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Multi-robot Field Exploration in Hex-Decomposed Environments for Dubins Vehicles

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***Abstract*— Multi-robot systems can be useful in applications such as map building, surveillance, and search and rescue. To be efficient in practice, the robotic team should cooperatively explore a region of interest. However, when the environment is unknown, it is challenging to plan collision-free paths in real-time under both non-holonomic mobility constraints and energy constraints. We propose the Multi-robot Hex Decomposition Exploration (M-HDE) method for multiple Dubins vehicles to explore unknown environments. M-HDE ensures robots return to the initial (departure) position before energy runs out. Furthermore, when available energy level allow, M-HDE can achieve complete exploration of the environment. The proposed approach generates smooth, continuous paths for Dubins vehicles to follow at constant velocities, and offers geometric closed-form solutions for both team formation and paths. The performance is evaluated with a team of Turtlebots in simulated environments with obstacles in Gazebo.**

***Index Terms*— multi-robot system, exploration, path planning**

I. INTRODUCTION

The paper addresses field exploration for a team of robots modeled as Dubins vehicles [1]. Efficient field exploration in unknown environments is essential for tasks such as map building [2], mine clearing [3], and search and rescue [4]. Even if in some cases the map can be available, the environment should still be considered unknown or partially-known prior to departure due to the potential existence of unexpected restricted areas. For instance, the environment can be completely different from the expected one after a natural disaster, such as earthquakes or floods. To this end, being capable of planning paths online in unknown environments with unvisitable areas to gather new information is important.

A key aim in field exploration is to maximize the total area visited by robots [5]. Use of multiple robots may reduce the overall task completion time by appropriate task allocation, and improve the robustness of the overall system to potential failures [6]. In centralized decision-making process [7], a "leader" or a control center exists to plan globally optimal paths. However, it prevents the algorithm from working

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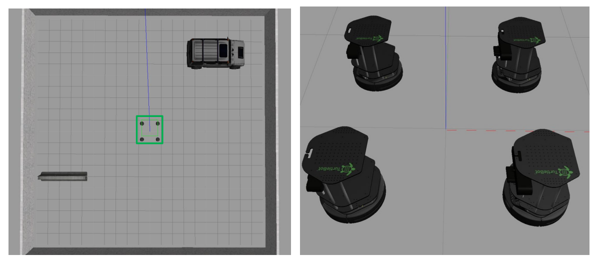


Fig. 1. (left) Simulation environment in Gazebo, and (right) Turtlebots used in experiments

for large robotics teams. On the other hand, decentralized approaches [8], [9] improve the robustness and can scale better with the number of robots in the team. Still, these methods may be challenged by redundant exploration [10] or in-team collision. Moreover, for strategies that require real-time information sharing [2], network connectivity may be hard to establish when massive information needs to be transmitted in cases of large robot teams.

To address the problems of redundancy and inter-robot collision, some methods [11], [12] conduct cellular decomposition and deploy each robot to explore different subregions. Region decomposition can be challenging without prior knowledge of boundaries. In this paper, we partition the environment into a series of hexagonal subregions. As hexagon enables regular tessellation [13], an unknown environment can be potentially filled by infinite duplicates of same hex shape with no gap or overlap until a boundary is hit. Comparing with other regular tilings, hexagons describe non-convex regions better [14]. Considering hexagonal subregions can provide the largest set of possible movement directions among adjacent subregions with same traversing distance, as a larger action space is a desired property in exploration.[1]

Robotic exploration can benefit by teams of unmanned vehicles, e.g., ground (wheeled/legged robots), aerial (fixed wing aircraft) and surface vehicles [15]–[18]. These vehicles are commonly under non-holonomic mobility constraints. One way to capture the constraints is by utilizing a Dubins vehicle model [1], which allows the vehicles to move in

[1]Compare 6 for hexagon with 3 for triangle grid and 4 for square grid.

paths consisting of straight lines and arcs at constant speed. Ensuring collision free paths—among robots and between the team and unexpected obstacles—becomes more challenging as Dubins vehicles cannot take sharp turns or move sideways.

In unknown environments, the expected time to finish exploring the whole space is unpredictable. To this end, it is essential to ensure that robots reach a designated base location for recharging or refueling before energy runs out. Several energy-aware methods for multi-robot teams [19]–[22] have been proposed, which either require prior knowledge of the environment, or assume convex environments without obstacles. In addition, these methods are not directly applicable to Dubins vehicles.

