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Real-time Dynamic Grasping Force Optimization of Multi-fingered Dextrous Hand*

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***Abstract* - A technique for real-time dynamic grasping force optimization is presented in this article. The force optimization is divided into online and offline two parts. The improved linear constrained gradient flow algorithm reduces the dimension of the description matrix based on the traditional gradient flow, and the non-negative linear combination method is introduced to optimize the initial force of the algorithm. Relatively optimal initial grasping force and dimensionality reduction gradient flow algorithm improve the calculating efficiency online. The corresponding illustrative examples are made to verify that the proposed method to obtain both optimal grasping force and faster convergence rate in the dynamic grasp process is feasible.**

Index Terms* – *Dextrous hand, Gradient flow, Linear combination, Dynamic force optimization

I. INTRODUCTION

As robotic dextrous hands apply in more fields, the research on the operation of robot multi-finger dextrous hands has received extensive attention. When multi-finger hand grabs an object, in order to achieve coordinated grasping, it is necessary to distribute the grasping force to apply an appropriate amount of force on the object, which is the premise of force control.

The optimization of the grasping force can be attributed to the force optimization under constrained conditions. In the early stage, NAKAMURA [1] introduces the nonlinear constraint of the friction cone and uses a nonlinear programming method to optimize the force. Then Cheng and Orin [2] approximated the friction cone to the rectangular pyramid, and proposed a Compact-Dual linear programming method in real time. Klein and Kittvatcharapong [3] approximated a hexagonal pyramid to linearize the friction cone, using linear programming method to solve the optimal grasping force.

Buss [4] proposed a linear constrained gradient flow method in 1996, which was equivalent to the friction cone constraint condition to the positive definite problem of a special symmetric matrix. LIU G F et al. [5] used the max-det method proposed by Han and the gradient algorithm to obtain the initial force, and introduced the European gradient and Riemann gradient in the standard gradient flow algorithm to select the step size. On the basis of Buss, Li [6] further analyzed the friction cone condition, transformed the positive definiteness problem of the description matrix into a linear matrix inequality (LMI) problem, converts the objective function in the Buss algorithm into the max-det objective function, and adopted the interior point method to solve the force optimization problem, which reduced the amount of calculation. Zheng conducted the research on the dynamic force distribution problem of multi-finger grasp deeply, and proposed decomposition and positive linear combination (DPC) [7] and GJK algorithm [8]. They both have little calculation and fast convergence rate. Cloutier and Yang [9] compared three optimization methods (linear, nonlinear, and nonlinear with linear matrix inequality friction constraints) and determined the robustness of the GFO methods [10]. V.Rakesh [11] formulated a modified genetic algorithm-based approach for the synthesis of high-quality grasps. But the reachability of the solution grasps by any specific hand model is not addressed. Hua Deng et al. [12] adopt the balance of internal grasping force on the thumb to plan the grasping force of other fingers by using the method of the linear constraint gradient flow. Yuki and Naomichi [13] proposed a hypothetical neuro-computational model to plan the fingertip forces.

For grasping force planning method, the solution of nonlinear programming is optimal but the calculation amount is large. The linear programming reduces the calculation amount but the obtained solution is conservative. The intelligent planning is mainly based on intelligent algorithm and the algorithm is complex. And in case of the dexterous hand grasping characteristics, most of the actual research focuses on the distribution of forces, not the optimization of forces. Therefore, it is necessary to study the optimization of the grasping force, improve the convergence rate of the force, and increase the real-time performance of the algorithm. In this paper, nonlinearized multi-finger grasping force planning problem is transformed into a Riemann manifold problem with linear constraints [14], and the improved gradient flow algorithm and linear combination method is introduced to achieve real-time dynamic grasp optimization.

The paper is organized as follows: In section II, the problem formulation for traditional gradient flow method and improved gradient flow method is developed and presented. Section III shows the optimization algorithm for iterative initial values are described and the grasping force optimization is divided into offline and online two parts. The simulation

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results will be given in Section IV. The conclusions are summarized in section V.

II. Grasping Force Optimization Method

A. Grasping Model and Problem Formulation

Consider a dextrous hand grasping an object with N (N=3,4,5) contact points between the object and the fingertips. In order to facilitate the discussion, the paper makes the following assumptions:

(1) The contact between fingers and object is point contact with friction. Namely that the friction between the fingertip and the object includes the normal force and the tangential force, and the force range is within the friction cone.

(2) There is a unique contact point between the finger and the object, regardless of the relative sliding.

(3) Finger is in the state of avoiding singularity.

(4) The direction of the force between the finger and the object points into the object.

