot of the step responses of unity feedback system. [10 marks]

2.2 Lag controller

6. Design a Lag controller for the transfer function $G(s)$ that you found in Lab 4, Question 2.1.3. to increase the open-loop phase margin to 60°.[20 marks]
Hint: You can follow subsequent steps to design a Lag controller.

(a) Suppose the the controller is of the form :

$$c_{lag}(s) = K_{lag}\frac{Ts + 1}{\alpha.Ts + 1}$$

(b) Since there is no steady-state requirement, set $K_{lag} = 1$.

(c) On the Bode diagram of $G(s)$, find the frequency w at which the phase equals $-180°+$ desired phase margin + 15°. Calculate the desired gain drop D by calculating the gain at the frequency w.

(d) Find the parameter α using the following equation:

$$\alpha = 10^{D/20}$$

(e) Fix the place of the zero of the controller by setting $T = \frac{10}{w}$, to find T.

(f) Plugin the parameters α and T in the formula to find the lag controller:

$$C_{lag}(s) = K_{lag}\frac{Ts+1}{\alpha Ts+1}$$

(g) Plot the Bode diagram of $C_{lag}(s)G(s)$, and verify the phase margin of the compensated system.

7. Plot the Bode diagram of $G(s)$ and $C_{lag}(s)G(s)$ and explain the effect of lag controller on the gain crossover frequency and phase margin. [5 marks]

8. Plot the unity feedback closed loop step responses of $G(s)$, $C_{lag}(s)G(s)$, and $C_{lead}(s)G(s)$. Explain the effect of lead and lag controllers on parameters of the step response.[10 marks]

9. Implement the lag controller for the position control of the cart system. Compare the performance of compensated and uncompensated controllers, by measuring the rise time and overshoot of the step response of unity feedback systems. [10 marks]

ECSE 403 Winter 2024
Lab assignment 8

Instructor: Prof. Caines
Lab TAs: Borna Sayedana, Jilan Samiuddin
Due date: March 15^{th}

August 21, 2024

1 Introduction

1.1 Lab Objectives

The main goal of this assignment is to (i) derive the non-linear dynamics of the inverted pendulum system using Lagrangian method, (ii) derive the linearized model of the system in the neighborhood of an unstable equilibrium (iii) derive the state space representation of the system.

1.2 Assignment

Students should answer all the questions and submit their answer in addition to screen shot of block diagrams and wave signals to MyCourses. The submitted assignment should be in PDF format.

1.3 System Model

The goal of this lab is to derive the non-linear dynamics of the inverted pendulum system using the Lagrangian mechanics. A schematic of the inverted pendulum system is depicted in Figure 1. Since we want to stabilize the inverted pendulum around the equilibrium point $\theta = 0°$, we linearize the non-linear dynamics around this point and design linear controllers to stabilize the system. In Fig 1, F_c is the force applied to the cart by the DC motor. Since the cart is only moving in one direction, the position of the cart is shown by x_c. The position of the pendulum, has two components (x_p, y_p).

In order to use Lagrangian method, we choose the generalized coordinates:

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} x_c \\ \theta_p \end{bmatrix}.$$

and the generalized force F_c. We calculate the total kinetic energy of the system T, and total potential energy of the system V to find he Lagrange function:

$$L = T - V.$$

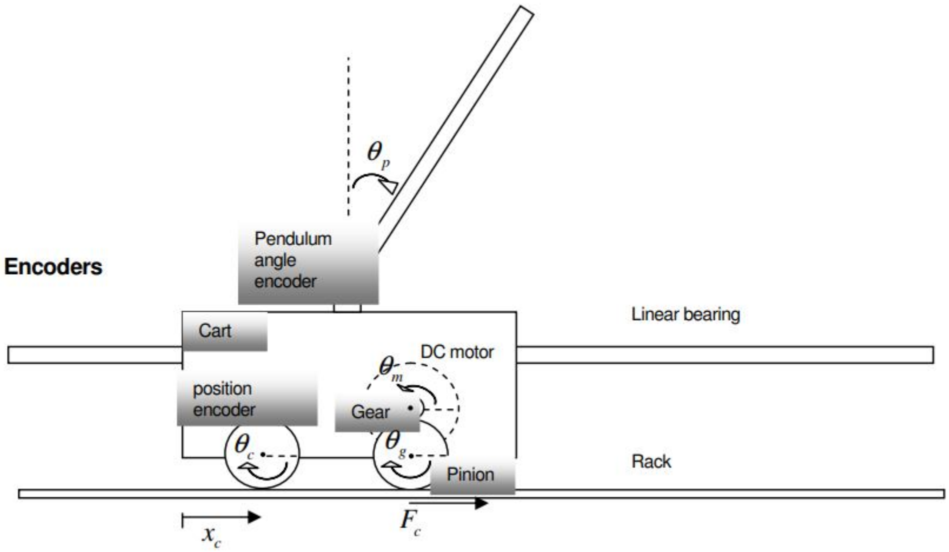

Figure 1: Model of the cart

Given the choice of generalized coordinate and generalized force for our model, the Euler-Lagrange equation for our model becomes:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_c}\right) - \frac{\partial L}{\partial x_c} = F_c \tag{1}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_p}\right) - \frac{\partial L}{\partial \theta_p} = 0. \tag{2}$$

The list of system variables and the constants is summarized as following:

- x_p : Pendulum's x-axis coordinate.
- y_p : Pendulum's y-axis coordinate.
- x_c : Cart's x-axis coordinate.
- θ_p : Pendulum's angle.
- $l_p = 0.168m$: Length of the rod.
- $m_c = 0.526Kg$: Mass of the cart.
- $m_p = 0.106Kg$: Mass of the pendulum.
- $g = 9.8m/s^2$: Acceleration due to gravity.

2 Questions

2.1 Nonlinear Model of the Inverted Pendulum

1. For the cart pole systems, find the Lagrangian of the system.[35 marks]
   Hint: You can follow subsequent steps:
   (a) Write the Cartesian position of the pendulum (x_p, y_p) in terms of $(x_c, l_p, \sin\theta_p, \cos\theta_p)$.
   (b) Differentiate the Cartesian position of the pendulum (x_p, y_p) to get the Cartesian velocities $(\dot{x}_p, \dot{y}_p)$.
   (c) Find the kinetic energy of the cart T_c, in terms of $(m_c, \dot{x}_c)$.
   (d) Find the Kinetic energy of the pendulum T_p, in terms of $(m_p, l_p, x_c, \theta_p)$.
   (e) Find the total kinetic energy $T = T_p + T_c$.
   (f) Find the potential energy of the pendulum V in terms of (m_p, g, l_p, θ_p).
   (g) Find the Lagrangian of the system $L = T - V$.

2. Using the Lagrangian that you found in Question 1, find the nonlinear dynamics of the system using Euler-Lagrange equation. [20 marks]
   Hint: You can follow subsequent steps:
   (a) Find $\partial L/\partial x_c$, $\partial L/\partial \dot{x}_c$, and $d/dt\big(\partial L/\partial \dot{x}_c\big)$.
   (b) Plugin these expressions in Equation (1).
   (c) Find $\partial L/\partial \theta_p$, $\partial L/\partial \dot{\theta}_p$, and $d/dt\big(\partial L/\partial \dot{\theta}_p\big)$.
   (d) Plugin these expressions in Equation (2).

2.2 Linearization of the Inverted Pendulum Model

The goal of this section is to find a linear approximation of the inverted pendulum system in the neighborhood of the equilibrium point $[x_c, \dot{x}_c, \theta_p, \dot{\theta}_p] = [0, 0, 0, 0]$. We then use this linearized model, and the linear system theory to design controllers to stabilize the system in the neighborhood of this equilibrium.

3. Find the linear approximation of the inverted pendulum system around $[x_c, \dot{x}_c, \theta_p, \dot{\theta}_p] = [0, 0, 0, 0]$.[20 marks]
   Hint: You can follow these steps:
   (a) Simplify the expression in Question 2 by using following approximations $\sin(\theta_p) \approx \theta_p$, $\cos(\theta_p) \approx 1$, $\theta^n \approx 0$ for $n > 1$, and $\dot{\theta}^n \approx 0$ for $n > 1$.
   (b) Find $\begin{bmatrix} \ddot{x}_c \\ \ddot{\theta}_p \end{bmatrix}$ as a function of F_c and θ_p.

