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Smooth Trajectory Tracking Using Longitudinal Distance Constraint for A 4WS4WD Unmanned Ground Vehicle*

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***Abstract*— The model-based trajectory tracking method of four wheel steering and four wheel drive (4WS4WD) unmanned ground vehicles (UGVs) needs to establish accurate vehicle kinematics and dynamics models, which is computationally complex thus functioning poorly in real-world scenarios. The geometry-based method, i.e., the pure pursuit method, uses the position information to regulate vehicle's steering angle, which has good real-time performance in practical applications. However, the existing pure pursuit method only considers the look-ahead distance constraint, ignoring the influence of the longitudinal distance which affects the smoothness of the vehicle trajectory. It may cause the vehicle to oscillate back and forth around the target trajectory. This work proposes an improved pure pursuit method with longitudinal distance constraint (LDC) to avoid the vehicle oscillation effectively. Additionally, to alleviate the cornering cutting problem, a fuzzy-logic based controller is proposed to dynamically adjust the look-ahead distance according to the curvature. A comparative experiment between the proposed method and the conventional pure pursuit method was conducted. Results show that the proposed method has better trajectory smoothness and tracking accuracy than the traditional pure pursuit method.**

***Index Terms*— Trajectory tracking; Pure pursuit; Unmanned ground vehicle.**

I. Introduction

The four wheel steering and four wheel drive unmanned ground vehicles (4WS4WD-UGVs) have excellent mobility, showing a good application prospect in future industrial applications, such as logistics handling and safety inspection. For the purpose of achieving autonomous operation, the navigation and control module needs to control the vehicle body to accurately track the predetermined trajectory. The existing trajectory tracking methods are roughly divided into three categories: model-based, feedback-error-based, and geometry-based methods. The model-based method usually requires to establish the accurate vehicle kinematics and dynamics model, considering the influence of the tire's lateral moment and slip rate on the trajectory tracking. Reference [1] calculates the wheel drive torque and steering angle based on model predictive control method. Reference [2] adopts a sliding mode controller to regulate the wheel steering angle, and a real-time particle swarm optimization controller to calculate the wheel drive torque. The feedback-error-based method does not need to establish a system model which may regard the controlled system as a black box [3].

The geometry-based controller calculates the steering angle based on the geometric relationship between the vehicle's pose and the target trajectory. The typical geometry-based trajectory tracking methods include pure pursuit [4], vector pursuit [5], CF-pursuit [6], and Stanley method [7]. The main problem of the geometry-based method is the cornering cutting problem [8], which means that the vehicle's actual trajectory can not fit the target one well at corners, resulting in a large error. Furthermore, if the initial lateral error is large, the vehicle body will oscillate back and forth around the target trajectory. To solve such problems, reference [8] estimates the look-ahead distance based on the vehicle speed and the lateral error. Reference [9] defines the look-ahead distance as the function of the vehicle speed and dynamically adjusts the look-ahead distance according to the speed. Reference [10] defines the look-ahead distance as the quadratic polynomial of the vehicle velocity and the shortest distance between the target trajectory and the vehicle position.

The existing research on pure pursuit method mainly focuses on how to dynamically adjust the look-ahead distance, ignoring the influence of the longitudinal distance on the tracking smoothness. This work introduces a longitudinal distance constraint to the pure pursuit method, where the look-ahead point needs to satisfy both the look-ahead distance constraint and the longitudinal distance constraint. To overcome the cornering cutting problem, this work builds a fuzzy controller to dynamically adjust the look-ahead distance according to the curvature of the target trajectory. The principle of the proposed method is that the look-

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ahead distance constraint guarantees the trajectory tracking accuracy, and the longitudinal distance constraint ensures the smoothness of the vehicle motion during the dynamic trajectory tracking process.

