

Energy Efficient Swing Leg Trajectory Planning For Quadruped Robots Walking On Rough Terrain*

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Abstract - When the quadruped robots perform some patrol and rescue missions, they have to stay outdoors for a long time. Therefore, energy consumption should be reduced as much as possible to improve the robot's endurance ability. In this paper, we analyzed the energy consumption of the electric quadruped robot in the swing leg phase and presented a swing leg trajectory optimization approach which is both collision-free and energy-efficient on rough terrain. The effectiveness of this trajectory has been verified by the experiments in which our quadruped robot Pegasus crossed obstacles such as steps, slopes, and stairs automatically. The experiments have been carried out both in computer simulation and the robot Pegasus.

Index Terms - quadrupedal robot, swing leg trajectory, energy-efficient.

I. INTRODUCTION

Energy efficiency is a key attribute that allows the successful use of legged robots in practical applications. As a quadruped robot used for patrol search and rescue tasks outdoors, the ability to travel long distances without charging is critical in the tasks. Therefore, many researchers studied the energy consumption and energy efficiency of the quadruped robots, such as the MIT Cheetah [1] [2]. They mainly reduced the energy consumption from the hardware design of motors, electric motor drivers, transmission and spine. The IIT's quadruped robot HyQ [3] minimized energy consumption by presenting an optimal COM motion trajectories and corresponding optimal foothold locations approach.

The most common movement legged robots do is swinging their legs. Whether the robot's movement is stable and power consumption is low is directly related to the design of the swing leg trajectory. By designing the energy-efficient trajectory of the swing leg, the energy consumption of the robot can be reduced.

When robots are walking outside, there are various obstacles on the ground, we call it rough terrain. The robot still needs to understand the terrain and to avoid obstacles. We want to design a swing leg trajectory that is both energy-efficient and collision-free. Under these requirements, only a few approaches have been presented. For example, the hydraulically driven quadruped robot SCalf [4] studied the



Fig. 1 The robot Pegasus is walking outside. The robot is about 1.0m long, 0.4m wide, and 0.75m tall when legs stand straight, with a weight of about 35.2 kg. The 3DOF legs are driven by electric motors at 100Hz by joint torque PD controller. We mount a rotating laser range sensor (Hokuyo UTM-30LX, update rate 0.5Hz for a full rotation) in front of the robot to perceive terrain. All the algorithms run on an onboard embedded computer (NVIDIA Jetson TX2) with Linux and ROS.

design of energy minimization of foot trajectory in planar motion, but did not consider avoiding obstacles. The ANYmal robot [5] from ETH Zurich, designed the shortest and collision-free swing leg trajectory and simplified these tasks to the problem of finding the foot trajectory only. The LittleDog robot [6] [7], manufactured by Boston Dynamics Inc, designed a smooth and collision-free swing leg trajectory.

None of the above robots' swing leg trajectory can save energy and avoid obstacles at the same time. In order to deal with this problem, in this paper, we propose a method for designing the swing leg trajectory in three-dimensional spaces, which is both collision-free and energy-efficient. After experimental verification, Our Pegasus robot is shown in Fig. 1. In addition to having a good ability to move on rough terrain, the energy efficiency of swing leg motion is far superior to other robots.

The organization of this paper is as follows: In Section II, we will introduce how robots can walk on rough terrain and construct the swing leg trajectory planning to an optimization problem. In Section III, we tell how the robot eliminates the collision between the leg and the terrain when the leg is

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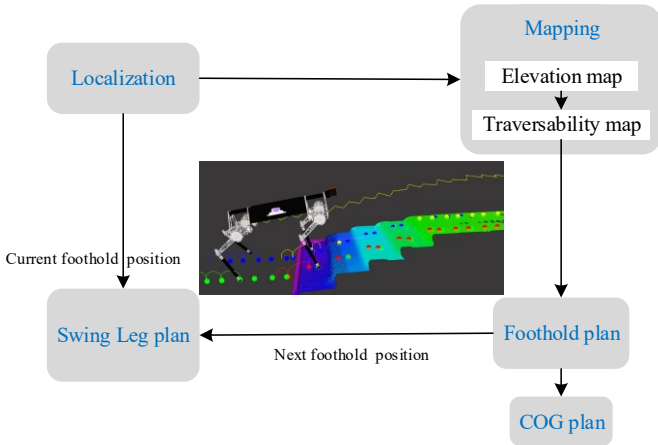


