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Reactive Deployment of Autonomous Drones for Livestock Monitoring Based on Density-based Clustering*

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***Abstract*— Previous research has established that drones could be an efficient tool for farming. In this paper, we study the problem of livestock tracking and monitoring using a group of drones. The main objective is to deploy a limited number of drones to track and monitor the maximum number of livestock, such as cattle and sheep, in a vast pasture while minimizing the average drone-animal distance. We assume the targeted livestock have been fitted with GPS collars, and the mobility of each animal cannot be neglected. We first introduce a procedure of performing sweep coverage by drones. By deploying drones to accomplish sweep coverage for the entire pasture, the initial locations of all targeted animals can be acquired. Then we applied a density-based clustering algorithm DBSCAN to find the deployment that places the drones to the centroids of animal clusters. Furthermore, based on the updated animals' locations, the deployment of the drones can be updated to follow the movement of animal clusters. We demonstrate that our solution can always yield a lower average drone-animal distance and a higher number of covered targeted animals, compared with the standard K-Means clustering algorithm.**

***Index Terms*— Autonomous drones, livestock monitoring, reactive deployment, density-based clustering.**

I. Introduction

Animal tracking and monitoring have long been topics and of interest for both wildlife and livestock researchers. For example, ecologists worldwide study the movements of animals around the landscape, the animal performance and behavior, the spatial heterogeneity of field occupancy by animals, the behavioral responses to heterogeneous environments, and the social affiliations within a herd by animal tracking [1]–[3]. Not only is an important component of natural areas management, animal tracking and monitoring is also a powerful technique for domestic livestock management [4], [5].

Satellite-based Global Positioning Systems (GPS) have the potential to collect large amounts of high-quality location data 24 hours per day and under all weather conditions. Thus, GPS-based systems have been used by ecologists to track animals since the 1990s [6]. For example, [7] developed and tested a low-cost (∼US$125), open-source GPS collar to track livestock based on Arduino. The ZebraNet project [8] innovatively applied opportunistic sensor networks for wildlife monitoring. Furthermore, some GPS collars can be integrated with the sensors that monitor the wellbeing of animals, producing health and behavioral data. Which can be collected and analyzed enables alerts sending when anomalies occur [9]. While the GPS collar technology has been well developed and can offer a position recording with relatively high accuracy, how to efficiently collect and update the recorded data from the animals equipped with GPS collars in large areas, however, can be challenging. [7] collect the positions data through reading the SD memory card integrated into the GPS collar thus cannot acquire the data in real-time. ZebraNet uses mobile vehicles as base stations, which periodically moved around in the savanna, and collect data from all encountered zebras. The mobility of the vehicle, however, could be limited by ground traffic conditions.

Due to the mobility and rapid deployment features, drones could be deployed to do the surveillance and follow the movement of animals in a herd, keep a relatively closer distance with the animals. Without been limited by ground traffic conditions, drones could be rapidly deployed to any suitable location. Besides, an architecture that consists of drones with switchable batteries and the ground drone bases for battery replacing can achieve persistent coverage to a specific location, by alternately deploying two or more drones. A number of studies have investigated the applications of UAV for surveillance purposes [10]–[12]. The updating data collected by drones can be then uploaded to the server through the wireless backhaul links to nearby ground base stations or satellites. Moreover, by attaching additional equipment such as a camera to the drones, drones as the data sink can provide additional functions such as real-time video monitoring to the livestock [13]. Additionally, using drones to track and monitor livestock could contribute to precision livestock farming that aims to manage individual animals by continuous and real-time monitoring of their welfare, health,

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production, and environmental impact [14].

Another factor that appears to be very important during animal tracking is how social the animals are. Many animals tend to live in a herd or cluster rather than staying alone, such as two-common livestock: cattle and sheep. The gregarious nature of sheep, exhibiting close spatial contact within the flock, has been extensively described in the previous literature [15]. Cattle are also gregarious animals, but the distance among animals on the pasture usually is larger than in the case of sheep. The location and the formation of the livestock herd, however, are often not fixed and can be influenced by factors such as the habitat characteristics, the grass availability, the season or the time of the day [16].

