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An extended stiffness model for 7 Dofs collaborative robots using the virtual joint method*

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Abstract **– In order to improve the stiffness modeling accuracy, an extended stiffness modeling and identification method of a 7 degrees of freedom(Dofs) collaborative robot is introduced in this paper. Unlike the traditional stiffness model which is assumed that each actuated joint is presented by a single one-dimensional virtual spring, the compliance of links, bearings and other structural components are considered and equivalent to two virtual joints in this method. These two additional virtual joints are orthogonal to the rotation axis of actuated joints. Including the equivalent virtual joint of joint torsional stiffness, each actuated joint of the robot has three virtual joints eventually. The method extends the traditional virtual joint method(VJM) and opens new prospects in terms of the stiffness optimal synthesis. Meanwhile, the accuracy of stiffness modeling is improved. Based on a 7 Dofs collaborative robot SHIR5, the static compliance simulations are performed to identify the elastic parameters of the robot, and the results show that more precise Cartesian stiffness characteristics are obtained from the extended stiffness model. Finally, the influence of compliance of links, bearings and other structural components on the stiffness model of the robot are analyzed.**

Index Terms - Collaborative robots Stiffness modeling Virtual joint method

I. INTRODUCTION

Collaborative robots (Cobots) can combine the repetitive performance of robots with the individual skills and ability of human [1,2]. However, due to the design concept of light-weight and high load/weight ratio in Cobots, integrated and modularized joints, compact structures and lightweight links are required [3]. Therefore, in addition to torsional compliance of joints, the compliance influence of links, bearings and other structural components on the stiffness performance of robots cannot be ignored [4, 5]. A lot of elastic factors are introduced to Cobots which is bring difficulties to improve the stiffness of robots and further affects the dynamic performance and accuracy of robots.

Stiffness modeling is an important method to improve the stiffness performance of robots. In the robotic manipulator stiffness modeling, there are three main methods that can be summarized as follows: the finite element analysis (FEA) [6], the finite element model of robots is established by finite element software, and the Cartesian stiffness of robots is obtained by calculating the elastic deformation of the robot under a certain pose and load. This method has the advantages of intuitive, reliable, and high precision. However, this method has the disadvantage of high computational expenses, each modeling and calculation consumes a lot of time, once the pose or load of robots is changed, it needs to be modeled again. The matrix structural analysis (MSA) [7] incorporates the main ideas of the FEA, but operates with rather large elements—3D flexible beams. And it can be thought of as a simplification of the FEA, as it brings about a reduction of the computational complexity at the expense of modeling accuracy. This modeling method is especially suitable for parallel robots. And finally, the virtual joint method(VJM) [8,9] is the most commonly used modeling method for traditional serial industrial robots. This method is based on the extension of the traditional rigid model by adding the virtual joints (localized springs) which describe the elastic deformations of links, joints and actuators. It provides reasonable trade-off between the modeling accuracy and computational complexity, and is easy to identify the elastic parameters and perform stiffness optimal synthesis. However, this method cannot comprehensive take into account the compliance of robot components, such as links, support bearings. It is suitable for the initial design stage of the robot, especially in the optimal synthesis of robot stiffness, and it has the characteristics of small amount of calculation and well real-time.

Based on the assumption that the link is a rigid body and only considering the linear torsional stiffness of joints, the VJM is used to establish the stiffness model of robots in [9, 10]. Then, in order to improve the modeling accuracy, the compliance influence of robot components, such as links, supporting bearings, on the stiffness of 6 Dofs industrial robots is considered in [4,11,12]. An extended 10 Dofs link model for