II. The 4WS4WD Unmanned Ground Vehicle

The experimental platform is shown in Fig. 1. The UGV prototype in this study consists of four independently steering wheels, each of which can be driven by in-wheel motors independently. The autonomous navigation system uses a single line light detection and ranging (LIDAR) for real-time localization and mapping considering the cost and real-time performance. The real-time point cloud data of LIDAR is matched with the local sub-map to calculate the vehicle pose. The sub-map consists of a certain number of point cloud frames, and the constructed sub-map participates in the loop close detection. According to the local pose constraint and global pose constraint between the scan and the sub-map, the global pose optimization is performed to reduce the cumulative error [11]. The navigation system architecture is shown in Fig. 2. The upper industrial computer deals with the point cloud data and obtains the current vehicle pose. The 4WS4WD vehicle trajectory tracking algorithm calculates the fitted trajectory radius based on the geometric relationship. The 4WS4WD vehicle kinematic model performs motion decomposition and calculates the steering angle of each wheel. The vehicle controller unit (VCU) controls the drive motor and the steering motor to generate the running speed and steering angle.

Fig. 1. 4WS4WD unmanned ground vehicle prototype.

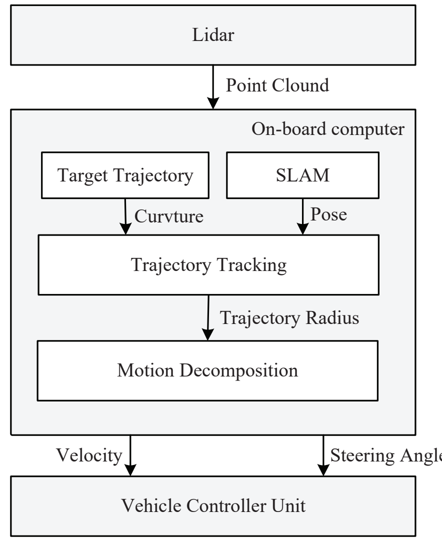


Fig. 2. UGV navigation control system architecture.

III. Trajectory Tracking Controller

A. The 4WS4WD vehicle kinematics model

In the conventional kinematics model, the steering center of the vehicle is on the extension line of the rear axle. However, the steering center of the four-wheel independent steering vehicle is not fixed. To build the kinematics model of the 4WS4WD vehicle, the instantaneous steering center of vehicle is on the center line of the front and rear axles. The vehicle body center of mass is taken as the vehicle reference point. The vehicle kinematics model is shown in Fig. 3.

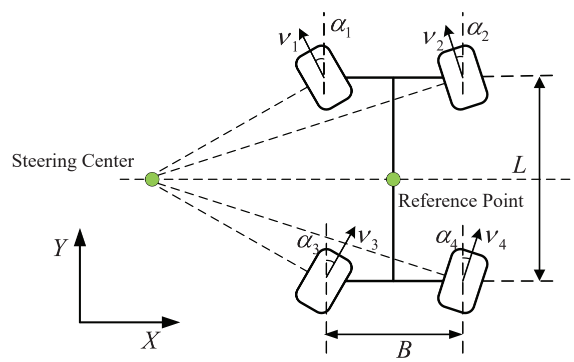


Fig. 3. 4WS4WD UGV kinematics model.

According to the kinematics model of the 4WS4WD vehicle, the wheel running speed and steering angle are obtained as follows.

$$|\alpha_1| = |\alpha_3| = \arctan\left(\frac{L}{2R - B}\right) \tag{1}$$

$$|\alpha_2| = |\alpha_4| = \arctan\left(\frac{L}{2R + B}\right) \tag{2}$$

$$V_1 = V_2 = \omega\sqrt{\left(R - \frac{B}{2}\right)^2 + \frac{L^2}{4}} \tag{3}$$

$$V_3 = V_4 = \omega\sqrt{\left(R+\frac{B}{2}\right)^2+\frac{L^2}{4}} \tag{4}$$

$$R = \frac{V}{\omega} \tag{5}$$

As in (1) - (5), V is the speed of the vehicle reference point, V_i indicates the linear speed of the i-th wheel and α_i represents the steering angle of the i-th wheel. ω is the vehicle's yaw rate, L is the distance between the shafts, B is the wheelbase length, and R is the steering radius at the vehicle reference point. The front and rear wheel steering angles of UGV are equal in magnitude and opposite in direction.

B. Pure pursuit trajectory tracking control

The pure pursuit method plans a circular path through the vehicle reference point and the look-ahead point. According to the curvature of the arc, the steering angle can be calculated. As shown in Fig. 4, the longitudinal distance is defined as the distance from the look-ahead point to the nearest way point, and the look-ahead distance is defined as the distance from the look-ahead point to the vehicle reference point. The lateral error is defined as the distance from the vehicle reference point to the nearest way point.

