

# Gait Analysis of the Mammal Quadruped Robot from the Perspective of Energy Efficiency

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**Abstract** - Because of the variety of parameters which are closely coupled, the mechanism design and control implementation of the bionic quadruped robot with higher performance is difficult. In this open research field, the energy efficiency of the motion system has become an important design and optimization goal. In this manuscript, a feasible simulation system framework is constructed in the case where the mechanism configuration, the length of the leg and the whole weight has been determined. Based on the step coefficient and the height coefficient, the motion features and the energy consumption of the quadruped robot in the gait sagittal plane are analysed, and the relationship between motion energy efficiency and gait parameters including gait type, step size, step height, step frequency, standing height are discussed in detail. The simulation results show that the gait that consistent with high energy efficiency is similar to animals with similar size.

**Index Terms** - legged robots, motion and path planning, dynamics, energy consumption.

## I. INTRODUCTION

Research on legged robots, especially quadruped robots, has made significant progress in recent years. However, whether it is a large quadruped robot with high load capacity or a small-sized electric quadruped robot for commercial use, the energy endurance is a key factor that limits the practical application of the legged robot. Therefore, the efficiency of motion energy is being studied more extensively to satisfy the application requirements.

On the other hand, legged robots originate from the biomimetic of the animal's body structure and flexible behavior of movement, while the animals' choice of their own motion behavior depends largely on energy conservation. Therefore, from the perspective of motion bionics and control, the research on the energy efficiency of legged motion is a basic and important work. A large number of studies [1]-[7] have been carried out on the energy consumption of different types of drive systems, including electric drive systems and hydraulic drive systems. These researchers adopted different methods to obtain stable efficient motion and energy analysis of the multi-legged robot.

This manuscript study the motion behavior of mammal quadruped robot from the perspective of energy consumption, and try to find a more reasonable gait trajectory, so as to provide effective basis and reference for motion control.

## II. DYNAMICS AND ENERGY EFFICIENCY

### A. Mechanism and Dynamics

The leg mechanism that discussed in this manuscript is a two-link structure to ensure high rigidity and strength of the leg. Each leg has three degrees of freedom, that is hip abduction-adduction, hip flexion-extension, knee flexion-extension, as shown in Fig. 1.

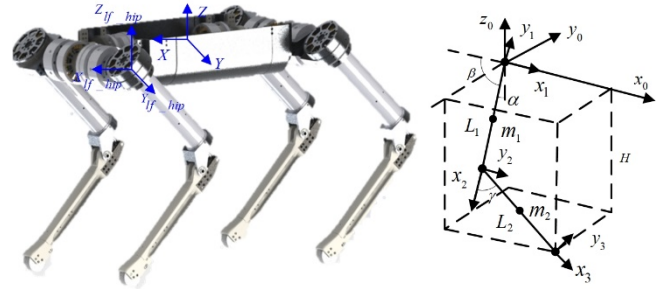


Fig.1 The kinematic model of the robot.

Motion analysis of the legged system can be simplified in the sagittal plane. Assume that the concentrated gravity of each link of the leg is located at the central position of each link, as shown in Fig. 1. Based on the Lagrangian dynamics, the motion equation can be obtained as

$$\tau = D(q)\ddot{q} + H(q, \dot{q}) + G(q) + \tau_f \quad (1)$$

where  $\tau = [\tau_{\text{Hip}}, \tau_{\text{Knee}}]^T$ ,  $q = [\beta, \gamma]^T$ ,  $\dot{q} = [\dot{\beta}, \dot{\gamma}]^T$ ,  $\ddot{q} = [\ddot{\beta}, \ddot{\gamma}]^T$ . And  $D(q)$  represents the inertia matrix,  $H(q, \dot{q})$  represents the centrifugal force and the coriolis force matrix,  $G(q)$  represents the gravity matrix.

When the leg is in the supporting phase, the general external force of the foot is set to  $F_p$ , and the moment of the joint can be obtained by

$$\tau_f = J_{(q)}^T F_p \quad (2)$$

where  $J_{(q)}$  is the Jacobian matrix.

This manuscript mainly analyze the motion performance of the quadruped robot that in trot gait. During the movement, the hip joint torque of the supporting leg ensures that the robot moves forward, and also causes the body to produce a certain pitch motion. While the reaction torque of the swinging leg can weaken the backward of the body to a certain extent,

similar to people balance their bodies by swinging their arms while walking. On the other hand, the quadruped robot will generate a turning moment around the line connecting the two diagonal legs in the trot gait, as shown in Fig. 2.