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the Mitsubishi PA-10 robot that comprises three additional virtual joints and seven actual joints is proposed in [4], the modeling error derived from link flexibility can be compensated by this method. The tilting rigidity of the bearing and the link deformations are considered and equivalent to two virtual joints in [11]. The torsional stiffness of links is introduced to stiffness model in [12] for robotic milling. A stiffness modeling method of industry robots with the gravity compensator is proposed in [13]. The typical virtual joint stiffness model is extended from 6 Dofs to 36 Dofs by considering 6 Dof deformations of joints in [14]. Based on the finite-element-and-analytical combined method, a new stiffness model for modular robots is established in [15], the 6 Dofs deformations of joints and links are considered in this method. In contrast to the conventional stiffness model, the enhanced stiffness modeling method, which implicitly assumes that the loading leads to the non-negligible changes of the manipulator posture and corresponding amendments of the Jacobian, is researched by [8,16]. However, the complementary stiffness matrix has less influence on the robot stiffness model and can be ignored with respect to the conventional model [9]. Different from traditional industrial robots, the elastic deformation of Cobots caused by links, bearings, and other structural components cannot be neglected besides joint torsional compliance [15,16]. So, the method, which simplifies the stiffness model of serial robots as an equal-degree-of-freedom virtual joint stiffness model, is no longer suitable.

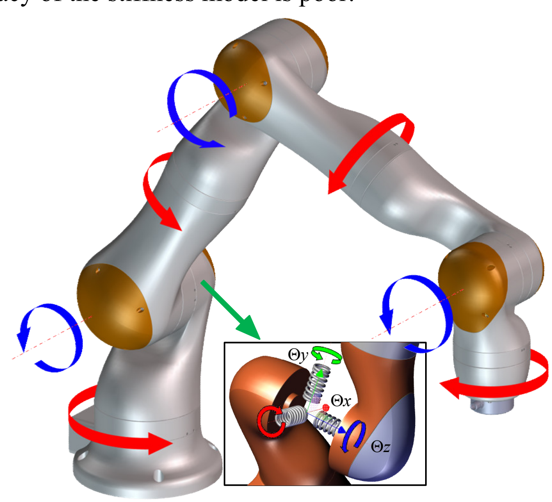
In this paper, a method of stiffness modeling and identification of a 7 Dofs Cobot is introduced. The compliance of links, bearings and other structural components are considered in this method, and are superposition equivalent to two virtual joints. These two additional virtual joints are orthogonal to the rotation axis of joints, therefore, another two (virtual) degrees of freedom per joint are introduced. Eventually, including the virtual joint of torsional stiffness, each actuated joint of robots has three virtual joints. This method extends the traditional virtual joint method, which not only considers the compliance of other components except actuated joint, but also reduces the amount of calculation. The Cartesian stiffness of Cobots in any pose can be obtained, and the stiffness modeling accuracy is improved. The method extends the traditional virtual joint method(VJM) and can be used in the optimal synthesis of robot stiffness. A set of measurement configurations are selected for stiffness identification, which can reduce the error sensitivity of the least-squares solution. The measurement configurations have higher dexterity which are determined from the joint space. Finally, a self-developed Cobot SHIR5 is selected for stiffness identification simulations in commercial finite element software, and the compliance effects of links, bearings, and actuated joints on the stiffness model SHIR5 are analyzed.

This paper is organized as follows: based on the VJM, the stiffness model of robots is built in Section II. Section III introduces the method of joint stiffness identification. Simulations and analysis are carried out in Section IV, the stiffness identification simulation is carried out and the compliance influence of each component on the stiffness model is analyzed. Conclusions are in Section V.

