Proceeding of the IEEE
International Conference on Robotics and Biomimetics
Dali, China, December 2019

A Continuous Robust Attitude Control Approach for Quadrotors Subject to Disturbance

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***Abstract*—In this paper, a continuous robust attitude control approach is proposed, which can achieve exponential stability even in the presence of external disturbance. In particular, on the basis of configuration space SO(3), the augmented dynamics of the system are transformed into a quasi-chained form. Subsequently, a continuous sliding mode approach is put forward, which is composed of a sliding mode differentiator and a sliding mode controller. Different from most reported approaches, the proposed control law is essentially continuous and the equilibrium point is exponentially stable even in the presence of complex unknowns. Finally, the simulation result illustrates the high tracking accuracy and strong robustness of the proposed method.**

***Index Terms*—Disturbances, augmented dynamics, continuous sliding mode control, quadrotor, attitude tracking**

I. Introduction

Over the past decades, quadrotors have been widely utilized and play dominant roles in many fields as typical unmanned aerial vehicles, such as exploration, package delivery, reconnaissance, and disaster responses. The advantages of quadrotors include not only vertical taking off and landing, but also rapid maneuvering which can realize efficient flight in different situations [1]-[10]. It is well known that the rapid maneuvering of quadrotors depends greatly upon the attitude control. Unfortunately, unmodeled dynamics are always retained in the model, which may include propeller flexibility, aerodynamic drag, and so on. In addition, the quadrotors are widely utilized in various air conditions where the unknown external disturbances are presented, further increasing the difficulty in solving the control problem. The robustness against these unknowns is hardly guaranteed, and there are much fewer available control methods which can achieve both exponential convergence of tracking errors and superior robustness in rejecting disturbances. From both practical and theoretical points of view, the design of attitude controller in the presence of complex unknowns, presents many challenges and deserves further investigations.

So far, a considerable amount of studies have been done to solve the attitude control problem. For example, based on the virtual angular velocity, a optimal control is proposed to achieve asymptotic convergence in [11]. For the sake of improving the reliability of data, Kalman filter and Moving-average are applied to filter out noise for the system in [12], on this basis, a cascade PID control is proposed to track desired angular velocity and angle. The attitude tracking of quadrotors is studied in [13], the feedback gains are tuned based on a heuristic approach. The above-mentioned approaches are designed based on exact model, and the robustness with respect to unknowns is not discussed. Although these control strategies achieve good performances in attitude tracking, there is no theoretical guarantee for the tracking accuracy under sustained disturbances. As a consequence, the control performance will be degraded when the system encounters external perturbations and unmodeled dynamics, thereby significantly restricting their ability to achieve complex flight maneuvers.

Until now, in order to improve the ability to resist unknowns, plenty of remarkable approaches are designed and applied to tackle the disturbance rejection problem. Generally, a disturbance observer can estimate the unknowns from measurable variables well, and then, on the basis of the estimation, a control action can be taken [14]. Toward this end, by combing disturbance observer and a state feedback controller, the anti-disturbance control methodology is proposed, and the stability analysis shows that all the signals will converge into a compact set in [15]. By introducing a wind gust model, a generalized extended state observer-based control strategy is proposed [16], to achieve precise attitude tracking even when the quadrotor is subject to wind disturbance. A disturbance rejection control strategy is put forward in [17], which consists of a robust disturbance observer and a nonlinear feedback controller, the control performance is guaranteed by H_∞ theory. The remarkable feature of these developed algorithms is disturbance rejection. However, due to the existence of unknowns, most of the reported results only guarantee that signals will converge into a compact set instead of desired equilibrium point.

