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A Multi-Robot Cooperative Confrontation Game with Limited Range of Motion *

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***Abstract*— With the development of robotics technology, it is possible for an autonomous robot to accomplish conflict tasks such as target reconnaissance in confrontation environment. Pursuit and evasion conflicts represent challenging problems with important applications in aerospace and robotics. In this paper, we address a novel multi-robot cooperative confrontation game which has three players: a Target, an Attacker and a Defender. The Attacker aims to capture the Target, while avoiding being captured by the faster Defender which is restricted to move in a circular area. The Target cooperates with the Defender to avoid being captured by the Attacker. We characterize some general properties of the multi-robot cooperative confrontation game. Based on the explicit policy method and geometric analysis, the optimal control strategies and the winning conditions for the Attacker are obtained.**

***Index Terms*— Multi-Robot pursuit-evasion game, Strategy switch, Cooperative policy.**

I. Introduction

Pursuit-evasion games provide a general framework that mathematically formalizes the important application for robots in different areas such as surveillance, navigation, analysis of biological behaviors, and conflict and combat operations [1]. For example, with the development of robotics technology, it is possible for an autonomous robot to accomplish conflict tasks such as target reconnaissance. When the maneuverability of the reconnaissance target is poor and cannot avoid being reconnoitred, an active interception is often requested to intercept the robot. This scenario is a typical multi-player cooperative confrontation game which has three players: a mobile target (Target), an active interception (Defender) and a robot (Attacker)— and this is called as Target-Attacker-Defender game.

The Target-Attacker-Defender game [2]–[4] exists widely in practical intelligent and autonomous unmanned system applications, e.g., a fired missile defending an evading aircraft against an incoming homing missile [5]–[8], a torpedo safeguarding a naval ship against a submarine [9], and an interceptor defending an asset against an intruder [10].

Recently, the authors of [11]–[15] make a lot of research on the Target-Attacker-Defender game. They first analyze the cooperation mechanism of a Target and a Defender when the Attacker implements typical guidance laws of Pure Pursuit and Proportion Navigation [11], [12], respectively. Then they consider the cooperation mechanism of a Target and a Defender in a differential game [13]–[15] where the Defender missile is launched by a Target or by a Target-friendly platform.

In those works above, the Attacker is regarded as a suicidal missile, which aims to get closer to the Target. Whether or not the Attacker will be captured by the Defender is rarely regarded. In many practical scenarios, the Attacker is often a recycleable agent and the Attacker should consider not being captured by the Defender. That is to say, the Attacker should avoid being intercepted by the Defender while pursuing the Target. In addition, the range of activities of three players in the current literature is not restricted. However, in the process of motion, agents will be affected by such factors as limited perception range or fuel, and their range of activities will be limited.

In this paper, we deal with a Target-Attacker-Defender game in which the Attacker aims to capture the Target, while avoiding being captured by the Defender, and the Defender tries to defend the Target from being captured by the Attacker while trying to capture the Attacker at an opportune moment. The Target and Defender cooperate in a team, and the Attacker is on the opposite side. The Defender is faster than the Attacker, and the Defender is restricted to move in a circular area. The main contributions of this work are as follows:

1) a novel Target-Attacker-Defender game with a bounded Defender is formulated;
2) both the maneuvering evasion and the active interception for the Attacker are considered;
3) the switching strategies and the corresponding conditions for the Attacker are developed.

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The rest of the paper is organized as follows. In Section 2, we formulate the Target-Attacker-Defender game into a multi-player game with a constraint of range of motion. In Section 3, we analyze the escape region for the Attacker when the Defender is restricted in a circular area. In Section 4, we present the winning conditions and corresponding control strategies for the Attacker. In Section 5, we use some simulation examples to illustrate the results. Finally, Section 6 gives some concluding remarks and a summary of planned future work.

