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# Any Curve Path Following of Snake-like Robots

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***Abstract*— Because of redundant freedoms, designing a robust any path following controller for snake-like robots is very challenging. In this paper, a novel path following controller, combining the robust following controller used in the unicycle robot and the fiber bundle model of snake-like robots, is proposed to follow any planar curve. More specifically, firstly, the kinematic model is established based on the fiber bundle theory which connects snake-like robots and unicycle robots. Then the robust path following controller for the unicycle robots is modified to be applied to the sake-like robots. Finally, to obtain a feasible controller, an estimation method for the orientation angle of the robot in the fiber bundle is proposed. To validate the proposed method, numerical simulations and experiments are performed. The results demonstrate that the proposed following controller is robust and valid.**

***Index Terms*— Snake-like robots, Path following, Geometric mechanics, Any planar curve.**

## I. Introduction

Since 1970s, the research about snake-like robots has attracted wide and lasting attentions of scholars from all over the world, because the snake-like robots can move in various environments such as land, ocean, forest, desert, etc. The main topics are focused on methods how to generate various gaits to adapt to different environments. Among these methods, three methods are commonly used: curve-based method [1], CPG-based method [2], and model-based method [3] [4]. These works are very useful and have brought about many achievements, such as swimming, climbing pipelines, searching and rescuing in disastrous sites. However, the control used in most of these work is either semi-automatic or completely manual, meaning the robots lack in autonomy. Generally speaking, path planning and path following methods are needed for a fully autonomous robot. Firstly, a desired path connecting the start point and the end point is derived by path planning methods such as graph search based methods [5] [6], sampling-based methods [7] [8], interpolating curve based methods[9] or numerical-optimization-based methods [10]. Then, a closed-loop path following controller is designed to enable the robot to track

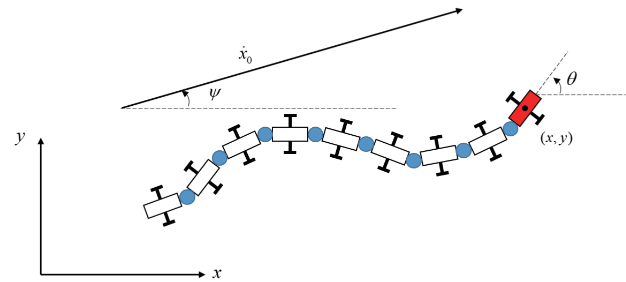


Fig. 1. Snake-like robots with passive wheels. The coordinates of the head is $(x, y, \theta)$. $\dot{x}_0$ denotes the velocity of the snake robots, and $\psi$ denotes the turning angle of the whole snake-like robot.

the desired path. Additionally, the path following method is also needed when a specified curve is given to finish some special task. In this paper, the topic about path following for snake-like robots is discussed.

In fact, path following methods have been deeply researched for the unicycle vehicle. The most popular one is the projection method such as LOS (Line-of-Sight) Guidance method[11] and Integral Line-of-Sight Guidance method [12], in which the projection point on the desired path is firstly computed together with the distance and angular errors being recalculated, then various nonlinear controller is designed based on Lyapunov function or sliding mode techniques [13]. For a simple curve following, they work very well. However, there is one critical drawback to these methods: the projection point must be unique which requires the initial point can not be too far from the desired curve and the desired curve can not be too complex. In order to solve the non-uniqueness of the projection method, a virtual target method is proposed, in which an explicit progression rate of the moving target method is tracked [14]. Unfortunately, the asymptotic convergence to the path can not be guaranteed. Morro proposes a new feedback control model which can guarantee asymptotic convergence to any 2-D curve neither requiring the projection nor considering a moving virtual target[15]. However, the controller is only applicable to unicycle robots.

