

# Control of Automotive Air-Spring Suspension System Using Z-Number Based Fuzzy System

Mohamed Essam Shalabi

*Department of Mecharonics and Robotics Engineering  
Egypt-Japan University for Science and Technology  
New Borg Al-Arab city, Alexandria, Egypt  
mohamed.elghwabi@ejust.edu.eg*

Haitham El-Hussieny

*Department of Electrical Engineering  
Faculty of Engineering (Shoubra), Benha University  
Cairo, Egypt  
haitham.elhussieny@feng.bu.edu.eg*

A. A. Abouelsoud

*Electronics and Communication Department  
Cairo University  
Cairo, Egypt  
aali711964@yahoo.com*

Ahmed M.R. Fath Elbab

*Department of Mechatronics and Robotics Engineering  
Egypt-Japan University for Science and Technology  
On leave from Mechanical Eng. Department  
Assiut University, Egypt  
New Borg El-Arab City, Alexandria, Egypt  
ahmed.rashad@ejust.edu.eg*

**Abstract**—This paper presents the control of an automotive air-spring suspension system using a Z-number based fuzzy inference system. One of automotive suspension functions is to isolate vehicle body from road disturbance which is described by transmissibility coefficient. There is a conflict between ride comfort and handling requirements. Lower transmissibility coefficient results in improving passengers' comfort. In this study, an air-spring suspension system is connected to two additional volumes of different sizes via ON-OFF valves that are controlled by the proposed control strategy. The stiffness of the suspension system is the control parameter that is controlled by changing the enclosed pressure and the total air volume inside the air-spring within one of four chosen levels. The proposed Z-number control scheme is characterized by two components; constraint and reliability. The interpolating reasoning of the Z-number fuzzy is constructed and applied. Genetic Algorithm is used to estimate the optimal control gains and switching levels of the air volumes. The proposed control system is applied to a simulated quarter car model to evaluate system performance. Then, the proposed controller is compared to a conventional fuzzy controller and a fuzzy controller merged with discrete volume control. The results show that the proposed controller improves the air suspension performance compared to the other controllers with a maximum acceleration of  $1.1 \text{ cm/s}^2$ .

**Index Terms**—Air Suspension, Air Stiffness, Fuzzy Control, Vibration Isolation, Z-number based Fuzzy System.

## I. INTRODUCTION

Different types of automotive suspension systems exist to afford comfort and road handling. These types are either passive, semi-active and active suspension systems. The function of the automotive suspension system is to carry the vehicle weight, isolate the vehicle body from road disturbance and to ensure road safety. So, the design of the suspension system is responsible for the passengers' comfort and vehicle handling. In the passive suspension system, the design is compromised between the comfort and handling requirements based on

vehicle type (race, luxury, sports car, ..etc) as there is a conflict between comfort and handling where soft suspension is required for better ride quality, while a stiff suspension is required for good handling behavior. [1] The advantage of the air suspension system is getting good ride comfort with maintaining vehicle safety. The air-spring suspension system carrying capacity depends on the enclosed air-spring pressure and its effective area. The control of the air suspension system is based on controlling the stiffness of the air-spring that depends on the enclosed air pressure, spring effective area and the total air volume [2].

The modeling of the dynamic behavior of the air-spring is done by Presthus [2] in 2002. This mathematical model enables the simulation application with no need for experimental work. Other research is presenting an interlinked air suspensions systems done by Wolf-Monheim [3] in 2008. They implemented controllable interlinked air suspension systems consisted of a conventional four-corner air suspension system including pneumatic interconnection lines between the two front and the two rear air spring modules to provide sufficient roll stiffness to counteract body roll under the influence of lateral acceleration. The connection lines can be switched on or off using solenoid valves. The system was tested by a servo-hydraulic test bench. The tests were carried out for different loading conditions and excitation input. The interlinked air suspension model was analyzed using Matlab-Simulink which consists of two thermodynamic air spring models and a pneumatic pipe model based on a concentrated parameter approach. The correlation between the simulation results and the measurement data was satisfying.

