Proceeding of the IEEE
International Conference on Robotics and Biomimetics
Dali, China, December 2019

Simultaneous Stabilization of Marine Dynamic Positioning System Based on PCH Model¯

Pei Zhou, Renming Yang*, Jiankuo Cui and Binghua Zhang

School of Information Science and Electrical Engineering
Shandong Jiaotong University, Jinan 250357, China
m18199050659@163.com (P Zhou), renmingyang0222@163.com (R Yang)
15066680467@163.com (J Cui), zhangbh1995@163.com (B Zhang)

***Abstract* - This paper studies two dynamic positioning ship systems' simultaneous stabilization problem. The Hamilton function method is used to study the issue of simultaneous stabilization with unknown disturbance, and the related controllers are designed. First of all, we transform the dynamic positioning ship model into port controlled Hamiltonian (PCH) model, and the general simultaneous stabilization conditions are developed. Based on which, by adding the external interference, the robust controller is designed. Finally, the simulation results are tested by an illustrative example, which is shown to meet the expected requirements, and prove the rationality of the designed controller.**

Index Terms - Dynamic positioning ship. Simultaneous stabilization. Port controlled Hamiltonian system .

I. INTRODUCTION

Dynamic positioning (DP) means that the marine vehicle does not rely on the mooring system, but through its self power system and self-installed thrusters to achieve accurate positioning on the sea surface [1]. At present, this technology is more and more applied in deep-sea exploration and development, as well as in marine engineering ships. It is an important technology to maintain the stability floating platform the marine development and related ship engineering operations.

The first generation DP goods emerged in the 1960s, which was used classical control theory to design controllers, namely conventional PID controllers. The second generation DP system came into being in the middle of 1970s, that is, Balchen et al. [2] [3] developed the DP control algorithm combined with modern theory-optimal control and Kalman filter theory. Simultaneously, the second generation DP system is also a widely used system. In recent years, some new control methods have emerged, mainly reflected in robust control, fuzzy control, nonlinear model predictive control and etcetera [4]. This kind of control theory and method is called intelligent control and is also known as the third generation DP system. In addition, the emergence of intelligent control has further promoted the development of DP technology.

Currently, the DP technology plays a leading role in the development, application and research of the marine field. At the same time, with the development of marine resources, the exploration area of human beings is gradually expanded, and further difficulty in development. So higher requirements are imposed on ship automation. Coordinated formation control of multiple power locating ships came into being [5]. As well known, for the multi-coordinated DP ships formation, in addition to improve operational efficiency, reduce costs and so on, its redundant nature also enhances the robustness of the entire formation system and has the advantage of protecting the operator from life threats when performing dangerous tasks. Moreover, several vessels formation systems are easier to upgrade and expand. Consequently, the coordination of multiple ship formations provides a new model for future joint operations at sea, which has aroused the attention of the majority of scholars [6] [7]. However, considering many factors of the ship itself and the uncertainty of the ocean, the research on the control of multi-DP ships is very difficult. Even if the study represented by multi-agent formation control has achieved good results in recent years [8] [9], most of the existing research results are linear systems or systems with linear main part. The study for nonlinear systems are relatively little due to the complexity of nonlinear systems and the challenges of simultaneous stabilization of multi-dynamic positioning ship systems. In fact, the so-called simultaneous stabilization refers to design single controller to stabilize multiple systems at the same time. Even for a group of linear systems, it is a very difficult task to design simultaneous stabilization controller, let alone nonlinear systems [10]. The emergence of Hamilton system control design method has opened a breakthrough for nonlinear system control research because it is a effective research tool for the studying of nonlinear systems [10]-[13].

This paper use Hamiltonian system method to study two DP ships' simultaneous stabilization problem, mainly aim at the problem of simultaneous stabilization with unknown

¯This work is supported by NSFC Grant G61773015 and Shandong Provice Key Research and development Project 2018GGX105003 to R. Yang.
*Corresponding Author: Renming Yang, School of Information Science and Electrical Engineering, Shandong Jiaotong University, Jinan 250357, China.

978-1-7281-6321-5/19//$31.00 ©2019 IEEE

disturbance, and the related controllers are designed. First of all, we transform the DP ship model into PCH one. Then, by using the dimension expansion technique, the extended dimension systems are established, and the related simultaneous stabilization of the systems are also studied. Finally, through the simulation results, the effectiveness of the designed controllers are verified. In the paper, the main contributions are as follows. 1) Unlike refs. [14][15], in which the results obtained are only single ship, while the stabilization problem studied in this present paper is about two ships system and get some simultaneous stabilization results. 2) Different from refs. [16]-[21], in which the research objects are a class of uncertain nonlinear systems, the research object of this paper is the ship DP system in practical application. 3) In this paper, we use energy method to study the simultaneous stabilization control problem of the DP system, which is different from the methods used in the existing literature, and illustrative example shows that the control one has better control effect than traditional one.

