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Topology Synthesis of Single-DOF Epicyclic Gear Trains Based on Graph Theory*

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***Abstract* - The research shows that the topology structure of gear kinematic chains can be represented by graphs. The topological synthesis of epicyclic gear trains based on graph theory is an effective system synthesis method. In this paper, a topology synthesis method is developed for single- degree of freedom (DOF) epicyclic gear trains. A new effective identification method Linkage Characteristic Polynomial-Fundamental Circuit Polygon (LCP-FCP) is proposed to solve isomorphism and rotationally isomorphism problem generated in topology synthesis process. The basic idea of proposed topology synthesis is to enumerate all candidate single-DOF gear train kinematic chains first. Then use the LCP-FCP algorithm to identify the isomorphism of the graphs and the rotation graphs respectively, and finally synthesize the kinematic chains that meets the corresponding fundamental requirements. All of the above steps can be automated by a computer program.**

Index Terms - Epicyclic gear trains; Topology synthesis; Graph theory; Isomorphism identification

I. INTRODUCTION

Structure synthesis is one of the most important stages in mechanical innovation design and conceptual design. It means to design new topology structure types according to the structural, kinematic and dynamic requirements of the mechanism. In order to study the topological structure of the mechanism systematically, the kinematic chains must be synthesized first. So far, researchers have proposed a number of methods for the synthesis of kinematic chains configurations. A viable approach of creating new configurations is to separate the structure of the mechanism from the function and use graph theory to systematically enumerate all candidate kinematic structures (i.e., the same number of degrees of freedom, the number of linkages, the desired functional properties). Then all kinematic structures that meet the functional requirements are identified. The introduction of graph theory makes it possible to digitize mechanical design through computer programming.

The topological synthesis method of gear kinematic chains using graph theory to derive the epicyclic gear trains was first proposed by Buchsbaum and Freufestein [1]. Buchsbaum and Freufestein proved that the gear kinematic chains can be represented by graphs. In addition, they have developed a topology synthesis procedure for the epicyclic gear trains. But the procedure is partially based on manual inspection and only applicable to gear kinematic chains with up to five links. Then, R. Ravisankar and Mrutyunjaya first proposed a fully computerized single-DOF kinematic chain synthesis method based on graph theory, and extended the number of links to six or more. Modern scholars' core ideas about the topological synthesis of gear kinematic chains are similar to those of these people [1, 2]. Since then, the isomorphic identification problem of graphs has attracted more and more attention. In the synthesis of planar kinematic chains, almost all synthesis methods produce a large number of isomorphic kinematic chains. Undetected isomorphic structures lead to duplicate solutions, while false isomorphism identification reduces the number of feasible solutions for new designs [3].

For the isomorphic identification problem of kinematic chains, many identification methods are derived at home and abroad, such as Linkage Characteristic Polynomial[1], Adjacency List [4], Eigenvector method [5], ant algorithm [6]and so on. However, these methods often cannot take into account both accuracy and efficiency. So far, the problem of isomorphism has not been solved well, and it is still a problem that limits the synthesis of the kinematic chains.

Therefore, this paper proposes a simple and effective isomorphism identification algorithm LCP-FCP based on the Linkage Characteristic Polynomial method. The algorithm is applied in the topology synthesis procedure of single-DOF epicyclic gear trains which can obtains the kinematic chains graphs with up to six links.

II. TOPOLOGICAL STRUCTURE OF GKCS

When some of the motion pairs in the kinematic chains represent geared pairs, the kinematic chains at this time is defined as geared kinematic chains (GKCs). The topological structure of the gear kinematic chains can be represented by graphs. Based on the concept of graph theory, a synthesis procedure for systematically creating a geared kinematic chains can be designed. In fact, there are many ways to represent the topological structure of the gear kinematic chain. The following describes representation methods of GKCs topological structure and some related concepts used in this paper.