In this paper, we propose the Multi-robot Hex Decomposed Exploration (M-HDE) approach for Dubins vehicles to explore an unknown environment. The robots form a team and visit a series of hexagon subregions. The selection of movement direction is based on observed obstacles and boundaries. Smooth, continuous Dubins paths, which require no acceleration or deceleration, are planned in a decentralized manner. Geometric closed-form solutions for team formation and feasible paths (e.g., straight line starting and ending positions, arcs angles) are given. The proposed approach is evaluated in Gazebo simulation with a team of Turtlebot robots, as shown in Fig 1. All robots are able to return to their initial (i.e. departure) position before battery runs out. In addition, when available energy levels suffice, M-HDE can achieve complete exploration of the unknown environment.

Contributions: The contribution of this work is on the multi-robot hex-decomposition-based exploration algorithm that has three key properties.

1) It guarantees the robot team returns to the departure location within energy constraints. When available energy levels suffice, M-HDE achieves complete exploration.
2) It ensures collision-free paths between team members as well as between robots and obstacles.
3) It generates smooth, continuous paths for Dubins vehicles to follow at constant speed.

II. Related Work

To maximize the unknown area explored by robots [5], multi-robot frontier-based exploration strategies are proposed [23]–[27]. Robots proceed according to the boundary line between explored and unexplored space. However, the time and computational cost to determine the boundary line increase as the map expands [28]. Another direction is to utilize randomized search strategies. One way is random walk search [29], which may be impractical in real-world applications because it can require large amount of energy to explore a reasonably large region, and unnecessarily go over the same space multiple times [30].

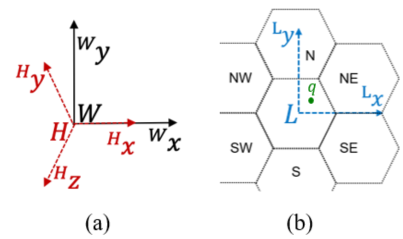


Fig. 2. (a) Hex frame H and world frame W, (b) local frame L with six adjacent cells of a subregion.

Sampling-based planners such as Rapidly-exploring Random Tree (RRT) [31] generate collision-free paths by adding randomly sampled points to a tree. RRT is fast and simple at the expense of optimality. In unknown, obstacle-dense environments, it can be challenging to sample a point that may be occluded by obstacles and still be able to connect to the tree. Variations of RRT, such as anytime RRTs [32], Multipartite RRTs [33], RRT* [34], SRT [35], and "next-best-view" planner [36] are proposed to improve efficiency of single robot exploration. A challenge for sampling-based approaches is the possibly high computational cost for large-scale planning, which may constrain real-time operation.

RRT-based approaches are also adapted into multi-robot exploration tasks [37]–[39]. These methods either require the environment to be known, or assume robots to be fully, losslessly connected to share the entire planned paths. Each robot departs from different positions in the environment and plans its own paths considering selected paths of other robots. However, the planned paths may lead robots to be far apart, even outside of communication range, where no information can be shared for next step path planning. In addition, sharp turns can exist in planned paths, which cannot be applied directly to Dubins vehicles.

III. Hexagon Region Decomposition And Data Representation

We use two coordinate systems, cube coordinates and Cartesian coordinates. A hex grid is formed by hexagon subregions. Location of subregions in hex grid can be referred using cube coordinates. On the other hand, Cartesian coordinates are necessary during the process of map building, target localization, on-board sensor data inference, and path planning. Therefore, we propose to use a two-layer environment map, in which a 2D hex grid plane is overlaid on top of a Cartesian plane.