II. PROBLEM FORMULATION AND PRELIMINARIES

The ship studied in this paper is the DP ship model proposed by Fossen in 1994[22].

$$\dot{\eta} = R(\psi)v, \tag{1}$$

$$M\dot{v} + C(v)v + D(v)v = \tau, \tag{2}$$

where $\eta = [n, e, \psi]^T \in R^{3\times1}$ is the position vector of the ship in the northeastern coordinate system, which represents the surge position, sway position and yaw angle of the ship respectively. $v = [u, v, r]^T \in R^{3\times1}$ is the velocity vector in the ship coordinate system, which represents the heave velocity, the yaw velocity and the bow angular velocity, respectively. $\tau \in R^{3\times1}$ indicates total torque, $M \in R^{3\times3}$ is hull additional mass matrix, $C(v) \in R^{3\times3}$ is coriolis centripetal matrix, $D(v) \in R^{3\times3}$ is damping matrix coefficient, $R(\psi) \in R^{3\times3}$ is a coordinate transformation matrix, satisfying

$$R(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

In present paper, we use the Hamiltonian system method to study DP ship simultaneous stabilization problem. Among them, the key step is to convert the ship model (1) (2) into the PCH model [23]. Therefore, first of all, let's introduce the relevant contents of the Hamiltonian system.

$$\dot{x} = [J(x) - R(x)]\frac{\partial H(x)}{\partial x} + g(x)u, \tag{4}$$

$$y = g^T(x)\frac{\partial H(x)}{\partial x}, \tag{5}$$

where $x \in R^n$ is a state vector, $H: R^n \to R$ a scalar function on state variables, named the Hamiltonian function, which indicates the total energy of the system. $u \in R^m, y \in R^q$ are input and output vectors. $J(x) \in R^{n\times n}$ is an skey-symmetric matrix, $R(x) \in R^{n\times n} \geq 0$ is a symmetric matrix, $g(x) \in R^{n\times m}$ is a weight coefficient matrix.

Next, transform system (1) (2) into its Hamiltonian form.

To do this, introduce a new state vector

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \triangleq \begin{bmatrix} Mv \\ \eta \end{bmatrix}, \tag{6}$$

and define Hamiltonian function

$$H(x) = \frac{1}{2}x_1^T M^{-1} x_1, \tag{7}$$

Then the system (1) (2) can be expressed as port-controlled Hamiltonian form [24].

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \left\{ \begin{bmatrix} 0 & -R^T(\psi) \\ R(\psi) & 0 \end{bmatrix} - \begin{bmatrix} C(v)+D(v) & 0 \\ 0 & 0 \end{bmatrix} \right\} \frac{\partial H(x)}{\partial x} + \begin{bmatrix} I \\ 0 \end{bmatrix}\tau \quad . \tag{8}$$

Without losing generality, the total control force is divided into two parts, namely

$$\tau = \tau_r + \tau_d, \tag{9}$$

where $\tau_r \in R^{3\times1}$ represents the control force matrix required by the path tracking control section, $\tau_d \in R^{3\times1}$ represents external environmental interference force matrix, $\tau \in R^{3\times1}$ represents total control force matrix.

Suppose we have designed τ_r (see below for details of the design process), and under residual control $\tau = \tau_d$, a new Hamilton model can be obtained.

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \left\{ \begin{bmatrix} 0 & -R^T(\psi) \\ R(\psi) & 0 \end{bmatrix} - \begin{bmatrix} C(v)+D_d(v) & 0 \\ 0 & 0 \end{bmatrix} \right\} \frac{\partial H_d(x)}{\partial x} + \begin{bmatrix} I \\ 0 \end{bmatrix}\tau_d, \tag{10}$$

where $D_d(v) \in R^{3\times3}$ is the system performance adjustment parameter, H_d is a new Hamilton function, its form is

$$H_d(x) = \frac{1}{2}x_1^T M^{-1} x_1 + \frac{1}{2}(x_2 - x_2^*)^T K_1 (x_2 - x_2^*), \tag{11}$$

$K_1 \in R^{3\times3}$ is a constant coefficient positive definite matrix, which is an adjusting parameter for system performance, x_2^* is the desired position.