Denote the contact wrench vector of the $\boldsymbol{i}-$th contact with object is $\boldsymbol{f}_i=[\boldsymbol{f}_{ni},\boldsymbol{f}_{oi},\boldsymbol{f}_{ti}]^{\mathrm{T}}$, where $\boldsymbol{f}_{ni}$ is normal force component, $\boldsymbol{f}_{oi}$ and $\boldsymbol{f}_{ti}$ are tangential force component. The friction factor is μ_i, $\mu_i>0$. Then the friction cone constraint and the force constraint in the case of FCWF are respectively expressed as

$$\boldsymbol{D}_i=\left\{\boldsymbol{f}_i\left|\sqrt{\boldsymbol{f}_{ni}^{\ 2}+\boldsymbol{f}_{oi}^{\ 2}}\le \boldsymbol{u}_i\boldsymbol{f}_{ti}\right.\right\} \tag{1}$$

The grasping force and external force keeping balance state can be described as

$$\boldsymbol{G}\boldsymbol{F}_c+\boldsymbol{F}_{od}=0 \tag{2}$$

where $\boldsymbol{G}\in\boldsymbol{R}^{6\times12}$ is grasping matrix related to the position of the contact point. $\boldsymbol{F}_{od}\in\boldsymbol{R}^6$ is the external force of the target object.

The dextrous hand force optimal issue can be described as

$$\begin{cases}\min\ \boldsymbol{F}_c\\ S.T\ \boldsymbol{G}\boldsymbol{F}_c+\boldsymbol{F}_{od}=0\\ \quad \boldsymbol{f}_i\in\boldsymbol{D}_i\\ \quad \boldsymbol{f}_{ti}\ge 0\end{cases} \tag{3}$$

B. Gradient flow algorithm

The friction cone constraint and the unidirectional force constraint are equivalent to the positive definiteness of a particular description matrix $\boldsymbol{P}_i$. $\boldsymbol{P}_i$ is expressed as

$$\boldsymbol{P}_i=\begin{bmatrix}\boldsymbol{u}_i\boldsymbol{f}_{ti} & 0 & \boldsymbol{f}_{ni}\\ 0 & \boldsymbol{u}_i\boldsymbol{f}_{ti} & \boldsymbol{f}_{oi}\\ \boldsymbol{f}_{ni} & \boldsymbol{f}_{oi} & \boldsymbol{u}_i\boldsymbol{f}_{ti}\end{bmatrix} \tag{4}$$

For four fingers hand, the description matrix $\boldsymbol{P}$ can be expressed as

$$\boldsymbol{P}=\mathrm{blockdiag}\left(\boldsymbol{P}_1\quad \boldsymbol{P}_2\quad \boldsymbol{P}_3\quad \boldsymbol{P}_4\right) \tag{5}$$

The description matrix $\boldsymbol{P}$ is a block diagonal matrix composed of each finger description matrix $\boldsymbol{P}_i(i=1,2,3,4)$.

The equality constraints of the description matrix $\boldsymbol{P}$ can be described as

$$\boldsymbol{A}_1\mathrm{vec}\left(\boldsymbol{P}\right)=0 \tag{6}$$

where vec represents the vectorization of the matrix, $\boldsymbol{A}_1\in\boldsymbol{R}^{24\times144}$ is constant matrix, $\boldsymbol{P}\in\boldsymbol{R}^{12\times12}>0$, $\mathrm{vec}(\boldsymbol{P})\in\boldsymbol{R}^{144}$.

In order to provide the desired force vector on the object, the contact force vector of the finger must satisfy the following force balance equation

$$\boldsymbol{G}\boldsymbol{F}_c=\boldsymbol{F}_{od} \tag{7}$$

The above form can be presented as a linearly constrained form.

$$\boldsymbol{A}_2\mathrm{vec}\left(\boldsymbol{P}\right)=\boldsymbol{F}_{od}\quad \boldsymbol{A}_2\in\boldsymbol{R}^{6\times144} \tag{8}$$

Combine (6) with (8),

$$\begin{gathered}\boldsymbol{A}\mathrm{vec}(\boldsymbol{P})=\boldsymbol{r}\\ \boldsymbol{A}=\begin{bmatrix}\boldsymbol{A}_1\\ \boldsymbol{A}_2\end{bmatrix}\in\boldsymbol{R}^{30\times144}\\ \boldsymbol{r}=\begin{bmatrix}\boldsymbol{O}_{24}\\ \boldsymbol{F}_{ext}\end{bmatrix}\in\boldsymbol{R}^{30}\end{gathered} \tag{9}$$

where $\boldsymbol{O}_{24}$ is 24-dimension vector.

Define the objective function [14] as

$$\boldsymbol{\Phi}=\mathrm{tr}\left(\boldsymbol{W}_{\mathrm{p}}\boldsymbol{P}+\boldsymbol{W}_{\mathrm{i}}\boldsymbol{P}^{-1}\right)=\mathrm{tr}\left(\boldsymbol{W}_{\mathrm{p}}\boldsymbol{P}\right)+\mathrm{tr}\left(\boldsymbol{W}_{\mathrm{p}}\boldsymbol{P}\right) \tag{10}$$

The geometric meaning of $\boldsymbol{Q}=\boldsymbol{I}-\boldsymbol{A}^{+}\boldsymbol{A}=\boldsymbol{I}-\boldsymbol{A}^{T}\left(\boldsymbol{A}\boldsymbol{A}^{T}\right)^{-1}$ is a linear projection operator projected onto a tangent space. The linear constrained gradient flow represented by the above equation can converge to a unique equilibrium point at an exponential velocity.

