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Optimal Reaction Control for the Flexible Base Redundant Manipulator System*

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***Abstract*— The null space of the Jacobian for the redundant manipulators can be utilized to reach dual control goals and optimal effects. For the flexible base redundant manipulator system, the reaction force from the motion of the manipulator should be carefully handled while the end effector is tracking a desired trajectory. Different optimal goals are discussed, including the minimization of base deflection force/torque and base energy dissipation path. Considering the base reaction dynamic, the reaction force/torque is treated as the input of the base and the deflection energy is treated as the output, an integrated optimal goal is built and solved based on the LQR control to get an optimal feedback for the closed-loop dynamics. We discuss the different optimal goals and implement trough the redefined acceleration. We test the different control laws on the three-link manipulators which mounted on a three-DoF flexible base. Simulation results demonstrate that optimal goal in LQR form can be advantageous in terms of flexible base manipulator.**

***Index Terms*— Null Space, Reaction Force, Base Energy, LQR, Output Redefinition.**

I. INTRODUCTION

The macro-micro manipulator which applied in the space station can be treated as a flexible base manipulator system (FBMS for short) [1], as shown in Fig.1, due to the flexibility of the long reach manipulator. The FBMS has complicated motion features due to the dynamic couple. The motion of the manipulator will generate reaction forces and torques on the base, and the base deflection will lead the end effector tracking accuracy declined and influence the operational performance.

The reaction null space control [1,2] which used the coupling inertial matrix to get the null space while the end-effector tracking the desired trajectory was proposed without inducing disturbances on the flexible base. Due to the orthogonal decomposition of joint space, the reaction null space operator provides extra DoFs to meet the other constraints such as zero reaction on the base or base vibration damping. Ref. [3] treated the first three joints of the JEMRMS system as a flexible base, and the rest of the structure is regarded as a nine-DOF redundant mini manipulator, integrated motion control such as reaction-less motion, vibration suppression and end path tracking was applied for the JEMRMS/SFA.

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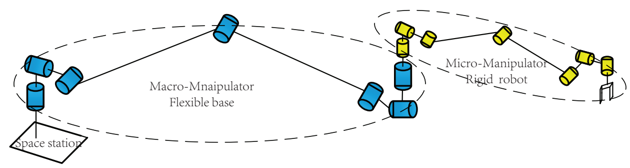

Fig. 1: Macro-Micro Manipulator is kind of FBMS

However, the inertia coupling matrix is a non-square matrix which means that DoF of the under-actuated part (flexible base) should be more than the actuated part (manipulator) and more redundancies are needed if getting better reaction-less motion. Besides, an initial stationary configuration is required, and it's not useful for controlling the vibration once it occurs.

For more common conditions, when the manipulator is redundant, which is a flexible base redundant manipulator(FBRMS for short), the null space of the Jacobian matrix can be utilized, which is so-called kinematic redundant. Ref. [4] analyzed different vibration damping algorithms through the local redundancy resolution, such as minimum base excitation, base energy dissipation path and a fixed weight multi-criteria optimizations.

Besides, many articles discussed the local and global optimal control for the redundancy manipulator, such as joint rates [5], control torque [6], however, for the flexible base manipulator, base deflection needs to be considered. The reaction force induced by the motion of the micro part is generated by input torque which represented as an external force to the flexible base [7,8]. The base deflection energy, including kinetic energy and elastic potential energy, is the scalar characterization of the base deflection. Optimal feedback control for energy dissipation in flexible structures was studied [9]. However, for the FBRMS, considering the dynamic features, the base information should be used to feedback and reduced, studies didn't pay much attention to the base acceleration for redundancy resolution at the acceleration level [10], the self-motion control should be specially designed.

We present a general reaction-force based method for FBRMS to get an optimal feedback control with the null space of the Jacobian. The base reaction dynamics are used to build a relationship with the reaction force and null space arbitrary vector. Different optimal goals are considered to deal with the base reaction, such as base reaction force minimization,

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base energy dissipation and a combination of the two factors. A comparison of different optimal goals were carried and showed the weighted multi-criteria integrated motion control has advantageous feedback effects.

This paper is organized as follows. Section II, the dynamics and kinematics of FBRMS are outlined. In Section III, we discuss three different optimal control methods. In Section IV, the Cartesian space tracking is specified in the acceleration level and implement based on the inverse dynamic control framework. In Section V, we evaluate three controllers through numerical simulations and discuss the implication/impact of the different controllers. Finally, the conclusions are given in Section VI.