By comparing equation (8) with equation (10), the following identities are obtained.

$$-[C(v)+D(v)]\frac{\partial H}{\partial x_1} + \tau_r = -[C(v)+D_d(v)]\frac{\partial H_d}{\partial x_1} - R^T(\psi)\frac{\partial H_d}{\partial x_2}, \tag{12}$$

$$R(\psi)\frac{\partial H}{\partial x_1} = R(\psi)\frac{\partial H_d}{\partial x_1}. \tag{13}$$

From which, the path tracking control part can be obtained

$$\tau_r = -R^T(\psi)K_1(x_2 - x_2^*) - [D_d(v) - D(v)]M^{-1}x_1. \tag{14}$$

Next, we give the relevant content of the L_2 interference suppression problem (see the literature [10] for details):

Consider the general system,

$$\dot{x} = f(x) + g_1(x)u + g_2(x)\omega \ , \tag{15}$$

where, $x \in R^n, u \in R^m$ are state and input, respectively. $\omega \in R^s$ is an interference, $g_1(x), g_2(x)$ is two weighted matrices of the appropriate dimensions.

The L_2 interference suppression problem is described as: Given a penalty signal $z = q(x)$, interference suppression level $\gamma > 0$, finding a feedback control law $u = k(x)$ and a positive definite storage function $V(x)$ such that the following $\gamma-$dissipative inequalities holds along the trajectory of the closed-loop system (15):

$$\dot{V} + Q(x) \le \frac{1}{2}\left\{\gamma^2 \|\omega\|^2 - \|z\|^2\right\}, \forall \omega \in L_2 \ , \tag{16}$$

where, $Q(x)$ is the non-negative function.

Remark From inequality (16), one can guarantee the following features:

P1. The L_2 gains from ω to $z \ \le \gamma$.

P2. When $\omega = 0$, the closed loop system is Lyapunov stable. Furthermore, if $Q(x) \ne 0 (\forall x \ne 0)$, that is $Q(x)$ positive definite, then the closed loop system is asymptotically stable.

Based on the above definition, we get the method to judge L_2 interference suppression problem in generalized Hamiltonian systems.

Consider a generalized Hamiltonian system as follow :

$$\begin{cases} \dot{x} = \left[J(x) - R(x)\right]\nabla H + g_1(x)u + g_2(x)\omega \\ z = h(x)g_1^{\ T}(x)\nabla H \end{cases} , \tag{17}$$

where, $R(x) \ge 0, H(x) > 0, h(x)$ is a weight matrix, ∇H is the gradient vector of $H(x)$, $x \in R^n, u \in R^m, \omega \in R^s$. Assume the interference suppression level $\gamma > 0$ is given , then we have the following Lemma[18]:

Lemma 1: Consider the system (17). Assuming that the interference suppression level $\gamma > 0$ is given and $z = h(x)g_1^{\ T}(x)\nabla H$ is the penalty signal, if

$$R(x) + \frac{1}{2\gamma^2}\left[g_1(x)g_1^{\ T}(x) - g_2(x)g_2^{\ T}(x)\right] \ge 0 \ , \tag{18}$$

then the interference suppression problem can be accomplished by designing control law as follow:

$$u = -\left[\frac{1}{2}h^T(x)h(x) + \frac{1}{2\gamma^2}I_m\right]g_1^{\ T}(x)\nabla H \ . \tag{19}$$

And the following inequality holds:

$$\dot{H} + \nabla H^T\left[R - \frac{1}{2\gamma^2}\left(g_2(x)g_2^{\ T}(x) - g_1(x)g_1^{\ T}(x)\right)\right]\nabla H \le \frac{1}{2}\left\{\gamma^2\|\omega\|^2 - \|z\|^2\right\}. \tag{20}$$

III. MAIN RESULTS

A. Simultaneous Stabilization Without the Disturbance

According to the derivation of (10), two PCH systems can be constructed [10] [25]:

$$\Sigma_1 : \begin{cases} \dot{x} = \left\{\begin{bmatrix} 0 & -R_1^{\ T}(\psi) \\ R_1(\psi) & 0 \end{bmatrix} - \begin{bmatrix} C_1(v) + D_{d1}(v) & 0 \\ 0 & 0 \end{bmatrix}\right\} \bullet \frac{\partial H_{d1}(x)}{\partial x} + \begin{bmatrix} I \\ 0 \end{bmatrix}\tau \\ y_1 = [I \quad 0]\frac{\partial H_{d1}(x)}{\partial x} \end{cases} , \tag{21}$$

$$\Sigma_2 : \begin{cases} \dot{\xi} = \left\{\begin{bmatrix} 0 & -R_2^{\ T}(\psi) \\ R_2(\psi) & 0 \end{bmatrix} - \begin{bmatrix} C_2(v) + D_{d2}(v) & 0 \\ 0 & 0 \end{bmatrix}\right\} \bullet \frac{\partial H_{d2}(\xi)}{\partial \xi} + \begin{bmatrix} I \\ 0 \end{bmatrix}\tau \\ y_2 = [I \quad 0]\frac{\partial H_{d2}(\xi)}{\partial \xi} \end{cases} , \tag{22}$$

where $x, \xi \in R^6$ and $y_1, y_2 \in R^6$ are the two systems' states and outputs , respectively; $\tau \in R^6$ is a control input; $H_i(x)$ are the Hamiltonian functions of two systems, respectively, and get a minimum at $x_e^{(i)}$, $i = 1,2$, and $x_e^{(1)} = x_0$, $x_e^{(2)} = \xi_0$.