In order to apply the algorithm in the computer, the linear constrained gradient flow represented by the above formula is discretized by Euler integration method.

$$\mathrm{vec}\left(\boldsymbol{P}_{k+1}\right)=\mathrm{vec}\left(\boldsymbol{P}_k\right)+\alpha_k\left(\boldsymbol{I}-\boldsymbol{A}^{+}\boldsymbol{A}\right)\mathrm{vec}\left(\boldsymbol{P}^{-1}\boldsymbol{W}_i\boldsymbol{P}_{\mathrm{p}}^{-1}-\boldsymbol{W}\right) \tag{11}$$

where the step factor α_k needs to guarantee the convergence of the objective function.

C. Gradient flow algorithm based on grasping force vector

Theorem 1: In the linear constrained gradient flow algorithm, if the initial description matrix $\boldsymbol{P}_0$ satisfies the affine constraint condition $\boldsymbol{A}\mathrm{vec}\left(\boldsymbol{P}\right)=0$, a discrete linear constrained gradient flow is used,

$$\mathrm{vec}\left(\boldsymbol{P}_{k+1}\right)=\mathrm{vec}\left(\boldsymbol{P}_k\right)+\alpha_k\left(\boldsymbol{I}-\boldsymbol{A}^{+}\boldsymbol{A}\right)\mathrm{vec}\left(\boldsymbol{P}^{-1}\boldsymbol{W}_i\boldsymbol{P}_{\mathrm{p}}^{-1}-\boldsymbol{W}\right) \tag{12}$$

$\boldsymbol{P}_k$ satisfies the constraint condition,

$$\boldsymbol{A}\mathrm{vec}\left(\boldsymbol{P}_k\right)=\boldsymbol{r} \tag{13}$$

The certification process is as follows:

Assume $\Delta\boldsymbol{P}_k=\boldsymbol{P}_{k+1}-\boldsymbol{P}_k$, then

$$\begin{aligned}\mathrm{vec}\left(\Delta\boldsymbol{P}_k\right)&=\mathrm{vec}\left(\Delta\boldsymbol{P}_{k+1}\right)-\mathrm{vec}\left(\Delta\boldsymbol{P}_k\right)\\ &=\alpha_k\left(\boldsymbol{I}-\boldsymbol{A}^{+}\boldsymbol{A}\right)\mathrm{vec}\left(\boldsymbol{P}^{-1}\boldsymbol{W}_i\boldsymbol{P}^{-1}-\boldsymbol{W}_{\mathrm{p}}\right)\end{aligned} \tag{14}$$

$$A\text{vec}\left(\Delta \boldsymbol{P}_k\right)=\alpha_k \boldsymbol{A}\left(\boldsymbol{I}-\boldsymbol{A}^{+}\boldsymbol{A}\right)\text{vec}\left(\boldsymbol{P}^{-1}\boldsymbol{W}_i\boldsymbol{P}^{-1}-\boldsymbol{W}_{\mathrm{p}}\right) \tag{15}$$

Assume $\boldsymbol{A}\in \boldsymbol{R}^{m\times n}$,

$$\boldsymbol{A}\left(\boldsymbol{I}-\boldsymbol{A}^{+}\boldsymbol{A}\right)=\boldsymbol{A}-\left(\boldsymbol{A}\boldsymbol{A}^{+}\right)\boldsymbol{A}=\boldsymbol{O}^{m\times n} \tag{16}$$

Then

$$A\text{vec}\left(\Delta \boldsymbol{P}_k\right)=\boldsymbol{O}^{m\times 1} \tag{17}$$

So $A\text{vec}\left(\boldsymbol{P}_1\right)=A\text{vec}\left(\boldsymbol{P}_0\right)+A\text{vec}\left(\Delta\boldsymbol{P}_0\right)=\boldsymbol{q}$. And we can get following conclusion: As long as $A\text{vec}\left(\boldsymbol{P}_0\right)=\boldsymbol{q}$, it is inevitable to get $A\text{vec}\left(\boldsymbol{P}_k\right)=\boldsymbol{q}$ and k is an arbitrary integer.

Taking the four-finger grasp as an example, the calculation formula of the linear constrained gradient flow is represented by the grasping force vector.

(1) reduction of dimensionality of $\text{vec}\left(\boldsymbol{P}_k\right)$.

According to Theorem 1, P_k obtained for each iteration satisfies the affine constraint (12).