Papers relating to coverage control of mobile robots include [17]–[22]. Where [17] addresses the problems of sweep coverage by a decentralized control algorithm based on a simple consensus algorithm. [18] proposed a decentralized solution for the problems of barrier coverage and sweep coverage by self-deployed mobile robots based on the nearest neighbor rule and formation consensus. In contrast to the work [17], [18], we consider the control of drones is a centralized control problem as we assume the drones have the full ability to communicate with the control centers and all other drones, not only the neighbor drones. Sweep coverage problems studied in [19]–[21] are related to complete coverage path planning problems. To solve this type of problem, a map of the operating region is required to be known or able to be constructed online for the path planner to generate robot paths that can completely cover the operating region.

In this paper, we study the following problem: deploying a limited number of drones to track and monitor the maximum number of the livestock such as cattle and sheep in a vast pasture while minimizing the average drone-animal distance. Suppose the targeted animals have been fitted with GPS collars, and the mobility of each animal cannot be neglected. We assume that there exist animal clusters in the operating area, contains most of the targeted animals. To solve this problem, a procedure of performing sweep coverage by drones is introduced. By deploying drones to achieving sweep coverage to the entire pasture, the initial locations of all targeted animals can be acquired. Then we applied a density-based clustering algorithm DBSCAN to find the deployment that places the drones to the centroids of animal clusters. Besides, the locations of all the targeted animals within the communication range of drones can be updated continuously. Based on the updated animals' locations, the deployment of the drones can be updated to the new animal clusters' centroids. Animals within the coverage of any drone can also have their health and behavioral data been uploaded through the drone to the farmers for monitoring purposes.

II. System Model

The scenario we consider is the livestock such as cattle and sheep in a relatively wide pasture need to be tracked and monitored. A limited number of m drones that labeled $1, 2, \ldots, m$ are deployed to track and monitor these animals. The drone's coverage is disk-like, with a coverage radius of R_d . Each of the targeted animals has been fitted with a GPS collar that can constantly transmit its current location, health, and behavioral data to the drone receiver when the animal is within the coverage area of the drone. The area of each drone's coverage region is $A_d = \pi R_d^2$. Throughout this paper, we assume

$$A_P >> mA_d \tag{1}$$

where A_P is the area of operating pasture. Suppose there are N uncovered targeted animals labeled $1, 2, \ldots, N$. Let the set $U^k = \left\{U_1^k, U_2^k, \cdots, U_N^k\right\}$ denotes the coordinates of the targeted animals' locations at time k, $D^k = \left\{D_1^k, D_2^k, \ldots, D_m^k\right\}$ denotes the coordinates of the drones' deployment locations at time k, where $D^k, U^k \in R^2$. A specific number of animal clusters are randomly distributed in the operating area. Note that targeted animals are not necessarily to be within the animal clusters. The number of targeted animals stayed out of any clusters u_o and the number of the clustered animals u_c, however, satisfy the following assumption:

$$u_o << u_c \tag{2}$$

Which means the targeted animal clusters contain most of the targeted animals within the operating area. Moreover, for all the targeted animal clusters, we assume

$$d_{i,j} \leq 2R_d, \forall i, j \in U_C, i \neq j \tag{3}$$

Where U_C is the set contains all targeted animals within the cluster, i and j are two different targeted animals belong to U_C, and $d_{i,j}$ donates the distance between them. Equation (3) indicates that a drone can entirely cover any single animal cluster. Some animals may be present out of the coverage of any drones due to the limited number of drones. The target of covering a maximum number of targeted animals can be realized by using drones to find locations of targeted animals and then deploying the drones to the optimal locations that can entirely cover each animal cluster while minimizing the average drone-animal distance of each drone. The updating data collected by drones is uploaded to the server through the wireless backhaul links to satellites or nearby ground base stations. We assume that the movement of some targeted animals cannot be ignored. An illustration of the considered scenario is shown in Fig. 1. According to [23], the optimal altitude and the corresponding coverage radius of the flying drone providing wireless connectivity can be numerically solved for a specific environment (urban, suburban) and a given path loss threshold. Therefore, in this paper, we only focus on finding the optimal two-dimensional (2D) deployment of drones and deploy all the drones at the same altitude with a fixed coverage radius.

