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A Novel Multi-scale Fourier Descriptor Based on Plane Orthogonal Vectors for Fast Shape Retrieval

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***Abstract*— Shape retrieval is an interesting and challenging issue in computer vision and robot vision. The recent works employing Fourier-based multi-scale descriptors achieve some success in shape retrieval, especially make real-time application possible by improving the efficiency. However, these methods are commonly adopting really complex shape descriptors to present the shape information which is not intuitive enough. In our work, we propose a new Fourier-based method with novel multi-scale descriptor which is a combination of contour feature and regional feature. This novel descriptor has two main characteristics: First, utilize six different invariant signatures which are based on plane orthogonal vectors; Second, leverage multiple scale structures to describe local and global information of shapes. Except the easiness in understanding, this descriptor approach also produces a more distinct topology of shapes than other approaches which means the property of stability on variations, linear or nonlinear. The method is tested on some popular databases that provides a promising retrieval rate(87.46% and 99.27%) with constant time complexity optimization. The results confirm that our method is more capable of representing the topology of shapes than most other methods based on Fourier descriptor.**

***Index Terms*— fast shape retrieval, multi-scale, plane orthogonal vectors, Fourier descriptor**

I. INTRODUCTION

Using shape geometric features for shape retrieval play an important role in computer vision and robot vision. Various work such as leaf shape analysis[1][17], captcha recognition[2], fish shape analysis[3] and germent image retrieval[7] has been explored by researchers. An excellent descriptor should fully reflect the topology and geometry information of the shape which is reflected in robustness to rigid transformation and non-rigid deformation[15]. Nowadays, we still can not find a perfect method to describe dissimilarity of shapes. Therefore, how to construct a shape descriptor for shape retrieval is still an open problem.

In recent years, many descriptors based on contours or region have been developed to improve accuracy of shape retrieval. The contour-based descriptors extract the edge information of shapes and the region-based descriptors consider the whole space. Shape contexts(SC)[4] is one of the most classical methods for shape retrieval which uses diagram of log-polar histogram bins to describe the relationship between corresponding points. Ling et al.[5] uses inner-distance shape contexts(IDSC) instead of Euclidean distance to improve robustness to articulation transformation. Xu et al.[10] proposes a hybrid shape descriptor(HD) which combines with area invariant, arc length invariant and central distance invariant to construct a complete shape features. In addition to explicit descriptors, some implicit features are used as shape descriptors. Shekar et al.[9] propose pattern spectrum combines with inner distance shape context(PS+IDSC) that obtain distribution of skeleton and contour respectively. Alwealy et al.[11] extract the spectral domain and propose an adaptive graph model based on threshold, which describes the shape feature from local view. Most of methods mentioned above are based on dynamic programming(DP). DP algorithm can effectively improve the accuracy of similarity estimation for descriptors. However, high time complexity($O(N^3)$) is unsuitable for shape retrieval in large database.

To improve shape retrieval efficiency, some descriptors combine Fourier transformations to compute dissimilarity, which reduced the dimension of shape features. The Fourier descriptors(FDs)[12][13] can quickly obtain the dissimilarity from overall contour and maintain the integrity of descriptor. El-ghazal et al.[14] put forward a new Fourier descriptor based on curvature(CBFD) to improve recognition performance. Perimeter area function(PAF)[8] is proposed to construct a Fourier feature based on triangular feature. This hybrid scheme is proved to have advantages over traditional Fourier descriptor in shape recognition. Jomma et al.[6] first proposed circle views(CVs) shape signature based on circular

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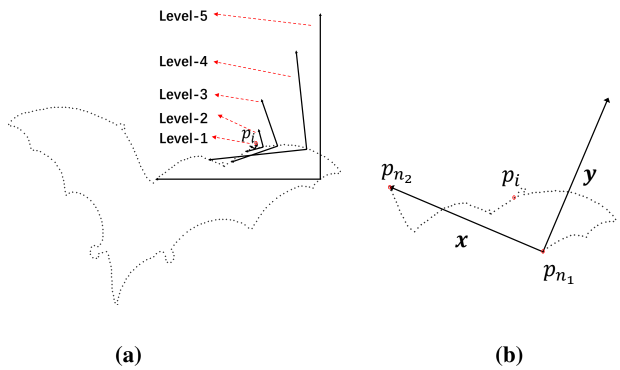

Fig. 1. Definition of the descriptor based on plane orthogonal vectors. In (a) we take 256 sample points.The figure (b) is specific definition in each level scale.

