# Effect of Compliant Passive Joint on Swimming Performance for a Multi-Joint Robotic Fish

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Abstract—In this paper, a novel compliant passive joint with two torsion springs is designed for a multi-joint robotic fish to improve propulsive performance. The motion of the multijoint robotic fish is governed by the central pattern generator (CPG) control method. Using this multi-joint robotic fish as a testbed, the effect of compliant passive joint on swimming speed with changes in spring constant and CPG parameters such as oscillation frequency and phase difference is explored. As for the multi-joint robotic fish with a compliant passive joint, the swimming speed increases directly with the increase of oscillation frequency and the swimming speed achieved at maximum oscillation frequency increases with the decrease of phase difference. In addition, the comparison of swimming performance between robotic fish with a compliant passive joint and a fixed passive joint is made. Remarkably, the maximum swimming speed after improvement increases approximately 9.43 cm/s, corresponding to 0.17 BL/s. Experimental results reveal that adding an extra compliant passive joint can not always bring an improvement in swimming performance. Appropriate stiffness of passive joint and control parameters are needed to achieve better swimming performance.

Index Terms—Multi-joint robotic fish, compliant passive joint, CPG, swimming performance.

# I. INTRODUCTION

Fish possesses superb skills in underwater locomotion with the evolution in nature, such as high maneuverability, high efficiency and low noise. With a wide range of application prospects such as aquatic environmental monitoring [1], underwater exploration, fish supervision, and so on [2], many researchers have devoted to develop robotic fish for decades. However, even a variety of robotic fish have been developed and some achievements have been made [3], [4], there are still wide disparities in locomotion performance between robotic fish and real fish in nature [5].

The biological studies indicate that real fish can modulate the stiffness of their bodies with muscle to improve swimming performance [6]. Inspired by this, many researchers have focused on the effect of stiffness on the propulsive

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performance of robotic fish. For example, Low et al. designed a robotic fish owning a compliant passive joint with a spring [7], [8]. Various design parameters including amplitude, spring constant, etc., were used to generate the maximum thrust. Experiments suggest that the usage of a passive spring joint shows the signs of drag reduction. Park et al. used a driving mechanism of a robotic fish to experimentally investigate the effect of passive mechanism of a compliant joint and flexible caudal fin on the thrust generation with changes of stiffness and frequency. The results show that the appropriate flexibility of caudal fin and stiffness of compliant joint can generate maximum thrust at different oscillating frequencies [9]. Liu et al. studied the hydrodynamic performance of a flexible caudal fin with two links through a fluidstructure interaction analysis. By changing the flexibility of two links and the stiffness of the hinge, the thrusts of the caudal fin were compared which points out that the thrust of a fish is not always getting benefits from the flexibility of a caudal fin [10]. Yeh et al. investigated the swimming performance of an actuated flexible plate with a passive attachment by computational simulation which shows that the passive attachment can effectively improve the swimming speed and efficiency [11]. Moreover, Leftwich et al. studied the wake structure of a passively flexible tail of a robotic lamprey [12]. Lauder et al. investigated the effects of the shape and stiffness of a caudal fin on swimming performance with a mechanically-actuated flapping foil model [13].

Those studies mentioned above all indicate that the passive properties with the stiffness of flexible fins and compliant joints play a significant role in propulsive performance. Accordingly, some researchers have devoted to the design of variable-stiffness mechanism for better propulsive performance [14]–[16]. However, owing to the complexity of variable stiffness mechanism, it is difficult to apply them on a robotic fish to pursue good performance. Although lots of works have been done to study the stiffness effect of the passive mechanism, most of them only investigate on a driving mechanism with flexible fins or compliant joints. Relatively little work focuses on the effect of passive properties based on a robotic fish platform, especially on a multi-joint robotic fish.

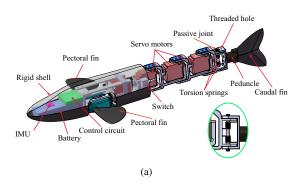
In this paper, we design a compliant passive joint with two torsion springs for a multi-joint robotic fish. This compliant passive joint serves to increase the degree-of-freedom (DOF) at lower cost in weight, size, and expense than adding an actuator. The effect of the passive mechanism on the swimming speed with changes in stiffness of passive joint and CPG parameters, such as oscillation frequency and phase difference is explored experimentally. According to the experimental results, both the improvement of swimming performance at maximum oscillation frequency and the comparison of compliant passive joint effects on the robotic fish with different number of active joints are analyzed.