2.1 Graph Representation

Functional representation. In essence, the functional representation of the GKCs is the mechanism schematic diagram that use lines and symbols to represent the links and motion pairs and show the relative motion between the various links of the mechanism. E.g. figure 1. (a) is a functional representation of a single-DOF epicyclic gear train. The

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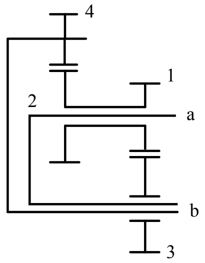
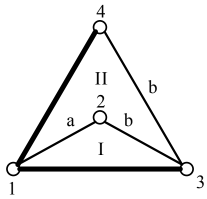
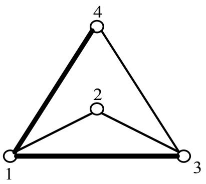
mechanism has two pairs of meshing gears 1-3, 1-4. In Fig1. (a), a and b represent the axes of rotation of the gear respectively, and 2 is a carrier for supporting the rotation of the gear.



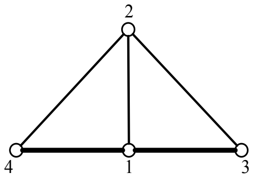
(a) Functional representation



（b）Fig1.(a) graph representation



（c）Unlabeled graph of Fig1.(a)



(d)Rotation graph representation of Fig1.(a)

Fig 1. Representation of GKCs

Graph representation. In the computer-aided structural synthesis of kinematic chains, the topological graph in graph theory is usually adopted to represent the topological structures of kinematic chains [6]. In the graph, the links are represented by vertices and the motion pairs by edges. The adjacency relationship between links is corresponding to the adjacency relationship between vertices. In order to distinguish the properties of the motion pair, the turning pair is represented by thin edge and the geared pair by heavy edge. And the turning edge is further labeled by the corresponding letter of the axis according to the spatial position of the axis of the turning pair. These letters are defined as the edge level. Fig. 1 (b) is a graph of the Fig. 1 (a). The vertices 1, 2, 3, 4 in Fig. 1(b) correspond to the links 1, 2, 3, 4 of Fig. 1(a); the heavy edges 1-3, 1-4 correspond to the gear pairs 1-3, 1-4; thin edges 1-2, 2-3, 3-4 corresponds to the turning pairs between links 1-2, 2-3, 3-4; the letters a, b correspond to the rotating axes. If the turning edges are not labeled, the result is called an unlabeled graph, as shown in Fig. 1(c).

There are some definitions in the graph that play an important role in the following description of the rotation graph and isomorphism identification. The subgraph obtained by deleting all the geared edges of the graph is called the tree. The tree plus any geared edge forms a fundamental circuit (f-circuit) that contains a geared edge and several turning edges. Each f-circuit has a vertex called the transfer vertex. The edge levels on the same side of the transfer vertex are the same, and the edges on either side of the transfer vertex are at different levels. E.g. there are two f-circuits in Fig. 1(b), and the transfer vertices of f-circuit I, II are both vertex 2.

Rotation graph representation. The rotation graph is a graph obtained by deleting all the turning edges and connecting the vertices at both ends of the geared edges with the corresponding transfer vertex. The purpose of the proposed rotation graph is that there is a mapping relationship between the rotation graph and the rotational displacement equations. The rotation displacement equations contain the motion information of the epicyclic gear train [7]. Fig. 1(d) is a rotation graph of Fig. 1(a).

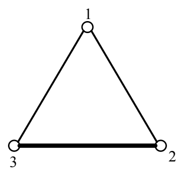
2.2 Adjacency Matrix Representation

In order to facilitate the storage and operation of graphs by computer, the graph is represented by matrix. One of the matrix representation methods is the adjacency matrix. The n vertices in the graph are labeled in the order of 1-n, and the adjacency matrix A is a square matrix of order n. The element values in the adjacency matrix depend on whether there is an adjacency relationship between the vertices. In order to distinguish the turning edges from the geared edges, weights 1 and g are assigned to the two types of edges respectively. The definition of matrix element is as follows: (1) if the vertex i and the vertex j are connected by the turning edge, $A_{ij}=1$; (2) if the vertex i and the vertex j are connected by the geared edges, $A_{ij}=g$; (3)if the vertex i and the vertex j are connected by no edge or i=j, $A_{ij}=0$.