In order to realize asymptotic regulation of the attitude tracking in the presence of complex unknowns, much effort has been devoted and many effective schemes have been obtained. In [18], a global exponential stability is achieved by hybrid control scheme with respect to a fixed disturbance. By using the robust integral of the signum of the error (RISE) method, a disturbance rejection control is proposed in [20], where the tracking error converges to zero exponentially fast. It is well known that sliding mode control approaches are powerful to address uncertain issues, and a lot of remarkable results have been obtained. For instance, a finite-time attitude controller is designed in [21] based on the improved super-twisting and the equivalent control algorithms, the unknown

This work is supported by National Natural Science Foundation of China under Grant 61873132.*(Corresponding author: Yongchun Fang.)*

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but bounded external disturbances are eliminated well. The finite time attitude controller is proposed in [22], with the aid of coordinate-free geometric approaches. In [23], a fractional-order controller is provided and exponential convergence of the tracking errors is guaranteed. The above mentioned strategies are robust to various disturbances. Unfortunately, a drawback of sliding mode control approaches is the undesirable chattering problem. In addition most reported works are designed based on Euler angles, which exhibits singularities.

To avoid these drawbacks, a continuous robust strategy is proposed in this paper by combing geometric and sliding mode control approaches. Different from most existing results, this approach is proposed based on the configuration space SO(3), which can achieve complex aerobatic maneuvers for attitude tracking. The remarkable feature of the developed algorithm is that a continuous sliding mode approach is proposed based on the augmented dynamics, which guarantees robustness with respect to a wide range of unknown dynamics and disturbance. More precisely, benefited from the augmented dynamics based design, the proposed method is essentially continuous, which effectively avoids the chattering problem and also indicates promising prospects for practical applications. The main contributions of this paper are summarized as follows

(1) On the basis of configuration space SO(3), the singularities are eliminated in an efficient manner. Specifically, the augmented dynamics are elegantly organized, which not only helps to avoid the chattering problem, but also rearranges the unknown disturbance term d_o in the first order dynamics. In addition, the sliding mode differentiators are applied to eliminate the influence of d_o.

(2) Benefited from the augmented dynamics, a derivative-based design is applied to the derivative of τ, which guarantees that both the control input and its time derivative are essentially continuous. Due to the specific design, the chattering problem is effectively avoided, which brings much convenience for practical implementation.

The remainder of this paper is structured as follows. Section II constructs the augmented dynamics of the system based on some carefully designed auxiliary variables. Then, controller design and stability analysis are provided in Section III. In section IV, simulation results are exhibited to verify the robustness and tracking accuracy of the proposed method. Concluding remarks are drawn in Section V.

II. System Model and Problem Formulation

In this paper, the attitude control of quadrotors is investigated. Generally, the equation of attitude model [3] can be presented as

$$\begin{aligned} \dot{R} &= R\hat{\Omega} \\ J\dot{\Omega} + \Omega \times J\Omega &= \tau + d_o \end{aligned} \tag{1}$$

where the operation $\hat{\star}$ is the transformation of a vector in $\mathbb{R}^3$ to a 3×3 skew-symmetric matrix such that $\hat{x}y = x \times y$, $\forall x, y \in \mathbb{R}^3$,and the $\times$ denotes cross product; $J \in \mathbb{R}^{3\times3}$ represents the inertia matrix; $R \in \mathbb{R}^{3\times3}$ denotes the rotation matrix from the body-fixed frame to the inertial frame, and $\Omega \in \mathbb{R}^3$ is the angular velocity; $d_o \in \mathbb{R}^3$ is the combining of external disturbance and unmodeled system dynamics; $\tau \in \mathbb{R}^3$ represents the control torque acting on the model. The control objective is to drive the attitude R tracking the desired signal R_d. The tracking error and angular velocity error are defined as

$$\begin{aligned} e_R &= \frac{1}{2}(R_d^\top R - R^\top R_d)^\vee \\ e_\Omega &= \Omega - R^\top R_d \Omega_d \end{aligned} \tag{2}$$

where $\star^\vee$ is the inverse operation of $\hat{\star}$ Then, the control objective can be mathematically summarized as follows

$$\begin{aligned} &e_R \to 0,\ e_\Omega \to 0, \\ &\text{s.t. unknown combined disturbance } d_o \end{aligned} \tag{3}$$

