Proceeding of the IEEE
International Conference on Robotics and Biomimetics
Dali, China, December 2019

An Indirect Control Method to Stabilize Tension in the Process of Towing Transfer *

Liqiao Shen, Zhongjie Meng and Panfeng Huang
Research Center for Intelligent Robotics and National Key Laboratory of Aerospace Flight Dynamics
Northwestern Polytechnical University
Xi'an, Shaanxi Province, China
mengzhongjie@nwpu.edu.cn

***Abstract* - This paper proposes an indirect control method to stabilize tension for the tethered space system in the process of towing transfer. In the paper, the Kalman filter is designed to estimate the length of the tether and the mass of the debris under the premise that the tension can be measured. The space platform and the debris are both regarded as mass points and the tether is regarded as the massless rigid rod. On the basis of the assumptions, the tension model and the centroid dynamics model are established. And the optimal tether retraction rate is obtained by using the nominal robust model predictive control. The simulation results of our method show that the Kalman filtering method is used for quality identification, and its estimation is close to the real value. The designed control scheme has good control performance and can effectively track the tension step command.**

Index Terms* - *Tethered space system, Kalman filter, Model predictive control

I. INTRODUCTION

A tethered space system (TSS) is a space system formed by connecting two or more space objects together with a flexible tether. The most typical of these is to connect a satellite to another satellite via a flexible tether to form a spatial combination. The tethered space towing system is a combination of a spacecraft and a target object connected by a tether, powered by a mission spacecraft, dragging a space object into a suitable space orbit and releasing it. Especially in the towing phase, the tether plays an important role in it, and due to the flexibility and elasticity of the tether, the system may collide during the towing process. Therefore, the research in this paper focuses on the towing phase. Fig. 1 illustrates that the TSS is consisted of a space platform, a debris satellite and a connected tether.

In the current research on tethered space systems, many studies on dynamics and control analysis are based on the assumption that the quality of the target satellite is negligible relative to the quality of the satellite that releases the tether. If the captured target is very small relative to the platform, the assumption is feasible. However, for the capture of close-range and large-size space targets, especially the capture of noncooperative targets, this assumption is not applicable. In this case, the quality characteristics of the captured targets cannot be ignored. The existed research is mostly based on the fact that the parameter information of target has been known. Zhang et al [1] studied the identification of the quality characteristics of the target satellite. Therefore, it is very necessary to identify the quality of the target satellite

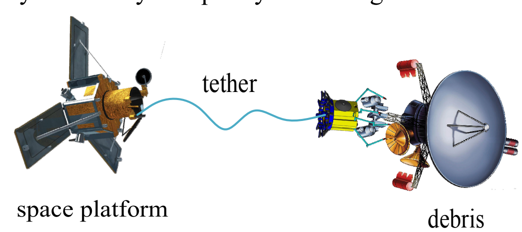

Figure 1. The tethered space system

In order to avoid collisions, there have been many methods to stabilize the tension of the tether. Mankala et al [2] used the micro-element method and the Hamilton mechanical variational principle to establish a dynamic model for the flexible tether and the winch-type release mechanism, and then used the Ritz method to solve the dynamic model and the unfolding process of the tether system which is short-distance space was simulated and analysed. In a paper by Jasper and Schaub [3], the authors studied the dynamics of the tether and the continuous open-loop thrust input, attenuating the dramatic dynamic effects of the TSS, thereby avoiding collisions between the end bodies. And the authors also highlighted the effect of tether-related parameters such as length, Young's modulus, and damping ratio on system dynamics. However, this approach is challenging given the ability of discrete on/off thrusters. Moreover, some works have also been aimed at designing closed-loop position control of the platform to maintain a safe distance relative to the debris, using tools from simple PID control [4-5], sliding mode control [6] and wave-based control [7].

Considering the fact that the elastic modulus of the tether is large, the tension will change drastically due to the inaccurate tension command. It is undoubtedly very difficult to track the tension of such a dramatic change. The problem of tracking tension can be translated into the problem of tracking the tether length information after analysis. Our approach focuses on the real implementation of tension via a reel. The works [8-9] point out that if the tension deviates from the

* This work is partially supported by the National Natural Science Foundation of China (Grant No: 61773317), Natural Science Basic Research Plan in Shaanxi Province of China (Program No: 2019JM-406) and the Fundamental Research Funds for the Central Universities (Grant No: 3102018JGC001).

978-1-7281-6321-5/19//$31.00 ©2019 IEEE

desired value, the reel should be triggered to change the tether natural length accordingly.

Wang et al [10] used the model predictive control to track the tension based on the quality identification of the debris, but there are still problems such as inaccurate quality identification. This paper optimizes and improves them based on the shortcomings of existing research. In the paper, the Kalman filter is designed to estimate the length of the tether and the mass of the debris under the premise that the tension can be measured. The space platform and the debris are both regarded as mass points and the tether is regarded as the massless rigid rod. On the basis of the assumptions, the tension model and the centroid dynamics model are established. And the optimal tether retraction rate is obtained by using the nominal robust model predictive control.

II. MODEL FORMULATION

The TSS is consisted of a space platform, a debris satellite and a connected tether. To simplify the model, several assumptions are employed as follows.

