Home work 1

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2.4 Excercises

1. For each of parts (a) through (d), indicate whether we would generally expect the performance of a flexible statistical learning method to be better or worse than an inflexible method. Justify your answer.

a) The sample size n is extremely large, and the number of predictors p is small.

Better! The flexible model will fit the data without overfiting, due to errors being minimized on the large number of observations.

b) The number of predictors p is extremely large, and the number of observations n is small.

Worse! A fewer data points will cause the overfit.

c) The relationship between the predictors and response is highly non-linear.

Better! The flexible model better expresses non-linearity in the data.

d) The variance of the error terms, i.e. σ2=Var(ϵ), is extremely high.

Worse! The flexible model will follow the noise/errors more precisely giving higher errors on different dataset.

2. Explain whether each scenario is a classification or regression problem, and indicate whether we are most interested in inference or prediction. Finally, provide n and p.

a) We collect a set of data on the top 500 firms in the US. For each firm we record profit, number of employees, industry and the CEO salary. We are interested in understanding which factors affect CEO salary.

- regression: the funtion of *Salary* from *qianitative Profit*, *quanttative Number of Employees*, and *categorical Industry* variables
- inference: the relationship between predictors in found parameters of fit
- *n*: 500
- *p*: 3

b) We are considering lauching a new product and wish to know whether it will be a success or a failure. We collect data on 20 similar products that were previously launched. For each product we have recorded whether it was a success or failure, price charged for the product, marketing budget, competition price, and the other variables.

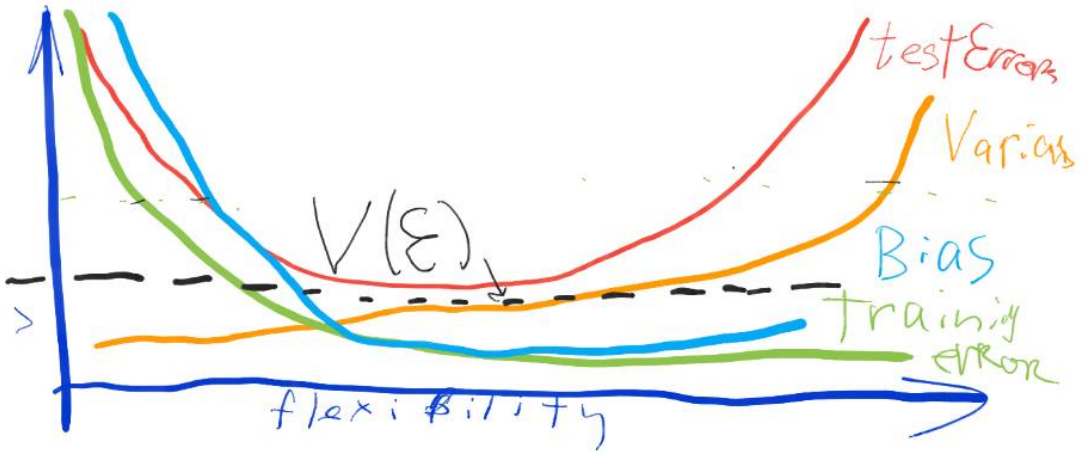
- classification: find whether the product succeed or fail given the *price*, *marketing budget*, *etc* ...
- prediction: measuring the probability of either for given values of the predictors
- *n*: 20
- *p*: 13

c) We are interesting in predicting the % change in the US dollar in relation to the weekly changes in the world stock markets. Hence we collect weekly data for all of 2012. For each week we record the % change in the dollar, the % change in the US market, the % change in the British Market, and the % change in the German market.

- regression: finding smooth function
- prediction: use the function to get outcome given the space of parameter values
- *n*: 52
- *p*: 5

3. We now revisit the bias-variance decomposition.

a) Provide a sketch of typical (squared) bias, variance, training error, test error, and Bayes (or irreducible) error curves, on a single plot, as we go from less flexible statistical learning methods towards more flexible approaches. The x-axis should represent the amount of flexibility in the method, and the y-axis should represent the values for each curve. There should be five curves. Make sure to label each one.



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b) Explain why each of the five curves has the shape displayed in part (a).

- *training error* declines as flexibility increases: the selected *f* curve able to follow the data closely
- *test error* intially declines when lexibility increases, it stops to fall and passing the valley starts to increase again cause the overfited *f* curve gives more erros on test data
- *irreducible error* is a constant hence the line is passing below the *test erros* curve: the expected *test erros* will be greater than *Var(ε)*
- *bias* decreases as the levels of flexibility creates more complex function that more precisely approximate the system but stopped being impacted at some point
- *variance* increases slowly with higher levels of flexibility, then increases rapidly causing *test errors* to rise

4. You will now think of some real-life applications for statistical learning.

a) Describe three real-life applications in which classification might be useful. Describe the response, as well as the predictors. Is the goal of each application inference or prediction? Explain your answer.

