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An improved D-H convention for establishing a link coordinate system

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***Abstract*—Robot kinematic modeling needs to be based on a clear physical concept. The widely used Denavit-Hartenberg(D-H) convention requires the coordinate system to be established on the extension of axis, which leads to three problems. First, in some cases, it is difficult to obtain the kinematic model parameters. Second, there can be a mismatch between the model and the object being modeled, leading to an incorrect analysis of local joint positions. Third, it is difficult to obtain the dynamic parameters. In this paper, a general convention to solve these problems is proposed. The main characteristic of ED-H is that the coordinate system fixes on the entity, which can establish unified, intuitive and accurate modeling. Then, a theorem is given to establish coordinate frame, with the process of obtaining the parameters. Both simulation and experimental results on a 6 DOF cooperative manipulator show that the proposed procedure is efficient and practical. Finally, the characteristics of traditional D-H and ED-H on kinematics and dynamics are comprehensively compared. The proposed convention has a good guiding significance for the research and design of joint manipulator arm products.**

Keywords—Traditional D-H; Enhanced D-H; Kinematics; Dynamics; MATLAB

I. INTRODUCTION

In order to meet the needs of simulation and computation for robots, it is very important to establish precise robot kinematic model based on clear physical concept. There are many methods to describe the relationship between the links of the mechanisms, one of the most famous is Denavit-Hartenberg notation [1]. In 1955, Denavit and Hartenberg proposed the definition of mechanisms by means of one joint variable and three link parameters. In the early 1970s, Kahn and Roth proposed a modified convention for open-loop articulated kinematic chains [2]. In 1982, another modified convention have been reported in literature [3]. The former modified notation is known as distal variant of D-H notation, which is called standard D-H and it is popularized by Paul [4]. The latter modified notation is known as proximal variant, which is called modified D-H, and is popularized by Craig [5]. A comparison of the original D-H notation and its two variants has been presented by Lipkin [6].

In the past several decades, many scholars have done a lot of theoretical and practical research on robots with D-H convention. Researchers used D-H method to study the degradation problem of D-H method matrix when applied on robots [7], the robot calibration method [8], use adjustable D-H parameters to solve the problem of variable structure [9], characteristic of parallel robot [10], kinematics and dynamics of service robot's manipulator [11], optimal design [12] and so on.

Even though the standard or modified D-H **convention** (collectively referred to **traditional D-H convention**) have a wide range of applications, it has technical shortcomings [13]. Traditional D-H convention allows the coordinate system to be established on the extension of axis, lead to three problems. First is that in some case the model's parameters are difficult to obtain; secondly, the established model does not match the entity which will lead wrong local joint position analysis; last, it increases the difficulty of obtaining dynamic parameters. Although the researchers have made some improvements to the traditional D-H representation, they have not proposed a general convention to solve the fundamental problems of D-H completely. In order to get a unified, intuitive and accurate method for model establishing, an enhanced D-H(ED-H for short) convention which the coordinate system fixes on the entity is proposed in this paper. The simulation and experimental results show that ED-H improves the traditional D-H convention and meets the needs of robot kinematics and dynamics modeling.

Particularly necessary to point out that Corke [14,15] added traditional D-H convention as basic tool for robot modeling in his Robotics Toolbox in MATLAB, great contributions for its ease of use. This paper is based on this toolbox and develops a series of works, include the modification of code for ED-H.

II. THE PROBLEMS OF D-H CONVENTION FOR ESTABLISHING LINK COORDINATE SYSTEM

The use of D-H convention in robotics has greatly facilitated all the modeling conditions (kinematics and dynamics). The traditional D-H convention (contains standard and modified D-H), powerful and useful as it is, however , is still hampered by certain difficulties. In fact,

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the application of traditional D-H notation to robots allows the linkage frame to be set on the extension cord of the axis. This situation raises the following three problems:

A. In some case the model's parameters are not accurate to be obtained

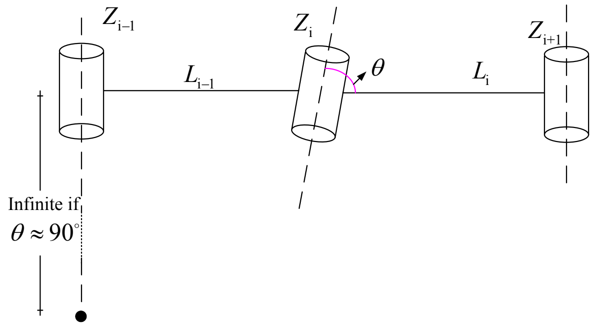


Fig. 1. Three DoFs serial link of almost 90° angle

Let's think about the case in Fig. 1, if the angle between the adjacent joints is nearly 90°, it's hard to get the parameters utilizing traditional D-H convention, because some of them are very large, which will lead to the reduction of calculation accuracy, is not conducive to numerical calculation.

In fact, this kind of circumstance often happens, such as robotic arm calibration, the inaccuracy of machining or assembly results in such subtle deviations. The same is true when adjacent joints are not coplanar.

B. The established model does not match the object which will lead wrong local joint position analysis

The establishment of robot model is inconsistent with the entity. In order to reduce the number of parameters, D-H method allows the coordinate system to be built on the extension cord of the axis, which makes the establishment of joint coordinate system to be far away from the entity, and the physical model is not intuitive and inconsistent with the physical entity. The Fig. 2 shows the link frames utilizing modified D-H convention when the angle between the adjacent joints is 45°.

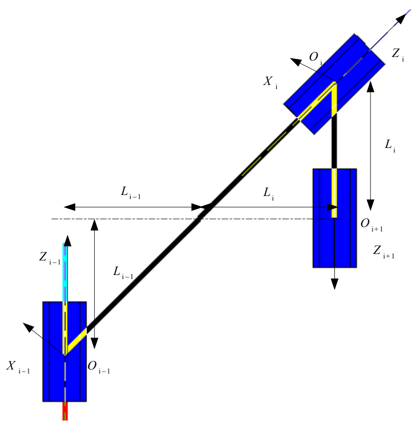


Fig. 2. The model using modified D-H in MATLAB with a 45° angle

It is impossible to analyze the position and orientation of local joints because the joint coordinate system can't be analyzed on the object. The following is a common and simple example to illustrate this problem. In Fig. 2, we can see that it's not intuitive when using standard D-H method to establish the multi-links model. Obviously, except the end joint, there are two joint coordinate systems are not on the object, a large gap between the model and the object is unwanted. It may lead to great difference between the motion space of joint model and that of joint. For example, Fig. 3 shows the solid motion space envelope (black line) of the joint and the D-H model space envelope (red line). This proves that the traditional D-H modeling method can only guarantee the position and orientation of the terminal joint.

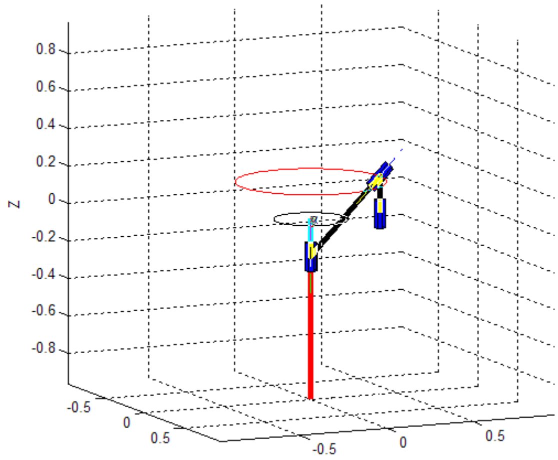


Fig. 3. The motion trajectory compare between D-H model and real-entity(for the second joint)

It is obvious that the influence of coordinate system not on the object, which will reduce the reliability of kinematic analysis of local joints. When the degree of freedom of the robot increases, the motion gap of each joint between model and object will be much larger, it is impossible to analyze the local joints of the robot. There's almost no effect for the robot motion in an unrestricted motion space, but on the contrary when the workspace of the robot is narrow and limited, it is necessary to consider whether the position of each joint is within the restrictive conditions. In this case, it is inappropriate to use the traditional D-H method to establish the robot model.