Note: $J_i(x) = \begin{bmatrix} 0 & -R_i^{\ T}(\psi) \\ R_i(\psi) & 0 \end{bmatrix} = -J_i^{\ T}(x)$, $R_i(x) = \begin{bmatrix} C_i(v) + D_{di}(v) & 0 \\ 0 & 0 \end{bmatrix} \ge 0$, $g_i(x) = \begin{bmatrix} I \\ 0 \end{bmatrix}$, $i = 1,2$.

Obviously, when $\tau = 0$, both system (21) and system (22) are stable, but not asymptotically stable.

An output feedback law $\tau = \tau(y_1, y_2)$ is designed, so that under the action of this control law, the system (21) and the system (22) can be simultaneously stabilize.

Theorem 1 *Assume there is a symmetric matrix $K \in R^{6\times 6}$ with appropriate dimensions such that the following inequalities*

$$\begin{cases} R_1(x) + K_{11}(x,x) > 0 \\ R_2(\xi) - K_{22}(\xi,\xi) > 0 \end{cases} , \tag{23}$$

where ,

$$K_{ij}(x,\xi) = g_i(x)Kg_j^T(\xi), \ i,j = 1,2 \ . \tag{24}$$

and $K = \begin{bmatrix} K_{11} & K_{12} \\ K_{12}^T & K_{22} \end{bmatrix}$, *then the output feedback controller*

$$\tau = -K(y_1 - y_2) \ , \tag{25}$$

can simultaneously stabilize the system (21) and the system (22).

Proof. Substituting (25) into the system (21) and the system (22), we obtain

$$\begin{cases} \dot{x} = \left\{\begin{bmatrix} 0 & -R_1^{\ T}(\psi) \\ R_1(\psi) & 0 \end{bmatrix} - \begin{bmatrix} C_1(v) + D_{d1}(v) & 0 \\ 0 & 0 \end{bmatrix}\right\}\frac{\partial H_{d1}}{\partial x} - \begin{bmatrix} I \\ 0 \end{bmatrix}K[I \quad 0]\frac{\partial H_{d1}}{\partial x} + \begin{bmatrix} I \\ 0 \end{bmatrix}K[I \quad 0]\frac{\partial H_{d2}}{\partial \xi} \\ \dot{\xi} = \left\{\begin{bmatrix} 0 & -R_2^{\ T}(\psi) \\ R_2(\psi) & 0 \end{bmatrix} - \begin{bmatrix} C_2(v) + D_{d2}(v) & 0 \\ 0 & 0 \end{bmatrix}\right\}\frac{\partial H_{d2}}{\partial \xi} - \begin{bmatrix} I \\ 0 \end{bmatrix}K[I \quad 0]\frac{\partial H_{d1}}{\partial x} + \begin{bmatrix} I \\ 0 \end{bmatrix}K[I \quad 0]\frac{\partial H_{d2}}{\partial \xi} \end{cases} ,$$

the above system can be represented as

$$\dot{X} = \left[J(X) - R(X)\right]\frac{\partial H(X)}{\partial X}, \tag{26}$$

where ,

$$X = \left[x^T, \xi^T\right]^T, H(X) = H_{d1}(x) + H_{d2}(\xi), \frac{\partial H(X)}{\partial X} = \begin{bmatrix} \frac{\partial H_{d1}(x)}{\partial x} \\ \frac{\partial H_{d2}(\xi)}{\partial \xi} \end{bmatrix} , \tag{27}$$

$$J(X) = \begin{bmatrix} \begin{bmatrix} 0 & -R_1^{\ T}(\psi) \\ R_1(\psi) & 0 \end{bmatrix} & K_{12}(x,\xi) \\ -K_{12}^{\ T}(x,\xi) & \begin{bmatrix} 0 & -R_2^{\ T}(\psi) \\ R_2(\psi) & 0 \end{bmatrix} \end{bmatrix} , \tag{28}$$

$$R(X)=\begin{bmatrix}\begin{bmatrix}C_1(v)+D_{d1}(v) & 0\\ 0 & 0\end{bmatrix}+K_{11}(x,x) & 0\\ 0 & \begin{bmatrix}C_2(v)+D_{d2}(v) & 0\\ 0 & 0\end{bmatrix}-K_{22}(\xi,\xi)\end{bmatrix}. \tag{29}$$

Obviously, $J(X)$ is anti-symmetric, and from formula (23), $R(X)$ is positive definite. Therefore, the system (26) is an extended strictly dissipative Hamiltonian system.

