Proceeding of the IEEE
International Conference on Robotics and Biomimetics
Dali, China, December 2019

A calibration method with anistropic weighting for LiDAR and stereo camera system

Wei Wang
Department of Mechanical Engineering
Tsinghua University
Beijing, China
wangweichuse@gmail.com

Yang Liu
Beijing Spacecrafts
Beijing, China
lylink@126.com

Rui Chen
Department of Mechanical Engineering
Tsinghua University
Beijing, China
callmeray@163.com

Jing Xu
Department of Mechanical Engineering
Tsinghua University
Beijing, China
jingxu@tsinghua.edu.cn

***Abstract*—Calibrating the extrinsic matrices between sensors is a significant pre-processing step of sensor fusion. Most of existing calibration methods use point-based rigid registration algorithm which considers the point coordinate error isotropic and uses the least square solution to estimate the extrinsic matrices. However, the error distribution of point coordinates is anisotropic due to the internal measurement properties of sensors, leading to decreased calibration accuracy. To solve this problem, we proposed an anisotropy weighting method: first we construct weighting matrices based on error distributions models of sensors; second we use surveying adjustment to further improve the calibration accuracy iteratively. We verified the effectiveness of our method through simulations. Compared with traditional methods, the accuracy is improved by about 45%. Moreover, our method can be applied in most of calibration methods to reduce the influence of anisotropic data and improve the accuracy.**

***Index Terms*—LiDAR, stereo camera, anisotropy, calibration method**

I. Introduction

As technology advances within a few years, lots of sensors of different types have been developed for computers to interact with the environment. So far, however, no sensor is prefect that can obtain all type of data from environment. Therefore, to perceive the environment precisely, multiple sensors are usually used together [1]–[3]. One of the common purposes is Simultaneous localization and mapping (SLAM), which is widely used in various tasks, such as indoor navigation [4], visual sensing [5], [6], and space exploration [7]. In automatic vehicle field, SLAM often requires high-accuracy and high-resolution depth data. To perceive the environment well, LiDARs and stereo cameras are usually used together as a visual system to handle the heavy traffic [8]. By time of flight (ToF) LiDARs obtain point clouds from environment in $360°$, yet the data is colorless and sparse. In contrast, stereo cameras obtain high-resolution and colorful data form the front view, but it is affected by object boundaries that are perpendicular to the epipolar lines [9]. It's worth noting that figuring out extrinsic matrices (rotational matrices, R, and translation vectors,T) between LiDARs and cameras is a significant pre-processing step in above methods which relates high accuracy outcome.

In general, most of extrinsic matrix calibration methods use point-based rigid registration method [10] to calibrate the sensors. By minimizing the fiducial registration error (FRE, the distance between nominal and measured feature points) on reference surface to figure out the RT matrix. Because point-based rigid registration needs fiducial points to calibrate, there are two types of calibration methods: one is extracting fudical points from the environment and the other is extracting points from the calibrations. The first type methods establish a RT matrix only by feature points extracted from the real world [11]–[13]. Normally, it's convenient to calibrate, but the accuracy extremely depends on the environment. [14]. Therefore, the stochastic noises from environment always decrease the accuracy of RT matrix. On the contrary, the second type is a reliable way to reduce external noises, since this method has set up calibrations objects which are designed to match the measurement property of sensors. There are many kinds of calibration objects, for instance, checkerboards [15] with black and white blocks corners can be simply detected by cameras. Boards with holes method [16], sensors can extract more fiducial points from the boards to calibrate. In QRcode cubes method [17], the edges, corners and QR code of the cube provide depth information and image features to calibrate. Although proposed solutions achieved a good result, the downside is that there still remain errors. The reason is above methods regard the error distribution of sensor data to be isotropic and homogenous. It's true that the

