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Normalized Neural Network for Energy Efficient Bipedal Walking Using Nonlinear Inverted Pendulum Model

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***Abstract*— In this paper, we present a novel approach for bipedal walking pattern generation. The proposed method is designed based on 2D inverted pendulum model. All control variables are optimized for an energy efficient gait. To obviate the need of solving non-linear dynamics on-line, a deep neural network is adopted for fast non-linear mapping from desired states to control variables. Normalized dimensionless data is generated to train the neural network, therefore, the trained neural network can be applied to bipedal robots of any size, without any specific modification. The proposed method is later verified through numerical simulations. Simulation results demonstrated that the proposed approach can generate feasible walking motions, and regulate robot's walking velocity successfully. Its disturbance rejection capability was also validated.**

I. INTRODUCTION

Legged robots have strong ground mobility; they can maneuver in unstructured, natural environments, where wheeled robots cannot. The locomotion mechanism of legged robots, especially under bipedal actuation, is hard to control due to the nature of discrete terrain contact and non-linear dynamics. The governing equations are periodically changing during motion, followed by large instability regions, thus warranting classical methods forfeiting desired performance.

There have been many methodologies proposed for bipedal locomotion, of which can be divided into two groups [1]. The first group bases bipedal locomotive design on precise knowledge of robots' dynamic parameters, therefore performance of these methods primarily depend on the accuracy of the robotic model [2] [3]. While for the second group, bipedal locomotion is based on a simplified model, and feedback control must be employed to stabilize this approach [4], [5]. The most popular simplified model from various literature, is the linear inverted pendulum model (LIPM) [6], in which the center of mass (CoM) is constrained at a constant height. The LIPM's equations of motion can be solved analytically due to its linear form. Kajita et al. [1] adopted preview control of a Zero Moment Point (ZMP) to generate a walking pattern, where the ZMP calculation is derived from a LIPM. Morisawa et al. [7] used a LIPM to simultaneously plan trajectories of CoM and ZMP, and modify foot trajectory according to the detection of disturbance. Pratt et al. [8] proposed the Capture Point (CP) concept, based on the LIPM to solve foothold positions to avoid falling. Although LIPM's are widely employed due to its simple mathematical representation, there are still some limitations, such as foot landing impact and the corresponding loss of mechanical energy is not considered within the model. Walking patterns designed from a LIPM is not the most energy efficient. Robots have to bend their knees to maintain constant CoM height during walking, therefore, unnecessary energy is inevitably consumed. Meanwhile, it is also likely to meet actuator saturation through a LIPM [9].

Another simplified model, is the inverted pendulum model (IPM). IPM's are usually constrain the leg length to be constant to simplify the robot's dynamics. Numerical integration is usually necessary for solving the IPM equations of motion due to the presence of non-linearity. IPM's are widely applied in passive dynamic walking [10] [11] and balancing [12], where the loss of kinetic energy is compensated by an increase in potential energy. Wight et al. [13] proposed the Foot Placement Estimator, which is based on the IPM as a dynamic measure of balance for bipedal robots. Non-linear equations introduced by the IPM were solved numerically. Srinivasan and Ruina's research [14] shows that the most energy efficient walking gait for a point-mass walker is the inverted pendulum walking with constant leg lengths, proving that the IPM is more energy efficient than the LIPM. However, IPM's are also coupled with limitations. In the IPM, large impact occurs at the foot-to-ground contact due to the constrained constant leg length, which makes the IPM approach more suitable for robots with soft robot-environment interaction capabilities [9]. Concurrently, the non-linear dynamics of IPM provides no analytical solution. Although it can be solved through numerical methods, the heavily required computational time and resources make the IPM not suitable for real-time applications.

In the past decades, the majority of robotic bipedal applications are designed based on LIPM's, leading to knee-bent walking in humanoid robots for kinematic singularity avoidance. However, anthropological walking is much more similar to the IPM as the support leg is always straight [15]. To improve the bipedal walking performance anthropomorphically and give a better understanding of human and animal locomotion, an IPM is chosen in this paper instead of the conventional LIPM. However, there still remains the one significant challenge of solving the IPM's non-linear dynamics in real time.

