Proceeding of the IEEE
International Conference on Robotics and Biomimetics
Dali, China, December 2019

Finite-time Attitude Stabilization Control of a Quadrotor with Parametric Uncertainties and Disturbances*

Zhi Li, Xin Ma* and Yibin Li
School of control science and engineering
Center for robotics
Shandong University
Jinan, Shandong 250061, China
maxin@sdu.edu.cn

***Abstract*— In practical applications, some system parameters of the quadrotor UAVs such as inertia moments and thrust coefficients are usually unknown or difficult to measure. Moreover, the quadrotor UAVs are suffering from bounded disturbances and model uncertainties. To address this problem, this paper proposes an adaptive non-singular terminal sliding mode controller (ANTSMC) is designed to stabilize the quadrotor in the presence of the unknown system parameters and the external disturbances. The adaptive laws are designed to estimate the unknown model parameters to avoid the requirement for the information of model dynamics. In addition, proper adaptive laws are designed to estimate the upper bounds of the external disturbances. Using the estimations, the need for large control gains is avoided. Lyapunov stability analysis have proven that the ANTSMC could guarantees the finite-time convergence of the states of the quadrotor. Experimental results are given to validate the robustness and effectiveness of the proposed controller.**

***Index Terms*— Adaptive control, Attitude stabilization, Finite-time convergence, Sliding mode control, Quadrotor.**

I. Introduction

Over the past recent years, the researchers have paid much attention to the quadrotor unmanned aerial vehicles (UAVs) due to their special characteristics such as low cost, vertical take-off and landing (VOTL) ability, flight flexibility etc. The quadrotor UAVs are widely used in civil and military fields.

In practical applications, the attitude of the quadrotor UAV is stabilized automatically using the on-board controller. The attitude control of a quadrotor UAV, which allows the quadrotor to maintain desired orientation and prevents the quadrotor from flipping over is still an open problem [4]. The quadrotor has complex dynamics and strong nonlinear characteristics such as strong coupling among the state variables and under-actuated characteristic etc. Such that the quadrotor attitude control is not an easy task. Moreover, the quadrotor UAV is very sensitivity to the external disturbances such as wind, nonlinear frictions and the model uncertainties which makes the quadrotor attitude control more challenging [5, 6]. For these reasons, it makes great sense to develop advanced quadrotor attitude controller with fast convergence, adaptive capability and strong robustness.

Over the past decades, many controllers have been developed to control the quadrotor [7-9]. Among these control approaches, the sliding mode control (SMC) has been widely applied in quadrotor control [10]. The SMC has advantages such as strong robustness to disturbances, model uncertainties and parametric variations. For example, in [11] the quadrotor UAV is divided into two subsystems: a full-actuated subsystem and a under-actuated subsystem. A robust sliding mode controller is designed to control the under-actuated subsystem to keep the roll and pitch angles zero. In [12], the SMC is combined with backstepping approach. Then a robust nonlinear controller is proposed to drive the quadrotor to desired position in the presence of disturbances and actuator faults. Considering that the calculation of the derivatives of the Coriolis matrix is difficult, a double-loop integral sliding mode controller is designed [13].

As is well known that, the discontinuous switching control of traditional SMC will cause the chattering problem. The chattering has damage to the quadrotor system and even lead to system breakdown. To solve this problem, a continuous saturation function is used to replace the discontinuous sign function and the experimental results is given to validated the effectiveness of the algorithm [14]. In [15], a fuzzy estimator is designed and by replacing the sign function by the estimator, the SMC law becomes continuous. The aforementioned traditional SMC has linear sliding mode hyperplanes and could enable the tracking error converge to zero asymptotically [16]. For the sake of finite-time convergence, high-order SMC method such as terminal sliding mode control (TSMC) has been widely applied in the trajectory tracking control of a quadrotor [17-20]. Furthermore, in [21] a non-singular terminal sliding mode controller (NTSMC) is first proposed for a class of nonlinear systems. The NTSMC technique not only enable the state variables converge to desired values in finite time, but also solved the singularity

*This work was supported in part by the Joint Funds of the National Natural Science Foundation of China-Shandong Province (Grant No. U1706228), Chinese National Natural Science Foundation (Grant No.61673245) and Chinese National Programs for High Technology Research and Development (Grant No.2015AA042307).

problem associated with TSMC.

In consideration of the disturbances and uncertain dynamics, the SMC based techniques could achieve satisfactory attitude or trajectory tracking performance, but the aforementioned SMC techniques are very conservative when the quadrotor model parameters and the information of upper bounds of the disturbances cannot be exactly achieved. Moreover, the quadrotor model parameter identification is not an easy task. It requires high accuracy equipment and repeated experiments. Furthermore, the upper bounds of the disturbances which are difficult to determine in advance requires high switching gains which may make the chattering problem more serious [22].