II. Model of the FBRMS

A. Dynamics of the FBRMS

The dynamic model of the FBRMS with a N_m DoFs manipulator and N_b Dofs base is set up by the Lagrange equation with the generalized coordinates of the joints, shown in (1).

$$\begin{aligned} \mathbf{H}_b\ddot{\mathbf{q}}_b + \mathbf{H}_{bm}\ddot{\mathbf{q}}_m + D_b\dot{\mathbf{q}}_b + \mathbf{K}_b\mathbf{q}_b + \mathbf{C}_b &= 0 \\ \mathbf{H}_{bm}^T\ddot{\mathbf{q}}_b + \mathbf{H}_m\ddot{\mathbf{q}}_m + \mathbf{C}_m &= \tau \end{aligned} \tag{1}$$

where $\mathbf{q}_b \in \Re^{N_b}$ denotes the positional and orientation deflection of the base with respect to the inertial frame, $\mathbf{q}_m \in \Re^{N_m}$ stands for the joint coordinates of the arm, $\mathbf{H}_b(\mathbf{q}_b, \mathbf{q}_m) \in \Re^{N_b \times N_b}$, $D_b \in \Re^{N_b \times N_b}$ and $\mathbf{K}_b \in \Re^{N_b \times N_b}$ denote base inertia, damping and stiffness, respectively. $\mathbf{H}_m(q_m) \in \Re^{N_m \times N_m}$ is the inertia matrix of the arm. $\mathbf{H}_{bm}(\mathbf{q}_b, \mathbf{q}_m) \in \Re^{N_m \times N_m}$ denotes the so-called inertia coupling matrix. $\mathbf{C}_b(\mathbf{q}_b, \dot{\mathbf{q}}_b, \mathbf{q}_m, \dot{\mathbf{q}}_m) \in \Re^{N_b}$ and $\mathbf{C}_m(\mathbf{q}_b, \dot{\mathbf{q}}_b, \mathbf{q}_m, \dot{\mathbf{q}}_m) \in \Re^{N_m}$ are velocity-dependent nonlinear terms, $D_m \in \Re^{N_m \times N_m}$ denotes arm joint damping and $\tau \in \Re^{N_m}$ is the joint torque. We do not consider external forces here, including the gravity force, having in mind a non-contact task in micro gravity environment.

B. Inverse Kinematics of the FBMS

Considering the deflection of the base, the dynamic kinematic based on the Jacobian on velocity and acceleration level is

$$\dot{\mathbf{x}} = \mathbf{J}_{bm}\dot{\mathbf{q}} = \mathbf{J}_b\dot{\mathbf{q}}_b + \mathbf{J}_m\dot{\mathbf{q}}_m \tag{2}$$

$$\ddot{\mathbf{x}} = \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}} + \mathbf{J}_{bm}\ddot{\mathbf{q}} \tag{3}$$

where $\dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$ are the velocity and acceleration of the end effector. $\mathbf{q} = [\mathbf{q}_b{}^T, \mathbf{q}_m{}^T]^T$ are the generalized variable, $\mathbf{J}_{bm}$ is the $N_t \times (N_b + N_m)$ Jacobian matrix.

$$\mathbf{J}_{bm} = [\mathbf{J}_b \vdots \mathbf{J}_m] \tag{4}$$

where N_t is the task space DoF, $\mathbf{J}_b$ is the Jacobian of the base which is the first N_b columns and the left part $\mathbf{J}_m$ is the Jacobian of the manipulator. Due to the flexible base are not directly controlled, the inverse kinematics in acceleration level is

$$\ddot{\mathbf{q}}_m = \mathbf{J}_m^+(\ddot{\mathbf{x}} - \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}} - \mathbf{J}_b\ddot{\mathbf{q}}_b) + \mathbf{S}\xi \tag{5}$$

where $\mathbf{J}_m^+$ is the Moore-Penrose pseudo-inverse of the Jacobian, $\mathbf{S} = (\mathbf{I} - \mathbf{J}_m^+\mathbf{J}_m)$ is idempotent matrix which has the same rank with the null space of $\mathbf{J}_m$, ξ is arbitrary vector, $\mathbf{S}$ projects ξ onto the null space of the Jacobian $\mathbf{J}_m$, ξ can be interpreted as a desired acceleration behavior that is only effective in the null space and does not interface with the task achievement.

III. Optimal Control Law Development

From the dynamic equation, it can be seen that the motion of the base and manipulators is coupled, so we concentrate primarily on the reactions of the base, the base dynamics can be separated as

$$\mathbf{H}_b\ddot{\mathbf{q}}_b + D_b\dot{\mathbf{q}}_b + K_b\mathbf{q}_b = -\mathbf{F}_{ext} \tag{6}$$

where $\mathbf{F}_{ext} = (\mathbf{f}^T, \mathbf{t}^T)^T$ is the disturbance reaction force/torque induced from the manipulator motion. Then the base reaction force/torque $\mathbf{F}_{ext}$ can be expressed as

$$\mathbf{F}_{ext} = \mathbf{H}_{bm}\ddot{\mathbf{q}}_m + \mathbf{C}_b \tag{7}$$

Then the base deflection dynamic is

$$\begin{aligned} \mathbf{H}_b\ddot{\mathbf{q}}_b + D_b\dot{\mathbf{q}}_b + K_b\mathbf{q}_b &= -\mathbf{F}_{ext} \\ \mathbf{F}_{ext} &= \mathbf{H}_{bm}\ddot{\mathbf{q}}_m + \mathbf{C}_b \end{aligned} \tag{8}$$

The base reaction has a different levels of expression, the first level is the magnitude of the base deflection, the second level is the base reaction force and the third level is the base deflection energy, however, the system state is hard to direct control. The reaction force is the characterization of the instant coupling source, which has a direct relationship with the joint acceleration. The base deflection energy is a scaler including the kinetic energy and potential energy. When the velocity changed, the energy may not change. However, when the energy changed, the velocity must have made a change. We made different optimal goals to deal with the base reaction.