C. Increase the complexity of dynamic model parameters acquisition

Due to the inconsistency with the object, the deviation of the center of mass and the careful calculation of the inertia tensor are required when constructing the robot dynamic model, which increases the complexity of the model establishment. And the dynamic parameters need special numerical treatment before they can be used, which increases the difficulty of designers and developers. Especially for the standard D-H convention, developers are easy to get confused about these dynamic parameters.

III. THE IMPROVEMENT OF D-H FOR ESTABLISHING COORDINATE SYSTEM

Although the predecessors have solved problem 1 by adding parameters, they have not put forward the reasons for the existence of the problem in essence, and the other two problems have not been discussed.

Because all the motions of D-H method are about x-axis and z-axis, and can't express the motion of y-axis, this method is not applicable as long as there is any motion about y-axis. This situation is very common, for example, because the accuracy of processing and installation is not enough to make the two joint axes close to flat. The essential reason for this problem is that the D-H method allows the coordinate system to be built on the extension line of the axis, which increases the difficulty of parameter acquisition.

In kinematics simulation, there's congenital disadvantage for the D-H convention compared with spherical coordinates method, Euler-Angle coordinate method and product of exponential (POE) methods[16]. Although the latter for robot modeling is direct and faster, the D-H convention(include the improvements) has wide application is that it has additional features, it has developed a lot of technology such as the Jacobi matrix calculation and dynamic analysis [17]. Therefore, the problems of traditional D-H must be solved to take advantage of its added value.

A. ED-H for serial link model

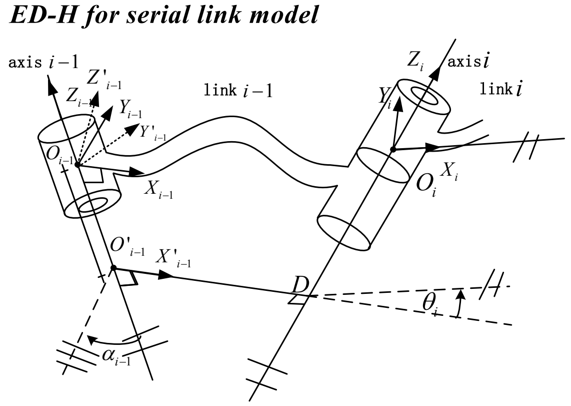


Fig. 4. Adjacent link coordinate system based on EDH

The problem with traditional D-H is that it allows the coordinate system to be set outside of the object, for example, an extension line can be built on an axis outside the object. Therefore, an enhanced D-H convention with coordinate system fixed on the object (ED-H for short) is proposed, as shown in Fig. 4. The procedure of ED-H for establishing coordinate system is:

1. Origin of coordinate system {i-1} O_{i-1} is set on the axis of joint i-1 on the physical object.

2. The z axis of coordinate system {i-1} Z_{i-1} is collinear with joint i-1, choose the direction at will.

3. Find the common perpendicular X'_{i-1} of axis Z_{i-1} and axis Z_i, based on point O_{i-1} make the parallel line of the common perpendicular above, the X_{i-1} has the same direction with X'_{i-1}. If axis Z_{i-1} and Z_i has intersection point, the direction of X_{i-1} is $X_{i-1} = Z_{i-1} \times Z_i$.

4. On O_{i-1} make a new Z'_{i-1} axis which is parallel with Z_i, $Z'_{i-1} // Z_{i-1}$.

5.The coordinate system{i-1} of axis Y is set using the right-hand rule, $Y_{i-1} = Z_{i-1} \times X_{i-1}$ 。

6. The base frame {0} is fixed to the base of the arm as a reference frame to describe the position and attitude of the other link frames. The axis of frame {0} should be oriented along joint axis 1 for simplicity.

B. Parameter interpretation

Based on the link coordinate system, the link parameters can be defined as：

1. α_{i-1} is the intersection angle between axis Z'_{i-1} and Z_i;

2. a_{i-1} is the position in the x direction of point O_i in the new coordinate system $\{O_{i-1} - X_{i-1} - Y'_{i-1} - Z'_{i-1}\}$.