(1) The space platform is equipped with an attitude controller, that is to say, its attitude is controllable, so the space platform can be regarded as a mass point.

(2) In the posture movement of the debris, the tension moment of the tether occupies the dominant factor. In order to study the influence of the tether on the tension at both ends, it is assumed here that the tether passes the centroid of the debris, and then the tension moment of the tether to the debris is zero. Regardless of the residual angular velocity of the debris, it can also be considered a mass point.

(3) Compared to the lumped-mass discretization model, the massless rigid rod model is a good approximation of the tether dynamics [8]. With this model, the effect of the tether on the end body is only the tension, and the tension is along the linear direction connecting the ends of the body. This method can simplify its mathematical model and obtain analytical solutions to study system motion.

The dumbbell model is shown in Fig. 2. The Earth-centered Inertial (ECI) *OXYZ* is inertia frame and Local Vertical Local Horizontal (LVLH) $Px_0y_0z_0$ is orbital frame with its origin on the platform. $\boldsymbol{R}$ is position vector of the platform in ECI frame. $\boldsymbol{\rho}$,expressed in LVLH frame, is the centroid distance vector which points from P to D and describes the heading of tether by in-plane angle θ and out-plane angle φ .

The complete model consists of three parts. First, the orbit model of the space platform is established. Here, the six elements of the classical orbit are used to represent the change of the orbit of the space platform with time.

$$\boldsymbol{G_e} = \{a, e, i_o, \Omega, \omega, M_0\} \tag{1}$$

where a is semi-major axis, e is eccentricity, i_o is inclination, Ω is longitude of the ascending node, ω is the argument of periapsis, M_0 is mean anomaly at epoch.

$\boldsymbol{R}$ is expressed as a form of track elements as follows:

$$\boldsymbol{R} = f[\boldsymbol{G_e}(t), t] \tag{2}$$

The time evolution of $\boldsymbol{R}$ is governed by gravitational force, platform thrust, tether tension and other perturbation forces, which requires Gauss's Variational Equation [6] to formulate the orbital dynamics.

Next, the establishment of a tether model is performed. The tether can only be stretched and incompressible, according to Hooke's law, so the tension in the tether is:

$$T = \begin{cases} \dfrac{EA(\rho - l_0)}{l_0} + \dfrac{\varsigma EA(\dot{\rho} - \dot{l}_0)}{l_0} & ,\rho > l_0 \\ 0 & ,\rho \le l_0 \end{cases} \tag{3}$$

where E is Young's modulus, A is the tether across area, ς is the damping coefficient, l_0 is the natural length of the tether, ρ is the actual length of the tether.

Finally, a relative motion model of the debris is established. The position of debris relative to the platform in LVLH frame is described as $\boldsymbol{\rho} = [x, y, z]^T$. And the dynamics equation can be expressed as:

$$\ddot{\boldsymbol{\rho}} = \frac{\mu \boldsymbol{R}}{|\boldsymbol{R}|^3} - \frac{\mu(\boldsymbol{R} + \boldsymbol{\rho})}{\left[\left(|\boldsymbol{R}| + x\right)^2 + y^2 + z^2\right]^{\frac{3}{2}}} + \frac{\boldsymbol{T}}{m_d} - \boldsymbol{d} \tag{4}$$

where μ is the Earth gravitational constant, m_d is the mass of debris, $\boldsymbol{d}$ is the nongravitational acceleration of the debris.

According to the rotation of coordinate systems, ω_{LE} can be expressed as:

$$\omega_{LE} = \begin{bmatrix} \dot{\Omega}\sin i_o \sin(\omega + f) + \dot{i}_o \cos(\omega + f) \\ \dot{\Omega}\sin i_o \cos(\omega + f) - \dot{i}_o \sin(\omega + f) \\ \dot{\Omega}\cos i_o + \dot{\omega} + \dot{f} \end{bmatrix} \tag{5}$$

Combine (4) and (5), the relative motion equation of debris in LVLH frame is described as:

$$\begin{cases} \ddot{x} - 2\dot{\psi}\dot{y} - \ddot{\psi}y - \dot{\psi}^2 x = -\dfrac{\mu(R + x)}{[(R + x)^2 + y^2 + z^2]^{\frac{3}{2}}} + \dfrac{\mu}{R^2} + d_x \\ \ddot{y} + 2\dot{\psi}\dot{x} + \ddot{\psi}x - \dot{\psi}^2 y = -\dfrac{\mu y}{[(R + x)^2 + y^2 + z^2]^{\frac{3}{2}}} + d_y \\ \ddot{z} = -\dfrac{\mu z}{[(R + x)^2 + y^2 + z^2]^{\frac{3}{2}}} + d_z \end{cases} \tag{6}$$

where $\psi = \omega + f$ is the orbital polar angle, f is the true anomaly, $\boldsymbol{d} = [d_x, d_y, d_z]^T$ is the nongravitational acceleration of the debris.