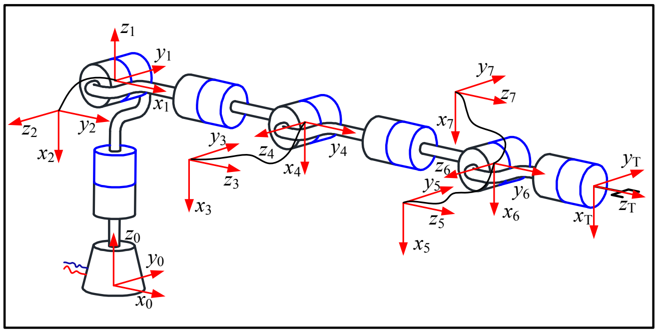
II. Stiffness Modeling of Cobots

In general, the stiffness model describes the resistance of a robotic manipulator to elastic deformations caused by an external force or torque. For relatively small deformations, the stiffness of a robot reflected at its end point is defined through the stiffness matrix K_X. The VJM is first introduced by Salisbury [8] which is assumed that the main influence factor of stiffness is the linear torsional stiffness of joints and each actuated joint is presented by a single one-dimensional virtual spring. In fact, due to the neglect of the link compliance, the traditional stiffness model only can predict about 80% of the displacement of the 6 Dofs industrial robot, which may be lower for the 7 Dofs lightweight robot [9]. To establish the extended stiffness model of Cobots by using the modified VJM, assuming that the main influence factors of the stiffness model are not only the linear torsional stiffness of joints, but also the compliance of links and supporting bearings.

The 7 Dofs redundant Cobots consist of eight links and seven joints, as shown in Fig.1a. The kinematics of each link and joint can be described by the modified Denavit-Hartenberg (DHm) parameters [5]. In the premise that the links, bearings and other structural components are assumed to be rigid bodies, the stiffness model of the serial robot is composed of several 1 Dof virtual joints which represent the torsional stiffness of the actuated joint. The stiffness model established by this method is inaccurate when the robot is in a singular pose. As shown in Fig.1b, the actual measurement results do not agree with the calculation results when the external force is applied to the robot end-point along x_0 direction, and the accuracy of the stiffness model is poor.



a) A 7 Dofs redundant robot SHIR5 and principles of virtual joints



b) The configuration of Cobot SHIR5

Fig.1 A 7 Dofs robot SHIR5 and its kinematics model

In order to improve the accuracy of the Cobot stiffness model, the compliance of links, bearings and other structural components should be considered. In this paper, based on the VJM, these components can be equivalent to rigid parts and virtual joints. Two virtual degrees of freedom per joint are introduced, and they are orthogonal to the virtual joint which represents the torsional stiffness of the actuated joint.

The two additional tilting virtual joints can be considered as an equivalent superposition of the compliance of the links, bearings and other structural components. As shown in Fig.1a, Θ_z represents the torsional virtual joint, Θ_x and Θ_y represent the tilting virtual joints. Unlike the extended stiffness model, the traditional stiffness model only have the torsional virtual joint Θ_z for each actuated joint.

Using this extended stiffness model, the number of virtual joints of the stiffness model increases from n (actuated joint) to $3\times n$ (virtual joint). As Fig.2, O_0 represents the base coordinate system of robots, $O_1,\cdots,O_7$ represent the coordinate systems of the actuated joints, $JV1,\cdots,JV14$ represent the coordinate systems of the additional virtual joints. The description of the extended robot kinematics is done by the well-known modified Denavit-Hartenberg convention [5]. To comply with the formalities of this convention and to take the asymmetric construction of the robot into account the additional coordinate systems is introduced. Three additional coordinate systems are added at the horizontal joints of the robot to construct an asymmetrical configuration, such as the coordinate systems O_{d1},O_{d2},O_{d3}. This results in a kinematic model with 21 Dofs which is described by 25 coordinate systems (including the base coordinate system), while the kinematic model (unextend) of the robot is shown in Fig.1b.