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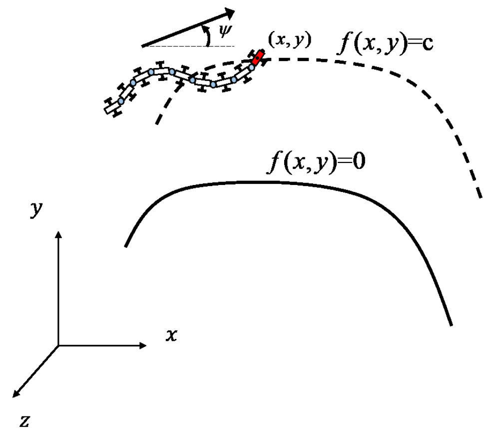


Fig. 2. The desired following curve is given by a generic planar curve equation $f(x, y) = 0$. The coordinate of the head is $(x, y)$ and the direction angle of the whole body is $\psi$.

There also exist some works about path following of snake-like robots which consist of methods with or without non-holonomic constraints. Liljeback proposes a cascaded approach based on LOS guidance law with simplified dynamic model which can guarantee the snake robot to converge to any desired straight path [16]. Wang proposes an adaptive path following method and a spline based curve path following method using a similar time-varying line-of-sight guidance law [17] [18]. As mentioned above, the LOS-based method can not deal with the non-uniqueness. Therefore the method can only be used to follow a simple curve. To solve this problem, Zhang proposes a framework of general curved path following control for planar eel robots [19]. However the modified feedback control law is based on the approximate kinematic model which is not accurate. For snake-like robots considering the nonholonomic constraints, Hitaka uses a reduced order model for the head position and makes the robot track a desired gliding curve[20]. Tanaka presents an approximate path-tracking control method for a snake-like robot with passive wheels [21]. However, these methods can be only used to track serpenoid curve which unfortunately makes it difficult for them to be applied for practical tasks.

In this paper, we aim to enable snake-like robots to follow curves with arbitrary curvatures. To this end, we propose a novel path following method combining the robust path following controller used in the unicycle robot and the fiber bundle model proposed in our previous work. Compared to the existing methods, the proposed method can follow any 2-D curve which only need its implicit equation. There are three main contributions in this paper:(1) A novel robust path following controller for snake-like robots is proposed which can track generic planar curve. (2) A novel estimation method of the head angle is proposed. (3) Real experiments are performed which validate the proposed method. In the rest of this paper, the kinematic model based on geometric mechanics is introduced in section II. The feedback controller is designed for path following in section III. Numerical simulations and experiments are given in section IV and conclusions are given in section V.

## II. Kinematic Model Based on Geometric Mechanics

The snake-like robot in this paper is one with passive wheels and the kinematic model based on the geometric mechanics is derived in our previous work [22] [23]. As shown in Fig.1, $(x, y, \theta)$ denote the head pose of the snake-like robot. Let the absolute orientation angle of module $i$ to module $i+1$ be $\phi_i$, then the general coordinate is $q = (x, y, \theta, \phi_1, \cdots, \phi_{n-1})$ for an n-module snake-like robot. The general velocity can be represented as:

$$\dot{q} = v^{\alpha} e_{\alpha} \tag{1}$$

where $v^{\alpha}$ denotes the quasi-velocity, $e_{\alpha}$ denotes the base, $\alpha = 1, 2$ . The set of bases is used in this paper as follows:

$$\begin{aligned}
e_1 = {} & \frac{\partial}{\partial x} - \frac{2}{l}\sin(\theta)\frac{\partial}{\partial \theta} + \frac{2}{l}\sum_{i=1}^{n-1}(-1)^{i-1} \\
& \times \sin(\theta + (-1)^k \sum_{k=0}^{i-1}\phi_k)\frac{\partial}{\partial \phi_i} + \frac{2}{l}\sum_{i=1}^{n-1}(-1)^{i-1} \\
& \times \sin(\theta + (-1)^k \sum_{k=0}^{i}\phi_k)\frac{\partial}{\partial \phi_i} \\
e_2 = {} & \frac{\partial}{\partial y} + \frac{2}{l}\cos(\theta)\frac{\partial}{\partial \theta} + \frac{2}{l}\sum_{i=1}^{n-1}(-1)^{i} \\
& \times \cos(\theta + (-1)^k \\
& \sum_{k=0}^{i-1}\phi_k)\frac{\partial}{\partial \phi_i} + \frac{2}{l}\sum_{i=1}^{n-1}(-1)^{i} \\
& \times \cos(\theta + (-1)^k \sum_{k=0}^{i}\phi_k)\frac{\partial}{\partial \phi_i}
\end{aligned} \tag{2}$$

where $\phi_0 = 0$ and $l$ representing the length of a single module of the robot. Taking this set of bases, the quasi-velocity equals the velocity of the head end.