Recently, researchers have introduced artificial neural networks in legged robots to explore the possibility of real-time implementation using non-linear models. Xin et al. [16] adopted a neural network to generate referential foot placements for a bipedal robot hopping and running based

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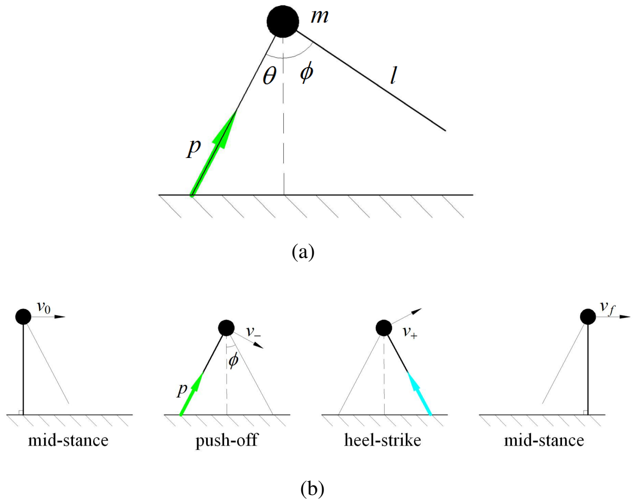


Fig. 1: **2D inverted pendulum walking model.** (a) The employed IPM. (b) A mandate of the walking sequence with a regulatory speed target at the mid-stance by properly choosing control variables.

on a spring-loaded inverted pendulum model. The neural network eliminates the need for solving non-linear equations on-line. Raj et al. [17] employed a neural network and Open Dynamics Engine to build a hybrid model for the locomotion control of a bipedal robot. Li et al. [18] adopted central pattern generators and neural networks to generate bipedal locomotion. However, the neural networks used in the following research investigations lack generality and work exclusively on specific robots.

In this paper, a novel approach for the locomotion control of bipedal robot walking is presented. The proposed method is designed based on a two dimensional IPM. All control variables are optimized to ensure an energy efficient walking gait. To eliminate the need for solving the non-linear equations on-line, a deep neural network is employed to realize fast non-linear mapping of the desired states to the control variables. The neural network is trained with normalized data. Therefore, the proposed approach can be applied to bipedal robots of any size without any specific modification. The proposed method is verified through various simulations, in which the results prove feasibility and robustness of the approach.

The paper is arranged as follows. Section II describes the dynamics of the IPM and feasible constraints. Section III gives details about how the control variables are optimized and how the neural network is trained. Finally, simulation results are presented in Section IV and conclusions are given in Section V.

II. 2D Inverted Pendulum Model

The walking bipedal robot model is considered to be a 2D IPM and only forward walking on a flat ground is discussed. The IPM consists of a single point-mass hip and two massless inextensible legs, as shown in Fig. 1. There are two control variables in this model: push-off impulse p along the stance leg that occurs just before foot-to-ground contact, and angle ϕ between the swing leg and vertical axis before impact. Push-off impulse determines the energy injected, and angle of the swing leg determines the time and location of the foot-to-ground contact.

For the sake of generality, normalization of the robots states and control variables are applied to make the proposed approach compliant for robots of differentiating illimitable sizes. If we don't do this, the trained neural network in the subsequent section will only work exclusively on specific robots. The robot mass m, leg length l and gravitational acceleration g are used as quantities for normalization. CoM velocity v can be normalized by $\sqrt{gl}$, push-off impulse p by $m\sqrt{gl}$, and time t by $\sqrt{l/g}$. Note that such normalization is effectively the equivalent of equating the values of m, l, and g equal to 1 [19].