In 2006, Deo and Suh [4] proposed a pneumatic suspension system design with independent control of damping, stiffness and ride height. This is done by connecting "N" additional

volumes to the air spring with ON/OFF valves. By adequately choosing  $N$  unequal volumes, they obtained  $2^N$  stiffness settings. A Matlab model was also introduced that describes the stiffness change and the road induced vibration isolation. As a result, they found that increasing the air volume reduces the system's natural frequency and transmissibility ratio. An adaptive pneumatic suspension system was proposed by Nito et. al. [5] based on excitation frequency. They proposed a control strategy to avoid the undesirable resonant frequencies, this is one by knowing the incoming vibration. The air spring characteristics estimation in an active vibration control system is proposed in reference [6] and then the results were compared with theoretical considerations estimation.

In 2014, Mirji and Arockia [7] applied a fuzzy logic control to half-car suspension model using Matlab. The half car suspension (four-degree of freedom) model was established and the equations of motion were derived. Fuzzy logic and PID control for suspension model were achieved using Matlab. The results showed that the fuzzy control resulted in more improvement in stability than PID control. Also, Zepeng et. al. [8]) attempt to solve the electrically controlled air suspension (ECAS) overshoot problem by using fuzzy logic. Finally, the problem is minimized. Gokul and Malar [9] proposed adaptive air suspension system design with LQR control strategy to improve ride comfort. The controller was optimized using particle swarm optimization technique. They design a mathematical model for both passive and adaptive air suspension systems. The results showed that the proposed strategy improves ride comfort by a percentage that reaches 31%.

In 2011, Zadeh [10] introduced the concept of Z-numbers to describe the uncertain information. In Z-numbers, there are some other variables that are idea of certainty or other closely related concept such as sureness, confidence, reliability, strength of truth, or probability. It should be noted that in everyday decision making most decisions are in the form of Z-numbers.

From the above paragraphs, They are divided into two main sections. The first is designing a model for the air spring to use it in a simulation for suspension system. While, the second is presenting techniques to improve the performance of vehicle suspension such as excitation frequency preknowledge, connecting  $N$  unequal additional volumes to the air spring via ON-OFF valves, apply fuzzy logic control or LQR control. The motivation of this work is to develop an adaptive air-spring suspension system capable to vary its stiffness to solve the conflict between the requirements of ride comfort and handling. This paper is concerned in air-spring suspension control using Z-number Fuzzy inference system merged with control algorithm responsible for changing the air volume to vary the spring stiffness through using two unequal volumes through ON-OFF valves.

This paper is organized as follows: the air-spring math-

ematical model is in section II. The control algorithm is discussed in section III, while the optimization of controller gains and levels are in Section IV. The simulation results for the proposed controller are presented and discussed in Section V. Finally, Section VI is the conclusion.

## II. MATHEMATICAL MODEL

The air spring stiffness,  $k_s$  can be described as force,  $F$  per unit deflection  $z$ . The air spring force or its load carrying capacity can be calculated as explained before in section I as the enclosed air pressure,  $p$  multiplied by the effective area,  $A_e$ .

$$F = p * A_e \quad (1)$$

$$k_s = \frac{F}{z} = \frac{dF}{dz} = \frac{d(p * A_e)}{dz} = A_e \frac{dp}{dz} + p \frac{dA_e}{dz} \quad (2)$$

It is assumed that the effective area change is neglected and the inside process is polytrophic process[11];  $pV^n = \text{constant}$ .

$$k_s = A_e \frac{dp}{dz} \quad (3)$$

$$p * V^n = \text{constant} \quad (4)$$

Where  $V$  is the volume and  $n$  is polytrophic coefficient. As assumed the change of the effective area is neglected, the rate of change of volume per unit deflection is the effective area but with negative sign. This is because decreasing the volume increases the deflection and vice versa.