Fig. 2 An overview frameworks for quadruped robot planning on rough terrain. Several functional modules such as localization, mapping, foothold planning, swing leg planning and COG trajectory planning are organized together to complete the robot's movement task on rough terrain.

swinging. In Section IV, we analyze the energy consumption of the swing leg motion of the motor-driven robot. In Section V, we will present our experimental design and implementation in detail and then show the experimental results. The last section is the conclusion.

II. PLANNING ON ROUGH TERRAIN

In order to deal with the movement tasks on the rough terrain of the quadruped robot, we give our planning frameworks in Fig. 2. Before the swing leg trajectory planning, we use the elevation map [8] [9] to structure terrain to helps the robot to understand its environment, then the foothold planner generates a sequence of footholds on traversability map [10]. Once the valid footholds are found, the body COG (center of gravity) trajectory and swing leg motion can be planned.

The static gaits are characterized to be stable at any point of time and hence preferred when it comes to slow walking or challenging terrain. Thereby, we employ a static gait when robots walk on the rough terrain which is based on the regular footfall sequencing (right-hind (RH), right-front (RF), left-hind (LH), left-front (LF)). During the static walking of our robot Pegasus, a full gait cycle also includes the COG turn left and the COG turn right, the schematic diagram as shown in Fig. 3.

Having a terrain map and given the current and target foothold position, the robot can be plan the swing leg trajectory. We reduce this task to the problem of finding a trajectory of the foot only. The swing leg trajectory in this paper refers to the foot trajectory (the execution trajectory of the end effector). We want to plan both energy-efficient and collision-free swing leg trajectory, inspired by the CHOMP trajectory planner [11] [12], we formulate the planning problem as an optimization. We parametrize the swing leg trajectories as Cubic Bézier curve and optimize directly over the two control

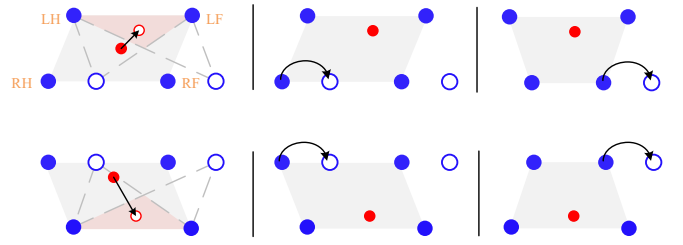


Fig. 3 An overview of our robot's full gait cycle, the robot continuously executes a stereotypic pattern COG turn left, swing right-hind (RH) leg, swing right-front (RF) leg, COG turn right, swing left-hind (LH) leg, swing left-front (LF) leg. The COG lies within the pink triangle center.

points of the Bézier curve (except for the start and target control points). Herewith, the optimization yields continuous paths that can be smoothly tracked by the robot. We stack the 3D knot positions as a vector u and the position on the curve for time t is given by $r(u, t)$. The optimization cost function is expressed as a weighted sum of collisions costs and energy consumption, $f_c[u]$ and $f_{en}[u]$, respectively (1).

$$\text{minimize} \quad f[u] = w_c f_c[u] + w_{en} f_{en}[u] \quad (1)$$

We first discuss in the next section how the robot's swing leg trajectory avoids collisions and how to construct the collision cost function.

III. AVOIDING COLLISIONS

Firstly, we choose some key points x from the robot's legs to check the collisions with the terrain. The distance $D(x)$ between the key points and the terrain is calculated by the signed Euclidean distance (SDF) method [13]. If the point x is inside of the terrain, $D(x)$ is negative, whereas if x is outside of the terrain, $D(x)$ is positive, and $D(x)$ is zero if the point x lies on the boundary of an obstacle.

