

# Less Conservative Delay-dependent Robust Stability Criteria for Uncertain Singular Time-delay Systems

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**Abstract** – Less conservative robust stability criteria for singular systems with parametric uncertainties are given in this paper. By developing a novel LKF, better results are achieved to pledge the robust stability of the target systems. Also, Integral inequalities proposed by Park et al. are also applied to reduce conservatism. At the end of the paper, simulation examples are provided to demonstrate that the results are validity and less conservativeness.

**Index Terms** – Singular systems; Time-delay; Parametric uncertainties; Less conservative.

## I. INTRODUCTION

In practical problems, uncertainties and time delays caused by system identification errors and device aging are inevitable. They are important factors causing system poor performance. Thence, researches on singular systems with parametric uncertainties and time-delay have significant theoretical and practical value.

In the past decade, more and more scholars have devoted themselves to the research of robust stability, which has promoted the further development of singular systems. Accordingly, In [3], the robust guaranteed cost control was studied by developing a new LKF. In [6], [7] and [10], problems of  $H_\infty$  control for singular systems with time-invariant delay was explored. [4,5] mainly studied the robust stabilization of the corresponding time-delay systems. In [11], new stability criteria for singular systems with time-delay was proposed. Then [12] mainly proposed less conservative stability criteria by delay-partitioning approach. Instead of time-invariant delay, [14] mainly investigated the robust stability of singular systems with time-varying delay. [19] mainly investigated the robust stabilization of singular systems by an improved approach.

Since concept of singular systems was proposed, many literatures have done a lot of research in this field, such as in [1,2,14-19] and so on. But there is still space for betterment in the goal of reducing conservatism. So in this paper, we will develop a new LKF and utilizing integral inequalities proposed by Park to investigate the robust stability of singular systems with parametric uncertainties, Then, two simulation examples are derived which prove the proposed theorem in this paper are valid and superior.

## II. PROBLEM STATEMENT

Continuous singular systems with time-delay and parameter uncertainties are generally described in the following form:

$$\begin{cases} E\dot{x}(t) = (A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t-d), \\ x(\theta) = \varphi(\theta), \theta \in [-d, 0], \end{cases} \quad (1)$$

Where  $x(t) \in R^n$  indicates the state vector of systems (1), state delay  $d$  is constant, and  $\varphi(t)$  is an compatible initial function.  $E \in R^n$  may be singular which is identified that  $\text{rank} E = r \leq n$ .  $A$  and  $A_d$  are known real constant matrices.  $\Delta A(t)$ ,  $\Delta A_d(t)$  are unknown matrices representing norm-bounded parametric uncertainties:

$$[\Delta A(t), \Delta A_d(t)] = GN(t)[H_1, H_2], \quad (2)$$

Where  $G, H_1$  and  $H_2$  indicate known real constant matrices with appropriate dimensions,  $N(t)$  is uncertain matrix which fulfills that:

$$N(t)^T N(t) \leq I.$$

(3)

When  $\Delta A(t) = 0$ ,  $\Delta A_d(t) = 0$ , singular systems (1) can be described as:

$$\begin{cases} E\dot{x}(t) = Ax(t) + A_d x(t-d), \\ x(\theta) = \phi(\theta), \theta \in [-d, 0], \end{cases} \quad (4)$$

**Definition 2.1**([1]):

(i) If  $\det(sE - A) \neq 0$ , then we can say that the pair  $(E, A)$  is regular.

(ii) If  $\text{rank}(E) = \deg(\det(sE - A))$ , then we can say  $(E, A)$  is impulse-free.

(iii) If all the roots of  $\det(sE - A) = 0$  have negative real parts then we can say  $(E, A)$  is stable.

(iv) If  $(E, A)$  is regular, impulse-free and stable, then we can say  $(E, A)$  is admissible.

**Definition 2.2**([2]):

(i) If  $(E, A)$  is regular and impulse-free, then we can get that singular systems described by (4) are regular and impulse-free.

(ii) If, for any  $\varepsilon > 0$ , there exists a scalar  $\delta(\varepsilon) > 0$ , for any function  $\varphi(t)$  having that:

$$\sup_{-d < t \leq 0} \|\varphi(t)\| < \delta(\varepsilon),$$

The state vector  $x(t)$  to (4) fulfills that

$$\|x(t)\| < \varepsilon,$$

And a scalar  $\delta > 0$  can be chosen such that

$$\sup_{-d < t \leq 0} \|\varphi(t)\| < \delta,$$

Implies

$$x(t) \rightarrow 0, \quad t \rightarrow \infty.$$

Then we can concluded systems (4) are stable.