The 2D inverted pendulum model's equation of motion is

$$\ddot{\theta} = \sin\theta, \tag{1}$$

where θ is angle between the stance leg and vertical axis. The foot-to-ground contact occurs and stance leg changes when $\theta = \phi$. Assume the initial velocity at a mid-stance is v_0, where mid-stance is the state when the stance leg is perpendicular to ground. According to the conservation of energy equations, velocity just before push-off is

$$v_- = \sqrt{v_0^2 + 2(1 - \cos\phi)}, \tag{2}$$

and velocity just after heel-strike can be expressed according to the conservation of angular momentum as [20]

$$v_+ = v_- \cos 2\phi + p \sin 2\phi. \tag{3}$$

Then the final velocity at the next mid-stance is

$$v_f = \sqrt{v_+^2 - 2(1 - \cos\phi)}. \tag{4}$$

Note that the final velocity v_f can be written as the function of v_0, p and ϕ as

$$v_f = \sqrt{(\sqrt{v_0^2 + 2(1 - \cos\phi)}\cos 2\phi + p\sin 2\phi)^2 - 2(1 - \cos\phi)}. \tag{5}$$

According to the above equation, the final velocity v_f can be regulated by properly choosing the push-off impulse p and swing leg angle ϕ.

To make the walking step feasible, there are several constraints which must be held. Firstly, to make sure no fight phase occurs during walking, the following inequality constraints are to be satisfied,

$$v_-^2 < \cos\phi, \tag{6}$$

$$v_+^2 < \cos\phi, \tag{7}$$

$$0 < p\cos 2\phi < \sin 2\phi v_-. \tag{8}$$

Equation (6) and (7) ensures the normal force on the foot is positive just before push-off and after the heel-strike. Equation (8) ensures radial speeds after push-off is zero. These equations can be converted into constraints of speeds

v_0 and v_f at the mid-stance, and control variables p and ϕ, as

$$\max(v_0^2, v_f^2) < 3\cos\phi - 2 \tag{9}$$

and

$$0 < p < \tan 2\phi\sqrt{v_0^2 + 2(1 - cos\phi)}. \tag{10}$$

Secondly, to prevent any slip motion, the inverted pendulum is constrained in the friction cone by

$$\tan\phi < \mu, \tag{11}$$

where μ is coefficient of friction. Meanwhile, to limit the walking pace, a fixed lower bound on step time t_{st} is imposed, which is the time from the mid-stance to the heel-strike, by

$$t_{st} > t_{st,\min} > 0. \tag{12}$$

Considering the small angle approximation assumption and integrating (1), t_{st} can be approximated by [9]

$$t_{st} = \ln\left(\frac{\phi + \sqrt{v_0^2 + \phi^2}}{v_0}\right). \tag{13}$$

Thus, (12) can be converted to a constrained ϕ, as

$$\phi > v_0 \sinh t_{st,\min}. \tag{14}$$

III. Neural network design with IPM

A. Control variables optimization

The target is to regulate the robot's velocity v_f at a mid-stance to reach the desired value v_{des}, by properly choosing the push-off impulse p and swing leg angle ϕ. While it can be seen from (5) that for a given desired velocity v_{des}, there are no analytical solutions of the control variables p and ϕ due to the non-linear dynamics of the IPM. Concurrently, there are countless combinations of p and ϕ which will all lead to the same v_f, but result in different stepping times, step distances, and energy costs. Current research indicates that when humans are walking and running, they tend to choose the most energy efficient gait [14], therefore it is quintessential to find a combination of p and ϕ which consumes the least amount of energy for the bipedal robot gait.

The energy consumed by a single step push-off can be calculated by

$$E_{po} = \frac{p^2}{2}. \tag{15}$$

As can be seen from Fig. 1, the step distance is $2\sin\phi$, which then determines the push-off energy consumed per unit distance as

$$E_{po,pd} = \frac{p^2}{4\sin\phi}. \tag{16}$$

Besides the push-off energy, the swinging leg simultaneously consumes energy throughout the human walking cycle. Although, this is not reflected in the simplified model employed, this energy is accounted for to formulate a higher degree of anthropomorphic walking behavior. Otherwise, the robot will tend to walk at an impractically fast pace. Here, the energy consumed by swinging the leg for one step is assumed as

$$E_{sl} = \frac{3\phi^2}{16t_{st}^2}, \tag{17}$$

which is approximated based on the anthropomorphic model [21] where the legs are considered to be half the total mass. The swinging-leg energy consumed per unit distance is

$$E_{sl,pd} = \frac{3\phi^2}{32t_{st}^2\sin\phi}. \tag{18}$$

Therefore, the total energy consumed per unit distance is

$$E_{total} = E_{po,pd} + E_{sl,pd} = \frac{p^2}{4\sin\phi} + \frac{3\phi^2}{32t_{st}^2\sin\phi}. \tag{19}$$

