An Augmented Reality Application with Hand Gestures for Learning 3D Geometry

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***Abstract*— Geometry is an interesting area of mathematics. It opens to many different approaches and closely relates to our everyday lives. However, when students recall their experiences of learning geometry, many of them regards it as not only unpleasant experiences but often also difficult experiences. The traditional materials and tools such as pens, papers, blackboards, textbooks and/or classical methodologies like drawing, narrative teaching stills cannot be regarded as a great support for students who learn geometry. In this paper, we propose a framework for learning geometry using a software tool based on augmented reality (AR) [1] and hand gestures recognition technologies [2]. These technologies are combined into a system that can address some current issues in geometry education and provide students with an easier way for studying geometry.**

Keywords*— *Augmented Reality; Hand Gesture based Interaction; 3D Objects Manipulation; Geometric Construction; Geometry Education; Computer-based Learning

I. INTRODUCTION

For decades, geometry has been regarded as one of the most difficult area in mathematics. It requires a lot of skills such as spatial thinking, visualization and imagination to deal with three-dimensional (3D) shapes. Nowadays, students still have many problems and troubles in learning geometry. Some of them admit that they are not confident enough about geometry knowledge. In many schools, teachers attempt to provide the best way to teach Geometry. They usually change the curriculums, republish the textbooks, upgrade the materials or tools and believe that it is easy for students to understand and retain the knowledge deeply [3]. However, these traditional teaching methods cannot adapt students to study geometry and cannot assist them to improve their knowledge of geometry.

Presently, Augmented Reality (AR) is getting more common and more popular in schools and classrooms. AR can enhance creative power and logical analysis of pupils and students. Furthermore, hand gestures provide students with an intuitive and natural way to interact with 3D shapes. The students can directly modify or control 3D structures of geometric shapes. This research primarily focuses on a goal to make an AR system and hand gesture based interaction that assists students to construct 3D shapes and interact with geometric shapes in 3D space.

II. RELATED WORK

A. AR and VR Applications for Education

Many researchers on AR and Virtual Reality (VR) have been done for more than a decade. In the past few years, due to the advancement of the technology, it becomes possible to build real applications for education.

The early works demonstrated that VR can help students to improve their own mathematical techniques, learning experiences and easy to remember that is the VRMath system [4]. The design of the Human Computer Interaction (HCI) components in VRMath brings out the main advantages for students which connect mathematical meanings of multiples resources. However, it requires students have to write some code to create 3D objects by themselves to facilitate learning of 3D geometry concepts. VRMath may expose new concepts for learning geometry; nevertheless, programming to generate the virtual environment and virtual objects is not a convenient way for students of all ages during dealing with geometry.

Besides, Construct3D [5] is a 3D dynamic geometry construction tool that can be used in high school and university education. It uses AR to provide the natural settings face-to-face collaboration of teachers and students. The main advantage is that students are able to see 3D objects which are constructed by some traditional methods. However, to work with Construct3D, students need to use a two-handed 3D interaction tool composed of instrumented hand-help props – a pen and a pad equipped with position and orientation trackers. By some ways, these additional devices may limit the comfort and the convenience of the students during studying.

B. Software for Geometry Education

To support teaching and learning geometry, there are some new technologies and applications which are brought into classrooms nowadays. In typical, Cabri3D [6] is an interactive geometry software in which a 3D environment containing objects such as points, lines, planes and polyhedral is represented on a 2D screen. The application allows students to animate geometrical object may easily be demonstrated how to get started to create geometric shapes and do some mathematics such as calculate the distance between two points or two objects. However, the students really need more operations and functions to understand concept and theory of geometry. In the meanwhile, Cabri3D provides too few of simple functions which are not enough for students to learn and explore new advanced features.

Another application, named GeoGebra [7] which is an interactive geometry, algebra, statistic and calculus application, intended for learning and teaching mathematics and science from primary school to university level. The main advantage is allowing for graphing of functions and manipulating the functions in all sorts of interesting ways. Students can add lines, circles, ellipses and all other sorts of geometric functions to the document. However, GeoGebra also has drawbacks which need to be improved. For instance, the students admit that they feel very hard to move and scale the objects in a 3D space. Also, they usually got confused when using some functions such as moving, scaling and rotating.