II. Problem formulation

Let us consider a Target-Attacker-Defender game that takes place in $\mathbb{R}^2$, with a Target, an Attacker and a Defender. The Target is a mobile agent that can move freely in the space to evade the Attacker. The Defender tries to defend the Target from being captured by the Attacker while trying to capture the Defender at an opportune moment. The Attacker aims to capture the Target while avoiding being captured by the Defender.

The Target, Attacker, and Defender have simple motions as encountered in the games of Isaacs [16], and three players move in the Euclidean plane. Their control variables are the instantaneous headings $\hat{\phi}, \hat{\chi}, \hat{\psi} \in \mathbb{R}$, respectively. In addition, the Target, Attacker, and Defender have constant speeds V_T, V_A, and V_D, respectively.

The states of the Target, Attacker, and Defender are specified by their Cartesian coordinates $\mathbf{x}_T = (x_T, y_T)$, $\mathbf{x}_A = (x_A, y_A)$, and $\mathbf{x}_D = (x_D, y_D)$, respectively; the dimension of the state space is six. The complete state of the Target-Attacker-Defender game is specified by $\mathbf{x} := (x_T, y_T, x_A, y_A, x_D, y_D) \in \mathbb{R}^6$. The game set is the entire space $\mathbb{R}^6$.

The dynamics $\dot{\mathbf{x}} = f(\mathbf{x}, \hat{\phi}, \hat{\chi}, \hat{\psi})$ in the realistic game space are given by:

$$\begin{aligned}
\dot{x}_T(t) &= V_T \cos\hat{\phi}(t), & x_T(t_0) &= x_{T_0}, \\
\dot{y}_T(t) &= V_T \sin\hat{\phi}(t), & y_T(t_0) &= y_{T_0}, \\
\dot{x}_A(t) &= V_A \cos\hat{\chi}(t), & x_A(t_0) &= x_{A_0}, \\
\dot{y}_A(t) &= V_A \sin\hat{\chi}(t), & y_A(t_0) &= y_{A_0}, \\
\dot{x}_D(t) &= V_D \cos\hat{\psi}(t), & x_D(t_0) &= x_{D_0}, \\
\dot{y}_D(t) &= V_D \sin\hat{\psi}(t), & y_D(t_0) &= y_{D_0}.
\end{aligned} \tag{1}$$

The initial state of the system is
$\mathbf{x}_0 := (x_{T_0}, y_{T_0}, x_{A_0}, y_{A_0}, x_{D_0}, y_{D_0}) = \mathbf{x}(t_0)$.

In our setting of the problem, the speed relation of the three players is $V_T < V_A < V_D$. Without loss of generality, the players' speeds are normalized so that $V_A = 1$. Let $\alpha = V_T/V_A$, and $\beta = V_D/V_A$ denote the speed ratios, $\alpha < 1, \beta > 1$, respectively. Both the state and the control variables of the Attacker and Target are unconstrained. The control variable of the Defender is also unconstrained. The state of the Defender is restricted in a circular area whose center is the initial position of the Defender and the radius is l. We define this circular area as Ω_D which is denoted by $\Omega_D := \{(x,y) | (x - x_{D_0})^2 + (y - y_{D_0})^2 \leqslant l^2\}$.

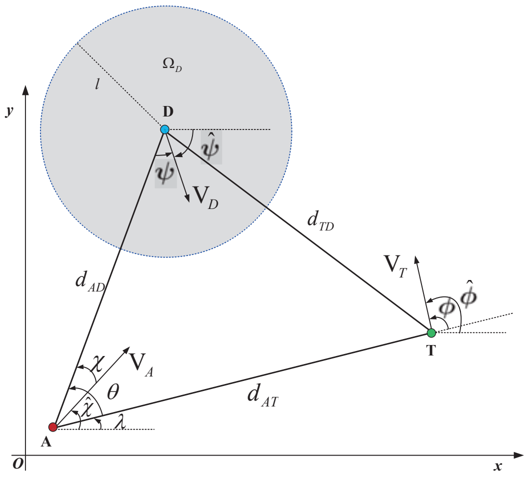

Fig. 1: The Target-Attacker-Defender game with a bounded Defender in the fixed reference system.