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least square problem converges in minuscule tolerance when the error distributions of fiducial points are isotropic. But, in fact it's difficult to converge, because the measurement principle of sensors always leads to anisotopy of the data [14]. LiDARs measure the distance by ToF which is the time cost of laser going from LiDAR's origin coordinate to the object, and transfer the distance and two angle values of encoders to obtain the object coordinates in Cartesian coordinates system. Therefore, resulting from the measurement property, the error distributions of distance and angles will be different. Such error distributions in three variables cause anisotropy data. This is similar in stereo cameras, a modular test [18] to evaluate the stereo camera depth accuracy in different location was proposed. Mikko Kyt.al set several checkerboards to gain the error distributions in the space. The experiments showed how stereo vision measurement property influences the error distributions in three axes [19]. For the error distribution issue, Craig Glennie.al used error adjusting moulds [20] to increase accuracy of calibration, but it showed that there are still some errors that cannot be erased when matching the fiducial points with least square method. Therefore, in order to improve accuracy the error distribution shouldn't be regarded as the same in different direction and distance. Obviously, regarding anisotropy data as isotropy data when calibrating will decrease the accuracy of extrinsic matrix [21]. For anisotropy issue, Ramya Balachandran.al used weighting iteration solution [22] to obtain high calibration accuracy, and equilibrated the error distribution in different axes by constructing weighting matrix. However, they assumed the weighting matrices were fixed, which is not for different locations. Also for anisotropy issue, to reduce error, the surveying adjustment algorithm [23] was used to deal with anisotropy error distribution issue.

To improve the accuracy of RT matrix, this paper aims at the anisotropy issue involved in LiDARs and stereo cameras, and proposes a high-accuracy extrinsic calibration method based on weighting matrices and surveying adjustment. The remainder of this paper is organizd as follows. Section II we introduce the measuring principles of LiDARs and stereo cameras, and construct the weighting matrices accordingly. In section III we illustrate our method of point-based rigid registration based on surveying adjustment and weighting matrix. In section IV, we perform simulations to compare the traditional singular value decomposition(SVD) method and our method. Finally, we conclude our work in section V.

II. Theory

A. LiDAR

To transform data into Cartesian coordinates, there are two steps in the operation of LiDAR. First, the LiDAR obtains the surrounding data in $360°$ with a laser emitter and encoders in spherical coordinates (R, ω, α), where R is the distance between environment and object and angles ω and α are the vertical and horizontal angle of the laser beam respectively. Then the LiDAR transforms data into Cartesian coordinate as follows:

$$\begin{cases} X = R * cos(\omega) * sin(\alpha) \\ Y = R * cos(\omega) * cos(\alpha) \\ Z = R * sin(\omega) \end{cases} . \quad (1)$$

Because the magnitude of noises are different in (R, ω, α), the error distributions of three axes (x, y, z) are also different, which causes anisotropy data.

B. Stereo camera

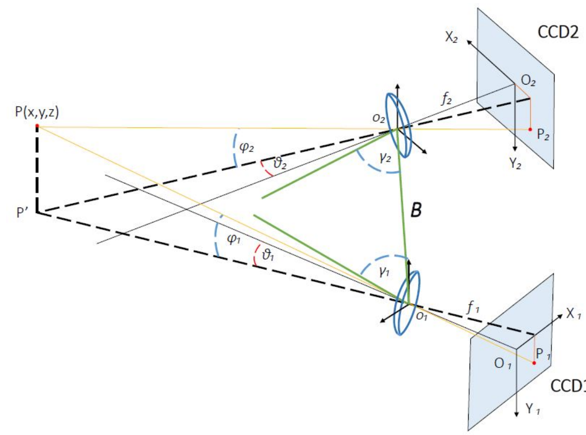

Fig. 1. Stereo camera measurement principle. Where CCD1 and CCD2 represent the imaging plane of two cameras. CCDs are used to get 3D coordinates of point P. $\{O_1, X_1, Y_1\}$ and $\{O_2, X_2, Y_2\}$ express the coordinates of two cameras. Coordinate of point P will be project to both of CCDs. $\{\theta, \phi\}$ indicate the angle on the horizon and vertical of point P. The focus of CCD1 and CCD2 are denoted by $\{f_1, f_2\}$. Baseline B is the distance between lens1 and lens2. $\{\gamma_1, \gamma_2\}$ represent the angles of B and lens.