C. *3D Modeling Software for Geometry Education*

Some of 3D modeling software are also equipped in many schools recently to provide a modern way for teaching and learning geometry. Among them, the use of commercial 3D computer-aided design (CAD) [8] or SketchUp (formerly Google SketchUp) [9] are wide spread in most of schools and considered to be popular application for teaching principles of 3D modeling. However, it is important to notice that while geometry education software shares many aspects with convenience and efficiency at a first glance, its aims and goals are fundamentally different. For instance, the Google SketchUp software includes variety of complex features and too much functionality. In contrast, students are interested in simple construction tools that expose the underlying process in a comprehensive way. This may help them to easily remember or recall the knowledge during studying.

Additionally, the stability and reliability are very important factors for educational software in general. In other words, when the software crashes or is unable to bring out the correct results, not only the learning will become no results but also students may lose their motivation.

D. *Hand Gestures*

Gesture recognition is a mathematical interpretation of a human motion by a computing device. Gesture recognition, along with facial recognition, voice recognition, eye tracking by commonly originate from faces or hands. Hand gestures are popular in many kinds of applications, such as education, entertainment, games, computer controls and so on.

Cabral, Morimoto and Zuffo [10] already discussed a study of hand gesture recognition. Several usability issues related to uses of hand gestures in multi-modal interfaces and virtual environment were addressed. Several usability issues related to the use of hand gestures in multi-modal interfaces and virtual environment were addressed. Despite some remaining drawbacks, these researchers have found that the use of hand gestures offers several advantages, such as quick access to computing resources, a natural and intuitive way that scales nicely to group and collaborative applications. Hand gestures can be used sporadically. In another recent study, hand gestures based user interfaces are also proved to enable the efficiency and flexibility for user to manipulate virtual objects by moving, rotating, scaling and selecting in 3D virtual environment [11]. These researchers also claimed that their own proposed hand gestures based interface could potentially be employed in place of a mouse or keyboard to interact with objects in a 3D environment.

III. AR FOR GEOMETRY

A. *Approach*

The objective of our research is to develop an AR and hand gesture based application for learning 3D geometry. This paper aims to present a novel approach for effective learning 3D geometry in school. In other words, it attempts to provide students with an easy and a convenient way to be able to learn 3D geometry.

The research basically focused on two main technologies: AR and hand gesture recognition to build up a hands-on learning method for students. With AR, the students can understand the basic concepts of 3D geometrical shapes, their relationships and ways to construct the 3D shapes and the objects in 3D space. Importantly, AR can provide a dynamic visualization of 3D structures of geometrical shapes. This feature helps the students to understand a comprehensive background of 3D geometrical shapes and improve the abilities of geometrical structures. Moreover, the hand gesture based interactions furnish an intuitive and convenient way for the students to directly control and interact with geometrical shapes in 3D space. With the experiences of interacting with the 3D shapes using their own hand gestures, the students can improve their own awareness of the relationships of the 3D shapes and easily remember or retain the knowledge about the 3D shapes.

B. *System Architecture*

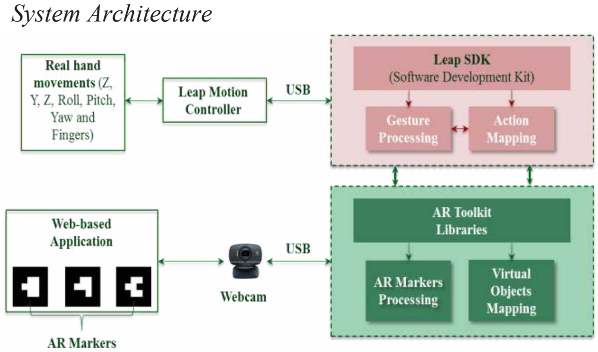


Fig. 1. An architectural overview of the proposed system.

The proposed system is designed to enable students to construct and manipulate 3D objects by using their hands gestures. With several AR markers, the students can visualize both 2D and 3D geometrical shapes at the same time, then directly modify and control them by hand gestures in the 3D AR environment. At this point, a webcam takes responsibility to detect and recognize the AR markers while the ARToolkit libraries overlays and displays exactly the 3D virtual objects on those physical AR markers in the AR environment. Fig.1

shows the architectural overview of the proposed system. Whenever the students try to interact with the 3D geometrical objects, the proposed system recognizes their hand gestures using the Leap Motion controller and maps them into appropriate commands of the system. Later, the application decides corresponding actions; invokes correct functions and then delivers geometrical information to the students.