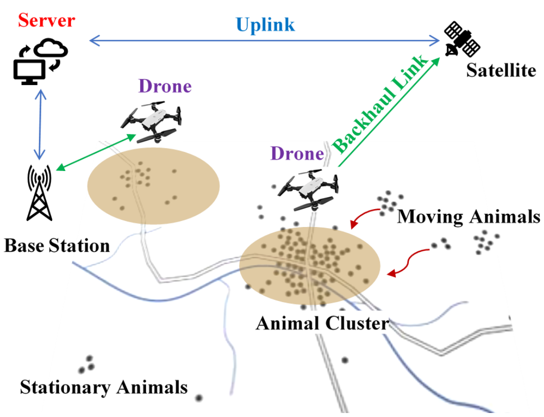

Fig. 1. An illustration of the considered scenario

III. Proposed Method

We now introduce how to find the optimal deployment of drones for covering animal clusters and minimizing the average drone-animal distance. We assume that no apriori targeted animals' location information is available. The first step is using drones to acquire the initial locations of the targeted animals. In the second step, determine the initial deployment of the drones, and update the deployment by density-based clustering.

A. Sweep Coverage and Animals Positioning

We assume that the map of the operating pasture is apriori and partly adopts the coverage path planning procedure proposed in [22] because of its generality and simplicity. Accordingly, we proposed a centralized control algorithm to achieve efficient sweep coverage by area decomposition and zigzag sweeping path, as shown in algorithm 1.

Algorithm 1 Sweep Coverage.

1: Determines the operating pasture for drones as a convex polygon;
2: Create the diameter function $d(\theta)$ of the polygon;
3: Find the optimal sweep direction from $d_{\min}$;
4: Slice the polygon with the divide lines parallel with the optimal sweep direction to match the pre-defined sub-area proportions;
5: Assign each generated sub-area to one drone;
6: Deploy each drone to sweep the assigned sub-area thoroughly, follows the zigzag pattern.

In detail, the inputs of the strategy are the convex polygon P modeled from the pasture and its area A_P, the number of sub-areas s that equals to the number of available drones m, and the proportions $\{p_1, p_2 \cdots p_s\}$ indicate the area of P that should be assigned to each sub-area. The outputs of the algorithm are the optimal sweep direction and a set of sub-areas $\{S_1, S_2 \cdots S_s\}$ with their areas equal to $\{A_1, A_2 \cdots A_s\}$, which satisfy

$$A_i = p_i A_P, i = 1, 2 \cdots s \tag{4}$$

$$\sum_{i=1}^{s} A_i = A_P \tag{5}$$

The proportions $\{p_1, p_2 \cdots p_s\}$ could be determined by the relative capabilities of drones, such as the remaining battery life. The diameter function $d(\theta)$ is created by rotating the polygon with the angle $\theta \in [0, 2\pi)$. Where diameter d refers to the height difference measured between the highest and lowest points on the boundary of the rotating polygon [22]. The angle that gives a minimum diameter $d_{\min}$ is θ_{opt}, i.e.

$$d_{\min} = d(\theta)|_{\theta=\theta_{opt}} \tag{6}$$

Since turning a drone is more time-consuming than letting it follows a straight line, we now compute the optimal sweep direction that minimizes the number of turns needed following the zigzag pattern, subsequently, reduce the total mission time.

According to [22], the optimal sweeping direction is vertical to the direction that gives the minimum polygon diameter $d_{\min}$. Finally, we divide the original polygon along the optimal sweep direction so that the area of each part is as same as calculated by (4). This can be finished by sweeping a line along the polygon with the direction that gives the minimum diameter, slicing the polygon once the area of the part on one side of the line equals to any that has not been cut yet [22]. Fig. 2 shows two examples of the polygon area decomposition and zigzag path planning results, with the number of the original polygon were divided into four and three sub-areas, respectively. After the drones finished the sweep coverage to the entire operating pasture. All the targeted animals' initial locations $U^0 = \{U_1^0, U_2^0, \ldots, U_N^0\}$ could be acquired.