In this paper, we only select the foot endpoint as the key point. The distance from the foot endpoint to the terrain can be expressed as $D(r_F(u, t))$ which is the SDF function of foot trajectory $r_F(u, t)$. And the piecewise function (2) is cost function for collisions defined based on the SDF, and ε is the threshold.

$$c(r_F(u, t)) = \begin{cases} -D(r_F(u, t)) + \frac{1}{2}\varepsilon, & \text{if } D(r_F(u, t)) < 0 \\ \frac{1}{2\varepsilon}(D(r_F(u, t)) - \varepsilon)^2, & \text{if } 0 < D(r_F(u, t)) \leq \varepsilon \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

We constructed the following collision cost function (3), which penalizes objects near the terrain to force the robot's swing leg trajectory to avoid collisions. T is the time of the swing leg phase.

$$f_c[u] = \int_0^T c(r_F(u, t)) \|r_F(u, t)\|_2 dt \quad (3)$$

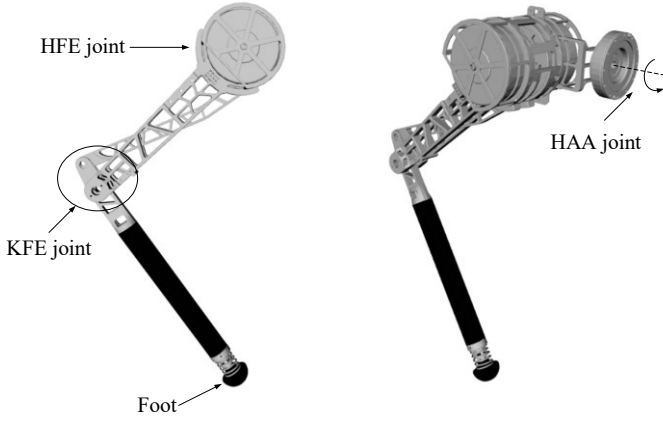


Fig. 4 Single leg of the quadruped robot.

IV. THE ENERGY CONSUMPTION OF ROBOTS

A. Robot energy consumption

The energy consumption in a legged robot is mainly due to the energy consumed by an actuator in each joint of the legs. For our robot, taking the left front leg as an example, we can see that the structure of the robot leg is as shown in Fig 4. The leg consists of three joints connected in series, each joint has a certain range of motion. The two joints in the leg-sagittal plane are called hip flexion/extension (HFE joint) and knee flexion/extension (KFE joint). The third joint is called hip abduction/adduction (HAA joint). There are three actuators, corresponding to 3 DOF, in each leg.

The research [14] [15] proved that power consumption measured at battery matches up with the sum of mechanical power and Joule heating, so this calculation is used for power-flow estimation from a motor, Joule heating dissipation P_j , Mechanical power P_m , and battery power measured during the swing leg phase has the following relationship (4).

$$P = P_j + P_m \quad (4)$$

We can get motor current I , joint angel q , torque constant K_t , armature resistance R . Then, according to the [16], we can get the Mechanical power and Joule heating dissipation, respectively, and the relationship between the actuator torque τ , like (5) (6).

$$P_j = \sum_{3\text{motors}} I^2 R = \sum_{3\text{motors}} \frac{\tau^2}{K_t^2} R = \sum_{3\text{motors}} \gamma \tau^2 \quad (5)$$

$$P_m = \sum_{3\text{motors}} K_t \times I \times \dot{q} = \sum_{3\text{motors}} \tau \dot{q} \quad (6)$$

For trajectory, the point u of each time on the trajectory, using inverse kinematics to find joint angle q , as follows (7).

$$q = ik(u) \quad (7)$$

Then, we need to analyze the kinematics and dynamics of the robot's swing leg movement to get τ . According to the geometry of the robot's leg Fig. 4, we use the modified DH