3. b_{i-1} is the position in the y direction of point O_i in the new coordinate system $\{O_{i-1} - X_{i-1} - Y'_{i-1} - Z'_{i-1}\}$.

4. c_{i-1} is the position in the z direction of point O_i in the new coordinate system $\{O_{i-1} - X_{i-1} - Y'_{i-1} - Z'_{i-1}\}$.

5. θ_i is the intersection angle between axis X_{i-1} and X_i.

C. Derivation of linkage transformation

Since the X-axis is the common perpendicular of Z_i and Z_{i-1}, rotation around the X-axis can make Z_{i-1} and Z_i parallel, translation in the three directions of X, Y and Z can make O_{i-1} and O_i coincide, then rotation around Z_{i-1} can make X_{i-1} and X_i coincide, thus completing the homogeneous transformation of coordinate system.

Coordinate {i-1} and {i} become joined through five parameters α_{i-1}, a_{i-1}, b_{i-1}, c_{i-1} an θ_i. So the transformation matrix ${}^{i-1}_{i}T_i$ of frame {i} with respect to frame is the function of these five parameters. The transformation matrix can be done in the following three simplified steps:

1. rotate axis X_{i-1} with angle α_{i-1} to let $Z_{i-1} // Z_i$.

2. shift O_{i-1} to O_i with vector ($a_{i-1}, b_{i-1}, c_{i-1}$), then the axis Z_{i-1} and axis Z_i is collinear.

3. rotate axis Z_{i-1} with angle θ_i to let the axis X_{i-1} and axis X_i to be coincident.

Since these changes are described with respect to the moving coordinate system, according to the principle of "from left to right", we can get:

$$\begin{aligned} {}^{i-1}_{i}T &= Rot(x,\alpha_{i-1}) \bullet Trans(a_{i-1},0,0) \bullet Trans(0,b_{i-1},0) \\ &\quad \bullet Rot(z,\theta_i) \bullet Trans(0,0,c_{i-1}) \\ &= Rotx(x,\alpha_{i-1}) \bullet Trans(a_{i-1},b_{i-1},c_{i-1}) \bullet Rot(z,\theta_i) \end{aligned} \tag{1}$$

The transformation form is given as：

$${}^{i-1}_{i}T_i = \begin{bmatrix} ct & -st & 0 & dx \\ ca \bullet st & ca \bullet ct & -sa & dy \bullet ca - dz \bullet sa \\ sa \bullet st & sa \bullet ct & ca & dz \bullet ca + dy \bullet sa \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

where $sa = \sin(\alpha_{i-1})$, $ca = \cos(\alpha_{i-1})$, $st = \sin(\theta_i)$, $ct = \cos(\theta_i)$, $a_{i-1} = dx$, $b_{i-1} = dy$, $c_{i-1} = dz$, the parameters of each joint were amplified to five.

IV. IMPLEMENTATION

The simulation object is the mechanical arm model shown in Fig. 5, TianSui-One cooperative manipulator. The traditional D-H method (modified D-H actually) is used for modeling, and then the ED-H method is used for modeling , kinematics analysis and dynamics analysis.

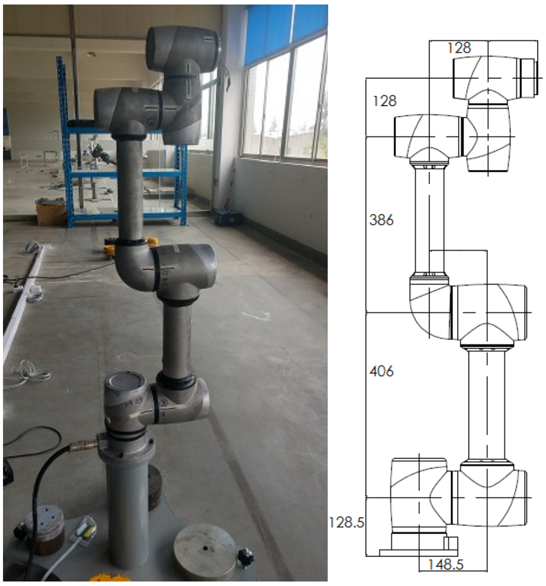
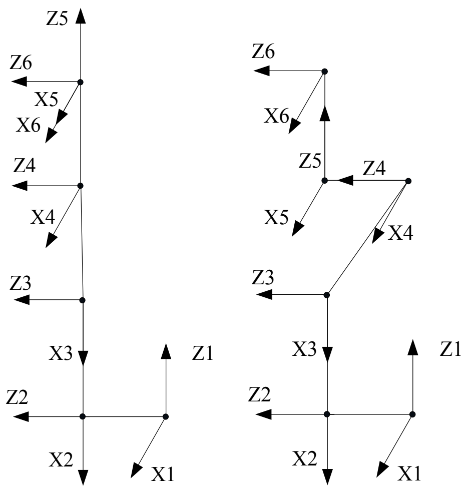