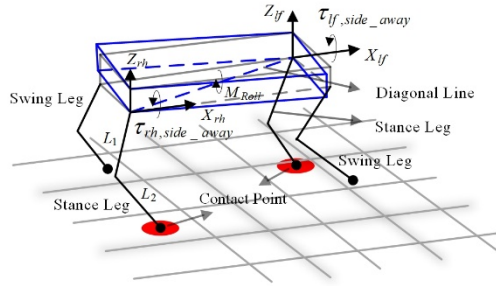


Fig. 2. Diagonal flip by the leg torque in trot gait

In Fig. 2,  $M_{roll}$  represents the flipping torque that rotates around the two diagonal legs, and  $\theta$  is the roll angle when the body rolls along the diagonal legs. There are two DOFs in the hip joint, that are hip abduction-adduction and hip flexion-extension, and the hip flexion-extension joint mainly provides the pushing moment for forward motion. In order to reduce the roll phenomenon, the side-sway joint motor at the hip abduction-adduction joint can be used, and the specific method is to add a spring damping structure in the direction of the roll angle  $\theta$  to suppress the change of  $\theta$ . The generated suppression torque include  $\tau_{lf,side\_sway}$  and  $\tau_{rh,side\_sway}$ .

$$\tau_{lf+rh,side\_sway} = -M_{roll} = -K_{\theta}(\theta_d - \theta) - B_{\theta}(\dot{\theta}_d - \dot{\theta}) \quad (3)$$

where  $K_{\theta}$  is the spring stiffness and  $B_{\theta}$  is the damping stiffness.  $\theta_d$  and  $\dot{\theta}_d$  are the desired angle and its angular velocity for suppressing the roll angle, respectively.  $\theta$  and  $\dot{\theta}$  are the roll angle and its angular velocity, respectively. And the roll angle and angular velocity generated by the body around the two diagonal legs are defined as a sinusoidal function that changes with time.

### B. Ground Reaction Force

In this manuscript, the contact between the feet and the ground is simulated as the rolling movement of the rigid sphere on the elastic ground. And we adopt the spring damping system model and Hertz contact theory to simulate the ground reaction force.

Consider the ground as a plane with a uniform distribution of nonlinear spring damping pairs, and the contact area  $A(z)$  represents the area of the circular cross section in contact with the sphere, as shown in Fig. 3.

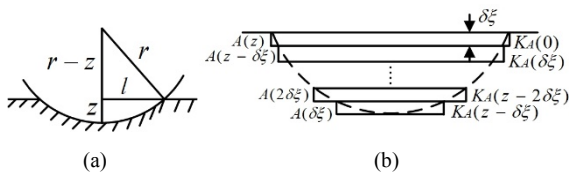


Fig. 3 Foot-ground contact model.

where  $r$  is the radius of the sphere,  $z$  is the compression depth

of the ground. Assuming  $z \ll 2r$ , then  $A(z) = 2\pi rz$  when  $z > 0$ . Therefore, the elastic force  $f_k$  can be calculated as:

$$f_k = \int_0^z A(\xi) K_A(z - \xi) d\xi \quad (4)$$

where  $K_A(z) = \frac{E^*}{2\pi\sqrt{r}} z^{-\frac{1}{2}}$ ,  $\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$ ,  $\nu_1$  and  $\nu_2$  are the Poisson's ratio of the sphere and the ground separately,  $E_1$  and  $E_2$  are the elastic modulus, respectively. Then the ground reaction force  $F_p$  can be obtained by the elastic force  $f_k$  and the damping force  $f_d$ .  $D_n$  is the damping coefficient.

$$\begin{cases} f_k = 4/3 * E^* \sqrt{r} z^{3/2} \\ f_d = D_n \dot{z} \\ F_p = f_k + f_d \end{cases} \quad (5)$$

In supporting phase, the horizontal friction is caused by normal contact force, i.e., the horizontal friction force  $F_f$  is calculated as follow, and  $\mu$  is the friction coefficient.

$$F_f = \mu F_p \quad (6)$$

### C. Mechanism Coefficient

The energy consumption of the robot depends on the gait parameters such as duty factor, step length, gait cycle, starting point of gait, and height. Considering that the leg length of the quadruped robot is fully determined, the main parameters related to this research include the step length  $S$ , step height  $h$ , gait cycle  $T$ , the gait starting point and body height  $H$ . And the foot trajectory model adopts the cycloidal type, and the duty factor is set to 0.5.