12 non-zero grasping force components in $\boldsymbol{P}\in \boldsymbol{R}^{12\times 12}>0$ are independent. Use a 12-dimensional grasping force vector $\boldsymbol{VP}_k$ to reduce the dimensionality of $\text{vec}\left(\boldsymbol{P}_k\right)$ as follow.

$$\boldsymbol{VP}_K=\left[\boldsymbol{u}_1\boldsymbol{f}_{t1} \quad \boldsymbol{f}_{n4} \quad \boldsymbol{f}_{o1} \quad \cdots \quad \boldsymbol{u}_4\boldsymbol{f}_{t4} \quad \boldsymbol{f}_{n4} \quad \boldsymbol{f}_{o4}\right]^{\mathrm{T}} \tag{18}$$

(2) dimensionality reduction of $\boldsymbol{Q}$

When the position of the contact point is invariable during the operation, $\boldsymbol{Q}=\boldsymbol{I}-\boldsymbol{A}^{+}\boldsymbol{A}\in \boldsymbol{R}^{144\times 144}$ in the traditional algorithm is a constant matrix.

Extract 12 row vectors $\boldsymbol{Q}_1,\boldsymbol{Q}_2,\cdots,\boldsymbol{Q}_{12}\in \boldsymbol{R}^{144}$ corresponding to $\boldsymbol{VP}_k$ from $\boldsymbol{Q}$, $\boldsymbol{Q}_i$ is the mth row vector of $\boldsymbol{Q}$.

$$\begin{gathered}\mathrm{m}=39\text{floor}\left(i-1,3\right)+2^{2^{i-3\text{floor}(i-1,3)-1}}-1\\ i=1,\cdots,12\end{gathered} \tag{19}$$

where function $\text{floor}\left(a,b\right)$ represents the largest integer not greater than a/b .

W_{p} and W_{i} are diagonal matrix, $\text{vec}\left(\boldsymbol{P}^{-1}\boldsymbol{W}_i\boldsymbol{P}^{-1}-\boldsymbol{W}_{\mathrm{p}}\right)$ is sparse matrix and they has the same structure with $\boldsymbol{P}_k$. Extract 12 vectors $\boldsymbol{Q}_{i,j}=\left[\boldsymbol{Q}\left(m,n\right) \quad \boldsymbol{Q}\left(m,n+1\right) \quad \boldsymbol{Q}\left(m,n+2\right)\right]\in \boldsymbol{R}^3$ related to non-zero block matrix of $\text{vec}\left(\boldsymbol{P}^{-1}\boldsymbol{W}_i\boldsymbol{P}^{-1}-\boldsymbol{W}_{\mathrm{p}}\right)$ from $\boldsymbol{Q}_i$, where $\boldsymbol{Q}\left(m,n\right)$ represents the mth row and nth column elements of $\boldsymbol{Q}$.

$$\begin{gathered}\mathrm{m}=39\text{floor}\left(i-1,3\right)+2^{2^{i-3\text{floor}(i-1,3)-1}}-1\\ i=1,\cdots,12\end{gathered} \tag{20}$$

$$\begin{gathered}n=39\text{floor}\left(j-1,3\right)+12\left[j-3\text{floor}\left(j-1,3\right)-1\right]+1\\ j=1,\cdots,12\end{gathered} \tag{21}$$

So, the new matrix $\bar{\boldsymbol{Q}}\in \boldsymbol{R}^{12\times 36}$ formed by $\boldsymbol{Q}_{i,j}$ is

$$\bar{\boldsymbol{Q}}=\begin{bmatrix}\boldsymbol{Q}_{1,1} & \boldsymbol{Q}_{1,2} & \cdots & \boldsymbol{Q}_{1,12}\\ \boldsymbol{Q}_{2,1} & \boldsymbol{Q}_{2,2} & \cdots & \boldsymbol{Q}_{2,12}\\ \vdots & \vdots & & \vdots\\ \boldsymbol{Q}_{12,1} & \boldsymbol{Q}_{12,1} & \cdots & \boldsymbol{Q}_{12,12}\end{bmatrix}$$

(3) Calculation of $\text{vec}\left(\boldsymbol{P}^{-1}\boldsymbol{W}_i\boldsymbol{P}^{-1}-\boldsymbol{W}_{\mathrm{p}}\right)$.

