# Optimum Electrode Path for EDM Manufacturing of Integral Shrouded Blisk

Ruining Huang and Huan Liu
School of Mechanical Engineering and Automation
Harbin Institute of Technology (Shenzhen)
Shenzhen, Guangdong, China
Email: hrn@hit.edu.cn.

Abstract—This paper reports an electrode feed path planning with a smooth trajectory for electrical discharge machining (EDM) manufacturing of integral shrouded blisk. This method is developed to offer a generic method that guarantees the smoothness of the feeding path, independently of the workpiece geometry and user-dependent features. An electrode design method for EDM machining of integral shrouded blisks from a primitive electrode by electrode size reduction and electrode division was first described. A new objective function with maximum motion range has been proposed and an optimization method for smooth path has been implemented, in which the position and attitude of a new electrode feeding path node was optimized by maximizing the motion range that the electrode is able to move along a predefined reference direction before it interferes with the blisk. Simulation results show that the proposed objective function generates a smooth path regardless of machined shape and available degrees of freedom.

*Index Terms*—Integral shrouded blisk, smooth trajectory, genetic algorithm, dynamic trajectory optimization.

# I. INTRODUCTION

Integral shrouded blisk is an important component of aerospace engine and a key factor in determining engine performance and maintenance because it improves the aerodynamic efficiency and reliability. The integral structure blisk, which combines both blades and shrouded on a single piece and eliminate the joint between them, address a variety of problems associated with the splicing parts in traditional turbine. This feature provides some extra benefits, such as higher turbine efficiency, higher reliability, lower aerodynamic resistance and lower weight [1], [2]. However, its complex shape and the low machinability of the materials make its manufacturing by traditional methods difficult or even impossible [3], [4]. Electrical discharge machining (EDM) is an alternative to manufacture this kind of parts. Due to the characteristics of the process, there is no contact between tool and workpiece, which means that the mechanical properties of the material do not have influence on the erosion process. Therefore, it can be used to machine materials with low machinability and high hardness values. Moreover, since

\*This work was partially supported by the National Natural Science Foundation of China (No.51475107), and Shenzhen Basic Research Program (No. JCYJ20170811160440239).

there is no contact, there are no machining forces, which helps improving accuracy [5], [6].

For integral shrouded blisk manufacturing, one fundamental challenges for the EDM processes is the definition of the optimum tool path to erode the cavity. There have been a wide range of efforts dedicated to investigating different approaches to obtain optimum tool path and improve machining accuracy. For example,

Liao et al. proposed electrode thickness reduction as the objective function to obtain the corresponding electrode feed path [7]. This method adopted a series of optimization solutions and can simplify the calculation, but the shroud and the hub surface are prone to generate errors and the electrode thickness reduction amount is large. Liu et al. proposed conjugate method, which requires the electrode feed path to keep the center line of the electrode and the center line of the flow channel tangent during the machining process. It can obtain highly stable feed trajectory [8], [9]. But for big twist blade with variable cross section, this method will fail because the center line is more than one and also difficult to gain. Ayesta et al. proposed an objective function that maximizes the minimum distance. According to the principle, the electrode is fed as much as possible along the center of the flow channel, and hybrid optimization is performed. The method can obtain a highly stable electrode feed path [10]. However, its application is mainly on relatively simpler surface. When encountering complex surfaces, its objective function cannot guarantee feeding along the center. Liang et al. proposed the trajectory planning method of maximum free travel. The maximum free travel of each point on the electrode is the objective function, which avoids the relationship between the trajectory search and flow channel [11]. But the solution involves the direction and prediction of the optimization variables, which difficult easy to implemented and increase computational complexity.

To address The smoothness of each axis feeding motion, this work investigates the optimum electrode path that uses trajectory smoothness and maximum motion range as multi-objective function. With the constraints of non-interference between the electrode and the turbine blade, an adaptive genetic algorithm is used to solve the problem. The electrode

and the flow channel are represented by discretized point set, which is suitable for the case where the blade surface equation is unknown, so the application field is wide.