By differentiating equation (4) with respect to deflection,  $z$ :

$$\frac{d(pV^n)}{dz} = pnV^{n-1} \frac{dV}{dz} + V^n \frac{dp}{dz} \quad (5)$$

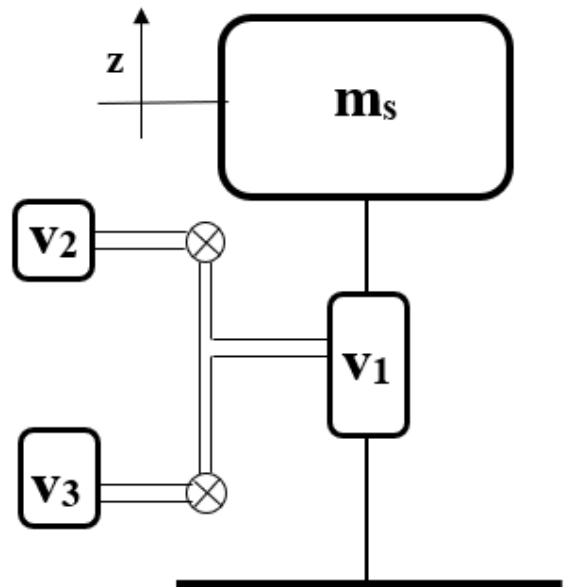


Fig. 1. Proposed schematic diagram for the air volume connected to two additional volumes via ON-OFF valves.

$$\frac{dV}{dz} = -A_e \quad (6)$$

Hence,

$$\frac{dp}{dz} = \frac{pnA_e}{V} \quad (7)$$

$$k_s = \frac{pnA_e^2}{V} \quad (8)$$

From equation(8), we can found that the air spring stiffness is directly proportional to the enclosed air pressure and inversely proportional with the air volume. The air-spring stiffness will be controlled by two controllers. The first will control the air pressure where increasing the pressure, increase the stiffness and vice versa. The second will be responsible of connecting two unequal additional volumes  $V_2$  and  $V_3$  ( $V_3 > V_2$ ) to the primary volume (air-spring volume),  $V_1$  as shown in Fig. 1. The maximum value for the air spring stiffness,  $k_{max}$  happens when the volume is minimum, So the two valves will be closed:

$$k_{max} = \frac{pnA_e^2}{V_1} \quad (9)$$

While the minimum value for the stiffness,  $k_{min}$  happens at the maximum value for the volume:

$$k_{min} = \frac{pnA_e^2}{V_1 + V_2 + V_3} \quad (10)$$

Fig. 2 shows the dynamic air spring suspension model which consists of two springs  $K_1$  and  $K_2$  in which their values vary with pressure and volume change. For volume control, the control is discrete, so there are four values of stiffness  $K_{11}$  and  $K_{21}$ ,  $K_{12}$  and  $K_{22}$ ,  $K_{13}$  and  $K_{23}$  or  $K_{14}$  and  $K_{24}$  according to volume connection described in Table I. Each stiffness of these four stiffness can vary depends on pressure change. The mass  $M$  represents the moving air inside the pipe and viscous damper  $b_z$  which represents the viscous damping inside the pipe. The equation of motion [11] for the model in Fig. 2 is:

$$M\ddot{z}_1 = (z - z_1)K_2 - b_z |\dot{z}_1| \dot{z}_1 \quad (11)$$

Where

$$K_2 = \frac{pnA_e^2}{V_1 + V_r} \frac{V_r}{V_1} \quad (12)$$

TABLE I  
STIFFNESS WITH VOLUMES CONFIGURATION.