Thus, the most energy efficient combination of p and ϕ for our model is the solution of the non-linear optimization

$$\min\ E_{total} = \frac{p^2}{4\sin\phi} + \frac{3\phi^2}{32t_{st}^2\sin\phi}, \tag{20}$$

subject to

$$\begin{aligned} p =& \frac{1}{\sin 2\phi}\Big(\sqrt{v_f^2 + 2(1-\cos\phi)} \\ &- \cos 2\phi\sqrt{v_0^2 + 2(1-\cos\phi)}\Big), \end{aligned} \tag{21}$$

$$t_{st} = \ln\left(\frac{\phi + \sqrt{v_0^2 + \phi^2}}{v_0}\right), \tag{22}$$

$$\max(v_0^2, v_f^2) < 3\cos\phi - 2, \tag{23}$$

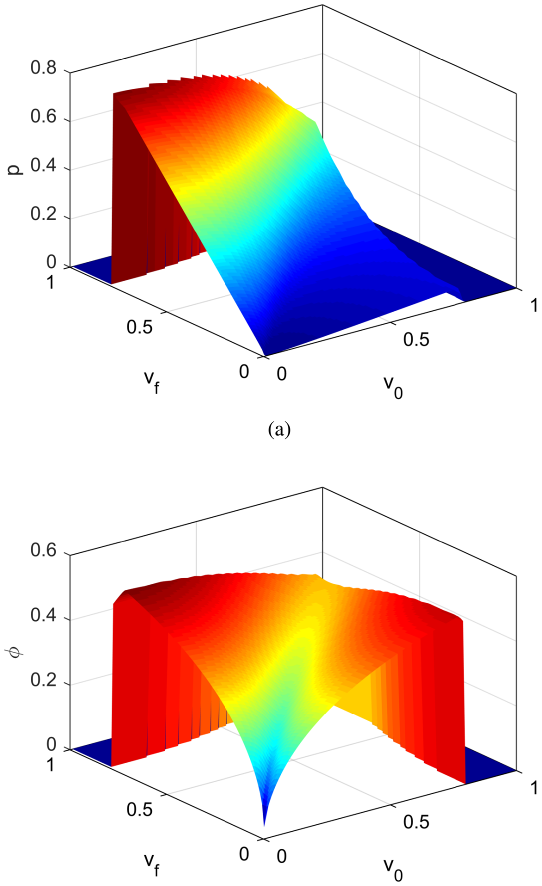
$$0 < p < \tan 2\phi\sqrt{v_0^2 + 2(1 - cos\phi)}, \tag{24}$$

$$\tan\phi < \mu, \tag{25}$$

$$v_0 \sinh t_{st,\min} < \phi, \tag{26}$$

where $\mu = 0.8$, $t_{st,\min} = 0.4$ [22], $v_0 \in [0.01, 1]$ and $v_f \in [0, 1]$. Noting all variables used in this optimization are normalised.