The adaptive control approach is a suitable choice to solve the parametric uncertainty problem. It provides a possible solution to overcome the above challenges [23]. Combining the terminal sliding mode control with the adaptive control approach, an adaptive sliding mode controller is designed for stability and tracking control of a quadrotor with parametric uncertainties [24]. The unknown terms are estimated by using the adaptive laws. Then a terminal sliding mode controller is designed to ensure the tracking error and estimate errors converge to zero in finite time. In [25], an adaptive non-singular terminal sliding mode controller is designed to eliminating tracking errors and certify Lyapunov stability. The adaptive tuning laws are designed to control the attitude of quadrotor and deal with the bounded disturbances. A disturbance observer based adaptive SMC controller is designed to control the position of the quadrotor. However, to our best knowledge, most adaptive terminal sliding mode controllers are only validated via numerical simulations and very few researchers has implemented the adaptive high-order SMC strategies on a real quadrotor. The main contributions of this paper are as follows:

1) An adaptive non-singular terminal sliding mode controller is designed to regulate the quadrotor attitude in the presence of parametric uncertainties and disturbances.
2) Proper adaptive tuning laws are well-designed to estimate the unknown inertia moments, aerodynamic damping factors and the upper bounds of the disturbances. By using these estimations, the requirement for the knowledge of model parameters and the upper bounds of disturbances is avoided.
3) The control performances of the proposed ANTSMC are obtained by conducting experiments on a self-made quadrotor testbed which make the results more convincing.

The rest of this paper is organized as follows: Section II presents the kinematics and dynamics models of a quadrotor. The ANTSMC design procedure and stability analysis are given in Section III. The experimental results are given in Section IV. Finally, Section V gives the conclusions.

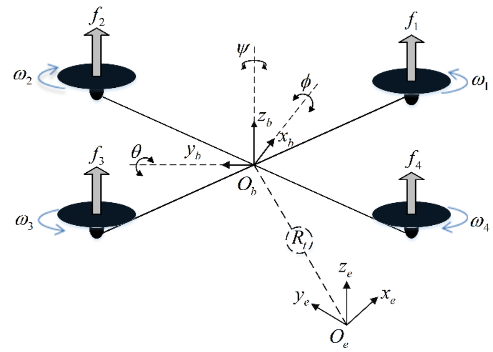

Fig. 1. Block diagram of a quadrotor

II. Dynamic model

The schematic of a quadrotor is shown in Fig.1. It can be seen that the quadrotor has a rigid cross-frame structure and four propellers. The longitudinal motions of a quadrotor can be achieved by increasing ω_3, ω_4 and decreasing ω_1, ω_2. The lateral movements are performed through rolling in desired direction by increasing ω_1, ω_4 and decreasing ω_2 , ω_3 . Finally the yaw movement can be achieved by the difference in the counter-torque between (ω_1, ω_3) and (ω_2, ω_4) , and vice versa.

Let $\xi = [x, y, z]^T$ and $\eta = [\phi, \theta, \psi]^T$ denote the position and the attitude of a quadrotor in the earth frame $(O_e - x_e y_e z_e)$. $v = [v_1, v_2, v_3]^T$ and $\Omega = [\Omega_1, \Omega_2, \Omega_3]^T$ denote the linear and the angular velocities of a quadrotor in the body-fixed frame $(O_b - x_b y_b z_b)$. Then the kinematics model is given as:

$$\begin{cases} \dot{\xi} = \mathbf{R_t} v \\ \dot{\eta} = \mathbf{R_r} \Omega \end{cases} \tag{1}$$

where the matrix $\mathbf{R_t}$ describes the linear velocity relationship between the body-fixed frame $(O_b - x_b y_b z_b)$ and the earth frame $(O_e - x_e y_e z_e)$. describes the angular velocity relationship between the two frames. $\mathbf{R_t}$ and $\mathbf{R_r}$ are given as [26]:

$$\begin{aligned} \mathbf{R_t} &= \begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \\ \mathbf{R_r} &= \begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi / c\theta & c\phi / c\theta \end{bmatrix} \end{aligned} \tag{2}$$

In order to implement our controller more easily, some reasonable assumptions and simplifications which are used in [3, 27, 28] are applied in this paper. By using the Newton-Euler formula, the simplified dynamic model is given as follows:

$$\begin{aligned} m\ddot{x} &= (c\phi s\theta c\psi + s\phi s\psi)\, u - K_{\xi 1}\dot{x} + d_{\xi 1} \\ m\ddot{y} &= (c\phi s\theta s\psi - s\phi c\psi)\, u - K_{\xi 2}\dot{y} + d_{\xi 2} \\ m\ddot{z} &= c\phi c\theta u - K_{\xi 3}\dot{z} - mg + d_{\xi 3} \end{aligned} \tag{3}$$

$$\begin{array}{l} J_1\ddot{\phi} = \tau_1 - K_{\eta 1}\dot{\phi} + d_{\eta 1} \\ J_2\ddot{\theta} = \tau_2 - K_{\eta 2}\dot{\theta} + d_{\eta 2} \\ J_3\ddot{\psi} = \tau_3 - K_{\eta 3}\dot{\psi} + d_{\eta 3} \end{array} \tag{4}$$