In the Target-Attacker-Defender game with a bounded Defender, we are interested in the following problems:

Q1: How does the size of the restricted area of the Defender affect the winning or losing of the two belligerent parties?

Q2: What initial states can ensure that the Attacker capture the Target, when the Defender is restricted to a circular area?

Q3: What are the optimal capture strategies of the Attacker?

To describe the dynamics of the Target-Attacker-Defender game with a bounded Defender more compactly, a reduced state space can be formed by the distances d_{AT}, d_{AD} and the included angle θ in which the dynamics are derived as follows:

$$\dot{d}_{AT} = \alpha \cos\phi - \cos(\theta - \chi), \qquad d_{AT}(t_0) = d_{AT_0}, \tag{2}$$

$$\dot{d}_{AD} = -\cos\chi - \beta\cos\psi, \qquad d_{AD}(t_0) = d_{AD_0}, \tag{3}$$

$$\begin{aligned}
\dot{\theta} = &-\frac{\alpha}{d_{AT}} \sin\phi + \frac{1}{d_{AT}} \sin(\theta - \chi) - \frac{\beta}{d_{AD}} \sin\psi \\
&+ \frac{1}{d_{AD}} \sin\chi, \qquad \theta(t_0) = \theta_0
\end{aligned} \tag{4}$$

where $\phi = \hat{\phi} - \lambda$, $\chi = \lambda + \theta - \hat{\chi}$, and $\psi = \hat{\psi} - \theta - \lambda + \pi$. λ denotes the included angle of the vector $\overrightarrow{AT}$ and the x axis. ϕ, χ, and ψ are the alternative control variables of the players, defined as the relative headings of the Target, Attacker, and Defender from the vectors $\overrightarrow{AT}$,$\overrightarrow{AD}$, and $\overrightarrow{DA}$, respectively. θ is the included angle $\angle DAT$. The full state space is denoted by $\Omega = \{d_{AT} \geq 0, d_{AD} \geq 0, -\pi < \theta \leq \pi\} \subseteq \mathbb{R}^3$.

In this study, we consider the Target-Attacker-Defender game with point capture. The objective of the Attacker is to make the Attacker-Target separation to become zero on the premise of the Defender-Attacker separation being greater than zero. The terminal set of the Attacker is $\mathbb{C}_1 : \{d_{AT} = 0, d_{AD} > 0\}$. On the contrary, the Target–Defender team aims to prevent the Attacker from achieving its goal. One intuitive way is that the Defender capture the Attacker before the Target is captured. In this case, the terminal set of the Target–Defender team is $\mathbb{C}_2 : \{d_{AD} = 0, d_{AT} > 0\}$. Therefore, the condition for termination is $\mathbb{C} = \mathbb{C}_1 \cup \mathbb{C}_2$.

In order to answer questions Q1-Q3, we need to solve the Target-Attacker-Defender game in games of kind. It is difficult to solve this problem by using Isaacs' classic approach because sketching the boundary of the terminal set is a challenge. Instead, we can address this problem by using the explicit policy method that decomposes a complex problem into a few simple sub-problems. In each sub-problem, we analyze the possibility of winning the game by giving specific strategies to the players.

III. Escape angle of Attacker

For a single pursuer and single evader game (PE game) without bounded region where both players move with simple motion, the complete solution is obtained by constructing an Apollonius circle [16]. In the unbounded region, the inner region of the Apollonius circle is the dominant region of E, and the external region is the dominant region of P. If $V_P > V_E$, then P will always be able to capture E, and the whole space is the capture area of P. That is to say, no matter where P and E are initially located, P can always catch E after a period of time.

Corollary 3.1: Assume that $V_P > V_E$ and P has a limited range of motion. Then, when the initial position of E is outside the limited range of P, E will not be captured as long as it does not enter the limited range of P.