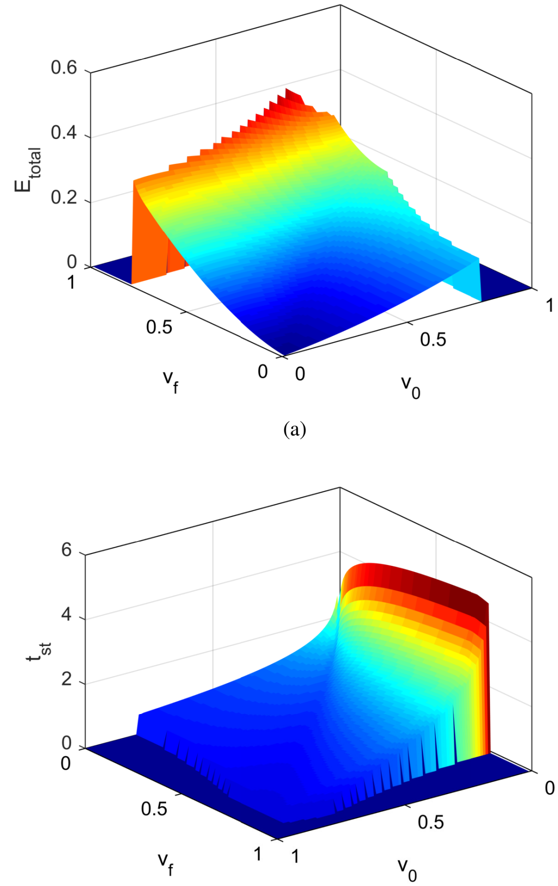
The above non-linear optimization is solved by the FMINCON function in MATLAB, and results of the optimized control variables are shown in Fig. 2. The total energy E_{total} consumed per distance and step time t_{st} corresponding to the optimized results are shown in Fig. 3. The regions where $p = \phi = 0$ is where the desired velocity cannot be achieved in a single step. A general trend of bipedal walking can be found from the optimized results. When the robot speeds up, it should take big steps and impose large push-off, while the push-off is saturated due to the no flight constraints if the speed is too fast. When the robot maintains speed, it should take small steps and impose moderate push-off, and when the robot slows down, it should take large steps with little push-off. These are consistent with what [20] concluded.



(a)

(b)

Fig. 2: **Optimized results of control variables.** (a) Optimized results of the push-off p. (b) Optimized results of the swing leg angle ϕ.



(a)

(b)

Fig. 3: **Optimized results of total energy and step time.** (a) Optimized results of total energy consumed per unit distance E_{total}. (b) Optimized results of step time t_{st}.

B. Neural network structure and training

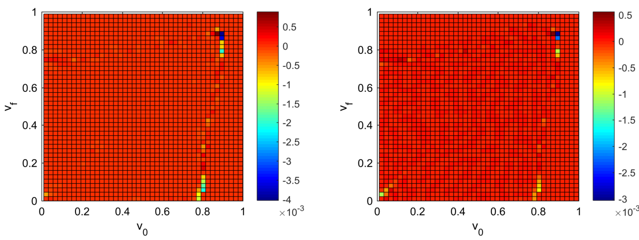
While intending to apply the optimized results from Section III-A to real-time control of bipedal robot walking, it is nearly impossible to pre-calculate all scenarios, and online calculation is extremely time consuming due to the non-linear dynamics of the IPM (calculation time for each scenario is about 0.0945 s in MATLAB). Thus, a neural network is designed to realize the fast non-linear mapping from v_0, v_{des} to p, ϕ, and obviate the need for solving the non-linear equations on-line. A feed-forward network is built in MATLAB, which has 3 hidden layers with 20, 50 and 20 units. The activation function for hidden layers is $tansig$, and the activation function for output layer is $purelin$. The network is trained via the Levenberg-Marquard method with a data size of 7244, which is taken from the $v_0 - v_f$ plane at a pitch of 0.01, and the scenarios where the desired velocity cannot be achieved in a single step are eliminated. The training process converges after 525 epochs. The trained networks are tested using grouped data of size 1389, taken from the $v_0 - v_f$ plane at a pitch of 0.023. Therefore, there is no overlap with the training data.

Mapping errors of the neural network are shown in Fig. 4. The maximum error is 8.886×10^{-4} for p and 5.608×10^{-4} for ϕ, and the minimum error is -4.023×10^{-3} and -3.027×10^{-3} respectively. It can be seen from the results that the trained neural network is sufficiently precise, so that it can be applied to real-time the control of bipedal robot walking.