Letting $X_0:=\left[x_0^T,\xi_0^T\right]^T$. Since $\nabla H_{d1}(x_0)=0,\nabla H_{d2}(\xi_0)=0$, so X_0 is the equilibrium of the system (26). On another hand, it is easy to know that $H(X)$ achieves local strict minima at X_0. According to the properties of the dissipative PCH system, the system (26) is asymptotically stable at X_0, which shows that $x\to x_0,\xi\to\xi_0$. Consequently, under the action of control (25), the system (21) and the system (22) are stabilized at the same time.

Remark In the Theorem, we design a output feedback controller (25) to simultaneously stabilize the systems (21) and (22) . It is well worth pointing out that, by applying the controller, one can obtain a keys-symmetric form in the closed-loop system. Thanks for the form, we can develop some concise condition. In addition, in the theorem, we use the expansion technology to obtain the augmented system (26). Based on (26) and existing Hamiltonian results, the paper develops the concise simultaneous result, which is the advantage of applying Hamiltonian method. This is also the reasons why the paper chooses the Hamiltonian method.

B. Robust simultaneous stabilization

In this section, based on the above results of simultaneous stabilization, the problem of robust simultaneous stabilization of systems (21) and (22) with external disturbances is studied.

Considering the situation in which the system (21) and the system (22) are subject to external interference, the system may be represented as

$$\Sigma_1:\begin{cases}\dot{x}=\left\{\begin{bmatrix}0 & -R_1^T(\psi)\\ R_1(\psi) & 0\end{bmatrix}-\begin{bmatrix}C_1(v)+D_{d1}(v) & 0\\ 0 & 0\end{bmatrix}\right\}\dfrac{\partial H_{d1}(x)}{\partial x}+\begin{bmatrix}I\\0\end{bmatrix}\tau+\bar{g}_1(x)\omega\\ y_1=[I\quad 0]\dfrac{\partial H_{d1}(x)}{\partial x}\end{cases}, \tag{30}$$

$$\Sigma_1:\begin{cases}\dot{\xi}=\left\{\begin{bmatrix}0 & -R_2^T(\psi)\\ R_2(\psi) & 0\end{bmatrix}-\begin{bmatrix}C_2(v)+D_{d2}(v) & 0\\ 0 & 0\end{bmatrix}\right\}\dfrac{\partial H_{d2}(\xi)}{\partial \xi}+\begin{bmatrix}I\\0\end{bmatrix}\tau+\bar{g}_2(\xi)\omega\\ y_2=[I\quad 0]\dfrac{\partial H_{d2}(\xi)}{\partial \xi}\end{cases}, \tag{31}$$

where , ω is external interference, $\bar{g}_i(x)(i=1,2)$ are weight matrices , other variables are the same as before.

Given the disturbance attenuation level $\gamma>0$, choose

$$z=\Lambda(y_1+y_2)\quad, \tag{32}$$

As the penalty signal of the system, Λ is the weight matrix with appropriate dimension.

In this section, the overall design goals are as follows:

The output feedback L_2 interference suppression controller is designed to meet the following requirements:

I. When $\omega=0$, the systems (30) and (31) can be simultaneously stabilization;

II. The L_2 gain (from ω to z) of the closed-loop system is not greater than the given γ .

Now, we present the main result.

Theorem 2 *Consider the systems* (30) *and* (31) , *with the penalty function* (32) *and the given level* $\gamma>0$. *If*

(i) *there exists a symmetric matrix* $K\in R^{6\times6}$ *such that*

$$\begin{cases}R_1(x)+K_{11}(x,x)>0\\ R_2(\xi)-K_{22}(\xi,\xi)>0\end{cases}, \tag{33}$$

where $K_{ij}(x,\xi)=g_i(x)Kg_j^T(\xi)$, $i,j=1,2$, $K=\begin{bmatrix}K_{11} & K_{12}\\ K_{12}^T & K_{22}\end{bmatrix}$, and

(ii) $\bar{g}_i(x)(i=1,2)$ *satisfy* $\bar{g}_1=\begin{bmatrix}I\\0\end{bmatrix},\bar{g}_2=\begin{bmatrix}I\\0\end{bmatrix}$, *then*

$$\tau=-K(y_1-y_2)-[\frac{1}{2}\Lambda^T\Lambda+\frac{1}{2\gamma^2}I_1](y_1+y_2)\ , \tag{34}$$

is an disturbance attenuation controller such that both I *and* II *hold true for the systems* (30) *and* (31).