In this Target-Attacker-Defender game, the Defender and Attacker form a pursuit-evasion game where the Defender is a pursuer and the Attacker is an evader. According to Corollary 3.1, we give the escape angle of the Attacker under the condition that a faster Defender is restricted in a circular area, which is described in the following theorem.

Theorem 3.1: If the Defender has a limited range of motion, when the initial position of the Attacker is inside the limited range of the Defender, the escape angle of the Attacker denoted by Θ is an angle of a sector area.

Proof: As shown in Fig. 2, the circle O_1 is the Apollonius circle of the Attacker and Defender. Ω_D is the motion range of the Defender. When the initial position of the Attacker is inside Ω_D, the circle O_1 intersects with Ω_D. The intersection points are denoted by P and Q, respectively.

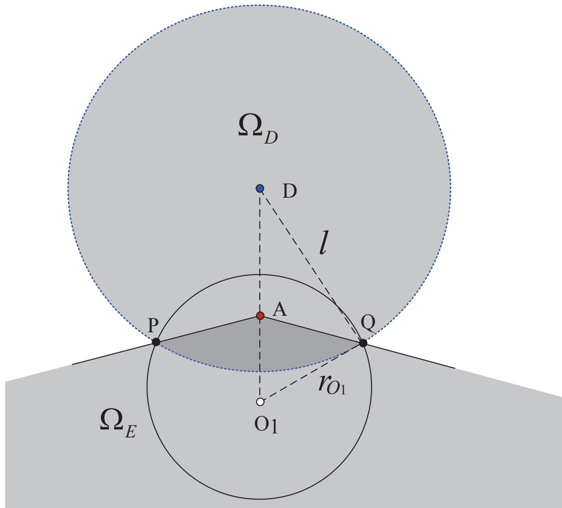

Fig. 2: The escape angle of the Attacker.

According to the definition of the Apollonius circle, we have

$$\frac{|AQ|}{|DQ|} = \frac{V_A}{V_D} = \frac{1}{\beta}, \tag{5}$$

that is,

$$|AQ| = \frac{l}{\beta}. \tag{6}$$

The center of the circle O_1 is

$$O_1 = (a_1, b_1), a_1 = \frac{\beta^2 x_A - x_D}{\beta^2 - 1}, b_1 = \frac{\beta^2 y_A - y_D}{\alpha^2 - 1}, \tag{7}$$

and the corresponding radius is

$$r_{O_1} = \frac{\beta d_{AD}}{\beta^2 - 1}. \tag{8}$$

Then, we can obtain

$$|O_1 A| = \frac{\beta d_{AD}}{\beta^2 - 1} - \frac{d_{AD}}{\beta + 1} = \frac{d_{AD}}{\beta^2 - 1}. \tag{9}$$

In $\triangle QAO_1$, we have

$$\cos\angle QAO_1 = \frac{(\beta^2 - 1)l^2 - \beta^2 d_{AD}^2}{2\beta l d_{AD}}. \tag{10}$$

Thus, the escape angle of the Attacker is

$$\Theta = 2\angle QAO_1 = 2 * \arccos(\frac{(\beta^2 - 1)l^2 - \beta^2 d_{AD}^2}{2\beta l d_{AD}}). \tag{11}$$

Therefore, the escape region of the Attacker is a sector area which is denoted by Ω_E (see Fig. 2). When the Attacker moves in a straight line towards this area, it will escape from the Defender. ■

Theorem 3.1 implies a condition that the circle O_1 intersects with Ω_D, that is

$$l - r_{O_1} < |O_1 D| < l + r_{O_1}. \tag{12}$$

Substituting and (7) and (8) into (12), we get

$$\frac{\beta}{\beta+1} \leq \frac{l}{d_{AD}} \leq \frac{\beta}{\beta-1}. \tag{13}$$

Therefore, according to the size of restricted area of Defender, we divide the game into three cases.

Case I): The size of the restricted area of the Defender is ordinary, $\frac{\beta}{\beta+1}d_{AD} \leqslant l \leqslant \frac{\beta}{\beta-1}d_{AD}$.