A. Minimization of Base Deflection Force/Torque

For the minimization of base deflection force/torque (MBDF for short), the optimal problem can be described as

$$\text{minimize } f(\ddot{\mathbf{q}}_m) = \mathbf{F}_{ext}^T\mathbf{F} = \|\mathbf{F}_{ext}\|^2 \tag{9}$$

$$\text{subject to } \mathbf{J}_m\ddot{\mathbf{q}}_m + \mathbf{J}_b\ddot{\mathbf{q}}_b + \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}} - \ddot{\mathbf{x}} = 0 \tag{10}$$

It can be turned into a non-constrained optimization problem based on the Lagrange multiplier method.

$$\begin{aligned} f^* &= \mathbf{F}_{ext}^T\mathbf{F} + \lambda^{\mathrm{T}}(\mathbf{J}_m\ddot{\mathbf{q}}_m + \mathbf{J}_b\ddot{\mathbf{q}}_b + \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}} - \ddot{\mathbf{x}}) \\ &= (\mathbf{H}_{bm}\ddot{\mathbf{q}}_m + \mathbf{C}_b)^T(\mathbf{H}_{bm}\ddot{\mathbf{q}}_m + \mathbf{C}_b) \\ &\quad + \lambda^{\mathrm{T}}(\mathbf{J}_m\ddot{\mathbf{q}}_m + \mathbf{J}_b\ddot{\mathbf{q}}_b + \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}} - \ddot{\mathbf{x}}) \end{aligned} \tag{11}$$

Solving the extremum of (11)

$$\frac{\partial f^*}{\partial \ddot{\mathbf{q}}_m} = 2\mathbf{H}_{bm}^T\mathbf{H}_{bm}\ddot{\mathbf{q}}_m + 2\mathbf{H}_{bm}^T\mathbf{C}_b + \lambda^{\mathrm{T}}\mathbf{J}_m = \mathbf{0} \tag{12}$$

$$\frac{\partial f^*}{\partial \lambda} = \mathbf{J}_m\ddot{\mathbf{q}}_m + \mathbf{J}_b\ddot{\mathbf{q}}_b + \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}} - \ddot{\mathbf{x}} = \mathbf{0} \tag{13}$$

Due to $\mathbf{H}_{bm}^T\mathbf{H}_{bm}$ is positive, equation (12) and (13) are sufficient and necessary condition for the existence of extremum of (11). Solving (12)

$$\ddot{\mathbf{q}}_m = -0.5(\mathbf{H}_{bm}^T\mathbf{H}_{bm})^{-1}(2\mathbf{H}_{bm}^T\mathbf{C}_b + \lambda^{\mathrm{T}}\mathbf{J}_m) \tag{14}$$

Substituting (14) into (13), then λ can be calculated as

$$\begin{aligned}\lambda = &-2(\mathbf{J}_m(\mathbf{H}_{bm}^T\mathbf{H}_{bm})^{-1}\mathbf{J}_m^T)^{-1}(\ddot{\mathbf{x}}\\ &- \mathbf{J}_b\ddot{\mathbf{q}}_b - \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}} + \mathbf{J}_m(\mathbf{H}_{bm}^T\mathbf{H}_{bm})^{-1}\mathbf{H}_{bm}^T\mathbf{C}_b)\end{aligned} \tag{15}$$

Finally substituting (15) into (14) and simplify as

$$\ddot{\mathbf{q}}_m = \mathbf{J}_w^+(\ddot{\mathbf{x}} - \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}} - \mathbf{J}_b\ddot{\mathbf{q}}_b) - (\mathbf{I} - \mathbf{J}_w^+\mathbf{J}_m)\mathbf{H}_{bm}^{-1}\mathbf{C}_b \tag{16}$$

where $\mathbf{J}_w^+$ is inertia-weighted pseudo-inverse with the weighting $(\mathbf{H}_{bm}^T\mathbf{H}_{bm})^{-1}$

$$\mathbf{J}_w^+ = (\mathbf{H}_{bm}^T\mathbf{H}_{bm})^{-1}\mathbf{J}_m^T[\mathbf{J}_m(\mathbf{H}_{bm}^T\mathbf{H}_{bm})^{-1}\mathbf{J}_m^T]^{-1} \tag{17}$$

A simple way is directly minimizing the norm of the deflect force/torque, described as:

$$\text{minimize } f(\ddot{\mathbf{q}}_m) = \mathbf{F}_{ext}^T\mathbf{F} = \|\mathbf{F}_{ext}\|^2 \tag{18}$$

$$\text{subject to } \ddot{\mathbf{q}}_m = \mathbf{J}_m^+(\ddot{\mathbf{x}} - \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}} - \mathbf{J}_b\ddot{\mathbf{q}}_b) + \mathbf{S}\xi \tag{19}$$