where m is the mass of the quadrotor. g is the gravity acceleration. $d_{\xi i}$ and $d_{\eta i}$ $(i = 1, 2, 3)$ are the disturbances such as the nonlinear frictions, unmodeled dynamics and wind effects etc. $[K_{\xi 1}\dot{x}, K_{\xi 2}\dot{y}, K_{\xi 3}\dot{z}]^T$ and $\left[K_{\eta 1}\dot{\phi}, K_{\eta 2}\dot{\theta}, K_{\eta 3}\dot{\psi}\right]^T$ represent the aerodynamic damping effects. u and τ_i $(i = 1, 2, 3)$ are the control inputs. The control inputs u , τ_1 , τ_2 and τ_3 are obtained as follows:

$$\begin{bmatrix} u \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -l & l & l & -l \\ -l & -l & l & l \\ -k_c & k_c & -k_c & k_c \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \tag{5}$$

where l is the distance between the mass center and the rotor. k_c is the force-to moment scaling factor. The thrusts f_i is proportional to the square of the propeller's angular speed ω_i $(i = 1, 2, 3, 4)$ [29]:

$$f_i = k_T\omega_i^2 \tag{6}$$

where k_T is the thrust coefficient which is determined by the blade rotor characteristics.

Assumption 1 The model parameters such as the mass m the inertia moments J_i and the aerodynamics damping factors $K_{\xi i}$, $K_{\eta i}$ are all unknown constants.

Assumption 2 The disturbances $d_{\xi i}$ and $d_{\eta i}$ are bounded by $D_{\xi i}$ and $D_{\eta i}$ $(i = 1, 2, 3)$ respectively. The upper bounds $D_{\xi i}$ and $D_{\eta i}$ are also unknown constants.

III. Controller Design

The control objective can be descripted as: design proper ANTSMC and adaptive tuning laws to ensure the quadrotor attitude converge to the origins in finite time in the presence of parametric uncertainties, nonlinear frictions and disturbances.

The attitude tracking errors are:

$$\begin{cases} e_{\phi 1} = \phi - \phi_d \\ e_{\phi 2} = \dot{\phi} - \dot{\phi}_d \end{cases} \tag{7}$$

$$\begin{cases} e_{\theta 1} = \theta - \theta_d \\ e_{\theta 2} = \dot{\theta} - \dot{\theta}_d \end{cases} \tag{8}$$

$$\begin{cases} e_{\psi 1} = \psi - \psi_d \\ e_{\psi 2} = \dot{\psi} - \dot{\psi}_d \end{cases} \tag{9}$$

where ϕ_d , θ_d and ψ_d are the desired roll, pitch and yaw angles respectively.

The sliding surfaces are chosen as:

$$s_1 = k_\phi e_{\phi 1} + \frac{1}{\alpha_1}e_{\phi 1}^{r+1} + \frac{1}{\beta_1}e_{\phi 2}^{\frac{p}{q}} \tag{10}$$

$$s_2 = k_\theta e_{\theta 1} + \frac{1}{\alpha_2}e_{\theta 1}^{r+1} + \frac{1}{\beta_2}e_{\theta 2}^{\frac{p}{q}} \tag{11}$$

$$s_2 = k_\theta e_{\theta 1} + \frac{1}{\alpha_2}e_{\theta 1}^{r+1} + \frac{1}{\beta_2}e_{\theta 2}^{\frac{p}{q}} \tag{12}$$

where k_ϕ , k_θ ,k_ψ , α_i , β_i are some positive constants. r is a positive even constant. p , q are positive odd constants and satisfy the following condition:

$$1 < \frac{p}{q} < 2 \tag{13}$$

The auxiliary functions $\Gamma_{\phi i}$, $\Gamma_{\theta i}$ and $\Gamma_{\psi i}$ $(j = 1, 2, 3)$ are defined as follows:

$$\begin{cases} \Gamma_{\phi 1} = k_\phi + \frac{r+1}{\alpha_1}e_{\phi 1}^r \\ \Gamma_{\phi 2} = \frac{\beta_1 q}{p}e_{\phi 2}^{2-\frac{p}{q}} \\ \Gamma_{\phi 3} = \frac{p}{\beta_1 q}e_{\phi 2}^{\frac{p}{q}-1} \end{cases} \tag{14}$$

$$\begin{cases} \Gamma_{\theta 1} = k_\theta + \frac{r+1}{\alpha_2}e_{\theta 1}^r \\ \Gamma_{\theta 2} = \frac{\beta_2 q}{p}e_{\theta 2}^{2-\frac{p}{q}} \\ \Gamma_{\theta 3} = \frac{p}{\beta_2 q}e_{\theta 2}^{\frac{p}{q}-1} \end{cases} \tag{15}$$

$$\begin{cases} \Gamma_{\psi 1} = k_\psi + \frac{r+1}{\alpha_3}e_{\psi 1}^r \\ \Gamma_{\psi 2} = \frac{\beta_3 q}{p}e_{\psi 2}^{2-\frac{p}{q}} \\ \Gamma_{\psi 3} = \frac{p}{\beta_3 q}e_{\psi 2}^{\frac{p}{q}-1} \end{cases} \tag{16}$$