Case II): The size of the restricted area of the Defender is large, $l > \frac{\beta}{\beta-1}d_{AD}$.

Case III): The size of the restricted area of the Defender is small, $l < \frac{\beta}{\beta+1}d_{AD}$.

The complete solution to all these cases is quite complicated. Due to space limit, in what follows, we focus on Case I and provide a detailed discussion.

IV. Different Strategies of the Attacker

In this section, we first analyze the winning conditions and the strategies for the Attacker. As shown in Fig. 3, when the size of restricted area of the Defender is ordinary, the initial position of the Attacker is inside Ω_D, and the circle O_1 intersects with Ω_D. The Attacker has two strategies to capture the Target and win the game. One is directly to pursue the Target without considering the interception of the Defender. The other is that the Attacker needs to escape from the Defender and then pursues the Target.

A. The Attacker directly chases the Target

According to the Theorem 3.1, it is obviously that if the Apollonius circle of the Attacker and Target is completely inside the escape region of the Attacker, then the Attacker directly pursues the Target. Thus, we have the following theorem.

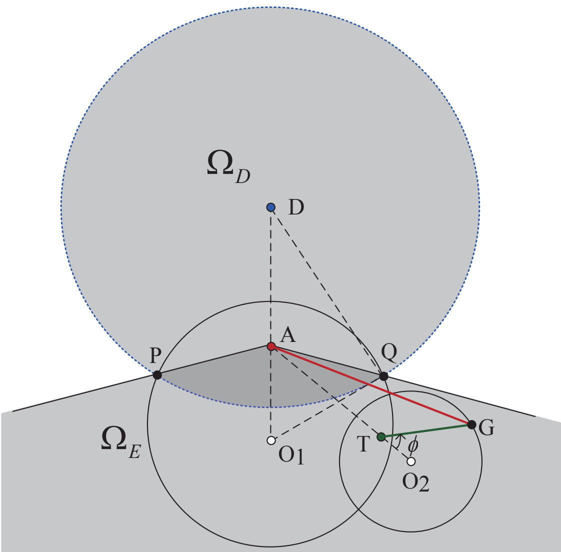

Fig. 3: The Attacker directly chases the Target.

Theorem 4.1: If the initial state of the Target-Attacker-Defender game with bounded Defender satisfies

$$\frac{\sqrt{4d_{AT}^2 d_{AD}^2 - {m_1}^2}}{2d_{AT}d_{AD}} \frac{\sqrt{4\beta^2 l^2 d_{AD}^2 - ((\beta^2-1)l^2 - \beta^2 d_{AD}^2)^2}}{2\beta l d_{AD}} > \sqrt{1-\alpha^2} - \frac{m_1}{2d_{AT}d_{AD}} \frac{(\beta^2-1)l^2 - \beta^2 d_{AD}^2}{2\beta l d_{AD}} \tag{14}$$

where $m_1 = d_{AT}^2 + d_{AD}^2 - d_{TD}^2$, then the Attacker can adopt the direct strategy

$$\chi^* = \theta - \arcsin(\alpha \sin(\phi)) \tag{15}$$

to pursue the Target and win the game.

Proof: As shown in Fig. 3, the circle O_1 is the Apollonius circle of the Attacker and Defender. The circle O_2 is the Apollonius circle of the Attacker and Target. According to the definition of the Apollonius circle, if the Target chooses the control angle ϕ and moves toward the point G, the Attacker also adopts control angle χ and moves toward the point G. By the sine theorem in $\triangle TAG$, we have

$$\frac{\sin\phi}{\sin(\theta-\chi)} = \frac{|AG|}{|TG|} = \frac{V_A}{V_T} = \frac{1}{\alpha}. \tag{16}$$

Thus, we can obtain that $\chi^* = \theta - \arcsin(\alpha\sin(\phi))$.