$K_2$	$V$
$K_{11}$ AND $K_{21}$	$V_1$
$K_{12}$ AND $K_{22}$	$V_1 + V_2$
$K_{13}$ AND $K_{23}$	$V_1 + V_3$
$K_{14}$ AND $K_{24}$	$V_1 + V_2 + V_3$

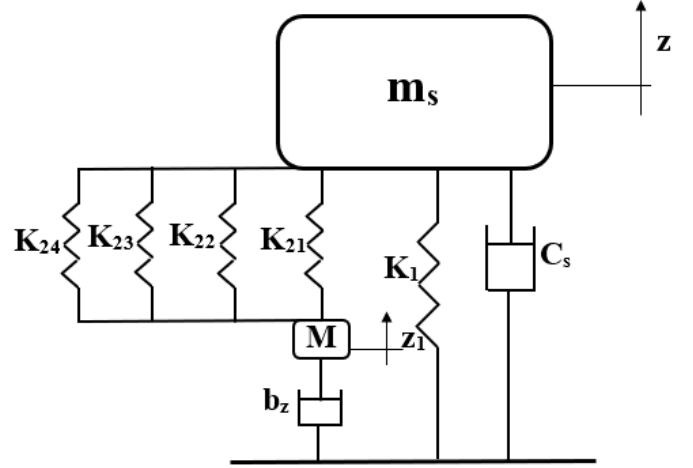


Fig. 2. Air spring model.

$$b_z = 0.5\rho k_t A_p \left( \frac{A_e}{A_p} \frac{V_r}{V_r + V_1} \right)^3 \quad (13)$$

$$M = \rho A_p l_p \left( \frac{A_e}{A_p} \frac{V_r}{V_r + V_1} \right)^2 \quad (14)$$

Where:

$z_1$ : air displacement inside the pipe

$V_1$ : air bag volume

$V_r$ : reservoir volume (0,  $V_2$ ,  $V_3$ ,  $V_2+V_3$ )

$\rho$ : air density

$k_t$ : total pressure drop in the pipeline

$A_p$ : pipe area

$l_p$ : pipe length

And the equivalent air suspension stiffness,  $K_s$  equation will be:

$$K_s = \frac{RMS(F_z)}{RMS(z)} \quad (15)$$

Where,  $F_z$  is the total force on the sprung mass:

$$F_z = pA_e + K_1 z + K_2 (z - z_1) \quad (16)$$

$$K_1 = \frac{pnA_e^2}{V_1 + V_r} \quad (17)$$

Fig. 3 shows two degree of freedom model (2-DOF) where the air spring system is entered to be tested where:

$m_s$ : Sprung mass, kg

$m_{us}$ : Unsprung mass, kg

$K_s$ : air spring stiffness, N/m

$K_{tr}$ : Tire radial stiffness, N/m

$C_s$ : Suspension damping coefficient, N.s/m

$C_{tr}$ : Tire equivalent damping coefficient, N.s/m

$z$ : Sprung mass vertical amplitude, mm

$z_2$ : Unsprung mass vertical amplitude, mm

$z_0$ : Road vertical excitation amplitude, mm

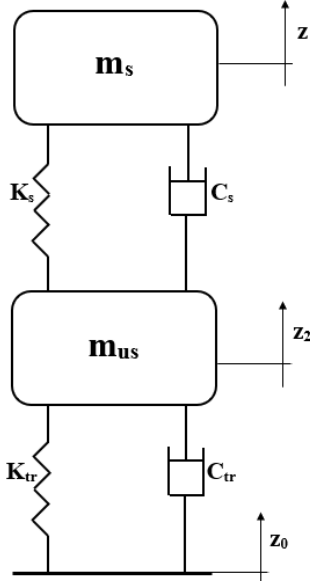


Fig. 3. Quarter car air spring model.

The equations of motion of the model shown in Fig. 3 are:  
For  $m_s$

$$m_s \ddot{z} + C_s(\dot{z} - \dot{z}_2) + K_s(z - z_2) = 0 \quad (18)$$

For  $m_{us}$

$$m_{us} \ddot{z}_2 + C_s(\dot{z}_2 - \dot{z}) + K_s(z_2 - z) + C_{tr}(\dot{z}_2 - \dot{z}_0) + K_{tr}(z_2 - z_0) = 0 \quad (19)$$

### III. CONTROL ALGORITHM

The control strategy will be divided into two subsystems; Discrete and Continuous controller.