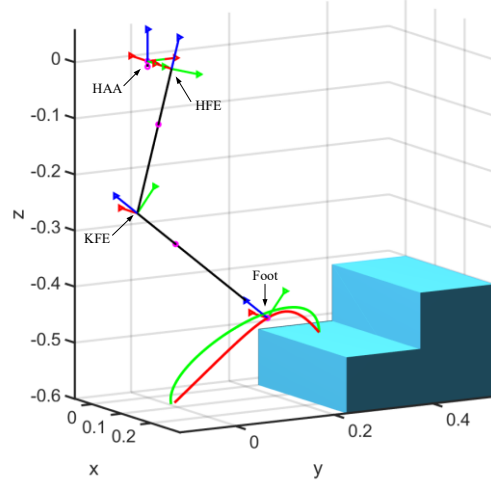


Fig. 5 Crossing the step obstacles, collision-free and shortest swing leg trajectory (red), collision-free and energy-efficient (green).

method to establish the kinematics model of the legged robot and Newton-Euler recursive method [17] and [18], we can construct the dynamics equation of the quadruped robot as follows (8), the mass matrix $M(q)$, $C(q, \dot{q})$ and $G(q)$ is a vector of gravity and Coriolis force terms.

$$\tau = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) \quad (8)$$

Combined with (7) (8), we can get the relationship between the point on the trajectory u and torque τ , and integrate the whole swing leg process to obtain the following energy consumption function (9). It penalizes a trajectory based on energy consumption. It parameterizes the relationship between foot trajectory and energy consumption.

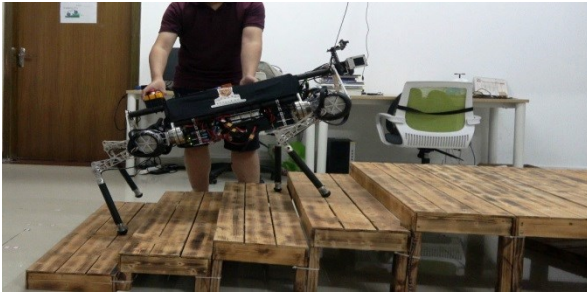
$$f_{en}[u] = \int_0^T P dt = \int_0^T \tau \dot{q} + \gamma \tau^2 dt \quad (9)$$

B. Constraints Objective Function

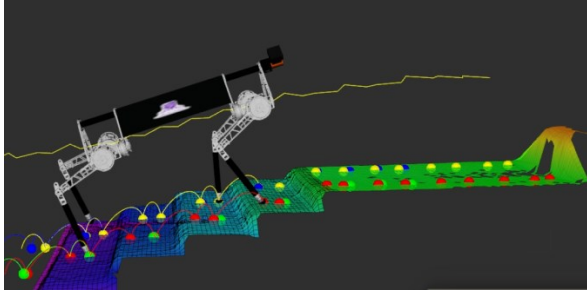
For different robots, the range of actual motion is limited due to the configuration of the robot, so the trajectory we plan also takes into account the working space of the actual robot. To this end, we impose constraints on the robot from the workspace to the joint space. In addition, the actual robot is driven by the motor. Due to the actual ability of the motor, we need to constrain the speed and acceleration of the robot to make it conform to the actual robotic ability situation.

$$C_{kine}(q, \dot{q}, \ddot{q}): \begin{cases} q(i)_{min} < q(i) < q(i)_{max} \\ \dot{q}(i)_{min} < \dot{q}(i) < \dot{q}(i)_{max} \\ \ddot{q}(i)_{min} < \ddot{q}(i) < \ddot{q}(i)_{max} \end{cases} \quad (10)$$