**Lemma 2.1**(Park,[20]): For arbitrary matrix  $T \in R^{n \times n}$  fulfills  $T > 0$ , vector function  $x: [a, b] \rightarrow R^n$ , it can be deduced that:

$$(b-a) \int_a^b \dot{x}^T(s) T \dot{x}(s) ds \geq \omega^T(t) E_e^T \hat{T} E_e \omega(t), \quad (5)$$

Which  $\hat{T} = \text{diag}\{T, 3T, 5T\}$  and  $\tau = b - a$ ,

$$E_e = \begin{bmatrix} I & -I & 0 & 0 \\ I & I & -2I & 0 \\ I & -I & 6I & -12I \end{bmatrix}$$

$$\omega(t) = [x^T(b), x^T(a), \frac{1}{\tau} \int_a^b x^T(s) ds, \frac{1}{\tau^2} \int_a^b \int_s^b x^T(u) du ds]^T.$$

**Lemma 2.2**(Finsler,[12]): If there exists a  $n$  dimensional vector  $\eta$ , a symmetric matrix  $J \in R^{n \times n}$  and a matrix  $K \in R^{m \times n}$  which is assumed that the rank of  $K$  is less than  $n$ , Then the statements (1) to (4) are equivalent:

(1)  $\eta^T J \eta < 0$ , for any  $\eta$  such that  $K\eta = 0, \eta \neq 0$ .

(2)  $K^\top J K < 0$ ,

(3)  $\exists \mu \in R: J - \mu K^\top K < 0$ ,

(4)  $\exists \zeta \in R^{n \times m}: J + \text{sym}(\zeta K) < 0$ ,

Where  $K^\top$  indicates a basis for the null-space of  $K$ .

**Lemma 2.3**([12]): If there exists matrices with appropriate dimensions:  $\Theta = \Theta^\top$ ,  $Y > 0$ ,  $H$ ,  $F(t)$  and  $E$ , then the statement (1) and statement (2) are equivalent:

(1)  $\Theta + HF(t)E + E^\top F^\top(t)H^\top < 0$ ,  $F^\top(t)F(t) \leq I$ .

(2)  $\Theta + HY^{-1}H^\top + E^\top YE < 0$ .

### III. RESULTS

In this section, a new LKF is established to give sufficient criteria for admissibility of systems (4) and systems (1).

**Theorem 3.1:** Singular systems (4) is said to be admissible if there exists matrices with appropriate dimensions :

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ * & P_{22} & P_{23} \\ * & * & P_{33} \end{bmatrix} > 0, \quad W > 0, Q > 0, R > 0, L_0, L_1, L_2$$

so that the following LMI holds:

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} \\ * & * & \Omega_{33} & \Omega_{34} & \Omega_{35} \\ * & * & * & \Omega_{44} & \Omega_{45} \\ * & * & * & * & \Omega_{55} \end{bmatrix} < 0, \quad (6)$$

Where

$$\Omega_{11} = E^\top P_{12} + P_{12}^\top E + E^\top P_{13} E + E^\top P_{13}^\top E$$

$$+ Q + dR - 9E^\top WE + L_0 A_d + A_d^\top L_0^\top,$$

$$\Omega_{12} = -E^\top P_{12} + 3E^\top WE + L_0 A_d + A_d^\top L_1^\top,$$

$$\Omega_{13} = -E^\top P_{13} E + dP_{22} + dE^\top P_{23}^\top - 24E^\top WE,$$

$$\Omega_{14} = P_{23} + E^\top P_{33} + (60/d)E^\top W,$$

$$\Omega_{15} = E^\top P_{11} + SV^\top - L_0 + A_d^\top L_2^\top,$$

$$\Omega_{22} = -Q - 9E^\top WE + L_1 A_d + A_d^\top L_1^\top,$$

$$\Omega_{23} = -dP_{22} + 36E^\top WE,$$

$$\Omega_{24} = -P_{23} - (60/d)E^\top W,$$

$$\Omega_{25} = -L_1 + A_d^\top L_2^\top,$$

$$\Omega_{33} = -dP_{23} E - dE^\top P_{23}^\top - dR - 192E^\top WE,$$

$$\Omega_{34} = -E^\top P_{33} + (360/d)E^\top W,$$

$$\Omega_{35} = dP_{12}^\top,$$

$$\Omega_{44} = (-720/(d \times d))W,$$

$$\Omega_{45} = P_{13}^\top,$$

$$\Omega_{55} = (d \times d)W - L_2 - L_2^\top.$$

And  $V$  is any column full rank matrix which fulfills that  $E^\top V = 0$ .