#### A. Discrete Controller

As explained before, the air spring stiffness is inversely proportional to air volume. So, the air spring volume is connected to two unequal additional volumes through ON-OFF valve. By controlling these valves, there are four different stiffness settings.

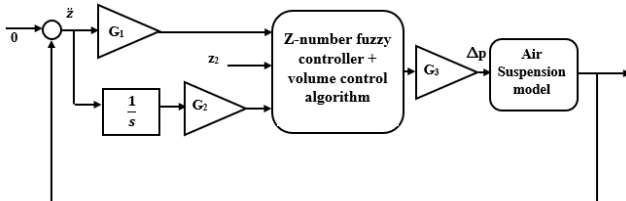


Fig. 4. Air-spring suspension control system block diagram.

#### Algorithm 1: Control Algorithm.

```

if  $z_2 \leq \text{Threshold 1 } (l_1)$  then
    Connect  $V_2 + V_3$  to  $V_1$ 
else if  $\text{Threshold 1} < z_2 \leq \text{Threshold 2 } (l_2)$  then
    Connect  $V_3$  to  $V_1$ 
else if  $\text{Threshold 2} < z_2 \leq \text{Threshold 3 } (l_3)$  then
    Connect  $V_2$  to  $V_1$ 
else
    Disconnect  $V_2$  and  $V_3$  from  $V_1$ 
end if

```

The controller starts with all valves open, so the volume is maximum and the stiffness is minimum resulting in better comfort. Until the unsprung mass displacement  $z_2$  increases gradually to the excitation amplitude, the volume will decrease to  $V_1$  by closing off the valves one after other to increase the spring stiffness. Algorithm 1 shown describes the discrete controller.

#### B. Continuous Controller

The main objective of this work is improve the performance of the air-spring in terms of comfort and handling. Z-number based fuzzy inference system is designed to reach this goal. Fig.4 shows the schematic diagram of Z-number fuzzy control system. From fig. 4 it clear that the acceleration,  $\ddot{z}$  output from the plant and compared by set point which is zero enter the controller multiplied by a gain,  $G_1$  as the first input and its velocity as a second input multiplied by a gain,  $G_2$ . The output of the controller is the change of air pressure,  $\Delta p$  multiplied by a gain,  $G_3$ . Table II shows the rule base of the controller.

From Table. II, there are three linguistic variables, P [positive], N [Negative], and Z [Zero]. While 'U' is usually. Z-number controller is applied for the construction of rule base for control of the suspension system. As shown in Table. II, each fuzzy term is represented by a pair of values. These values correspond to the components of the Z-number which are constraint and reliability parameters. As explained, the values of acceleration, velocity and the pressure change signal are Z-numbers are presented by two fuzzy values including constraint and reliability degrees of information. Fuzzy rule interpolation that will be explained later is used for designing an inference engine of the fuzzy rule-based system. In the rule base, the constraint and reliability values of input signals are

TABLE II  
FUZZY RULE BASE.

$\ddot{z} \backslash \dot{z}$	P, U	Z, U	N, U
P, U	P, U	P, U	Z, U
Z, U	P, U	Z, U	N, U
N, U	Z, U	N, U	N, U

represented by triangular membership functions. Fig. 5 shows the membership function of reliability values. Where, 'S' is 'sometimes', 'U' is 'usually', and 'A' is 'always'. The range of input and output variables are taken from -1 to 1 while, the range of the values of reliability from 0 to 1.

