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# Global Output-Feedback Stabilization for a Class of Stochastic Nonlinear Systems

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***Abstract*— This paper aims to solve the global output-feedback stabilization problem for a class of stochastic nonlinear systems by combining observer construction and backstepping technique. Considering the fact that the system state is unmeasurable except for the output, a parameter gain is introduced to dominate nonlinearities and diffusion terms in the design of linear observer, and the backstepping is used to construct the actual output-feedback controller. A stochastic simulation example is provided to illustrate the effectiveness of the control strategy.**

***Index Terms*— Stochastic nonlinear systems, Global output-feedback stabilization, Observer construction, Backstepping.**

## I. Introduction

Global output-feedback stabilization has attracted more and more attention in nonlinear control domain, the major reason lies in the most states in actual systems are unmeasurable. Meanwhile, there is no unified method to design the state observer [1]-[4]. Luckily, with the development of the backstepping method [5] and nonseparation principle paradigm [6], lots of meaningful outcomes have studied the output-feedback stabilization for nonlinear systems with multiple structures and uncertainties, such as [7]-[21] and the references.

Note that stochastic factors are widespread in real industrial systems in the form of external environment variations, stochastic errors, parameter perturbations and so on. Thus, it is of great significance to study the stabilization of stochastic nonlinear systems [19]-[21]. With the excellent transient response performance in mind, [3] developed the backstepping technique well into stochastic nonlinear control, which greatly promotes the development of stochastic nonlinear control by means of observer construction.

This paper solves the global stabilization for a class of stochastic nonlinear systems by output-feedback. The main strategy of this paper are highlighted twofold. On the one hand, a parameter gain is drawn into the observer to dominate uncertain nonlinearities and diffusion terms for a class of s-tochastic nonlinear systems. On the other hand, the traditional backstepping method is combined with observer design in the construction of the output-feedback controller to overcome the cost of large control gain.

## II. Problem Formulation and Preliminaries

In this paper, for $i = 1, \dots, n-1$, consider the following stochastic nonlinear system

$$\begin{cases} d\xi_i(t) = \xi_{i+1}(t)dt + f_i(t,\xi,u)dt + g_i^T(t,\xi,u)d\omega, \\ d\xi_n(t) = u(t)dt + f_n(t,\xi,u)dt + g_n^T(t,\xi,u)d\omega, \\ y(t) = \xi_1(t), \end{cases} \tag{1}$$

where $\xi(t) = [\xi_1(t), \dots, \xi_n(t)]^T \in \mathbb{R}^n$ and $u(t) \triangleq \xi_{n+1}(t) \in \mathbb{R}$ are system state and control input, the initial conditions $\xi(0) = \xi_0$, and $y(t) \in \mathbb{R}$ is system output. $f_i : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ and $g_i : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^r$ are continuous and satisfy locally Lipschitz conditions. For $\forall t \geq 0$, $f(0) = g(0) = 0$. $\omega$ is an $r$-dimensional standard Wiener process defined on a probability space $(\Omega, \mathcal{F}, P)$ with $\Omega$ being a sample space, $\mathcal{F}$ being a $\sigma$-field, and $P$ being a probability measure. Only the output $y(t)$ is known.

The following assumption is given.

*Assumption 1: For each $j = 1, \dots, n$, there exists a constant $c \geq 0$ such that*

$$|f_j(t,\xi,u)| + \|g_j(t,\xi,u)\| \leq c \sum_{k=1}^{j} |\xi_j|. \tag{2}$$

Assumption 1 is general and also be used in [1], [3], [6], which shows that nonlinearities $f_i$ and diffusion terms $g_i$ are related to states $\xi_1, \dots, \xi_i$ in a linear growth, and indicates the lower-triangular form of the system (1).

Now, four key Lemmas are given as follows.