The control inputs τ_i $(i = 1, 2, 3)$ are designed as:

$$\tau_1 = \hat{K}_{\eta 1}\dot{\phi} + \ddot{\phi}_d\hat{J}_1 - \Gamma_{\phi 1}\Gamma_{\phi 2}\hat{J}_1 - \hat{D}_{\eta 1} - k_{s1}sgn\left(s_1\right) - k_1 s_1 \tag{17}$$

$$\tau_2 = \hat{K}_{\eta 2}\dot{\theta} + \ddot{\theta}_d\hat{J}_2 - \Gamma_{\theta 1}\Gamma_{\theta 2}\hat{J}_2 - \hat{D}_{\eta 2} - k_{s2}sgn\left(s_2\right) - k_2 s_2 \tag{18}$$

$$\tau_3 = \hat{K}_{\eta 3}\dot{\psi} + \ddot{\psi}_d\hat{J}_3 - \Gamma_{\psi 1}\Gamma_{\psi 2}\hat{J}_3 - \hat{D}_{\eta 3} - k_{s3}\mathrm{sgn}(s_3) - k_3 s_3 \tag{19}$$

where $k_{s\phi}$, $k_{s\theta}$, $k_{s\psi}$ and k_i $(i = 1, 2, 3)$ are positive controller gains. $\hat{K}_{\eta i}$, $\hat{J}_i$ and $\hat{D}_{\eta i}$ are the estimations of $K_{\eta i}$, J_i and $D_{\eta i}$ $(i = 1, 2, 3)$ respectively. The adaptive tuning laws are designed as follows:

$$\begin{cases} \dot{\hat{J}}_1 = s_1\left(\Gamma_{\phi 1}e_{\phi 2} - \Gamma_{\phi 3}\ddot{\phi}_d\right) \\ \dot{\hat{K}}_{\eta 1} = -s_1\Gamma_{\phi 3}\dot{\phi} \\ \dot{\hat{D}}_{\eta 1} = s_1\Gamma_{\phi 3} \end{cases} \tag{20}$$

$$\begin{cases} \dot{\hat{J}}_2 = s_2\left(\Gamma_{\theta 1}e_{\theta 2} - \Gamma_{\theta 3}\ddot{\theta}_d\right) \\ \dot{\hat{K}}_{\eta 2} = -s_2\Gamma_{\theta 3}\dot{\theta} \\ \dot{\hat{D}}_{\eta 2} = s_2\Gamma_{\theta 3} \end{cases} \tag{21}$$

$$\begin{cases} \dot{\hat{J}}_3 = s_3\left(\Gamma_{\psi 1}e_{\psi 2} - \Gamma_{\psi 3}\ddot{\psi}_d\right) \\ \dot{\hat{K}}_{\eta 3} = -s_3\Gamma_{\psi 3}\dot{\psi} \\ \dot{\hat{D}}_{\eta 3} = s_3\Gamma_{\psi 3} \end{cases} \tag{22}$$

Remark 1 It can be seen that the ANTSMC laws (17)-(19) are only related to the adaptive estimations and attitude errors. Adaptive tuning scheme are only related to the attitude errors such that the requirement for the information of quadrotor dynamic model has been avoided.

Theorem 1 Given the rotational motion model (4) and assuming the sliding surfaces are chosen as in (10)-(12). If the adaption laws are chosen as in (20)-(22), and the control gains satisfy the condition in (23), then the attitude errors defined in (7)-(9) converge to zero in finite time under the ANTSMC laws (17)-(19).

$$k_{si} > D_{\eta i} - d_{\eta i},\ (i = 1, 2, 3)\,. \tag{23}$$

Proof: The positive Lyapunov functions are chosen as:

$$V_1(t) = \frac{1}{2}J_1 s_1^2 + \frac{1}{2}\tilde{J}_1^2 + \frac{1}{2}\tilde{K}_{\eta 1}^2 + \frac{1}{2}\tilde{\mathrm{D}}_{\eta 1}^2. \tag{24}$$

$$V_2(t) = \frac{1}{2}J_2 s_2^2 + \frac{1}{2}\tilde{J}_2^2 + \frac{1}{2}\tilde{K}_{\eta 2}^2 + \frac{1}{2}\tilde{\mathrm{D}}_{\eta 2}^2. \tag{25}$$

$$V_3(t) = \frac{1}{2}J_3 s_3^2 + \frac{1}{2}\tilde{J}_3^2 + \frac{1}{2}\tilde{K}_{\eta 3}^2 + \frac{1}{2}\tilde{\mathrm{D}}_{\eta 3}^2. \tag{26}$$

where $\tilde{J}_i$, $\tilde{K}_{\eta i}$ and $\tilde{\mathrm{D}}_{\eta i}$ $(i = 1, 2, 3)$ are the estimation errors defined as $\tilde{J}_i = J_i - \hat{J}_i$, $\tilde{K}_{\eta i} = K_{\eta i} - \hat{K}_{\eta i}$, $\tilde{\mathrm{D}}_{\eta i} = \mathrm{D}_{\eta i} - \hat{\mathrm{D}}_{\eta i}$.