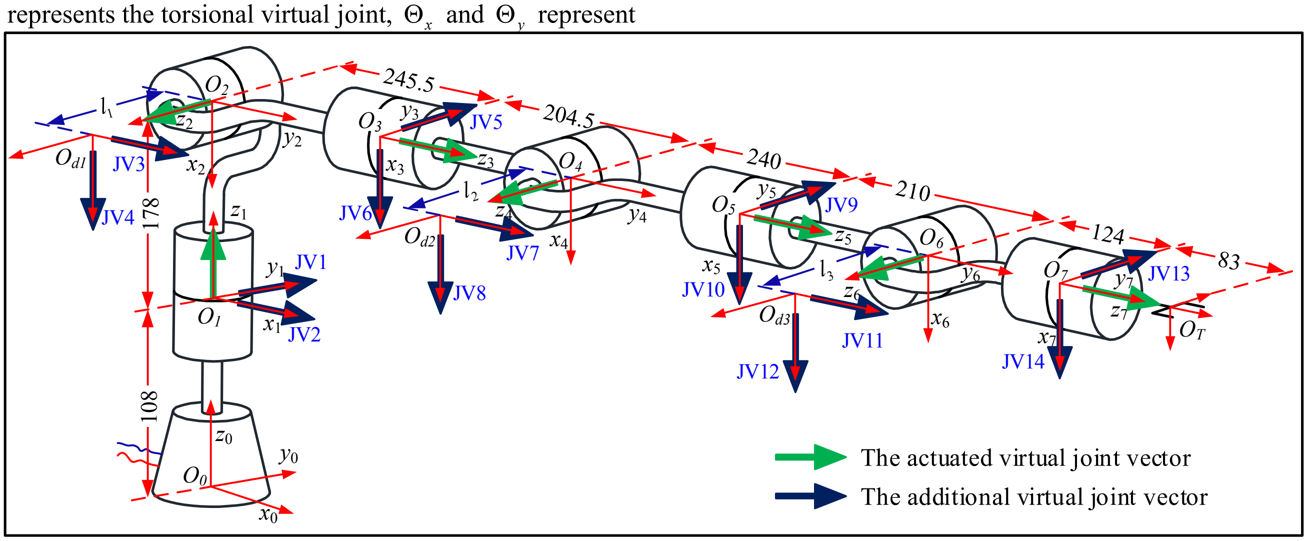


Fig.2 Extended stiffness model of Cobot SHIR5

Using the duality between the generalized relationships for motion and force transfer between the joint and Cartesian spaces, the following force relationship is obtained:

$$\Gamma = J^T \cdot F \tag{1}$$

Where F represents the wrench acting at the end-point of robots, J represents Jacobian matrix of robots, Γ represents the external torque exerting at the robot joints.

The elastic deformation of robots caused by the external wrench satisfies the differential motion condition, so the relationship between the elastic deformation of the virtual joint and the external torque can be derived as:

$$\Gamma = K_\Theta \cdot \Delta\Theta \tag{2}$$

Where $K_\Theta = diag.[k_1,k_2,\cdots k_n]$ represents the stiffness matrix of joints, $\Delta\Theta = \left[\delta\theta_1,\delta\theta_2,\cdots,\delta\theta_n\right]^T$ represents the elastic deformation vector of joints caused by Γ.

Meanwhile, the relationship between the elastic displacements of the robot end-point and the external wrench can be written as:

$$F = K_X \cdot \Delta X \tag{3}$$

Where K_X represents the stiffness matrix of robots which is a symmetric and semi-positive definite 6×6 matrix. $\Delta X = \left[\delta x,\delta y,\delta z,\delta\alpha,\delta\beta,\delta\gamma\right]^T$ represents the elastic deformation of the robot's end-point in Cartesian space.

$\delta\alpha, \delta\beta, \delta\gamma$ are the deviations in Euler angles due to the external wrenches.

The partial differentiation of (2) with respect to Θ leads to the following relationship:

$$K_\Theta = \frac{\partial \Gamma}{\partial \Theta} = \frac{\partial (J^T F)}{\partial \Theta} = \left(\frac{\partial J^T}{\partial \Theta}\right) F + J^T \frac{\partial F}{\partial X}\frac{\partial X}{\partial \Theta} = K_C + J^T K_X J \quad (4)$$

Where $K_C = \left(\frac{\partial J^T}{\partial \Theta}\right) F = \left[\frac{\partial J^T}{\partial \theta_1} F, \frac{\partial J^T}{\partial \theta_2} F, \cdots, \frac{\partial J^T}{\partial \theta_n} F\right]$ is the complementary stiffness matrix of the stiffness model [8], and stands for the case when the manipulator is externally loaded and/or the manipulator Jacobian changes with its configuration.