Let $\boldsymbol{P}_k=\text{blockdiag}\left(\boldsymbol{P}_{1k},\boldsymbol{P}_{2k},\boldsymbol{P}_{3k},\boldsymbol{P}_{4k}\right)$, the inverse matrix is $\boldsymbol{P}_k^{-1}=\text{blockdiag}\left(\boldsymbol{P}_{1k}^{-1},\boldsymbol{P}_{2k}^{-1},\boldsymbol{P}_{3k}^{-1},\boldsymbol{P}_{4k}^{-1}\right)$.

$$\boldsymbol{WP}_k=\boldsymbol{P}^{-1}\boldsymbol{W}_i\boldsymbol{P}^{-1}-\boldsymbol{W}_p=\text{blockdiag}\left(\boldsymbol{WP}_{1k},\boldsymbol{WP}_{2k},\boldsymbol{WP}_{3k},\boldsymbol{WP}_{4k}\right) \tag{22}$$

Let $\boldsymbol{W}_p^{'}=\omega\boldsymbol{I}_3$, $\boldsymbol{W}_i^{'}=\lambda\boldsymbol{I}_3$, ω and λ are weight factor,

$$\boldsymbol{WP}_{ik}=\boldsymbol{P}_{ik}^{-1}\boldsymbol{W}_i\boldsymbol{P}_{ik}^{-1}-\boldsymbol{W}_p^{'} \qquad i=1,2,3,4 \tag{23}$$

Take 12 non-zero blocks of $\boldsymbol{WP}_k$ to form $\boldsymbol{VWP}_k\in \boldsymbol{R}^{36}$

$$\boldsymbol{VWP}_k=\left[\boldsymbol{VWP}_{k1},\boldsymbol{VWP}_{k2},\cdots,\boldsymbol{VWP}_{k12}\right] \tag{24}$$

$$\begin{gathered}\boldsymbol{VWP}_{ki}=\left[\boldsymbol{WP}_{mk}\left(1,n\right),\boldsymbol{WP}_{mk}\left(2,n\right),\boldsymbol{WP}_{mk}\left(3,n\right)\right]\in \boldsymbol{R}^3\\ n=i-3\text{floor}\left(i-1,3\right)\\ m=\text{floor}\left(i-1,3\right)+1\end{gathered} \tag{25}$$

(4) Linear constrained gradient flow based on grasping force vector and its calculation analysis.

The linear constrained gradient flow based on the grasping force vector is

$$\boldsymbol{VP}_{k+1}=\boldsymbol{VP}_k+\alpha_k\bar{\boldsymbol{Q}}\boldsymbol{VWP}_k \tag{26}$$

If the calculation of $\boldsymbol{WP}_k$ is not considered, we can easily know that in each iterative calculation, the traditional gradient flow algorithm includes 144×144=20376 times multiplication and 144×144=20376 times addition; the improved gradient flow algorithm only includes 12×36=432 times multiplication and 12×35+12=432 times addition, the calculation of addition and multiplication is reduced by 20736/432=48 times, greatly improving the efficiency of the algorithm.

(5) Calculation of objective function $\boldsymbol{\Phi}\left(\boldsymbol{P}_k\right)$.

According to the equation (12), it can be easily obtained

$$\boldsymbol{\Phi}\left(\boldsymbol{P}_k\right)=\text{tr}\left(\boldsymbol{W}_{\mathrm{p}}\boldsymbol{P}_k+\boldsymbol{W}_{\mathrm{i}}\boldsymbol{P}_k^{-1}\right)=\text{tr}\left(\boldsymbol{W}_{\mathrm{p}}\boldsymbol{P}_k\right)+\text{tr}\left(\boldsymbol{W}_{\mathrm{p}}\boldsymbol{P}_k^{-1}\right) \tag{27}$$

Let $\boldsymbol{W}_p^{'}=\omega\boldsymbol{I}_3$, $\boldsymbol{W}_i^{'}=\lambda\boldsymbol{I}_3$, ω 和 λ are weight factor, and

$$\text{tr}\left(\boldsymbol{W}_{\mathrm{p}}\boldsymbol{P}_k\right)=3\omega\sum_{i=1}^{4}\mu_i c_{i,1} \tag{28}$$

$$\begin{gathered}\text{tr}\left(\boldsymbol{W}_{\mathrm{p}}\boldsymbol{P}_k^{-1}\right)=\lambda\sum_{i=1}^{4}\left(3\mu_i^2 c_{i,1}^{\ 2}-c_{i,2}^{\ 2}-c_{i,3}^{\ 2}\right)/d_i\\ d_i=\left(\mu c_{i,1}\right)^3-\mu c_{i,1}c_{i,2}^2-\mu c_{i,1}c_{i,3}^2\end{gathered} \tag{29}$$

Use the force vector express the equation (29),

$$\text{tr}\left(\boldsymbol{W}_{\mathrm{p}}\boldsymbol{P}_k\right)=3\omega\sum_{i=1}^{4}VP_k\left(3i-2\right) \tag{30}$$

$$\begin{gathered}\text{tr}\left(\boldsymbol{W}_{\mathrm{p}}\boldsymbol{P}_k^{-1}\right)=\lambda\sum_{i=1}^{4}\left\{3\left[VP_k\left(3i-2\right)\right]^2-\left[VP_k\left(3i-1\right)\right]^2-\left[VP_k\left(3i\right)\right]^2\right\}/b_i\\ b_i=a_i VP_k\left(3i-2\right)\\ a_i=\left[VP_k\left(3i-2\right)\right]^2-\left[VP_k\left(3i-1\right)\right]^2-\left[VP_k\left(3i\right)\right]^2\end{gathered} \tag{31}$$

III. Linear Combination Algorithm of Initial Force

In the grasping force optimization algorithm, the selection of the initial force of the grasping force is very important. On the basis of satisfying the force balance condition and the contact constraint condition, selecting a better initial force of the grasping force can make the convergence rate of the force optimization faster. This paper introduces a non-negative linear combination method to find the initial force.