Equation (6) is conveniently expressed in the spherical coordinate system. The conversion relationship between the Cartesian coordinate system and the spherical coordinate system is as follows:

$$\begin{cases} x = -|\boldsymbol{\rho}|\cos\varphi\sin\theta \\ y = -|\boldsymbol{\rho}|\cos\varphi\cos\theta \\ z = -|\boldsymbol{\rho}|\sin\varphi \end{cases} \tag{7}$$

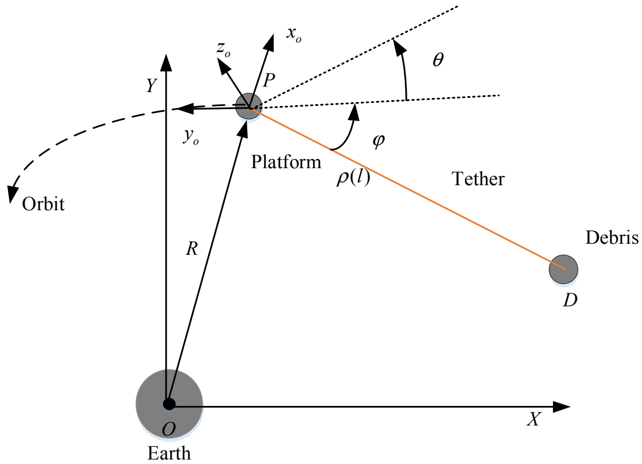

Figure 2. The simplified model of the TSS

Connect (7) with (6), the dumbbell can be derived :

$$\begin{cases} \ddot{\rho} = \rho[\dot{\varphi}^2 + (\dot{\psi}+\dot{\theta})^2\cos^2\varphi + \dfrac{\mu(3\cos^2\varphi\cos^2\theta - 1)}{R^3}] \\ -\dfrac{(m_p+m_d)T}{m_p m_d} + \dfrac{F\cos\varphi\cos\theta}{m_p} \\ \ddot{\theta} = -2(\dfrac{\dot{\rho}}{\rho} - \dot{\varphi}\tan\varphi)(\dot{\theta}+\dot{\psi}) - \dfrac{3\mu\sin 2\theta}{2R^3} \\ +\dfrac{F\sin\theta}{m_p\rho\cos^2\varphi} - \ddot{\psi} \\ \ddot{\varphi} = -2\dfrac{\dot{\rho}\dot{\varphi}}{\rho} - [\dfrac{1}{2}(\dot{\psi}+\dot{\theta})^2 + \dfrac{3\mu\cos^2\theta}{2R^3}]\sin 2\varphi \\ +\dfrac{F\cos\theta\sin\varphi}{m_p\rho} \end{cases} \tag{8}$$

where F is platform thrust, θ is in-plane angle and φ is out-plane angle.

We expect the direction of the tether and the platform thrust is along the y_0 axis of LVLH frame. Thus, tether heading angles are small enough, namely $\theta = 0$ and $\varphi = 0$ .So the above relative motion equation can be simplified as:

$$\ddot{\rho} = -\frac{(m_p+m_d)T}{m_p m_d} + \frac{F}{m_p} + \Delta d \tag{9}$$

where $\Delta d = \rho(\dot{\psi}^2 + \dfrac{2\mu}{R^3})$ is the disturbance.

Connect (3) and (9), the centroid distance model is written as:

$$\ddot{\rho} + \frac{(m_d+m_p)EA}{m_d m_p l_0}\varsigma\dot{\rho} + \frac{(m_d+m_p)EA}{m_d m_p l_0}\rho = \frac{F}{m_p} + \frac{(m_d+m_p)EA}{m_d m_p} \tag{10}$$

III. Analysis of Dynamics

The purpose of this section is to analyse the response relationship between the length and the tension of the tether. All analyses are performed on the basis of system (10). Here we study the effect of different damping coefficients on the dynamics.

we analyse the case without tension control, and if the system is stable, the length of the tether will eventually stabilize near a constant value. The parameters of the system and the thrust of the space platform are set as: $m_d = 3000\text{kg}$, $m_p = 2000\text{kg}$, $EA = 10^5\,\text{N}$, $l_0 = 100\text{m}$, $\dot{l}_0 = 0\text{m/s}$, $F = 100\,\text{N}$.

Figs. 3-6 show that when different damping coefficients are taken respectively, the corresponding phase plane and the actual length changes of the system show different characteristics. When the damping coefficient of the tether is larger, the actual length converges faster under the action of the tether damping, and vice versa.

According to the system (10), the characteristic root is expressed as $\lambda_{1,2} = \dfrac{-\varsigma c \pm \sqrt{\varsigma^2 c^2 - 4c}}{2}$, where $c = \dfrac{(m_d+m_p)EA}{m_d m_p l_0}$.

The real part of it is negative, and it can be seen that the system is asymptotically stable.