The solution becomes

$$\begin{aligned}L_{\mathrm{MBDF}} &= \|\mathbf{H}_{bm}\ddot{\mathbf{q}}_m + \mathbf{C}_b\|^2\\ &= \left\|\mathbf{H}_{bm}\mathbf{J}_m^+(\ddot{\mathbf{x}} - \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}}) + \mathbf{H}_{bm}\mathbf{S}\xi + \mathbf{C}_b\right\|^2\end{aligned} \tag{20}$$

Letting the norm of the deflect force/torque to be zero, the ξ can be chose as

$$\xi_{\mathrm{MBDF}} = -[\mathbf{H}_{bm}\mathbf{S}]^+[\mathbf{H}_{bm}\mathbf{J}_m^+(\ddot{\mathbf{x}} - \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}}) + \mathbf{C}_b] \tag{21}$$

substituting (21) into (5)

$$\begin{aligned}\ddot{\mathbf{q}}_m = &\mathbf{J}_m^+(\ddot{\mathbf{x}} - \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}} - \mathbf{J}_b\ddot{\mathbf{q}}_b)\\ &-\mathbf{S}[\mathbf{H}_{bm}\mathbf{S}]^+[\mathbf{H}_{bm}\mathbf{J}_m^+(\ddot{\mathbf{x}} - \dot{\mathbf{J}}_{bm}\dot{q} - \mathbf{J}_b\ddot{\mathbf{q}}_b) + \mathbf{C}_b]\end{aligned} \tag{22}$$

According to the properties of idempotent matrices

$$\mathbf{S}[\mathbf{H}_{bm}\mathbf{S}]^+ = [\mathbf{H}_{bm}\mathbf{S}]^+ \tag{23}$$

Then the reference joint acceleration which meet the tracking requirement and reduce the base reaction force is

$$\begin{aligned}\ddot{\mathbf{q}}_m = &\mathbf{J}_m^+(\ddot{\mathbf{x}} - \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}} - \mathbf{J}_b\ddot{\mathbf{q}}_b)\\ &-[\mathbf{H}_{bm}\mathbf{S}]^+[\mathbf{H}_{bm}\mathbf{J}_m^+(\ddot{\mathbf{x}} - \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}} - \mathbf{J}_b\ddot{\mathbf{q}}_b) + \mathbf{C}_b]\end{aligned} \tag{24}$$

The solutions presented in two ways of (16) and (24) have different forms. However, the results should be equivalent since the performance is the same in both cases. A similar mathematical proof of the equivalent between the weighted Jacobian pseudo-inverse and the constrained least-squares solution was proven in Ref. [11], where the object of the study is the space manipulator with a local minimization of the reaction torque.

B. Base Energy Dissipate Path

If we consider the base vibration from the view of the energy, the base energy should be dissipated through the motion of the manipulator. Another choice is the BEDP (Base Energy Dissipation Paths) [3], the energy of the base consist of kinetic and potential energy, which expressed as:

$$E_b = \frac{1}{2}\dot{\mathbf{q}}_b^T\mathbf{H}_b\dot{\mathbf{q}}_b + \frac{1}{2}\mathbf{q}_b^T K_b\mathbf{q}_b \tag{25}$$

The goal is to reduce the base energy while the manipulator is moving, the derivative of the base energy must be negative definite.

The differential form of (25) is

$$\begin{aligned}\dot{E}_b &= \frac{1}{2}\dot{\mathbf{q}}_b^T\dot{\mathbf{H}}_b\dot{\mathbf{q}}_b + \dot{\mathbf{q}}_b^T\mathbf{H}_b\ddot{\mathbf{q}}_b + \dot{\mathbf{q}}_b^T K_b\mathbf{q}_b\\ &= \frac{1}{2}\dot{\mathbf{q}}_b^T\dot{\mathbf{H}}_b\dot{\mathbf{q}}_b + \dot{\mathbf{q}}_b^T(\mathbf{H}_b\ddot{\mathbf{q}}_b + K_b\mathbf{q}_b)\end{aligned} \tag{26}$$

Submitting (1) into (26),

$$\dot{E}_b = -\dot{\mathbf{q}}_b^T D_b\dot{\mathbf{q}}_b + \dot{\mathbf{q}}_b^T(\frac{1}{2}\dot{\mathbf{H}}_b\dot{\mathbf{q}}_b - \mathbf{C}_b - \mathbf{H}_{bm}\ddot{\mathbf{q}}_m) \tag{27}$$

considering D_b is positive and the variation of $\mathbf{H}_b$ changed little $\dot{H}_b \approx \mathbf{0}$. Then we can set (28) to make sure $\dot{E}_b \leq 0$.

$$\mathbf{H}_{bm}\ddot{\mathbf{q}}_m + \mathbf{C}_b = -\mathbf{G}\dot{\mathbf{q}}_b \tag{28}$$

Then the free vector ξ can be defined as

$$\xi_{\mathrm{BEDP}} = [\mathbf{H}_{bm}\mathbf{S}]^+[P\dot{\mathbf{q}}_b - \mathbf{C}_b - \mathbf{H}_{bm}\mathbf{J}_m^+(\ddot{\mathbf{x}} - \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}} - \mathbf{J}_b\ddot{\mathbf{q}}_b)] \tag{29}$$