## III. Feedback Controller Design for Path Following

### A. Formulation of Path Following in Fiber Bundle

Based on the kinematic model (1) and (2), serpentine gaits can be generated by planning the motion of the head. For a snake, it winds its body periodically to move forward, implying that the quasi-velocity $v^2$ should be periodic. Inspired by this idea, we propose a novel gait generating method:

$$\begin{bmatrix} v^1 \\ v^2 \end{bmatrix} = \begin{bmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{bmatrix} \begin{bmatrix} \dot{x}_0 \\ A\cos(\omega t) \end{bmatrix} \tag{3}$$

where $\dot{x_0}$ is the forward velocity, and $A$ and $w$ are the amplitude and the angular velocity of oscillator. $\psi$ is the turning angle. More details can be derived in our previous work. The motion planning problem for the snake-like robot can be solved by planning the motion in the fiber space $(x, y, \psi)$, where the kinematic model for can be described as follows:

$$\left\{\begin{array}{c}\dot{x} = \dot{x}_0 \cos(\psi) \\ \dot{y} = \dot{x}_0 \sin(\psi) \\ \dot{\psi} = u\end{array}\right. \quad (4)$$

where $u$ is the control input. The motion model of the snake-like robot in the fiber space $(4)$ is the same as the unicycle robot's.

As shown in Fig.2, the desired curve to be followed is a generic one which is given as $f(x, y) = 0$. $c = f(x_{head}, y_{head})$ denotes a general signed distance from the curve.

### B. Controller for the Path Following

A controller which can guarantee asymptotic convergence can be derived :

$$\begin{array}{l}\dot{x}_0 = \dot{x}_0(t) \\ u = K_1(-\|\nabla f\| \dot{x}_0 S(f) - f_x |\dot{x}_0| \cos\psi - f_y |\dot{x}_0| \sin\psi) + \dot{\psi}_c\end{array} \quad (5)$$

where $K_1 > 0$ and $S(f)$ is the sigmoid function.

$$S(f) = \frac{K_2 f}{\sqrt{1+f^2}} \quad (6)$$

where $0 < K_2 \le 1$. $\dot{\psi}_c$ is given as:

$$\dot{\psi}_c = \frac{(f_x f_{xy} - f_y f_{xx})\dot{x}_0 \cos\psi + (f_x f_{yy} - f_y f_{xy})\dot{x}_0 \sin\psi}{\|\nabla f\|^2 + \sigma} \quad (7)$$

where $\sigma$ is a infinitesimal to avoid denominator being zero, $f_x$, $f_y$, $f_{xx}$,$f_{xy}$ and $f_{yy}$ are the first- and second-order partial derivatives of $f$. To ensure the convergence, there are four assumptions:

a) $f$ is differentiable

b) $\lim_{t\to\infty}\dot{x}_0 \neq 0$

c) $f$ is twice differentiable, and $f_x$, $f_y$, $f_{xx}$, $f_{xy}$, and $f_{yy}$ are bounded in any bounded domain $D \subset \mathcal{R}^2$, where $f$ is bounded.

d) $\dot{x_0}$ and $\ddot{x}_0$ are bounded.

The details of the stability analysis can be derived in reference [15]. In this paper, we set $\dot{x_0}$ as a positive constant and the feedback $\psi$ is estimated in next subsection.

### C. Estimation of Direction Angle in Fiber Bundle

The difference between unicycle robots and snake-like robots is the turning angle $\psi$. For a unicycle robot, the turning angle denotes the robot's orientation which can be measured by the sensor on board. However, for the snake robot, the turning angle $\psi$ means the angle in the fiber space which is a abstract angle and can not be measured by the sensor. In previous work, we have prove that the orientation angle of

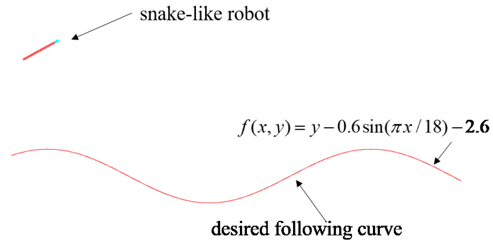


Fig. 3. The desired following curve is described as $f(x, y) = y - 0.6sin(\pi x/18) - 2.6$.