- recommend the mortgage products according to the potential customer atributes: age, work status, income, assets, loan amount, etc: inference.
- estimate the default for the debt products based on age, income, occupation, industry of employment, family status, life events: prediction.
- make decision to stop the production process and tune the mill based of the variation of sizes of the produced part: inference - classify the situation as the need for the adjustments.

b) Describe three real-life applications in which regression might be useful. Describe the response, as well as the predictors. Is the goal of each application inference or prediction ? Explain your answer.

- provide decompression time given certain gas mixure needed for SCUBA diver to avoid bends as function of time spent at certain deps, using particular breathing mixture with several fixed percentages of *Oxigen* vs other gasses: *N* and *He*. It's inferense: the model is set in dive computer
- Perdict sesonal flow of the water streams based on the data collected each day, over years, on levels of rain, temperature, depth of snow, air and water temperatures.
- feed optimal charding current for the batteries based of model, describing optimal charging rate for batery chemistry type, the adsorbed capacity, actual voltage, time. It is inference: different battery chemistry uses different models of charging.

c) Describe three real-life applications in which cluster analysis might be useful.

- find what factors contibutes to certain disease: age, gender, food, sleap time, execise time, recreational drugs: alcohol, coffe, etc. Perdiction: find what illness are common for the group with similar habits.
- recommend the music based on the lisened and liked songs, discover the concentration of songs around collaborationg artists for the songs one likes.
- find the list of topics for the news letter to maximise the click rates based on similarity of interests of the targets.

5. What are the advantages and disadvantages of a very flexible (versus a less flexible) approach for regression or classification? Under what circumstances might a more flexible approach be preferred to a less flexible approach? When might a less flexible approach be preferred?

Advantages: Produces less bias representing complex and non-linear systems.
Disadvantages: May overfits training data, have higher variance.

More flexible approachs have to be used to get the reasonable fit, espesially when data contains substantial non-linearity. The less flexible aproaches shall be used when the dataset has fewer observations, for more interpretability, for limiting the solution space to the linear system for computational power reasons.

6. Describe the differences between a parametric and a non-parametric statistical learning approach. What are the advantages of a parametric approach to regression or classification (as opposed to a non-parametric approach)? What are its disavantages?

The Parametric approach makes assumptions about the form the function: linear or polinomial, more or less flexible. The choice will be choice of minimun errors of a fitted function. This model based approach is called parametric approach. Linear/Logistical Regressions are among these.

The non-parametric approach does not make assumptions of shape of functions. Instead we estimate the function which fits closely to the data. SVM and various spline method use non-parametric approach.

Advantages:

- Parametric approach can fit any kind of data choosing wide range of functions
- can fit something with lower size of sample compared to Non-Parametric, where the bigger sample size is needed to achive comparable performance.

Disadvantages:

- Parametric approach is prone to overfiting when too flexible model is selected during training, that leads to large test errors
- The errors will be higher for less flexible models with fewer parameters.

7. The table below provides a training data set containing six observations, three predictors, and one qualitative response variable.

Suppose we wish to use this data set to make a prediction for Y when X1=X2=X3=0 using K-nearest neighbors.

a) Compute the Euclidean distance between each observation and the test point, X1=X2=X3=0

The Euclidean distance between points p and q is the lenght of the line segment connecting them.

observation #	formula	distance
1	sqrt(3^2)	3
2	sqrt(2^2)	2
3	sqrt(1^2 + 3^2)	3.16
4	sqrt(1^2 + 2^2)	2.24
5	sqrt(1^2 + 1^2)	1.41
6	sqrt(1^2 + 1^2 + 1^2)	1.73

b) What is our prediction with K=1? Why?

The closest to 1 is the value for observation 5 hence it is *Green*.

c) What is our prediction with K=3? Why?