A frame H is placed in hex grid plane as the frame of reference for cube coordinates. 2D cube coordinates correspond to three axes $({}^H x, {}^H y, {}^H z)$ in frame H, as shown in Fig. 2(a) with dashed red lines. The origin of frame H is set at the center of the subregion that robots depart. Axis ${}^H x$ is directed East, axis ${}^H y$ points 30 degrees from North and 60 degrees from West, and axis ${}^H z$ points 30 degree from South and 60 degree from West. In the hex grid plane, a hexagon

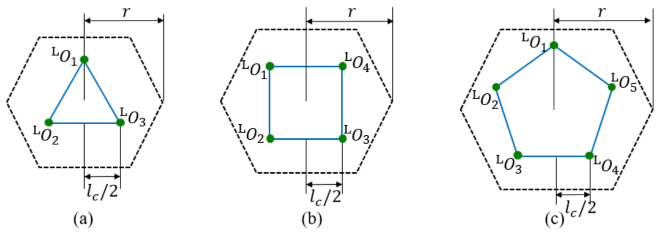


Fig. 3. Formation of multi-robot team with (a) $M = 3$, (b) $M = 4$, and (c) $M = 5$. Regular polygons and subregions are shown in blue solid and black dashed lines, respectively. Center points ${}^{L}O_m$ are shown as green dots.

subregion is referred to as S_k, with its cube coordinate being $({}^{H}x_k, {}^{H}y_k, {}^{H}z_k)$. Each S_k has six adjacent cells (Fig. 2(b)).

A world frame W (Fig. 2(a) in solid black lines) is placed in the Cartesian plane, with the axes ${}^{W}x$, ${}^{W}y$ and ${}^{W}z$ directed East, North, and opposite to the center of earth, respectively. For 2D planning tasks, we omit ${}^{W}z$ and assume a flat earth model. The origin of frame W matches frame H. ${}^{W}x$ coincides with the ${}^{H}x$ expressed in cube coordinates, and ${}^{W}y$ is normal to ${}^{W}x$, with its positive direction pointing toward the semiplane spanned by $+{}^{H}y$ and $-{}^{H}z$ (Fig. 2(a)).

In a subregion S_k, the local frame L_k is placed to represent points inside a subregion with Cartesian coordinates, as shown with dashed blue lines in Fig. 2(b). Frame L_k orients identically to the world frame with its origin at the center of each subregion, that is ${}^{W}_{L}R = I_{2\times 2}$. Let the hexagon side length to be r. For a point q inside subregion S_k, its position in local frame $({}^{L}x_q, {}^{L}y_q)$ can be mapped to world frame as

$$\begin{bmatrix} {}^{W}x_q \\ {}^{W}y_q \end{bmatrix} = \begin{bmatrix} \frac{3}{2}r & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2}r & \frac{-\sqrt{3}}{2}r \end{bmatrix} \begin{bmatrix} {}^{H}x_k \\ {}^{H}y_k \\ {}^{H}z_k \end{bmatrix} + {}^{W}_{L}R \begin{bmatrix} {}^{L_k}x_q \\ {}^{L_k}y_q \end{bmatrix} . \qquad (1)$$

IV. Team Formation and Multi-robot Deployment

Without full knowledge of the environment, the robot team aims to reach as many unexplored locations as possible, and return to departure position with limited energy. Feasible plans should also satisfy constraints such as avoiding restricted areas and in-team collisions. In this section, we first discuss team formation. Then, we continue with how to move between subregions. Next, we propose M-HDE approach to determine the sequence of subregions to visit.

A. Communication-Range-Based Team Formation

A team consisting of M robots is deployed to explore an unknown region. In dangerous and hazardous environments, obstacles or restricted area that is unsafe to visit can exist frequently. It is beneficial that all robots maintain communication with at least one of other robots in the team. Robot failure can be detected in time by other team members, and position of observed obstacles can be shared among robots.

We consider a Dubins vehicle model for each robot, i.e. ${}^{W}\dot{x} = v\cos\theta$, ${}^{W}\dot{y} = v\sin\theta$, $\dot{\theta} = v/r_t$, where $({}^{W}x, {}^{W}y)$ is the robot's Cartesian position, θ is the heading, v is a constant speed, and r_t is radius of arcs paths. Upon departure, each robot creates a hex grid map locally by setting the current position as the origin of frame H. Suppose that the hex grid plane consists of subregions $\bigcup_{k=1}^{\infty} S_k$. In subregion S_k, a robot $m \in [1, M]$ follows an arc path, which is a portion of the circumference of a circle centered at ${}^{L_k}O_m({}^{L_k}x_m, {}^{L_k}y_m)$, and of radius r_t.