Define the foot step coefficient as the ratio of  $L_s$ , which is the distance between the starting point of the foot trajectory and the coronal plane of the hip, to the step length of the robot  $S$ , i.e.  $\xi_s = L_s / S$ . Define height coefficient to  $\xi_H = H / (L_1 + L_2)$ , that is the ratio of the height of the hip joint  $H$  to the total length of the legs, as shown in Fig. 4.

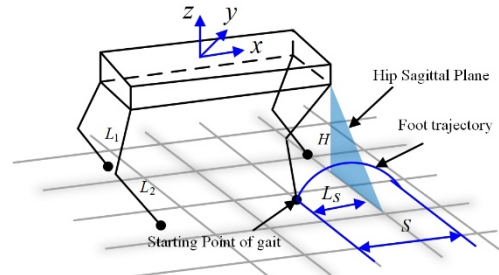


Fig. 4 Foot step coefficient and Height coefficient.

### D. Energy Consumption Model

The mechanical efficiency of the legged robot during walking is an important indicator for evaluating mechanical design. By comparing the energy efficiency of different

motion systems at different speeds, specific resistance  $\varepsilon = P_w / (W_g V)$  is introduced as a dimensionless indicator [8].

The power consumption of the robot constantly changes during motion. By numerical integration of the power in stance phase and swing phase, the energy consumption of the system in the whole period can be obtained. The power is the product of the force/torque of the joint actuator and the speed of the actuator in the whole gait period.

$$P_w = \sum_{i=1}^m \sum_{j=1}^n \tau_{ij} \dot{\theta}_{ij} \quad (7)$$

where  $m$  is the number of robot's legs,  $n$  is the number of joints of each leg.

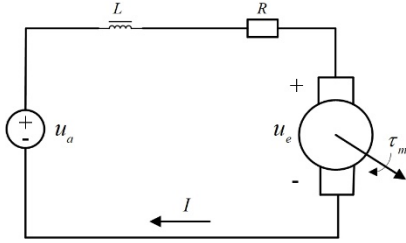


Fig. 5 Simplified model of the permanent magnet synchronous motor.

The robot driving system designed in this manuscript uses a high power density permanent magnet synchronous motor (PMSM). And the energy consumption model of the motor reducer power system in the process of energy consumption is established. According to the PMSM energy model,  $u_a$  is the system input voltage and  $I$  is the armature current, then the energy calculation model is:

$$E = \int_0^T u_a I dt \quad (8)$$

The torque and voltage of the PMSM can be calculated as:

$$\begin{cases} u_a = u_e + RI + L \frac{dI}{dt} \\ u_e = K_e \dot{\theta}_m \\ \tau_m = K_t I \end{cases} \quad (9)$$

where,  $\tau_m$  is the rotor torque of PMSM,  $K_t$  is motor torque constant,  $K_e$  is back electromotive force constant,  $\dot{\theta}_m$  is the angular velocity of the rotor,  $R$  is armature resistance (line resistance),  $L$  is armature inductance,  $u_e$  is the induced voltage in an armature winding relative to the applied voltage.

The relationship between rotor angular velocity  $\dot{\theta}_m$  and reducer output angular velocity  $\dot{\theta}$  is  $\dot{\theta}_m = \dot{\theta} / G_s$ , where  $G_s$  is the gear motor speed ratio. The relationship between rotor torque  $\tau_m$  and output torque  $\tau$  is  $\tau_m = \tau G_s$ , and the influence of winding inductance is small and is ignored. Then, the energy calculation model can be calculated as:

$$E = \int_0^T \tau_N \dot{\theta}_N dt + \int_0^T \frac{3RG_s^2}{K_t^2} \tau_N^2 dt \quad (10)$$

The first term is the mechanical energy of the PMSM, and the second term is the energy loss caused by the thermal

radiation of the PMSM. The subscript  $N$  represents the joint index. The mechanical energy of the PMSM acts as the input energy of the external actuator, and drives the robot to perform the motion, in which the thermal energy directly evaporates as a loss.

According to the specific resistance model, the energy consumption model can be written as:

$$\varepsilon = \frac{E}{MgS_T} \quad (11)$$

where  $M$  is total mass of robot,  $g$  is the gravity acceleration,  $S_T$  is the distance traveled by the robot during a period.

### III. SIMULATION AND ANALYSIS

#### A. Simulation Diagram

By studying three horses in electric treadmill performed walk, trot, gallop gait, and use the oxygen consumption rate as the evaluation index of energy consumption, it can be confirmed that horses and people can change the gait, and choose the speed in the gait to minimize energy consumption [9]. Moreover, the conversion between the gaits usually occurs in the intersection where there is same energy consumption value between speed curve and oxygen consumption rate.