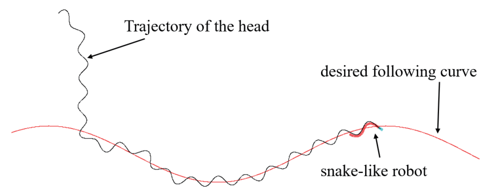


Fig. 4. The trajectory of the head when the snake-like robot follows the sinusoidal curve.

the head converges to the $\psi$ by periodic with (1), (2), (3). In fact we can estimate the angle in the fiber space by the orientation angle of each modular and the control command.

Let $\theta_i$ denote the orientation angles of each modular, then $\theta_i$ can be divided into two parts:

$$\theta_i = \theta_{ip} + \int_{-T}^{-T+i\cdot dt} \dot{\psi} dt \quad (8)$$

where $\theta_{ip}$ is periodic because of the periodic gait generating method (3). Let $\theta_c$ denote the orientation angle of the center of mass for a snake-like robot and assume that there are $N$ modulars, then $\theta_c$ is given as follows:

$$\theta_c = \frac{1}{N}\sum_{i=1}^{N}\theta_i = \frac{1}{N}\sum_{i=1}^{N}\left(\theta_{ic} + \int_{-T}^{-T+i\cdot dt}\dot{\psi} dt\right) \quad (9)$$

Let $\phi_{-T}$ denote the angle in fiber bundle a period ago, then:

$$\psi_{-T} = \frac{1}{N}\sum_{i=0}^{N}\theta_{ip} \quad (10)$$

Submitting (10) into (9), we have:

$$\begin{array}{l}\theta_c = \psi_{-T} + \frac{1}{N}\sum_{i=1}^{N}\int_{-T}^{-T+i\cdot dt}\dot{\psi} dt \\ \approx \psi_{-T} + \frac{N+1}{2}\dot{\psi} dt \\ \approx \psi_{-T} + \int_{-T}^{-\frac{1}{2}T}\dot{\psi} dt\end{array} \quad (11)$$

Additionally, the current angle in the fiber bundle can be computed as follows:

$$\begin{array}{l}\psi = \psi_{-T} + \int_{-T}^{0}\dot{\psi} dt \\ = \psi_{-T} + \int_{-T}^{-\frac{1}{2}T}\dot{\psi} dt + \int_{-\frac{1}{2}T}^{0}\dot{\psi} dt\end{array} \quad (12)$$

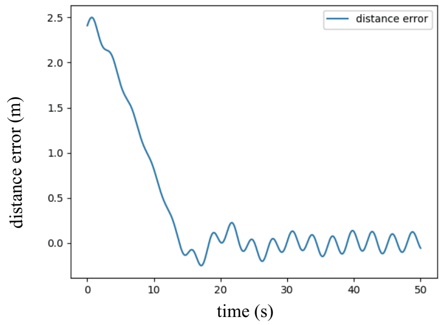


Fig. 5. The distance error when the robot follows the sinusoidal curve.

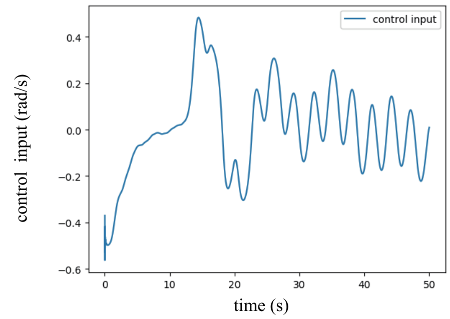


Fig. 6. The control input $\dot{\psi}$ when the robot follows the sinusoidal curve.