#### II. DESIGN OF THE ELECTRODE

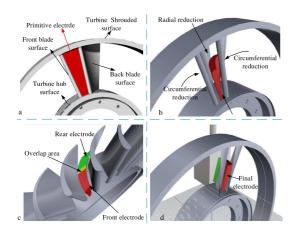


Fig. 1. Electrode design: (a) primitive electrode; (b) circumferential and radial reduction; (c) electrode division; (d) final electrodes.

In the integral shrouded blisk EDM manufacturing, the design of the machining electrode and is the key steps to ensure the machining accuracy. This process requires the machining electrode consistent with the flow channel of the turbine. That is: the upper, lower, left and right surface of the electrode should coincide with the adjacent turbine shrouded surface, turbine hub surface, back blade and front blade surface respectively, which is shown in Fig. 1(a).

To obtain the corresponding movement space, a proper division and reduction operation are implemented on the initial electrode. A circumferential reduction operation is performed on the electrode to get circumferential movement range. The circumferential reduction means that the front blade or back blade rotates a certain angle around the central axis of the integral shrouded blisk (X axis), so the electrode can obtain circumferential motion space. A radial reduction operation means that shrouded surface is contracted in the radial direction, and the initial electrode is split to obtain a height motion space. Fig. 1(b) shows the reduction operations on the electrode. When the blade twist severely, it is necessary to divide the electrode into some sub-part which is called division operation. Divided into front and rear two small electrodes with setting a certain overlap area at the junction of the two electrodes to eliminate the error of the gear mark, the electrode now has more motion space showing in Fig. 1(c).

However, the reduction and division of the electrode also cause a large increase in the number of processing electrodes, thereby increasing production costs. So an integrated consideration of reasonable reduction of the electrode and the proper number of splits are important aspects of electrode design. According to the complexity of the actual processing blade, one of the final electrodes is shown in Fig. 1(d), and the reduction amount of the electrode is optimized to obtain a larger electrode thickness and improve its processing efficiency.

#### III. ELECTRODE FEED TRAJECTORY PLANNING

# A. Trajectory planning strategy

Due to the great randomness of the initial position and the determination of the final processing position, shown in Fig. 1(d), so generally, the backward trajectory planning method is adopted to obtain the exiting trajectory without interference with channel. And the inverse trajectory is the non-interference feed trajectory.

According to the copy processing method, the end position and posture information of the electrode have been determined. The electrode at the end position rotates a corresponding angle around the X axis to remove the material, which is called the processing procedure. The copying motion after the tool electrode reaches the end position is already clear and simple. The key is how the tool electrode feeds from an initial position to the end processing position.

This paper takes a five-axis machine as an example. Just as shown in Fig. 2, it consists of X, Y, Z linear axes, A-axis rotating around the X-axis, and C-axis rotating around Z-axis. Generally, X axis is taken as the reference axis direction. Along the direction of the reference axis, the total distance from the end position to the initial position is divided into a series of nodes. The electrode move along the reference axis by one step, the movement of other axes of the machine tool is adjusted so that the electrode does not interfere with the blisk. Meanwhile, the smoothness of the feed trajectory for each axis is required and the movement range of the electrode at the motion node is optimized to be as large as possible. It is sequentially circulated until the electrode moves out of the flow channel, so the motion node of the electrode exiting the flow channel without interference can be obtained.

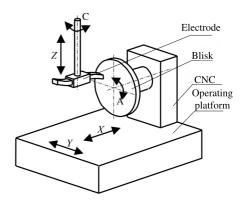


Fig. 2. CNC machine tool and its motion axes

## B. Establishment of motion trajectory optimization model

1) Determination of independent variables: According to the strategy, the end position of the electrode is taken as the starting point, and the position where the electrode moves out of the flow channel with a certain safety distance is set as the end point, and the node division is performed in this interval.

$$\delta: x_b = x_0 < x_1 < x_2 < \dots < x_n = x_{\varepsilon} \tag{1}$$

While the X-axis coordinates of the electrode are at each node (i=0,1,2...n), as long as the motion of the auxiliary axes are adjusted to make the electrode non-interference with the turbine blade, the solution is acceptable. The smooth connection of these series of discrete points is the feed trajectory without interference.