*Lemma 1: Consider stochastic nonlinear system with the form*

$$dx = f(\xi)dt + g^T(\xi)d\omega, \quad \forall \xi(0) = \xi_0 \in \mathbb{R}^n, \tag{3}$$

*where $\xi \in \mathbb{R}^n$ is the state vector; $\omega$ is defined as in (1); $f(\xi)$: $\mathbb{R}^n \to \mathbb{R}^n$ and $g^T$: $\mathbb{R}^n \to \mathbb{R}^{n\times r}$ are continuous, and*



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$f(0)=g(0)=0$, $\forall t\geq 0$. *For a* $\mathcal{C}^2$ *function* $V(\xi)\in\mathbb{R}^n$, *the differential operator* $\mathcal{L}$ *acting on system (3) is defined as*

$$\mathcal{L}V(\xi)=\frac{\partial V(\xi)}{\partial \xi}f(\xi)+\frac{1}{2}Tr\left\{g^T(\xi)\frac{\partial^2 V(\xi)}{\partial \xi^2}g(\xi)\right\}. \quad (4)$$

*Lemma 2: For a given constant* $r>0$ *and all* $z_i\in\mathbb{R}, i=1,\ldots,n$, *there is*

$$(|z_1|+\cdots+|z_n|)^r\leq C_r(|z_1|^r+\cdots+|z_n|^r), \quad (5)$$

*where* $C_r=1$ *if* $0<r\leq 1$ *and* $C_r=n^{r-1}$ *if* $r>1$.

*Lemma 3: Given a real matrix* $K=(k_{ij})_{m\times n}$, *there is* $\|K\|_\infty\leq\sqrt{n}\|K\|_F$. *If* $n=m$, *then* $\mathrm{Tr}(K)\leq m\|K\|_\infty$.

*Lemma 4: Let* $c,d$ *be positive real numbers. For* $a\in\mathbb{R}, b\in\mathbb{R}$, *and any positive real-valued function* $\mu(a,b)$, *there holds*

$$|a|^c|b|^d\leq\frac{c}{c+d}\mu(a,b)|a|^{c+d}+\frac{d}{c+d}\mu^{-\frac{c}{d}}(a,b)|b|^{c+d}.$$

The proofs of Lemmas 1-4 can be seen from [2], [22], [23], and [5], respectively.

## III. Main Results

Now, the main results of this paper are presented in the following.

**Theorem 1.** If the stochastic nonlinear systems (1) satisfy Assumption 1, then there exists an output-feedback controller (29) which can ensure that the equilibrium point at the origin of the closed-loop systems is stochastically asymptotically stable, in addition, $P\{\lim_{t\to\infty}\xi(t;t_0,\xi_0)=0\}=1$ for all $\xi_0\in\mathbb{R}^n$.

**Proof.** We give the proof step by step.

**Step** $0$**.** First of all, a linear observer is constructed in the following

$$\begin{cases}\dot{\hat{\xi}}_j(t)=\hat{\xi}_{j+1}(t)+L^j\kappa_j(\xi_1(t)-\hat{\xi}_1(t)), j=1,\ldots,n-1,\\ \dot{\hat{\xi}}_n(t)=u(t)+L^n\kappa_n(\xi_1(t)-\hat{\xi}_1(t)),\end{cases} \quad (6)$$

where $L\geq 1$ is a constant which will be determined later, and parameters $\kappa_1,\ldots,\kappa_n$ are positive coefficients of Hurwitz polynomial $H_1(s)=s^n+\kappa_1 s^{n-1}+\cdots+\kappa_{n-1}s+\kappa_n$.