C. LQR Based Base Energy and Reaction Force Minimization

From (25), it can be seen that the kinetic energy and potential energy of the base is a function of the base states. If we defined the base states as

$$\mathbf{x}_b \triangleq \begin{bmatrix}\mathbf{q}_b\\ \dot{\mathbf{q}}_b\end{bmatrix} \in \Re^{2N_b} \tag{30}$$

According to (7), the base states can be express as

$$\dot{\mathbf{x}}_b = \tilde{\mathbf{A}}\mathbf{x}_b + \tilde{\mathbf{B}}u \tag{31}$$

where

$$\begin{aligned}\tilde{\mathbf{A}} &= \begin{bmatrix}0 & \mathbf{I}\\ -\mathbf{H}_b^{-1}K_b & -\mathbf{H}_b^{-1}D_b\end{bmatrix}\\ \tilde{\mathbf{B}} &= \begin{bmatrix}\mathrm{O}\\ -\mathbf{H}_b{}^{-1}\end{bmatrix}\\ \mathbf{u} &= \mathbf{F}_{\mathrm{ext}}\end{aligned} \tag{32}$$

Then we can build the optimal goal as

$$r = \frac{1}{2}\int_{t_0}^{\infty}\left[\mathbf{x}_b^{\mathrm{T}}\mathbf{Q}\mathbf{x}_\mathbf{b} + \mathbf{u}^{\mathrm{T}}W\mathbf{u}\right]dt \tag{33}$$

where

$$\mathbf{Q} \triangleq \begin{bmatrix}K_b & \\ & \mathbf{H}_b\end{bmatrix} \in \Re^{2N_b\times 2N_b} \tag{34}$$

where $\mathbf{x}_b^{\mathrm{T}}\mathbf{Q}\mathbf{x}_\mathbf{b}/2$ is the base energy and the integration form represent the total energy, $W \in \Re^{N_b\times N_b}$ is a positive definite symmetric matrix which represents the trade-off between minimization of the base reaction force and minimization of the base energy. The desired base states at the steady-state is zero and the final time when the base stop is indeterminable cause the self-motion fo the manipulator won't stop in a short time unless the residual base energy is consumed away. So an infinite time state regulator is suitable, and this optimal goal can be solved by Rikati differential equation

$$\mathbf{u}^* = -W^{-1}\tilde{\mathbf{B}}^{\mathrm{T}}(t)\mathbf{P}\mathbf{x}_b(t) \tag{35}$$

$$\mathbf{P}\tilde{\mathbf{A}}(t) + \tilde{\mathbf{A}}^{\mathrm{T}}(t)\mathbf{P} - \mathbf{P}\tilde{\mathbf{B}}(t)W^{-1}\tilde{\mathbf{B}}^{\mathrm{T}}(t)\mathbf{P} + \mathbf{Q}(t) = 0 \tag{36}$$

where $\mathbf{P}(t) \in \Re^{2N_b \times 2N_b}$ is non-positive definite symmetric matrix.

Submitting(35) into(7),

$$\mathbf{H}_{bm}\ddot{\mathbf{q}}_m + \mathbf{C}_b = -W^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}\mathbf{x}_b \tag{37}$$

the free-vector can be solved as

$$\begin{aligned}\xi_{\mathrm{LQR}} = [\mathbf{H}_{bm}\mathbf{S}]^{+}\Big[&-W^{-1}\tilde{\mathbf{B}}^{\mathrm{T}}\mathbf{P}\mathbf{x_b} - \mathbf{C}_b \\ &-\mathbf{H}_{bm}\mathbf{J}_m^{+}(\dot{\mathbf{v}}_{\mathrm{E}} - \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}} - \mathbf{J}_b\ddot{\mathbf{q}}_b)\Big]\end{aligned} \tag{38}$$

IV. Tracking Control in the Task Space

The acceleration information of the base is hard to obtain, and a controller should try to reject the acceleration feedback without a well-equipped sensor. We redefine the output [12] as

$$\ddot{\mathbf{y}} \triangleq \ddot{\mathbf{q}}_{\mathbf{m}} + \mathbf{\Gamma}\ddot{\mathbf{q}}_{\mathbf{b}} \tag{39}$$

where $\Gamma = \mathbf{J}_m^{+}\mathbf{J}_b$, then the of (5) can be modified as

$$\ddot{\mathbf{y}} = \mathbf{J}_m^{+}(\ddot{\mathbf{x}} - \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}}) + \mathbf{S}\xi \tag{40}$$

A resolved acceleration control is used when the end effector tracking a desired task space trajectory $\mathbf{x}_d, \dot{\mathbf{x}}_d, \ddot{\mathbf{x}}_d$.

$$\ddot{\mathbf{x}}_r = \ddot{\mathbf{x}}_d + \mathbf{K}_d(\dot{\mathbf{x}}_d - \dot{\mathbf{x}}) + \mathbf{K}_p(\mathbf{x}_d - \mathbf{x}) \tag{41}$$

where $\ddot{\mathbf{x}}_r$ is the reference task space acceleration command, $\mathbf{K}_p$ and $\mathbf{K}_d$ are PD gain matrices. The control law is given by acceleration based inverse dynamic control.