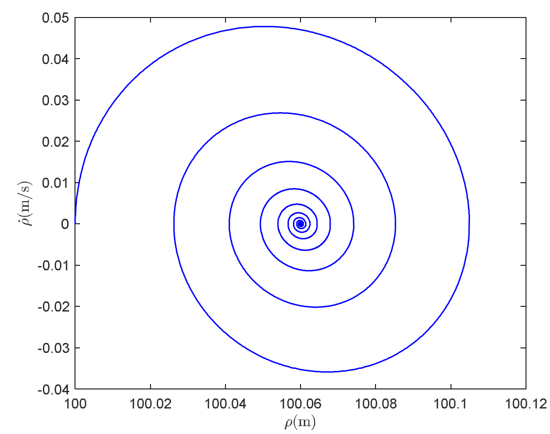

Figure 3. The phase plane— ς=0.2

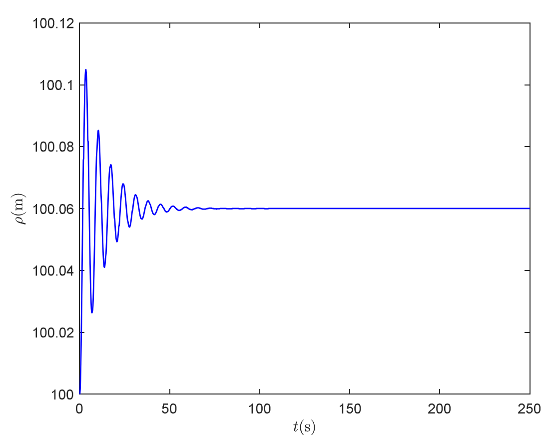

Figure 4. The actual length of the tether— ς=0.2

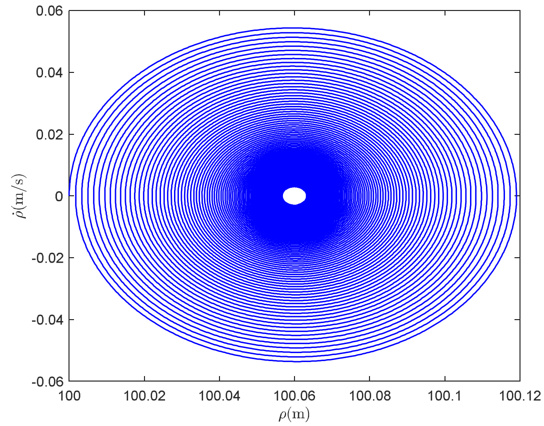

Figure 5. The phase plane— ς=0.01

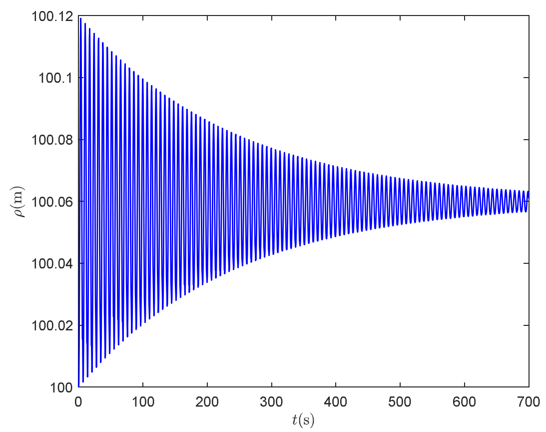

Figure 6. The actual length of the tether— ς=0.01

Referring to the material of the actual tether, the damping coefficient is relatively small ($<=10^{-2}$), it takes a long time to stabilize the tension only relying on the tether itself, and there may be entanglement of the debris and the tether, so it is necessary to design a tension controller on the space platform to quickly stabilize the tension.

IV. TENSION CONTROL DESIGN

In this section, our goal is to control the tension based on the system (10). Under the premise that the tension can be measured, a Kalman filter is designed to estimate the actual length of the tether and determine the mass of the debris. Here we use the rotational speed of the reel mechanism as the control input of the system. We designed a nominal robust model predictive controller to generate the optimal sequence of control inputs. When control is applied, the system becomes:

$$\ddot{\rho}=\frac{F}{m_p}-\frac{(m_d+m_p)EA}{m_d m_p}\left[\frac{\rho+\varsigma(\dot{\rho}-u)}{l_0+\Delta l}-1\right] \tag{11}$$

where u is the tether retraction rate, $\Delta l=\int u$ is the length change of the tether.

It can be seen from (3) that the tension of the tether is substantially inversely proportional to the natural length of the tether. That is to say, when the tension of the tether is greater than the given reference command, the reel mechanism should release the tether according to the deviation of the tension, otherwise the tether should be recovered. So we design a simple proportional control law:

$$l_{n+1}=l_n+k(T-T_{ref}) \tag{12}$$

where k is a positive proportional gain, l_n and l_{n+1} represent the natural length in previous and current time respectively. T_{ref} is the tension command which is defined below to (13).

$$T_{ref}=\frac{m_d}{m_d+m_{p0}-\int |F|/I_{sp}g_0}F \tag{13}$$

where m_{p0} is the initial mass of the space platform, I_{sp} is the specific impulse, g_0 is the gravitational acceleration of the Earth's surface.

A. Kalman filter

We first discretize the system using the 1-order Euler method. In order to be able to identify the mass of the debris and estimate the actual length, $\theta=1/m_d$ will be regarded as one of the state of the system, then the states at kth sampling time is defined as $\hat{\boldsymbol{x}}_k=\begin{bmatrix}\hat{\rho}_k & \hat{\dot{\rho}}_k & \hat{\theta}_k\end{bmatrix}^T$, the input as $\boldsymbol{u}_k=\begin{bmatrix}T_k & F & 0\end{bmatrix}^T$. So the discrete model can be expressed as:

$$\begin{cases}\hat{\boldsymbol{x}}_{k+1}=\hat{\boldsymbol{A}}_{k+1}\hat{\boldsymbol{x}}_k+\hat{\boldsymbol{B}}_{k+1}\hat{\boldsymbol{u}}_k+\hat{\boldsymbol{\omega}}_k\\ \hat{\boldsymbol{y}}_k=\hat{\boldsymbol{H}}_k\hat{\boldsymbol{x}}_k+\hat{\boldsymbol{v}}_k\end{cases} \tag{14}$$

where $\hat{\boldsymbol{y}}_k\in\mathbb{R}^{1\times1}$ is the output at kth time, and $\hat{\boldsymbol{A}}_{k+1}=\begin{pmatrix}1 & \Delta t & 0\\ 0 & 1 & -T_k\Delta t\\ 0 & 0 & 1\end{pmatrix}$ is the system transfer matrix, Δt is sampling time, $\hat{\boldsymbol{B}}_{k+1}=\begin{pmatrix}0 & 0 & 0\\ \frac{\Delta t}{m_p} & \frac{\Delta t}{m_p} & 0\\ 0 & 0 & 0\end{pmatrix}$ is the system input matrix, $\hat{\boldsymbol{H}}_k=\begin{bmatrix}\frac{EA}{l_k} & \frac{\varsigma EA}{l_k} & 0\end{bmatrix}$ is the system output matrix, $\hat{\boldsymbol{\omega}}_k,\hat{\boldsymbol{v}}_k$ represent Gauss white noise of the process and measurement respectively, and the covariance is $\boldsymbol{Q}\in\mathbb{R}^{3\times3}$ and $\boldsymbol{R}\in\mathbb{R}^{1\times1}$.

Therefore, the discrete Kalman filter equation can be expressed as:

$$\begin{aligned}&\hat{\boldsymbol{x}}_k^-=\hat{\boldsymbol{A}}_k\hat{\boldsymbol{x}}_k+\hat{\boldsymbol{B}}_k\hat{\boldsymbol{u}}_k\\ &\boldsymbol{P}_k^-=\hat{\boldsymbol{A}}_k\boldsymbol{P}_{k-1}\hat{\boldsymbol{A}}_k^T+\hat{\boldsymbol{Q}}_k\\ &\boldsymbol{K}_{k+1}=\boldsymbol{P}_k^-\hat{\boldsymbol{H}}_k^T(\hat{\boldsymbol{H}}_k\boldsymbol{P}_k^-\hat{\boldsymbol{H}}_k^T+\hat{\boldsymbol{R}}_k)^{-1}\\ &\hat{\boldsymbol{x}}_{k+1}=\hat{\boldsymbol{x}}_k^-+\boldsymbol{K}_{k+1}(T_{k+1}-\hat{T}_{k+1})\\ &\boldsymbol{P}_k=(\boldsymbol{I}-\boldsymbol{K}_k\hat{\boldsymbol{H}}_k)\boldsymbol{P}_k^-\end{aligned} \tag{15}$$

where $\hat{\boldsymbol{x}}$ is the estimated state, T_{k+1} is the tension measurement at $k+1$th time, $\hat{T}_{k+1}$ is calculated using the estimated state at kth time, which is determined by:

$$\hat{T}_{k+1}(\hat{x}_k,\Delta l_k)=\begin{cases}\dfrac{[\hat{\rho}_k-(l_0+\Delta l_k)]EA}{l_0+\Delta l_k}+\varsigma\dfrac{(\hat{\dot{\rho}}_k-u_k)EA}{l_0+\Delta l_k} & ,\hat{\rho}_k>l_0+\Delta l_k\\ 0 & ,\hat{\rho}_k\le l_0+\Delta l_k\end{cases} \tag{16}$$

B. Nominal robust model predictive controller

Regardless of the disturbance and model error, taking the 1-order model as the tether retraction model, the MPC model [10] is:

$$\hat{\boldsymbol{x}}_{k+1}=\boldsymbol{A}_{k+1}\hat{\boldsymbol{x}}_k+\hat{\boldsymbol{B}}_{k+1}\hat{\boldsymbol{u}}_k \tag{17-a}$$

$$\Delta l_{k+1}=\Delta l_k+u_k\Delta t_{MPC} \tag{17-b}$$

Where $\hat{\boldsymbol{x}}_k=\begin{bmatrix}\hat{\rho}_k & \hat{\dot{\rho}}_k & \dfrac{1}{\hat{m}_d(k)}\end{bmatrix}^T$, and $\hat{m}_d(k)$ is updated at a given sampling time, it remains unchanged during the subsequent predictive horizon, Δt_{MPC} is the sampling time.

In order to avoid the collision of objects at both ends of the tether, the slack of the tether and the large retraction of the tether, the length change and the length of the tether can be defined as follows:

$$\boldsymbol{X}=\left\{\hat{\boldsymbol{x}}_k\left|\begin{matrix}l_0+\Delta l_k<\hat{\rho}_k\le\rho_{\max}\\ -v_{\rho\max}\le\hat{\dot{\rho}}_k\le v_{\rho\max}\end{matrix}\right.\right\} \tag{18}$$

$$L=\left\{\Delta l_k\left|-\Delta l_{\max}\le\Delta l_k\le\Delta l_{\max}\right.\right\} \tag{19}$$

As a control input, the retraction rate of the tether meets the following constraints:

$$\boldsymbol{U}=\left\{u_k\left|-u_{\max}\le u_k\le u_{\max}\right.\right\} \tag{20}$$