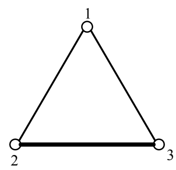
The elements in the matrix reflect the adjacency relationship between the vertices. As can be seen from the definition of the matrix elements, the adjacency matrix A is a symmetric matrix. E.g. the adjacency matrix of the graph shown in Fig. 1(b) is

$$A=\begin{bmatrix} 0 & 1 & g & g \\ 1 & 0 & 1 & 0 \\ g & 1 & 0 & 1 \\ g & 0 & 1 & 0 \end{bmatrix} \tag{1}$$

2.3 Derivation and Analysis of Isomorphic Problem



（a）



(b)

Fig 2. Two isomorphic graphs

The graph mentioned in this paper are essentially a graphical representation of the relationship. The two graphs have the same information as long as they do not change the adjacency relationship between the vertices. They are called isomorphic graphs. E.g. Fig. 2(a) and (b) are two isomorphic graphs. The vertices 1, 2 in Fig. 2(a) are swapped to obtain Fig. 2(b), but the adjacency relationship between the vertices and edges of the two graphs are the same. The result of isomorphism will lead to repeated solutions. Therefore, it is necessary to identify isomorphism of obtained graphs.

Based on further analysis, three conclusions can be drawn about isomorphism :(1) if two graphs are isomorphic, one graph can always be transformed into another graph by several vertex exchanges;(2) the exchange of vertex i, j in the graph equivalents the exchange of row i, j and column i, j of corresponding adjacency matrix;(3) the two adjacency matrices are similar if two graphs are isomorphic. Conclusions (1), (2) can be directly by observing graphs and matrices. A brief proof of Conclusion (3) will be given below:

Suppose that the two adjacency matrices of two isomorphic graph H_1, H_2 are A_1 and A_2 respectively. For one vertex exchange, exchange the vertex i, j and then the row i, j and column i, j of corresponding adjacency matrix will be exchanged too. Then there is an equation:

$$M = P_1 A_1 P_1 \tag{2}$$

In the equation, P_1 represents elementary row/column transformation matrix in the first vertex exchange, and M represents the adjacency matrix of the graph after the exchange. It can be seen from the conclusion (2) that after n vertex exchanges H_2 will be transformed from H_1.The following equation can be obtained.

$$A_2 = P_n \cdots P_2 P_1 A_1 P_1 P_2 \cdots P_n \tag{3}$$

Because $(P_n \cdots P_2 P_1) \bullet (P_1 P_2 \cdots P_n) = E$, it can be derived from the similar definition of the matrix that the matrix A_1 is similar to A_2.

III. FUNDAMENTAL REQUIREMENTS FOR SINGLE-DOF GKCS GRAPHS

The fundamental requirements of the GKCs graphs are established to derive epicyclic gear trains.

R1. For an n-link, the single-DOF GKCs, the quantitative relation among the vertex, the turning edges and the geared edges is n, n-1and n-2.

R2. The number of f-circuits is equal to the number of geared edges.

R3. There is no f-circuit consisting only of turning edges. Otherwise, the rotation of the links in the circuit is limited.

R4. All vertices are adjacent to at least one turning edge.

R5. The degree of freedom of any circuit must be at least one; for an f-circuit, it's equal to the number of vertices in the circuit minus two.

R6. The turning edges at the same level in the graphs must intersect at a common vertex.

R7. Each f-circuit has only one transfer vertex. All edges on the side of the transfer vertex are at the same level, and the edges on the other side of the transfer vertex are at different levels.