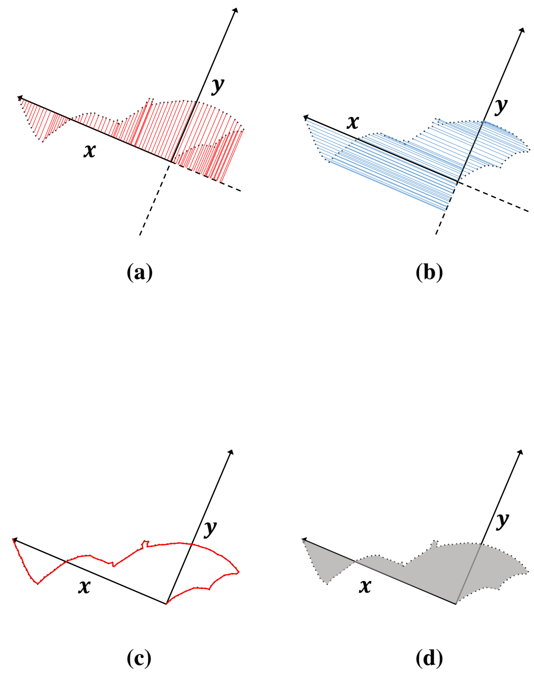

Fig. 2. The explanation of features based on plane orthogonal vectors. The red lines and blue lines in (a) and (b) represent the distance from point to the vector **x**,**y** respectively. The red curve in (c) is the curve segment. The gray area in (d) denotes area closed by vector **x** and the curve segment.

orbit. The simple and efficient feature makes CVs more practical for Various application scenarios. These Fourier descriptors can help us find similar shapes from database quickly. But they describe shapes in a single signature and can not express integrity of shape structure. So far, using the FDs for high accuracy retrieval is still a problem worth challenging.

In general, multiple Fourier descriptors can make up for the defect of single scale Fourier descriptor. Some multi-scale features such as string cut[15], circle features[22], curvature features[14][17], triangle features [8] and ellipse features[25] are proposed to improve retrieval speed. On the one hand, the combination of different features can make up for the limitation of single feature. On the other hand, more comprehensive shape information can be obtained by describing shape features from both local and global perspectives. Global level features capture the overall information and local level features acquire details. However, Most multi-scale descriptors cause it challenge to visualize the intrinsic meaning of topological diagnosis which make descriptor difficult to understand. Many descriptors also lack of region-based features, which makes the structure of multi-scale descriptors incomplete.

After the discussion above,we propose a novel multi-scale descriptor based on plane orthogonal vectors(POVs) including six invariants which combine with contour features and region features. The proposed POVs provides complete and visualized shape features and distinguish different shapes quickly with Fourier descriptor. Some excellent properties of our descriptor such as translation invariance, rotation invariance, scale invariance and robustness for inter-class changes are proved on famous databases:MPEG-7[14][15][16][17][18] and Kimia's 99[4][5]. The experimental results express that our descriptor can distinguish similarity measures between complex shapes quickly and efficiently.

II. Descriptors Based on Plane Orthogonal Vectors

A. *Definition of the Descriptors*

A set of uniform spaced sample points can effectively represent a closed shape, which decrease the computation time of descriptor. [4][5][15][16][17][22]. Therefore, a closed shape contour C can be expressed by N anticlockwise simple points sequence $P = \{p_i | i = 1, 2, ..., N\}$, where i is index of points sequence and N is length of contour. In order to facilitate the definition of the descriptor, N should be in the power of 2. In fact, the contour is closed, there are $p_{-i} = p_{N-i}$ and $p_{N+i} = p_i$. These simple points can be regard as a discrete periodic sequence.