With the aid of the coordinate transformations of

$$e_j(t)=\frac{\xi_j(t)-\hat{\xi}_j(t)}{L^{j-1}},\ j=1,\ldots,n, \quad (7)$$

combining (1), (6) with (7), one has

$$\begin{aligned}de_j(t)&=\frac{d\xi_j(t)-d\hat{\xi}_j(t)}{L^{j-1}}\\&=\frac{\xi_{j+1}(t)dt+f_j(t,\xi(t),u(t))dt+g_j^T(t,\xi(t),u(t))d\omega}{L^{j-1}}\\&\quad\frac{-\hat{\xi}_{j+1}(t)dt-L^j\kappa_j(\xi_1(t)-\hat{\xi}_1(t))dt}{L^{j-1}}\\&=-L\kappa_j e_1(t)dt+Le_{j+1}(t)dt+\frac{f_j(t,\xi(t),u(t))dt}{L^{j-1}}\\&\quad+\frac{g_j^T(t,\xi(t),u(t))d\omega}{L^{j-1}},\quad j=1,\ldots,n.\end{aligned}$$

With above formula in mind, there holds

$$de(t)=\Big(LK_e e(t)+f(t,\xi(t),u(t))\Big)dt+g^T(t,\xi(t),u(t))d\omega, \quad (8)$$

where

$$K_e=\begin{bmatrix}-\kappa_1 & 1 & 0 & \cdots & 0\\ -\kappa_2 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ -\kappa_{n-1} & 0 & 0 & \cdots & 1\\ -\kappa_n & 0 & 0 & \cdots & 0\end{bmatrix},\ \kappa=\begin{bmatrix}\kappa_1\\ \kappa_2\\ \vdots\\ \kappa_n\end{bmatrix},$$

$$f=\begin{bmatrix}f_1\\ \frac{f_2}{L}\\ \vdots\\ \frac{f_n}{L^{n-1}}\end{bmatrix},\ g^T=\begin{bmatrix}g_1^T\\ \frac{g_2^T}{L}\\ \vdots\\ \frac{g_n^T}{L^{n-1}}\end{bmatrix},\ e=\begin{bmatrix}e_1\\ e_2\\ \vdots\\ e_n\end{bmatrix}.$$

Note that $K_e$ is Hurwitz, there exists a symmetric and positive definite matrix $P_e$ such that $K_e^TP_e+P_eK_e=-I$. Then, for system (8), from Lemma 1, the differential operator acting on $W(e)=(n+1)e^TP_ee$ shows

$$\begin{aligned}\mathcal{L}W(e)&=\frac{\partial V}{\partial e}\big(LK_ee+f\big)+\frac{1}{2}\mathrm{Tr}\Big\{g\frac{\partial^2 V}{\partial e^2}g^T\Big\}\\&=2(n+1)e^TP_e\big(LK_ee+f\big)+(n+1)\mathrm{Tr}\{gP_eg^T\}. \quad (9)\end{aligned}$$

Next, each term on the right side of the above equation (9) is estimated separately.

Firstly, it can be deduced that

$$\begin{aligned}2(n+1)e^TP_eLK_ee&=(n+1)Le^T(P_eK_e+K_e^TP_e)e\\&=-(n+1)L\|e\|^2. \quad (10)\end{aligned}$$

Later, for each $j=1,\ldots,n$, from Assumption 1 and $L\geq 1$, one obtains

$$|f_j(t,\xi,u)|\leq|f_j(t,\xi,u)|+\|g_j(t,\xi,u)\|\leq c\sum_{i=1}^{j}|\xi_i|,$$

moreover,

$$\Big|\frac{f_j(t,\xi,u)}{L^{j-1}}\Big|\leq\frac{c}{L^{j-1}}\sum_{i=1}^{j}|\xi_i|.$$

Think about

$$f=\big[f_1,\frac{f_2}{L},\cdots,\frac{f_n}{L^{n-1}}\big]^T.$$

Lemma 2 shows

$$\begin{aligned}\|f\| &= \Big(\sum_{j=1}^{n}\frac{f_j^2}{L^{2(j-1)}}\Big)^{\frac{1}{2}}\leq\sum_{j=1}^{n}\Big|\frac{f_j}{L^{j-1}}\Big|\\ &\leq \sum_{j=1}^{n}\Big(\frac{c}{L^{j-1}}\sum_{i=1}^{j}|\xi_i|\Big)\leq c\sum_{j=1}^{n}\frac{n-j+1}{L^{j-1}}|\xi_j|.\end{aligned}$$