Therefore, this manuscript analyzes the quadruped robot gait when switching energy consumption with a typical speed to improve the energy efficiency of the robot. Through the simulation analysis, the Velocity-Energy Consumption scatter diagram of the quadruped robot walk and trot is obtained. It can be seen that the speed of the gait transition point located roughly 0.5m/s from Fig. 6. In this manuscript, we analyze the relationship between gait parameters and energy consumption under typical speed conditions.

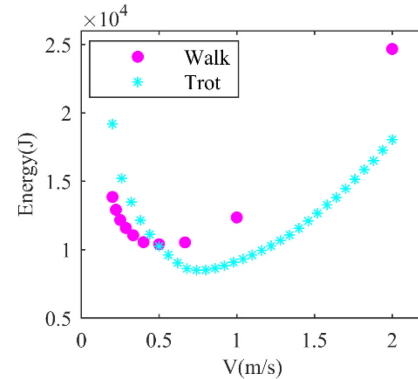


Fig. 6 Energy consumption diagram of quadruped robot moving at different speed and different gait.

This section mainly discusses the kinematic performance of the robot as shown in Fig. 1 under certain parameters, in which the angle range of hip flexion extension and knee flexion extension cannot achieve 360 ° rotations due to the limitation of the leg structure. The main physical parameters of the robot are list in Table I.

The simulation framework in this manuscript is shown in Fig. 7. And the default gait parameters are shown in Table II.

TABLE I  
PARAMETERS OF QUADRUPEL ROBOT IN SIMULATION

Body parameters		value
Dimensions (m)	Length	1.55
	Width	0.88
Mass of the body including payload (kg)	M	120
Link parameters		value
Mass(kg)	Thigh	Shank
	$m_i$	4.1 3.2
Length(m)	Thigh	Shank
	$L_i$	0.6 0.6
Joint angle		value
Angle Range	Hip Joint	20°-160°
	Knee Joint	120°-120°

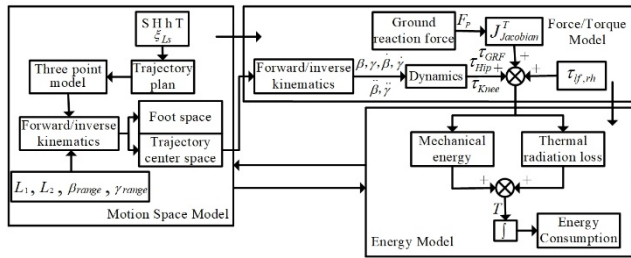


Fig. 7 The simulation diagram.

TABLE II  
DEFAULT GAIT PARAMETERS

Gait parameters	Default value
Step length S(mm)	700
Step height h(mm)	200
Gait cycle T(s)	2.5
Height coefficient $\xi_H$	0.8
Step coefficient $\xi_S$	-0.6

### B. Analysis of Step Coefficient and Height Coefficient

Take three special points in the trajectory from the cycloid model of the foot trajectory: start point  $P_0$ , foot trajectory peak  $P_1$ , stance point  $P_2$ , as shown in Fig. 8. And point  $P$  is the center of the trajectory.

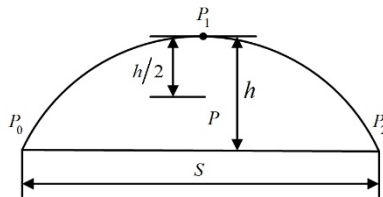


Fig. 8 The cycloid model of the foot trajectory.

The joint angles corresponding to the foot position at the above three points can be obtained by inverse kinematics. According to the joint angle constraint range, we can get the reachable area of the trajectory center. In Fig. 9, the blue region represents the reachable area of the center of the trajectory, and the cyan region represents the reachable area of the foot endpoint.

Each P coordinate point in Fig. 9 corresponds to a foot trajectory. And the variation trend of specific resistance  $\varepsilon$

corresponding to the reachable region of the center trajectory can be obtained, as shown in Fig. 10.

Fig. 10 shows the relationship between step coefficient, height coefficient and  $\varepsilon$ , other gait parameters keep default values. The darker of the blue color is, the lower the energy consumption is. At the position with the smallest energy consumption, the step coefficient is -0.617 and the absolute value of the height coefficient is 0.814.