Firstly, considering the unit force rotation component of the external force, the external force of the object can be decomposed into the following form.

$$\boldsymbol{F}_{ext} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \boldsymbol{\lambda} = \boldsymbol{E\lambda} \quad (32)$$

where, $\boldsymbol{\lambda} \in R^{6\times12}$ $\boldsymbol{E} \in R^{12}$ $\boldsymbol{E}_i, \boldsymbol{E}_{i+6} \geq 0, i = 1,...,6$

We can draw a conclusion that for any external force such as $\boldsymbol{F}_{ext} = [\boldsymbol{F}_1 \ \ \boldsymbol{F}_2 \ \ \boldsymbol{F}_3 \ \ \boldsymbol{F}_4 \ \ \boldsymbol{F}_5 \ \ \boldsymbol{F}_6]^{\mathrm{T}}$, it can be easily factorized into a unit force/torque matrix and a coefficient matrix, that is, namely a linear combination of unit external force.

The coefficient matrix of the unit force matrix in the combination is a non-negative real matrix, and there must be one zero in the coefficient of the positive and negative relative unit external force vectors.

Assume that the corresponding grasping force values of each column of λ are $\boldsymbol{f}_i$, $i = 1,...,12$, and the force balance equations of 12 external forces can be connected together to obtain

$$\begin{cases} \boldsymbol{G} \cdot \boldsymbol{f}_1 + [-1\ 0\ 0\ 0\ 0\ 0]^{\mathrm{T}} = 0 \\ \qquad \cdots \\ \boldsymbol{G} \cdot \boldsymbol{f}_{12} + [0\ 0\ 0\ 0\ 0\ 1]^{\mathrm{T}} = 0 \end{cases} \quad (33)$$

Then the external force can be used to obtain the corresponding grasping force.

$$\boldsymbol{F}_c = [\boldsymbol{f}_1 \ \ \boldsymbol{f}_2 \ \ \cdots \ \ \boldsymbol{f}_{12}] \cdot \lambda \quad (34)$$

Then it can be easily obtained equation (2). Therefore, a simple method of calculating the initial force of the grasping force when any external force $\boldsymbol{F}_{ext}$ applied to the object is obtained. The optimal grasping force under the external force of each unit is calculated offline as the basic solution system of the linear combination.

According to the composition relationship between the grasping external force $\boldsymbol{F}_{ext}$ and the unit external force $\boldsymbol{E}$, the corresponding linear combination of the basic grasping force $\boldsymbol{f}_i$ is performed, and the obtained result automatically satisfies the external force balance constraint. Its calculation only requires simple addition and multiplication, and the amount of calculation is small.

The flow chart of improved linear constrained gradient flow algorithm based on linear combination method to calculate initial value is shown in Figure 1.