Take

$$\begin{aligned}&2\|e\|\cdot\|P_e\|c\frac{n-j+1}{L^{j-1}}|\xi_j|\\ \le\ & 2(n-j+1)c\|P_e\|\cdot\|e\|^2+\frac{(n-j+1)c}{2L^{2j-2}}\|P_e\|\xi_j^2\end{aligned}$$

into consideration with $j=1,\dots,n$, it renders

$$\begin{aligned}&2(n+1)e^TP_ef\\ &\le 2(n+1)\|e\|\cdot\|P_e\|c\sum_{j=1}^{n}\frac{n-j+1}{L^{j-1}}|\xi_j|\\ &\le n(n+1)^2c\|P_e\|\|e\|^2+\sum_{j=1}^{n}\frac{(n-j+1)(n+1)c}{2L^{2j-2}}\|P_e\|\xi_j^2. \quad (11)\end{aligned}$$

Then, by Assumption 1 and Lemma 2, we have

$$\begin{aligned}\|g\|_F^2\ &=\|[g_1,\frac{g_2}{L},\dots,\frac{g_n}{L^{n-1}}]\|_F^2\le\sum_{j=1}^{n}\Big(\sum_{i=1}^{j}\frac{c|\xi_i|}{L^{j-1}}\Big)^2\\ &\le\sum_{j=1}^{n}\frac{jc^2}{L^{2j-2}}\sum_{i=1}^{j}x_i^2\le\sum_{j=1}^{n}\frac{(n+j)(n-j+1)c^2}{2L^{2j-2}}\xi_j^2,\end{aligned}$$

from Lemma 3, one leads to

$$\begin{aligned}&(n+1)\mathrm{Tr}\{gP_eg^T\}\\ \le\ & r\|gP_eg^T\|_\infty\le(n+1)r\sqrt{r}\|gP_eg^T\|_F\\ \le\ & r\sqrt{r}\|g\|_F\cdot\|P_eg^T\|_F\le(n+1)r\sqrt{r}\|P_e\|_F\cdot\|g\|_F^2\\ \le\ & (n+1)r\sqrt{r}\|P_e\|_F\sum_{j=1}^{n}\frac{(n+j)(n-j+1)c^2}{2L^{2j-2}}\xi_j^2. \quad (12)\end{aligned}$$

Substituting (10)-(12) into (9) yields

$$\mathcal{L}W(e)\le-(n+1)L\|e\|^2+d_1\|e\|^2+\sum_{j=1}^{n}\frac{d_{j1}}{2L^{2j-2}}\xi_j^2, \quad (13)$$

where $d_1=n(n+1)^2c\cdot\|P_e\|$, and for each $j=1,\dots,n$,

$$d_{j1}=(n+1)(n-j+1)c\|P_e\|+(n+j)(n-j+1)c^2r\sqrt{r}\|P_e\|_F,$$

and it is obviously to see that these constants are independent of $L$ and nonnegative.

Considering equation (7), one has $\xi_j=\hat{\xi}_j+L^{j-1}e_j$, so that

$$\begin{aligned}\sum_{j=1}^{n}\frac{d_{j1}}{2L^{2j-2}}\xi_j^2\ &\le\ \sum_{j=1}^{n}\frac{d_{j1}}{2L^{2j-2}}2(\hat{\xi}_j^2+L^{2j-2}e_j^2)\\ &\le\ \sum_{j=1}^{n}(\frac{d_{j1}}{L^{2j-2}}\hat{\xi}_j^2+d_{j1}e_j^2), \quad (14)\end{aligned}$$

substituting (14) into (13), there holds

$$\begin{aligned}\mathcal{L}W(e)\ \le\ & -(n+1)L\|e\|^2+(d_1+\sum_{j=1}^{n}d_{j1})\|e\|^2\\ &+\sum_{j=1}^{n}\frac{d_{j1}}{L^{2j-2}}\hat{\xi}_j^2. \quad (15)\end{aligned}$$