Taking the derivative of (24), we can get:

$$\dot{V}_1(t) = s_1 J_1 \dot{s}_1 - \tilde{J}_1 \dot{\hat{J}}_1 - \tilde{K}_{\eta 1}\dot{\hat{K}}_{\eta 1} - \tilde{\mathrm{D}}_{\eta 1}\dot{\hat{\mathrm{D}}}_{\eta 1}. \tag{27}$$

Using the dynamic model in (4) and substituting the control law τ_1 (17), one can get:

$$\begin{aligned}\dot{V}_1(t) = {} & s_1(\Gamma_{\phi 1}e_{\phi 2} - \Gamma_{\phi 3}\ddot{\phi}_d)\tilde{J}_1 + s_1\Gamma_{\phi 3}(\hat{D}_{\eta 1} - d_{\eta 1}) \\ & - k_1\Gamma_{\phi 3}s_1^2 - \Gamma_{\phi 3}k_{s1}\left|s_1\right| - s_1\Gamma_{\phi 3}\dot{\phi}\tilde{K}_{\eta 1} \\ & - \tilde{J}_1\dot{\hat{J}}_1 - \tilde{K}_{\eta 1}\dot{\hat{K}}_{\eta 1} - \tilde{\mathrm{D}}_{\eta 1}\dot{\hat{\mathrm{D}}}_{\eta 1}\end{aligned} \tag{28}$$

It can be seen that the use of the adaptive tuning scheme in (20) ensures terms $s_1(\Gamma_{\phi 1}e_{\phi 2} - \Gamma_{\phi 3}\ddot{\phi}_d)\tilde{J}_1$ and $-s_1\Gamma_{\phi 3}\dot{\phi}\tilde{K}_{\eta 1}$ cancel out. Then $\dot{V}_1(t)$ can be rewritten as follows:

$$\begin{aligned}\dot{V}_1(t) &= s_1\Gamma_{\phi 3}(d_{\eta 1} - D_{\eta 1}) - \Gamma_{\phi 3}k_{s1}\left|s_1\right| - k_1\Gamma_{\phi 3}s_1^2 \\ &\le -k_1\Gamma_{\phi 3}s_1^2 - \Gamma_{\phi 3}k_{s1}\left|s_1\right| + \Gamma_{\phi 3}(D_{\eta 1} - d_{\eta 1})\left|s_1\right|\end{aligned} \tag{29}$$

If the control gain k_{s1} satisfies the condition in (23), we have:

$$-\Gamma_{\phi 3}k_{s1}\left|s_1\right| + \Gamma_{\phi 3}(D_{\eta 1} - d_{\eta 1})\left|s_1\right| < 0 \tag{30}$$

Finally,

$$\dot{V}_1(t) < 0 \tag{31}$$

By using the Lyapunov theory, we can get that the attitude control system is asymptotically stable. When the sliding surface $s_1 = 0$ is reached, the equation (10) is:

$$k_\phi e_{\phi 1} + \frac{1}{\alpha_1}e_{\phi 1}^{r+1} + \frac{1}{\beta_1}e_{\phi 2}^{\frac{p}{q}} = 0 \tag{32}$$

That is:

$$\begin{aligned}\dot{e}_{\phi 1} &= -\beta_1{}^{\frac{q}{p}}(k_\phi e_{\phi 1} + \frac{1}{\alpha_1}e_{\phi 1}^{r+1})^{\frac{q}{p}} \\ &= -e_{\phi 1}^{\frac{q}{p}}(\beta_1(k_\phi + \frac{1}{\alpha_1}e_{\phi 1}^{r}))^{\frac{q}{p}}\end{aligned} \tag{33}$$

TABLE I

CONTROLLER PARAMETERS

Symbols	Values	Symbols	Values
α_1, α_2	16	r	2
α_3	20	k_ϕ, k_θ	12
β_1, β_2	10	k_ψ	13
β_3	20	k_1, k_2	7
p	5	k_3	9
q	3	$k_{s\phi}, k_{s\theta}, k_{s\psi}$	0.02

Assuming the finite time which is taken from $e_{\phi 1}(t_{\phi 0}) \neq 0$ to $e_{\phi 1}(t_{\phi 0} + t_{\phi s}) = 0$ is $t_{\phi s}$, we can get that:

$$\begin{aligned}\int_{e_{\phi 1}(t_{\phi 0})}^{e_{\phi 1}(t_{\phi 0}+t_{\phi s})} e_{\phi 1}^{\frac{q}{p}}de_{\phi 1} &= -\int_{t_{\phi 0}}^{t_{\phi s}}\left(\beta_1\left(k_\phi + \frac{1}{\alpha_1}e_{\phi 1}^{r}(\sigma)\right)\right)^{\frac{q}{p}}d\sigma \\ &\le -\int_{t_{\phi 0}}^{t_{\phi s}}(\beta_1 k_\phi)^{\frac{q}{p}}d\sigma\end{aligned} \tag{34}$$