### C. Fuzzy Interpolative Reasoning

The interpolation approach [10] for inference mechanism is used for designing Z-number based fuzzy system to control air suspension. This inference mechanism is based on  $\alpha$ -cuts. By applying  $\alpha$ -cut, the lower  $d_L$  and upper  $d_U$  distances between two fuzzy sets can be calculated from:

$$d_{ij}^L = d^L(A_{ij}^\alpha, X_j^\alpha) = \inf(A_{ij}^\alpha) - \inf(X_j^\alpha) \quad (20)$$

$$d_{ij}^U = d^U(A_{ij}^\alpha, X_j^\alpha) = \sup(A_{ij}^\alpha) - \sup(X_j^\alpha) \quad (21)$$

$$d_j^L = d^L(B_j^\alpha, Y_j^\alpha) = \inf(B_j^\alpha) - \inf(Y_j^\alpha) \quad (22)$$

$$d_j^U = d^U(B_j^\alpha, Y_j^\alpha) = \sup(B_j^\alpha) - \sup(Y_j^\alpha) \quad (23)$$

Where  $A$  is a restriction on the values of fuzzy variable  $X$ ,  $\inf$  is for infimum,  $\sup$  is for supremum,  $B$  is a measure of reliability for the  $A$  component.  $\alpha$ -cut for any fuzzy sets  $A$  or  $B$  is expressed as  $A^\alpha$  or  $B^\alpha$ .

$$d_{ij}(A_{ij}^\alpha, X_j^\alpha) = |A_{ij}^\alpha - X_j^\alpha|; d_i^\alpha = \sum_j^m d_{ij}(A_{ij}^\alpha, X_j^\alpha) \quad (24)$$

$$d_i^\alpha = dc_i^\alpha + dr_i^\alpha \quad (25)$$

$dc_i^\alpha$ ,  $dr_i^\alpha$ ,  $d_i^\alpha$  are distances for constraint and reliability and sum of both respectively. Hence, the output value,  $Y$  and reliability,  $R_Y$  are determined from:

$$Y^\alpha = \frac{\sum_{i=1}^n \frac{1}{\sum_j^m d_L(A_{X_{i,j}}^\alpha, X_j^\alpha)} B_{Y,i}^\alpha}{\sum_{i=1}^n \frac{1}{\sum_j^m d_L(A_{X_{i,j}}^\alpha, X_j^\alpha)}} \quad (26)$$

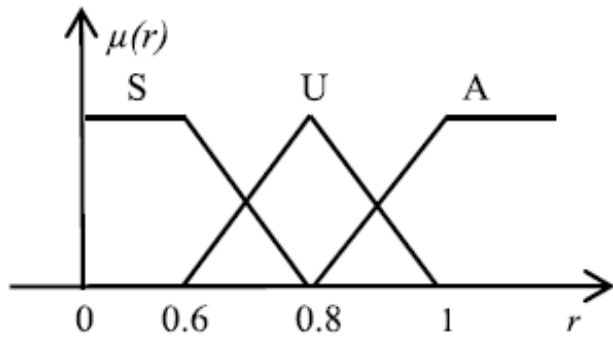


Fig. 5. Reliability membership function.[10]

$$R_Y^\alpha = \frac{\sum_{i=1}^n \frac{1}{\sum_j^m d_L(A_{X_{i,j}}^\alpha, X_j^\alpha)} R_{Y,i}^\alpha}{\sum_{i=1}^n \frac{1}{\sum_j^m d_L(A_{X_{i,j}}^\alpha, X_j^\alpha)}} \quad (27)$$

$$Y = ((Y_l + 4*Y_m + Y_r)/6) * ((R_{Y_l} + 4*R_{Y_m} + R_{Y_r})/6) \quad (28)$$

### IV. OPTIMIZATION

From the previous section, the controller is divided into two parts; discrete and continuous controller. For the discrete controller, the thresholds ( $l_1, l_2$  and  $l_3$ ) used in Algorithm 1 is responsible for volume change. While for the continuous controller, the Z-number controller has two inputs and one output on which each input or output has a gain  $G_1, G_2$  and  $G_3$ . These Thresholds and gains are optimized using Genetic Algorithm (GA) optimization technique using Matlab R2019a. The optimization objective function;  $q$  is as follows:

$$q = \text{minimum} \sum \ddot{z}^2 \quad (29)$$

The optimization is done to minimize the sum of square of sprung mass acceleration. This is done by minimizing the air-spring stiffness,  $K_s$  while limiting the unsprung mass displacement to a value not more than the road excitation to ensure vehicle safety. Also, the threshold levels  $l_1, l_2$  and  $l_3$  should be in sequence, so the inequality constraint is added to the optimization as follows:

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} < \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

### V. SIMULATION RESULTS

The 3-DOF model described in section II is simulated using Matlab R2019a at different operating conditions and optimized using GA optimization to get the values of the control gains and volume threshold levels. The gains and threshold levels  $G_1, G_2, G_3, l_1, l_2$  and  $l_3$  values are 25.63, 32.88, 8140, 0.98, 0.985, 0.995.

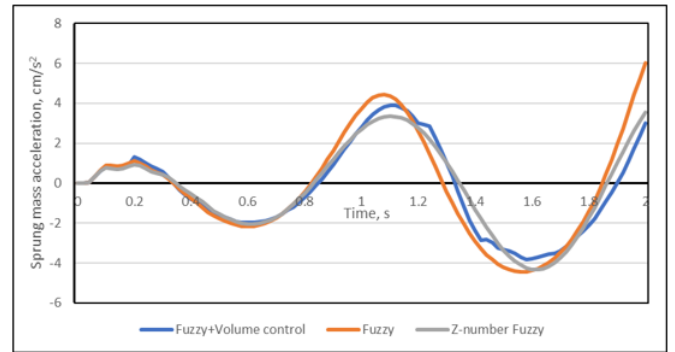


Fig. 6. Sprung mass acceleration versus time for sinusoidal input excitation of 2 cm and 1 Hz frequency

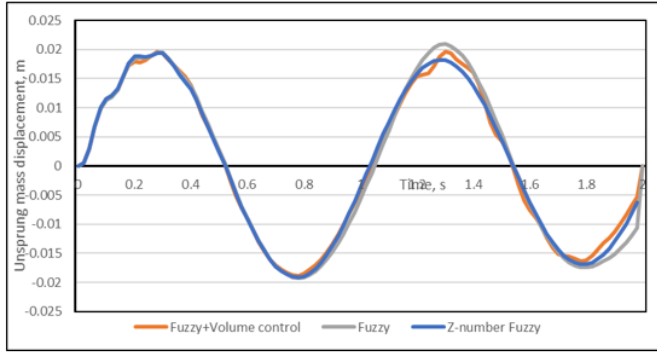


Fig. 7. Unsprung mass displacement versus time for sinusoidal input excitation of 2 cm and 1 Hz frequency

Fig. 6, 7, 8 and 9 show the comparison between the air suspension under different control systems which are conventional fuzzy controller, fuzzy control merged with discrete volume control and Z-number fuzzy control merged with volume control. It can be found from fig. 6 and 7 that at frequencies near system natural frequency. The Z-number based fuzzy controller gives the better performance in terms of comfort (sprung mass acceleration) and handling (unsprung mass displacement). But, at high frequencies the air suspension performance change is very small. From the above, it is clear that implementing the Z-number control improves the air suspension performance at frequency near system natural frequency.