So, the stiffness model of robots can be expressed as:

$$K_X = J^{-T}\left(K_\Theta - K_C\right) J^{-1} \quad (5)$$

III. Joint Stiffness Identification

Assume that the Jacobian matrix of robots does not change with the external wrench, that is, the influence of complementary stiffness matrix on the stiffness model of robots can be negligible [8,9], the stiffness model can be derived as follows:

$$K_X = J^{-T} K_\Theta J^{-1} \quad (6)$$

$$C_X = J \cdot C_\Theta \cdot J^T \quad (7)$$

Where C_X, C_Θ represent the compliance matrix of robots in Cartesian space and joint space respectively.

Equation (3) and (7) can be rewritten as

$$\Delta X = J C_\Theta J^T F \quad (8)$$

Equation (8) also can be expressed as

$$\Delta X = \begin{bmatrix} \sum_{j=1}^{n}\left(c_j J_{1j} \sum_{i=1}^{6} J_{ij} F_i\right) \\ \vdots \\ \sum_{j=1}^{n}\left(c_j J_{6j} \sum_{i=1}^{6} J_{ij} F_i\right) \end{bmatrix} \quad (9)$$

where c_j represents the jth diagonal element of matrix C_Θ, i.e. $c_j = k_j^{-1} (j = 1, 2, \cdots, n)$, and F_i being the ith component of vector F which is exerted to the end-point of robots. By isolating the components of vector c in (9), the joint compliances can be expressed with respect to the robot end-point displacements as follows:

$$A(J, F) \cdot c = \Delta X \quad (10)$$

Where $A(J,F) = \begin{bmatrix} J_{11}\sum_{i=1}^{6} J_{i1} F_i & \cdots & J_{1n}\sum_{i=1}^{6} J_{in} F_i \\ \vdots & \ddots & \vdots \\ J_{61}\sum_{i=1}^{6} J_{i1} F_i & \cdots & J_{6n}\sum_{i=1}^{6} J_{in} F_i \end{bmatrix}$.

Because the posture of robots is difficult to measure, only the first three rows of the A matrix is used to solve it in this paper. p times of experiments are performed in k configurations for a total of $k \times p$ groups. So, the size of the matrix A is changed from $3 \times n$ to $3kp \times n$. Equation (10) cannot be obtained by inversion of the matrix. In that case, the joint stiffness values are obtained by minimizing the Euclidean norm of the approximation error Δ of the overdetermined linear-equation system (10), namely,

$$\min \quad \Delta = \frac{1}{2}\left\|Ac - \Delta X\right\|_2^2 \quad \text{over} \quad c \quad (11)$$

Based on a generalized inverse of $A(J,F)$, the value $c_{\min}$ of c that minimizes the Euclidean norm of the approximation error Δ is

$$c_{\min} = (A^T A)^{-1} A^T \Delta X \quad (12)$$

IV. Simulations and Analysis

Based on a self-developed 7 Dofs Cobot SHIR5 [15,17] as shown in Fig.1, the static compliance simulations are performed to identify the elastic parameters of the robot in a commercial finite element package. The SHIR5 robot has a 7 Dofs anthropomorphic configuration with the weight is about 25 Kg, the load is 5 Kg, and the workspace is 900mm. The SHIR5 robot is composed of smooth surface links and integrated joints in series. Three kinds of joints, Joint32, Joint25, Joint17, adopt modular and integrated design concept and have the same structural layout. The smooth surface links of the SHIR5 robot reduce contact and collision pressure between operators and robots, and then improve the comfort and safety of human-robot-collaboration.

A. Simulations

Fig.3 shows the finite element model of SHIR5 robot for stiffness identification. In order to mesh the models smoothly, the robot model should be reduced using CAE software. In the FEM of the robot, the mechanical interface of robot modules is reduced to a rigid connection. The components and structures, the compliance effect of which on the stiffness of modules can be neglected, are simplified as quality points and connected with the structural parts rigidly, like brakes, encoders, and so on. Assuming that the mechanical assembly of robot joints has no effect on the stiffness of transmission components and supporting bearings, so the stiffness of them are set to nominal value in the finite element model. The output end flange of robots is rigidly connected with the mass point unit by using MPC, and the mass point is regarded as the robot end-point. The FEM is divided by using Tetrahedral elements. The input flange of the base is fixed and the external wrench is applied to the end-point of the robot under the base coordinate system. After the calculation is completed, the elastic deformation of the robot is obtained by measuring the displacement of the end-point in the base coordinate system.

