

Mészáros effect

In cosmology, the development of initial perturbations which eventually give rise to structures such as galaxies, clusters of galaxies, etc. can be described in the δ -approximation by perturbing the Friedmann equations. The relative density contrast δ (where ρ is the matter-energy density and $\delta\rho$ is the small excess in any given region) obeys the perturbation equation

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} + v_s^2 k^2 \delta - 4\pi G \rho_m \delta = 0. \quad (1)$$

e.g. [1, 2, 3], where a is the expansion scale factor of the Universe, $v_s = (P/\rho)^{1/2}$ is the sound speed, k is wavenumber, G is Newton's constant. For perturbations inside the horizon this leads to well-known results for the Jeans mass and growth times for non-relativistic collisionless perturbations (cold matter $P = 0$) and for relativistic collisional perturbations (e.g. a radiation gas $P = \rho c^2/3$). In the early Universe the $P = 0$ case corresponds to what is known today as cold dark matter, and the radiation case corresponds to photons and neutrinos. After radiation-matter decoupling $z < z_{dec} \approx 10^3$ the perturbation growth is given by the $P = 0$ matter-dominated solution, while before the matter-radiation equilibrium epoch $z < z_{eq} \approx 10^4$ the growth is given by the radiation-dominated solution.

However, in a combined picture of collisionless matter in a radiation background one has a mode of perturbations inside the horizon where the collisionless (non-relativistic) matter component of density ρ_m is perturbed relative to the relativistic radiation component of density ρ_r (which for $z > z_{dec}$ oscillates and on average can be considered as unperturbed, just following the Universe's expansion). This was first considered in [4], leading to the perturbation equation

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G(\rho_m + \rho_r)\delta = 0, \quad (2)$$

where now $(\dot{a}/a)^2 = 8\pi G(\rho_m + \rho_r)/3$, and $k = 0$ is appropriate for early times. Changing variables to $y = \rho_m/\rho_r = (a/a_{eq})^2$ and using Friedman's equation leads to

$$y\ddot{\delta} + \frac{2+3y}{2y(1+y)}\dot{\delta} - \frac{3}{2y(1+y)}\delta = 0. \quad (3)$$

This equation has closed analytical solutions, the growing mode of which can be seen to be

$$1 + \frac{3}{2}y. \quad (4)$$

That is, for $a < a_{eq}$ the (cold) matter perturbation remains frozen $\delta = \text{constant}$, while for $a > a_{eq}$ the matter perturbation grows linearly with $y = a/a_{eq}$. This is referred to as the Mészáros effect (or equation); e.g. Google these keywords, or see references above (also [6, 7, 8, 9, 10],...). This effect is important for the initial perturbations in the cold dark matter to achieve non-linearity, leading to the observed galaxies and clusters at the present epochs; it adds extra growth time between z_{eq} and z_{dec} .

References

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