C. Recognition of Hand Gestures

Fundamentally, the Leap Motion device is used to recognize the hand gestures of the students as the input for the AR based application. The Leap Motion device is connected to computer via a USB 3.0 port. In the computer side, the Leap Software Development Kit (SDK) includes a set of functions such as `Connector` and `Gestures Dispatcher`, which can recognize hand gestures and processes them. Whenever the students perform some hand gestures, the device's `Connector` processes these gestures and then dispatches messages through `Gestures Dispatcher` to core functions (e.g., `Gesture Listener`) of the application to map the suitable actions.

TABLE I. THE SYSTEM FUNCTIONALITY FOR CONTROLLING OBJECTS

Functions/Features	Hand Gestures Descriptions
Rotation	User performs this actions by using two fingers – index and middle fingers to rotate the objects in 6 DoF (six degrees of freedom)
Zooming	User performs this action by using both hands and scale the objects in three axes of object's coordinates – x, y and z.
Moving	User performs this action by using the index finger to pick up 3D object and move it around.

The `Action Mapping` function is an interface that consists of a set of functions for controlling geometrical objects: `RotateObject`, `ScaleObject`, `ScaleVertices` and `PanObject`. Whenever these functions are invoked, the application updates the positions of student's hands (or fingers) and the changes of the objects (e.g., position, orientation and coordinates) for each frame rate of the Leap Motion device. These control functions are defined by calculating the physical distance between the appropriate AR markers. If the distance `d` is in the range of `[30…70]`, then each function is called based on its properties. Table I describes all of functions which are used for controlling the objects in our application.

To control the object, students can use both hands, up to ten fingers to interact with virtual 3D objects. For this purpose, our proposed system specifies several hands gestures to manipulate geometrical objects and ensure outcomes of the application. Table II presents some basic and explicit functions and their corresponding hand gestures provided by proposed system.

D. Recognition of the AR Markers

The recognition process is started whenever the application is successfully loaded. At that time, the core libraries of ARToolkit such as `arScene`, `arController` and `arCamera` are configured and loaded for the marker recognition.

The `arController` function sets up some parameters about video orientation and barcode marker recognition (with markerUID and markerWidth). The `arScene` function includes all of parameters which are used to set up the scene for virtual object visualization, for example, renderer, camera, spotlight and objects. Moreover, the camera is configured using a physical webcam we use in our application. After calibrating the camera, the ARToolkit calibration function generates a `.dat` file as the camera configuration which is located in the application directory and will be loaded for marker tracking and recognition.

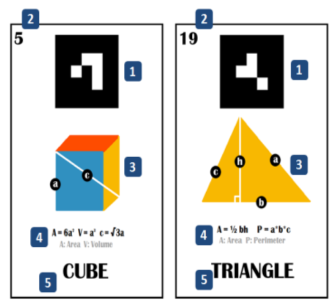


Fig. 2. Physical markers for both 2D and 3D geometrical shapes. Each marker includes: (1) marker ID, (2) marker barcode, (3) geometric shape, (4) shape formulas and (5) shape name.

After successfully loading, the process of drawing 2D and 3D virtual objects is rendered whenever the webcam captures the markers. With the ARToolkit libraries, multiple markers can be recognized and visualized at the same time. Each marker presents one kind of a geometrical shape. Fig. 2 shows examples of the markers which are used in our application. Using the ARToolkit library, physical positon, size and orientation of each visible marker can be detected and recognized. The Virtual Object Mapping function of the ARToolkit library takes responsibility to display overlaid virtual objects onto these markers. Inside these core functions, there are some callbacks which are registered to supply an