In the extended stiffness model, there are 21 elastic parameters need to be identified. So, at least 7 tests are needed to performed to complete the stiffness identification. In order to reduce the error sensitivity of the least-squares solution, a set of measurement configurations, which have higher dexterity, is selected for stiffness identification [18]. The selected configurations for stiffness identification and its condition number are shown in Table I. In addition, when selecting the configurations and applying external forces for stiffness identification, the torque of each joint should not equal to zero, and form a large joint torque as much as possible.

TABLE I
CONFIGURATIONS FOR STIFFNESS IDENTIFICATION AND ITS CONDITION NUMBER

Poses	The displacement of joints（°）							κ_F^{-1}
	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	
Configurations for stiffness identification								
1	-30	30	-30	-100	-160	110	0	0.621
2	60	30	30	100	-140	-100	0	0.584
3	80	-30	-20	110	0	110	0	0.699
4	-60	-70	30	110	0	115	0	0.803
5	30	-45	30	110	20	110	0	0.723
6	-60	-60	-30	100	-30	100	0	0.644
7	-45	30	-50	-115	0	-110	0	0.664
8	0	45	0	-100	160	110	0	0.678
Configurations for verify the extended stiffness model								
9	-30	30	-30	90	160	-90	0	0.492
10	-45	45	-100	90	160	-110	0	0.609
11	0	0	0	0	0	0	0	0
12	0	-30	0	-90	0	-60	0	0.352
13	0	30	0	-90	0	-120	0	0.643

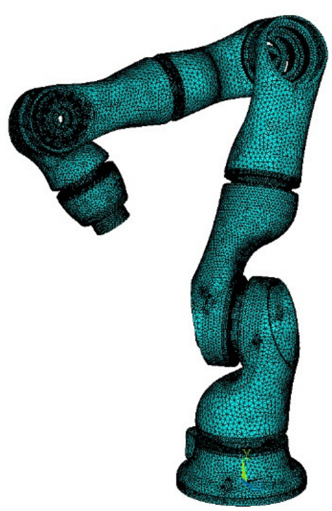
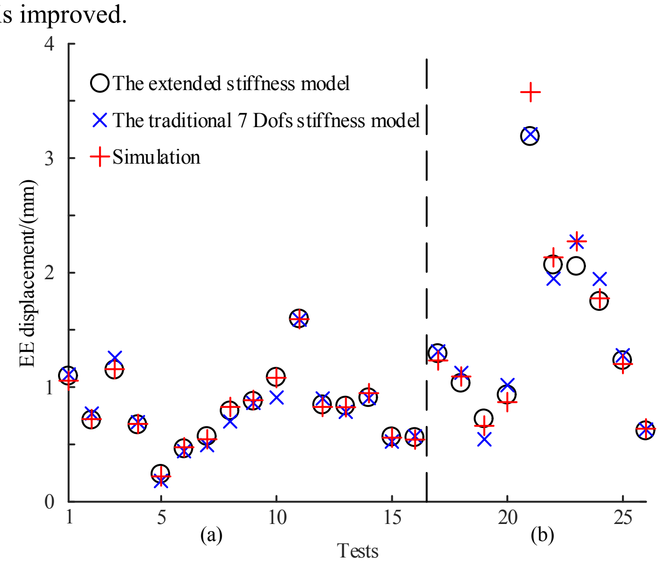


Fig.3 Finite element model of SHIR5 robot

Eight configurations are selected for stiffness identification, and two sets of external force are applied in each configuration. Therefore, 16 sets of measurement results are obtained, and the elastic parameters of SHIR5 Cobot can be solved by (12).