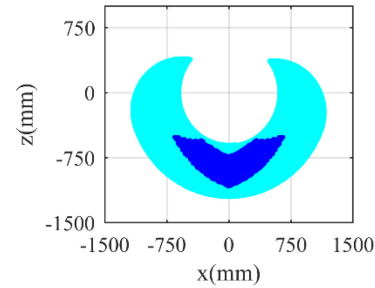


Fig. 9 The reachable region of the trajectory center and the foot endpoint.

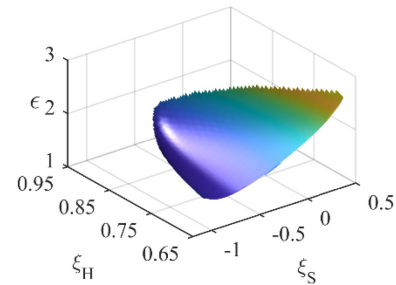


Fig. 10 The corresponding relationship of foot step coefficient, height coefficient and specific resistance.

According to the research of Milton Hildebrand [10] and A. Barstow [11], the horses in the motion process use special strategy to optimize energy consumption. By analyzing framing video of horses at slow speed and medium speed, the step coefficient of the horse in the movement process is approximately -0.6, showing a certain asymmetry. Compared with the step coefficient of robot, the robot and horse have similar motion characteristics under the lowest energy consumption.

### C. Height Analysis

Fig. 11(a) shows the Speed-Height-Energy Consumption diagram of the robot. It can be seen that, at a certain speed, the increase of height will reduce the energy consumption to a certain extent. The ratio of asymmetric step coefficient,  $R$ , is

$$R = \frac{|\varepsilon_{asymmetric} - \varepsilon_{symmetric}|}{\varepsilon_{asymmetric}}$$

When the body height ranges from 0.7 to 1.0m, the step coefficient is -0.6, the energy consumption reduction ratio is 4.2%~5.7%. Meanwhile, when the energy consumption is same, the center of gravity (COM) of asymmetric step is lower than that of symmetric step, which means the robot would be more stable.



#### D. Step Length Analysis

As the step length increases, the energy consumption of the robot motion also increases. Then the asymmetric step and the symmetric step are compared, and the energy consumption reduction ratio is equal to 3%~7% when the step length is equal to 0.5~0.9m, as shown in Fig. 12. Due to the step length is mainly determined based on the speed requirements, which is set to 0.5m/s to meet the two movements of walking and trot in this manuscript, so the step size is equal to 0.7m.

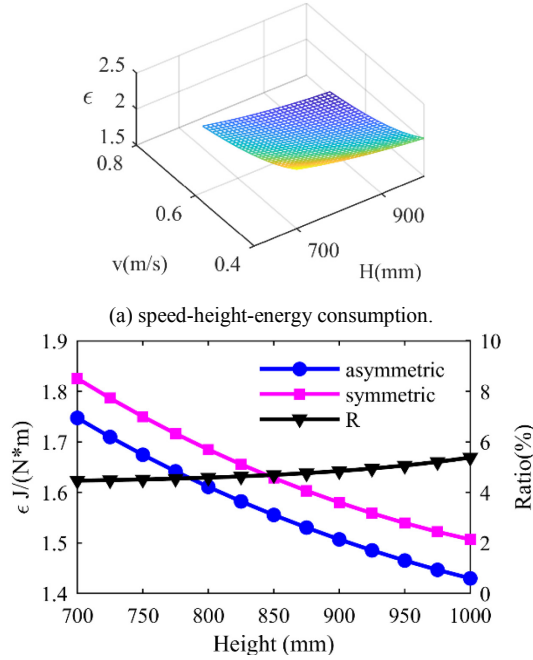


Fig.11 Specific resistance with different height.

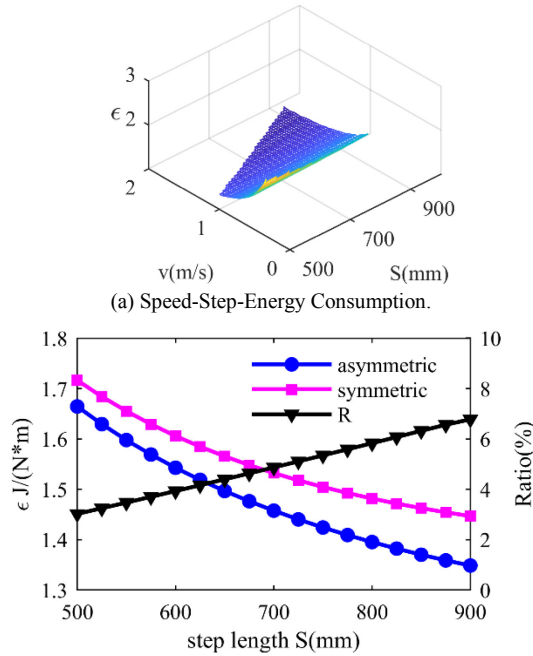


Fig.12 Specific resistance with different step.