For sample point p_i, we take neighboring points p_{n_1},p_{n_2} where the level scale is λ. Let $\lambda = 1, 2, ..., logN - 1$ denote the level of scale size where the neighboring points $p_{n_1} = p_{i-2^\lambda}$, $p_{n_2} = p_{i+2^\lambda}$ respectively(see Fig. 1 (a)). It is obvious that lower level distance describes local feature, and higher level distance represents global feature. Construct a pair of plane orthogonal vectors $\mathbf{x}, \mathbf{y}$ intersecting at p_{n_1} where $\mathbf{x}$ is going in the direction from p_{n_1} to p_{n_2} and $\mathbf{y}$ is going in the clockwise direction of **x** and is perpendicular to **x**. The length of **x** and **y** is equal to the straight line distance from p_{n_1} to p_{n_2}. Let points sequence $L_i = \{p_{n_1}, p_{n_1+1}, ..., p_{n_2-1}, , p_{n_2}\}$ denote a piece of slice sequence of the contour C, where begins with p_{n_1} and ends at p_{n_2}. Normally, L_i pass through the orthogonal vectors **x**,**y** and fall to the both sides of them(see Fig. 1 (b)). We defined that the set of points that fall to the left of vector is $P_r(P_r = \{p_{r1}, p_{r2}, ..., p_{r3}, , p_{rk}\})$. Equally, the set of points that fall to the left of vector is $P_l(P_l = \{p_{l1}, p_{l2}, ..., p_{l3}, , p_{lj}\})$.

B. Plane Orthogonal Vectors Features

A suitable graphical feature is not only robust to intra-class expression but also can effectively distinguish topological differences between inter-class. There are many advanced topological features including curvature, distribution contour points, and area that are proved effective. We combine existing geometric features with property of orthogonal vectors and propose a new multi-scale measure in terms of six invariant features.

In each segmented level, the topological properties of the points sequence L_i can be described as the horizontal distribution deviations (f_1 and f_2), vertical distribution deviations (f_3 and f_4), degree of bending(f_5), and cross-cut area(f_6) as defined below.

$$f_1^{\lambda}(i) = \max\left(\frac{1}{M_{P_r}} D(P_r, \mathbf{x}), \frac{1}{M_{P_l}} D(P_l, \mathbf{x})\right) \qquad (1)$$

$$f_2^{\lambda}(i) = \min\left(\frac{1}{M_{P_r}} D(P_r, \mathbf{x}), \frac{1}{M_{P_l}} D(P_l \mathbf{x})\right) \qquad (2)$$

$$f_3^{\lambda}(i) = \max\left(\frac{1}{M_{P_r}} D(P_r, \mathbf{y}), \frac{1}{M_{P_l}} D(P_l, \mathbf{y})\right) \qquad (3)$$

$$f_4^{\lambda}(i) = \min\left(\frac{1}{M_{P_r}} D(P_r, \mathbf{y}), \frac{1}{M_{P_l}} D(P_l, \mathbf{y})\right) \qquad (4)$$

$$f_5^{\lambda}(i) = \frac{d(p_{n_1}, p_{n_2})}{Length_{\lambda}(i)} \qquad (5)$$

$$f_6^{\lambda}(i) = Area_{\lambda}(i) \qquad (6)$$

where M_{P_r}, M_{P_l} are the numbers of points in P_r and P_l, **x** and **y** are the horizontal and vertical vectors mentioned above, $Length_{\lambda}(i)$ is the length of the curve segment L_i, $d(p_{n_1}, p_{n_2})$ is the Euclidean distance between p_{n_1} and p_{n_2}, and $Area_{\lambda}(i)$ is the area enclosed by the L_i and **x**. The general form of function $D(P, \mathbf{v})$ means sum of perpendicular distance from the points of set P to the line represented by vector **v** which can be calculated by

$$D(P, \mathbf{v}) = \sum_{p_j \in P} \frac{|(x_p - x_i)(y_j - y_i) - (y_p - y_i)(x_j - x_i)|}{||\mathbf{v}||_2} \qquad (7)$$

where (x_i, y_i) is start point of **v** and (x_j, y_j) is the end point.