The rest of this paper is organized as follows. In Section II, mechanical design and prototype of the multi-joint robotic fish with a compliant passive joint are described. Section III shows the applied CPG network for the locomotion control of the multi-joint robotic fish. The experimental design for measuring swimming performance is detailed in Section IV. Section V provides the experimental results as well as detailed analyses. Finally, conclusions and future work are presented in Section VI.

#### II. DESIGN OF COMPLIANT PASSIVE MECHANISM

The multi-joint robotic fish used in this study is a modified version which was developed in [17]. Fig. 1(a) shows the mechanical structure of the robotic fish. Specially, it is designed with a well-streamlined body shape inspired by an Esox lucius with a length of 550 mm and a weight of 1.73 kg. The robotic fish consists of two parts: a rigid head and a self-propelled body with a caudal fin. In the part of the rigid head, some electrical modules are equipped, such as a micro-controller, an inertial measurement unit (IMU), and lithium batteries. In addition, there is also a pair of pectoral fins with four DOFs, which can generate both the pitching motion and the heaving motion. Depending on the four-DOFs pectoral mechanism, the robotic fish can realize three-dimensional motion with high maneuverability. In this paper, we only discuss the effect of compliant passive joint in a two-dimensional plane motion, so the pectoral fins of the robotic fish are kept still.

In essence, the mechanism of the self-propulsive body is a multi-link hinge structure, which is composed of three active joints driven by servomotors and one passive joint with two torsion springs. The last link of the robotic fish is a rigid caudal fin which is fixed to the passive joint by a caudal peduncle. In order to ensure the symmetric swing of the caudal fin, two of the same torsion springs are implemented on the passive joint symmetrically. The last joint would rotate passively under the action of frontal link and water. Torsion springs with different spring constants can be selected to change the stiffness of the passive joint. In order to fix the passive joint in some situations, a threaded hole is designed as shown in Fig. 1(a). Thus, we can easily keep the passive



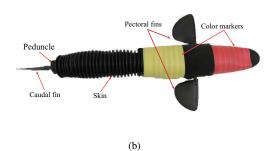


Fig. 1. Multi-joint robotic fish with a compliant passive joint. (a) Mechanical structure. (b) Prototype with color markers.

joint still through a screw.

Fig. 1(b) depicts the prototype of the multi-joint robotic fish. The overall body of the robotic fish is covered by an elastic waterproof skin made of emulsion. Red and yellow color tapes are attached to the head of the robotic fish, which can be used as markers to detect and track the position of robotic fish continuously by a motion measurement system.

# III. CPG-BASED LOCOMOTION CONTROL

In our previous work [18], the CPG model was utilized to realize the locomotion control of a multi-joint robotic fish. Inspired by vertebrates rhythmic movement with the periodic signals, the CPG model can generate rhythmic signals with phase locking under the interaction of neuron oscillators. Taking into account the strong robustness and the explicit parameters with regard to the oscillation frequency and amplitude, a Hopf oscillator-based CPG model is applied to govern the motion of multi-joint robotic fish in this paper. The topological structure of the adopted CPG network is shown in Fig. 2. The CPG model is presented as below:

$$\begin{cases} \dot{x}_{i} = -\omega_{i}(y_{i} - b_{i}) + x_{i}(A_{i} - x_{i}^{2} - (y_{i} - b_{i})^{2}) \\ + h_{1}(x_{i-1}\cos\varphi_{i} + (y_{i-1} - b_{i-1})\sin\varphi_{i}) \\ \dot{y}_{i} = \omega_{i}x_{i} + (y_{i} - b_{i})(A_{i} - x_{i}^{2} - (y_{i} - b_{i})^{2}) \\ + h_{2}(x_{i+1}\sin\varphi_{i+1} + (y_{i+1} - b_{i+1})\cos\varphi_{i+1}) \\ \theta_{i} = c_{i}y_{i} \end{cases}$$
(1)

where the subscript i denotes the ith oscillator (i = 1, 2, 3) of the CPG network.  $x_i$  and  $y_i$  stand for the state variables

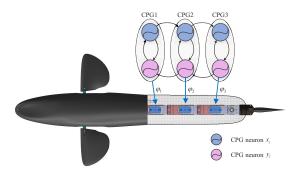


Fig. 2. Topology structure of the adopted CPG network.