When the electrode moves from the previous node to the next node, for a five-axis CNC machine, the X, Y, and Z coordinate axes need to move  $m_x$ ,  $m_y$ , and  $m_z$  millimeter respectively, and the C axis should rotate  $r_c$  radians. When entering the machining position, the A axis rotation amount is used for copy processing. The movement of the electrode can be regarded as the movement of all the shape points on the electrode. In the workpiece coordinate system (denoted as W), the electrode left and right surfaces are firstly reduced circumferentially, and rotated around the A axis by a certain angle h. So the point  $B_0(B_{0x}, B_{0y}, B_{0z}, 1)$  on the left surface becomes  $B_1(B_{1x}, B_{1y}, B_{1z}, 1)$ . Then each shape point on the electrode needs to be converted to the tool coordinate system (denoted as T), so  $B_1$  becomes  $B_{1t}$ . Afterwards, shape points in T should follow the movement axes to move to the corresponding next position, so  $B_{1t}$ becomes  $B_{2t}$ . Finally the transformed shape points in T are converted to W to make an interference judgment with the turbine blade, so  $B_{2t}$  becomes  $B_2$ . In this way, the electrode continues to advance, and the trajectory search is continuously performed until the flow channel is exited. Then the relationships between these points are:

$$B_{1} = H_{h} * B_{0}$$

$$B_{1t} = H_{wt}^{-1} * B_{1}$$

$$B_{2t} = H_{x} * H_{y} * H_{z} * H_{rc} * B_{1t}$$

$$B_{2} = H_{wt} * B_{2t}$$
(2)

$$H_x = \begin{bmatrix} \mathbf{I} & \mathbf{T}_x \\ \mathbf{0} & 1 \end{bmatrix} H_z = \begin{bmatrix} \mathbf{I} & \mathbf{T}_z \\ \mathbf{0} & 1 \end{bmatrix} H_c = \begin{bmatrix} \mathbf{R}(r_c) & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix}$$

$$H_y = \begin{bmatrix} \mathbf{I} & \mathbf{T}_y \\ \mathbf{0} & 1 \end{bmatrix} H_h = \begin{bmatrix} 1 & \mathbf{0} \\ \mathbf{0} & \mathbf{R}(h) \end{bmatrix} H_{wt} = \begin{bmatrix} \mathbf{I} & \mathbf{T}_d \\ \mathbf{0} & 1 \end{bmatrix}$$

Where  $H_h, H_x, H_y, H_z, H_c, H_{wt}$  are homogeneous transformation matrices, and I is a 3 by 3 unit matrix, T is a 3 by 1 column vector,  $\mathbf{T}_x = [m_x, 0, 0]^\mathrm{T}$ ,  $\mathbf{T}_y = [0, m_y, 0]^\mathrm{T}$ ,  $(T)_z = [0, 0, m_z]^\mathrm{T}$ , respectively,  $\mathbf{R}(\bullet)$  is a 3 by 3 rotation matrix function around the A or C axis, and the  $\bullet$  is the value, which is needed to rotate with A axis or C axis.  $T_d = [d_x, d_y, d_z]$ 

represents the positional difference between the origin of T and the origin of W at the initial position. Since T moves with the electrode, after the one-step search is completed, the conversion relationship between T and W needs to be modified in real time which is shown as  $H_{wt} = H_x * H_y * H_z * H_{rc} * H_{wt}$ .