As shown in Fig. 5, after selecting the look-ahead point in the discrete target trajectory point set, the trajectory radius R is calculated according to the projection length of the look-ahead point on the Y-axis.

$$R = \frac{L_{\mathrm{a}}^2}{2y} \tag{6}$$

In (6), L_a is the look-ahead distance and y is the projection length of the look-ahead point on the Y-axis. The wheel's steering angle and speed can be calculated as follows.

$$|\alpha_1| = |\alpha_3| = \arctan\left(\frac{L\cdot y}{L_a^2 - B\cdot y}\right) \tag{7}$$

$$|\alpha_2| = |\alpha_4| = \arctan\left(\frac{L\cdot y}{L_{\mathrm{a}}^2 + B\cdot y}\right) \tag{8}$$

$$V_1 = V_3 = \frac{2V\cdot y}{L_a^2}\sqrt{\left(\frac{L_a^2}{2y}-\frac{B}{2}\right)^2+\frac{L^2}{4}} \tag{9}$$

$$V_2 = V_4 = \frac{2V\cdot y}{L_a^2}\sqrt{\left(\frac{L_a^2}{2y}+\frac{B}{2}\right)^2+\frac{L^2}{4}} \tag{10}$$

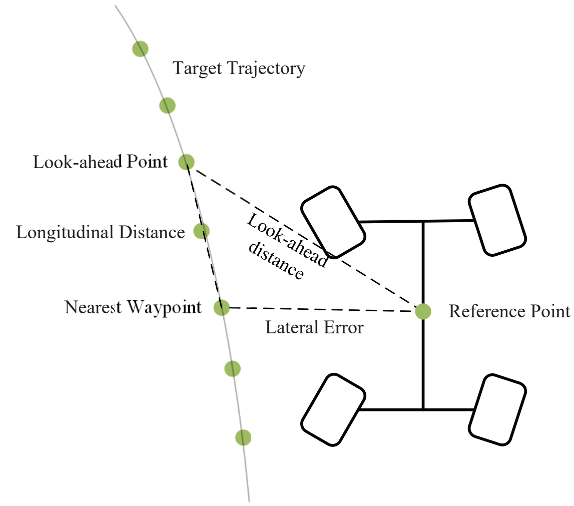


Fig. 4. Pure pursuit method related variable definition.

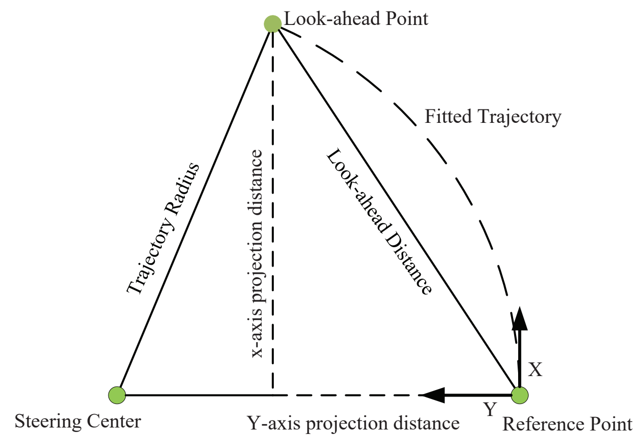


Fig. 5. Schematic diagram of pure pursuit method.

C. Pure pursuit with longitudinal distance constraint

The trajectory tracking performance of the pure pursuit method mainly depends on the selection of the look-ahead point. However, the existing pure pursuit method selects the look-ahead point only considering the tracking accuracy, while ignoring the smoothness of the vehicle motion. When the lateral error is large, the traditional pure pursuit method will result in a large yaw rate. The vehicle oscillates back and forth around the target trajectory, resulting in an increased trajectory tracking error (see Fig. 6).

To make the vehicle body converge smoothly onto the target trajectory, this paper defines the longitudinal distance constraint (LDC). As shown in Fig. 4, the longitudinal distance is defined as the linear distance between the nearest way point and the look-ahead point. In the process of lateral error convergence, the reason for the vehicle oscillation is that the longitudinal distance is too small.