Fig. 5. Experimental arm

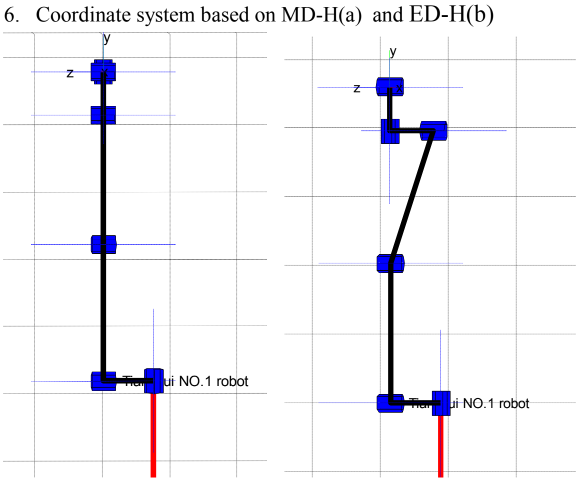
A. Comparison of D-H and ED-H on kinematical model

Utilize the traditional modified D-H (MD-H) and enhanced D-H (ED-H)to establish the robot coordinate system, as the Fig. 6 illustrates.



(a) (b)

Fig. 6. Coordinate system based on MD-H(a) and ED-H(b)



(a) (b)

Fig. 7. Simulation in MATLAB for MD-H(a) and ED-H(b)

The model utilizes MATLAB robotics toolbox with MD-H is illustrated in Fig.7(a) with parameters in Table I.

TABLE I. MD-H PARAMETERS

Link number	$\alpha_{i\text{-}1}$	a_{i-1}	d_i	θ_i	Joint variables
1	0	0	0	0	θ_1
2	π/2	0	0.1485	-π/2	θ_2
3	0	-0.406	0	0	θ_3
4	0	-0.386	0.002	π/2	θ_4
5	-π/2	0	0.128	0	θ_5
6	π/2	0	0	0	θ_6

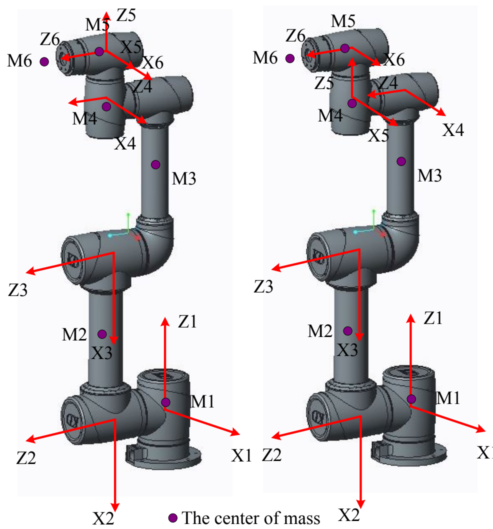
We modified the MATLAB robotics toolbox in ED-H. The coordinate system is shown in Fig. 6(b). The model is illustrated in Fig. 7(b) with parameters in Table II.