In general, the angles γ_1, γ_2 of stereo cameras are normally equal to $90°$. Thus, according to Fig. 1, the geometrical relationship in triangle $o_1, o_2, P^{'}$ we can obtain baseline equations as follows:

$$\begin{cases} x_1 = z \cdot cot(\iota) \\ B = z \cdot cot(\iota) + z \cdot cot(\kappa) \end{cases} , \quad (2)$$

where $\iota = \gamma_1 + \theta_1$, $\kappa = \gamma_2 + \theta_2$, and z is the depth of the object point which is the value between object point and CCD1 along z-axis. In triangle $P, P^{'}, o_i$, the angle ϕ can be derived as follows:

$$tan\phi_i = \frac{y \cdot sin(\gamma_i + \theta_i)}{z}; (n = 1, 2). \quad (3)$$

In summary, with (2) and (3) the coordinates of a object point in Cartesian coordinate will be denoted as follows:

$$\begin{cases} x = \frac{B \cdot cot(\iota)}{cot(\iota) + cot(\kappa)} \\ y = \frac{z \cdot tan(\phi_1)}{sin(\iota)} = \frac{z \cdot tan(\phi_2)}{sin(\kappa)} \\ z = \frac{B}{cot(\iota) + cot(\kappa)} \end{cases} , \quad (4)$$

where $\phi_i = arctan\frac{Y_i \cdot cos(\theta)}{f_i}$, $\theta_i = arctan\frac{X_i}{f_i}$ where $i = 1, 2$.

C. Weighting matrix

According to the analysis of LiDARs and stereo cameras, now we can construct weighting matrix. For LiDAR(R, α, ω), measurement error of R is from the laser emitter, the error of angle α is from the encoder, and error of ω is fixed when the LiDAR was produced. Thus we let these errors be $\{\sigma_R, \sigma_\alpha, \sigma_\omega\}$ as:

$$\begin{cases} \sigma_R = K_1 \cdot \sigma_\alpha \\ \sigma_\alpha = K_2 \cdot \sigma_\omega \end{cases}. \tag{5}$$

And the weighting matrix W_{Li} as follows:

$$\mathrm{W}_{Li} = \begin{bmatrix} \sigma_R^2 & 0 & 0 \\ 0 & \sigma_\alpha^2 & 0 \\ 0 & 0 & \sigma_\omega^2 \end{bmatrix}. \tag{6}$$

Equation (5) means the relationship between error distribut in spherical coordinate (R, α, ω). However, LiDAR w in Cartesian coordinate (x, y, z). By covariance propagat we can transform (1) to obtain the covariance matrix in Cartesian coordinate as:

$$\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} cos(\alpha)cos(\omega) & -Rcos(\alpha)cos(\omega) & -Rcos(\alpha)sin(\omega) \\ sin(\alpha)cos(\omega) & Rcos(\alpha)cos(\omega) & -Rsin(\alpha)sin(\omega) \\ sin(\omega) & 0 & Rcos(\omega) \end{bmatrix} \cdot$$

Let:

$$J = \begin{bmatrix} cos(\alpha)cos(\omega) & -Rcos(\alpha)cos(\omega) & -Rcos(\alpha)sin(\omega) \\ sin(\alpha)cos(\omega) & Rcos(\alpha)cos(\omega) & -Rsin(\alpha)sin(\omega) \\ sin(\omega) & 0 & Rcos(\omega) \end{bmatrix}, \tag{8}$$

be the covariance propagation, we can transfer W_{Li} into Cartesian coordinate.

$$\mathrm{W}_{Li}^{'} = \mathrm{J} \cdot \mathrm{W}_{Li} \cdot \mathrm{J}^T \tag{9}$$

When the angles γ_1 and γ_2 equal to $90°$, we take (4) partial derivative, then we get as follows:

$$\begin{cases} \frac{\partial x}{\partial X_1} = -\frac{B}{2f_1} \cdot \frac{cos^2\theta_1 \cdot sin2\theta_2}{sin^2(\Lambda)} \\ \frac{\partial x}{\partial X_2} = -\frac{B}{2f_2} \cdot \frac{cos^2\theta_2 \cdot sin2\theta_1}{sin^2(\Lambda)} \\ \frac{\partial z}{\partial X_1} = -\frac{B}{f_1} \cdot \frac{cos^2\theta_1 \cdot sin^2\theta_2}{sin^2(\Lambda)} \\ \frac{\partial z}{\partial X_2} = -\frac{B}{f_2} \cdot \frac{cos^2\theta_2 \cdot sin^2\theta_1}{sin^2(\Lambda)} \\ \frac{\partial y}{\partial Y_1} = \frac{B}{f_1} \cdot \frac{cos\theta_1 \cdot sin\theta_2}{sin^2(\Lambda)} \\ \frac{\partial y}{\partial Y_2} = \frac{B}{f_2} \cdot \frac{cos\theta_2 \cdot sin\theta_1}{sin^2(\Lambda)} \\ \frac{\partial y}{\partial X_1} = -\frac{B}{f_1} \cdot \frac{tan\phi_1 cos\theta_1 sin\theta_2}{sin(\Lambda) sin\theta_1} (\frac{cos\theta_1 sin\theta_2}{sin(\Lambda)} + cos\gamma_1) \\ \frac{\partial y}{\partial X_2} = -\frac{B}{f_2} \cdot \frac{tan\phi_2 cos\theta_2 sin\theta_1}{sin(\Lambda) sin\theta_2} (\frac{cos\theta_2 sin\theta_1}{sin(\Lambda)} + cos\gamma_2) \end{cases}, \tag{10}$$