Property 1: Define the configuration function Ψ_R as follow

$$\Psi_R = \frac{1}{2}[I - R_d^\top R] \tag{4}$$

where the function Ψ_R can be expressed as

$$\Psi_R = 1 - \cos|x| \tag{5}$$

with $|x|$ being the norm of x. According to the tracking error defined in (2), it is easy to show that

$$|e_R|^2 = \sin^2|x| = (1 + \cos|x|)\Psi_R = (2 - \Psi_R)\Psi_R \tag{6}$$

Ψ_R can be bounded by

$$\frac{1}{2}|e_R|^2 \le \Psi_R \le \frac{1}{2 - \psi_R}|e_R|^2 \tag{7}$$

where $\Psi_R \le \psi_R < 2$ denotes a positive parameter [19]. Then the following relationship can be obtained for $\dot{\Psi}_R$

$$\dot{\Psi}_R = e_R e_\Omega \tag{8}$$

Before stating the control problem, some auxiliary variables are given. One first constructs the following first-order sliding variable

$$x_1 = Je_\Omega + k_1 e_R \tag{9}$$

where $k_1 \in \mathbb{R}$ is a positive gain parameter. Further, let the following auxiliary function x_2 be defined

$$x_2 = \dot{x}_1 - d_o \tag{10}$$

Then, by substituting (1), (2) and (9) into (10), one can represent the attitude dynamic equation as follow

$$x_2 = \tau + f_s \tag{11}$$

where $f_s = k_1\dot{e}_R - \Omega \times J\Omega + J\left(\hat{\Omega}R^\top R_d\Omega_d - R^\top R_d\dot{\Omega}_d\right)$. Taking the time derivative of x_2 and making some mathematical arrangements yields

$$\dot{x}_2 = f_o + f_\tau + f_d + \dot{\tau} \tag{12}$$

where the following properties $\frac{\mathrm{d}}{\mathrm{dt}}(R_d^\top R) = (R_d^\top R)\hat{e}_\Omega$, $\hat{e}_\Omega^\top = -\hat{e}_\Omega$ are used. In order to facilitate the subsequent description,

$R_c = R^\top R_d$ and $\Xi = \text{tr}[R^\top R_d]I - R^\top R_d$ are defined, then one has

$$
\begin{aligned}
f_o =& -\frac{k_1}{2}\Xi[J^{-1}(\Omega \times J\Omega)] + \frac{k_1}{2}\Xi\left(\hat{\Omega}R_c\Omega_d - R_c\dot{\Omega}_d\right) \\
&+ \frac{k_1}{2}(\hat{e}_\Omega R_c - \text{tr}[\hat{e}_\Omega R_c]I)e_\Omega - \Omega \times (-\Omega \times J\Omega) \\
&+ J[J^{-1}(\widehat{-\Omega \times} J\Omega)]R_c\Omega_d - JR_c\ddot{\Omega}_d - J\hat{\Omega}\hat{e}_\Omega R_c\Omega_d \\
&+ J\hat{e}_\Omega R_c\dot{\Omega}_d + J\hat{\Omega}R_c\dot{\Omega}_d - J^{-1}(-\Omega \times J\Omega) \times J\Omega \\
f_\tau =& k_1\Xi(J^{-1}\tau)/2 + J(\widehat{J^{-1}\tau})R_c\Omega_d - J^{-1}\tau \times J\Omega \\
&- \Omega \times M \\
f_d =& k_1\Xi(J^{-1}d_o)/2 + J(\widehat{J^{-1}d_o})R_c\Omega_d - J^{-1}d_o \times J\Omega \\
&- \Omega \times d_o
\end{aligned}
\tag{13}
$$

where f_o, f_τ and f_d denote the feedforward terms, the first term f_o contains only system states, which can be compensated directly, and the rest parts f_τ and f_d are coupled with the input signal τ and unknown combined disturbance d_o, respectively. Correspondingly, the system can be derived in the following manners

$$
\begin{aligned}
\dot{x}_1 &= x_2 + d_o \\
\dot{x}_2 &= f_o + f_M + f_d + \dot{\tau}
\end{aligned}
\tag{14}
$$

Remark 1: By combining the angle and angular velocity signals, the first-order sliding variable is designed in (9). On this basis, the augmented dynamics are organized more clearly and concisely. Actually, when the states converge to the manifold $x_1 = 0$, the tracking error will reach the equilibrium point exponentially fast, and it will be further analyzed in the stability analysis part in Section III. Based on (14), a direct design can be applied to the derivative of τ, which helps to solve the chatting problem.