IV. TOPOLOGY SYNTHESIS PROCEDURE

4.1 Graph Enumeration

We start the enumeration with the known unlabeled graphs as the genetic graph G. The singe-DOF kinematic chains graphs of n+1 vertices are enumerated from the graphs with n vertices. According to the R1, the number of the turning edges and geared edges increases by one respectively when adding a new vertex to the graph. The new turning edge is connected between one of the existing n vertices of the genetic graphs and the added vertex. The new geared edge is connected between one of the remaining n-1 vertices and the added vertex. Then, n(n-1) graphs with n+1 vertices will be obtained. From the perspective of the adjacency matrix, the changes are as follows when adding a new vertex, turning edge and geared edge: the order of the matrix is increased by one, i.e. the 0 element of one row and one column is added; assign 1 to any value of n-1 new elements except diagonal element, and g to one of the remaining n-2 elements in row n+1; the elements in column n+1 are processed similarly to ensure that the adjacency matrix is a symmetric matrix.

According to R1, the number of links of single-DOF GKCs starts from 3. Based on the enumeration method above, fundamental requirements 1-6 will be meet automatically. However, there may be graphs that are isomorphic or violate R7. It's necessary to identify isomorphism and R7 to screen the graphs that meet the requirements.

4.2 LCP-FCP Isomorphism Identification Algorithm

The isomorphism identification of the graphs and corresponding rotation graphs is a very important part of the topology synthesis. The adjacency matrix can uniquely determine the topological structure of the mechanism. It theoretically can be used to identify isomorphism and rotationally isomorphism of the graphs. However, it is inefficient to use adjacency matrix directly. Some scholars have proposed the Linkage Characteristic Polynomial to identify isomorphism by comparing the values of the characteristic polynomial. This algorithm is simple and improve the computational efficiency to some extent. But the calculation amount of this algorithm is still not small when the number of links increases and the isomorphism and rotationally isomorphism of the graphs need to be identify at the same time. Based on the traditional Linkage Characteristic Polynomial method, the Linkage Characteristic Polynomial-Fundamental Circuit Polygon (LCP-FCP) algorithm is proposed in this paper. The Linkage Characteristic Polynomial is applied to identify the isomorphism of graphs, and Fundamental Circuit Polygon is used to the rotationally isomorphism.

Isomorphism identification. In Section2.3, the conclusion is drawn: if two graphs are isomorphic, whose adjacency matrices are similar, i.e. $A_1 \sim A_2$. It can be proved that$|xE\text{-}A_1|=|xE\text{-}A_2|$. Define the linkage characteristic polynomial as the value of the matrix e.g.

$$p = |xE - A| \tag{4}$$

Where, E is the unit matrix with the same order as adjacency matrix A, and x is a random variable. In order to improve the computational efficiency, this paper uses the computer to generate two random numbers, which are assigned to the variable x and edge weight g mentioned above. By using the values of the linkage characteristic polynomial, the isomorphism of the graph can be effectively identified.

Rotationally isomorphism identification. Since the rotation isomorphism of the genetic graph (the number of vertices is n) has been identified, it is only necessary to study what impact the added vertex n+1 has on rotationally isomorphism of graphs. According to the R2, the number of f-circuit is increased by one as the graph is added a new vertex. The problem is converted into the study on the new f-circuit.

When the new f-circuit is converted into a rotation graph, it can be divided into two cases: the f-circuit is a triangle or a polygon. If the new f-circuit is a triangle, after transformed to the rotation graph, the result is still itself. If the new f-circuit is a polygon, the f-circuit will become a triangle after the transformation. That means it may be isomorphic to the graph generated from the same genetic graph, whose new f-circuit is a triangle. Therefore, we choose the graphs with f-circuit polygon as a potential rotationally isomorphic graph.

For example, Fig. 3(a) and 3(b) are graphs derived from the same genetic graph. The new f-circuit in Figure.3 (a) is 4-3-1, which is a triangle with a transfer vertex 1. When transformed to a rotation graph, it is still itself. The new f-circuit in Fig. 3(b) is 4-2-1-3, which is a polygon with the transfer vertex 1. Fig. 3(a) is obtained by transforming Fig. 3(b) to a rotation graph. Obviously, Fig. 3(b) and Fig. 3(a) are rotationally isomorphic.