We place the centers ${}^{L_k}O_m, m \in [1, M]$ on vertices of a regular M-polygon, as shown in Fig. 3. For each robot m, the position of ${}^{L_k}O_m$ is the same in different subregions. Therefore, we can drop the subscript k for local frame L_k, and the coordinate becomes ${}^{L}O_m({}^{L}x_m, {}^{L}y_m)$. Let the high-confidence communication distance for a robot be l_c, which is set to be the side length of regular M-polygon. The position ${}^{L}O_m$ in each local frame L is

$$({}^{L}x_m, {}^{L}y_m) = \left(\frac{l_c/2}{\sin(\frac{\pi}{n})} \cos((2k-1)\frac{\pi}{n}) \ , \ \frac{l_c/2}{\sin(\frac{\pi}{n})} \sin((2k-1)\frac{\pi}{n}) \right).$$

Note that $r_{min} < r_t < l_c/2$, where r_{min} is the minimum turning radius of a Dubins Vehicle. The side length of the hexagon subregion is

$$r = \frac{l_c/2}{\sin(\pi/n)} + l_c/2 \ .$$

If each robot departs from the same relative position on the circle with respect to ${}^{L}O_m$, e.g., $({}^{L}x_m - r_t, {}^{L}y_m)$, and moves at the same constant speed v toward the same direction, the distance between any two adjacent robots remains l_c. Inter-robot collisions can be avoided while robots are following arc paths. In addition, each robot is always within the communication range with two adjacent robots. We assume that the combination of sensor footprints of all robots in the team can fully cover a subregion. The exploration efficiency increases with the growth of team size n in the sense that, with a larger hexagon radius r, more area can be covered by visiting a subregion.

B. Movement Between Subregions

Suppose the team is currently in subregion S_a, and intends to move to an adjacent subregion S_b. We discuss how to select S_b in Section IV-C. For now, let us assume that S_b is given.

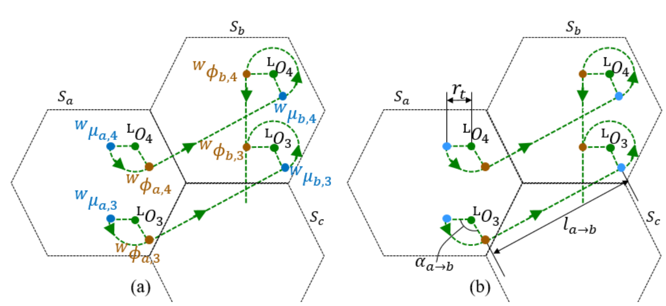


Fig. 4. Paths (green dashed) of robots $m = 3, 4$ in a team of 4 robots when moving $S_a \to S_b \to S_c$. Paths of robots $m = 1, 2$ are omitted for clarity. (a) and (b) show same paths, and are separated for better view of variables.

The team formation in S_b is a translation of its formation in S_a. Constrained by vehicle model, the path from S_a to S_b for each robot contains an arc of angle $\alpha_{a\to b}$, radius r_t, and followed by a straight line segment of length $l_{a\to b}$, as shown in Fig. 4(b) with green dashed curves. For robot m, the starting point ${}^W\mu_{a,m}$ and ending point ${}^W\phi_{a,m}$ of the arc path are shown in Fig. 4(a) as blue and orange dots, respectively. The straight line is chosen to be an outer tangent line of the two circles of radius r_t centered at ${}^LO_m$ in S_a and S_b. Following a robot's current moving direction, only one tangent line can be selected for a given S_b. The position ${}^LO_m$ in local frame can be converted to world frame frame for S_a and S_b using Eq. (1), and represented as ${}^WO_{a,m}$ and ${}^WO_{b,m}$, respectively. Let the tangent point on circle centered at ${}^WO_{a,m}$ be ${}^W\phi_{a,m}$, then the corresponding tangent point on circle centered at ${}^WO_{b,m}$ will become ${}^W\mu_{b,m}$. The geometric closed-form solutions for outer tangent points are

$$\begin{aligned}
{}^W\phi_{a,m}(x) &= {}^WO_{a,m}(x) \pm r_t \frac{({}^WO_{a,m}(y) - {}^WO_{b,m}(y))}{\sqrt{w}} \ , \\
{}^W\phi_{a,m}(y) &= {}^WO_{a,m}(y) \pm r_t \frac{({}^WO_{b,m}(x) - {}^WO_{a,m}(x))}{\sqrt{w}} \ , \\
{}^W\mu_{b,m}(x) &= {}^WO_{b,m}(x) \pm r_t \frac{({}^WO_{a,m}(y) - {}^WO_{b,m}(y))}{\sqrt{w}} \ , \\
{}^W\mu_{b,m}(y) &= {}^WO_{b,m}(y) \pm r_t \frac{({}^WO_{b,m}(x) - {}^WO_{a,m}(x))}{\sqrt{w}} \ ,
\end{aligned} \tag{2}$$