Fig.4 depicts the calculating results using the extended stiffness model, the calculating results using the traditional stiffness model [9] and simulation results. Besides eight configurations for stiffness identification, it also includes five configurations for verifying the extended stiffness model as shown in Table I. Two sets of external force are applied in each configuration and 26 sets of measurement results are obtained in the base coordinate system. On the total deformation, the simulation results are close to the calculated results of the extended stiffness model, especially in the tests used for the joint stiffness identification. Hence, the effectiveness of the modelling method is verified. Compare to the traditional 7 Dofs stiffness model, the mean relative error of the extended stiffness model reduced from 7.75% to 3.74%. For the tests of verification, the maximum relative error of the extended stiffness model is 10.9%, appearing in 21 test. Compared with the traditional 7 Dofs stiffness model, this value reduced by 6.1% and the accuracy of stiffness modeling is improved.



a) Validation with the tests used for the joint stiffness identification; b) Validation with the other tests.

Fig.4 Calculation and simulation results of robot end-point translational displacements

B. *Results Analysis*

In addition to the actuated joint torsional stiffness, the elastic deformation of the Cobot caused by structural components such as links and joint support bearings cannot be ignored. The influence of structural components and joint torsional stiffness on robot stiffness model is evaluated by the displacement of the end-point of robots. An external force $F = [25N, 35N, 15N, 6Nm, 8Nm, 9Nm]^T$ in the base coordinate system is applied to the robot end-point to calculate the total deformation, the deformation caused by joint torsional stiffness and the deformation caused by links, joint support bearings and other structural components are shown in Fig.5.

As we can see, the configurations of the robot have a great influence on the elastic deformation of the robot, the actuated joint torsional compliance is still the main influence factor of the stiffness model. Over the 13 tests, the average elastic deformation caused by actuated joint torsional compliance is 76.5% of the total deformation. In the ninth test, the deformation caused by actuated joint torsional compliance accounts for 88.1% of the total compliance, this is the maximum value in all tests. The minimum value, occurs in the eleventh test, is 44.9%, which means that the compliance of structure components affects the stiffness model of robots more than the compliance of joints. The reason is that in the eleventh test, the robot is in a singular configuration with displacement of each joint is 0°.

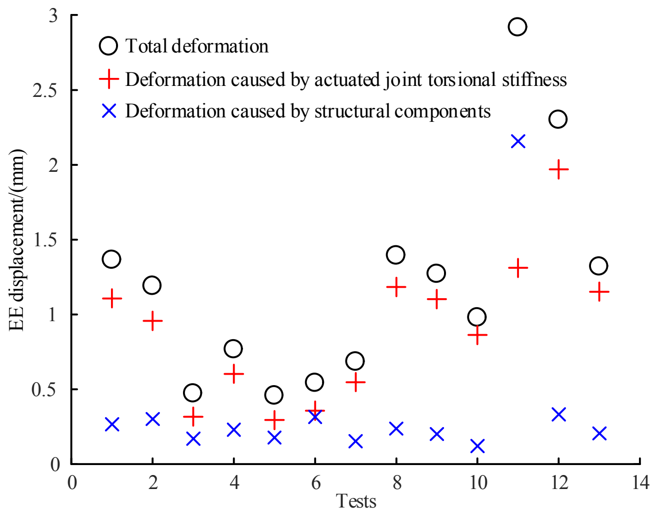


Fig.5 The results are calculated in the F loaded case

In summary, the influence of structural components and joints on the stiffness model of Cobots varies according to the configurations. In general, the influence of the joint torsional compliance on the stiffness model is more than the compliance of structure components except for the robot in singular configuration.

V. Conclusions

This paper introduces an extended stiffness modeling method of a 7 Dofs Cobot. The method extends the traditional VJM, and each actuated joint of robots has three virtual joints eventually. The stiffness modeling accuracy is improved, compare to the traditional 7 Dofs stiffness model, the mean relative error of the extended stiffness model reduced from 7.75% to 3.74%. The influence of structural components and joint torsional stiffness on robot stiffness model is analyzed, and the influence of the joint torsional compliance on the stiffness model is more than the compliance of structure components except for the robot in singular configuration.