TABLE II. THE SYSTEM FUNCTIONALITY FOR CONTROLLING OBJECTS

Functions	Functions Descriptions
function **updateFrame**(frame, object, _camera, scaleX, scaleY, scaleZ, rotateY, rotateZ, flag, tick){}	Update an object in each frame rate of Leap Motion controller. The function also tracks the position of virtual _camera for moving the object. There are some parameters such as scaleX, scaleY, scaleZ, rotateY, rotateZ for updating the scale and rotation ratios. If a flag is enabled, then a user can control the object directly. If a tick is enabled, then a user can rotate an extrusion model only.
function **rotateObject**(frame, object){}	Rotate an object in each frame rate. This allows students to rotate any object in 6 DoF (degrees of freedom).
function **scaleObject**(frame, object){}	Scale an object in each frame rate. This allows students to scale any object to zoom-in/out along all of three axes (x, y and z).
function **scaleVertices**(frame, object){}	Scale vertices of an object in each frame rate. This is applied for a circle only.
function **panObject**(frame, object, camera){}	Move an object in each frame rate. The virtual camera position is tracked to display the object with a new position which is corresponded with the camera position. This is applied to a 3D geometrical object only.

application specific listener to invoked corresponded functions whenever users control the virtual objects in client-side. In the client-side, students (who want to learn geometry) can visualize both 2D and 3D structures of geometrical shapes. They can see that the virtual objects are correctly placed on the physical markers through the webcam. Each marker presents different shapes. Then, the students can manipulate those virtual objects by hand gestures and see the changes of the objects immediately.

E. Functionality

Currently, our proposed system offers several functions for constructing two-dimensional geometrical primitives and three-dimensional geometrical shapes.

1) Construction of 3D geometrical shapes from a set of 2D shapes:

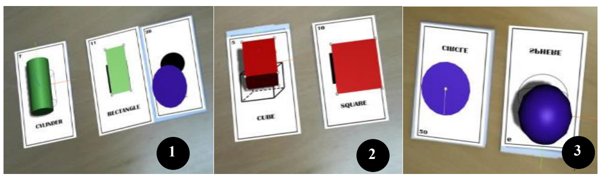


Fig. 3. The 3D geometrical shapes can be constructed from a single 2D shape and/or from a set of 2D shapes. (1) a cube is created from a square; (2) a sphere is constructed using a circle; (3) a cylinder is created by the combination of a circle and a rectangle.

The students can construct 3D geometrical shapes based on a 2D shape or a set of 2D shapes. Basically, the students only need to provide some kinds of 2D shapes, and then the system will automatically generate corresponding 3D shapes from them. Fig. 3 shows an example of this function where a cube, a circle and cylinder are constructed by transforming 2D shapes such as a square, a circle and a cylinder, respectively.

2) Constructive Solid Geometry: Union, Subtraction and Intersection:

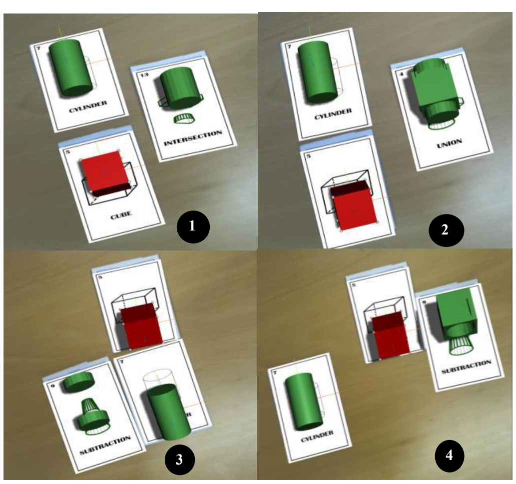


Fig. 4. Construct Solid Geometry (CSG). (1) union of a cube and a cylinder; (2) intersection of the cube and the cylinder; (3) the cylinder is subtracted from the cube; (4) the cube is subtracted from the cylinder.

Constructive solid geometry [12] allows a modeler to create a complex surface or object by using CSG operators to combine objects. In our system, we use the simplest solid objects which are called primitive, such as a cube, a cylinder, a sphere and a cone. One object is constructed from primitives by means of the CSG operations: "union", "intersection" and "subtraction". These primitives can be combined into compound objects using such CSG operations like Fig. 4.