where $\Lambda = \theta_1 + \theta_2$, and by law of covariance propagation the error distributions along x, y and z axes can be expressed as follows:

$$\begin{cases} \sigma_x = \sqrt{(\frac{\sigma_x}{\sigma_{x1}})^2 \sigma_{x1} + (\frac{\sigma_x}{\sigma_{x2}})^2 \sigma_{x2}} \\ \sigma_y = \sqrt{(\frac{\sigma_y}{\sigma_{x1}})^2 \sigma_{x1} + (\frac{\sigma_y}{\sigma_{x2}})^2 \sigma_{x2} + (\frac{\sigma_y}{\sigma_{y1}})^2 \sigma_{y1} + (\frac{\sigma_y}{\sigma_{y2}})^2 \sigma_{y2}} \\ \sigma_z = \sqrt{(\frac{\sigma_z}{\sigma_{x1}})^2 \sigma_{z1} + (\frac{\sigma_z}{\sigma_{x2}})^2 \sigma_{z2}} \end{cases}. \tag{11}$$

And the weighting matrix is constructed as follows:

$$\mathrm{W}_{Ca} = \begin{bmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_z^2 \end{bmatrix}. \tag{12}$$

III. Method

A. Weighting algorithm

This paper proposed a weighting iterative solution based on anisotropy weighting and surveying adjustment. The basic concept of point-based rigid registration is to minimize fiducial point localization error in LiDAR and stereo camera coordinate, which is also called fiducial registration errors [24] as shown in Fig. 2.

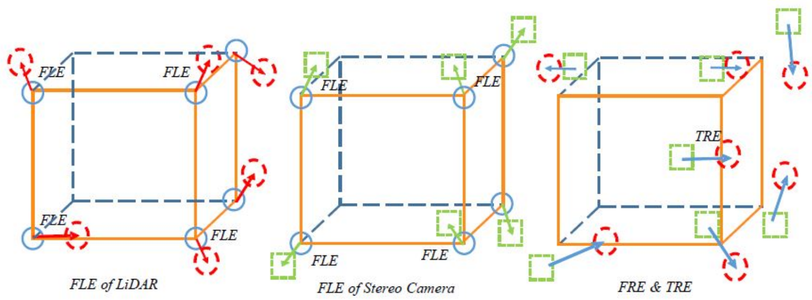

Fig. 2. Error of rigid registration.

And FRE equation is expressed as follows:

$$FRE^2 = \frac{1}{N} \sum_{i=1}^{N} [\mathrm{W}(\mathrm{R} \cdot P_{Li} + \mathrm{T} - P_{Ci})]^2, \tag{13}$$

where P_L and P_C represent the point sets of LiDAR and stereo camera respectively. W is a weighting matrix, and $[\mathrm{R}, \mathrm{T}]$ are exactly the extrinsic matrix. We use weighting matrices (5) and (12) both for LiDAR and stereo camera as follows:

$$\begin{cases} P_L(x_i, y_i, z_i) = \mathrm{W}_{Li} \cdot P_L(x_i, y_i, z_i) \\ P_C(x_i, y_i, z_i) = \mathrm{W}_{Ca} \cdot P_C(x_i, y_i, z_i) \end{cases}, \tag{14}$$

where P is the point set in certain coordinate. As we constructed the weighting matrix W which is based on the measurement properties of LiDAR or camera, it will compensate the error in different directions.

In each iteration, we do the following steps. We need linear constraints to iterate, thus transfer (13) with an approximated matrix $\mathrm{I} + \Delta\Theta^n$. By the least square solution, we get a new iterative solution, $\mathrm{I} + \Delta\Theta^n$, and transfer back into matrix R^n form. With the new matrix R^n, we transfer the LiDAR coordinate to camera coordinate which is supposed to be an optimization solution. The following steps will be explained in LiDAR, as it's the same in stereo camera.