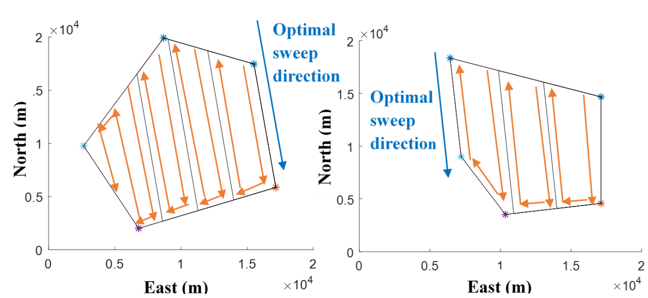

Fig. 2. Sweep coverage examples

B. Find the optimal deployment by density-based clustering

We now determine the optimal deployment that places the drones to the centroids of animal clusters to minimize the average drone-animal distance based on obtained targeted animals' location information. The goal of covering a maximum number of targeted animals can be achieved by find and cover the animal clusters with more animals. This is a non-convex, non-smooth and NP-hard optimization problem. Many publications have shown that some heuristic algorithms are capable of finding approximate solutions for such a problem. Since we suppose that the density of clustered targeted animals is much higher than non-clustered targeted animals, we adopt a density-based clustering algorithm DBSCAN to solve the problem.

The idea is first to define a radius parameter ε so that the ε-neighborhood of a selected point indicates a disk-like area with the selected point as the center and ε as the radius. Call the points within the ε-neighborhood as the ε-neighbors, and the average point density of ε-neighbors as the ε-neighbor density of the selected point. Then select a neighbor density threshold ρ_{th} so that the points with their ε-neighbor density less than ρ_{th} are called low-density points, and the points with their ε-neighbor density higher than ρ_{th} are called high-density points or the core points. After that, randomly visit a data point, record the ε-neighbor density of the point. If the point is a high-density point and in the ε-neighborhood of another high-density point, connect these two points together. If the point is a low-density point and in the ε-neighborhood of a high-density point, connect it to the nearest high-density point and call it a boundary point. Then repeat until all points have been visited. The detailed algorithm of DBSCAN is as shown in algorithm 2. We apply the DBSCAN algorithm by taking the acquired initial locations of all targeted animals as the input data set. The output is a set of targeted animal clusters. After that, deploy the drones to the centroids of the animal clusters containing more targeted animals.

After finishing the initial deployment, the drones remain static at their assigned locations, i.e., animal clusters' centroids. Then, acquire and update the locations of covered targeted animals that have changed their locations as well as the targeted animals newly joined or left the covered clusters. After that, compute the new clusters' centroids based on updated animal locations and deploy the drones to the new centroids. By continually doing this way, the drone could autonomously move to follow the moving animal cluster. Fig. 3 shows an example of the deployment generated by DBSCAN with two drones.

Algorithm 2 DBSCAN

Input:
  $U^0 = \left\{U_1^0, U_2^0, \ldots, U_N^0\right\}$, ε, ρ_{th}
Output:
  A set of density-based targeted animal clusters

```
mark all data points in U^0 as unvisited;
repeat
   randomly select an unvisited point U_i^0
   mark U_i^0 as visited
   if the ε-neighbor density of U_i^0 is larger than ρ_th then
      create a new cluster C_n, and add U_i^0 to C_n
      let N_p be the set of ε-neighbors of U_i^0;
      for each point p' in N_p do
         if p' is unvisited then
            mark p' as visited;
         end if
         if p' is not yet a member of any cluster then
            add p' to C_n;
         end if
         output C_n;
      end for
   else
      mark U_i^0 as noise;
   end if
until no point in U^0 is unvisited.
```

IV. Simulation Results

In this section, we present simulation results for deploying drones by our DBSCAN based solution and the standard K-Means clustering based solution. We simulated the deployment of a team of m drones in a 5km $\times$ 5km quadrangle pasture. Theoretically, the proposed sweep control method in section 3.1 can guarantee a 100% sweep coverage to the operating area. Therefore, the simulation is conducted based on the assumption that all the initial animal locations have been obtained by sweep coverage, and the estimated locations of all the targeted animals in the covered clusters can be updated continuously. Moreover, based on these updated locations, we assume that drones' 2D placement can be maintained at the centroids of covered animal clusters with DBSCAN based solution. Whereas the standard K-Means clustering based solution cannot reactively adapt to moving animals' scenario because it requires the location information of all the targeted animals within the operating area. This contradicts our considered scenario that the limited number of drones cannot cover the entire operating area, and the reality that only covered targeted animals' location can be estimated. For simplicity, we only show the simulation results for the initial deployment results by DBSCAN based solution and the standard K-Means clustering based solution.