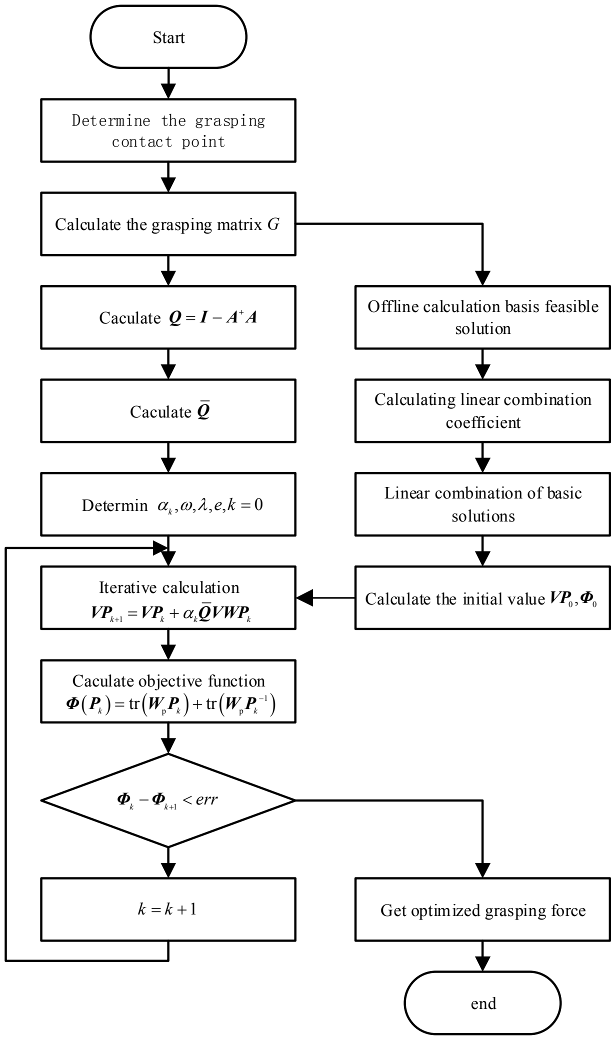

Figure 1 Block diagram of improved gradient flow algorithm

IV. NUMERICAL EXAMPLE

A. *Constant force grasp of dextrous hand*

Build a four-finger grasping cuboid model, as shown in figure 2, a uniform material cuboid. a=0.1m, H=0.1m. The coefficient is same at each friction point, $\mu = 0.4$. Assume that the finger and the object are rigid, the contact point is unchanged, and the grasping matrix $\boldsymbol{G}$ does not change.

The coordinate system of the object is $\boldsymbol{O}_0\boldsymbol{x}_o\boldsymbol{y}_0\boldsymbol{z}_0$, the coordinate system of the contact point is $\boldsymbol{O}_i\boldsymbol{x}_i\boldsymbol{y}_i\boldsymbol{z}_i$, the grasping matrix $\boldsymbol{G}$ is

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & H/2 & 0 & 0 & H/2 & 0 & 0 & H/2 & 0 & 0 & H/2 & 0 \\ H/2 & 0 & 0 & -H/2 & 0 & a & -H/2 & 0 & 0 & -H/2 & 0 & -a \\ 0 & 0 & 0 & 0 & -a & 0 & 0 & 0 & 0 & 0 & a & 0 \end{bmatrix}$$

First, take the constant force grasp as an example to verify the validity of the linear combination initial force algorithm. Taking the external force as [1 0.5 -0.6 -0.1 0.2 0], the single value optimization method, the Lagrange method and the linear combination method proposed in this paper are used to calculate the grasping force respectively, and the improved algorithm is used to calculate the optimal force finally.

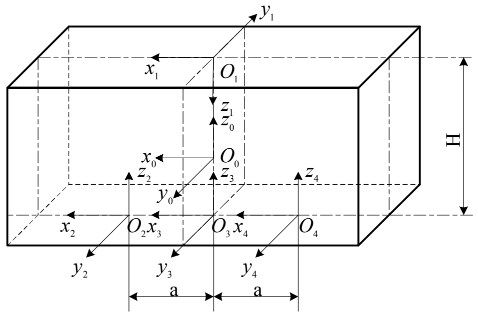

Figure 2 Grasp model of four fingers

Since the main direction of force application during the grasping process is the z-direction, the main direction z-direction is taken as an example. The initial grasping force between three algorithm and the optimal force based on the initial force are as show in TABLE I.

TABLE I
COMPARISON OF INITIAL FORCE AND OPTIMAL FORCE

	algorithm	$Fz1$	$Fz2$	$Fz3$	$Fz4$
Initial force	single value optimization method	8.3	2.73	2.57	2.40
	Lagrange method	5.19	1.59	1.50	1.51
	linear combination method	4.15	1.13	1.15	1.26
Optimal force		3.25	0.66	0.79	1.21

The single value optimization method converges around 160 steps. Lagrange method converges around 120 steps. The linear combination method converges around 80 steps. The results of iteration are shown in the figure 3. The calculating time of three methods are 0.536s, 0.465s, 0.435s respectively. This shows that the linear combination method of this paper has higher efficiency.

B. Dynamic force grasp of dextrous hand

In order to verify the dynamic force optimization effect of the algorithm, the linear combination method proposed in this paper is used to solve the initial force, and the improved gradient flow algorithm is used to solve the optimal grasping force.Given a continuously changing external force W_{ext} as follow.

$$W_{ext} = \begin{bmatrix} \cos(\pi t) \\ \cos^2(\pi t) \\ 8\cos^2(0.5\pi t) - 5 \\ 0.2\sin(\pi t) \\ 0.2\cos(\pi t) \\ \sin^2(0.5\pi t) \end{bmatrix}$$

The simulation time is 2s and the number of sampling points is 200. Results are shown in Figure 4.

In Fig. 4, it can be seen from the contact force of each finger that the force optimization is mainly in the direction of the normal force F_z, and the initial value of the contact force in the three directions is very close to the optimal value, and the result shows that the equation is satisfied the equation (2). Furthermore, combined with the grasping force vector and finger friction coefficient, the grasping force of each finger satisfies the friction cone constraint condition (1), which shows that the force optimization algorithm proposed in this paper is effective.

This grasping force optimization algorithm divides the process into online and offline process: the initial value obtained by the combination method is already suboptimal grasping force, and in the improved algorithm, the calculation of the multiplication and addition is both reduced by 48 times compared with the traditional gradient flow algorithm respectively.