According to the Figs. 3-6, the actual length is asymptotically stable in the neighbourhood near the equilibrium position $\boldsymbol{x}_e=[\rho_{\mathrm{ref}},0]^T$, and the terminal region of the MPC can be defined as follows:

$$\boldsymbol{\Omega}=\left\{\hat{\boldsymbol{x}}_{k+N|k}\left|\begin{matrix}\left\|\hat{\rho}_{k+N|k}-\rho_{ref}\right\|\le\delta_\rho\\ \left\|\hat{\dot{\rho}}_{k+N|k}\right\|\le\delta_{\rho v}\end{matrix}\right.\right\} \tag{21}$$

where N is the predictive horizon, δ_ρ and $\delta_{\rho v}$ are the radiuses of the terminal region. When the system state is in the terminal region, it can be considered that the original length of the tether is close to the expected value, and the following formula is established:

$$l_{ex}=l_0+\Delta l_{k+N/k} \tag{22}$$

where $l_{ex}=\mathrm{EA}\rho_{ref}\left[\dfrac{\hat{m}_d(k)F}{\hat{m}_d(k)+m_p}+\mathrm{EA}\right]^{-1}$ is the expected natural length of the tether.

The system is asymptotically stable when the system state is in the terminal region, so we can set control input as zeros after Nth prediction, namely $u_{k+n/k}=0, n\ge N$, substituting it into (17-a) and subtracting the equation from the equilibrium position of the system can obtain the following error model.

$$\Delta\boldsymbol{x}_{ek+1}=\boldsymbol{\Phi}_k\boldsymbol{x}_{ek} \tag{23}$$

where $\Delta\boldsymbol{x}_{ek}=\hat{\boldsymbol{x}}_k-\boldsymbol{x}_e=\begin{pmatrix}\Delta\rho_k\\ \Delta\dot{\rho}_k\end{pmatrix}$ is the state error and

$$\boldsymbol{\Phi}_k=\begin{bmatrix}1 & \Delta t_{MPC}\\ -\dfrac{\left(m_p+\hat{m}_d(k)\right)\mathrm{EA}}{m_p\hat{m}_d(k)l_{ex}}\Delta t_{MPC} & 1-\dfrac{\varsigma\left(m_p+\hat{m}_d(k)\right)\mathrm{EA}}{m_p\hat{m}_d(k)l_{ex}}\Delta t_{MPC}\end{bmatrix} \tag{24}$$

Therefore, based on the control objective of $\hat{x}_k\to x_e$ and the error model (23), the cost function with the terminal cost term is designed as:

$$\boldsymbol{J}_k=\sum_{n=0}^{N-1}\left(\left\|\hat{\boldsymbol{x}}_{k+n|k}-\boldsymbol{x}_e\right\|^2_{\boldsymbol{Q}_{MPC}}+\left\|u_{k+n|k}\right\|^2_{\boldsymbol{R}_{MPC}}\right)+\left\|\hat{\boldsymbol{x}}_{k+N|k}-\boldsymbol{x}_e\right\|^2_{\boldsymbol{P}} \tag{25}$$

where $\boldsymbol{Q}_{MPC}$ and $\boldsymbol{R}_{MPC}$ are weighting matrices of states and control respectively, $\boldsymbol{P}$ is the weighting matrix of the terminal cost.

In summary, the NRMPC designed for the system (11) is used to solve the following optimization problem:

$$\boldsymbol{J}_k^*=\min_{u_{k+n|k}}\boldsymbol{J}_k\left(\hat{\boldsymbol{x}}_{k+n|k},u_{k+n|k}\right)$$

$$\begin{cases}\hat{\boldsymbol{x}}_{k+1}=\boldsymbol{A}_{k+1}\hat{\boldsymbol{x}}_k+\hat{\boldsymbol{B}}_{k+1}\hat{u}_k\\ \Delta l_{k+n+1|k}=\Delta l_{k+n|k}+u_{k+n|k}\Delta t_{MPC}\\ \hat{\boldsymbol{x}}_{k+n|k}\in\boldsymbol{X}\\ \hat{\boldsymbol{x}}_{k+N|k}\in\boldsymbol{\Omega}\\ u_{k+n|k}\in\boldsymbol{U}\\ \Delta l_{k+n|k}\in L\end{cases} \tag{26}$$

The proof of the stability of the proposed control method is detailed in [11].

The final tension control structure is shown in Fig.7.

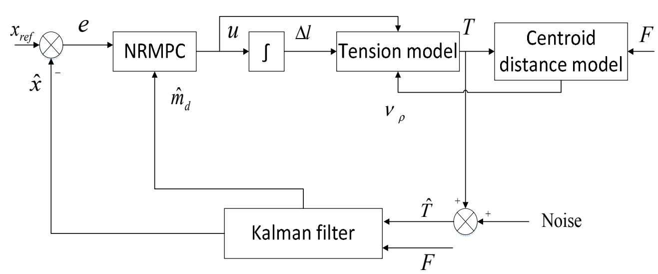

Figure 7. The tension control diagram

V. Simulation Results

In this section, it is assumed that the thrust of the platform is along the direction of the tether. In this paper, the initial mass of the space platform is chosen to be $m_{p0}=2000$kg ,the specific impulse be $I_{sp}=325$s , and the standard gravitational acceleration is $g = 9.80665 \mathrm{m/s^2}$. Assume that the true mass of the debris is $m_d = 3000\mathrm{kg}$, and the initial estimated value is taken as $\hat{m}_{d0} = 2500\mathrm{kg}$. The thrust of the platform is a step function and is not taken as a constant, so that the designed controller can be tested for its ability to respond to different thrusts. In this case, the simulation time of the thrust is set as $300\mathrm{s}$. Let the change of the platform thrust as follows:

$$F = \begin{cases} 80\mathrm{N}, & t \le 100\mathrm{s} \\ 40\mathrm{N}, & 100\mathrm{s} < t \le 200\mathrm{s} \\ 20\mathrm{N}, & t > 200\mathrm{s} \end{cases} \tag{27}$$

With the thrust and (27), the tension reference is as follows:

$$T_{ref} = \begin{cases} 48\mathrm{N}, & t \le 100\mathrm{s} \\ 24\mathrm{N}, & 100\mathrm{s} < t \le 200\mathrm{s} \\ 12\mathrm{N}, & t > 200\mathrm{s} \end{cases} \tag{28}$$

Parameters of the Kalman filter and NRMPC are shown in Table Ⅰ and Table Ⅱ. Results are shown in Figs. 8-12.

It can be seen from Fig. 8 that the mass of the platform is decreased by 4.5kg under the conditions, and the change is small enough, so it is reasonable to calculate the reference tension command by taking the same mass.

Fig. 9 shows that the mass of the debris oscillates in the first few seconds, but then the real value can be reached very quickly. It can be seen that the use of Kalman filter for quality identification has achieved good results.

TABLE I
THE PARAMETERS OF KALMAN FILTER

Parameters	Value
Q	$diag([0,0,10^{-20}])$
R	0.01
x_0	$[99.999,0]^T$
$\hat{m}_{d0}(\mathrm{kg})$	2500
P_0	$1 \cdot eye(3)$
$\Delta t(\mathrm{s})$	0.01

TABLE II
THERMPC PARAMETERS OF NRMPC

Parameters	Value
Q_{MPC}	$diag([25,15])$
R_{MPC}	2
$\lvert u \rvert(\mathrm{m/s})$	0.05
$\rho_{\max}(\mathrm{m})$	100.5
$v_{\rho\max}(\mathrm{m/s})$	0.2
$\Delta l_{\max}(\mathrm{m})$	0.1
$\delta_\rho(\mathrm{m})$	10^{-3}
$\delta_v(\mathrm{m/s})$	10^{-4}
N	20
$u_{0/0}$	$zeros(1,20)$
$\Delta t(\mathrm{s})$	0.1

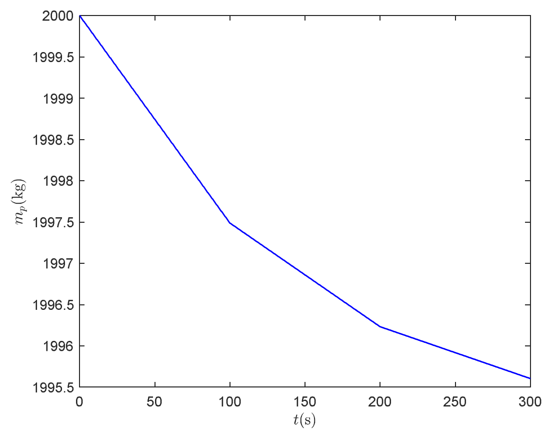

Figure 8. The change of mass of the platform

Fig. 10 shows the tracking effect of the tension. At the 142s, we can see that the expected value of the tension is 24N and the actual tension fluctuates around 24N. So it can be concluded that the tension can always converge to the expected command quickly in each thrust phase. The three phases with different thrusts can be found that the smaller the thrust, the smaller the amplitude of the tension oscillation and the tension converges faster. This shows that the designed control scheme has achieved good results.

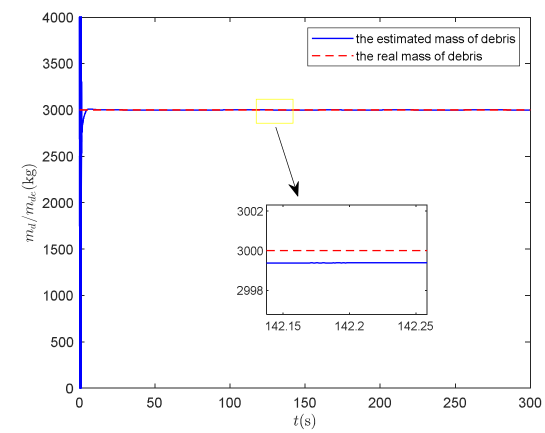

Figure 9. The estimated mass of the debris

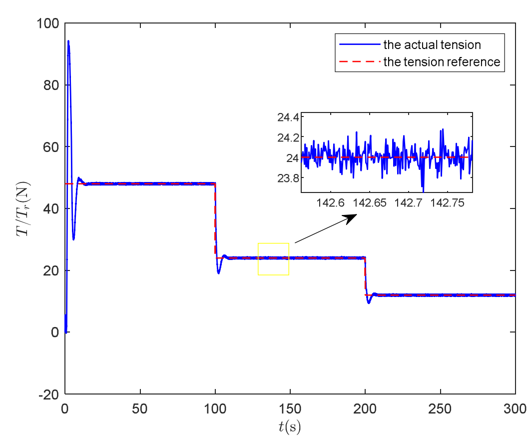

Figure 10. The actual tension and the tension reference

The control efforts are presented in Fig. 11 where the length change decreases from $-0.02m$ to $-0.0045m$ and finally stays at $-0.0045m$, while the optimal control input only expresses oscillation at the beginning of the thrust change in Fig. 12.