To generate the targeted animals' distribution in the operating area, we first generate animal clusters' center locations through a two-dimensional uniformly random process. After that, we generate the locations of the clustered animals and the locations of non-clustered animals by two independent Poisson point processes (PPPs), with the clustered animal density of λ_c and the non-clustered animal density of λ_{nc}.

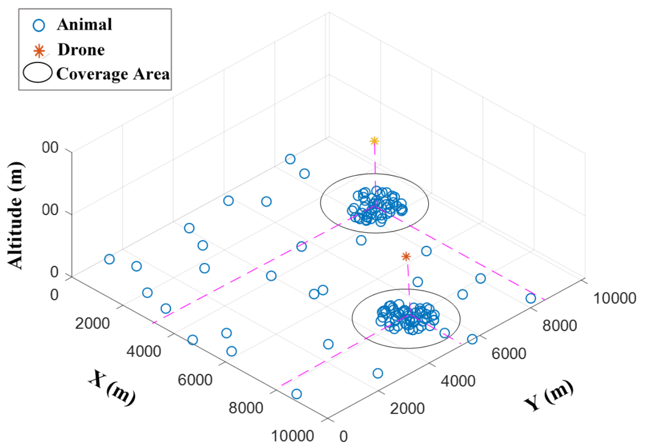

Fig. 3. Example of a generated deployment of two drones

The number of animal clusters is N_c. The number of animal clusters is N_d. Animal clusters are set to be disk-like, with their radius no larger than the maximum radius R_{max}. The coverage radius of the drone is R_d. The drone-BSs have no apriori knowledge on targeted animals' locations. The neighborhood density threshold is ρ_{th}. The neighborhood radius parameter is ε. ρ_{th} and ε are inputs of DBSCAN. The detailed simulation parameters are as shown in Table 1.

TABLE I
SIMULATION PARAMETER VALUES

Parameters	Values	Parameters	Values
R_{max}	500 m	R_d	700 m
λ_c	1e-4 animals/m^2	λ_{nc}	1e-6 animals/m^2
ρ_{th}	1e-5 animals/m^2	ε	400 m

The comparison of the deploying two drones to track and monitor livestock by the proposed DBSCAN based method and K-Means clustering based method is as illustrated in Fig. 4. Where (a) stands for one animal cluster exists in the operating area, and (b), (c) stands for two and three animal clusters exist in the area, respectively. The dots stand for the targeted animals while the hollow rings stand for drones' coverage areas. It can be obviously seen from Fig.4 that the DBSCAN based method always leads to the deployments that are closer to the centroids of the covered animal clusters, compare with the K-Means clustering based method. Thus, the DBSCAN based method leads to a lower average drone-animal distance. Moreover, as shown in Fig.4 (c), in the case of more animal clusters than available drone-animals, K-Means cluster could lead to a worse deployment that some drones failed to cover an entire animal cluster, thus leads to a lower number of covered targeted animals and higher average drone-animal distance. This is due to DBSCAN treats the non-clustered animals' locations as noise, while the result of K-Means clustering could be easily influenced by non-clustered animals.

Fig. 5 compares the number of covered targeted animals and the average drone-animal distance versus the number of drones by DBSCAN and K-Means clustering based method, with four animal clusters exist in the operating area. Fig.5 indicates that our proposed DBSCAN based method always performs better in terms of a higher number of covered targeted animals and lower average drone-animal distance. Note that the number of covered targeted animals for the K-means clustering based method does not increase follow the number of available drone-animals. This is attributed to the interference of the non-clustered animals during K-means clustering.