where $w = ({}^WO_{b,m}(x) - {}^WO_{a,m}(x))^2 + ({}^WO_{b,m}(y) - {}^WO_{a,m}(y))^2$. Quantities $\alpha_{a\to b}$ and $l_{a\to b}$ can be found as

$$\begin{aligned}
&\alpha_{a\to b} = \cos^{-1}((2r_t^2 - \|{}^W\mu_{a,m} - {}^W\phi_{a,m}\|^2)/2r_t^2) \ , \\
&l_{a\to b} = \|{}^W\mu_{a,m} - {}^W\phi_{a,m}\| \ .
\end{aligned} \tag{3}$$

Note that $\alpha_{a\to b}$ and $l_{a\to b}$ are the same for all robots in the team since the movement is performed as translation. As the paths for robots do not intersect when moving from one subregion to another, inter-robot collision avoidance can be achieved. Each robot can plan its own paths in a decentralized manner given S_b. One advantage of the planned paths are their smoothness and continuity, which allow robots to move at constant velocity without acceleration and deceleration. Fixed-speed movements are more energy efficient, and can also enable more reliable sensor measurements.

C. Energy-aware Path Planning for Exploration

Our Multi-robot Hex Decomposition Exploration (M-HDE) approach for an unknown environment with potential obstacles is given in Algorithm 1. M-HDE plans a sequence of subregions for the team to visit in the hex grid plane. The goal is to visit as many hexagon subregions as possible and be able to return to the departure position within energy limits. We show that without energy constraints, M-HDE guarantees that all obstacle-free subregions are visited.

A robot's operation time can be constrained in terms of fuel level or battery level. The planned paths in Section IV-B require no acceleration and deceleration, and the energy consumption for same traveling distance remains the same. Therefore, in our approach, the energy constraint can be interpreted as the total distance that a robot can travel.[2]

Algorithm 1 M-HDE

initialize $S_0 \leftarrow (0,0,0)$, $S_a \leftarrow S_0$, $\mathcal{U} \leftarrow \{\}$, $\mathcal{V} \leftarrow \{S_0\}$. For robot m, select as ${}^W\mu_{a,m}$. Set ${}^LO_m, r_t, d_t$ globally.
repeat
  $h \leftarrow 1$, $\Phi_1 \leftarrow \{\}$, $\Phi_2 \leftarrow \{\}$
  repeat
    for all $S^* \in$ RangeH(S_a, h) **do**
      if RangeH$(S^*, 1) \cap \mathcal{V} \neq \emptyset$ **then**
        $(d_1, P_1) \leftarrow$ MovD$(S^*, S_0, \mathcal{V}, {}^W\mu_{*,m})$
        $(d_2, P_2) \leftarrow$ MovD$(S_a, S^*, \mathcal{V}, {}^W\mu_{a,m})$
        if $S^* \notin \mathcal{V} \cup \mathcal{U}$ & $d_t - d_2 \geq d_1$ **then**
          $\Phi_1 \leftarrow \Phi_1 \cup (S^*, d_2, P_2)$
        else if $S^* \notin \mathcal{U}$ & $d_t - d_2 \geq d_1$ **then**
          $\Phi_2 \leftarrow \Phi_2 \cup (S^*, d_2, P_2)$
        end if
      end if
    end for
    if $\Phi_1 \neq \emptyset$ **then**
      $(S_b, d_2, P_2) \leftarrow \underset{(S,d,P)\in\Phi_1}{\arg\max}$ (RangeH$(S, 1) \setminus \{\mathcal{V} \cup \mathcal{U}\}$), $d_t \leftarrow d_t - d_2$
    else if $\Phi_2 \neq \emptyset$ **then** $h \leftarrow h+1, \Phi_2 \leftarrow \{\}$
    else $S_b \leftarrow S_0$, $(d_2, P_2) \leftarrow$ MovD$(S_a, S_0, \mathcal{V}, {}^W\mu_{a,m})$
    end if
  until S_b is determined
  Follow P_2 until robot m reach ${}^W\mu_{b,m}$. Update $\mathcal{U}$.
  $\mathcal{V} \leftarrow \mathcal{V} \cup S_b, S_a \leftarrow S_b, {}^W\mu_{a,m} \leftarrow {}^W\mu_{b,m}$.
until $S_b = S_0$