The closest to 3 are observations 1, 3 and 4: 2/3 for Red and 1/3 for Green, hence: *Red*

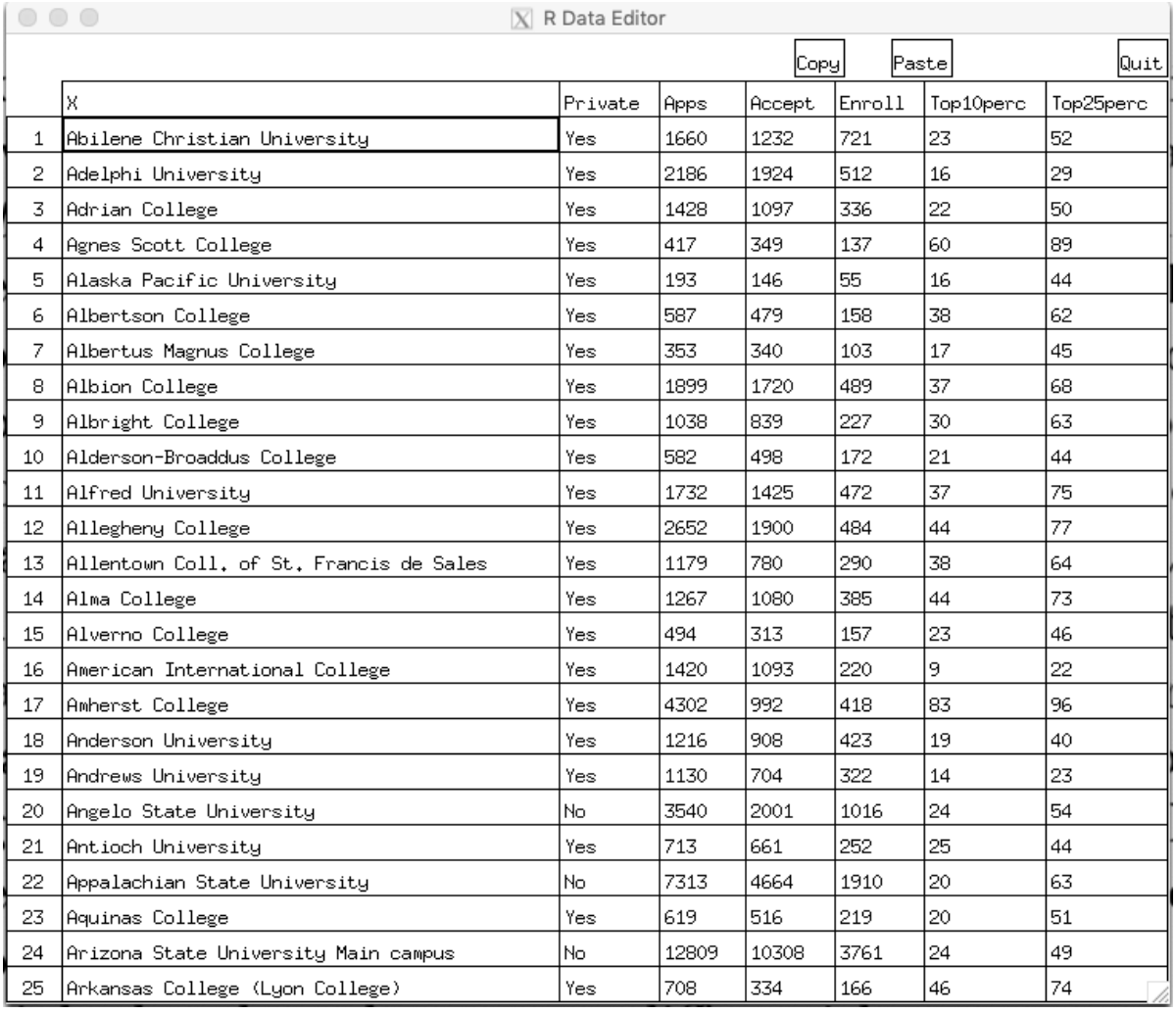
d) If the Bayes decision boundary in this problem is highly non-linear, then would we expect the best value for K to be large or small? Why?

Bigger K causes more linear, less curvy boundaries. The better value for K is small. The level of flexibility is inversely depends on the number of K.

8. This exercise relates to the “College” data set, which can be found in the file “College.csv”. It contains a number of variables for 777 different universities and colleges in the US.

a) Use the read.csv() function to read the data into R. Call the loaded data “college”. Make sure that you have the directory set to the correct location for the data.

```
> college <- read.csv("College.csv")
> fix(college)
> rownames(college)=college[,1]
> college <- college[, -1]
> head(college[, 1:5])
                           Private Apps Accept Enroll Top10perc
Abilene Christian University     Yes 1660   1232    721        23
Adelphi University               Yes 2186   1924    512        16
Adrian College                   Yes 1428   1097    336        22
Agnes Scott College              Yes  417    349    137        60
Alaska Pacific University        Yes  193    146     55        16
Albertson College                Yes  587    479    158        38
```