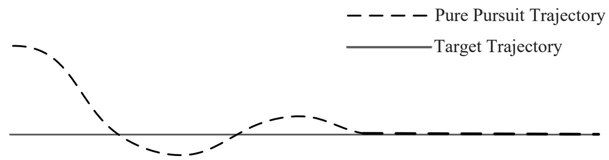


Fig. 6. The vehicle oscillation.

The idea of the proposed method is that the look-ahead distance constraint guarantees the trajectory tracking accuracy, and the longitudinal distance constraint ensures the smoothness of the vehicle body. The look-ahead distance constraint is dynamically adjusted according to the curvature information of the target trajectory. Then considering the lateral error, the longitudinal distance constraint is added to the look-ahead point. The way point needs to satisfy with both the look-ahead distance constraint and the longitudinal distance constraint. The longitudinal distance as a function of lateral error is:

$$L_r = K_{\mathrm{r}} \cdot d \tag{11}$$

where, K_r is the LDC coefficient and d is the lateral error. The LDC coefficient can be determined via the actual vehicle experiment. The adaptive pure pursuit method with longitudinal distance constraint is shown in Fig. 7, where the blue dashed box module is the main contribution of this paper.

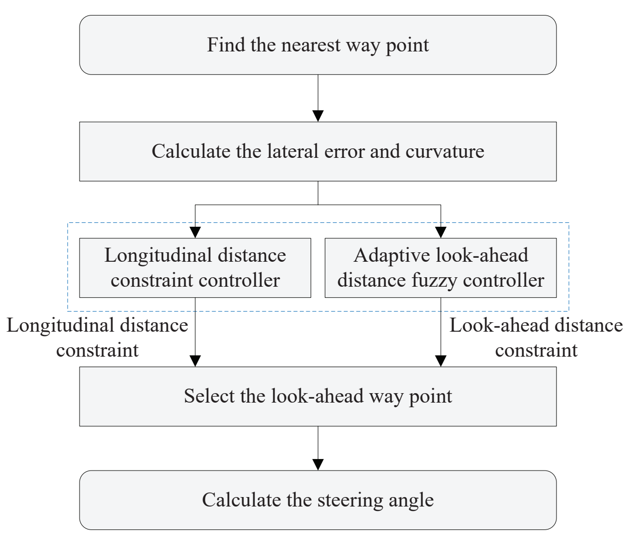


Fig. 7. The flow chart of the proposed method.

In order to ensure the trajectory tracking accuracy of the pure pursuit method, it is necessary to adjust the look-ahead distance according to the curvature information of the target trajectory. However, there is no exact functional relationship between the curvature and the look-ahead distance. Through the real vehicle test, it is only known that in the straight line section, the controller needs to increase the look-ahead distance to ensure the smoothness of the vehicle movement. In the curve trajectory segment, the look-ahead distance needs to be correspondingly reduced, ensuring the accuracy of the trajectory tracking. Fuzzy controller does not need to establish an accurate system model, only requiring to set rules for fuzzy reasoning. In this paper, a fuzzy controller is proposed. The input of the fuzzy controller is the curvature of the nearest way point, and the curvatures at 1.5m and 3m in front of the vehicle. The output of the fuzzy controller is the look-ahead distance. The range of the input curvature is [0m^{-1}, 0.4m^{-1}]. The corresponding fuzzy variable is small, middle and large. The scope of the output look-ahead distance is [0.6m, 1.5m]. The membership function of curvature is shown in Fig. 8.

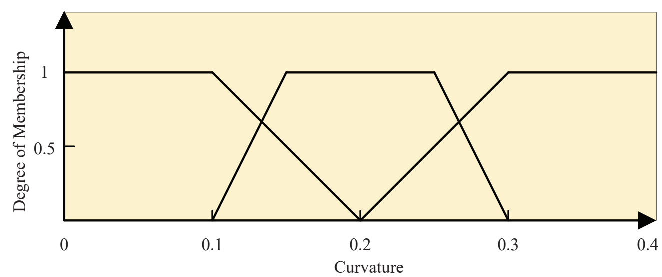


Fig. 8. Membership function of waypoint curvature.