 $\label{eq:TABLE} TABLE\ I$  Parameters of The Hopf Oscillator-based CPG Model

-	$A_1$	$A_2$	$A_3$	$h_1$	$h_2$	$c_i$	$b_i$
	8.70	19.08	25.50	4.0	5.0	6.0	0.0

of the ith oscillator, respectively.  $\omega_i$  and  $A_i$  indicate the intrinsic oscillation frequency and amplitude, respectively.  $b_i$  is the directional bias.  $\varphi_i$  represents the phase difference between the (i-1)th and ith oscillators.  $h_1$  and  $h_2$  denote the coupling coefficients.  $c_i$  is a constant scale coefficient.  $\theta_i$  is the output signal of the ith oscillator. The locomotion of the robotic fish can be controlled by adjusting parameters of the CPG model. For simplicity, the same intrinsic oscillation frequency  $\omega_i = \omega$  and phase difference  $\varphi_i = \varphi$  are used for all oscillators. Furthermore, the turning swimming motion is not considered and the directional bias  $b_i = 0$  is adopted. In this paper, the intrinsic oscillation frequency and phase difference are chosen as the adjustable parameters to control the output signals of the CPG model. Other parameters of the CPG model are tabulated in Table I.

#### IV. DESIGN OF EXPERIMENTS

Generally, many indicators can be used to evaluate the swimming performance of the robotic fish, such as swimming speed, propulsive efficiency, and acceleration time. Considering the swimming speed is easily and directly measured, we finally choose it as the main indicator. Thus, extensive experiments are designed to measure the swimming speed of the robotic fish with both a compliant passive joint and a fixed passive joint.

As presented in Fig. 3, the experiments were carried out in a pool filled with water. A global vision camera connected to a host computer is installed above the pool, which can record the planar motion of the robotic fish in real time. Meanwhile, a motion measurement system is particularly developed to analyze the swimming performance of the robotic fish according to the recorded videos [19], and we can easily draw the curves of the swimming speed varying with time.

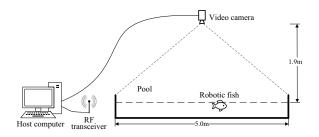


Fig. 3. Illustration of the motion measurement system.

TABLE II Adjustable Parameters of Experiments

Phase difference (°)	10	30	50	70		
Oscillation Frequency	12	16	20	24	28	
Spring constant (g·mm/deg)	50	100	150	200	250	300

Extensive experiments were carried out to investigate the effect of compliant passive joint with different spring constants and CPG parameters on swimming speed. Definitely, we focus on the effect exerted by the oscillation frequency and phase difference in the CPG model. Therefore, three groups of adjustable parameters were taken into account: spring constant  $(K_s)$ , oscillation frequency  $(\omega)$ , and phase difference  $(\varphi)$ .

The stiffness of the passive joint is govern by spring constants of two torsion springs. Considering the size of the passive joint, spring constants of various specifications of torsion springs are calculated according to outside diameter, wire diameter, body length, and material. Finally, six different specifications of torsion springs with the range of spring constant from 50 g·mm/deg to 300 g·mm/deg are adopted to change the stiffness of the passive joint. Additionally, the situation of the fixed passive joint as infinite stiffness is also taken into consideration.

According to our previous work [18], the oscillation frequency and phase difference are two important parameters of the CPG model to affect the propulsive performance. Therefore, we select five oscillation frequencies and four phase differences tabulated in Table II to explore how the CPGs' parameters affect the swimming performance.

In this paper, the effect of passive joint stiffness on swimming performance with different number of active joints is also studied. The multi-joint robotic fish with two active joints and three active joints are considered, respectively. In terms of multi-joint robotic fish with two active joints, the first active joint is fixed and the rest active joints are still controlled with the same CPG parameters. Finally, with the adjustable parameters and the situation of fixed passive joint, a total of 280 cases are conducted.

# V. RESULTS AND ANALYSIS

The experiments of the robotic fish with two active joints and three active joints are all considered. For simplicity, we

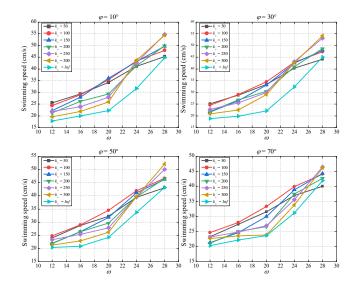


Fig. 4. Variation of swimming speed with  $\omega$  and  $K_s$  at four phase differences (Case I).

take the situation of robotic fish with two active joints as Case I, and the situation of robotic fish with three active joints is regarded as Case II.