Remark 2: Based on the configuration space SO(3), the augmented dynamic is organized in this section. It can be seen from (14) that the expression separates the combined disturbance d_o into the first order in (14) and even though the variable f_d contains the function about d_o, one can estimate d_o exactly through the first line in (14), and further, f_d will be compensated completely if d_0 is approximated precisely, which will be discussed later.

III. Control Design and Stability Analysis

In this section, one will first develop a sliding mode differentiators applicable to estimate d_o exactly, and then, a continuous sliding mode approach will be proposed based on (14), and further, the exponential stability analysis of the augmented system will be presented.

Based on the expressions of (14), one can construct sliding mode differentiator as follow [24], which estimates the unknown d_{oi} in finite time t_{1i}. i.e., $\widetilde{d}_{oi} = \bar{d}_{oi} - d_{oi} \to 0$, $\widetilde{x}_{1i} = \bar{x}_{1i} - x_{1i} \to 0$.

$$
\begin{aligned}
\dot{\bar{x}}_{1i} &= x_{2i} + v_{oi} \\
v_{oi} &= -k_{ai}L_{2i}^{\frac{1}{3}}|\bar{x}_{1i} - x_{1i}|^{\frac{2}{3}}\text{sgn}\ (\bar{x}_{1i} - x_{1i}) + \bar{d}_{oi} \\
\dot{\bar{d}}_{oi} &= v_{1i} \\
v_{1i} &= -k_{bi}L_{2i}^{\frac{1}{2}}|\bar{d}_{oi} - v_{oi}|^{\frac{1}{2}}\text{sgn}(\bar{d}_{oi} - v_{oi}) + e_i \\
\dot{e}_i &= v_{2i} \\
v_{2i} &= -k_{ci}L_{2i}\text{sgn}(e_i - v_{1i})
\end{aligned}
\tag{15}
$$

where $i = 1, 2, 3$ denotes the i-th element of the target vector, i.e., $\bar{d}_o = [d_{o1}\ d_{o2}\ d_{o3}]^\top$. $\bar{d}_{oi}$ and $\bar{x}_{1i}$ denote the estimate values of d_{oi} and x_{1i}, respectively. From now on, the subscript i is assumed to be $i = 1, 2, 3$ for notation simplicity. k_{ai} and k_{bi} are positive gains. L_{2i} is the Lipshitz constant of $\ddot{d}_{oi}$. The proof of finite time estimation can be found in [25], which is omitted here for brevity. To proceed, one further constructs the following second-order sliding variables as follows

$$
s_{oi} = (x_{2i} + \bar{d}_{oi})^{\frac{a_i}{b_i}} + k_{oi}x_{1i},\ i = 1, 2, 3 \tag{16}
$$

where $1 < \frac{a_i}{b_i} < 2$ and a_i, b_i are positive odd integers. The control law is proposed as follows

$$
\dot{\tau} = -f_o - f_\tau - \bar{f}_d + U \tag{17}
$$

where $\bar{f}_d$ is the estimate value of f_d. $U = [U_1\ U_2\ U_3]^\top$ is the sliding mode control law defined as follows

$$
\begin{aligned}
U_i = &-\frac{k_{oi}b_i}{a_i}(\tau_i + f_{si} + \bar{d}_{oi})^{2-\frac{a_i}{b_i}} \\
&- k_{oi}|s_{oi}|^{\xi_i}\text{sgn}(s_{oi}) - v_{1i},\ i = 1, 2, 3
\end{aligned}
\tag{18}
$$

where ξ_i denotes a positive parameter satisfying $0 < \xi_i < 1$.