*Proof:* considering that  $\text{rank} E = r \leq n$ . then it will be two nonsingular matrices  $F$  and  $J$  so that

$$\bar{E} = FEJ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{A} = FAJ = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$

$$\bar{S} = J^\top S = \begin{bmatrix} S_{11} \\ S_{21} \end{bmatrix},$$

$$\bar{R} = F^{-\top} V = \begin{bmatrix} 0 \\ V_{22} \end{bmatrix}.$$

It can be deduced from (6) that

$$\begin{bmatrix} \Omega_{11} & \Omega_{15} \\ * & \Omega_{55} \end{bmatrix} < 0 \quad (7)$$

Left-multiplying and right-multiplying (7) by  $\begin{bmatrix} I & A^T \end{bmatrix}$  and its transposition, yields that

$$\begin{aligned} & \text{sym}(E^T P_{12} + E^T P_{13} E + E^T P_{11} A + A^T V S^T) \\ & + (d \times d) A^T W A + Q + dR - 9E^T W E < 0 \end{aligned} \quad (8)$$

Left-multiplying and right-multiplying (8) by  $J^T$  and  $J$ , yields  $\text{sym}(S_{21} V_{22}^T A_{22}) < 0$ , which shows that  $A_{22}$  is nonsingular matrix, by Definition 2.1 and 2.2, it means that system (4) is regular and impulse-free.

Construct the following LKF

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) + V_4(x_t) \quad (9)$$

Where

$$V_1(x_t) = \xi_1^T(t) P \xi_1(t),$$

$$V_2(x_t) = \int_{t-d}^t x^T(\alpha) Q x(\alpha) d\alpha,$$

$$V_3(x_t) = \int_{-d}^0 \int_{t+\theta}^t x^T(\alpha) R x(\alpha) d\alpha d\theta,$$

$$V_4(x_t) = d \int_{-d}^0 \int_{t+\theta}^t \dot{x}^T(\alpha) E^T W E \dot{x}(\alpha) d\alpha d\theta,$$

$$\xi_1^T(t) = \begin{bmatrix} x^T(t) E^T & \int_{t-d}^t x^T(\alpha) d\alpha & \frac{1}{d} \int_{-d}^0 \int_{t+\theta}^t x^T(\alpha) E^T d\alpha d\theta \end{bmatrix}.$$

Then deriving the function  $V(x_t)$ , yields that

$$\dot{V}_1(x_t) = 2\xi_1^T(t) P \dot{\xi}_1(t), \quad (10)$$

Where

$$\dot{\xi}_1^T(t) = \begin{bmatrix} \dot{x}^T(t) E^T & x^T(t) - x^T(t-d) & x^T(t) E^T - \frac{1}{d} \int_{t-d}^t x^T(\alpha) E^T d\alpha \end{bmatrix}.$$

$$\dot{V}_2(x_t) = x^T(t) Q x(t) - x^T(t-d) Q x(t-d), \quad (11)$$

$$\begin{aligned} \dot{V}_3(x_t) &= dx^T(t) R x(t) - \int_{t-d}^t x^T(\alpha) R x(\alpha) d\alpha \\ &\leq dx^T(t) R x(t) - \frac{1}{d} \left( \int_{t-d}^t x(\alpha) d\alpha \right)^T R \left( \int_{t-d}^t x(\alpha) d\alpha \right), \end{aligned} \quad (12)$$

From Lemma 2.1,

$$\begin{aligned} \dot{V}_4(x_t) &= d^2 \dot{x}^T(t) E^T W E \dot{x}(t) - d \int_{t-d}^t \dot{x}^T(\alpha) E^T W E \dot{x}(\alpha) d\alpha \\ &\leq d^2 \dot{x}^T(t) E^T W E \dot{x}(t) - \xi_2^T(t) \hat{\Omega} \xi_2(t). \end{aligned} \quad (13)$$