First of all, initialize $\mathrm{R}^0, \mathrm{T}^0$ and P_L^0. To get the first approximate matrix R^0 and a translation vector T^0, we use SVD method to match point sets:

$$P_L^0 = \mathrm{R}^0 \cdot P_C + \mathrm{T}^0. \tag{15}$$

Then at each stage of iteration, we perform the following codes:

Algorithm 1 Anisotropy weighting algorithm

for $i = 1, \ldots, 10000$ **do**
  Set $error = 1$;
  while $error > 1.0e^{-}5$ **do**
    if $i == 1$ **then**
      Get initial RT matrix $\mathrm{I} + \Delta\Theta^{(0)}, \Delta\mathrm{T}^{(0)}, and \mathrm{P}_{Li}^{(0)}$.
      $i = i + 1$;
    else
      Minimize $[\mathrm{I} + \Delta\Theta^{(n)}, \Delta\mathrm{T}^{(n)}]$ by least square method.
      To avoid oscillations:
      Let $\Delta\Theta^{(n-1)} \leftarrow \left(\Delta\Theta^{(n-1)} + \Delta\Theta^{(n)}\right)/2$;
      Let $\Delta\mathrm{T}^{(n-1)} \leftarrow \left(\Delta\mathrm{T}^{(n-1)} + \Delta\mathrm{T}^{(n)}\right)/2$;
      Update R_i, T_i, P_{Li}:
      $\mathrm{U}\Lambda\mathrm{V}^T \leftarrow \mathrm{I} + \Delta\Theta^{(n-1)}$;
      $\mathrm{R}_i^n \leftarrow \mathrm{U}\mathrm{V}^T$;
      $P_{Li}^n \leftarrow \mathrm{R}_i \cdot P_{Li}^{n-1} + \mathrm{T}_i$;
      Check the value of error.
      $e1 \leftarrow \sum_{i=1}^{N} \left[p_{Li}^{(n)} - p_{Li}^{(n-1)}\right]^2$
      $e2 \leftarrow \sum_{i=1}^{N} \left[p_{Li}^{(n-1)} - \bar{p}^{(n-1)}\right]^2$
      $error \leftarrow e1/e2$;
      $n \leftarrow n + 1$;
    end if
  end while
end for
Output: $\mathrm{R}_i^n, T^{(n)}$

IV. Simulation

We assume the calibration object is a rigid cube, and set the fiducial points N is $4, 5, 6$. Then the theoretical matrix between LiDAR and camera which consists of three degrees $\{\theta, \phi, \gamma\}$ are $\{30°, -40°, 50°\}$ and distance vector $[D_x, D_y, D_z]^T$ are [5000mm, 8000mm, 9500mm $]^T$. Thus, the RT matrix to transform LiDAR coordinate to camera coordinate is performed as follows:

$$P_{camera} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} cos\phi & 0 & sin\phi \\ 0 & 1 & 0 \\ -sin\phi & 0 & cos\phi \end{bmatrix} \cdot \begin{bmatrix} cos\gamma & -sin\gamma & 0 \\ sin\gamma & cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot P_{LiDAR} + \begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} \tag{16}$$

Then, improve accuracy with surveying adjustment. The contradictions between nominal and measured points can be represented as:

$$K_{ij} = D_{ij} - \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}, \tag{17}$$

where K is a $[2N \times 1]$ matrix. And the error equation is expressed as follows:

$$\mathrm{CQ} + K_{ij} = 0, \tag{18}$$

where C is a coefficient matrix which is obtained by taking the partial derivative. And Q is the correction matrix of the surveying adjustment. The surveying adjustment is based on least square method by minimizing $\mathrm{Q}^T \cdot \mathrm{W} \cdot \mathbf{Q}$ to get the optimal Q. In this case, typically the indefinite Lagrange multiplier method is used to solve. The final equation is expressed as:

$$P\mathrm{Q} - \mathrm{C}^T L = 0, \tag{19}$$

where P is invertible, so we have

$$\mathrm{Q} = P^{-1}\mathrm{C}^T L. \tag{20}$$

Now both (18) and (19) have the same number of unknowns to solve which is the optimal solution. To verify the effectiveness of our proposed method, are carried out two simulations.