In Eq. (1)-(4), we consider the point average distance distribution in both horizontal and vertical directions(see in Fig.2 (a) and (b)), which shows the topological characteristics of curves in plane space more comprehensively. We eliminate the effects of mirror transformation by taking the maximum values of the average distance between the left and right sides. In Eq. (5), using ratio of the length of vector **x** to the length of curve segment denotes degree of bend of a curve(see in Fig.2 (c)). The length of **x** determines the size of the ratio. The Eq. (6) represents the area enclosed by curve segment and vector **x**, which denotes fullness of curve(see in Fig.2 (d)). In these formulas, we use the region feature area to compensate for the locality of contour, making our descriptor more robust.

These six features maintain invariant to translation of the graphic and also tolerate slight deformation. To solve plane orthogonal vectors signatures invariant to scaling, we normalize them by dividing the maximum of features themselves. Next, we use Fourier transform to keep signatures stable with rotation transform and improve computing efficiency. The magnitudes of Fourier transform coefficients are represented as

$$F_{\alpha}^{\lambda}(m) = \frac{1}{N}\left|\sum_{i=0}^{N-1} f_{\alpha}^{\lambda}(i) exp\left(\frac{-j2\pi im}{N}\right)\right| \qquad (8)$$

where $\alpha = 1,2,3,4,5,6$; $m = 0, ..., N-1$ is order of $F_{\alpha}^{\lambda}(m)$(specific description see in [15]). We choose the first $M(M \ll N)$ coefficients of Fourier to reduce noise interference and calculated amount of dissimilarity. Ultimately, we get plane orthogonal vectors descriptor(POVs) $\zeta_{\alpha}^{\lambda} = \{F_{\alpha}^{\lambda}(m), \sigma_{\alpha}^{\lambda} | m = 0, ..., M-1\}$ processed by Fourier transform where $\sigma_{\alpha}^{\lambda}$ is corresponding standard deviation of f_{α}^{λ} designed to enhance the ability of distinctiveness.

C. Shape Dissimilarity Measure

Given two shapes A and B from database, we get POVs $\zeta(A) = \{F_{\alpha}^{(A)\lambda}(m), \sigma_{\alpha}^{(A)\lambda}\}$ and $\zeta(B) = \{F_{\alpha}^{(B)\lambda}(m), \sigma_{\alpha}^{(A)\lambda}\}$. The shape dissimilarity is based on Euclidean distance which can be computed by

$$D(A, B) = \sum_{\lambda=1}^{logN-2} \sum_{m=0}^{M-1} \sum_{\alpha=1}^{6} \left| w_{\alpha}(\zeta_{\alpha}^{(A)\lambda}(m) - \zeta_{\alpha}^{(B)\lambda}(m)) \right| \qquad (9)$$

where w_{α} is weight of those features of POVs.

It is noteworthy that when $\lambda = logN - 1$, The start point and the end point of the level points sequence p_{n_1} , p_{n_2} overlap each other which is unsuitable for Eq. (9). In order to keep integrity of our descriptors, we use global features eccentricity and rectangularity[19] to replace the dissimilarity measure of this level. The eccentricity is represented as

$$E(S) = \frac{Minor(S)}{Major(S)} \qquad (10)$$

where S is contour of shape, $Minor(S)$ and $Major(S)$ are major axis and minor axis of fitting ellipse of S respectively.

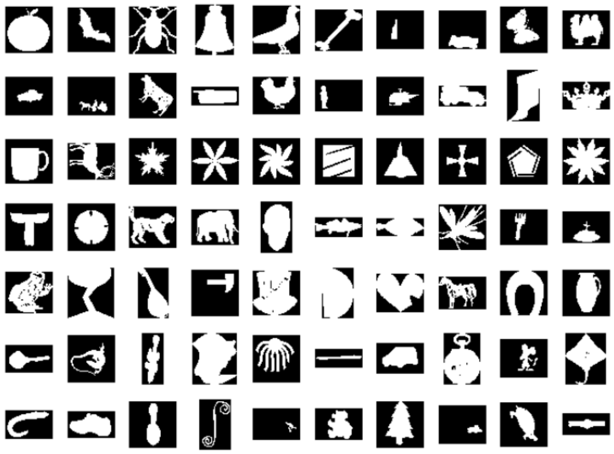

Fig. 3. Part shapes from each class of MPEG-7 database

The rectangularity can be calculated as

$$R(S) = \frac{Area(S)}{Rect(S)} \tag{11}$$

where $Rect(S)$ is minimum circumscribed rectangle of S and $Area(S)$ is area closed by S. Finally, the dissimilarity between shape A and B can be expressed as