Where

$$\begin{aligned} \xi_2^T(t) &= [x^T(t) \quad x^T(t-d) \\ &\quad \frac{1}{d} \int_{t-d}^t x^T(\alpha) d\alpha \quad \frac{1}{(d \times d)} \int_{-d}^0 \int_{t+\theta}^t x^T(\alpha) d\alpha d\theta], \\ \hat{\Omega} &= \begin{bmatrix} I & I & 0 & 0 \\ I & I & -2I & 0 \\ I & -I & 6I & -12I \end{bmatrix}^T \text{diag}\{E^T W E, 3E^T W E, 5E^T W E\} \end{aligned}$$

$$\times \begin{bmatrix} I & I & 0 & 0 \\ I & I & -2I & 0 \\ I & -I & 6I & -12I \end{bmatrix}.$$

Noting that  $E^T V = 0$ , we can get

$$x^T(t) S V^T (E \dot{x}(t)) = 0. \quad (14)$$

From (10), (11), (12), (13), (14), we can see that

$$\dot{V}(t) \leq \xi^T(t) \Omega \xi(t) - \xi^T(t) (L B + B^T L^T) \xi(t), \quad (15)$$

Where

$$\xi^T(t) = [x^T(t) \quad x^T(t-d) \quad \frac{1}{d} \int_{t-d}^t x^T(\alpha) d\alpha$$

$$\quad \frac{1}{d} \int_{-d}^0 \int_{t+\theta}^t x^T(\alpha) E^T d\alpha d\theta \quad \dot{x}^T(t) E^T],$$

$$L = [L_0^T \quad L_1^T \quad 0 \quad 0 \quad L_2^T]^T,$$

$$B = [A \quad A_d \quad 0 \quad 0 \quad -I].$$

Obviously, we can find that  $B \xi(t) = 0$ . Then applying Lemma

2.2, let  $\xi^T(t) \Omega \xi(t) < 0$ , which means  $\dot{V}(t) < 0$ . Thus, the singular systems (4) can be stable. So far, the proof is completed.

**Theorem 3.2:** singular systems (1) is called admissible if there exist several matrices with appropriate dimensions:

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ * & P_{22} & P_{23} \\ * & * & P_{33} \end{bmatrix} > 0, \quad W > 0, \quad Q > 0, \quad R > 0, \quad L_0, L_1, L_2$$

make the following LMI true:

$$\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} & \Lambda_{15} & L_0 G \\ * & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} & \Lambda_{25} & L_1 G \\ * & * & \Lambda_{33} & \Lambda_{34} & \Lambda_{35} & 0 \\ * & * & * & \Lambda_{44} & \Lambda_{45} & 0 \\ * & * & * & * & \Lambda_{55} & L_2 G \\ * & * & * & * & * & -Z \end{bmatrix} < 0, \quad (16)$$

Where

$$\begin{aligned} \Lambda_{11} &= E^T P_{12} + P_{12}^T E + E^T P_{13} E + E^T P_{13}^T E + Q \\ &\quad + dR - 9E^T W E + L_0 A + A^T L_0^T + H_1^T Z H_1, \end{aligned}$$

$$\Lambda_{12} = -E^T P_{12} + 3E^T W E + L_0 A_d + A^T L_1^T + H_1^T Z H_2,$$

$$\Lambda_{13} = -E^T P_{13} E + dP_{22} + dE^T P_{23}^T - 24E^T W E,$$

$$\Lambda_{14} = P_{23} + E^T P_{33} + (60/d) E^T W,$$

$$\Lambda_{15} = E^T P_{11} + S V^T - L_0 + A^T L_2^T,$$

$$\Lambda_{22} = -Q - 9E^T W E + L_1 A_d + A_d^T L_1^T + H_2^T Z H_2,$$

$$\Lambda_{23} = -dP_{22} + 36E^T W E,$$

$$\Lambda_{24} = -P_{23} - (60/d) E^T W,$$

$$\Lambda_{25} = -L_1 + A_d^T L_2^T,$$

$$\Lambda_{33} = -dP_{23} E - dE^T P_{23}^T - dR - 192E^T W E,$$

$$\begin{aligned}\Lambda_{34} &= -E^T P_{33} + (360/d)E^T W, \\ \Lambda_{35} &= dP_{12}^T, \\ \Lambda_{44} &= (-720/(d \times d))W, \\ \Lambda_{45} &= P_{13}^T, \\ \Lambda_{55} &= (d \times d)W - L_2 - L_2^T.\end{aligned}$$