C. Feasibility check

We can see from the optimized results, there are some cases where the desired velocity cannot be reached in a single step due to the satisfied constraints of bipedal walking. The region where desired velocity can be reached in one step is the feasible region, and the remaining is unfeasible. In Fig. 5, the feasible and unfeasible regions are coloured white and red respectively. If any v_0 and v_{des} values from the unfeasible region are adopted as an input to the neural network, out-of-range control efforts may be generated, and may cause unexpected movements such as flight, or slip motions during walking, resulting in falling of the robot.

In order to prevent falling of the bipedal robot during walking, a feasibility check is used. Before being inputted



(a) (b)

Fig. 4: **Mapping errors of the neural network.** (a) Mapping errors of push-off impulse p. (b) Mapping errors of swing leg angle ϕ.

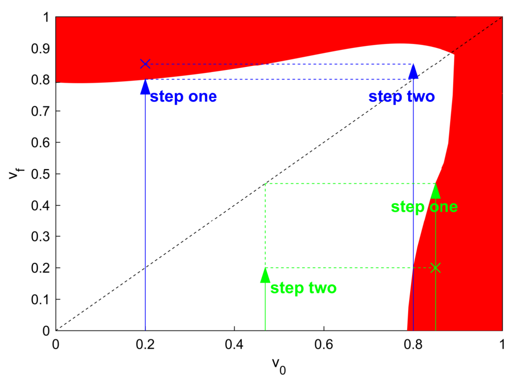
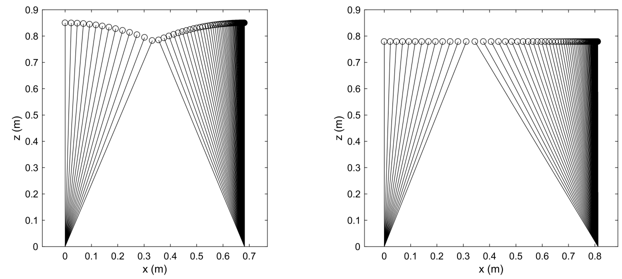


Fig. 5: **Feasibility check.** The feasible region for biped walking is coloured white and the unfeasible zone is coloured red.

into the neural network, all desired states are checked, whether it belongs to the feasible region or not. If the desired state belongs to an unfeasible region, it will be replaced by an intermediate state closest to the desired state in the feasible region. This process is shown in Fig. 5 by two examples. The blue lines show the robot speeding up in the unfeasible region, and the green lines show the robot slowing down in the unfeasible region. Subsequent to this, the robot's walking motion is always feasible and error between the actual and desired state is minimized.

IV. Simulations

The proposed normalized neural network based approach's performance in bipedal walking were validated by three simulation studies in an increasing order of complexity. Firstly, a comparison study with the LIPM in a one-step balancing scenario is performed. Secondly, continuous walking under disturbances to show the robustness of the proposed approach is completed. Then finally, commanding the desired states to the unfeasible region during continuous walking is carried out to verify the proposed strategy for feasibility check. Note that the proposed neural network is trained with normalized data. Therefore, the proposed approach can be applied to bipedal robots of any size, without specific adaptations. To demonstrate this advantage, differentiating sizes of robots



(a) (b)

Fig. 6: **One-step balancing** using (a) IPM and (b) LIPM. The step distance using LIPM is 1.1913 times bigger than using IPM.

are employed. The first two simulations uses a bipedal robot with a mass and leg length of $m = 55$ kg and $l = 0.85$ m respectively. The third simulation uses $m = 90$ kg and $l = 1.5$ m respectively.

A. One-step balancing

One-step balancing of the bipedal robot is simulated with the initial condition $v_0 = 0.4$ $(1.1545$ m/s$)$ to achieve the desired velocity $v_{des} = 0$ at the next mid-stance. The number inside the brackets is the actual value corresponding to the normalised number outside the brackets. The push-off impulse $p = 0.0289$ $(4.5905$ kg·m/s$)$ and the swing leg angle $\phi = 0.4126$ rad are provided by the trained neural network. The time elapsed $(0.02$ s$)$ of one-step walking using the proposed method is shown in Fig. 6 (a). In this simulation, the robot comes to a complete stop at the second mid-stance. The total energy consumed per distance is $E_{total} = 0.0492$ $(26.5211$ J$)$, the step time is $t_{st} = 0.9042$ $(0.2663$ s$)$ and the step distance is 0.8020 $(0.6817$ m$)$. From the simulation results, it can be seen that the proposed method successfully balances the robot in one step when a proper initial velocity is applied.