Submitting (11) into (12), the angle in the fiber bundle can be estimated:

$$\begin{aligned} \psi = & \theta_c + \int_{-\frac{T}{2}}^{0} \dot{\psi} dt \\ = & \frac{1}{N} \sum_{i=0}^{N} \theta_i + \sum_{t=-T/2}^{0} \dot{\psi}_t dt \end{aligned} \tag{13}$$

## IV. SIMULATIONS AND EXPERIMENTS

To validate the proposed method, we build a 9-module snake-like robot in the simulated environment and the parameters are shown in table 1. In order to prove that the proposed method is robust and general, a sinusoidal curve and a conic are chosen as the desired generic curve which are both difficult to follow using the previous methods because of the non-uniqueness of the projection point.

TABLE I
TYPE SIZE FOR PAPERS

| Parameter name | Symbol | Value | Unit |
|---|---|---|---|
| Number of modules | n | 9 | |
| Half length of each module | l | 0.05 | m |
| velocity | $\dot{x}_0$ | 0.2 | m/s |
| parameter in controller | $K_1$ | 1 | |
| parameter in controller | $K_2$ | 1 | |
| infinitesimal | $\sigma$ | 0.00001 | |

### A. *Sinusoidal curve following*

As shown in Fig. 3, the desired following curve is a sinusoidal curve which can be described as $f(x,y) = y - 0.6\sin(\pi x/18) - 2.6$. The initial head pose of the snake-like robot is $(2.0m, 5.6m, 0.5rad)$ and all of the joint angles are set as zero. The feedback controller can be given by (1), (3), (5), and (13). The trajectory of the head is shown in Fig. 4 and the general distance $f$ can be shown in Fig.5 which converges to zero by period. The control input in (7) is shown in Fig. 6 and the joint angles from joint 1 to joint 8 are shown in Fig.7.

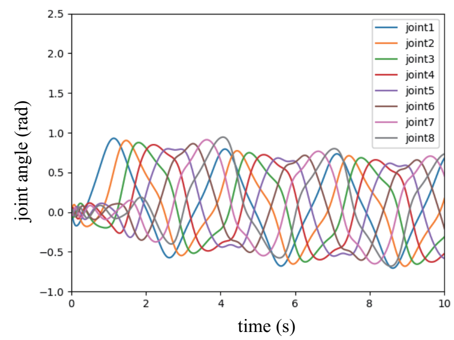


Fig. 7. The joint angles when the robot follows the sinusoidal curve.

### B. *Conic following*

As shown in Fig. 8, the desired following curve is an elliptic curve which can be described as $f(x,y) = \frac{x^2}{9} + y^2 - 1$. The initial pose of the head is $(0.0m, 0.0m, 0.5rad)$ and all of the joint angles are set as zero as shown in Fig. 8. The trajectory of the head is shown in Fig. 9 and the general distance $f$ is shown in Fig. 10 which is negative when the snake-like robot is in the elliptic curve and then positive when the snake-like robot is out of the elliptic curve, finally converges to zero by period. The feedback controller is shown in Fig.11 and the joint angles from joint 1 to joint 8 are shown in Fig. 12.

### C. *Experiment*

A real snake-like robot system which consists of a 9-module snake-like robot, and a capture system is built as

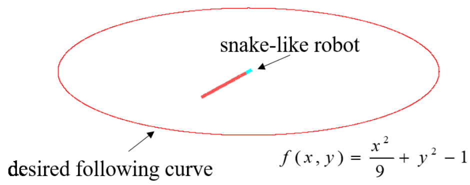


Fig. 8. The desired following curve is described as $f(x,y) = \dfrac{x^2}{9} + y^2 - 1$.

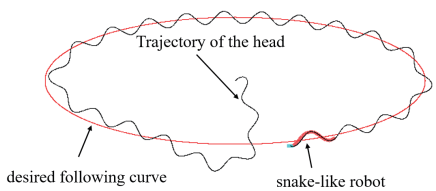


Fig. 9. The trajectory of the head when the snake-like robot follows the elliptic curve.

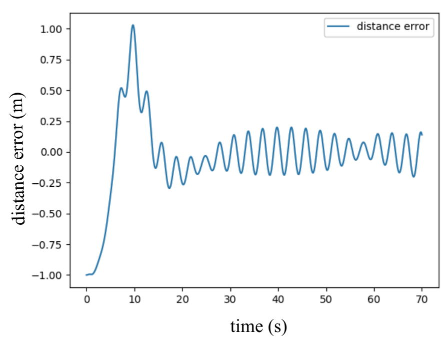


Fig. 10. The distance error when the snake-like robot follows the elliptic curve.