The motion axis and the circumferential reduction are used as optimization variables. Since the X-axis motion amount  $m_x$  is already known as the step size selected in advance, the optimization variable can be set to :  $X = \begin{bmatrix} m_y & m_z & r_c & h \end{bmatrix}$ .

2) The construction of the constraints: Using the idea of point-to-surface, that is, each type of surface of the electrode is shaped to obtain a finite number of points for each surface. As long as each point of the electrode is judged to be within the feasible region enclosed by the adjacent back blade surface, front blade surface, turbine shrouded and hub surface, it can be considered that the electrode does not interfere with the flow channel. Since the turbine shrouded and hub are torus, it is easy to judge whether the point is in the region surrounded by turbine shrouded and hub, which can be formulated in (3).

Let  $P(x_p, y_p, z_p)$  denote a shape point on a surface of the electrode, then, as mentioned above, the formula that judge whether P is within the turbine shrouded and hub can be written as following:

$$(R_h + \Delta_1)^2 < x_p^2 + y_p^2 < (R_c - \Delta_1)^2$$
 (3)

Where  $R_h$  and  $R_c$  are the radius of the turbine shrouded and hub, and  $\Delta_1$  is the safety factor.

The key is to determine whether the value point be within the flow channel between the left and right surfaces. As shown in Fig. 3, assuming that the shape points  $M_1, M_2, M_3, M_4$  and  $M_5$  on a surface of the flow channel are arranged in an orderly manner and the outer normal of each type point can be calculated. Point  $E_0$  is a shape point on the electrode. Firstly, vectorize the lines from each shape point of the flow channel surface to  $E_0$ , and then find the shortest of these vector lengths. Assuming that the  $\overline{M_1E_0}$  is the shortest, the vector  $\overline{N_1}$  is a normal vector of the surface through point  $M_1$  and point to the flow channel side. Point  $M_5$  is the closest shape value to point  $M_1$  on the same surface. Meanwhile, point  $M_5$  is also required to be on the same side (left or right) as point  $E_0$  relative to point  $M_1$ . Then, calculate the angle  $\theta$  between the vector  $\overline{M_1M_5}$ and the normal vector  $\overline{N}'_1$ , the angle  $\alpha$  between the vector  $\overline{M_1E_0}'$  and the normal vector  $\overline{N_1}'$ , and compare the size of the two. And the conclusion is: only if  $\alpha$  is smaller than  $\theta$ , the point  $E_0$  on the electrode can be judged non-interfere with the flow channel surface.

Compared with the traditional method, the proposed method can more fully consider the bending and torsion of the blade surface. As shown in Fig. 3, traditional method judge the feasibility of each shape point by comparing the dot product of the vector  $\overline{M_1E_0}$  and the normal vector  $\overline{N_1}$  with zero. That is, 90 degree is the criterion. But when the surface is quite twisted, as the point  $E_1$ , although the angle between  $\overline{M_4E_1}$  and  $\overline{N_4}$  is greater than 90 degree, it is still non-interference with the flow channel surface. Obviously, if utilize the traditional method, there would be a misjudgment. So, it is necessary to take the neighbors into consideration, which is also a consideration of the bending and twisting extent of the surface.

In conclusion, if we take all points  $(E_1, E_2, ..., E_i, ..., E_n)$ 

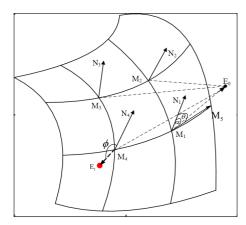


Fig. 3. Constraint illustration

on the left surface of the electrode for example, the corresponding angle is denoted as  $\alpha_i$ ,  $\theta_i$ , the safety margin is  $\Delta_2$ . Taking the notations  $Con_i$ ,  $Con_L$ ,  $Con_R$ ,  $Con_U$ ,  $Con_D$  to represent the judgement conditions of a single point, the left surface, right surface, upper surface and lower surface respectively. Then, the constraints can be written as:

$$con_i = \begin{cases} 1 & \text{if } \alpha_i - \theta_i - \Delta_2 > 0 \\ 0 & \text{if } \alpha_i - \theta_i - \Delta_2 \le 0 \end{cases} \quad i = 1, 2, 3 \dots N$$

So, the total judgment condition of the left surface is:  $Con_L = \prod_1^N con_i$ . In the same way,  $Con_R$ ,  $Con_U$  and  $Con_D$  can be obtained.