Algorithm 2 RangeH $(S'(x', y', z'), h)$

initialize $\Omega_h \leftarrow \{\}$
for each integer $a \in [-h, h]$ **do**
  for each integer $b \in [\max(-h, -a-h), \min(h, -a+h)]$ **do**
    $S'' \leftarrow (a + x', b + y', -a - b + z')$
    if $f(S', S'') = h$ **then** $\Omega_h \leftarrow \Omega_h \cup S''$
    end if
  end for
end for
return Ω_h

Two types of distance are considered in the planning algorithm in different coordinate systems. In 2D hex grid plane, the distance between two subregions S_a and S_b can

[2]Integrating the influence of natural factors such as wind, waves, etc., falls outside of the scope of this present work; future work will identify how to best integrate those effects within the M-HDE framework proposed herein.

Algorithm 3 MovD $(S_c, S_d, \mathcal{V}, {}^W\mu_{a,m})$

initialize $P' \leftarrow \{\}, P''\{\}, d' \leftarrow 0$
repeat
  $S_{c''} \leftarrow \operatorname*{arg\,min}_{S_{c'} \in \{\text{RangeH}(S_c,1) \cap \mathcal{V}, S_d\} \setminus P''} f(S_{c'}, S_d)$
  Calculate ${}^W O_{c'',m}, {}^W O_{c,m}$ using (1), ${}^W\phi_{c,m}, {}^W\mu_{c'',m}$ using (2), $\alpha_{c \to c''}, l_{c \to c''}$ using (3).
  $d' \leftarrow d' + \alpha_{c \to c''} r_t + l_{c \to c''}$, $P'' \leftarrow P'' \cup S_{c''}$
  $S_c \leftarrow S_{c''}, P' \leftarrow P' \cup (S_{c''}, {}^W\phi_{c,m}, {}^W\mu_{c'',m})$
until $S_d = S_c$
return (d', P')

be calculated using function

$$f(S_a, S_b) = (|x_a - x_b| + |y_a - y_b| + |z_a - z_b|)/2 \ .$$

Function *RangeH* in Algorithm 2 returns a set Ω_h which contains subregions that are h away from S_a in hex grid, i.e. $\forall S_{a'} \in \Omega_{a,h}, f(S_a, S_{a'}) = h$. The distance in Cartesian plane represents the actual robot path length when following arcs and straight lines. Suppose that upon departure, the total distance that each robot can travel at constant velocity v is d_t. To move from S_a to S_b, starting from ${}^W\mu_{a,m}$—the starting point of the arc for robot m in subregion S_a, the robot follows the arc path to ${}^W\phi_{a,m}$, then continues with the straight line path to reach ${}^W\mu_{b,m}$ in subregion S_b. Using (3), the total movement length is $\alpha_{a \to b} r_t + l_{a \to b}$.

While moving, robots observe their surroundings and determine if an adjacent subregion is visitable, i.e. contains no obstacle. In this work, we assume that the robots have no prior knowledge about the position of obstacles and boundaries. However, once a robot is in close proximity to an obstacle or a boundary, it is capable of recognizing that area as unvisitable with its on-board sensors. Since planning is decentralized, the only information that needs to be shared by the team members is a list of observed unvisitable subregions, denoted as $\mathcal{U} = \{S_k | S_k \text{ is unvisitable}\}$. If all robots run Algorithm 1 with a common $\mathcal{U}$, they will select the same subregion S_b to visit in the next step, therefore, no further communication is required.