#### E. Gait Cycle Analysis

The energy consumption under the multiple gait cycle is analyzed, as shown in Fig. 13(a), it can be seen that under the condition of certain default parameters, the energy consumption of the robot increases as the period increases. Fig. 13(b) shows that the energy consumption reduction ratio is roughly in the range of 4.3% to 8%, in the case of the symmetric step coefficient and the asymmetric step coefficient is used separately. Taking into account the step size and speed of movement, the gait cycle is set to 2.5s.

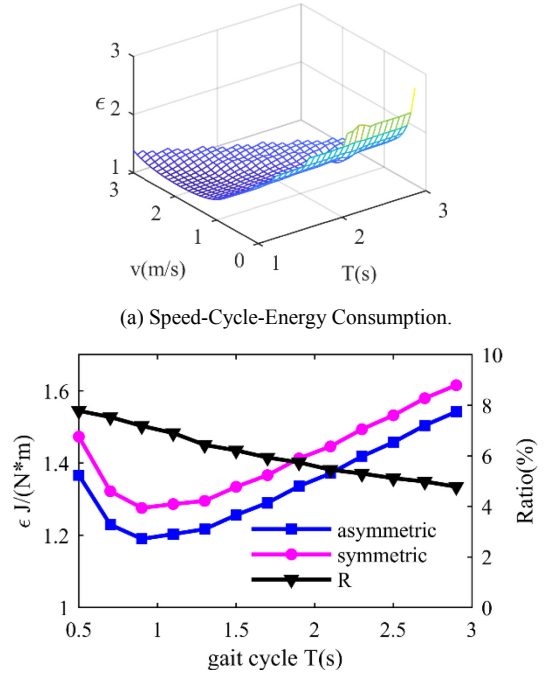
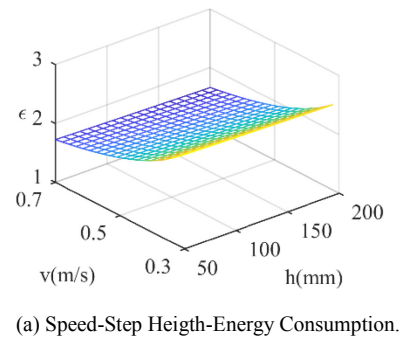
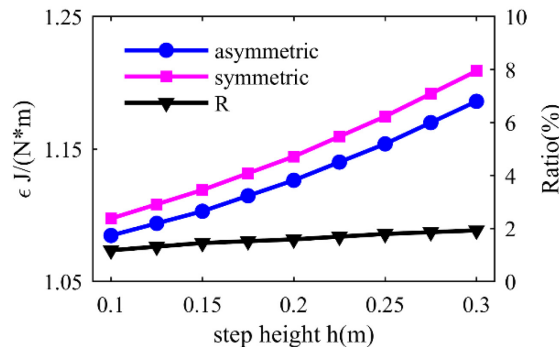


Fig.13 Specific resistance with different cycle.

#### F. Step Height Analysis

The analysis shows that the impact of step height on the energy consumption of the robot is mainly reflected in the swinging phase. For a larger step height, the motion speed and torque of the leg joints are also larger. In symmetric and asymmetric steps, the step height has little effect on energy consumption reduction, so the choosing of step height mainly considers the height of obstacles.





(b) The influence of two kinds of step coefficients on energy consumption.

Fig.14 Specific resistance with different step height.

#### IV. CONCLUSION

In this manuscript, a feasible simulation system framework is constructed in the case where the mechanism configuration, the length of the leg and the whole weight has been determined. Based on the step coefficient and the height coefficient, the motion features and the energy consumption of the quadruped robot in the gait sagittal plane are analyzed, and the relationship between motion energy efficiency and gait parameters including gait type, step size, step height, step frequency, standing height are discussed in detail. The simulation results show that the gait that consistent with high energy efficiency is similar to animals with similar size. According to the research on the motion space and gait parameters of the heavy electricity-driven quadruped robot in the foot end, this manuscript provides some parameter guidance for the following motion control of the whole robot.

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