To further show the advantages of the IPM, one-step balancing using LIPM [9] is introduced to compare with the results above. Initial velocity v_0 and angle of the stance leg at heel-strike are kept the same in both the IPM and LIPM. The time elapsed $(0.02$ s$)$ of one-step walking using the LIPM is shown in Fig. 6 (b). The energy consumed per distance is $E_{total} = 117.0074$ J (consumed energy is calculated based on the work done by leg force), the step time is $t_{st} = 0.2577$ s and the step distance is 0.8121 m. It's seen from the comparison that the IPM consumes much less energy and takes a smaller step than the LIPM. The approach using the IPM is subsequently more energy efficient and more realistic.

B. Continuous walking with disturbance

To verify the robustness of the proposed method, continuous walking with a disturbance is simulated for six steps. The initial condition for the first step is $v_0 = 0.1$ $(0.2886$ m/s$)$, and the final velocity of previous step becomes the initial velocity of the latter step upon conclusion of each

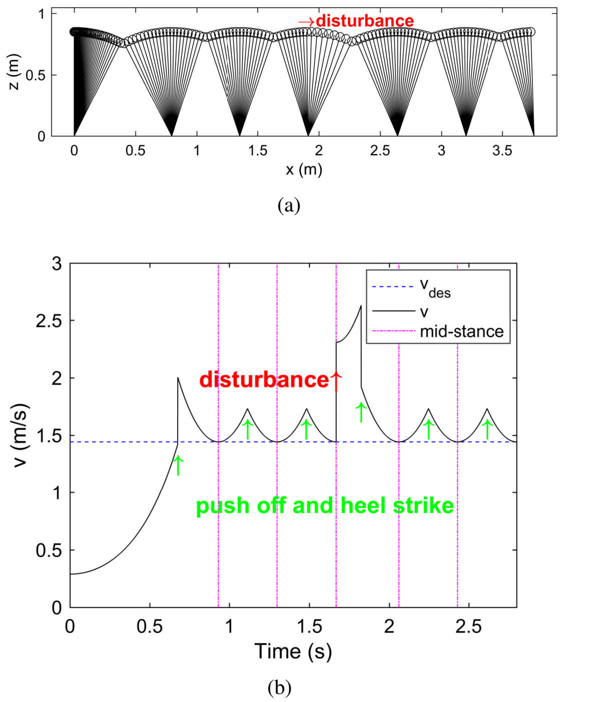


Fig. 7: **Continuous walking with disturbance.** (a) The time elapsed $(0.02\ \mathrm{s})$. (b) The velocity of CoM. A disturbance is appended to the hip before the fourth step.