**Step** 1**.** Consider the Lyapunov function

$$V_1(e,\hat{\xi}_1)=W(e)+\frac{\hat{\xi}_1^2}{2}, \quad (16)$$

from Lemma 4, a direct calculation shows

$$\begin{aligned}\mathcal{L}V_1\ \ =\ \ & \mathcal{L}W+\hat{\xi}_1(\hat{\xi}_2+L\kappa_1e_1)\\ \le\ & \mathcal{L}W+\hat{\xi}_1\hat{\xi}_2+\frac{L}{4}\kappa_1^2\hat{\xi}_1^2+L\|e\|^2\\ \le\ & -(nL-(d_1+\sum_{j=1}^{n}d_{j1}))\|e\|^2+\sum_{j=2}^{n}\frac{d_{j1}}{L^{2j-2}}\hat{\xi}_j^2\\ &+\hat{\xi}_1\hat{\xi}_2+\frac{L}{4}\kappa_1^2\hat{\xi}_1^2+d_{11}\hat{\xi}_1^2. \quad (17)\end{aligned}$$

Let $L\ge d_{11}$ and $\eta_2=\hat{\xi}_2-\alpha_1$ with $\alpha_1$ is the first virtual controller, then

$$d_{11}\hat{\xi}_1^2\le L\hat{\xi}_1^2,\qquad \frac{d_{21}}{L^2}\hat{\xi}_2^2\le\frac{2d_{21}}{L^2}\eta_2^2+\frac{2d_{21}}{L^2}\alpha_1^2, \quad (18)$$

with this in mind, we have

$$\begin{aligned}\mathcal{L}V_1\le&-(nL-(d_1+\sum_{j=1}^{n}d_{j1}))\|e\|^2+\sum_{j=3}^{n}\frac{d_{j1}}{L^{2j-2}}\hat{\xi}_j^2\\ &+\frac{2d_{21}}{L^2}\eta_2^2+\frac{2d_{21}}{L^2}\alpha_1^2+\hat{\xi}_1\eta_2+\hat{\xi}_1\alpha_1+L(\frac{\kappa_1^2}{4}+1)\hat{\xi}_1^2. \quad (19)\end{aligned}$$

Choosing the first virtual controller

$$\alpha_1=-Lb_1\hat{\xi}_1,\qquad b_1=n+\frac{\kappa_1^2}{4}+1>0 \quad (20)$$

results in

$$\begin{aligned}\mathcal{L}V_1\le&-(nL-(d_1+\sum_{j=1}^{n}d_{j1}))\|e\|^2-(nL-2d_{21}b_1^2)\hat{\xi}_1^2\\ &+\sum_{j=3}^{n}\frac{d_{j1}}{L^{2j-2}}\hat{\xi}_j^2+\frac{2d_{21}}{L^2}\eta_2^2+\hat{\xi}_1\eta_2. \quad (21)\end{aligned}$$

For notation convenience, we define $\eta_1=\hat{\xi}_1$ from now on.

**Step** $k(2\le k\le n-1)$**.** Suppose that there exist a positive definite and proper Lyapunov function $V_k(e,\eta_1,\dots,\eta_k)$, and a series of virtual controllers $\alpha_1,\dots,\alpha_k$, with the coordinate transformations

$$\begin{aligned}&\alpha_{j-1}=-Lb_{j-1}\eta_{j-1},\\ &\eta_j=\hat{\xi}_j-\alpha_{j-1},\qquad j=2,\dots,k+1, \quad (22)\end{aligned}$$

where $b_j$ are positive constants being independent of the gain $L$, such that

$$\begin{aligned}\mathcal{L}V_k\ \le&-((n+1-k)L-(d_1+\sum_{j=1}^{n}d_{j1}))\|e\|^2\\ &-\sum_{j=1}^{k}\frac{1}{L^{2j-2}}((n+1-k)L-2d_{21}b_j^2)\eta_j^2\\ &+\sum_{j=k+2}^{n}\frac{d_{j1}}{L^{2j-2}}\hat{\xi}_j^2+\frac{2d_{21}}{L^{2k}}\eta_{k+1}^2+\frac{1}{L^{2k-2}}\eta_k\eta_{k+1}. \quad (23)\end{aligned}$$