Then,

$$t_{\phi s} \le \frac{p}{(\beta_1 k_\phi)^{\frac{q}{p}}(p - q)}e_{\phi 1}^{1-\frac{q}{p}}(t_{\phi 0}) \tag{35}$$

Summing up, the Lyapunov function $V_1(t) > 0$ while $\dot{V}_1(t) < 0$. We can also obtain that $V_2(t) > 0$, $V_3(t) > 0$ and $\dot{V}_2(t) < 0$, $\dot{V}_3(t) < 0$. By using the Lyapunov theory, it can be see that the attitude system is asymptotically stable. According to (32)-(35), it can be seen that the attitude error converge to zero in finite time. Similarly, the finite time convergence of the attitude errors $e_{\theta 1}$ and $e_{\psi 1}$ can be ensured. Then the proof is finished.

Remark 2 In the real-time experiment, a continuous function $\tanh(\cdot)$ is used to replace the discontinuous switching function $sign(\cdot)$ to eliminate the chattering phenomenon.

IV. EXPERIMENTAL RESULTS

In order to validate the effectiveness and robustness of the proposed controller, the real-time experimental results which are conducted using an indoor testbed named HILS [30] is presented. The test consists a typical X450 quadrotor helicopter and a DSP board. The quadrotor is fixed on a ball joint. The quadrotor has unrestricted yaw movement and degree motion range for the roll and pitch motion. The attitude angles and angular velocities of the quadrotor are measured by an MTi attitude heading reference system (AHRS). The controller gains obtained by trail and error are listed in Table 1. The initial values of the estimations $\hat{K}_{\eta i}(t_0)$, $\hat{J}_i(t_0)$ and $\hat{D}_{\eta i}(t_0)$ are zero. In order to further validate the robustness of the proposed controller, wind disturbance has been considered. In the experiment, the wind disturbance is instantaneous wind which is made artificially.

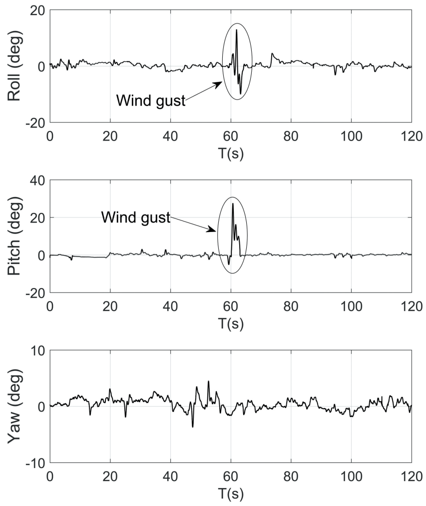

Fig. 2. Attitude response of quadrotor system in the presence of wind gust

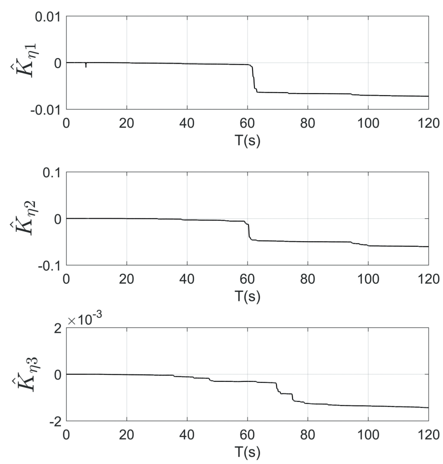

Fig. 3. Estimations of the aerodynamic damping coefficients

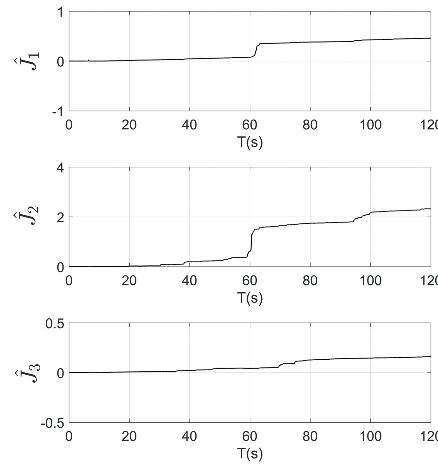

Fig. 4. Estimations of the moments of inertia

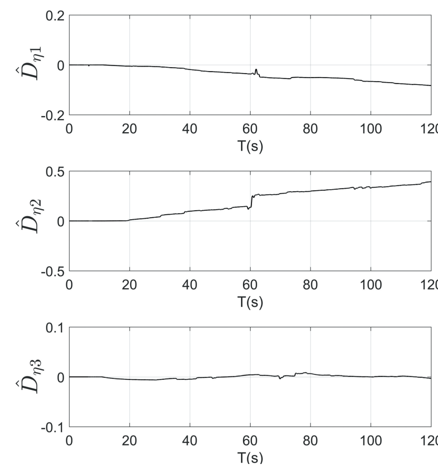

Fig. 5. Estimations of the disturbance upper bounds

The experiment lasts for 120 seconds, providing enough time to comply the attitude stabilization.