Proof. Rewrite (34) as follows:

$$\begin{cases}\tau=-K(y_1-y_2)+\mu\\ \mu=-[\frac{1}{2}\Lambda^T\Lambda+\frac{1}{2\gamma^2}I_1](y_1+y_2)\end{cases}, \tag{35}$$

Substituting the first part of equation (35) into systems (30) and (31), from the proof of Theorem 1, one can obtain

$$\dot{X}=[J(X)-R(X)]\frac{\partial H}{\partial X}+G(X)\mu+\bar{G}(X)\omega, \tag{36}$$

where X, $J(X)$, $R(X)$ and $H(X)$ are given in (27) (28)and(29), $G(X):=\left[[I\quad 0],[I\quad 0]\right]^T,\bar{G}(X):=\left[[I\quad 0],[I\quad 0]\right]^T$.

On the other hand, the penalty function (32) can be rewritten as

$$z=\Lambda G^T(X)\frac{\partial H}{\partial X}\ . \tag{37}$$

Moreover , using conditions (i) and (ii), we have

$$R(X)+\frac{1}{2\gamma^2}[G(X)G^T(X)-\bar{G}(X)\bar{G}^T(X)]=R(X)>0\ . \tag{38}$$

Therefore, the system (36) and the penalty signal (37) satisfy all the conditions of Lemma 1. From Lemma 1, the interference suppression controller of system (36) can be designed as

$$\mu=-[\frac{1}{2}\Lambda^T\Lambda+\frac{1}{2\gamma^2}I_1]G^T(x)\frac{\partial H}{\partial X}\ , \tag{39}$$

and $\gamma-$ dissipation inequality holds

$$\dot{H}+\frac{\partial^T H}{\partial X}R(X)\frac{\partial H}{\partial X}\le\frac{1}{2}\left\{\gamma^2\|\omega\|^2-\|z\|^2\right\}\ . \tag{40}$$

Note that equation (39) is the second part of equation (35). Therefore, the control law (34) is an L_2 interference suppression controller of systems (30) and (31). Moreover, since $\frac{\partial^T H}{\partial X}R(X)\frac{\partial H}{\partial X}>0$, the system (36) is asymptotically stable. That is, $x\to x_0,\xi\to\xi_0$ (as $t\to\infty$). Therefore, the control of (34) is the simultaneous stabilization one of the system (30) and (31).

IV. SIMULATION

In order to test the effectiveness of the designed control algorithm, a dynamic positioning Guarantee vessel [15] and a supply ship model [26] are simulated, the mass matrix and damping matrix of the two ship models are as follows :

$$M_1 = \begin{bmatrix} 1.8564\times10^6 & 0 & 0 \\ 0 & 1.3850\times10^6 & 0 \\ 0 & 0 & 2.9804\times10^8 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 2.4373\times10^4 & 0 & 0 \\ 0 & 2.6316\times10^5 & 0 \\ 0 & 0 & 1.0616\times10^8 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 120\times10^3 & 0 & 0 \\ 0 & 177.9\times10^3 & 0 \\ 0 & 0 & 636\times10^5 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 215\times10^2 & 0 & 0 \\ 0 & 117\times10^3 & 0 \\ 0 & 0 & 802\times10^4 \end{bmatrix}.$$

The working speed of dynamic positioning ship is small (Ship speed is less than 2 knots), $C(v)\approx 0$. The initial position and orientation of the two ships are set to (0m,0m,0°), the expectation location point is set to (-5m,5m,3°)and (-5m,10m,3°).

The parameters required for the controller are selected as follows: $K = Diag\{1,-1,2,1,-1,2\}$, $K_1 = Diag\{700,700,400\}$, $D_d = Diag\{0.0191, 0.1228, 0.0210\}$, $\Lambda = Diag\{0.2,0.3,0.4\}$, $\gamma = 0.5$.

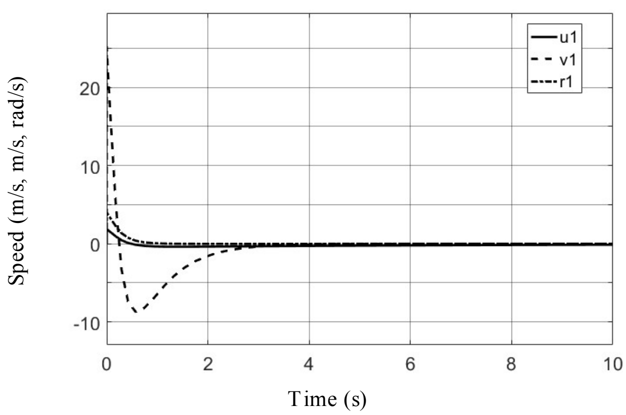

Fig. 1 Speed response of ship 1.