4. Let $x = [x_c, \dot{x}_c, \theta_p, \dot{\theta}_p]^T$ denote the state of the system and $u = F_c$ denote the input of the system. Find the system matrices $\mathbf{S}_1 = (A, B, C, D)$ for the state-space representation of the system. [15 marks]

$$\mathbf{S}_1 = \begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

5. Write the definition of controllability of the system. Check if the system $\mathbf{S}_1$ is controllable. [10 marks]
   Hint: You can use the Matlab command *ctrb(A,B)*.

ECSE 403 Winter 2024
Lab assignment 9

Instructor: Prof. Caines
Lab TAs: Borna Sayedana, Jilan Samiuddin
Due date: March 22^{nd}

August 21, 2024

1 Introduction

1.1 Lab Objectives

The main goal of this assignment is to (i) derive the state space representation of the system with voltage as the input, (ii) design a linear state feedback controller based on pole placement method.

1.2 Assignment

Students should answer all the questions and submit their answer in addition to screen shot of block diagrams and wave signals to MyCourses. The submitted assignment should be in PDF format.

1.3 System Model

From Lab 4, we had a simplified electrical model of the DC motor of the cart-pole system in the lab is shown in Figure 1.

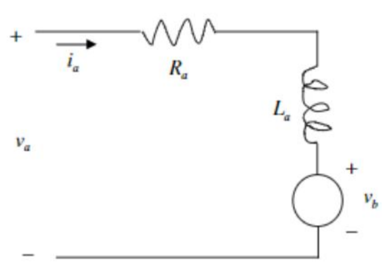

Figure 1: Simplified model of the DC motor

By applying KVL and substituting back electromotive force, we have :

$$v_a = R_a i_a + L_a \frac{d}{dt} i_a + v_b, \qquad v_b = K_e \omega_m \tag{1}$$

where v_a is the armature voltage, i_a is the armature current, R_a is armature resistance, L_a is armature inductance, and ω_m is the angular velocity of the motor shaft. Since $L_a << R_a$, we simplify this equation to get:

$$v_a = R_a i_a + K_e \omega_m \tag{2}$$

For the motor torque, we have:

$$\tau_m = K_m i_a \tag{3}$$

The model of the cart is shown in Figure 2. Recall that we had:

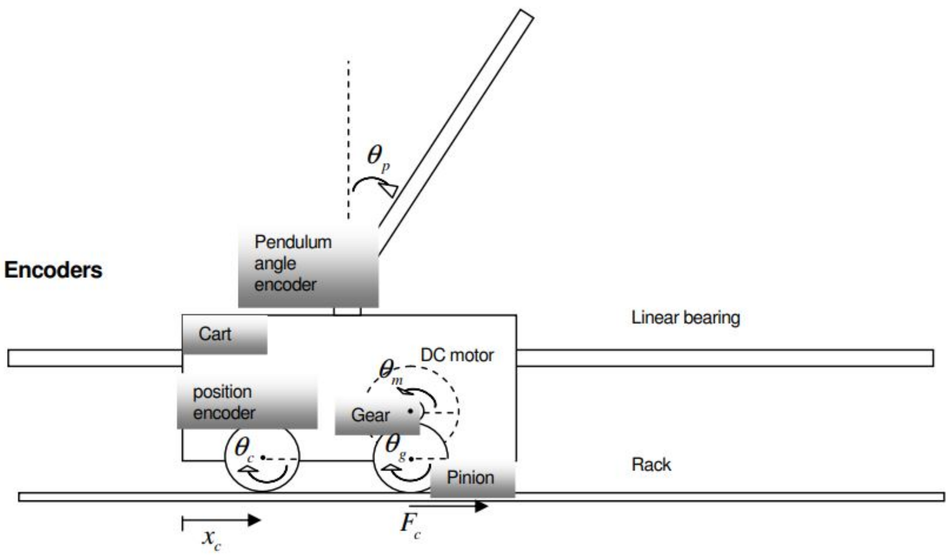

Figure 2: model of the cart

$$x_c = r_g \theta_g, \quad \omega_g = \frac{d\theta_g}{dt} \tag{4}$$

where r_g is the radius of the pinion, θ_g is the angle of the pinion, and ω_g is the angular velocity of the pinion. The angular velocity and torque of the pinion and the motor are related as following:

$$\omega_g = \frac{1}{K_g} \omega_m, \qquad \tau_m = \frac{1}{K_g} \tau_g \tag{5}$$

where K_g is the gear ratio. We can write the force applied to the cart as:

$$F_c = \frac{1}{r_g} \tau_g. \tag{6}$$

Following chart summarizes the system variables and the the constants.

- $K_m = K_e = 0.0077 N.m/amp$: Motor and back EMF constants
- $R_a = 2.6 ohms$: Armature resistance
- $K_g = 3.7/1$: Internal gear ratio
- $r_g = 0.0064m$: Motor gear radius
- x_p : Pendulum's x-axis coordinate.
- y_p : Pendulum's y-axis coordinate.
- x_c : Cart's x-axis coordinate.
- θ_p : Pendulum's angle.
- $l_p = 0.168m$: Length of the rod.
- $m_c = 0.526Kg$: Mass of the cart.
- $m_p = 0.106Kg$: Mass of the pendulum.
- $g = 9.8m/s^2$: Acceleration due to gravity.

2 Questions

1. Use the system model's section, and find F_c as a function of input voltage v_a and $\dot{x}_c$. [15 marks]
   Hint: You can follow subsequent steps:
   (a) Write i_a as a function of v_a and ω_m using Equation (2).
   (b) Use Equation (3), and write τ_m as a function of v_a and ω_m.
   (c) Use Equations (5) and (6) to find F_c as a function of v_a and ω_g.
   (d) Use Equations (5) and (4) to find F_c as a function of v_a and $\dot{x}_c$.

2. Use the state space model that you found in Lab 8 Question 2.2.4, and the ODE you found in Question 1, find the state space representation of the system with following definitions [15 marks].

$$\mathbf{S}_1 = \begin{cases} \dot{x} = A_1 x + B_1 u \\ y = C_1 x + D_1 u \end{cases}$$

   where:

$$x = \begin{bmatrix} x_c \\ \dot{x}_c \\ \theta_p \\ \dot{\theta}_p \end{bmatrix}, \quad y = \begin{bmatrix} x_c \\ \theta_p \end{bmatrix}, \quad u = v_a$$

3. Write the definition of controllability, and observability. Check if the system $\mathbf{S}_1$ is controllable, and observable. Find the transfer function between the input and each of the outputs individually. [15 marks]
   Hint: You can use the command $sys = ss(A_1, B_1, C_1, D_1)$ to define the state space of the systems. You can use $ctrb(sys)$ and $obsv(sys)$ to find controllability and observability Gramians. You can use $tf(sys)$ to find the transfer function of a state space model.

4. (i) Find the state space model $\mathbf{S}_2 = (A_2, B_2, C_2, D_2)$ where x, u have the same definitions as Question 2 and $y = x$. (ii) Explain how we can compute the vector x for the cart pole system when the measurements are only the vector $\begin{bmatrix} x_c \\ \theta_p \end{bmatrix}$.[10 marks]

5. (i) Define internal stability and BIBO stability. (ii) Check the BIBO stability of the system $\mathbf{S}_2$ and internal stability of $\mathbf{S}_2$. Are the poles of the system same as eigenvalues of the matrix A_2? (iii) Explain intuitively if you expect the linearized model of the inverted pendulum around $\theta_p = 0$ be a stable system or not. [15 marks]
   Hint: You can use commands $pzplot()$ for the pole-zero plot of a system and $eig()$ to find the eigenvalues of a matrix.