As is shown in Figs.3-5, the estimations $\hat{K}_{\eta i}$, $\hat{J}_i$ and $\hat{D}_{\eta i}$ converge to some constants before the wind gust being introduced. During the experiment, the system parameters of the quadrotor is not changed. At $t = 60s$, the wind breaks the stable status of the quadrotor so that the updating laws are excited to change the values of the adaptive estimators. When the quadrotor returns to the desired attitude, the estimations stop changing and converge to different values. The adaptive estimations $\hat{K}_{\eta i}$, $\hat{J}_i$ and $\hat{D}_{\eta i}$ cannot be guaranteed to converge to their true values because they do not meet the persistent-excitation condition. It can be seen that the disturbances could regarded as the influence of different parameters. Taking the advantages of the sliding mode control such as strong robustness to model uncertainties, the wind-induced effects have been well compensated such that the attitude could be stabilized.

The control inputs τ_i $(i = 1, 2, 3)$ are shown in Fig. 6. We can see that the control input waves up and down due to the small attitude errors and disturbances. It also can be seen that the wind not only cause the attitude and adaptive estimation changes but also effect the control inputs. Despite these changes, the system still keeps stable.

V. Conclusion

In this paper, an adaptive non-singular terminal sliding mode controller is proposed to stabilize a quadrotor in the presence of unknown system parameters and disturbances. Adaptive laws are well-designed to estimate the unknown model parameters such as the inertia moments, the aero-

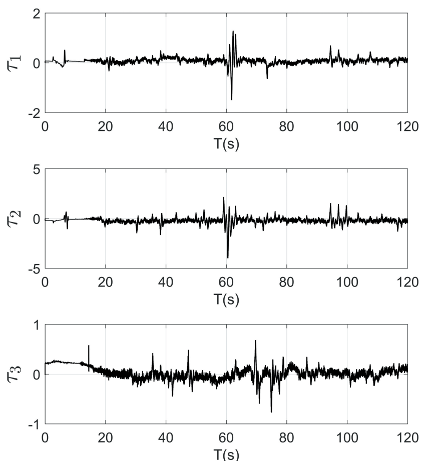

Fig. 6. Control inputs

dynamic damping factors and the upper bounds of disturbances. The stability has been proved via Lyapunov theory. Experimental results performed on HILS testbed is presented which have demonstrated the effectiveness of the proposed controller. Future works will focus on the designing of the position controller and combines the proposed attitude controller to complete robust trajectory tracking task.

REFERENCES

[1] K. L. Cook, "An evaluation of the effectiveness of low-cost UAVs and structure from motion for geomorphic change detection," Geomorphology, vol. 278, pp. 195-208, 2017.

[2] J. Valente, J. Del Cerro, A. Barrientos, and D. Sanz, "Aerial coverage optimization in precision agriculture management: A musical harmony inspired approach," Computers and Electronics in Agriculture, vol. 99, pp. 153-159, 2013.

[3] Z. Li, X. Ma, and Y. Li, "Robust tracking control strategy for a quadrotor using RPD-SMC and RISE," Neurocomputing, vol. 331, pp. 1-6, 2019.

[4] A. Tayebi and S. McGilvray, "Attitude stabilization of a VTOL quadrotor aircraft," IEEE Transactions on Control Systems Technology, vol. 14, no. 3, pp. 562-571, 2006.

[5] R. Mahony, V. Kumar, and P. Corke, "Multirotor Aerial Vehicles: Modeling, Estimation, and Control of Quadrotor," IEEE Robotics & Automation Magazine, vol. 19, no. 3, pp. 20-32, 2012.

[6] F. K. Yeh, "Attitude controller design of mini-unmanned aerial vehicles using fuzzy sliding-mode control degraded by white noise interference," IET Control Theory & Applications, vol. 6, no. 9, pp. 1205-1212, 2012.

[7] B. J. Emran and H. Najjaran, "A review of quadrotor: An underactuated mechanical system," Annual Reviews in Control, vol. 46, pp. 165-180, 2018.

[8] B. Tian, J. Cui, H. Lu, Z. Zuo, and Q. Zong, "Adaptive Finite-Time Attitude Tracking of Quadrotors With Experiments and Comparisons," IEEE Transactions on Industrial Electronics, vol. 66, no. 12, pp. 9428-9438, 2019.

[9] D. W. Kim, "Fuzzy model-based control of a quadrotor," Fuzzy Sets and Systems, vol. 371, pp. 136-147, 2019.

[10] L. Besnard, Y.B.Shtessel and B. OLandrum, "Quadrotor vehicle control via sliding mode controller driven by sliding mode disturbance observe," Journal of the Franklin Institute-Engineering and Applied Mathematics, vol 349, no.2, pp. 4957-4962, 2012.

[11] R. Xu and U. Ozguner, "Sliding Mode Control of a Quadrotor Helicopter," in IEEE Conference on Decision and Control, 2006, pp. 4957-4962.