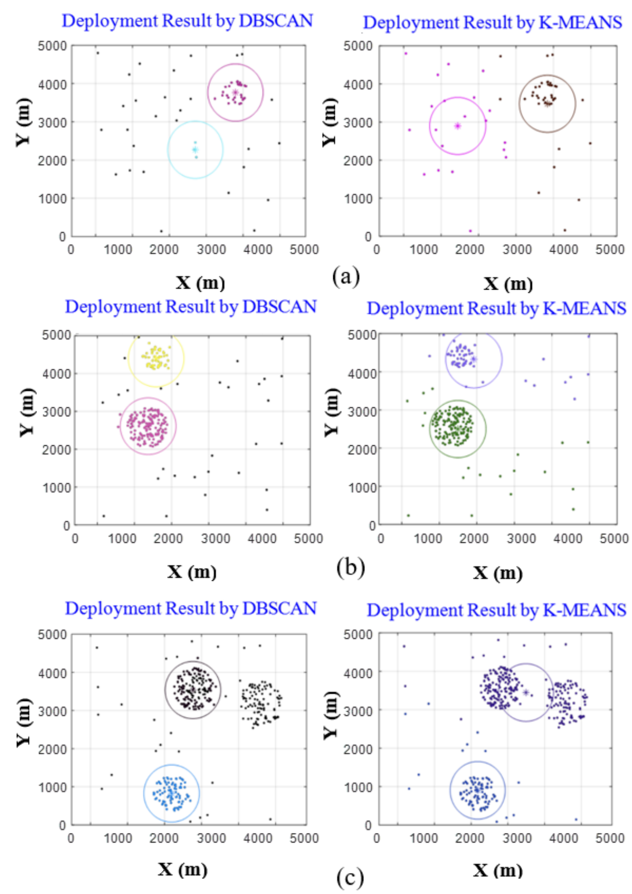

Fig. 4. Comparison of deploying two drones by DBSCAN and K-Means clustering to serve 1,2 and 3 animal clusters respectively

V. Conclusion

In this paper, we considered the problem of livestock tracking and monitoring using a group of drones, with the target of maximizing the total number of livestock covered by a limited number of drones in a vast pasture, while minimizing the average drone-animal distance. We first introduce a

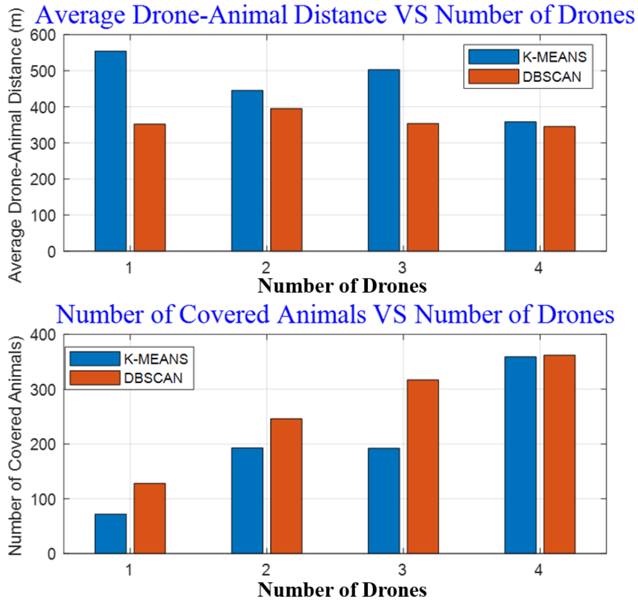

Fig. 5. Comparison of the number of covered targeted animals and average drone-animal distance versus the number of drones by DBSCAN and K-Means clustering based method with 4 animal clusters exits.

procedure of performing sweep coverage by drones to acquire the initial locations of all targeted animals. Then we applied a density-based clustering algorithm DBSCAN to find the deployment that places the drones to the centroids of animal clusters. In addition, the deployment of the drones can be updated to follow the movement of animal clusters based on the updated animals' locations. The proposed method relies on accurate location information of targeted animals. One future research direction is to extend the current method to the case where more precise position estimation can acquired by some estimation tools such as robust Kalman filter [24]–[27].

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