## VI. CONCLUSION

Z-number based fuzzy controller is proposed in this work in order to improve ride comfort and handling by using additional volumes connected to the air-spring volume and switching between them and compare its performance with Mamdani fuzzy logic control and fuzzy control merged with volume control. The Z-number based fuzzy control is

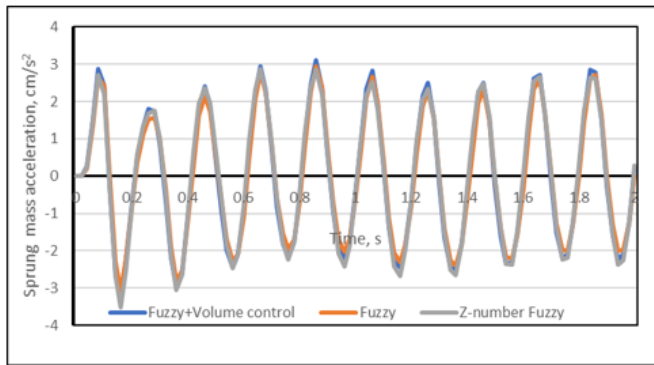


Fig. 8. Sprung mass acceleration versus time for sinusoidal input excitation of 2 cm and 5 Hz frequency.

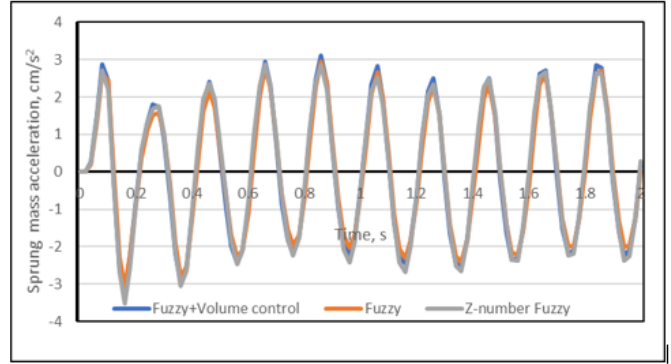


Fig. 9. Unsprung mass displacement versus time for sinusoidal input excitation of 2 cm and 5 Hz frequency

responsible of controlling the enclosed air-spring pressure. While the discrete volume control is used to switch between volumes. The control parameters are estimated using Genetic algorithm. It was found that implementation of the proposed controller improves the performance of ride comfort and handling behavior especially near system natural frequency. The sprung mass body acceleration decreases by a value that reaches 1.1 cm/s<sup>2</sup> which means improving the passenger ride comfort as well maintain the passenger safety. In future, we could compare the proposed controller with model predictive control (MPC).

## REFERENCES

- [1] Wong, Jo Yung, "Theory of ground vehicles." John Wiley & Sons, 2008.
- [2] Presthus, M., "Derivation of air spring model parameters for train simulation," 2002.
- [3] Wolf-Monheim, F., et al. "Modeling, testing and correlation of inter-linked air suspension systems for premium vehicle platforms." 32nd FISITA World Automotive Congress, Munich, Germany. 2008.
- [4] Deo, Hrishikesh, and Nam P. Suh. "Pneumatic suspension system with independent control of damping, stiffness and ride-height." Proceedings of ICAD2006, 4th International Conference on Axiomatic Design, Firenze. 2006.
- [5] Nieto, A. J., et al. "An adaptive pneumatic suspension based on the estimation of the excitation frequency." Journal of sound and vibration 330.9 (2011): 1891-1903.
- [6] Ballo, Igor. "Properties of air spring as a force generator in active vibration control systems." Vehicle System Dynamics 35.1 (2001): 67-72.B.
- [7] Mirji, S. G., and A. S. Arockia. "Fuzzy logic control for half car suspension system using matlab." International Journal of Engineering Research & Technology (IJERT) 3.3 (2014).B.
- [8] Zepeng, Gao, et al. "Research on air suspension control system based on fuzzy control." Energy Procedia 105 (2017): 2653-2659.
- [9] Gokul P. S., and Malar M. K.B., "A contemporary adaptive air suspension using LQR control for passenger vehicles," ISA Transactions, Elsevier, 2019.
- [10] Zadeh L. A., "A note on a Z-number," Inf. Sci., vol. 181, no. 14, pp. 2923-2932, 2011.
- [11] Gavriloski, Viktor, et al., "Development of a new air spring dynamic model," FME Transactions 42.4 (2014): 305-310.B.