$$Dis(A, B) = D(A, B) + |E(A) - E(B)| + |R(A) - R(B)| \tag{12}$$

D. Time Complexity of Shape Matching

The time complexity of shape retrieval includes two parts: calculation of descriptor and shape matching. In large databases, the time of shape matching is much larger and more principal than calculation of descriptor which is almost negligible. Here, we only analyze the time complexity of shape matching. In Eq. (9), the complexity of calculating dissimilarity POVs is equal to

$$O(6Mlog(N - 2)) = O(MlogN) \tag{13}$$

The time complexity of calculating eccentricity and rectangularity difference is $O(1)$, which is lower than $O(MlogN)$ consumption, therefore, the overall computational cost is $O(MlogN)$ in Eq. (12).

III. EXPERIMENTS

In our experiments, to estimate the accuracy and timeliness of our descriptor we selected the famous MPEG-7 shape dataset as the experimental data and compare with some classic and advanced methods. The parameters($N = 256, M = 13, w_1 = 1.7, w_2 = 1, w_3 = 0.1, w_4 = 1, w_5 = 0.5, w_6 = 1$) is set for our experiments.

TABLE I

"BULLS EYE" SCORE UNDER DIFFERENT KINDS OF FEATURE EXTRACTION BASED ON MPEG-7 DATASET

Situations	Score
contour-based features of POVs	84.80%
region-based features of POVs	78.62%
POVs	87.46%

TABLE II

"BULLS-EYE" SCORES BASED ON THE MPEG-7 DATASET

Shape descriptors		Score	Complexity
Non-Fourier based methods	CSS[20]	75.44%	$O(N)$
	SC[4]	76.51%	$O(N^2)$
	MCC[21]	84.93%	$O(N^3)$
	IDSC+DP[5]	85.40%	$O(N^3)$
	HD[10]	90.25%	$O(N^3)$
	IMD+LP[26]	94.51%	$O(N^3)$
Fourier based methods	FD[23]	67.94%	$O(N)$
	TSA+TCD+TASL[16]	83.94%	$O(MlogN)$
	CVs+SGF[6]	83.71%	$O(N^2)$
	HSC[15]	87.31%	$O(MlogN)$
	proposed POVs	**87.46%**	$\boldsymbol{O(MlogN)}$

A. MPEG-7 Dataset

MPEG-7 is a popular dataset widely used in image retrieval and shape match. It consists of 1400 shape images, which are equally divided into 70 classes. Part of the shapes is shown in Figure 3. As we can see, there are many similar shapes that belong to different categories. So, testing on MPEG-7 is a very challenging matter for proving the robustness of descriptors.The results of experiments are determined by well-known test methods "bulls-eye" score, in this measurement, each shapes of the dataset is served as a query images to get 40 best matching shapes from MPEG-7. Counting the number of shapes which belong to the same category, the maximum number of matching shapes is 20. Therefore, the retrieval of "bulls eye" score is the ratio of total similar shapes to the highest possible numbers (1400 $\times$ 20).

The Table I shows results of experiments under different kinds of feature extraction. The best score is obtained when we mix contour-based and region-based features. Considering the above three descriptors, contour-based descriptor shows

Fig. 4. Shapes from Kimia's 99 database

TABLE III

EXPERIMENTAL RESULTS BASED ON KIMIA'S 99 DATASET

Shape descriptors	**Score**
SC[4]	76.36%
CSPH+EMD[24]	81.61%
HD[10]	94.46%
proposed POVs	**99.27%**

higher accuracy which reflect the overall part of the contour. However, the insufficient detail of contour-based descriptor is effectively compensated by region-based features, with a 2.66% increase in score.