3) Transformation of 2D shapes to make another type of 3D shapes by Extrusion:

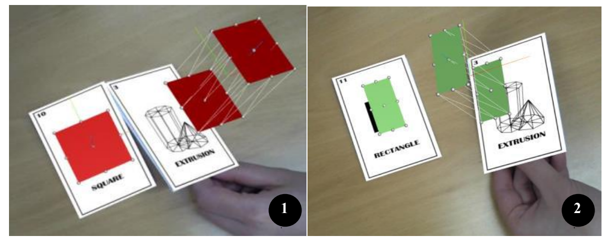


Fig. 5. Extrusion of a square (1) and a rectangle (2). The vertices of 2D shapes are illustrates by the number of spheres. To realize the extrusion operation, every vertex of the 2D shape is extruded from the top to the bottom. The lines are drawn to show the direction of the extrusion operation.

The "extrusion" technique or "extrusion" modeling is a method to create 3D shapes from 2D shapes. To make it easy for the students to understand the extrusion modeling, with every 2D shapes (except a circle), we firstly generate the top and the bottom which has the properties of the 2D shapes, such as size, coordinates, geometry and materials. Then, we extrude every vertex of the top to the bottom and draw the lines between them. Fig. 5 illustrates how a "simple" extrusion operation of 2D shapes works.

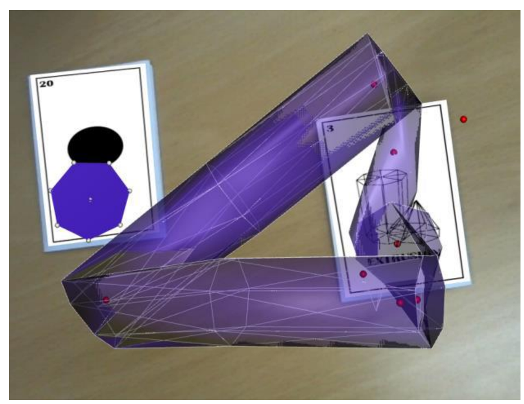


Fig. 6. Applying the extrusion modeling using a circle to make a tube-based object with seven different directions. The path of the extrusion is illustrated by the direction of red spheres.

With a circle, a more "complex" way to perform the extrusion transformation exists. By moving hands of the students up and down, they can transform a circle into another shape by changing the number of its vertex. Moreover, with this technique, the students can create a complex shape such as a tube. Whenever they change the direction of the extrusion, a "joint" of the tube can be created. Fig. 6 demonstrates how the students can create a tube-based "complex" object using the extrusion technique.

4) Interaction and Manipulation:

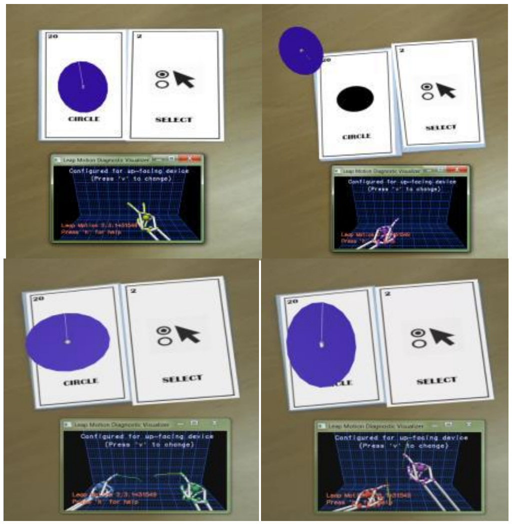


Fig. 7. Basic interaction methods using hand gestures with geometrical shapes.

Three major interaction methods of geometrical transformations are specified: rotation, scaling and panning, which can be performed by using both hands and fingers of the students. The students can use two hands for scaling, the index and middle fingers to rotate the objects and use the index finger for panning or move the object around. Fig. 7 demonstrates sample interactions using hand gestures which consist of rotation, moving, horizontally and vertically scaling. The gestures are specified as simple as possible for the students to remember and perform easily and conveniently.

F. Scenarios

Three scenarios are designed to explain clearly how the proposed system can be used and in which situation our application can be preferred.

- **Scenario 1 – Properties and relationships of geometric shapes.** In this scenario, using some predefined hand gestures, the students can interact with 2D and 3D geometrical shapes. Then, the students can easily understand the properties of the 2D and 3D geometrical shapes and their relationships. Moreover, the students can directly explore by themselves for better awareness of the basic knowledge of geometry.
- **Scenario 2 – Geometric shapes construction using CSG operators.** This scenario allows the students to construct some combination of geometrical shapes with the three primary CSG operators: union, subtraction and intersection. This enables the students to visualize the complex structures of geometrical shapes in an intuitive and natural way.
- **Scenario 3 – Transforming Geometric shapes to a different type by the extrusion technique.** This technique supports another way to construct 3D geometrical shapes from 2D geometrical shapes. Basically, by providing one 2D geometrical shape and the path, the students directly create the 3D shape based on the type of the original shape along that path.