$$\begin{aligned}\tau &= \tilde{\mathbf{H}}\ddot{\mathbf{y}} + \tilde{\mathbf{D}}\dot{\mathbf{q}}_b + \tilde{\mathbf{K}}\mathbf{q}_b + \tilde{\mathbf{C}} - D_{mc}\dot{\mathbf{q}}_m \\ &= \tilde{\mathbf{H}}(\mathbf{J}_m^{+}(\ddot{\mathbf{x}}_r - \dot{\mathbf{J}}_{bm}\dot{\mathbf{q}}) + \mathbf{S}\xi) + \tilde{\mathbf{K}}\mathbf{q}_b \\ &+ \tilde{\mathbf{D}}\dot{\mathbf{q}}_b + \tilde{\mathbf{C}} - D_{mc}\dot{\mathbf{q}}_m\end{aligned} \tag{42}$$

where

$$\begin{aligned}\tilde{\mathbf{H}} &= \mathbf{H}_m - (\mathbf{H}_{bm}^T - \mathbf{H}_m\Gamma)(\mathbf{H}_b - \mathbf{H}_{bm}\Gamma)^{-1}\mathbf{H}_{bm} \\ \tilde{\mathbf{D}} &= (\mathbf{H}_{bm}^T - \mathbf{H}_m\Gamma)(\mathbf{H}_b - \mathbf{H}_{bm}\Gamma)^{-1}D_b \\ \tilde{\mathbf{K}} &= (\mathbf{H}_{bm}^T - \mathbf{H}_m\Gamma)(\mathbf{H}_b - \mathbf{H}_{bm}\Gamma)^{-1}K_b \\ \tilde{\mathbf{C}} &= \mathbf{C}_m - (\mathbf{H}_{bm}^T - \mathbf{H}_m\Gamma)(\mathbf{H}_b - \mathbf{H}_{bm}\Gamma)^{-1}\mathbf{C}_b\end{aligned} \tag{43}$$

The damping term D_{mc} in joint space are used for reducing the self-motion of the joints when suppressing the residual oscillation from the base.

This is the acceleration-based control with the redefinition output $\mathbf{y}$ and $\mathbf{q_b}$ which yields the task space tracking error dynamics

$$\ddot{\mathbf{e}} + \mathbf{K_d}\dot{\mathbf{e}} + \mathbf{K_p}\mathbf{e} = 0 \tag{44}$$

which can be shown that this controller achieves asymptotic tracking in operational space, ie. $\mathbf{e} \to \mathbf{0}$ as $t \to \infty$. It should be noticed that the reversible of $\mathbf{H}_{bm}\mathbf{S}$ is important, kinematic singularity and dynamic singularity [13] will lead instability since it's sensitive to ill conditioning, care should be taken to avoid neighbourhoods of such locations.

The control law results in the following closed loop dynamics:

$$\mathbf{H}_b\ddot{\mathbf{q}}_b + (\mathbf{D}_b + \mathbf{G}_b)\dot{\mathbf{q}}_b + \mathbf{K}_b\mathbf{q}_b = 0 \tag{45}$$

However, we can see that (21), (29) and (38) have different forms. Contrasting with the MBDF, BEDP and LQR, the MBDF doesn't add any damping into the base cause $\mathbf{F}_{ext} = 0$, which leads $\mathbf{G}_b = 0$. The BEDP leads $\mathbf{F}_{ext} = \mathbf{G}\dot{\mathbf{q}}_b$ and adds a fixed damping into the base. The LQR method leads the time variable Rikati feedback gains $W^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}\mathbf{x}_b$ which are relative to the base states and base states change rate. It can be seen that the deflection of the base are reflected in kinetic energy and potential energy, which can get an optimal feedback with the motion of the manipulators. The choice of W is important to the base energy optimal, the less of weighting parameters W, the more damping is added into the base. But W should not be very small cause the feedback gains will become larger and will be sensitive to the errors.

V. Evaluations and Discussions

The 3-link flexible base manipulator was built as Fig.2. The links are mounted at the remote end of the base; the springs, including X, Y and rotary in Z directions, are connected with the ground and the root of the flexible base. Without considering the orientation of the end effector, the motion of the 3-link along a given trajectory is redundant under the planar motion in the x-y plane and no gravity force is needed to calculate under micro-gravity environment.