Remark 3: It is worth to point out that the unknown combined disturbance d_o is well estimated by the sliding mode differentiator (15), which means $\bar{d}_{oi} = d_{oi}$ in finite time t_{1i}. In (17), $\bar{f}_d$ is designed to compensate the coupled term f_d in the following form:

$$
\begin{aligned}
\bar{f}_d =& \frac{1}{2}k_1\Xi(J^{-1}\bar{d}_o) + J(\widehat{J^{-1}\bar{d}_o})R_c\Omega_d - J^{-1}\bar{d}_o \times J\Omega \\
&- \Omega \times \bar{d}_o
\end{aligned}
\tag{19}
$$

Obviously, the estimate value of $\bar{d}_o$ converges to d_o in finite time $t_1 = \max\{t_{1i}\}, i = 1, 2, 3$, and then

$$
\begin{aligned}
\bar{f}_d =& k_1\Xi(J^{-1}d_o)/2 + J(\widehat{J^{-1}d_o})R_c\Omega_d - J^{-1}d_o \times J\Omega \\
&- \Omega \times d_o
\end{aligned}
\tag{20}
$$

which indicates that f_d is well compensated as $\bar{f}_d = f_d$.

Theorem 1: The proposed continuous sliding mode control law (17), along with the differentiator (15), guarantees that the tracking error converges to the equilibrium point exponentially fast.

Proof 1: On the basis of (14) and (17), the closed-loop system is derived as

$$
\begin{aligned}
\dot{x}_{1i} &= x_{2i} + d_{oi} \\
\dot{x}_{2i} &= U_i
\end{aligned}
\tag{21}
$$

where $i = 1, 2, 3$ denotes the i-th element of the target vector. The proof will be completed in three steps. In ***step1***, it will be shown that both the control input and its time derivative are continuous. Subsequently, one will prove the fact that all the states will converge to $s_{oi} = 0$ in finite time in ***step2***. Finally, based on the former two steps, the exponential convergence of tracking error is obtained in ***step3***.

Step1: continuity analysis:

One will first analyze the continuity of control input τ and its derivative $\dot{\tau}$ with respect to time. First, consider the equation (15), it is evident that

$$|v_{2i}| = |k_c L_{2i} \mathrm{sgn}(e_i - v_{1i})| \leq k_c L_{2i} \tag{22}$$

which means that v_{2i} is always bounded. Then, it follows from (15) that v_{1i} is always bounded and continuous the same as e_i. Due to the fact $|\dot{d}_{oi}| \in L_\infty$ and

$$\bar{d}_{oi} = \int_0^t v_{1i} \mathrm{dt},\ \widetilde{d}_{oi} = \bar{d}_{oi} - d_{oi} \tag{23}$$

One can conclude from (23) that both $\bar{d}_{oi}$ and $\widetilde{d}_{oi}$ are continuous. Then, one proceed to define an auxiliary signal $A_i = \tau_i + f_{si} + \bar{d}_i$. Taking the derivative of A with respect to time, inserting for (18), (17), (12), (11), and making some mathematical arrangements yields

$$\dot{A}_i = -\frac{k_{oi} b_i}{a_i} A_i^{2-\frac{a_i}{b_i}} - k_{oi}|s_{oi}|^{\xi} \mathrm{sgn}(s_{oi}) - \widetilde{f}_{di} \tag{24}$$

where $\widetilde{f}_{di} = \bar{f}_{di} - f_{di}$ is the estimate error of f_{di}, and the same as $\widetilde{d}_{oi}$, $\widetilde{f}_{di}$ is continuous. Since $k_{oi}|s_{oi}|^{\xi}\mathrm{sgn}(s_{oi})$ is continuous, it is straightforward to derive that $\dot{A}_i$ and A_i are continuous, which further implies from (17) and (18) that τ_i and $\dot{\tau}_i$ are both continuous.

Step2: finite time convergence to manifold: Consider Lyapunov candidate functions as follows

$$V_{oi} = \frac{1}{2} s_{oi}^2 + \widetilde{d}_{oi}^2 \tag{25}$$

Differentiating V_{oi} with respect to time and inserting the control law (17) and (18) into (25) yields

$$\dot{V}_{oi} = -k_{oi}\rho_i |s_{oi}|^{\xi_i+1} - \rho_i s_{oi} \widetilde{f}_{di} - k_{oi} s_{oi} \widetilde{d}_{oi} + \widetilde{d}_{oi} \dot{\widetilde{d}}_{oi} \tag{26}$$