If the Attacker can adopt the strategy (15) to directly pursue the Target, then the Apollonius circle O_2 must be completely inside the escape region of the Attacker, that is, $\angle QAT > \arcsin\alpha$.

Due to $\angle QAT = \theta - \angle DAQ$, we have

$$\cos\angle QAT = \cos(\theta - \angle DAQ) < \sqrt{1-\alpha^2}. \tag{17}$$

According to the triangle cosine theorem in $\triangle DAT$ and $\triangle DAQ$, we have

$$\cos\theta = \frac{m_1}{2d_{AD}d_{AT}}, \cos\angle DAQ = \frac{d_{AD}^2 + \frac{l^2}{\beta^2} - l^2}{2d_{AD}\frac{l}{\beta}}. \tag{18}$$

Substituting (18) into (17), we can obtain (14). Therefore, if the condition (14) is satisfied, the Attacker can adopt (15) to capture the Target and win the game. ■

If the initial state of the Target-Attacker-Defender game with bounded Defender does not satisfy the condition (14) (see Fig. 4), the Attacker should first reach to the point Q or P and escape from the Defender. Then the Attacker adopts the corresponding strategy to pursue the Target. Whether the Attacker can win the game or not is analyzed in the following subsection.

B. The Attacker switches strategy to pursue the Target

When $\theta < \arcsin\alpha + \angle DAQ$, the Attacker should first escape and then maneuver around the Defender to intercept the Target. If the Attacker can prevent the Target from entering the restricted area of the Defender, the Attacker can always capture the Target. In other words, if the Attacker can reach the point where the Target enters Ω_D earlier than the Target, the Attacker will win the game. Thus, we give the following theorem.

Theorem 4.2: If the initial state of the Target-Attacker-Defender game with bounded Defender satisfies

$$\arccos(K) < \frac{\beta d - \beta l - \alpha l}{\alpha \beta l}, \tag{19}$$

where

$$\begin{aligned} K = {} & \frac{m_3}{2d_{TD}d_{AD}} \frac{d_{AD}^2 + l^2 - \frac{l^2}{\beta^2}}{2ld_{AD}} + \\ & \frac{\sqrt{4d_{TD}^2 d_{AD}^2 - m_3^2}}{2d_{TD}d_{AD}} \frac{\sqrt{4l^2 d_{AD}^2 - (d_{AD}^2 + l^2 - \frac{l^2}{\beta^2})^2}}{2ld_{AD}}, \end{aligned} \tag{20}$$

$m_3 = d_{AD}^2 + d_{TD}^2 - d_{AT}^2$, then the Attacker can capture the Target before the latter reaches to Ω_D and the Attacker wins the game.

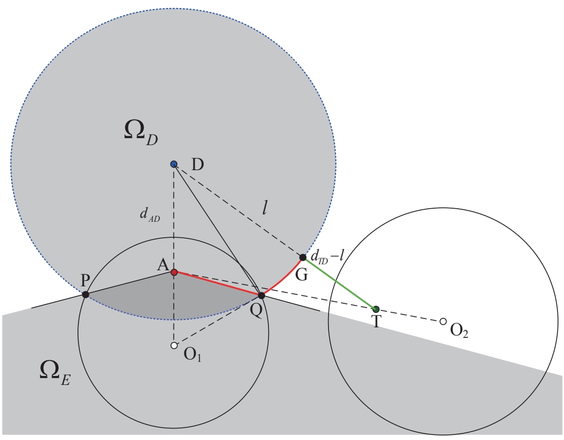

Fig. 4: The Attacker bypasses Ω_D to intercept the Target.

Proof: In order to enter Ω_D as soon as possible, the Target moves directly towards the Defender. Assume the line TD intersects with Ω_D at the point G (see Fig. 4), t_3 indicates the time spent for the Target to arrive at the point G. Then, we get $t_3 = (d_{TD} - l)/V_T$.

Note that Ω_D intersects the circle O_1 at the point Q. According to the nature of the Applonius circle, the Attacker and Defender will arrive at the point Q at the same time which is denoted by $t_1 = l/V_D$.