*Proof:* Let  $A = A + \Delta A(t)$ ,  $A_d = A_d + \Delta A_d(t)$  in (6), we can get that

$$\Omega + MN(t)Y + (MN(t)Y)^T < 0, \quad (17)$$

and

$$M = \begin{bmatrix} G^T L_0^T & G^T L_1^T & 0 & 0 & G^T L_2^T \end{bmatrix}^T,$$

$$Y = \begin{bmatrix} H_1 & H_2 & 0 & 0 & 0 \end{bmatrix}.$$

From (17) and Lemma 2.3, there exists  $Z > 0$  such that

$$\Omega + Y^T ZY + MZ^{-1}M^T < 0. \quad (18)$$

Then by Schur Complement, we can get (16), the proof is completed.

#### IV. NUMERICAL SIMULATION EXAMPLE

*Example 4.1([13]):* Considering singular system (4) with

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 0.5 & 0 \\ -1 & -1 \end{bmatrix}, A_d = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}.$$

The results of the maximum allowable time-delay are compared in TABLE I. It implies that the derived theorems in this paper are less conservative.

For simulation, let  $d = 1.2091$  and  $\varphi(t) = [-1.2874 \quad 0.6437]$ , the state response of  $x(t)$  are shown in Fig. 1, which implies that theorem 3.1 is correct.

*Example 4.2([12]):* Considering the following singular system [1] with the following parameters:

$$E = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}, A_d = \begin{bmatrix} -2.4 & 2 \\ 0 & 1 \end{bmatrix},$$

$$G = \lambda I, H_1 = H_2 = 0.5I, \lambda > 0$$

For different  $\lambda$ , The results of the maximum allowable time-delay are compared in TABLE II. which implies that the conservatism is reduced.

TABLE I  
Maximum Delay

Reference theorems	Maximum allowable delay
Reference [14]	1
Reference [15]	1.1547
Reference [16]	1.1547
Reference [17]	1.1547
Reference [18]	1.1547
Reference [19]	1.1547

Reference [13](N=2)	1.1954
Reference [13](N=10)	1.2060
Theorem 3.1	1.2091

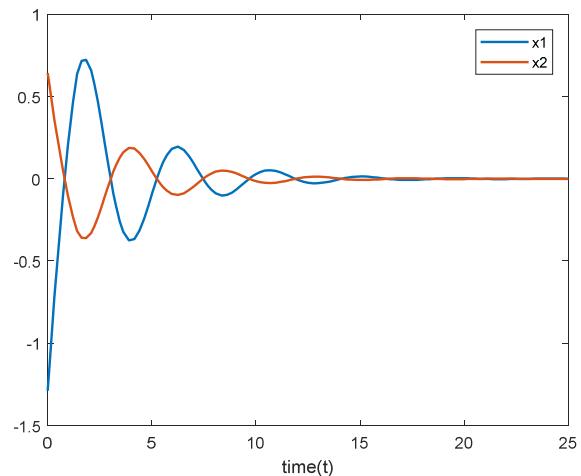


Fig. 1 the state response of  $x(t)$ .

TABLE II  
Maximum Delay

$\lambda$	0.25	0.30	0.35	0.40	0.45	0.5
[20]	0.4209	0.3939	0.3637	0.3279	0.2817	0.2106
[21]	0.8087	0.7942	0.7689	0.7262	0.6521	0.5054
[22]	0.8514	0.8249	0.7924	0.7438	0.6641	0.5110
[23]	0.8962	0.8787	0.8616	0.8448	0.8283	0.8121
[24]	0.8962	0.8787	0.8616	0.8448	0.8283	0.8121
[13](N=1)	0.9425	0.9232	0.9043	0.8858	0.8676	0.8496
[13](N=3)	0.9427	0.9234	0.9045	0.8860	0.8677	0.8498
Theorem3.2	0.9429	0.9237	0.9048	0.8862	0.8679	0.8500

#### V. CONCLUSION

Throughout the full paper, new admissibility criterions for singular systems with parametric uncertainties and time-delay are provided. Through using novel LKFs and integral inequalities, the results are less conservative. Finally, the superiority and validity of the results are proved by simulation examples.

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