In addition, the general force optimization method takes it as a linear programming problem, and its feasible solution is obtained at the vertices of the convex polyhedron, which easily leads to the discontinuity of the contact force of each finger. The dynamic force optimization method is also continuous for the optimal contact force of the continuously varying external force, so that the sudden change of the grasping force can be avoided.

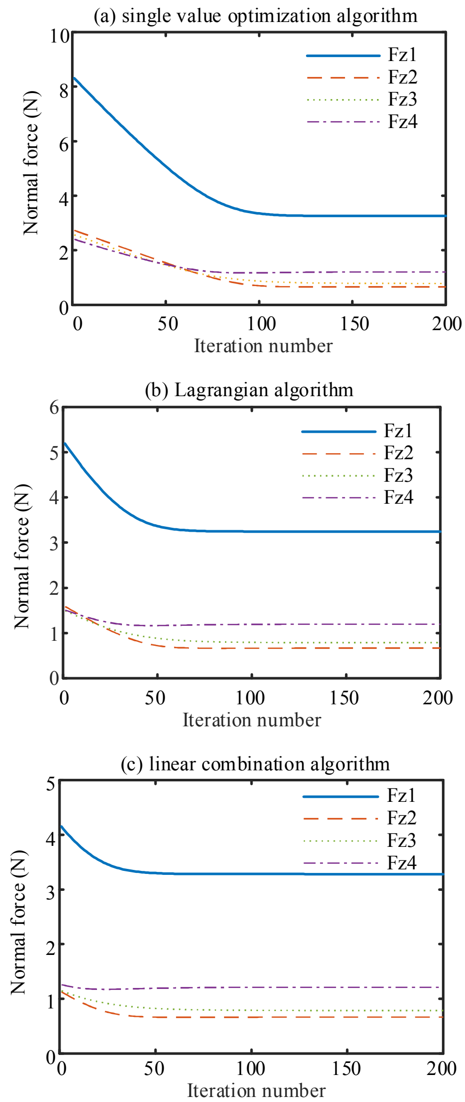

Fig. 3 Force optimization iteration

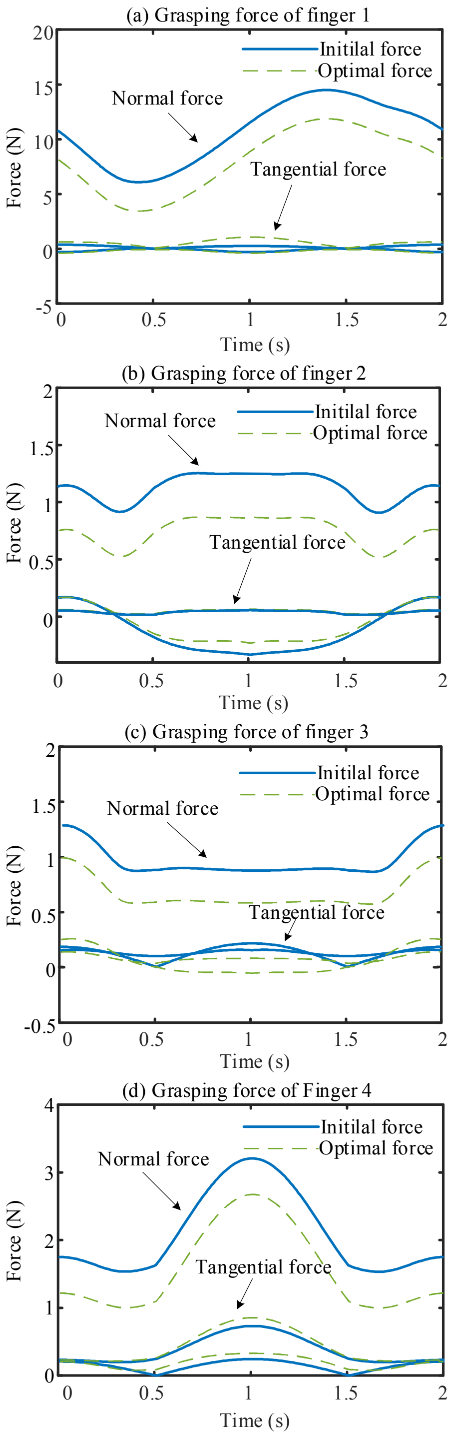

Fig 4 Force optimization results

V. CONCLUSION

On the basis of the traditional gradient flow optimization algorithm, the dimension of the linear constraint gradient flow expression is reduced. And the linear combination method is introduced to obtain the initial force, the online calculation amount is decreased, and the real-time performance of the grasping force optimization algorithm is significantly improved when the number of the fingers is large, and the dynamic allocation of the grasping power is realized to meet the requirements of the online application. In the actual simulation process, the weight coefficient and step size of the objective function will have a great impact on the convergence speed and convergence result. The next step will continue to study its influence law.

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