3) The design of the objective function: The purpose of the trajectory optimization in this paper is to make the feeding of each axis of the machine tool smooth, and to maintain a large safety margin for the movement space of the electrode.

For each motion axis of the machine tool, its feed motion curve is actually a two-dimensional space curve about time. According to the definition of advanced mathematics, the smooth curve represents that it has a first-order continuous derivative. In fact, in the feed of each motion axis of the machine tool, the smoothing of the feed curve mainly means that the feed is stable, and the trajectory does not appear to be pulsating and sharp. So the smooth objective function is

designed as shown in formula (4).

$$Objfun_{1} = a_{1}|(z2 - z1)/(\Delta_{t} * \Delta_{z})| + b_{1}|(y2 - y1)/(\Delta_{t} * \Delta_{y})| + c_{1}|(C2 - C1)/(\Delta_{t} * \Delta_{C})|$$
(4)

In the above formula  $a_1$ ,  $b_1$  and  $c_1$  are the weight coefficients of the optimization targets, where  $a_1+b_1+c_1=1$ . And  $z_2$  and  $z_1$  represent the z axis translation amount in the current and last search step respectively.  $\Delta_z$  represents the total feasible scope of the z axis. The same is true for the y and C axis. And  $\Delta_t$  is the time interval between two steps. Since the feed rate of the x-axis is actually a straight line with a certain slope, no smooth optimization is needed.

As for the safety margin requirement, the statistical method is applied because of the application of discretizing point sets. Just as the following Fig. 4 shown, in the picture(a), due to this electrode is copied from the corresponding profile, the minimum distance of all shape points on the electrode left surface to the back blade (denoted as  $D_L$ ) is quite close to the minimum distance of all shape points on the electrode right surface to the front blade (denoted as  $D_R$ ). And the standard deviation of the distance distribution (denoted as  $S_L$ ) from each shape point on the electrode left surface to the back blade is almost zero. The same is true for the electrode right surface (denoted as  $S_R$ ). And as a comparison, in the picture(b), the electrode has moved to a bad position with small safety margin, its distance distribution is very uneven which causes its big  $S_R$  and  $S_L$ . Furthermore, the calculation of the standard deviation needs to centralize the distance distribution first. So, in conclusion, the objective function can be written as:

$$Objfun_2 = a_2 (D_L - D_R)^2 + b_2 S_L + c_2 S_R$$
 (5)

In the formula above,  $a_2$ ,  $b_2$ ,  $c_2$  are also called the weight factors, where  $a_2 + b_2 + c_2 = 1$ . To conclude, with  $a_3$  and

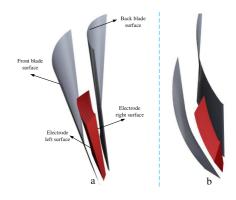


Fig. 4. Electrode position comparison

 $b_3$  denoting the weight coefficients, the entire optimization

question can be written as:

min 
$$Objfun(X) = a_3Objfun_1(X) + b_3Objfun_2(X)$$
  
s.t  $Con_L > 0, Con_B > 0, Con_U > 0, Con_D > 0.$  (6)

#### C. Genetic algorithm

The optimization problem in this paper is a complex nonlinear constraint problem, and the objective function is an implicit function. The traditional optimization algorithm is difficult to solve such problems. Genetic algorithm has high precision and good global convergence, and it does not require the derivative of the objective function. It is very suitable for the mathematical model of this paper. Therefore, the genetic algorithm is selected for optimization.

