- **Set up a matrix of alphabets. You can change the elements to be pixels or numbers.**

In[26]:=
```
M = {{2, 1}, {1, 1}};
n = 5;
m1 = map[0] = Table[FromCharacterCode[97 + (i - 1) * n + j - 1], {i, n}, {j, n}];
m2 = Table[0, {i, n}, {j, n}];
g[x_, y_] := Mod[M.{{x}, {y}}, n, 1];
For[t = 1, m2 ≠ map[0], t++, {For[i = 1, i < n + 1, i++,
    For[j = 1, j < n + 1, j++, m2[[g[i, j][[1, 1]], g[i, j][[2, 1]]]] = m1[[i, j]]]],
   map[t] = m1 = m2}]
Print["n=", n, ", Period = ", prd[n] = t - 1]
Print[MatrixForm[map[0]], "=>", MatrixForm[map[1]], "=>",
 MatrixForm[map[2]], "=>", MatrixForm[map[3]], "=>", MatrixForm[map[4]], "=>",
 MatrixForm[map[5]], "=>", MatrixForm[map[6]], "=>", MatrixForm[map[7]], "=>",
 MatrixForm[map[8]], "=>", MatrixForm[map[9]], "=>", MatrixForm[map[10]]]
```

```
n=5, Period = 10
```

$$\begin{pmatrix} a & b & c & d & e \\ f & g & h & i & j \\ k & l & m & n & o \\ p & q & r & s & t \\ u & v & w & x & y \end{pmatrix} => \begin{pmatrix} u & r & o & g & d \\ e & v & s & k & h \\ i & a & w & t & l \\ m & j & b & x & p \\ q & n & f & c & y \end{pmatrix} => \begin{pmatrix} q & b & l & v & g \\ d & n & x & i & s \\ k & u & f & p & a \\ w & h & r & c & m \\ j & t & e & o & y \end{pmatrix} => \begin{pmatrix} j & r & a & n & v \\ g & t & c & k & x \\ i & q & e & m & u \\ f & s & b & o & w \\ h & p & d & l & y \end{pmatrix} => \begin{pmatrix} h & b & u & t & n \\ v & p & o & i & c \\ k & j & d & w & q \\ e & x & r & l & f \\ s & m & g & a & y \end{pmatrix} => \begin{pmatrix} s & r & q & p & t \\ n & m & l & k & o \\ i & h & g & f & j \\ d & c & b & a & e \\ x & w & v & u & y \end{pmatrix}$$

$$=> \begin{pmatrix} x & b & j & m & p \\ t & w & a & i & l \\ k & s & v & e & h \\ g & o & r & u & d \\ c & f & n & q & y \end{pmatrix} => \begin{pmatrix} c & r & h & w & m \\ p & f & u & k & a \\ i & x & n & d & s \\ v & l & b & q & g \\ o & e & t & j & y \end{pmatrix} => \begin{pmatrix} o & b & s & f & w \\ m & e & q & i & u \\ k & c & t & g & x \\ n & a & r & j & v \\ l & d & p & h & y \end{pmatrix} => \begin{pmatrix} l & r & x & e & f \\ w & d & j & k & q \\ i & o & p & v & c \\ t & u & b & h & n \\ a & g & m & s & y \end{pmatrix} => \begin{pmatrix} a & b & c & d & e \\ f & g & h & i & j \\ k & l & m & n & o \\ p & q & r & s & t \\ u & v & w & x & y \end{pmatrix}$$

- **So we can define a function called Arnold[N], which will calculate the period of an N× N map**

In[34]:=
```
Arnold[N_] := {M = {{2, 1}, {1, 1}}; n = N;
   m1 = map[0] = Table[FromCharacterCode[97 + (i - 1) * n + j - 1], {i, n}, {j, n}];
   m2 = Table[0, {i, n}, {j, n}];
   g[x_, y_] := Mod[M.{{x}, {y}}, n, 1];
   For[t = 1, m2 ≠ map[0], t++, {For[i = 1, i < n + 1, i++,
       For[j = 1, j < n + 1, j++, m2[[g[i, j][[1, 1]], g[i, j][[2, 1]]]] = m1[[i, j]]]],
      map[t] = m1 = m2}];
   (*Print["n=",n];
   Print[MatrixForm[map[0]]];For[i=1,i<t,i++,Print["=>"MatrixForm[map[i]]]];
   Print["Period = ",prd[n]=t-1];*)};
```