# A. Effect with Spring Constant and CPG Parameters

For Case I, variation of swimming speed with oscillation frequency  $\omega$  and spring constant  $K_s$  are shown in Fig. 4, which presents results with phase differences  $\varphi = 10^{\circ}$ ,  $30^{\circ}$ , 50°, and 70°, respectively. When the compliant passive joint is fixed, the spring constant can be taken as infinite, that is the situation of  $K_s = Inf$ . Fig. 4 suggests that swimming speed increases with the increase of oscillation frequency  $\omega$  under the condition of all kinds of stiffness of the passive joint and phase differences. The effect of spring constant on swimming speed is inexplicit. In most cases, a more compliant passive joint can contribute to a better performance when the robotic fish swims at a smaller oscillation frequency. On the contrary, when the robotic fish swims at a larger oscillation frequency, a higher swimming speed can be obtained with a more rigid passive joint. In addition, under the same CPG parameters, the robotic fish with a compliant passive joint swims faster than the robotic fish with a fixed passive joint.

As for Case II, the results are presented in Fig. 5. Similarly, swimming speed increases with the increase of oscillation frequency  $\omega$ . However, because of one more active joint, the effect of passive joint stiffness on swimming speed becomes more complicated. With given CPG parameters, there exists an optimal stiffness of the passive joint to achieve best swimming performance. Compared with the robotic fish with a fixed passive joint, the swimming performance is reduced when an inappropriate stiffness of the passive joint is selected.

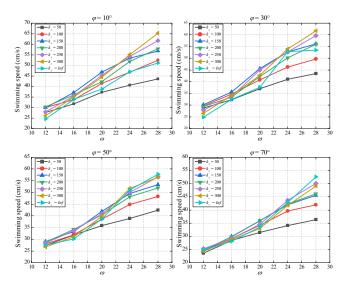


Fig. 5. Variation of swimming speed with  $\omega$  and  $K_s$  at four phase differences (Case II).

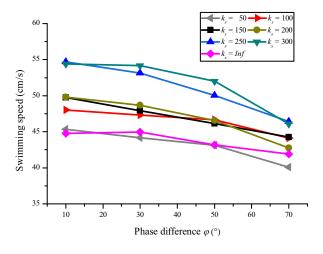
B. Effect of Compliant Passive Joint on Swimming Speed Obtained at  $\omega_{\rm max}=28$ 

The swimming speed increases directly with oscillation frequency. Accordingly, we pay more attention to the improvement of swimming speed at the maximum oscillation frequency. In this subsection, we take the swimming speed as the swimming speed achieved at  $\omega_{\rm max}=28$  to further analyze the effect of compliant passive joint on swimming performance. The swimming speed varying with phase differences  $\varphi$  and spring constants  $K_s$  are presented in Fig. 6.

For Case I, as shown in Fig. 6(a), the maximum swimming speeds with a given spring constant are all obtained when  $\varphi=10^\circ$ . Swimming speed of the robotic fish with a passive joint decreases with the increase of phase difference. For the robotic fish with a fixed passive joint, the maximum swimming speed is achieved at  $\varphi=30^\circ$ . Considering the six given spring constants, the maximum swimming speeds achieved at  $\varphi=10^\circ$  are all larger than the maximum swimming speed of robotic fish with a fixed passive joint.

In Case II, Fig. 6(b) suggests that the maximum swimming speeds with different spring constants are all also obtained when  $\varphi=10^\circ$ . Swimming speed of the robotic fish with a passive joint decreases with the increase of phase difference. Moreover, the maximum swimming speed with different phase differences is achieved when the spring constant is maximum. In comparison to the robotic fish with a fixed passive joint, the robotic fish with some certain stiffness of passive joint swims more slowly. For example, when  $K_s=50~\mathrm{g\cdot mm/deg}$ , no matter how to adjust the parameters of CPG model, the robotic fish always swims slower than the robotic fish with a fixed passive joint.

In sum, two conclusions can be drawn from the analysis



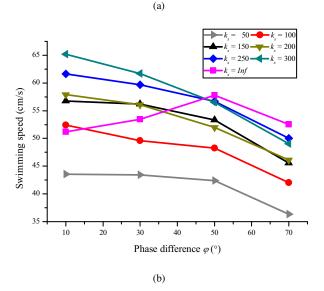


Fig. 6. Variation of swimming speed achieved at  $\omega=28$  with  $\varphi$  and  $K_s$ . (a) The robotic fish with two active joints. (b) The robotic fish with three active joints.

of swimming speed varying with  $\varphi$  and  $K_s$ :

- 1) When the stiffness of compliant passive joint is given in this paper, maximum swimming speed of the robotic fish with two active joints and three active joints can be obtained at minimum phase difference  $\varphi=10^\circ$  and maximum oscillation frequency  $\omega=28$ .
- 2) Adding an extra compliant passive joint can not always bring an improvement in swimming performance. Appropriate stiffness of passive joint and control parameters are needed to achieve better swimming performance.