The curvature of the nearest way point characterizes the curvature information at the trajectory where the UGV is currently located. However, it is not enough to consider only the curvature of the nearest way point. In order to alleviate the cornering cutting problem, it is also necessary to involve the curvatures at 1.5m and 3m in front of the vehicle. The inference rules for dynamically adjusting the look-ahead distance are as Table I.

IV. Experimental Test

A. Tracking Straight Trajectory

In order to verify the effectiveness of the proposed LDC based pure pursuit method, a straight line trajectory tracking experiment was performed in the experimental area shown in Fig. 9. The look-ahead distance was set to 1.5m. The initial lateral deviation was 1.5m, and the vehicle speed was 0.6m/s. The LDC coefficients of the proposed method are set to 0.6, 0.9, and 1.2, respectively.

The experimental results in Fig. 10 indicate that the traditional pure pursuit method and the method with LDC coefficient 0.6 have a certain degree of oscillation. This is because the longitudinal distance is too small. The method of LDC coefficient 0.9 and 1.2 can smoothly converge to the target trajectory. The time for the method with coefficients 0.9 and 1.2 to enter the steady state is shorter than the conventional method, which is 7.5s and 8.0s respectively. In addition, it can be seen form Table II that the maximum steering angle of the method with coefficient 0.9 and 1.2 is also smaller than the traditional pure pursuit method. It means that the method with the coefficients 0.9 and 1.2 can converge to the target trajectory more smoothly and quickly than the conventional method. The proposed LDC can effectively improve the accuracy and smoothness of the actual trajectory tracking performance.

TABLE I

FUZZY CONTROLLER INFERENCE RULE

Om curvature	1.5m curvature	3m curvature	Look-ahead distance
small	small	small	large
small	small	middle	middle
small	small	large	middle
small	middle	none	middle
large	none	none	small
middle	small	none	middle
middle	middle	none	middle
middle	large	none	small

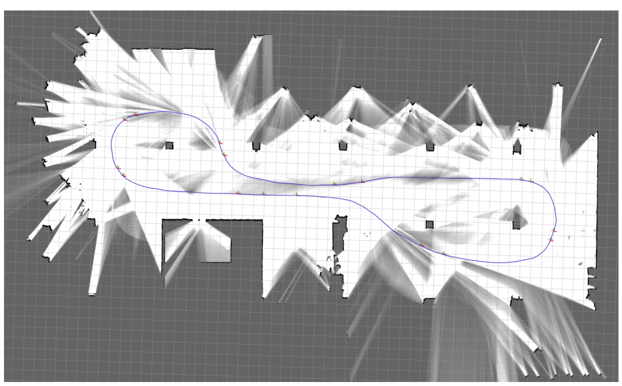
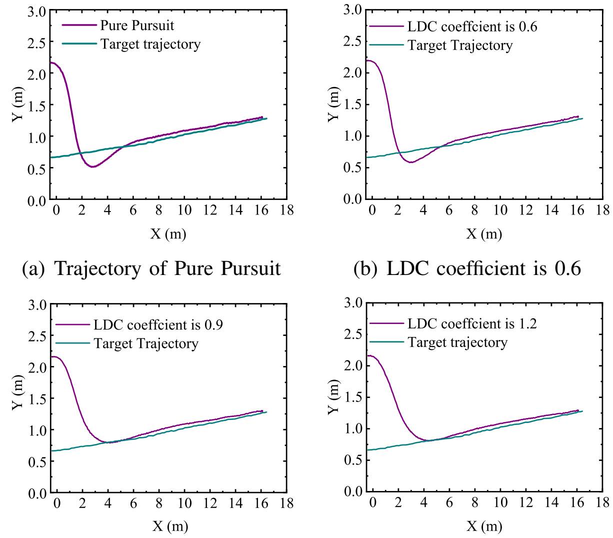


Fig. 9. The grid map of the experimental area.

TABLE II

STRAIGHT LINE EXPERIMENTAL RESULT

	Average error	Steady time	Maximum steering angle
Pure pursuit	0.234m	15.3s	56.9°
Kr is 0.6	0.242m	14.8s	42.7°
Kr is 0.9	0.209m	7.5s	30.2°
Kr is 1.2	0.215m	8.0s	16.7°



(a) Trajectory of Pure Pursuit (b) LDC coefficient is 0.6

(c) LDC coefficient is 0.9 (d) LDC coefficient is 1.2

Fig. 10. Trajectory comparison with different LDC coefficients.