Simulation 1: Select $N = 4, 5, 6$ fiducial points. Assume the error obeys Gaussian distribution, and is isotropic and homogenous along three axes.

Simulation 2: Select $N = 4, 5, 6$ fiducial points. Assume the error distribution according to the measurement properties of sensors.

In both simulation 1 and simulation 2, first we fixed the coordinates of LiDAR and stereo camera with RT matrix.

$$P_C = \mathrm{R} \cdot P_L + \Delta\mathrm{T}. \tag{21}$$

Second, we get the coordinate of calibration objects by LiDAR and stereo camera, and add measurement errors to the calibration coordinate to create 10,000 point sets. To verify the theory, finally we register two groups of fiducial point sets both by SVD method and anisotropic weighting algorithm, and obtain the FRE of the two methods, and the results are shown in Fig. 3:

From Fig. 3, it is found that for isotropy data the

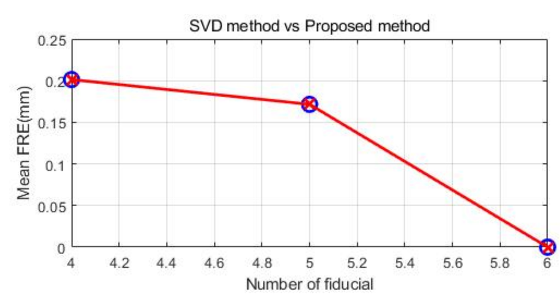

Fig. 3. Mean FRE of two methods in isotropy data.

accuracy of SVD method and anisotropy weighting method are almost the same. The order of magnitude is less than 10^{-4}mm. To compare the two methods in anisotropy data, we perform simulation 2 and the result is shown in Fig. 4.

The accuracy improved along with the number of fiducial points, in addition the proposed method was always better

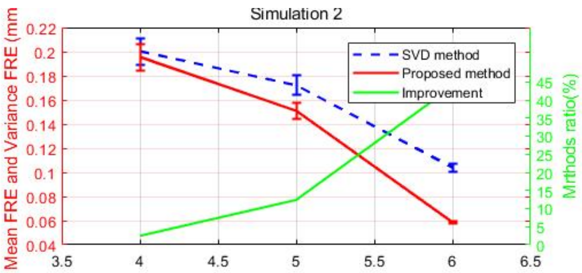

Fig. 4. Mean of FRE and Variance FRE of two methods in anisotropy data.

than SVD method. We compared the accuracy of the proposed method to SVD method, and found it increased almost 45% when the number of fiducial points is six.

The simulation shows the result of traditional SVD method is as good as proposed method in isotropy data. However in either anisotropy data, the registration accuracy of proposed method is always higher than SVD method.

V. CONCLUSION

We proposed a new extrinsic calibration method to solve the anisotropy error distribution problem. The method constructed weighting matrices based on measurement properties of sensors, and reduced the registration error by weighting the coordinate of different axes. Moreover, we also used surveying adjustment method to increase the accuracy. The improvement of registration accuracy was remarkable in simulation 2, and no matter how many fiducial points were the proposed method was always better than SVD method in anisotropy data. Furthermore, the accuracy of proposed method almost improved 45% compared with SVD method.

This proposed extrinsic calibration method had a very well result in the LiDAR and stereo camera system. Besides, most of calibration methods can use this method to increase the accuracy. Nevertheless, to apply the method in the real world needs to combine the analysis of error distribution model and actual sensor data to construct weighting matrices. In the future, we aim to apply our method on various calibration methods, to prove its robustness and to complete the calibration model.

REFERENCES

[1] Asvadi, Alireza, et al. ”Multimodal vehicle detection: fusing 3D-LIDAR and color camera data.” Pattern Recognition Letters 115 (2018): 20-29.

[2] Hwang, Jae Pil, et al. ”Multi-classifier based LIDAR and camera fusion.” 2007 IEEE Intelligent Transportation Systems Conference. IEEE, 2007.

[3] Nguatem, William, and Helmut Mayer. ”Modeling urban scenes from pointclouds.” Proceedings of the IEEE International Conference on Computer Vision. 2017.

[4] Fallah, Navid, et al. ”Indoor human navigation systems: A survey.” Interacting with Computers 25.1 (2013): 21-33.