Consider Lyapunov function $V_{k+1} = V_k + \frac{1}{2L^{2k}}\eta_{k+1}^2$, and observe that

$$\eta_k = \hat{\xi}_k + Lb_{k-1}\hat{\xi}_{k-1} + L^2 b_{k-1}b_{k-2}\hat{\xi}_{k-2} + \cdots + L^{k-1}b_{k-1}b_{k-2}\cdots b_1\hat{\xi}_1, \tag{24}$$

then, it is straightforward to show that

$$\begin{aligned} d\Big(\frac{\eta_{k+1}^2}{2L^{2k}}\Big) &= \frac{\eta_{k+1}}{L^{2k}}(\hat{\xi}_{k+2} + L^{k+1}\kappa_{k+1}e_1 \\ &\quad + Lb_k\sum_{j=1}^{k}\frac{\partial\eta_k}{\partial\hat{\xi}_j}(\hat{\xi}_{j+1} + L^j\kappa_i e_1))dt \\ &= \frac{\eta_{k+1}}{L^{2k}}(\hat{\xi}_{k+2} + L^{k+1}\kappa_{k+1}e_1 + \sum_{j=1}^{k}L^{k-j+1}b_k\cdots b_j \\ &\quad \cdot(\eta_{j+1} - (b_j\eta_j + L^j\kappa_i e_1))dt \\ &= \frac{\eta_{k+1}}{L^{2k}}(\hat{\xi}_{k+2} + L^{k+1}q_0e_1 + L^{k+1}q_1\eta_1 + L^kq_2\eta_2 \\ &\quad + \cdots + Lq_{k+1}\eta_{k+1})dt, \end{aligned} \tag{25}$$

where the coefficients

$$\begin{aligned} q_0 &= \sum_{j=1}^{k}(b_k\cdots b_j\kappa_j) + \kappa_{k+1}, \\ q_1 &= -b_k\cdots b_1^2, \\ q_j &= b_k\cdots b_{j-1} - b_k\cdots b_j^2, \quad j = 2,\cdots,k, \\ q_{k+1} &= b_k^2 > 0 \end{aligned} \tag{26}$$

are independent of gain constant $L$. Putting (23) and (25) together, and consider $\hat{\xi}_{k+2}^2 = (\eta_{k+2} + \alpha_{k+1})^2 \le 2(\eta_{k+2}^2 + \alpha_{k+1}^2)$, we have

$$\begin{aligned} \mathcal{L}V_{k+1} &= \mathcal{L}V_k + \frac{\eta_{k+1}}{L^{2k}}d\eta_{k+1} \\ &\le -((n+1-k)L - (d_1 + \sum_{j=1}^{n}d_{j1}))\|e\|^2 \\ &\quad - \sum_{j=1}^{k}\frac{(n+1-k)L - 2d_{21}b_j^2}{L^{2j-2}}\eta_j^2 \\ &\quad + \sum_{j=k+3}^{n}\frac{d_{j1}}{L^{2j-2}}\hat{\xi}_j^2 + \frac{2d_{k+2,1}}{L^{2k+2}}(\eta_{k+2}^2 + \alpha_{k+1}^2) \\ &\quad + \frac{\eta_{k+1}}{L^{2k}}(\eta_{k+2} + \alpha_{k+1}) \\ &\quad + \eta_{k+1}\Big(\frac{q_0}{L^{k-1}}e_1 + \frac{q_1}{L^{k-1}}\eta_1 + \frac{q_2}{L^{k-2}}\eta_2 + \cdots \\ &\quad + \frac{q_{k-1}}{L^{2k-3}}\eta_{k-1} + \frac{q_k+1}{L^{2k-2}}\eta_k + \frac{q_{k+1} + \frac{2d_{21}}{L}}{L^{2k-1}}\eta_{k+1}\Big) \\ &\le -((n-k)L - (d_1 + \sum_{j=1}^{n}d_{j1}))\|e\|^2 \\ &\quad - \sum_{j=1}^{k+1}\frac{(n-k)L - 2d_{21}b_j^2}{L^{2j-2}}\eta_j^2 + \sum_{j=k+3}^{n}\frac{d_{j1}}{L^{2j-2}}\hat{\xi}_j^2 \\ &\quad + \frac{2d_{k+2,1}}{L^{2k+2}}(\eta_{k+2}^2 + \alpha_{k+1}^2) + \frac{1}{L^{2k}}\eta_{k+1}\eta_{k+2} \\ &\quad + \frac{1}{L^{2k}}\eta_{k+1}\alpha_{k+1} + \eta_{k+1}^2\frac{1}{L^{2k-1}}\Big(\frac{q_0^2}{4} + \frac{q_1^2}{4} + \cdots \\ &\quad + \frac{q_{k-1}^2}{4} + \frac{(q_k+1)^2}{4} + q_{k+1} + 1\Big). \end{aligned} \tag{27}$$