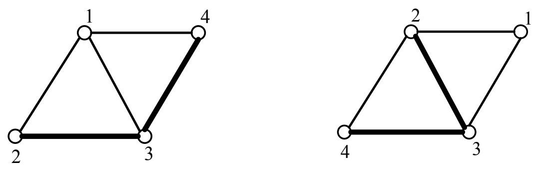


Fig. 3 Two rotationally isomorphic graphs

Then, it is necessary to further identify the isomorphism of the potential rotationally isomorphic graphs by setting condition. First, after determining the transfer vertex of the new f-circuit, it is judged whether the transfer vertex is adjacent to the added vertex n+1. If not adjacent to the added vertex, the graph is excluded. Then, it is judged whether the transfer vertex is connected to the vertex at one end of the added geared edge. If yes, the graph is also excluded.

Based on the above analysis, the specific steps of the LCP-FCP isomorphism identification algorithm in this paper are as follows:

Step1 Determine the linkage characteristic polynomial. The isomorphism is checked by comparing the values of the characteristic polynomial of the adjacency matrix.

Step2 Determine the existence of the f-circuit polygon of the graph. Look for a new f-circuit to determine if the circuit is a polygon. If it is, it is a potential isomorphic graph, otherwise it is a non-rotationally isomorphism graph.

Step3 Determine the new transfer vertex. The function $x_{i,k,j}$ is presented in [6] to determine the new transfer vertex. In this paper, the number of function $x_{i,k,j}$ =1 in the new f-circuit is set as variable *h*. There are three cases for the determination of the transfer vertex: (1) *h*>1, it violates R7 and is excluded; (2) *h*=1, the f-circuit has only one transit vertex and the graph is retained; (3) *h*=0, choose the vertex connected with the added vertex by turning edge as the transfer vertex.

Step4 Judge whether the transfer vertex is adjacent to the added vertex n+1, and whether it is connected to the vertex at one end of the added geared edge, not the vertex n+1.

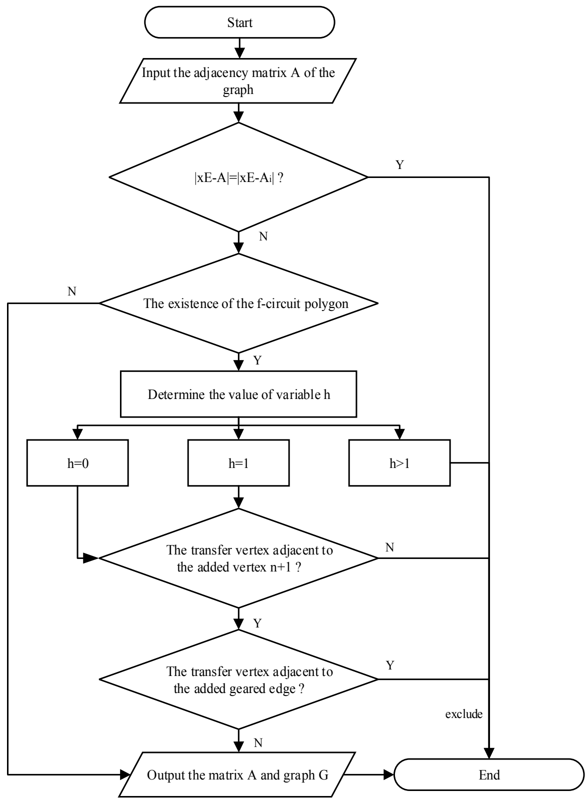


Fig 4. Flowchart of LCP-FCP isomorphism identification algorithm

4.3 Synthesis Procedure

This paper develops a GKCs topology synthesis procedure to derive single-DOF epicyclic gear trains. The procedure steps are simplified as follows:

1. Start the enumeration with the unlabeled graphs of n vertices as genetic graphs. There are totally of n (n-1) possible solutions.

2. Apply the LCP-FCP algorithm to identify isomorphism and rotationally isomorphism of the graphs. The duplicate solutions and the graphs that don't satisfy the R7 can be eliminated. The result will be a series of non-rotationally isomorphic graphs.