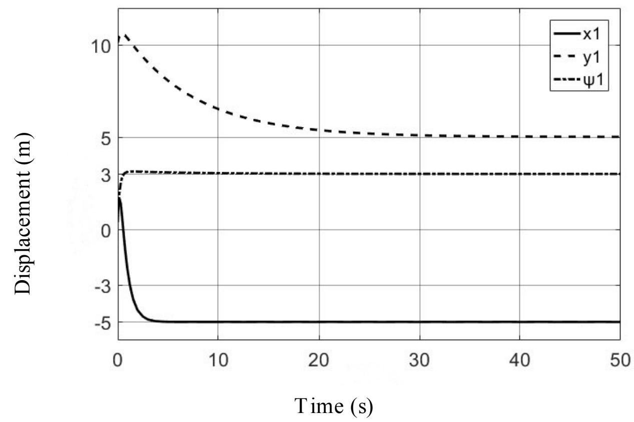

Fig. 2 Actual position response of ship 1 (x_1, y_1) and heading angle ψ_1.

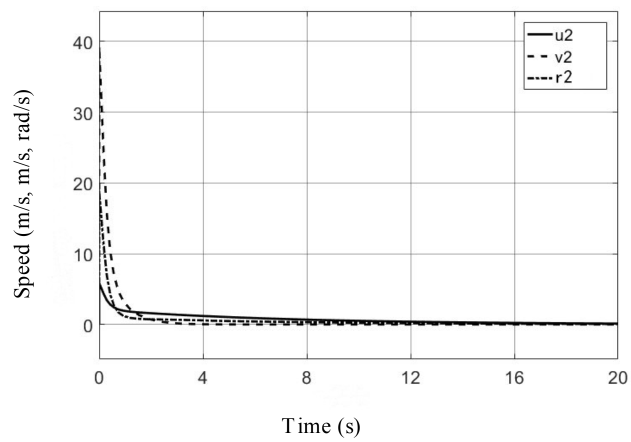

Fig. 3 Speed response of ship 2.

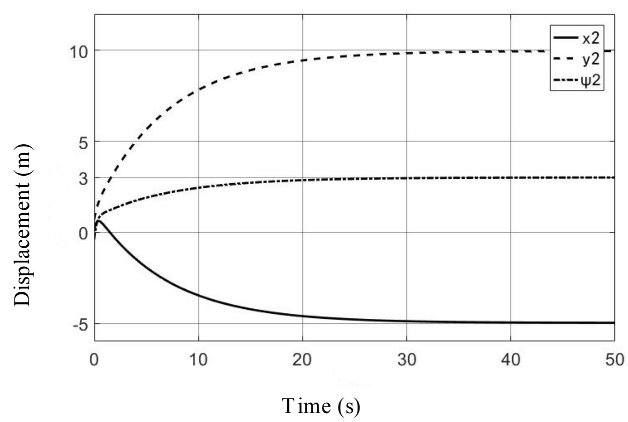

Fig. 4 Actual position response of ship 2 (x_2, y_2) and heading angle ψ_2.

Figs. 1 and 3 are longitudinal velocity response curves, transverse velocity response curves and heading angular velocity response curves of ship 1 and ship 2, respectively. By simulation diagram, we find out that the velocity response curves are smooth and reach the equilibrium state in a short time. Fig. 2 and Fig. 4 describe the actual position (x, y) and heading angle ψ curves for the two ships. From the simulation results, we know that the clntroller which is designed can make the ship resist external interference and Gradually realize fixed point stability step by step; In the whole dynamics, first the expected heading angle has quickly reached, then the desired longitudinal displacement has achieved. In the end, the lateral propulsion gradually achieve desired position and stay calm; Driven by the controller, the ship gradually converges to the target position. Moreover, the position and heading are tending towards stability in 40 seconds.

V. CONCLUSION

In this article, we study the problem of robust simultaneous stabilization of two DP ships. First of all, the DP ship model is transformed into PCH model, and the simultaneous stabilization controller for unperturbed systems have designed based on Hamilton method. Then the problem of robust simultaneous stabilization for systems with external disturbances is studied, and the corresponding output feedback controller is designed. Different from the existing literature, Hamilton method and spatial expansion technique are adopted in this paper. The simulation results show the effectiveness of the controller designed in the paper. The research content of this paper is complementary to the actual marine engineering application, and the designed controller can be used for supply ship.

REFERENCES

[1] X. Bian, M. Fu, and Y. Wang, *Ship Dynamic Positioning*, Beijing : Science Press, 2011, pp.1-13.