IV. IMPLEMENTATION

There are two main phrases of implementation of the system.

A. Shape Construction

This phrase aims to construct some core functionality for constructing 3D geometrical shapes through combing 2D geometrical shapes, the CSG operations such as union, subtraction and intersection, and transforming 2D shapes through the extrusion operation.

B. Shape Manipulation/ Interaction

This phrase is to implement three basic interaction methods such as rotation, scaling and panning (or moving).

To correctly implement the proposed system, we need to set up some basic properties such as the camera calibration, coordinates transformation between camera, the ARToolkit markers and the Leap Motion controller.

V. EXPERIMENT

To perform a comparative study for the proposed AR application, the following applications of learning geometry were selected:

- Cabri3D: a 3D geometry application which runs on a desktop computer. It is executed through interactions using a mouse and a keyboard. Its output is a conventional 2D monitor.
- GeoGebra: a dynamic mathematic application for learning and teaching geometry. It runs on an iPad tablet. It is based on touch screen based gestures.

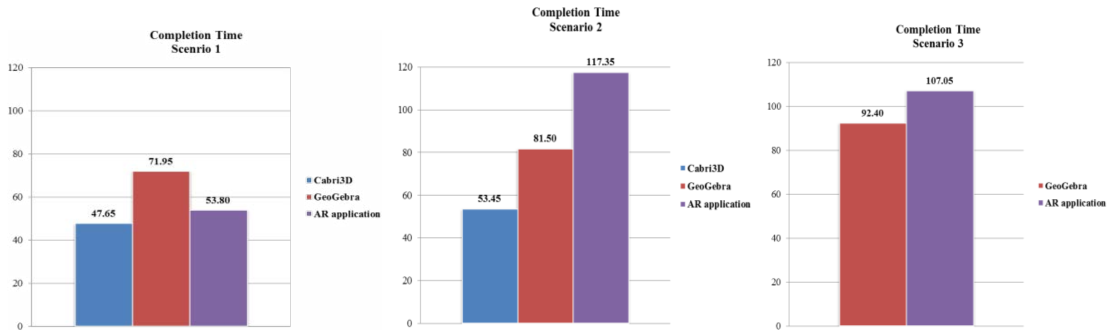
The proposed AR application is executed through hand gestures which are recognized by the Leap Motion controller. A webcam is used to recognize the AR markers. Its output device is a 2D monitor.

A. Subjects

For the user evaluation, 27 subjects were invited. They include high school students and university students. All subjects were instructed and trained how to use the applications. The subjects were allowed to take a break at any time during the experiment trails. The total experiment time per subject including questionnaires, instructions, training, experiments, breaks and debriefing was about one hour.

B. Methods

The subjects were instructed to perform tasks based on the scenarios described in Section IV. All the three scenarios were independently executed. After each experiment, the subjects were asked to fill out the questionnaires to check the user



(1) (2) (3)

Fig. 8. The task completion times: (1) Scenario 1 (Relationship among geometrical shapes) between three applications. (2) Scenario 2 (Geometrical shapes construction using the CSG operators). (3) Scenarion 3 (Transforming geometrical shapes using the extrusion technique). The subjects were asked to perform tasks of the three scanarios using the three applications: Cabri3D using a mouse and a keyboard, GeoGebra using a touch screen based tablet, and the proposed AR application using hand gestures and the AR markers. The third chart does not show the result of using Cabri3D for Scenario 3 because Cabri3D does not currently support the extrusion function.

satisfaction of the applications. There are four main factors of the questionnaires of the proposed AR based education application – "Intuitiveness", "Ease of Use", "Ability of Understand" and "Dynamic Modification and Interaction". These factors focus on user experiences of the proposed system, the ability to understand the geometrical knowledge, and the capability to interact with the 3D geometrical shapes. Table III describes the requirements and their associated of the questionnaires (which are used during the user evaluation) in detail.