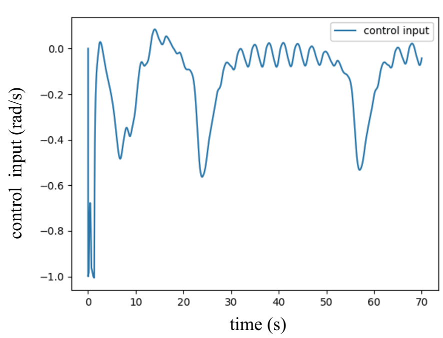


Fig. 11. The control input $\dot{\psi}$ when the snake-like robot follows the elliptic curve.

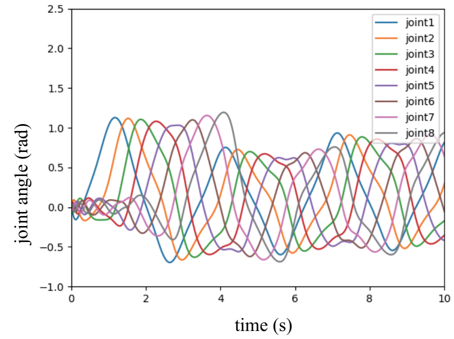


Fig. 12. The joint angles when the snake-like robot follows the elliptic curve.

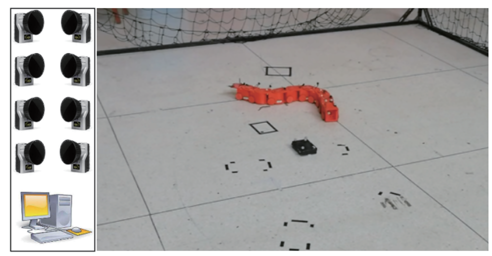

Fig. 13. The experiment platform which contains a snake-like robot and a capture system.

shown in Fig.13. Markers are installed at the head of the snake-like robot to measure the pose and at a black box to define the origin. The capture system is "Qualisys" which contains 8 cameras and one software. The position and orientation of a rigid body can be captured by "Qualisys" with the markers installed at the rigid body. The length of each module is $0.1m$ and other parameters are the same as those in the simulations.

As shown in Fig. 14, in the experiment, the snake-like robot follows an elliptic curve which is described by $f(x,y) = x^2 + \dfrac{y^2}{0.25} - 1$. The initial position is $(0.51, 0.07)$ which is random and the head trajectory of the real snake-like robot is shown in Fig. 14. The red dotted line represents the desired path and the black solid curve represents the trajectory of the head in the experiment. At the beginning of the experiment, the snake-like robot is far from the desired following curve, then, the controller is used and the snake-like robot gradually gets close to the desired path. Finally, the snake-like robot converges to the desired curve.

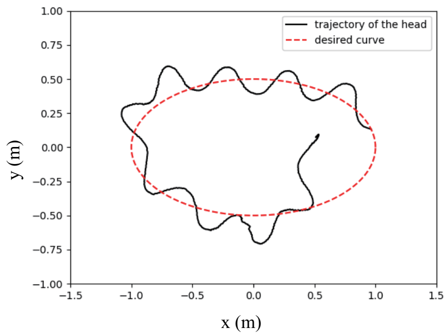


Fig. 14. The trajectory of the head for the snake-like robot when it follows an elliptic curve.

## CONCLUSIONS

In this paper, a novel path following controller which can follow any planar curve represented by $f(x, y) = 0$ is developed based on geometric mechanics and robust controller used for an unicycle robot. To bridge the snake-like robot and the unicycle robot, an estimation method for the angle in the fiber bundle is proposed. Finally the simulations for the sinusoidal curve following and conic following and the experiment for the real snake-like robot are performed.The results prove that our proposed method is valid and robust. In the future, we will consider the varying $\dot{x_0}$ which can reduce the sliding and increase the following accuracy.

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