[12] F. Chen, R. Jiang, K. Zhang, B. Jiang, and G. Tao, "Robust Backstepping Sliding-Mode Control and Observer-Based Fault Estimation for a Quadrotor UAV," IEEE Transactions on Industrial Electronics, vol. 63, no. 8, pp. 5044-5056, 2016.

[13] S. Li, Y. Wang, J. Tan, and Y. Zheng, "Adaptive RBFNNs/integral sliding mode control for a quadrotor aircraft," Neurocomputing, vol. 216, pp. 126-134, 2016.

[14] I. Gonzlez, S. Salazar, and R. Lozano, "Chattering-Free Sliding Mode Altitude Control for a Quad-Rotor Aircraft: Real-Time Application," Journal of Intelligent & Robotic Systems, vol. 73, no. 1, pp. 137-155, 2014.

[15] J. Zhang, Z. Ren, C. Deng, and B. Wen, "Adaptive fuzzy global sliding mode control for trajectory tracking of quadrotor UAVs," Nonlinear Dynamics, vol. 97, no. 1, pp. 609-627, 2019.

[16] C. Edwards and S. Spurgeon, Sliding mode control: theory and applications. CRC Press, 1998.

[17] F. Munoz, I. Gonzlez-Hernndez, S. Salazar, E. S. Espinoza, and R. Lozano, "Second order sliding mode controllers for altitude control of a quadrotor UAS: Real-time implementation in outdoor environments," Neurocomputing, vol. 233, pp. 61-71, 2017.

[18] F. Munoz, E. S. Espinoza, I. Gonzlez-Hernndez, S. Salazar and R. Lozano, "Robust Trajectory Tracking for Unmanned Aircraft Systems using a Nonsingular Terminal Modified Super-Twisting Sliding Mode Controller," Journal of Intelligent & Robotic systems, vol. 93, no. 1, pp. 55-72, 2019.

[19] H. Chen, B. Zhang, T. Zhao, T. Wang, and K. Li, "Finite-time tracking control for extended nonholonomic chained-form systems with parametric uncertainty and external disturbance," Journal of Vibration and Control, vol. 24, no. 1, pp. 100-109, 2018.

[20] J.J. Xiong and G.B. Zhang, "Global fast dynamic terminal sliding mode control for a quadrotor UAV," ISA Transactions, vol. 66, pp. 233-240, 2017.

[21] Y. Feng, X. Yu, and Z. Man, "Non-singular terminal sliding mode control of rigid manipulators," Automatica, vol. 38, no. 12, pp. 2159-2167, 2002.

[22] G. Hoffmann, H. Huang, S. Waslander, and C. Tomlin, "Quadrotor Helicopter Flight Dynamics and Control: Theory and Experiment," in AIAA Guidance, Navigation and Control Conference and Exhibit, 2013, pp.1-20.

[23] M. Imran Rashid and S. Akhtar, "Adaptive control of a quadrotor with unknown model parameters," in 9th International Bhurban Conference on Applied Sciences and Technology, 2012, pp. 8-14.

[24] O. Mofid and S. Mobayen, "Adaptive sliding mode control for finite-time stability of quad-rotor UAVs with parametric uncertainties," ISA Transactions, vol. 72, pp. 1-14, 2018.

[25] A. Modirrousta and M. Khodabandeh, "Adaptive robust sliding mode controller design for full control of quadrotor with external disturbances," in International Conference on Robotics and Mechatronics, 2014, pp. 870-877.

[26] F. Chen, W. Lei, K. Zhang, G. Tao, and B. Jiang, "A novel nonlinear resilient control for a quadrotor UAV via backstepping control and nonlinear disturbance observer, " Nonlinear Dynamics, pp. 1-15, 2016.

[27] A. Chamseddine, Y. Zhang, C. A. Rabbath, C. Join, and D. Theilliol, "Flatness-Based Trajectory Planning/ Replanning for a Quadrotor Unmanned Aerial Vehicle," IEEE Transactions on Aerospace and Electronic Systems, vol. 48, no. 4, pp. 2832-2848, 2012.

[28] B. Zhao, B. Xian, Y. Zhang, and X. Zhang, "Nonlinear Robust Adaptive Tracking Control of a Quadrotor UAV Via Immersion and Invariance Methodology," IEEE Transactions on Industrial Electronics, vol. 62, no. 5, pp. 2891-2902, 2015.

[29] C. Wang, B. Song, C. Tang, and C. Tang, "Trajectory Tracking Control for Quadrotor Robot Subject to Payload Variation and Wind Gust Disturbance," Journal of Intelligent & Robotic Systems, vol. 83, no. 2, pp. 315-333, 2016.

[30] Z. Bo, X. Bin, Z. Yao, and Z. Xu, "Nonlinear Robust Adaptive Tracking Control of a Quadrotor UAV Via Immersion and Invariance Methodology," Industrial Electronics, IEEE Transactions on, vol. 62, no. 5, pp. 2891-2902, 2015.