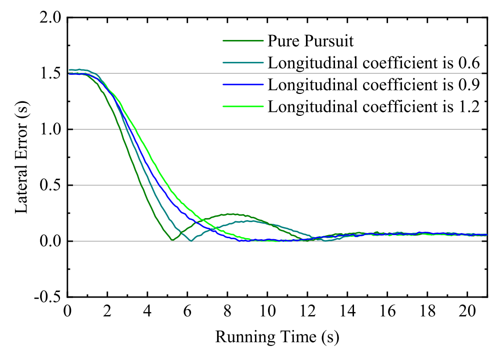
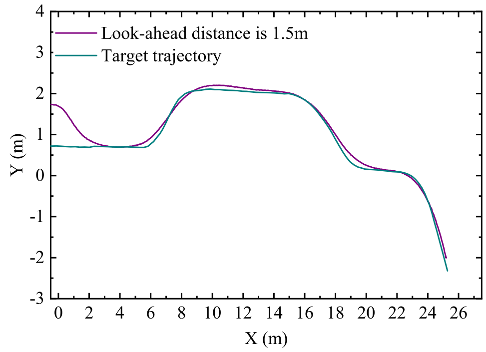


Fig. 11. Comparison of lateral errors with different longitudinal distance coefficients.

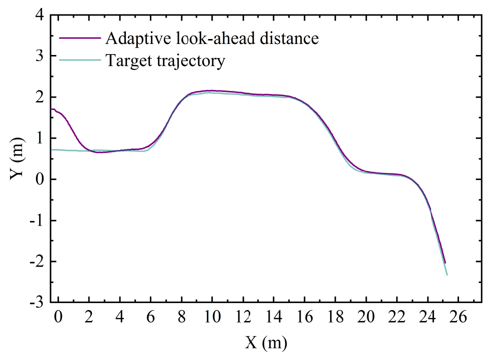
B. Tracking Curve Trajectory

To further verify the trajectory tracking performance of the proposed method, a curve trajectory tracking experiment was carried out as well. The initial lateral error was 1 m, the vehicle speed was 0.6 m/s. The target trajectory was tracked using pure pursuit method and the proposed method respectively.

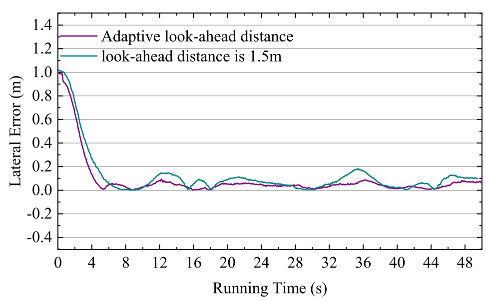
The curve tracking experiment results are shown in Fig. 12. Results show that the adaptive look-ahead distance pure pursuit method can better fit the target trajectory on the curve. The average trajectory tracking error of the traditional pure pursuit method is 0.121m, and the average error of the proposed method is 0.085m. It indicates that the method of adaptive look-ahead distance can improve the tracking accuracy of the pure pursuit method.



(a) Trajectory of pure pursuit



(b) Trajectory of the proposed method



(c) Comparison of the lateral error

Fig. 12. Experimental results of tracking curve.

V. Conclusion

This paper proposes a trajectory tracking method with a longitudinal distance constraint (LDC) for a 4WD4WS unmanned ground vehicle. For the traditional pure pursuit method, if the initial lateral error is large, the vehicle will oscillate back and forth around the target trajectory, resulting in a large tracking error. To solve the problem, this paper introduces the longitudinal distance constraint. The straight-line trajectory tracking experiments have verified that the longitudinal distance constraints can improve the smoothness of trajectory tracking. In addition, a fuzzy controller is proposed for improving the trajectory tracking accuracy, which dynamically adjusts the look-ahead distance according to the curvature. From the curve trajectory tracking experiment results, it can be seen that the adaptive look-ahead distance constraint can effectively improve the trajectory tracking accuracy at corners.

In the trajectory tracking process, the slip rate of the wheels inevitably affects the trajectory tracking accuracy. Therefore, the wheel slip model will be considered to dynamically compensate the wheel speed in future study.

Acknowledgment

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