VI. Results

A. Task Completion Time

Fig. 8-(1) illustrates the average completion time of three geometry learning applications to complete tasks of Scenario 1, which makes the subjects understand 2D and 3D geometrical shapes and the relationships of the shapes. It can be clearly seen that the AR application required 53.8 seconds to finish the task, while GeoGebra required 71.95 seconds. In contrast, Cabri3D required 47.67 seconds because this application is quite simple. Also, the subjects are familiar with the mouse-and-keyboard based interaction. Notice that the touch screen based interaction (GeoGebra) showed the worst performance.

In Fig. 8-(2), the average completion times in all of three geometry learning applications for Scenario 2, where the subjects perform operations of union, subtraction and intersection, are presented. The subjects with Cabri3D spent 53 seconds and the subjects with GeoGebra spent 80 seconds to complete the task. However, the proposed AR application required longer than a minute (117.35 seconds) for doing the task. Why did the AR application take longer time than those of other applications? It is because the type of the interaction (i.e., hand gestures) for the CSG operations was totally new to them. The subjects reported that performing the CSG operations in the proposed AR application using their hand gestures was quite new and made them more engaged into the task. Moreover, the virtual 3D geometric objects in the AR application looked more realistic to interact with. The more engaged students were, the longer time they took to finish the task.

Similarly, Fig. 8-(3) for Scenario 3 shows the average completion time of GeoGebra and the AR application. All subjects were asked to perform the extrusion operation. The experiment was not conducted on Cabri3D since this application does not support the extrusion function currently. The results show clearly that GeoGebra required 92.45 seconds, while the AR application required 107.05 seconds to achieve the task, as illustrated in Fig. 8-(3). For some subjects, the extrusion modeling was an interesting and useful technique to deeply understand geometry and how the 2D shapes transform into a three-dimensional model. The GeoGebra application could make them understand the basic knowledge and show the step-by-step for subjects to complete the task. While, the proposed AR application not only showed the details of the extrusion modeling, but also allowed the subjects to use hand gestures to control the objects in a natural way of the extrusion. This makes the subjects easier to remember and retain the knowledge of geometry.

B. Subjective Satisfaction

1) Intuitiveness: Fig. 9 shows the results of the user evaluation on subjective satisfaction about the “Intuitiveness” requirement of the proposed application. The 56% of the subjects responded that it was “Easy for them to visualize and imagine” the 3D structures of geometrical shapes. The 26% of them said that it was “O.K”. The 11% of them responded that the proposed application was “Very Easy” and had enough information for them to learn geometry. Moreover, more than a half of the subjects (67%) answered that the application was “Easy” and the 30% of the subjects felt “Very Easy” to explore

Table III. The main factors as the application’s objectives which are used to define the questionnaire

Factors	Questions
Intuitiveness	1. Could you visualize and imagine the 3D structures of geometrical shape? Does it have enough information for you or not? 2. Is it easy for you to discover new features and understand the knowledge of 3D shapes and their operations?
Ease of use	1. How do you feel when you are getting started? 2. How much easy for you to remember all kinds of hand gestures and use them?
Ability to understand	1. How much easy for you to understand the concept of geometrical shapes? 2. By using our application, learning advanced features about 3D geometrical shapes is difficult or easy for you?
Dynamic modification and interaction	1. Is it easy to perform the actions and interact to 3D geometrical shapes? 2. Is it good enough to simulate the 3D geometrical shapes during interaction?

new things and understand the knowledge of geometry.

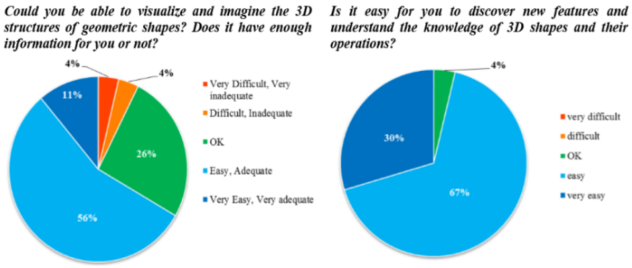


Fig. 9. The reactions to the "Intuitiveness" requirement of the proposed AR application.