6. For a standard 2^{nd} order system, the place of the poles are given by:

$$p_{1,2} = -\zeta\omega_n \pm i\omega_n\sqrt{1-\zeta^2}.$$

   where ζ is the damping factor, and ω_n is the natural frequency. For the maximum overhoot Mp and the settling time t_s we have:

$$M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}, \quad t_s \approx \frac{3.9}{\zeta\omega_n}.$$

   (i) Find the place of the poles $p_{1,2,3,4}$ such that $p_{1,2}$ are dominant poles, the real parts of $p_{3,4}$ are 10 times further compared to the dominant poles, i.e.:

$$p_{3,4} = 10\big(-\zeta\omega_n \pm i\omega_n\sqrt{1-\zeta^2}\big).$$

   and $M_p = 0.1$, $t_s = 2.0$. [15 marks]

7. (i) Use the pole placement method and find the state feedback gain which places the poles of the closed loop system on the desired poles in Question 6. (ii) Use the pole-zero plot and verify if the poles of the closed loop system are equal to $p_{1,2,3,4}$. (iii) Find the transfer function between the input and each of the outputs individually for the closed loop system. [15 marks]
   Hint: You can use the the command $place(A, B, p)$ where $p = [p_1, p_2, p_3, p_4]$ is the vector of all the desired poles.

ECSE 403 Winter 2024
Lab assignment 10

Instructor: Prof. Caines
Lab TAs: Borna Sayedana, Jilan Samiuddin
Due date: April 5^{th}

August 21, 2024

1 Introduction

1.1 Lab Objectives

The main goal of this assignment is to (i) Design state feedback controllers using the dominant pole placement method. (ii) Implement the state feedback controller for the cart-pole system. (iii) Implement an automatic starting mechanism which closes the feedback loop whenever $\theta_p \approx 0$.

1.2 Assignment

Students should answer all the questions and submit their answer in addition to screen shot of block diagrams and wave signals to MyCourses. The submitted assignment should be in PDF format.

1.3 System Model

Recall that in the last two labs, we derived the following state space model for the inverted pendulum system in Figure 1.

$$\mathbf{S} = \begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

where:

$$x = \begin{bmatrix} x_c \\ \dot{x}_c \\ \theta_p \\ \dot{\theta}_p \end{bmatrix}, \quad y = \begin{bmatrix} x_c \\ \dot{x}_c \\ \theta_p \\ \dot{\theta}_p \end{bmatrix}, \quad u = v_a$$

Using this state space representation, we designed a state feedback controller using the dominant pole placement method. In this lab, we try to implement these controller in the cart-pole system.

In the inverted pendulum system, the measurement of both x_c and θ_p are relative to the initial condition of these states in which the simulation is started. This creates problems since we have to start the simulation each time from $\theta_p \approx 0$. An alternative way is to start the simulation from $\theta_p = \theta_{\min}$ and correct the measurement of θ_p in the Simulink by adding the correct measurement offset. With having access to the correct measurement of θ_p, we design a mechanism which closes the feedback loop the first time θ_p is close to 0.