TABLE II. ED-H PARAMETERS

Link number	$\alpha_{i\text{-}1}$	a_{i-1}	$b_{i\text{-}1}$	$c_{i\text{-}1}$	θ_i	Joint variables
1	0	0	0	0	0	θ_1
2	π/2	0	0	0.1485	-π/2	θ_2
3	0	-0.406	0	0	0	θ_3
4	0	-0.386	0	-0.126	π/2	θ_4
5	-π/2	0	-0.128	0	0	θ_5
6	π/2	0	0.128	0	0	θ_6

From the comparison in Fig. 7(a) and Fig. 7(b), it can be seen that the traditional D-H method, due to the problems in the section 2 of the paper, leads to the phenomenon of joint origin overlap in the established model. Although it has no influence on the control of the end joint, it is unfavorable to the position analysis of the overlap joint. ED-H fully solves this problem and has the same control effect as D-H method. Meet the needs of robot kinematics modeling.

B. Comparison of D-H and ED-H on dynamics



(a) (b)

Fig. 8. Comparison of DH and EDH in dynamic parameters acquisition: (a)D-H , (b)ED-H

It is obviously that the ED-H is much more intuitional than the D-H in dynamic parameters acquisition. Traditional D-H often leads to confusion of dynamic parameters because transformation of the coordinate system is needed(especially for the standard D-H). In Fig. 8, the influence of the last 3 joints(joint 4, joint 5 and joint 6) is obvious.

V. DISCUSSION

Using the simulation results in section 4, the computational complexity of D-H method and ED-H method is analyzed, and the time consumed by the two methods to solve the analytical solution can be calculated respectively.

Utilizing Newton-Raphson algorithm to calculate the numerical inverse kinematics. Confirm the test parameters: robot initial position and orientation of end-effecter is :

$$ {}_{6}^{0}T = \begin{bmatrix} -0.7598 & 0.4151 & 0.5004 & -0.3655 \\ -0.4391 & 0.2399 & -0.8658 & -0.3712 \\ -0.4794 & -0.8776 & 0.0000 & -0.0245 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{(unit:m)} $$

Along the Z+ direction motion 100mm, and the interpolation interval is 1mm, that is, 100 inverse solutions are calculated. Set the convergence error=10^{-10}, test environment is MATLAB release 7.11.0，CPU：Intel i5-6200, main frequency is 2.3GHZ with 4GB RAM, MATLAB version R2010b, Toolbox is the version of 2002. The test code is in the m file of " All_compare.m " in the Appendix section.

The test result is illustrated in Table III with 100 times inverse calculation.

TABLE III. COMPARISON OF DH AND EDH FOR TIME LOSS WITH 100 TIMES INVERSE KINEMATICS CALCULATION

num	1	2	3	4	5	Average time(unit:s)
D-H	0.460	0.444	0.446	0.448	0.482	0.4560
ED-H	0.488	0.557	0.496	0.489	0.491	0.5042

ED-H method can also use Newton-Raphson algorithm (actually the Jacobian computation has been tested at the same time) to solve the inverse solution, in spite of the increase of the number of parameters which will lead more computation, the computational complexity is still in the same order of magnitude, has almost the same computational efficiency.

The overall comparison between D-H method and ED-H method is shown in Table IV.

TABLE IV. OVERALL COMPARISON BETWEEN DH METHOD AND EDH METHOD

Details	DH	EDH
Clear physical concepts	Yes	Yes
Standard modeling methods	Yes	Yes
Modeling effect	May not be consistent with the object	Object consistent
Joints that cannot be modeled	Yes	No
Number of Parameters	4	5
Computational complexity	Normal	Normal

VI. CONCLUSION

The traditional D-H method allows the coordinate system to be established on the extension line of the axis, which will lead to problems such as in some case the model's parameters are difficult to obtain, the established model does not match the object which will lead wrong local joint position analysis and it increases the difficulty of obtaining dynamic parameters. Based on the traditional D-H convention, this paper proposes an improved convention, D-H representation with coordinate system fixed on object (enhanced D-H, ED-H method for short). Compared with the traditional D-H method, this method can model all joint linkages and establish the model is consistent with the entity. At the same time the disadvantage is that the complexity of kinematics calculation is slightly increased due to the increase of parameters. In summary, ED-H improves the standard D-H convention, which can make the robot model simulation more intuitive and accurate, and has a good guiding significance for engineering practice, and provides a good theoretical guidance basis for the research and design of joint manipulator arm products.

ACKNOWLEDGMENT

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APPENDIX

Source code for this article is available via the following links.

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