The parameters of the model are shown in Table I, the base stiffness is $[1748\mathrm{N/m}, 751\mathrm{N/m}, 39321\mathrm{N/rad}]$. We removed the damping term D_b to be more intuitive to verify the vibration control from the input damping. The initial configuration is $[19.71, 45, -60.08]$ deg. The starting point of the end is $[5.553, -0.305]$, the planned trajectory shown in Fig.3 is one cycle of "figure-eight (8)", the planned time is 40s

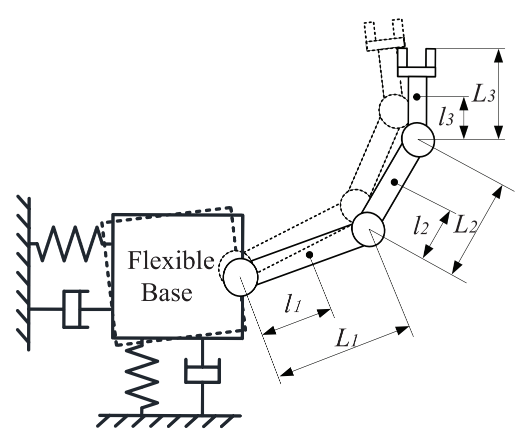

Fig. 2: 3-link Manipulator Mounted on the 3-DoF Flexible Base

TABLE I: Parameters of the 3-Link FBRMS

	Base	Link1	Link2	Link3
Mass(kg)	490.35	55.67	91.93	456.53
Internalkgm^2	87.56	49.52	74.91	80.11
Length L(m)	1.006	2.080	2.080	0.862
CoM l (m)	0.700	1.040	0.780	0.475

TABLE II: Root Mean Squared Tracking Error Achieved by the Different Control Laws

	MBDF	BEDP	LQR
RMS error (mm)	0.3455	0.3622	0.3852

and total simulations time is 100s. The tracking gain matrices were set as $\mathbf{K}_p = 200\mathbf{I}$, $\mathbf{K}_d = 14.14\mathbf{I}$, the damping term $D_{mc} = [195, 130, 104]\text{Nsm}^{-1}$, $\mathbf{G} = [1500, 1000, 500]\text{Nsm}^{-1}$, the weighting term $W = 10^{-3}$.

We evaluated different control methods mentioned in Section III and recorded the base deflection and base energy when the end effector is tracking the given trajectory. Fig.4 shows the L_2 norm of the base deflection in the translation direction and Fig.5 shows the deflection in the rotation direction. Fig.6 shows the instantaneous base energy and Fig.7 shows the total energy in 100s. The dotted blue line is the MBDF control, the dashed red line is the BEDP control and the solid black line is the LQR control. The simulations showed similar results in the first 15s due to the smooth path planning and small joint acceleration. However, a distinct difference appeared at the rest motion period when the joint velocity reaches the peak, and the control algorithm worked well to deal with the base vibration.

The root mean squared (RMS) tracking error for each of the controllers are shown in Table II. All the controllers have similar tracking effects, this probably due to full states feedback and accurate model parameters. But as expected, the LQR based base energy and reaction force/torque minimization have the most obvious advantage contrast with other controllers when dealing with the base vibration suppression. The LQR shows the global optimization effect no matter tracking(first 40s in yellow) and self-motion period(last 60s in green).

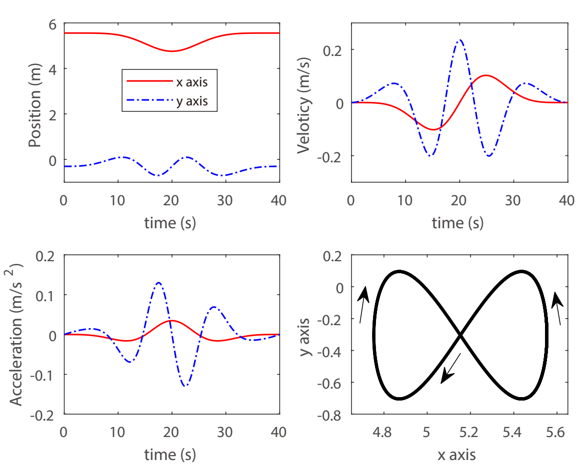

Fig. 3: Planned Trajectory in X-Y Plane

The D_{mc} is important and make a difference during the self-motion of the manipulator. If no damp is imposed into the base and manipulator, the system may be more easily to move to the singularity conditioning without the mechanical damping. However, the MBDF appears to have a slight advantage here for tracking accuracy. The reason is most likely due to the increase of damping will increase feedback gains and amplify the tracking errors.

It's clear from the contrast of different simulation results that considerable improvement is achieved in performance by using the LQR based base deflection energy and reaction force minimization. A time variable damping term is imposed into the flexible base, and make a relative better effect from the view of base vibration suppression. The redefined output changed the inverse kinematics of the system which can be replaced without the base acceleration information, that make it possible to be applied in the real system with only measuring joint angles, velocities and base deflections. However, the rate of change of the flexible base is still needed, it can be measured by using the observer [14].

Fig.8 shows the stroboscopic plots of the configurations for the LQR method. The blue line is the 3-link; the solid red