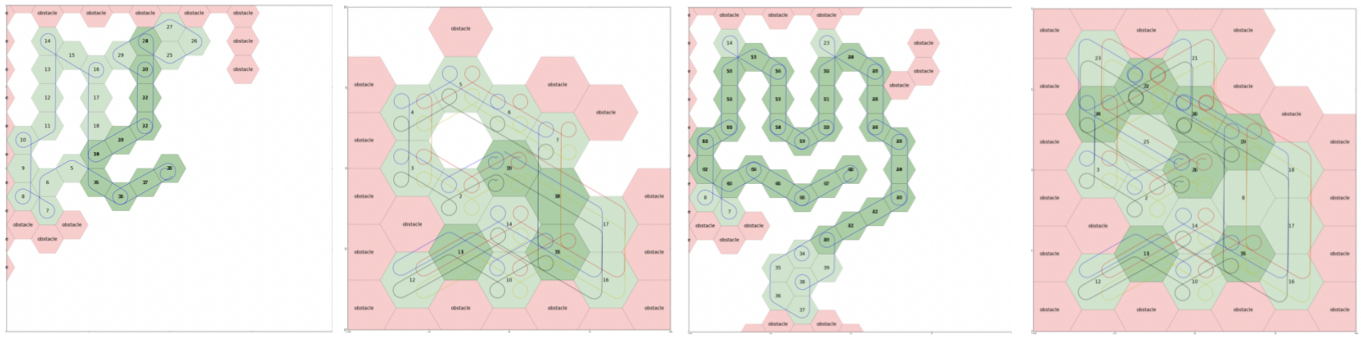
Departing from a random position in the environment, robots form a team as discussed in Section IV-A. The departure position is set to be $S_0(0,0,0)$ in hex grid plane. At each step, consider the team is currently at S_a. Lines 3-21 in Algorithm 1 describes the approach to determine the next subregion S_b to explore. Let $\mathcal{V}$ be the set containing subregions that have been visited. A subregion S^* is considered as a candidate of S_b if it satisfies three criteria. 1) S^* has not yet been visited or marked as an obstacle. 2) At least one subregion adjacent to S^* has been visited, i.e. belongs to set $\mathcal{V}$. 3) The sum of the movement distance in Cartesian plane from S_a to S^* (d_2) and from S^* to S_0 (d_1), is less than a robot's remaining possible travel distance, d_t.

The second criterion is necessary since the estimate of d_1 and d_2 requires the existence of at least one valid path from S_a to S^*. If a candidate S^* is an unvisited subregion such that all of its adjacent subregions are not yet visited, any potential path from S_a to S^* could contain unvisitable subregions, as we cannot guarantee the existence of visitable adjacent subregions of S^*. Moreover, it is possible that S^* itself is unvisitable. The third criterion ensures that after visiting a selected S^*, robots are able to return to departure position within battery limits. All candidates S^* are stored in the set Φ_1. Note that all elements in Φ_1 have the same distance h to S_a. Among all candidates, the one that has the most unexplored adjacent subregions will be selected as S_b for two reasons. First, the selected subregions provides the highest possibility to collect the largest amount of new information. Second, the chance that the team needs to alter its direction immediately and visit another far-away unexplored subregion is the least. If multiple candidates in Φ_1 have the same number of unexplored adjacent subregions, S_b will be selected based on a pre-set priority sequence, which indicates the order to attempt visiting each neighboring subregion. A priority sequence consists of six digits, representing six adjacent subregions of a hex cell. An example of priority sequence is {North, Northeast, Southeast, South, Southwest, Northwest}. Note that all robots hold the same priority sequence at the time of departure.

The required movement distance in Cartesian plane from S_a to S_b and from S_b to S_0 can be calculated following function *MovD* in Algorithm 3. To determine a valid path between two subregions, for instance, from S_c to S_d, we examine all adjacent subregions of S_c and select the one, $S_{c''}$, that is 1) visitable and 2) the closest to S_d in terms of distance h in hex grid plane. Then calculate the required movement distance in Cartesian plane d' using (1),(2),(3), and add the chosen $S_{c''}$ into a path P'. Next, set $S_{c''}$ to be S_c and repeat the process until S_d is reached. Once S_b is selected, each robot follows its planned path P_2 to visit S_b. Meanwhile, robots share information about unvisitable subregions $\mathcal{U}$. The distance d_2 will be deducted from the remaining distance d_t, and S_b will be set as new S_a. The process terminates when the departure subregion S_0 is selected as S_b.

Remark 1 If d_t is infinite and the environment is unknown but bounded, M-HDE guarantees *complete exploration* in hex grid plane. M-HDE can return a path $\mathcal{P}_c$ that starts and ends at S_0, so that all subregions in the environment are explored.