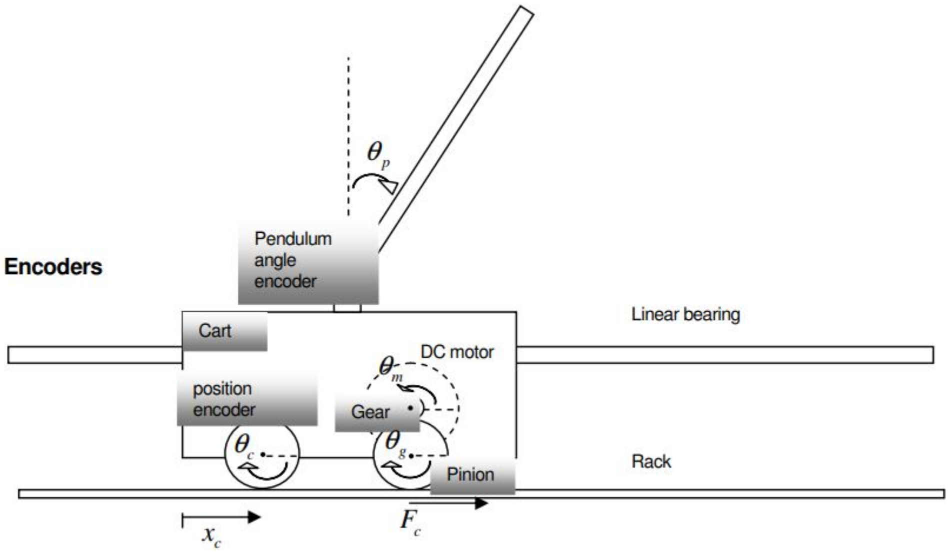

Figure 1: model of the cart

2 Questions

1. By measuring the open loop response of the system, change and calibrate the measurement gains of parameters x_c and θ_p. Make sure that the positive direction of both the parameters are matching.[15 marks]
   Hint: You can follow subsequent steps:
   (a) Set the position gain and angle gain to the values specified in the lab manual.
   (b) Simulate the open loop system and by changing the sign of angle's gain, make sure that the positive direction of x_c and θ_p are matching.
   (c) Measure the entire range of $\theta_p \in [\theta_{\min}, \theta_{\max}]$ and find the offset constant C_θ which corrects the measurement reading in the Simulink model. Add this offset constant to your Simulink model.

2. Design a *starting* mechanism which only closes the state feedback loop when θ_p is in the range of $[-0.01, 0.01]$. [25 marks]
   Hint: You can follow subsequent steps:
   (a) Design a logic circuit which returns the output 0 if $|\theta_p| > 0.01$, and returns the output 1 whenever $|\theta_p| < 0.01$, and remains 1 irrespective of the value of θ_p.
   (b) Use the signal of the previous circuit as the threshold criteria for a switch in the feedback loop which selects between 0 and the feedback signal.
   (c) Before implementing the mechanism in the feedback loop, test your system in the open loop setup and make sure it's working as desired.

3. (i) Implement the state feedback controller for the inverted pendulum system with the feedback gains you found in Lab 9, Question 7. (ii) Set the input to 0 and plot all the states trajectories for a disturbance in the pendulum's angle. [20 marks]

4. (i) Repeat the steps of Question 6 and 7 in Lab 9 with parameters $M_p = 0.1$, and $t_s = 1.8$. (ii) Set the input to 0 and implement the state feedback controller for the inverted pendulum system with these gains. (iii) Plot all the states trajectories for a disturbance in the pendulum's angle. [20 marks]

5. Repeat the steps of Question 4 with parameters $M_p = 0.15$, and $t_s = 2.5$. to design a state feedback controller for the longer bar. Assume that the mass of the new bar is $2m_p$ and length of the new bar is $2l_p$. [20 marks]

ECSE 403 Winter 2023
Lab assignment 11

Instructor: Prof. Caines Lab TA: Borna Sayedana
Due date: April 13^{th}

August 21, 2024

1 Introduction

1.1 Lab Objectives

The main goal of this assignment is to (i) Design state feedback controllers using the LQR method. (ii) Implement the LQR state feedback controller for the cart-pole system.

1.2 Assignment

Students should answer all the questions and submit their answer in addition to screen shot of block diagrams and wave signals to MyCourses. The submitted assignment should be in PDF format.

1.3 System Model

Recall that in the last two labs, we derived the following state space model for the inverted pendulum system in Figure 1.

$$\mathbf{S} = \begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

where:

$$x = \begin{bmatrix} x_c \\ \dot{x}_c \\ \theta_p \\ \dot{\theta}_p \end{bmatrix}, \quad y = \begin{bmatrix} x_c \\ \dot{x}_c \\ \theta_p \\ \dot{\theta}_p \end{bmatrix}, \quad u = v_a$$

Using this state space representation, we designed a state feedback controller using the dominant pole placement method. In this lab, we try to implement these controller in the cart-pole system.