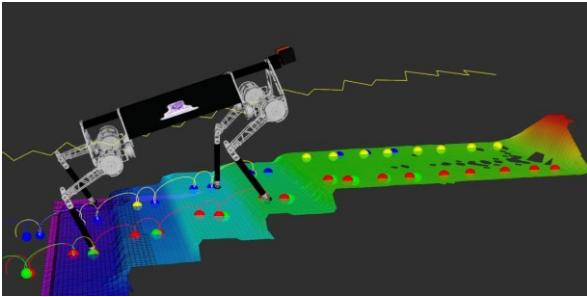
For the robot, in addition to the kinematics limitation, the actual torque needs to be calculated according to the motion requirements in the actual planning process [19], but the torque must conform to the actual physical constraints, that is, the torque output of the robot must be within the actual motor



(a) Experimental scene



(b) Experiment without energy optimization



(c) Experiment with energy optimization

Fig. 6 Quadruped robot climbing stairs.

performance constraints. So we have to constrain the torque as (11).

$$C_{motor}(\tau(i)) : \tau(i)_{\min} < \tau(i) < \tau(i)_{\max} \quad (11)$$

C. Optimization Objective Function

We solve the objective function (1) as a nonlinearly constrained minimization problem with NLOpt, an open-source library for nonlinear optimization, providing a common interface for many different optimization algorithms. We use a time resolution of $\Delta t = 10\text{ms}$ to provide a dense resolution for collision checking. In order to compare with the other robot's swing leg trajectory planning method such as ANYmal, in the same situation, we used the shortest and collision-free swing leg trajectory as (12), the optimization cost function is expressed as a weighted sum of the trajectory length $f_l[u]$ (13) and collisions costs for comparison. The simulation result of the comparison is shown in Fig. 5. And the shortest trajectory requires 17.8 J of energy, while the energy-efficiency trajectory requires 14.2 J of energy, which can save 20.2% of energy.

$$\text{minimize } f[u] = w_c f_c[u] + w_l f_l[u] \quad (12)$$

$$f_l[u] = \int_0^T \|\dot{r}(u, t)\|_2 dt \quad (13)$$

The computer simulation proves that our swing leg trajectory does not collide with the terrain, and saves the robot's energy compared to other trajectory planning methods.

V. EXPERIMENT

We validate our trajectory optimization method on our Pegasus robot. The experimental site is the five-step staircase we built and each step is 8cm high, 120cm wide and 30cm long. Under the same experimental conditions and guaranteed the same foothold position. We conducted a comparative experiment with optimized and without optimization of energy, as shown in Fig. 6. We use the shortest and collision-free swing leg trajectory as without optimization of energy trajectory. The parameters of gait parameters showed in the following TABLE I.

TABLE I
PEGASUS GAIT PARAMETERS

Gait Parameters							
Step Length	Cog turn left	Swing RH	Swing RF	Cog turn right	Swing LH	Swing LF	Gait cycle
0.2 m	2 s	2.2 s	2.2 s	2 s	2.2 s	2.2 s	13 s

In the experiment, we collected information on the real-time speed, current, and working status of each leg of 12 motors, calculate energy consumption according to Equation (5) and (6). The sampling period of current and speed is 1 millisecond, and the relevant parameters of the actuator are shown in Table II.

TABLE II
PEGASUS ACTUATOR PARAMETERS

Actuator Parameters		
Gear ratio	Resistance	Torque constant
51:1 G_s	2.28 Ω	0.217 K_t

For comparison, we calculated the sum of mechanical power and Joule heat power of each leg (including three joints) in the same gait cycle. The results are shown in Fig. 7. And the single leg swing phase energy consumption as shown in Table III. The trajectory of energy optimization saves at least 20% of energy.

TABLE III
MECHANICAL+HEAT ENERGY CONSUMPTION DURING A SINGLE LEG SWING PHASE

	RH leg	RF leg	LH leg	LF leg
Without optimization	15.5694J	22.8012J	15.7306J	15.4903J
With optimization	11.6889J	16.1166J	9.8597J	10.3770J
Energy reduction	24.92%	29.32%	37.32%	33.01%