3. Repeat steps 1, 2 and adopt the unlabeled graphs generated in step 2 as the genetic graphs.

4. Establish single-DOF GKCs atlas.

V. SYNTHESIS RESULTS OF THE GRAPH

The proposed synthesis procedure is used to synthesize the single-DOF epicyclic gear trains with up to 6 links. The synthesis results of the graph are described below.

3-link GKCs graphs. The 3-link graph is the simplest single-DOF GKCs graph. There is only one case for the 3-link GKCs, as shown in Fig. 2(b).

4-link GKCs graphs. Start the enumeration with Fi. 2(b) as the genetic graph. There are3×(3-1)=6 possible solutions as shown in Fig5. Graph1 and 2, graph3 and 4, graph5 and 6 have the same value of the characteristic polynomial. The two graphs in each group is isomorphic to each other. Therefore, there are only three nonisomorphic graphs. For the first group of graphs such as graph 1, the new f-circuit is 4-3-1-2. The value of h is 1. However, the transfer vertex 1 of the new f-circuit is not adjacent to the added vertex 4. So it can be excluded. The synthesis results of 4-link GKCs are only two, as shown in Fig.6.

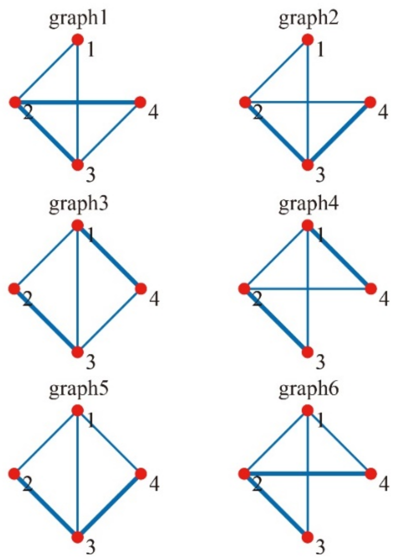


Fig. 5 4-link GKCs graphs

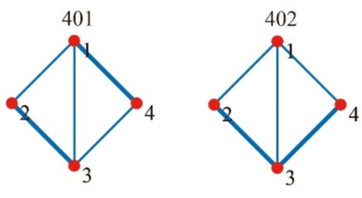


Fig. 6 4-link non-rotationally isomorphic graphs

5-link GKCs graphs. Graph 401 and 402 in Fig. 6 are used as the genetic graphs. There are 4× (4-1) =12 possible solutions for one genetic graph. So there are totally 2×12=24 ways of enumeration. Eleven of the 24 graphs are excluded because of isomorphism. And only 6 graphs are preserved due to the rotationally isomorphism and R7. It is shown in Fig. 7.

6-link GKCs graphs. All possible ways of enumeration are 120. After the isomorphism identification and the rotationally isomorphism, R7 identification, the results are 63 and 26. It is shown in Fig. 8. By comparison, it is found that the synthesize results with up to six links are consistent with the results of [1, 2].

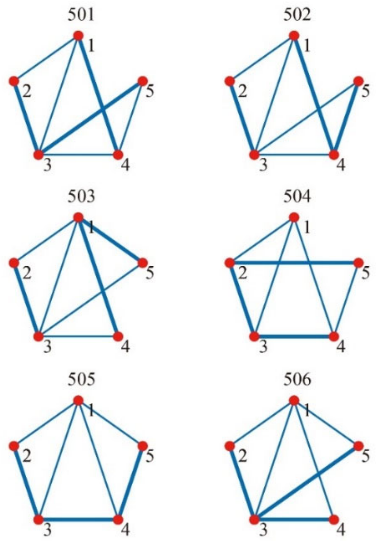


Fig. 7 5-link non-rotationally isomorphic graphs

VI. SUMMARY

The results prove that the proposed topology synthesis procedure is an effectively synthesize method to derive more epicyclic gear trains. The synthesis results are consistent with the results of [1, 2]. Compared to traditional algorithms, LCP-FCP algorithm can improve the computational efficiency to a certain extent. And ant algorithm has a defect that its parameters are instability while solving the small-scale problem. By contrast, LCP- FCP algorithm has more accurate results when there are fewer vertices in the graph. However, the isomorphism identification based on the Linkage Characteristic Polynomial is not sufficient. As the number of links increases, the results will be wrong. In addition, the future work of this paper is to apply the topology synthesis procedure to the mechanism with specific functional requirements. And establish the mechanism evaluation system to creatively design new and valuable mechanisms.