Suppose that d_t is unlimited. Starting from S_0, in each subregion S_a, function *RangeH* searches for a subregion S_b that is h away from S_a and has not been explored. For the searching process to terminate, S_b has to be selected; otherwise, h will keep increasing. Since the environment is bounded, there must be a moment such that all returned candidates from function *RangeH* are unvisitable, in which case Φ_1 and Φ_2 are both empty sets. Then, 1) all subregions



(a) (b) (c) (d)

Fig. 5. Exploration paths for (a) Single robot given travel distance d_{t1}, (b) Multi-robot team given travel distance d_{t1}, (c) single robot given travel distance d_{t2}, and (d) Multi-robot team given travel distance d_{t2}.

in the environment are in $\mathcal{V}$, 2) for current h, function *RangeH* returns a list of subregions which form a ring and completely surround the environment. Then, the robot team will return to S_0 and the exploration process terminates.

V. Simulation and Results

The proposed approach is tested in Gazebo simulation. A team of robots is deployed to explore a region without knowledge of the environment. The performance is compared with single-robot baseline using the proposed approach.

A. Simulation Setup

Figure 1 (left) shows a 20 m $\times$ 20 m 2D Gazebo simulation environment with two randomly placed obstacles. The green square indicates a randomly selected departure position. A team of four Turtlebot robots is deployed to explore the area without prior knowledge of the environment map. Information about obstacles and boundaries is acquired via on-board depth cameras. Subregions that contain detected obstacles are marked as red, and visited subregions are marked as green. The communication range l_c is set to be 1.6 m, and the radius of arc path r_t is 0.5 m. Therefore, in four-robot scenario, the hexagon radius r is approximately 2 m, and in single-robot scenario, r is 1 m. Each Turtlebot moves at constant velocity of 1 m/s.

In the first set of experiments, the initial battery level allows each robot to travel a total distance of $d_{t1} = 100$ m at constant velocity 1 m/s. In the second set of experiments, the total possible travel distance is $d_{t2} = 200$ m for each robot.

B. Results and Discussion

Figures 5(a) and (b) show the exploration paths under constraints d_{t1} for a team of four robots and single robot, respectively. In both scenarios, robots are able to follow generated paths to explore the environment, and return to the departure position before energy runs out. The generated paths are smooth and continuous, hence avoiding sudden acceleration and deceleration. At the same time, no inter-robot or robot-obstacle collision happens, as the planned paths for each robot do not intersect at any time step. Figure 5(a) suggests more area is covered compared with Fig. 5(b), and reveals the advantage of using multiple robots.

When a longer travel distance d_{t2} is allowed, the exploration paths are given in Fig. 5(c) and (d) for a team of four robots and single robot, respectively. In Fig. 5(c) the robot team visits all subregions in the environment, then returns to the departure position. However, in Fig. 5(d), with the same departure energy, a single robot cannot finish exploring the entire environment. More importantly, Fig. 5(c) suggests that M-HDE achieves complete exploration when energy allows.

If an obstacle is detected by robots, the subregion that contains the obstacle will be marked as unvisitable. When the number of robots in the team or the communication range l_c increase, the subregion size increases. The existence of a small portion of an obstacle can cause a big area not to be explored. In Fig. 5(d), the white space in the northeast corner happens due to the fact that, the obstacle is considered to occupy all subregions in that direction and causes the region to be unvisitable. This is one weakness of the proposed approach, and can be improved by setting side length of the regular M-polygon for team formation to be less than l_c, that is, allowing the circle of communication range to intersect.

VI. Conclusions

The paper contributes to energy-aware multi-robot field exploration. Research on this vein for unknown and non-convex regions with potential restricted areas is limited. Specifically, our proposed approach, called M-HDE, is best targeted when in need to explore an environment of unknown size and shape, with unpredictable restricted areas. Our approach decomposes a region into a set of hexagonal subregions, and then plans smooth, continuous paths to visit a sequence of subregions.

We provide geometric closed-form solutions for path planning in terms of starting and ending position in each subregion, which enables efficient, online path planning. In addition, we provide closed-form solutions for team formation for different number of robots.

M-HDE can guarantee that non-holonomic robots can return to their departure position under energy constraints. When energy allows, M-HDE ensures complete exploration. Our approach scales well as the environment size grows, and as the number of robots in the team increases. The proposed approach is resolution complete in the size of the hex cell.

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