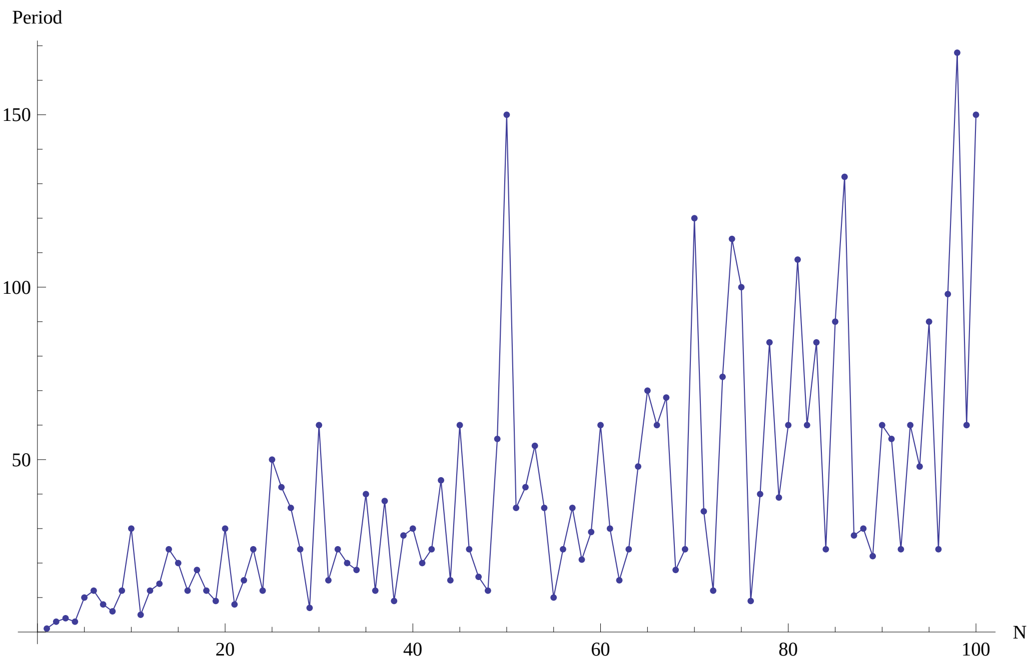
In[35]:=
```
Table[{Arnold[i], Print["n=", n, ", Period = ", prd[n] = t - 1];}, {i, 1, 5}];
```

```
n=1, Period = 1

n=2, Period = 3

n=3, Period = 4

n=4, Period = 3

n=5, Period = 10
```

- **We can plot out the graph of Period vs N**

```
Table[{Arnold[i], prd[n] = t - 1;}, {i, 1, 100}];
Table[{n, prd[n]}, {n, 1, 100}]
ListLinePlot[Table[{n, prd[n]}, {n, 1, 100}], Mesh → All, AxesLabel → {"N", "Period"}]
```

```
{{1, 1}, {2, 3}, {3, 4}, {4, 3}, {5, 10}, {6, 12}, {7, 8}, {8, 6}, {9, 12}, {10, 30}, {11, 5},
 {12, 12}, {13, 14}, {14, 24}, {15, 20}, {16, 12}, {17, 18}, {18, 12}, {19, 9}, {20, 30},
 {21, 8}, {22, 15}, {23, 24}, {24, 12}, {25, 50}, {26, 42}, {27, 36}, {28, 24}, {29, 7},
 {30, 60}, {31, 15}, {32, 24}, {33, 20}, {34, 18}, {35, 40}, {36, 12}, {37, 38}, {38, 9},
 {39, 28}, {40, 30}, {41, 20}, {42, 24}, {43, 44}, {44, 15}, {45, 60}, {46, 24}, {47, 16},
 {48, 12}, {49, 56}, {50, 150}, {51, 36}, {52, 42}, {53, 54}, {54, 36}, {55, 10}, {56, 24},
 {57, 36}, {58, 21}, {59, 29}, {60, 60}, {61, 30}, {62, 15}, {63, 24}, {64, 48}, {65, 70},
 {66, 60}, {67, 68}, {68, 18}, {69, 24}, {70, 120}, {71, 35}, {72, 12}, {73, 74}, {74, 114},
 {75, 100}, {76, 9}, {77, 40}, {78, 84}, {79, 39}, {80, 60}, {81, 108}, {82, 60}, {83, 84},
 {84, 24}, {85, 90}, {86, 132}, {87, 28}, {88, 30}, {89, 22}, {90, 60}, {91, 56}, {92, 24},
 {93, 60}, {94, 48}, {95, 90}, {96, 24}, {97, 98}, {98, 168}, {99, 60}, {100, 150}}
```



- **If we take n = 74, as the example on wiki, we can find its period is right 114.**
  http://en.wikipedia.org/wiki/File:Arnold%27s_Cat_Map_animation_(74px,_zoomed,_labelled).gif