The use of genetic algorithm is within a certain interval, so it is necessary to determine the range of motion of each axis. The specific formula is:

$$\Delta z \in [-(R_c - R_h), R_c - R_h] 
\gamma \in 1/2 [R_c + R_h] \cdot \sin(\theta/2) 
\Delta y \in [-\gamma, +\gamma] 
\beta \in acos (\overrightarrow{AO} \cdot \overrightarrow{BO} / |\overrightarrow{AO}| \cdot |\overrightarrow{BO}|) 
\Delta_C \in [-\beta, +\beta]$$
(7)

Where  $\Delta_z, \Delta_y, \Delta_C, R_c$  and  $R_h$  have mentioned above and  $\theta$  is the center angle between adjacent blades of a turbine. The point O is the origin of the tool coordinate system. The point A is on the electrode left surface which is farthest from the origin O. The point B is the closest to A on the blade surface. They are easy to know form the 3-D model.

Genetic algorithm is mainly performed in the following steps: coding, initial population generation, competition, crossover, mutation, and decoding. It is briefly explained below.

Encoding: After determine the number N of binary bits calculated by formula (8), it is convenient to use a random function to generate a binary figure to represent an individual like 01001001101.

$$D = (U - L)/ \text{ precision}$$

$$2^m < D < 2^N$$
(8)

Where L and U are the lower and upper bounds of the range of motion of a certain axis of the machine. The precision is the processing accuracy of the corresponding axis.

Initial population generation: This procedure eliminates the individuals that produce interference until the initial population of individuals meets the required number.

Competition: All individuals in the population carry out roulette selection according to the principle of natural selection, and the excellent individuals with high fitness are chosen to proliferate.

Crossover: Partial chromosome binary coding between excellent individuals is exchanged under certain probability.

Mutation: A positional variation of the excellent individual chromosome binary coding is implemented under certain probability.

Decoding: Convert the binary-coded individual to the corresponding decimal number to get the solution for each axis. This formula can be written as following:

Dec = 
$$\sum_{0}^{N-1} 2^{i}$$
  
S =  $L + \text{Dec} * \frac{U - L}{2^{N} - 1}$  (9)

Where a, b and N are mentioned above.

The flow chart below is the overall flow chart of the algorithm. According to the reduction information of the electrode, the range of each optimization variable is obtained, and then the optimization solution is performed. The overall algorithm is a dynamic planning process.

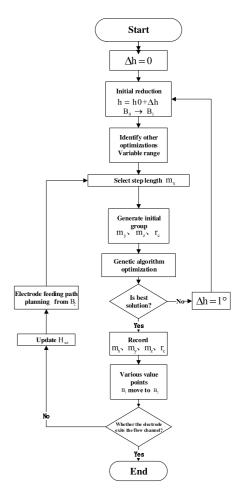


Fig. 5. Flow chart

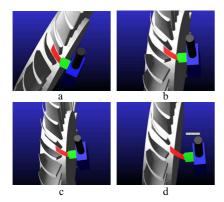


Fig. 6. Electrode exits flow channel diagram

## IV. MOTION TRAJECTORY SIMULATION ANALYSIS

## A. Formation of motion trajectories

After the above solution obtains the information of each motion node, it is necessary to interpolate the motion nodes to obtain a smooth motion trajectory. Cubic spline interpolation has the advantages of relatively simple calculation, good stability, certain convergence and good smoothness. So it is chosen as the interpolation method and simply the Matlab built-in function spline can do this very well.

## B. Adams software simulation

This paper uses ADAMS software for simulation. Taking the electrode shown in Fig. 1 as an example, the electrode and the blisk are introduced into the software, and the drivings of the X, Y, Z, A, and C axes are loaded onto the electrode. For the motion reference origin, the motion trajectory is imported, and the motion simulation is performed. As shown in Fig. 6, the electrode exit from flow channel from picture (a) to picture(d) with non-interfrence with the blisk, which verifies the correctness of trajectory planning.