where $\rho_i = \frac{a_i}{b_i}(x_{2i} + \bar{d}_{oi})^{\frac{a_i}{b_i}-1} \geq 0$ and $\rho_i = 0$ is not a attractor [26]. Since s_{oi}, $\widetilde{d}_{oi}$ and $\dot{\widetilde{d}}_{oi}$ are bounded, it is indicated from (26) that V_{oi} will not escape to infinity in finite time. Further, if $t > t_{1i}$, one has $\widetilde{d}_{oi} = \widetilde{f}_{di} = 0$, which indicates that

$$\dot{V}_{oi} = -k_{oi}\rho_i |s_{oi}|^{\xi_i+1} \tag{27}$$

The Lyapunov candidate for such a system (11) is chosen as

$$V_o = \sum_{i=1}^{3} \frac{1}{2} s_{oi}^2 + \widetilde{d}_{oi}^2 \tag{28}$$

The derivative of (28) with respect to time can be written as

$$\dot{V}_o = -\sum_{i=1}^{3} k_{oi}\rho_i |s_{oi}|^{\xi_i+1} \leq 0 \tag{29}$$

It is concluded from (28) and (29) that the states of augmented system (21) converges to $s_{oi} = 0$ in finite time t_{2i}.

Step3: exponential stability: To complete the proof, one will discuss the dynamic of states in the manifold $s_{oi} = 0$. It is noticed from (16) that

$$(x_{2i} + \bar{d}_{oi})^{\frac{a_i}{b_i}} + k_{oi} x_{1i} = 0,\ \forall t > t_{2i} \tag{30}$$

By inserting (21) into (30), one can rearrange (30) into

$$(\dot{x}_{1i} + \widetilde{d}_{oi})^{\frac{a_i}{b_i}} = -k_{oi} x_{1i} \tag{31}$$

where $\widetilde{d}_{oi} = 0$ in finite time t_{1i}. Hence, it is concluded that x_{1i} converges to 0 in finite time t_{3i}, which indicates that

$$J e_\Omega + k_1 e_R = 0,\ \forall t > t_3 \tag{32}$$

where $t_3 = \max\{t_{3i}\}$. Then, by using the relationships of (8), one obtains

$$\dot{\Psi}_R = -J^{-1} k_1 e_R^2 \tag{33}$$

Substituting (7) into (33) and making some mathematical arrangements leads to

$$\dot{\Psi}_R \leq -k_2 \Psi_R \tag{34}$$

where $k_2 = J^{-1} k_1 (2 - \psi_R)$ is a positive parameter. It is then implied from (8), (32) and (34) that Ψ_R, together with e_R and e_Ω converges to zero exponentially fast.

IV. Simulation Results and Analysis

In this section, some simulation results are presented to illustrate the tracking accuracy and robustness against sustained disturbance in the MATLAB/Simulink software.

The parameters of model described in (1) are configured as $J = \mathrm{diag}[0.1\ 0.1\ 0.15]$. The desired and initial attitudes are set as

$$R_d = \begin{bmatrix} 1 & 0 & 0.5 \\ 0 & 0.707 & 0.5 \\ 0 & -0.5 & 0.707 \end{bmatrix}, R_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{35}$$

The control gains are chosen as

$$\begin{aligned} &k_1 = 1.2,\ k_{oi} = 8,\ a_i = 11,\ b_i = 7,\ \xi_i = 0.5, \\ &k_{ai} = 2.5,\ k_{bi} = k_{ci} = 0.01,\ i = 1, 2, 3 \end{aligned} \tag{36}$$

The combined disturbance d_{oi} is set as $d_{oi} = 0.1\sin(\pi t)$, which will be applied into the system after 10 seconds, and L_{2i} is set to be 1 in the observer. Subsequently, one will verify the effective of proposed method. The corresponding simulation results are detailed in Fig. 1-4. The tracking error are recorded in Fig. 1. The solid line represents simulation results and the dotted line denotes the desired attitude. It can be seen from the results in Fig. 1 that the proposed approach exhibits high control accuracy and strong robustness. More precisely, the tracking errors converge to zero rapidly, and further there are hardly any performance degradation when the sustained disturbance is applied to system during the last 10 seconds as shown from Fig. 1. It is well known that a drawback of sliding mode control approaches is the undesirable chattering problem. The control input τ and its derivative are exhibited