Choose $\alpha_{k+1} = -Lb_{k+1}\eta_{k+1}$, with $b_{k+1} = n - k + \frac{q_0^2}{4} + \frac{q_1^2}{4} + \cdots + \frac{q_{k-1}^2}{4} + \frac{(q_k+1)^2}{4} + q_{k+1} + 1 > 0$ is independent of $L$, then

$$\begin{aligned} \mathcal{L}V_{k+1} &\le -((n-k)L - (d_1 + \sum_{j=1}^{n}d_{j1}))\|e\|^2 \\ &\quad - \sum_{j=1}^{k+1}\frac{(n-k)L - 2d_{21}b_j^2}{L^{2j-2}}\eta_j^2 \\ &\quad + \sum_{j=k+3}^{n}\frac{d_{j1}}{L^{2j-2}}\hat{\xi}_j^2 + \frac{2d_{k+2,1}}{L^{2k+2}}\eta_{k+2}^2 \\ &\quad + \frac{1}{L^{2k}}\eta_{k+1}\eta_{k+2}. \end{aligned} \tag{28}$$

**Step** $n$**.** Design the linear controller

$$\begin{aligned} u &= -Lb_n\eta_n \\ &= -Lb_n(\hat{\xi}_n + Lb_{n-1}(\hat{\xi}_{n-1} + \cdots + Lb_2(\hat{\xi}_2 + Lb_1\hat{\xi}_1)\cdots)), \end{aligned} \tag{29}$$

with $b_j > 0, \ \ j = 1,\ldots,n$ being real constants independent of $L$, there holds

$$\mathcal{L}V_n \le -(L - (d_1 + \sum_{j=1}^{n}d_{j1}))\|e\|^2 - \sum_{j=1}^{n}\frac{L - 2d_{21}b_j^2}{L^{2j-2}}\eta_j^2, \tag{30}$$

where $V_n(e,\eta_1,\ldots,\eta_n) = (n+1)e^TP_ee + \sum_{j=1}^{n}\frac{\eta_j^2}{2L^{2j-2}}$ is positive definite, decrescent and radially unbounded.

Choose

$$L > L^* = \max\{1, d_1 + \sum_{j=1}^{n}k_{j1}, 2d_{21}b_1^2, \ldots, 2d_{21}b_{n-1}^2\},$$

then

$$\mathcal{L}V_n < 0.$$

Thus, the closed-loop system is stochastically asymptotically stable. This completes the proof. ■

*Remark 1:* The main contributions of this paper are summerized as follows. (i) A parameter gain is introduced in the observer to dominate uncertain nonlinearities and diffusion terms. (ii) The traditional backstepping approach is combined with observer design in the construction of the output-feedback controller to overcome the cost of large control gain.