In the inverted pendulum system, the measurement of both x_c and θ_p are relative to the initial condition of these states in which the simulation is started. This creates problems since we have to start the simulation each time from $\theta_p \approx 0$. An alternative way is to start the simulation from $\theta_p = \theta_{\min}$ and correct the measurement of θ_p in the Simulink by adding the correct measurement offset. With having access to the correct measurement of θ_p, we design a mechanism which closes the feedback loop for the first time whenever θ_p crosses 0.

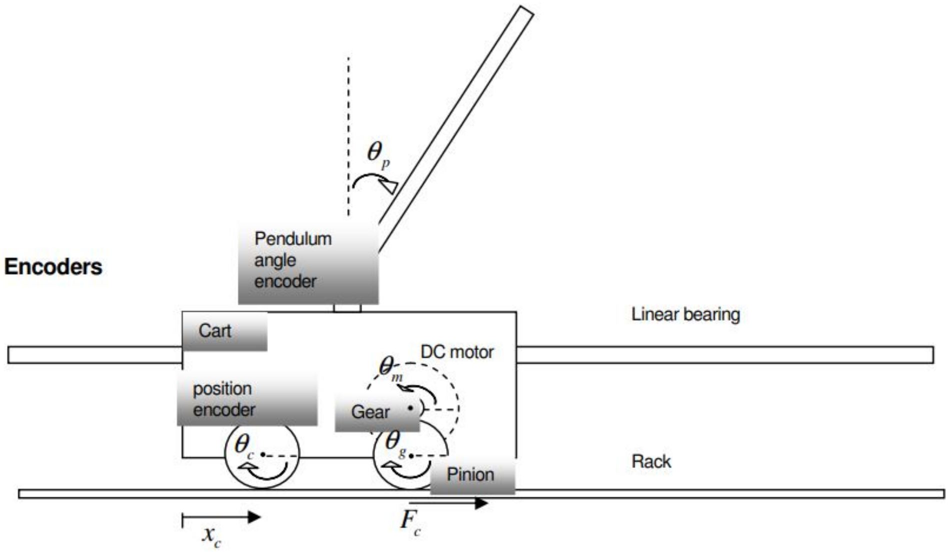

Figure 1: model of the cart

2 Questions

1. Define the stabilization of the inverted pendulum problem as an infinite horizon Linear quadratic regulation problem.
   Hint: Define the cost function, dynamics, optimization problem, cost to go function, and the optimal policy.

2. For the short rod, (i) calculate the state feedback gain and place of the poles of the closed loop system for the following cost matrices, (ii) implement the optimal state feedback gains for the inverted pendulum and plot the response of the system to a disturbance. (iii) Let

$$Q = \begin{bmatrix} q_{11} & 0 & 0 & 0 \\ 0 & q_{22} & 0 & 0 \\ 0 & 0 & q_{33} & 0 \\ 0 & 0 & 0 & q_{44} \end{bmatrix}$$

   By comparing the response of the system for cost matrices in part (a) with other items, explain the effect of increasing elements q_{22}, q_{44} and R. (iv) Explain intuitively the effect of changing each of these parameters on the behaviour of the system.
   Hint: You may use the Matlab command *[K,S,P] = lqr(A,B,Q,R)*

   (a)

$$Q_1 = \begin{bmatrix} 50 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R_1 = 0.01$$

(b)

$$Q_2 = \begin{bmatrix} 50 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R_2 = 0.005$$

(c)

$$Q_3 = \begin{bmatrix} 50 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R_3 = 0.01$$

(d)

$$Q_3 = \begin{bmatrix} 50 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad R_3 = 0.01$$

3. For the long rod, (i) calculate the state feedback gain and place of the poles of the closed loop system for the following cost matrices, (ii) implement the optimal state feedback gains for the inverted pendulum and plot the response of the system to a disturbance. (iii) By comparing the response of the system for cost matrices in part (*a*) with other items, explain the effect of decreasing elements q_{11}, q_{33} and R. (iv) Explain intuitively the effect of changing each of these parameters on the behaviour of the system.

(a)

$$Q_1 = \begin{bmatrix} 50 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R_1 = 0.01$$

(b)

$$Q_1 = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R_1 = 0.01$$

(c)

$$Q_1 = \begin{bmatrix} 50 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R_1 = 0.01$$