2) Ease of Use: The reactions to the "Ease of Use" requirement are shown in Fig. 10. Many subjects admitted that they felt "Easy" (41%) and "Very Easy" (26%) to get started with the proposed AR based education. Moreover, the 59% of them responded "Easy", the 22% said "O.K" and the 19% reacted "Easy" to remember the hand gestures and use them to control 3D structures of geometrical shapes. This means the proposed application is very easy to get started and the hand gestures are as simple as possible for the subjects to interact with the 3D geometrical shapes and learn geometry about the 3D shapes.

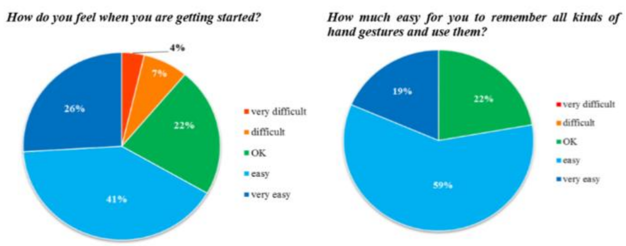


Fig. 10. The reactions to the "Ease of Use" requirement.

3) Ability to Understand: Fig. 11 shows the "Understand" requirement. The 52% of the subjects responded that it was "Easy" to understand the concept of geometrical shapes using the AR based application. The almost 20% of them felt "O.K" to understand the application. The 52% of the subjects said "Easy" for them to learn and the 26% of them reported "Very Easy" to learn the features.

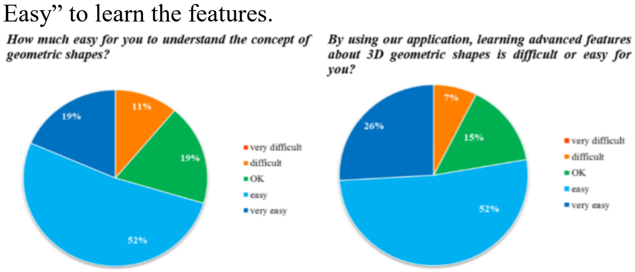


Fig. 11. The reactions to the "Understand" requirement.

4) Dynamic Modification and Interaction: Fig. 12 shows the reactions from the subects about the "Dynamic Modification and Interaction" requirement. The 70% of the subjects performed the hand gestures and interacted with the 3D geometrical shapes. Moreover, the 22% of them reported that it was "Very Easy" for them to control the 3D shapes. As for the simulation, the 56% of the subjects said that it was "Good" and the 26% of them reported that it looked "O.K" to control the 3D structure of the geometrical shapes.

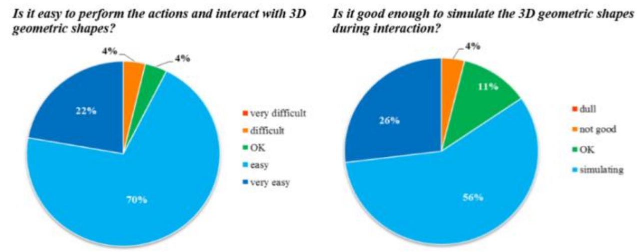


Fig. 12. The reactions to the "Dynamic Manipulation and Interaction" requirement.

VII. Conclusions

This paper presents design, implementation and evaluation of a geometry learning application based on AR and hand gesture based interactions.

It was suggested that hand gestures could provide with an easy and convenient way for students to learn 3D geometry. Considering the results from the user studies, it could be suggested that the proposed application can be used to study 3D geometry in easier, more convenient and more efficient manners than traditional ways of using pens and paper based on imagination. On the other hand, it was also noted that the hand gesture based interaction for manipulating 3D objects should be enhanced in terms of the performance in completing tasks. Since most users are not familiar with the hand gestures of 3D geometrical objects in an AR environment, their usability issues should be researched and resolved.

The proposed application can be extended to provide students with more functionality to learn 3D geometry. For instance, the application could offer a collaborative method of the study so that students would be able to cooperate with other students and teachers during learning and teaching in a classroom or at home. They can remotely collaborate for their learning and teaching.

We could also expand the functionalities of the application for a multi-device environment where students and teachers can study and teach using multiple devices such as mobile phones, tablets or desktop computers. Most of the devices can execute AR applications. The various types of devices and their ubiquitous usages could motivate students to study 3D geometry wherever they are.

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