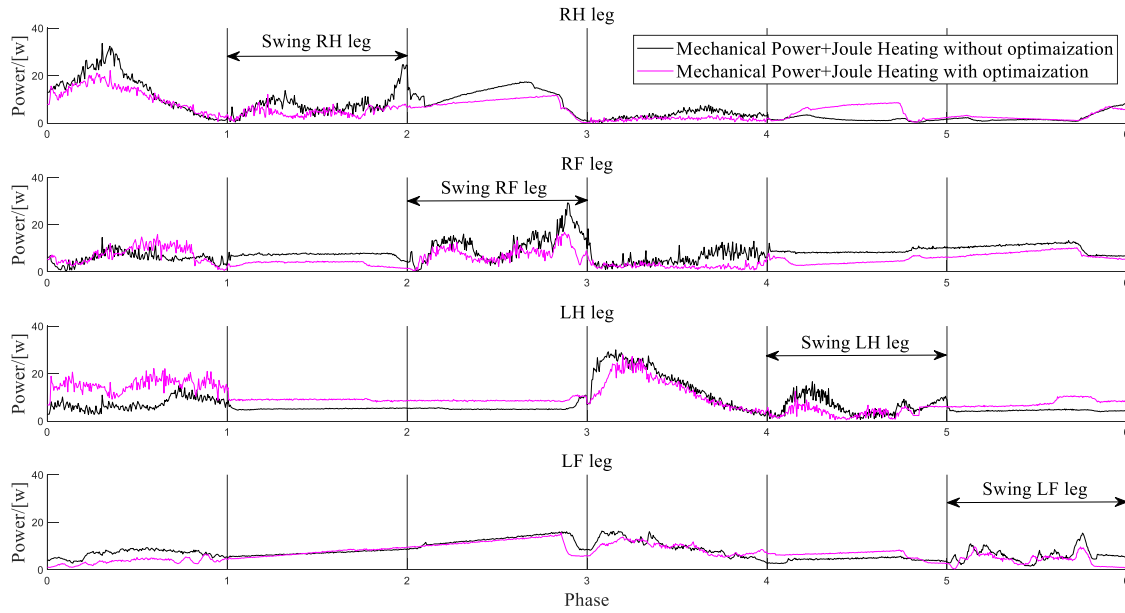


Fig. 7 Energy consumption during the movement of the robot's swinging leg under different trajectories.

VI. CONCLUSION

In this paper, we reduce the energy consumption of the robot's swing leg motion by planning a collision-free and energy-efficient swing leg trajectory. We show the planning framework to illustrate how our robots walk on the terrain. After the energy consumption and collision avoidance analysis during the execution of the swing leg trajectory, we expressed the trajectory as a cost function which is the weighted sum of collisions costs and energy consumption, and then turn this problem to a nonlinear optimization problem. Finally, the effectiveness of the proposed method is verified by the Pegasus robot climbing stairs experiments. The experiments have shown that energy is reduced by more than 20% compared with other methods of trajectory planning.