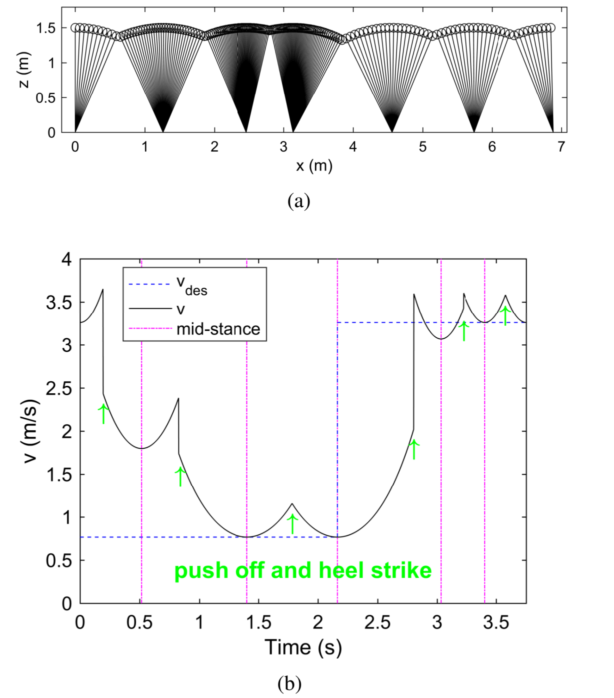


Fig. 8: **Continuous walking in unfeasible region.** (a) The time elapsed $(0.02\ \mathrm{s})$. (b) The velocity of CoM. The desired states of step one and step four are in the unfeasible region.

step. The desired velocity for the mid-stance is set to be $v_{des} = 0.5$ (1.4431 m/s). A disturbance $\Delta v = 0.3$ (0.5772 m/s) is added before the fourth step. Control variables for all the steps are provided by the neural network controller. The push-off impulses p of each step are 0.5053 (80.2138 kg·m/s), 0.2087 (33.1259 kg·m/s), 0.2087 (33.1259 kg·m/s), 0.1156 (18.3467 kg·m/s), 0.2087 (33.1259 kg·m/s) and 0.2087 (33.1259 kg·m/s) respectively, and the swing leg angles ϕ for each step are 0.4873 rad, 0.3344 rad, 0.3344 rad, 0.4438 rad, 0.3344 rad and 0.3344 rad respectively. The time elapsed $(0.02\ \mathrm{s})$ of the continuous walking is shown in Fig. 7 (a), and velocity of the robot's CoM is shown in Fig. 7 (b). It is seen that the proposed method compensates for the imposed disturbance successfully and ensures the robot achieves desired velocity at every mid-stance.

C. Continuous walking in unfeasible region

To verify the feasibility check strategy, a six-step continuous walking simulation in the unfeasible region is performed. Parameters of the robot in this simulation are $m = 90$ kg and $l = 1.5$ m respectively. Initial velocity is $v_0 = 0.85$ (3.2589 m/s) for the first step. Desired velocity is set to be $v_{des} = 0.2$ (0.7668 m/s) for the first three steps, and $v_{des} = 0.85$ (3.2589 m/s) for the last three steps. It can be seen from Fig. 5 that the desired states of step one and step four are in the unfeasible region, thus an intermediate velocity is applied to replace the desired velocity and inputted into the neural network. The intermediate velocity is the closest to the desired velocity in the feasible region, as shown in Fig. 5. In this simulation, the push-off impulses p of each steps are 0.0262 (9.0330 kg·m/s), 0.0436 (15.0514 kg·m/s), 0.0696 (24.0057 kg·m/s), 0.7749 (267.4074 kg·m/s), 0.4447 (153.4464 kg·m/s) and 0.3851 (132.8838 kg·m/s) respectively, and the swing leg angles ϕ for each steps are 0.4335 rad, 0.4116 rad, 0.2267 rad, 0.4947 rad, 0.4040 rad and 0.3908 rad respectively. The time elapsed $(0.02\ \mathrm{s})$ of the continuously walking model is shown in Fig. 8 (a), and velocity of robot's CoM is shown in Fig. 8 (b). In this simulation, the bipedal robot reaches desired velocity in two steps, which means that the feasibility check strategy is efficient and functional, and the proposed approach has overall good performance. Although, the model parameters in this simulation are different from the first two, the proposed method still works effectively, succeeding a demonstration that the proposed method inherits generality.

V. Conclusion

In this paper, a novel approach for locomotion control of a bipedal robot walking using a neural network is presented. The proposed method is designed using a 2D IPM, where the robots are controlled to walk in an energy efficient gait. The neural network assists with the fast non-linear mapping realization of desired states to control variables, and averts the need for solving non-linear equations on-line. The neural network is trained with normalised data, so it can be applied to bipedal robots of any size, without any individual refinement. The feasibility and robustness of the proposed approach is verified by simulation results. Future work entails the same approach to be extended into a 3D inverted pendulum model, and verified on a real robot.

ACKNOWLEDGMENT

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