Aimed at the practical application, experiments will be carried out to verify the method proposed in this paper, and the method of elastic deformation compensation for Cobots will be researched.

References

[1] ISO, "Robots and robotic devices — Safety requirements for industrial robots Part 2: Robot systems and integration," vol. ISO 10218-2, ed. Geneva, Switzerland: International Standards Organization, 2011.

[2] M. Hu, H. Wang, X. Pan, Y. Tian and Y. Chang, "Analysis and simulation on kinematics performance of a collaborative robot," *CAAI transactions on intelligent systems*, vol.12, pp. 75-81, Feb, 2017.

[3] E. H. Ostergaard, "Lightweight Robot for Everybody," *Robotics and Automation Magazine,* vol. 19, pp. 17-18, December 2012.

[4] T. Tsumugiwa, Y. Fukui, and R. Yokogawa, "Compliance measurement for the Mitsubishi PA-10 robot," *Advanced Robotics,* vol. 28, pp. 919-928, July 2014.

[5] B. Sciliano and O. Khatib, *Handbook of Robotics*. Switzerland: Springer, Cham, 2016.

[6] A. Mekaouche, F. Chapelle, and X. Balandraud, "FEM-Based Generation of Stiffness Maps," *IEEE Transactions on Robotics,* vol. 31, pp. 217-222, Feb 2015.

[7] C. Dong, H. Liu, W. Yue, and T. Huang, "Stiffness modeling and analysis of a novel 5-DOF hybrid robot," *Mechanism and Machine Theory,* vol. 125, pp. 80-93, July 2018.

[8] G. Alici and B. Shirinzadeh, "Enhanced stiffness modeling, identification and characterization for robot manipulators," *IEEE Transactions on Robotics,* vol. 21, pp. 554-564, Aug 2005.

[9] C. Dumas, S. Caro, phane, M. Cherif, S. Garnier, bastien, *et al.*, "Joint stiffness identification of industrial serial robots," *Robotica,* vol. 30, pp. 649-659, July 2012.

[10] A. Olabi, M. Damak, R. Bearee, and O. Gibaru, "Improving the accuracy of industrial robots by offline compensation of joints errors," presented at the 2012 IEEE International Conference on Industrial Technology, Athens, Greece, 2013.

[11] E. Abele, S. Rothenbücher, and M. Weigold, "Cartesian compliance model for industrial robots using virtual joints," *Production Engineering,* vol. 2, pp. 339-343, July 2008.

[12] M. F. Zaeh and O. Roesch, "Improvement of the machining accuracy of milling robots," *Production Engineering,* vol. 8, pp. 737-744, December 2014.

[13] A. Klimchik, Y. Wu, S. Caro, B. Furet, and A. Pashkevich, "Accuracy Improvement of Robot-Based Milling Using an Enhanced Manipulator Model," *Mechanisms and Machine Science,* vol. 22, pp. 73-81, 2014.

[14] U. Schneider, M. Momeni-K, M. Ansaloni, and A. Verl, "Stiffness modeling of industrial robots for deformation compensation in machining," presented at the 2014 IEEE/RSJ International Conference on Intelligent Robots and Systems, Chicago, IL, USA, 2014.

[15] M. Hu, H. Wang, X. Pan, and Y. Tian, "Research on Elastic Deformation Modeling of Collaborative Robots," presented at the 2017 IEEE International Conference on Robotics and Biomimetics Macau, China, 2017.

[16] A. Pashkevich, A. Klimchik, and D. Chablat, "Enhanced stiffness modeling of manipulators with passive joints," *Mechanism and Machine Theory,* vol. 46, pp. 662-679, May 2011.

[17] M. Hu, H. Wang, X. Pan, and Y. Tian, "Optimal synthesis of pose repeatability for collaborative robots based on the ISO 9283 standard," *Industrial Robot,* 2019.

[18] K. Yang, W. Yang, G. Cheng, and B. Lu, "A new methodology for joint stiffness identification of heavy duty industrial robots with the counterbalancing system," *Robotics and Computer-integrated Manufacturing,* vol. 53, pp. 58-71, October 2018.