When the Attacker arrives at point the Q, in order to avoid being captured by the Defender, the Attacker should intercept the Target along the periphery of the area Ω_D. Let t_2 indicate the time spent for the Attacker to move from the point Q to G. Then, it can be expressed as $t_2 = \widehat{QG}/V_A$, where $\widehat{QG} = \angle QDG \times l$. The corresponding strategy for the Attacker is

$$\hat{\chi} = \arctan \frac{y_A - y_{D_0}}{x_A - x_{D_0}} + \pi/2. \tag{21}$$

According to the triangle cosine theorem in $\triangle DAT$ and $\triangle DAQ$, we have

$$\cos\angle ADT = \frac{k_3}{2d_{TD}d_{AD}}, \cos\angle ADQ = \frac{d_{AD}^2 + l^2 - \frac{l^2}{\beta^2}}{2ld_{AD}}. \tag{22}$$

Sustituting (22) into $\cos\angle QDG = \cos(\angle ADT - \angle ADQ)$, we can obtain

$$\angle QDG = \arccos(K). \tag{23}$$

If the condition (19) holds, then we have

$$\frac{d_{TD} - l}{V_T} > \frac{l}{V_D} + \frac{\widehat{QG}}{V_A}, \tag{24}$$

that is, $t_1 + t_2 < t_3$.

Therefore, the Attacker arrives at the point G before the Target, and the Attacker can successfully intercept the Target. ■

Theorem 4.2 includes three stages of the Attacker. The first stage is that the Attacker moves to the crossover points. The crossover points of the circle O_1 and Ω_D are denoted by $P(x_P, y_P)$ and $Q(x_Q, y_Q)$, which satisfy the following equations:

$$\begin{aligned} (x_i - x_{D_0})^2 + (y_i - y_{D_0})^2 = l^2 \\ (x_i - a_1)^2 + (y_i - b_1)^2 = {r_{O_1}}^2 \end{aligned} \tag{25}$$

where the index $i = P, Q$.

The length of the segment DO_1 is denoted by $L_1 = \sqrt{(x_{D_0} - a_1)^2 + (y_{D_0} - b_1)^2}$. The slopes of the segments DO_1 and PQ are k_1 and k_2, respectively:

$$k_1 = \frac{b_1 - y_{D_0}}{a_1 - x_{D_0}}, k_2 = -1/k_1. \tag{26}$$

Define $L_2 = l^2 - (1 + k_1^2)(\frac{(a_1 - x_{D_0})(l^2 - r_{O_1}^2 + L_1^2)}{2L_1^2})^2$, and then

$$\begin{aligned} x_P &= x_{D_0} + \tfrac{(a_1 - x_{D_0})(l^2 - r_{O_1}^2 + L_1^2)}{2L_1^2} - \sqrt{L_2/(1 + k_2^2)} \\ y_P &= y_{D_0} + \tfrac{k_1(a_1 - x_{D_0})(l^2 - r_{O_1}^2 + L_1^2)}{2L_1^2} - k_2\sqrt{L_2/(1 + k_2^2)} \\ x_Q &= x_{D_0} + \tfrac{(a_1 - x_{D_0})(l^2 - r_{O_1}^2 + L_1^2)}{2L_1^2} + \sqrt{L_2/(1 + k_2^2)} \\ y_Q &= y_{D_0} + \tfrac{k_1(a_1 - x_{D_0})(l^2 - r_{O_1}^2 + L_1^2)}{2L_1^2} + k_2\sqrt{L_2/(1 + k_2^2)}. \end{aligned} \tag{27}$$

Therefore, the Attacker can arrive at point Q by adopting the following strategy

$$\begin{aligned} \sin\hat{\chi} &= \frac{y_Q - y_A}{\sqrt{(x_Q - x_A)^2 + (y_Q - y_A)^2}}, \\ \cos\hat{\chi} &= \frac{x_Q - x_A}{\sqrt{(x_Q - x_A)^2 + (y_Q - y_A)^2}}. \end{aligned} \tag{28}$$

The second stage is that the Attacker moves along the periphery of the area Ω_D until $\theta > \arcsin\alpha + \angle DAQ$, and the corresponding control strategy is (21).