[5] Zhou, Guyue, et al. ”Guidance: A visual sensing platform for robotic applications.” Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition Workshops. 2015.

[6] Ravi, Radhika, et al. ”Simultaneous System Calibration of a Multi-LiDAR Multicamera Mobile Mapping Platform.” IEEE Journal of selected topics in applied earth observations and remote sensing 11.5 (2018): 1694-1714.

[7] Cheng, Yang, Mark Maimone, and Larry Matthies. ”Visual odometry on the Mars exploration rovers.” 2005 IEEE International Conference on Systems, Man and Cybernetics. Vol. 1. IEEE, 2005.

[8] Guindel, Carlos, et al. ”Automatic extrinsic calibration for lidar-stereo vehicle sensor setups.” 2017 IEEE 20th International Conference on Intelligent Transportation Systems (ITSC). IEEE, 2017.

[9] Roy, Sbastien, and Ingemar J. Cox. ”A maximum-flow formulation of the n-camera stereo correspondence problem.” Sixth International Conference on Computer Vision (IEEE Cat. No. 98CH36271). IEEE, 1998.

[10] Balachandran, Ramya, J. Michael Fitzpatrick, and Robert F. Labadie. ”Fiducial registration for tracking systems that employ coordinate reference frames.” Medical Imaging 2005: Visualization, Image-Guided Procedures, and Display. Vol. 5744. International Society for Optics and Photonics, 2005.

[11] Shi, Chenghao, et al. ”Extrinsic Calibration and Odometry for Camera-LiDAR Systems.” IEEE Access (2019).

[12] Xiao, Zhipeng, et al. ”Accurate extrinsic calibration between monocular camera and sparse 3D lidar points without markers.” 2017 IEEE Intelligent Vehicles Symposium (IV). IEEE, 2017.

[13] Pandey, Gaurav, et al. ”Automatic targetless extrinsic calibration of a 3d lidar and camera by maximizing mutual information.” Twenty-Sixth AAAI Conference on Artificial Intelligence. 2012.

[14] Horaud, Radu, et al. ”An overview of depth cameras and range scanners based on time-of-flight technologies.” Machine vision and applications 27.7 (2016): 1005-1020.

[15] Fuersattel, Peter, et al. ”Accurate laser scanner to camera calibration with application to range sensor evaluation.” IPSJ Transactions on Computer Vision and Applications 9.1 (2017): 21.

[16] Fremont, Vincent, and Philippe Bonnifait. ”Extrinsic calibration between a multi-layer lidar and a camera.” 2008 IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems. IEEE, 2008.

[17] Pusztai, Zoltan, and Levente Hajder. ”Accurate calibration of LiDAR-camera systems using ordinary boxes.” Proceedings of the IEEE International Conference on Computer Vision. 2017.

[18] Kyt, Mikko, Mikko Nuutinen, and Pirkko Oittinen. ”Method for measuring stereo camera depth accuracy based on stereoscopic vision.” Three-Dimensional Imaging, Interaction, and Measurement. Vol. 7864. International Society for Optics and Photonics, 2011.

[19] Liu, Qiong, et al. ”Structural parameters optimal design and accuracy analysis for binocular vision measure system.” 2008 IEEE/ASME International Conference on Advanced Intelligent Mechatronics. IEEE, 2008.

[20] Glennie, Craig, and Derek D. Lichti. ”Static calibration and analysis of the Velodyne HDL-64E S2 for high accuracy mobile scanning.” Remote Sensing 2.6 (2010): 1610-1624.

[21] Maier-Hein, Lena, et al. ”Convergent iterative closest-point algorithm to accomodate anisotropic and inhomogenous localization error.” IEEE transactions on pattern analysis and machine intelligence 34.8 (2011): 1520-1532.

[22] Balachandran, Ramya, and J. Michael Fitzpatrick. ”Iterative solution for rigid-body point-based registration with anisotropic weighting.” Medical Imaging 2009: Visualization, Image-Guided Procedures, and Modeling. Vol. 7261. International Society for Optics and Photonics, 2009.

[23] Akyilmaz, O. ”Total least squares solution of coordinate transformation.” Survey Review 39.303 (2007): 68-80.

[24] Crocetto, Nicola, and Auatonio Vettore. ”Best unbiased estimation of variance-covariance components: From condition adjustment to a generalized methodology.” Journal of Information and Optimization Sciences 22.1 (2001): 113-122.