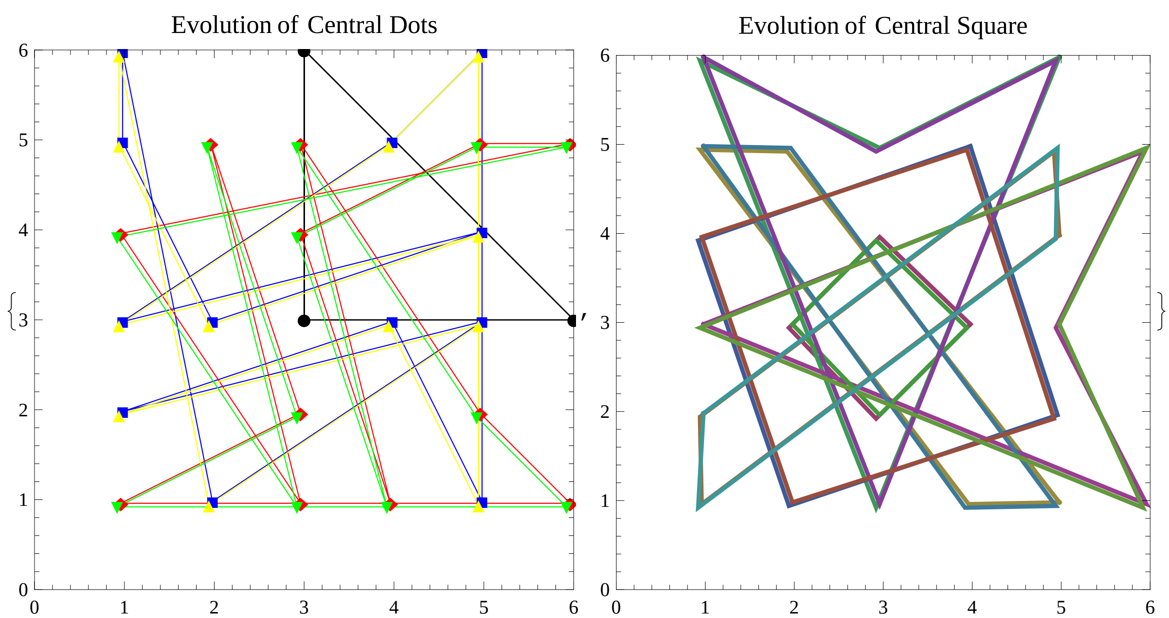
```
In[36]:= n = 74;
         {Arnold[n]; Print["n=", n, ", Period = ", prd[n] = t - 1];};
         (*MatrixForm[map[0]]*)
```

```
n=74, Period = 114
```

Central symmetry of points around (n/2,n/2)

In[38]:=
```
n = 6; m = 1;
traj[x_, y_] := {Arnold[n]; prd[n] = t - 1;
  tr[0] = {i, j} = {Floor[n / 2] + x, Floor[n / 2] + y};
  For[t = 1, t < prd[n] + 1, t++, tr[t] = {i, j} = {g[i, j][[1, 1]], g[i, j][[2, 1]]}];}
traj[0, 0]; t0 = Table[tr[i], {i, 0, prd[n]}];
traj[m, 0]; t1 = Table[tr[i] - {0.02, 0.02}, {i, 0, prd[n]}];
traj[0, m]; t2 = Table[tr[i] - {0.04, 0.04}, {i, 0, prd[n]}];
traj[-m, 0]; t3 = Table[tr[i] - {0.06, 0.06}, {i, 0, prd[n]}];
traj[0, -m]; t4 = Table[tr[i] - {0.08, 0.08}, {i, 0, prd[n]}];
{ListLinePlot[{t0, t1, t2, t3, t4}, PlotStyle → {Black, Blue, Red, Yellow, Green},
  PlotMarkers → Automatic, AspectRatio → Automatic, Frame → True,
  PlotRange → {{0, n}, {0, n}}, PlotLabel → "Evolution of Central Dots"],
 ListLinePlot[Table[{t1[[i]], t2[[i]], t3[[i]], t4[[i]], t1[[i]]}, {i, 0, prd[n]}],
  AspectRatio → 1, Frame → True, PlotStyle → {Thick},
  PlotRange → {{0, n}, {0, n}}, PlotLabel → "Evolution of Central Square"]}
```

Out[45]=



- **we can see the central symmetric points also share a symmetry in their trajectories. in certain cases, they even overlap with each other. (lines are slightly shifted for better observation.)**

- ### Animations

In[48]:=
```
{Animate[Graphics[
   {{Black, PointSize[Large], Point[t0[[i]]], {Blue, PointSize[Large], Point[t1[[i]]]},
     {Red, PointSize[Large], Point[t2[[i]]]}, {Yellow, PointSize[Large], Point[t3[[i]]]},
     {Green, PointSize[Large], Point[t4[[i]]]}}},
   AspectRatio → Automatic, Frame → True, PlotRange → {{0, n}, {0, n}},
   PlotLabel → "Animation of Central Dots"], {i, 1, prd[n] + 1, 1}],
 Animate[ListLinePlot[{t1[[i]], t2[[i]], t3[[i]], t4[[i]], t1[[i]]},
   AspectRatio → 1, Frame → True, PlotStyle → {Thick}, PlotRange → {{0, n}, {0, n}},
   PlotLabel → "Animation of Central Square"], {i, 1, prd[n] + 1, 1}]}
```