In Table II, our proposed descriptors compare with existing methods on "bulls eye" score and time complexity of shape match, which achieves promising results. The shape descriptors in Table II are divided into two parts which are based on Fourier and non-Fourier respectively. We can observe that our descriptors is better than other descriptors based on Fourier and reach a high level in Fourier-based methods. In the Fourier-based descriptors, TSA+TCD+TASL[16](83.94%) and HSC[15](87.31%) are both based on multi-scale Fourier descriptors which are related to our descriptor. But our method(87.46%) performs more robust to complex shapes in MPEG-7 than other methods. In addition, some descriptors such as ellipse features[25] and curved shape features[27] also get satisfactory results, but the topological structure of features is not intuitive enough and the construction features in ellipse make the expression of descriptors limited. In contrast, our method makes the descriptors more flexible and expandable by constructing features in two-dimensional plane, which makes it possible to match and retrieve the shape of 3D objects. In the methods other than Fourier descriptors, the descriptor IMD+LP[26](94.51%) has a slightly higher scores than POVs with a 7.05% increase, however, the high time complexity is unsuitable for fast shape retrieval.

B. Kimia's 99 Dataset

The Kimia's 99 dataset[5] is also a standard dataset used for shape retrieval(see in Figure 4). The database includes 99 shapes from nine different categories. each of which contains some occlusions, articulations change, which causes challenge for shape retrieval. Our method adopts the same test manner as "bulls eye" score on the MPEG-7 database and compares with other descriptors(see in Table III). Our descriptor shows great power in the Kimia's 99 dataset(99.27%), which indicates the robustness of POVs to various intra-class variation.

IV. CONCLUSIONS

In our paper, we proposed a novel multi-scale Fourier descriptor based on plane orthogonal vectors for fast shape retrieval. There are two main contributions of our method. On the one hand, we use orthogonal vectors to calculate the distribution of contour segment from multiple scales, which is more intuitive than other multi-scale Fourier descriptor. On the other hand, POVs combines with contour-based feature and region-based feature which improves the robustness of descriptor to complex shapes. The results of experiments in MPEG-7 dataset and Kimia's 99 dataset demonstrate that our method is superior to other multi-scale Fourier methods and dose not perform unfavorably comparing with non-Fourier based methods. Low-cost and efficient shape matching capability enables POVs to be applied to large databases for fast shape retrieval.

REFERENCES

[1] F. Ni,B. Wang, "Integral contour angle: an invariant shape descriptor for classification and retrieval of leaf images," IEEE International Conference on Image Processing, pp.1223-1227, 2018.

[2] G. An and W. Yu, "CAPTCHA recognition algorithm based on the relative shape context and point pattern matching," International Conference on Measuring Technology and Mechatronics Automation (ICMTMA), 2017, pp. 168-172.

[3] E. Milios and E. G. M. Petrakis, "Shape retrieval based on dynamic programming," in IEEE Transactions on Image Processing, vol. 9, no. 1, pp. 141-147, Jan. 2000.

[4] S. Belongie, J. Malik and J. Puzicha, "Shape matching and object recognition using shape contexts," in IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 24, no. 4, pp. 509-522, April 2002.

[5] H. Ling and D. W. Jacobs, "Shape classification using the inner-distance," in IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 29, no. 2, pp. 286-299, Feb. 2007.

[6] H. D. Jomma and A. I. Hussein, "Circle views signature: a novel shape representation for shape recognition and retrieval," in Canadian Journal of Electrical and Computer Engineering, vol. 39, no. 4, pp. 274-282, Fall 2016.

[7] A. L. Hou, L. Q. Zhao and D. C. Shi, "Garment image retrieval based on multi-features," International Conference on Computer, Mechatronics, Control and Electronic Engineering, 2010, pp. 194-197.

[8] B. Wang, "Shape retrieval using combined Fourier features," Optics Communications, Volume 284, Issue 14, 2011, Pages 3504-3508.