# C. Effect of Compliant Passive Joint on Different Number of Active Joints

The last analysis focuses on how the compliant passive joint affect the swimming performance of robotic fish with

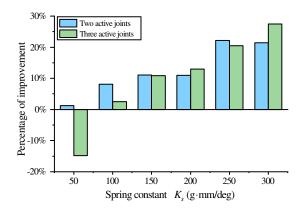


Fig. 7. Comparison of performance improvement between multi-joint robotic fish with different number of active joints.

different number of active joints. The experimental results are analyzed in two aspects. First, we take the improvement in maximum swimming speed as an indicator. For Case I, maximum swimming speed of the robotic fish with a compliant passive joint is about 54.41 cm/s (0.99 BL/s) when  $K_s = 300 \text{ g}\cdot\text{mm/deg}, \ \varphi = 10^{\circ}, \ \text{and} \ \omega = 28. \ \text{Maximum}$ swimming speed of the robotic fish with a fixed passive joint is about 44.98 cm/s (0.82 BL/s) when  $\varphi = 30^{\circ}$  and  $\omega = 28$ . The maximum swimming speed increases about 9.43 cm/s (0.17 BL/s). Similarly, with regard to Case II, maximum swimming speed of the robotic fish with a compliant passive joint is about 65.20 cm/s (1.19 BL/s) when  $K_s = 300$ g·mm/deg,  $\varphi = 10^{\circ}$ , and  $\omega = 28$ . Maximum swimming speed of the robotic fish with a fixed passive joint is about 57.77 cm/s (1.05 BL/s) when  $\varphi = 50^{\circ}$  and  $\omega = 28$ . The increase of maximum swimming speed is about 7.43 cm/s (0.14 BL/s). Thus, Case I gets a little better improvement from a compliant passive joint.

Second, as for different spring constants, the maximum improvement of swimming speed is compared between Case I and Case II. The maximum improvement of swimming speed is defined as follows:

$$\Delta u_i = \max(\frac{u_{ip} - u_f}{u_f} \times 100\%) \tag{2}$$

where subscript i means ith spring constant.  $u_{ip}$  and  $u_f$  denote the swimming speed of the robotic fish with a compliant passive joint and a fixed passive joint, respectively.  $u_{ip}$  and  $u_f$  are obtained with the same phase difference when  $\omega = 28$ .

Fig. 7 presents the comparison results from which we can conclude that in most case, Case I has a better improvement when the stiffness of the passive joint is smaller. When the stiffness of the passive joint is larger, Case II can obtain more improvement in swimming performance. The passive joint has a more significant influence on the robotic fish with three active joints.

# D. Discussion

It is observed from the experiment results that the swimming speed of a robotic fish is affected complicatedly when three parameters including spring constant, oscillation frequency, and phase difference are considered simultaneously. Although the effect of spring constant and phase difference is analyzed under the condition of maximum oscillation frequency, the relationship between swimming speed and various parameters is still not revealed effectively. To some extent, a robotic fish can get benefits from the addition of compliant passive joint in swimming performance. Nevertheless, the optimization of passive joint stiffness with partly given control parameters is still unknown.

Another issue to note is that the comparison analysis of passive joint effects on the robotic fish with different number of active joints is made with limited situation. However, the swimming speed of a robotic fish is influenced by many factors when adding an extra active joint, such as the position of assembly and the amplitude of the rotation. Hence, more studies especially the dynamic model are needed to explore the passive properties effectively.

# VI. CONCLUSIONS AND FUTURE WORK

In this paper, a novel compliant passive joint with two torsion springs has been designed for a multi-joint robotic fish to improve propulsive performance. Based on the robotic fish controlled by CPG network, extensive experiments with different spring constants and CPG parameters are carried out to investigate the effect of passive joint stiffness on propulsive performance. The swimming speeds of the robotic fish with different spring constants and phase differences all increase directly with the oscillation frequency. Compared with the robotic fish owning a fixed passive joint, the maximum swimming speed of robotic fish with a compliant passive joint is improved. However, the effect of passive joint stiffness and CPG parameters on swimming speed is complicated, and adding an extra compliant passive joint can not always bring an improvement in swimming performance. Appropriate stiffness of the passive joint and parameters of the CPG model are needed to achieve a better swimming performance. This study provides some insights into how to regulate the control parameters and stiffness of passive joint for optimizing swimming performance and suggests the benefits of compliant passive joint in a multi-joint robotic fish.

The ongoing and future work will focus on developing the dynamic model of the robotic fish with a compliant passive joint, which will be utilized to further optimize the propulsive performance including propulsive efficiency, swimming speed, and so on. Considering the yaw problem caused by the compliant passive joint in experiments, the yaw control will be studied.

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