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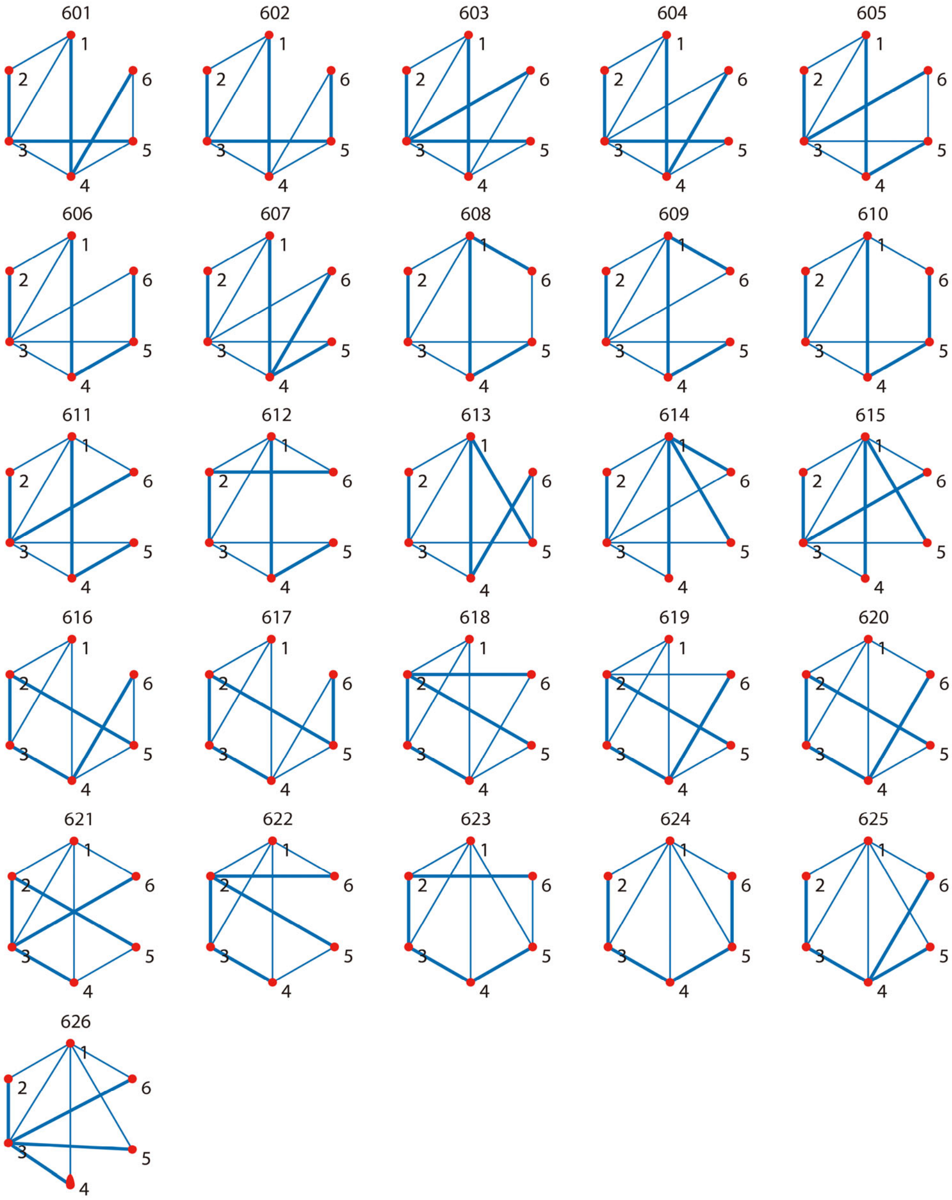

Fig. 8 6-link non-rotationally isomorphic graphs