The third stage is that the Attacker adopts the strategy (15) to directly pursue the Target.

V. Simulation results

According to the two strategies of the Attacker winning game described in the previous section, we give two examples in this section. Assume that the initial state of the Defender and Attacker are $\mathbf{x}_D = (0, 5)$, $\mathbf{x}_A = (0, 0)$, respectively. The speed ratios $\alpha = 0.6$, $\beta = 1.2$. The initial location of the Target is set in each example. The region of the blue curve is Ω_D. The red curve is the Appolonius circle of the Defender and Attacker.

Fig. 5 illustrates one instance of the Attacker that directly chases the Target and wins the game. At the beginning, the Target lies in the escape region of the Attacker, that is, $\mathbf{x}_T \in \Omega_E$. The initial state satisfies the condition (14), the Attacker can adopt the strategy (15) to pursue the Target and win the game.

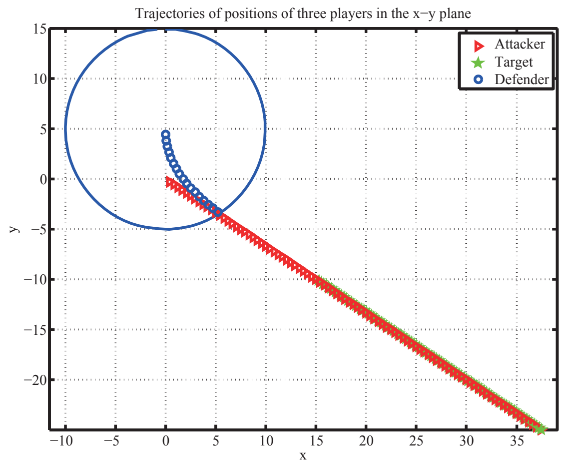

Fig. 5: The Attacker adopts strategy (15) to directly pursue the Target, where $l = 10$ and $\mathbf{x}_T = (15, -10)$.

Fig. 6 illustrates one instance of the Attacker that switches strategies to capture the Target. The Attacker first evades the Defender and moves to the crossover points. Then, the Attacker moves along the periphery of area Ω_D to increase θ. Finally, when $\theta > \arcsin\alpha + \angle DAQ$, the Attacker adopts the strategy (15) to directly pursue the Target.

VI. Conclusion

In this paper,we study a multi-robot cooperative confrontation game, which is composed of a target robot, an attacker robot and a defender robot with limited range of motion. First, we analyze the influence of the restricted area on the multi-robot cooperative confrontation game. Then, by employing the explicit policy method, we provide the winning conditions and corresponding strategies for the Attacker.

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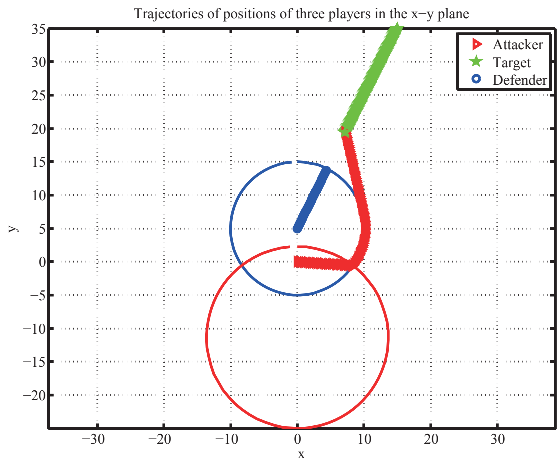

Fig. 6: The Attacker adopts strategy (28), (21) and (15) to pursue the Target, where $l = 10$ and $\mathbf{x}_T = (15, 35)$.