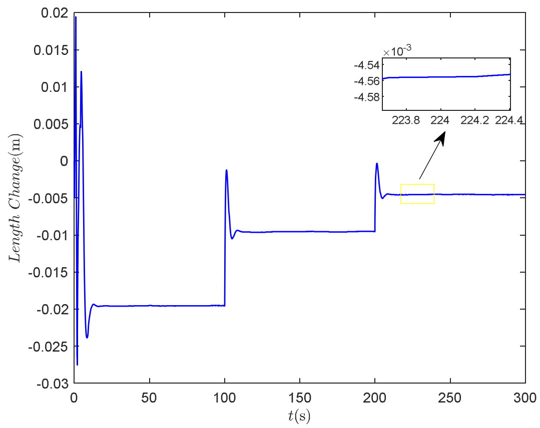

Figure 11. The optimal control input

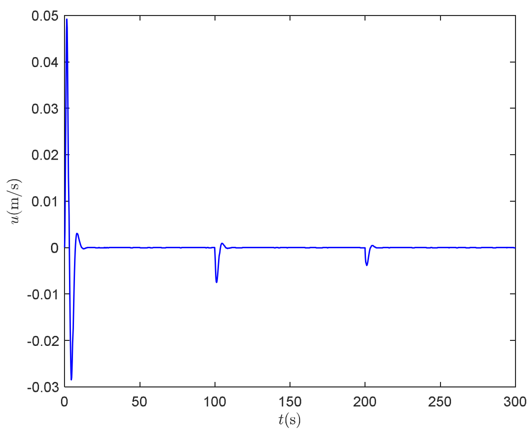

Figure 12. The length change of the tether by the reel

VI. Conclusions

The purpose of this paper is to solve the tension control problem of the tethered space system, otherwise there may be a risk of entanglement or even collision at the later stage. We first establish the tension model and the centroid distance model of TSS. It is concluded that the centroid distance is asymptotically stable near the equilibrium point. In addition, under the premise that the tension can be measured, a Kalman filter is designed to estimate the actual length of the tether and determine the mass of the debris. The nominal robust model predictive control is used to obtain the optimal retraction rate of the tether. It can be seen from the simulation results that this approach can accurately track the step command of the tension and embodies the excellent control performance.

Acknowledgment

This work is supported by the National Natural Science Foundation of China (Grant No: 61773317), Natural Science Basic Research Plan in Shaanxi Province of China (Program No: 2019JM-406) and the Fundamental Research Funds for the Central Universities (Grant No: 3102018JGC001).

References

[1] F. Zhang, I. Sharf, A. Misra, "On-line estimation of inertia parameters of space debris for its tether-assisted removal," *Acta Astronautica*, vol. 107, pp. 150-162, 2015.

[2] Mankala. K, AgrawalSK, "Dynamics modeling and simulation of satellite tethered systems," *Transaction of the ASME*, vol. 127, no. 2, pp. 144-156, 2005.

[3] L. Jasper, H. Schaub, "Tether design considerations for large thrust debris de-orbit burns," *AAS/AIAA Space Flight Mechanics Meeting*, 2014.

[4] R. Benvenuto, M. Lavagna, S. Salvi, "Multibody dynamics driving GNC and system design in tethered nets for active debris removal," *Advances in SpaceResearch*, vol. 58, pp. 45-63, 2016.

[5] R. Benvenuto, S. Salvi, M. Lavagna, "Dynamics analysis and GNC design of exible systems for space debris active removal," *Acta Astronautica*, vol. 110, pp. 247-265, 2015.

[6] H.T.K. Linskens, "Tether Dynamics Analysis and Guidance and Control Design for Active Space Debris Removal," Master Thesis, Delft University of Technology, 2015.

[7] S. Cleary, W.J.O. Connor, J. William, "Control of Space Debris Using an Elastic Tether and Wave-Based Control," *Joural of Guidance, Control and Dynamics*, vol. 39, no. 6, pp. 1392-1406, 2016.

[8] M. Sabatini, P. Gasbarri, G.B. Palmerini, "Elastic issues and vibration reduction in a tethered deorbiting mission," *Advances in Space Research*, vol. 57, pp. 1951-1964, 2016.

[9] Z.J. Meng, B.H. Wang, P.F. Huang, "Twist suppression method of tethered towing for spinning space debris," *Journal of Aerospace Engineering*, vol. 30, no. 4, 2017.

[10] B.H. Wang, et al, "Tension control of tethered space robots," unpublished.

[11] Z.J. Meng, B.H. Wang, P.F. Huang, "MPC-based anti-sway control of tethered space robots," *Acta Astronautica*, vol. 152, pp. 146-162, 2018.