REFERENCES

- [1] S. Seok, A. Wang, M. Y. Chuah, D. Otten, J. Lang, and S. Kim, "Design principles for highly efficient quadrupeds and implementation on the MIT Cheetah robot," *IEEE International Conference on Robotics and Automation (ICRA)*, pp. 3307-3312, 2013.
- [2] S. Seok, A. Wang, M. Y. Chuah, D. Otten, J. Lang, and S. Kim, "Design principles for energy-efficient legged locomotion and implementation on the MIT cheetah robot," *IEEE/ASME Transactions on Mechatronics*, pp. 1117-1129, 2015.
- [3] C. Mastalli, M. Focchi, I. Havoutis, A. Radulescu, S. Calinon, and J. Buchli, "Trajectory and foothold optimization using low-dimensional models for rough terrain locomotion," *IEEE International Conference on Robotics and Automation (ICRA)*, pp. 1096-1103, 2017.
- [4] K. Yang, L. Zhou, X. Rong, and Y. Li, "An Energy Optimal Foot Trajectory for the Hydraulic Actuated Quadruped Robot," *IEEE 8th Annual International Conference on CYBER Technology in Automation, Control, and Intelligent Systems (CYBER)*, pp. 329-333, 2018.
- [5] P. Fankhauser, M. Bjelonic, C. D. Bellicoso, T. Miki, and M. Hutter, "Robust Rough-Terrain Locomotion with a Quadrupedal Robot," *IEEE International Conference on Robotics and Automation (ICRA)*, pp. 5761-5768, 2018.
- [6] M. Kalakrishnan, J. Buchli, P. Pastor, M. Mistry, and S. Schaal, "Learning, planning, and control for quadruped locomotion over challenging terrain," *The International Journal of Robotics Research*, vol. 30, pp. 236-258, nov 2010.
- [7] M. Zucker, N. Ratliff, M. Stolle, J. Chestnutt, J. A. Bagnell, C. G. Atkeson, and J. Kuffner, "Optimization and learning for rough terrain legged locomotion," *The International Journal of Robotics Research*, vol. 30, pp. 175-191, jan 2011.
- [8] P. Fankhauser, M. Bloesch, and M. Hutter, "Probabilistic Terrain Mapping for Mobile Robots with Uncertain Localization," in *IEEE Robotics and Automation Letters (RA-L)*, vol. 3, no. 4, pp. 3019-3026, 2018.
- [9] P. Fankhauser, M. Bloesch, C. Gehring, M. Hutter, and R. Siegwart, "Robot-Centric Elevation Mapping with Uncertainty Estimates," in *International Conference on Climbing and Walking Robots (CLAWAR)*, 2014.
- [10] M. Wermelinger, P. Fankhauser, R. Diethelm, P. Krüsi and M. Hutter, "Navigation planning for legged robots in challenging terrain," *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pp. 1184-1189, 2016.
- [11] M. Zucker, N. Ratliff, a. D. Dragan, M. Pivtoraiko, M. Klingensmith, C. M. Dellin, J. a. Bagnell, and S. S. Srinivasa, "CHOMP: Covariant Hamiltonian optimization for motion planning," *The International Journal of Robotics Research*, vol. 32, no. 9-10, pp. 1164-1193, 2013.
- [12] N. Ratliff, M. Zucker, J. A. Bagnell, and S. Srinivasa, "CHOMP: Gradient optimization techniques for efficient motion planning," *IEEE International Conference on Robotics and Automation (ICRA)*, pp. 489-494, 2009.
- [13] P. F. Felzenszwalb, and D. P. Huttenlocher, "Distance Transforms of Sampled Functions," *Theory of Computing* 8.19(2004):415-428.
- [14] S. S. Roy and D. K. Pratihar, "Dynamic modeling and energy consumption analysis of crab walking of a six-legged robot," *IEEE Conference on Technologies for Practical Robot Applications*, pp. 82-87, 2011.
- [15] S. Seok, A. Wang, M. Y. Chuah, D. Otten, J. Lang, and S. Kim, "Design principles for highly efficient quadrupeds and implementation on the MIT Cheetah robot," *IEEE International Conference on Robotics and Automation (ICRA)*, pp. 3307-3312, 2013.
- [16] S. Seok, A. Wang, M. Y. Chuah, D. Otten, J. Lang, and S. Kim, "Design principles for energy-efficient legged locomotion and implementation on the MIT cheetah robot," *IEEE/ASME Transactions on Mechatronics*, pp.

- 1117-1129, 2015.
- [17] R. Featherstone, *Rigid Body Dynamics Algorithms*. Boston, MA: Springer US, 2008.
 - [18] A. Winkler, C. Mastalli, L. Havoutis, and M. Focchi, "Planning and execution of dynamic whole-body locomotion for a hydraulic quadruped on challenging terrain," *IEEE International Conference on Robotics and Automation (ICRA)*, pp. 5148-5154, 2015.
 - [19] C. Mastalli, I. Havoutis, M. Focchi, D. G. Caldwell, and C. Semini, "Motion planning for quadrupedal locomotion: coupled planning, terrain mapping and whole-body control," 2017, working paper or preprint.