[9] B. H. Shekar and B. Pilar, "A decision level fusion of morphology based Pattern Spectrum and IDSC for shape representation and classification," International Conference on Contemporary Computing and Informatics (IC3I), 2014, pp. 908-914.

[10] H. Xu, J. Yang, Y. Tang and Y. Li, "A hybrid shape descriptor for object recognition," IEEE International Conference on Robotics and Biomimetics (ROBIO), 2015, pp. 96-101.

[11] B. Alwaely and C. Abhayaratne, "Graph spectral domain shape representation," European Signal Processing Conference (EUSIPCO), 2018, pp. 598-602.

[12] C. T. Zahn and R. Z. Roskies, "Fourier descriptors for plane closed curves," in IEEE Transactions on Computers, vol. C-21, no. 3, pp. 269-281, March 1972.

[13] D. Zhang, G. Lu, "Study and evaluation of different Fourier methods for image retrieval," Image Vis. Comput. vol. 23, no. 1, pp. 33–49, 2005.

[14] A. El-ghazal, O. Basir, S. Belkasim, "Invariant curvature-based Fourier shape descriptors," Journal of Visual Communication and Image Representation. vol. 23, no. 4, pp. 622-633, 2012.

[15] B. Wang and Y. Gao, "Hierarchical String Cuts: A translation, rotation, scale, and mirror invariant descriptor for fast shape retrieval," in IEEE Transactions on Image Processing, vol. 23, no. 9, pp. 4101-4111, Sept. 2014.

[16] C. Z. Yang, Q. Yu, "Multiscale Fourier descriptor based on triangular features for shape retrieval," in Signal Processing: Image Communication, vol.71, pp. 110-119, 2018.

[17] J. Zeng, M. Liu, X. Fu, R. Gu and L. Leng, "Curvature bag of words model for shape recognition," in IEEE Access, vol. 7, pp. 57163-57171, 2019.

[18] C. Grigorescu and N. Petkov, "Distance sets for shape filters and shape recognition," in IEEE Transactions on Image Processing, vol. 12, no. 10, pp. 1274-1286, Oct. 2003.

[19] C. Martinez-Ortiz, D. Žunić and J. Žunić, "Measuring shape rectangularities," 2011 IEEE International Symposium on Signal Processing and Information Technology (ISSPIT), Bilbao, 2011, pp. 369-374.

[20] S. Abbasi, F. Mokhtarian, J. Kittler. "Curvature scale space image in shape similarity retrieval." Multimedia Systems, 1999, vol. 7, no. 6, pp. 467-476.

[21] T. Adamek and N. E. O'Connor, "A multiscale representation method for nonrigid shapes with a single closed contour," in IEEE Transactions on Circuits and Systems for Video Technology, vol. 14, no. 5, pp. 742-753, May 2004.

[22] X. Yang, S. Koknar-Tezel and L. J. Latecki, "Locally constrained diffusion process on locally densified distance spaces with applications to shape retrieval," IEEE Conference on Computer Vision and Pattern Recognition, Miami, FL, 2009, pp. 357-364.

[23] T. P. Wallace, P. A. Wintz. "An efficient three-dimensional aircraft recognition algorithm using normalized fourier descriptors," Computer Graphics & Image Processing, vol. 13, no. 2 pp. 99-126, 1980.

[24] X. Shu, X. J. Wu, "A novel contour descriptor for 2D shape matching and its application to image retrieval." Image and Vision Computing, 2011, vol. 29, no. 5, pp: 286-294.

[25] X. Zhang, J. Xiang and S. Xiong, "Shape retrieval using multiscale ellipse descriptor," 2017 IEEE International Conference on Image Processing (ICIP), Beijing, 2017, pp. 1042-1046.

[26] J. Y. Yang, H. X. Wang, J. S. Yuan, Y. F. Li, J. Y. Liu, "Invariant multi-scale descriptor for shape representation, matching and retrieval," Computer Vision and Image Understanding, vol. 145, pp. 43-58, 2016.

[27] G. G. Demisse, D. Aouada and B. Ottersten, "Deformation based curved shape representation," in IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 40, no. 6, pp. 1338-1351, 1 June 2018.