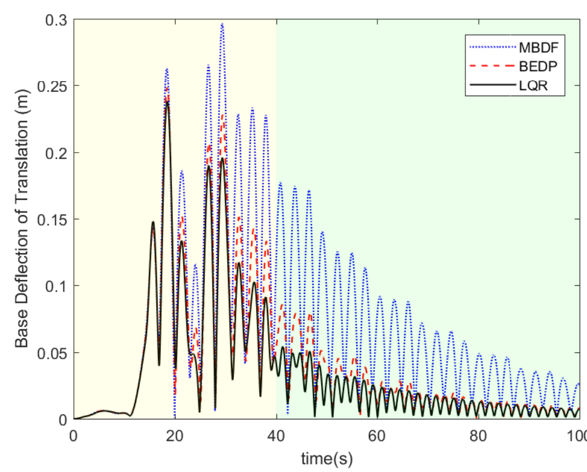

Fig. 4: The L_2 Norm Base Deflection in the Translation Direction

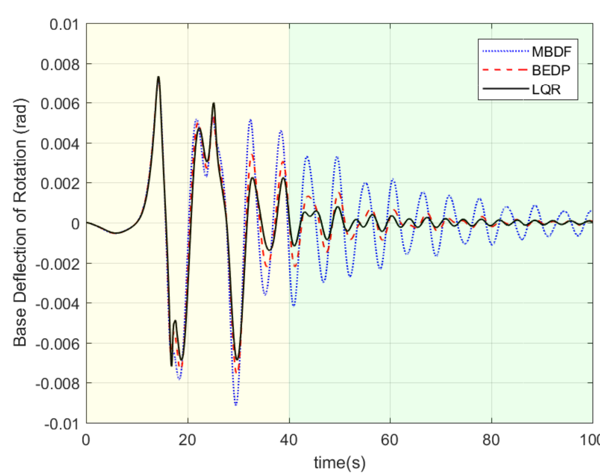

Fig. 5: The Base Deflection in the Rotation Direction

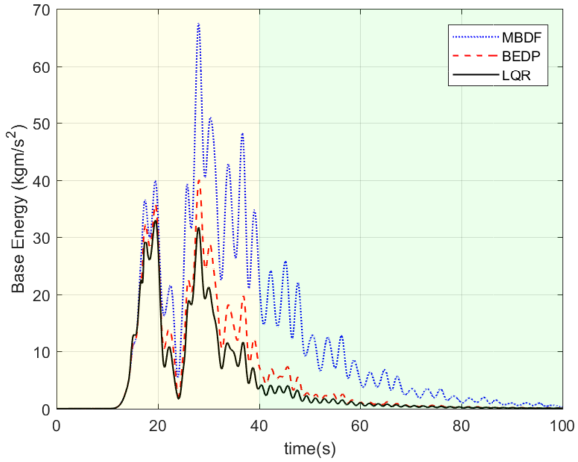

Fig. 6: The Instantaneous Base Energy Changes over Time

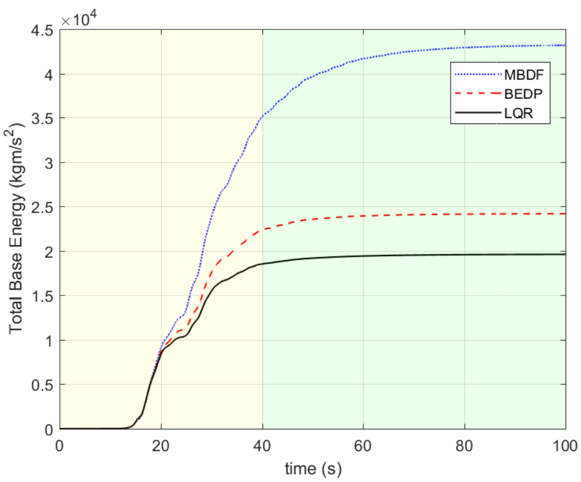

Fig. 7: The Total Base Energy Changes over Time

line is the real trajectory; the dash red circles shows the root of the 3-link. It can be seen that even the base has apparent deflection, the end of the manipulator can still get an accurate tracking task.

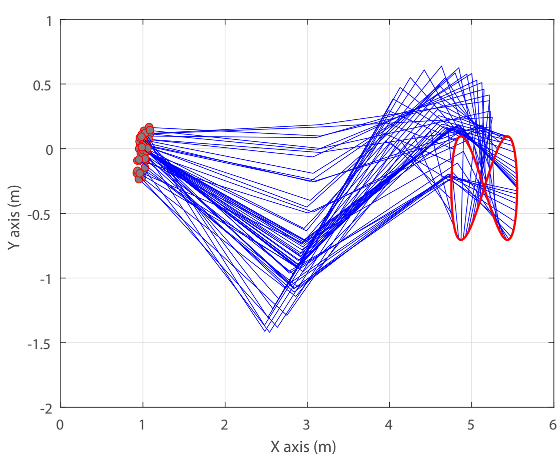

Fig. 8: Stroboscopic Plots of the Configurations for the LQR Method

VI. Conclusion

In this paper, we have presented a novel framework of closed-form optimal solution for the flexible base redundant manipulator system which can suppress the base vibration while performing task space tracking. The control law is designed based on the reaction force/torque, and we have shown how we build the relationship with the reaction force/torque and joint null space arbitrary vector. We have discussed different optimal goals and made a contrast analysis for the feedback gains. An integrated multi-criteria optimization is proposed, which combines the base reaction force/torque and base energy had been shown an effective enhancement of vibration suppression. Under this framework, other optimal methods can also be applied such Pontryagin minimum principle. Besides, the output redefinition of the acceleration level inverse kinematic avoided the feedback of the base acceleration, the trade-off of tracking accuracy and stable motion control under the base deflection needed to be paid attention with the redefined output and we will discuss in the future.

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