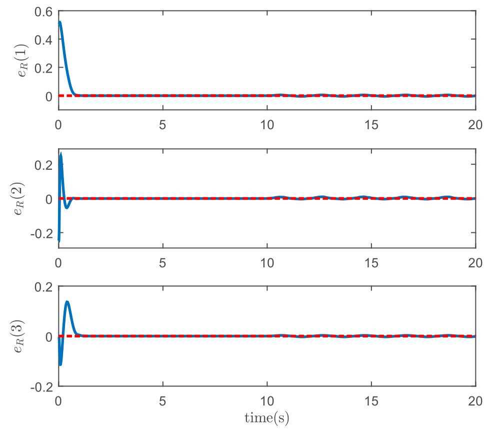

Fig. 1. Simulation results of e_R.

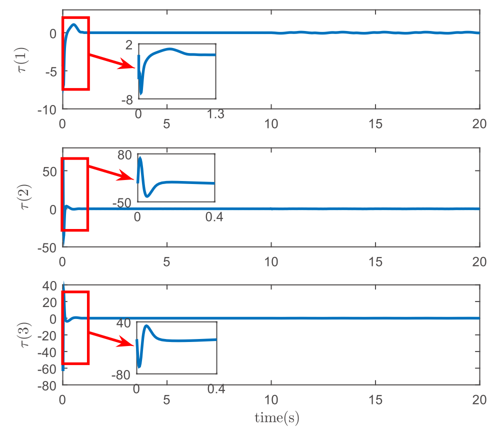

Fig. 2. Simulation results of τ.

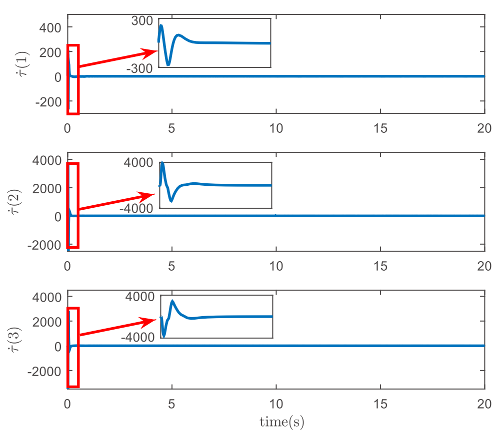

Fig. 3. Simulation results of $\dot{\tau}$.

in Fig. 2 and Fig. 3, respectively. Based on the simulation results shown in the Fig. 2 and Fig. 3, it is concluded that, the proposed method is continuous, which effectively avoids the chattering problem. The combined disturbance and observer outputs are recorded in Fig. 4. It is is clearly shown that the combined disturbance is well estimated in Fig. 4.

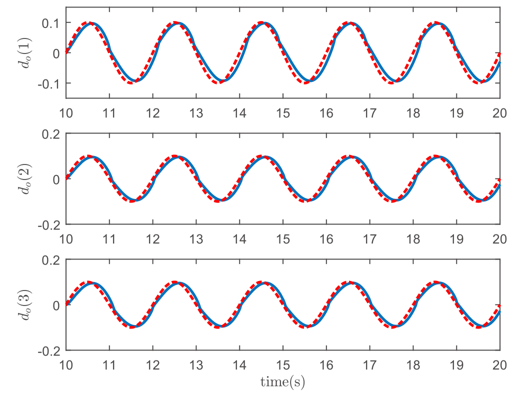

Fig. 4. Simulation results of the observer (solid line: simulation results; dotted line: the applied disturbance).

V. Conclusion

In this paper, a continuous robust attitude control approach is proposed for quadrotors. On the basis of configuration space SO(3), the singularities are eliminated in an efficient manner, meanwhile the augmented dynamics are organized, which separates the combined disturbance, and further ensures a derivative level design. More precisely, by combining the sliding mode differentiator and the controller designed in the derivative level, the proposed scheme drives the system to the equilibrium point exponentially fast even in the presence of external disturbance and unmodeled system dynamics. In addition, the designed control input and its time derivative are continuous, which effectively avoids the chattering problem. Finally, the simulation results show strong robustness and high tracking accuracy of the proposed control scheme.

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