## IV. Simulation Example

Consider the global output-feedback stabilization of the following stochastic nonlinear system:

$$\begin{cases} d\xi_1 = \big(\xi_2 + \sin(t)\xi_1\big)dt + \sin(\xi_1)d\omega, \\ d\xi_2 = \big(u + \cos(t)\ln(1+\xi_1^2)\big)dt + 0.1\cos(\xi_2)d\omega, \\ y = \xi_1. \end{cases}$$

It's obviously to see that Assumption 1 can be ensured with $c = 1$. Based on the design process of the controller, we select $\kappa_1 = 5, \kappa_2 = 1$. Therefore, we construct the output-feedback controller and observer as $u = -Lb_2\hat{\xi}_2 - L^2 b_2 b_1 \hat{\xi}_1$ and $\dot{\hat{\xi}}_1 = \hat{\xi}_2 + 5L(y - \hat{\xi}_1), \dot{\hat{\xi}}_2 = u + L^2(y - \hat{\xi}_1)$. In addition, initial values are chosen as $[\xi_1(0), \xi_2(0), \hat{\xi}_1(0), \hat{\xi}_2(0)]^T = [-2, 3, 0, 0]^T$ and $L = 3$. Figs. 1-3 show that the considered system can be globally stabilized by the controller.

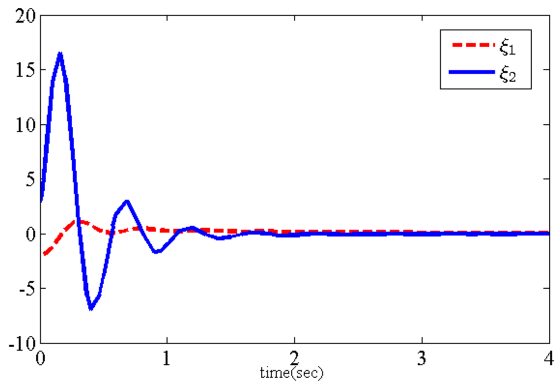


Fig. 1. The trajectories of $\xi_1, \xi_2$.

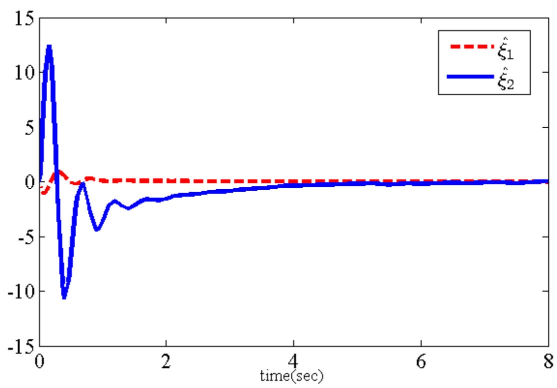


Fig. 2. The trajectories of $\hat{\xi}_1, \hat{\xi}_2$.

## V. Conclusion

This paper has addressed the global stabilization problem for a class of stochastic nonlinear systems by output-feedback. Take into account the fact that all the information in the system is unmeasurable except for the output, a parameter gain has been introduced in the construction of linear observer to dominate nonlinearities and diffusion terms, and the actual output-feedback controller has been constructed with the aid of the backstepping approach.

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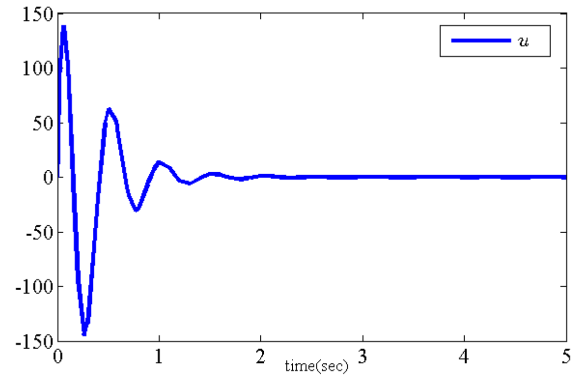


Fig. 3. The trajectory of $u$.


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