[2] J G. Balchen,N A. Jenssen and S. Sælid, "Dynamic positioning using Kalman filtering and optimal control theory." *IFAC/IFIP Symposium on Automation in Offshore Oil Field Operation*, pp. 183-186,1976.

[3] J G. Balchen,N A. Jenssen and S. Sælid, "Dynamic positioning of floating vessels based on Kalman filtering and optimal control. *19th IEEE Coonference on Decision and Control*, pp. 852-864,1980.

[4] X. Ding, *Ship Dynamic Positioning Control System*, Dalian : Dalian Maritime University Press, 2017, pp.1-30.

[5] J. Jiao, G. Wang, Y. Gu and X. Sun, *Coordinated Formation Control for Dynamic Positioning Ships*, Beijing : Electronic Industry Press, 2017, pp.1-20.

[6] B. Wang, "Research on Robust Adaptive Formation Control of Multiple Dynamic Positioning Ships," *D. Harbin Engineering University*, 2017.

[7] S. Wang, "Research on Consistency Theory of Multi-dynamic Positioning Ship," *D. Harbin Engineering University*, 2014.

[8] Y. Li, F. Xu, G. Xie, and X. Huang, "Survey of the development and application of multi-agent technology," *Computer Engineering and Applications*, vol. 54, no. 9, pp. 13-21, 2018.

[9] G. Xu, "The Research on Control Design Consensus and Formation in Multi-agent System," *D. Taiyuan University of Technology*, 2018.

[10] Y. Wang, *Generalized Hamiltonian Control System Theory: Implementation, Control and Application*, Beijing : Science Press, 2007.

[11] R. Yang , Y. Wang , Stability for a class of nonlinear time-delay systems via Hamiltonian functional method, Sci. China Inf. Sci. 55 (5) (2012) 1218–1228 .

[12] R. Yang , Y. Wang , Finite-time stability analysis and H ∞ control for a class of nonlinear time-delay Hamiltonian systems, Automatica 49 (2) (2013) 390–401 .

[13] R. Yang, R. Guo, Adaptive finite-time robust control of nonlinear delay Hamiltonian systems via Lyapunmov–Krasovskii method, Asian J. Control 20 (2) (2018) 1–11.

[14] G. Chen, "Research on the Stabilization Control of Underactuated Surface Vessels," *D. Dalian Maritime University*, 2014.

[15] Z. Kang, J. Zou, L. Zan, and J. Ma, "Design of Motion Stabilization Controller for Fully-driven Ships," *SHIP & BOAT*, vol. 29, no. 5, pp. 51-58, 2018.

[16] J. Mao, "Simultaneous Stabilization and Simultaneous H^{∞} Control for Uncertain Nonlinear Systems," *D. Zhengzhou University*, 2011.

[17] X. Cai, H. Gao, and Y. Liu, "Simultaneous H^{∞} Stabilization for a Class of Multi-input Nonlinear Systems," *Acta Automatica Sinica*, vol. 38, no. 3, pp. 473-478, 2012.

[18] Y. Wang, Y. Liu and G. Feng, "Simultaneous stabilization of nonlinear port-controlled Hamiltonian systems via output feedback," *Journal of Shandong University (Engineering Science)*, vol. 39, no. 2, pp. 52-63, 2009.

[19] J. Zhang, "Simultaneous Stabilization and Control Stydy of Time-delay Systems," *D. Inner Mongolia Normal University*, 2016.

[20] R. Li, "Simultaneous Stabilization and Control for Singular Systems with Time-delay," *D. Inner Mongolia Normal University*, 2016.

[21] Y. Liang, "Simultaneous Stabilization and Control for Inter-connected Large-scale System with Time-delay," *D. Inner Mongolia Normal University*, 2016.

[22] TI. Fossen, *Guidance and control of ocean marine vehicles*, New York: John Wiley and SonsLtd, 1994.

[23] A. van der Schaft,"Port-Hamiltonian systems: An introductory survey," *C. In Proceedings if International Congress of Mathematicians*, 2006.

[24] Y. Wang, and X. Yang, "Controller Design for Dynamic Positioning of Ships Based on PCH Model," *Computer Simulation*, vol. 35, no. 3, pp. 10-14, 2018.

[25] Y. Wang, F. Gang and D. Cheng, "Simultaneous stabilization of a set of nonlinear port-controlled Hamiltonian systems," *Automatica*, vol. 43, no. 3, 2006.

[26] G. Xia, and Q. Zhou, "Nonlliner adaptive backstepping controller design of dynamic positioning system," *Applied Science and Technoology*, vol. 41, no. 3, pp. 27-30, 2014.