In addition, the motion trajectories of various axes of the machine tool can be drawn inside the data spline. Compared with literature [11], as shown in Fig. 7,the feed curves along the y, z and C axis with no smoothness have many sharp points which make the curve undifferentiable and the machine pulsate to some extent. And the movement curve of the electrode centroid is also very tortuous. And the method proposed can improve the smoothness of the feed trajectory obviously. Less sharp points and trajectory pulsations improve the safety of processing to some extent.

#### V. CONCLUSION

For the large twist integral shrouded blisk EDM machining, The paper proposes trajectory optimization to make the feeding of each axis of the machine tool smooth, and to maintain a large safety margin for the movement space of the electrode. Based on this goal, this paper improves the traditional constraint construction method, and designs the

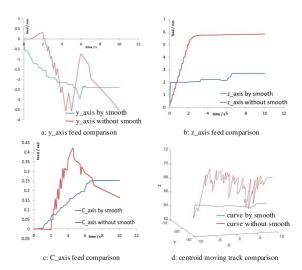


Fig. 7. smoothness comparison

corresponding objective function, and then uses the adaptive genetic algorithm to solve the problem. Finally, in software Adams, the trajectories of the electrodes were interpolated, simulated and compared with the method in literature [11]. The results prove the proposed method is feasible and can improve the smoothness of the trajectory to some extent.

#### REFERENCES

- [1] Yoon S, Curtis E, Denton J, et al. The Effect of Clearance on Shrouded and Unshrouded Turbines at Two Levels of Reaction[J]. 2014, 136(2):1231-1241.
- [2] Gao J, Zheng Q, Zhang H, et al. Comparative investigation of tip leakage flow and its effect on stage performance in shrouded and unshrouded turbines[J]. Proceedings of Institution of Mechanical Engineers Part G Journal of Aerospace Engineering, 2013, 227(8):1265-1276.
- [3] M'Saoubi R, Nobel C, Sim W M. High performance cutting of advanced aerospace alloys and composite materials[J]. CIRP Annals- Manufacturing Technology, 2015, 64(2):557-580.
- [4] Jing J, Zhao H, Liu Y, et al. Tool path planning in finish-milling process for integrally-shrouded impeller channels with rings[C]// International Conference on Electronic and Mechanical Engineering and Information Technology. IEEE, 2011:1635-1638.
- [5] Klocke F, Zeis M, Klink A, et al. Technological and economical comparison of roughing strategies via milling, sinking-EDM, wire-EDM and ECM for titanium and nickel-based blisks[J]. Cirp Journal of Manufacturing Science & Technology, 2013, 6(3):198-203.
- [6] Zhan H, Zhao W, Wang G. Manufacturing turbine blisks[J]. Aircraft Engineering & Aerospace Technology, 2000, 72(3):247-252.
- [7] Liao P, Sun G, Yang Z. The Method Research of Feed Path for Molding Electrode in Rimed Turbine Blisks Manufacturing[J]. Machine Tool & Hydraulics, 2007.
- [8] Liu X, Kang X, Zhao W, et al. Electrode Feeding Path Searching for 5-Axis EDM of Integral Shrouded Blisks[J]. Procedia Cirp, 2013, 6(8):107-111.
- [9] Liu X, Kang X, Xi X, et al. Electrode feed path planning for multi-axis EDM of integral shrouded impeller[J]. International Journal of Advanced Manufacturing Technology, 2012, 68(5-8):1697-1706.
- [10] Ayesta I, Izquierdo B, Sanchez J A, et al. Optimum electrode path generation for EDM manufacturing of aerospace components[J]. Robotics and Computer-Integrated Manufacturing, 2016, 37(C):273-281.
- [11] Kang X, Liang W, Yang Y, et al. Maximum free distance method for electrode feeding path planning in EDM machining of integral shrouded blisks[J]. Precision Engineering, 2017, 51.