R Data Editor

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	X	Private	Apps	Accept	Enroll	Top10perc	Top25perc
1	Abilene Christian University	Yes	1660	1232	721	23	52
2	Adelphi University	Yes	2186	1924	512	16	29
3	Adrian College	Yes	1428	1097	336	22	50
4	Agnes Scott College	Yes	417	349	137	60	89
5	Alaska Pacific University	Yes	193	146	55	16	44
6	Albertson College	Yes	587	479	158	38	62
7	Albertus Magnus College	Yes	353	340	103	17	45
8	Albion College	Yes	1899	1720	489	37	68
9	Albright College	Yes	1038	839	227	30	63
10	Alderson-Broaddus College	Yes	582	498	172	21	44
11	Alfred University	Yes	1732	1425	472	37	75
12	Allegheny College	Yes	2652	1900	484	44	77
13	Allentown Coll. of St. Francis de Sales	Yes	1179	780	290	38	64
14	Alma College	Yes	1267	1080	385	44	73
15	Alverno College	Yes	494	313	157	23	46
16	American International College	Yes	1420	1093	220	9	22
17	Amherst College	Yes	4302	992	418	83	96
18	Anderson University	Yes	1216	908	423	19	40
19	Andrews University	Yes	1130	704	322	14	23
20	Angelo State University	No	3540	2001	1016	24	54
21	Antioch University	Yes	713	661	252	25	44
22	Appalachian State University	No	7313	4664	1910	20	63
23	Aquinas College	Yes	619	516	219	20	51
24	Arizona State University Main campus	No	12809	10308	3761	24	49
25	Arkansas College (Lyon College)	Yes	708	334	166	46	74

i. summary()

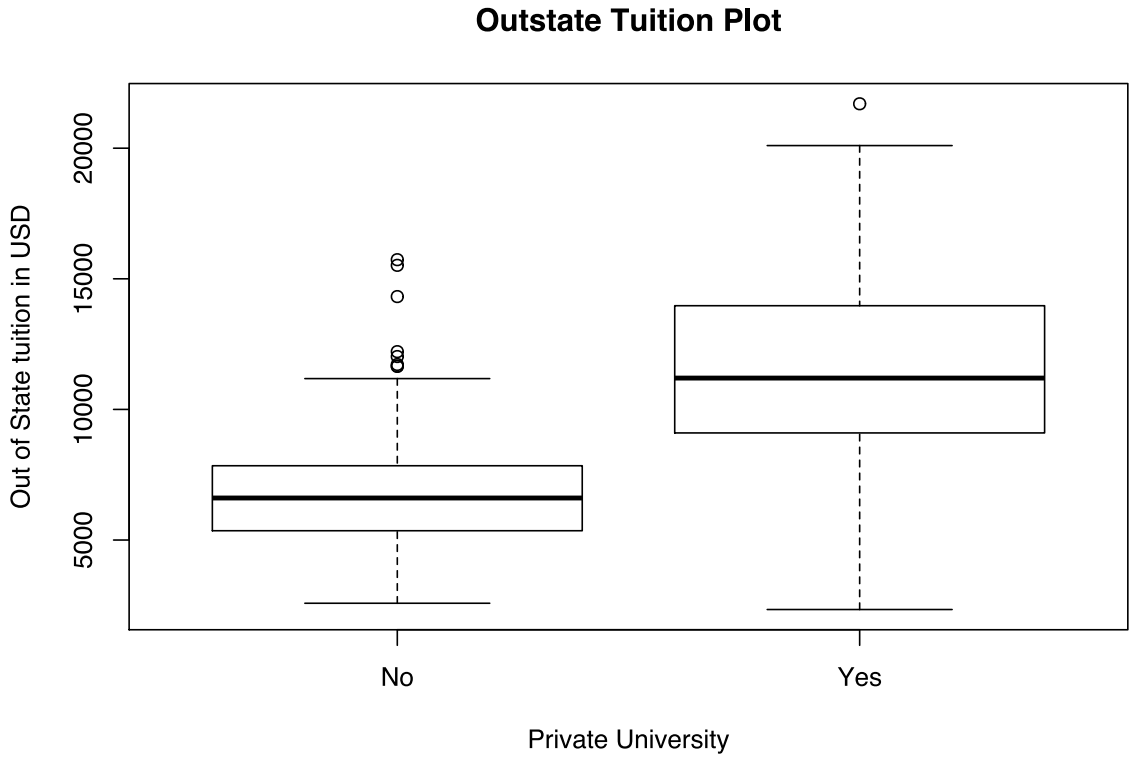
```
> summary(college)
 Private        Apps           Accept          Enroll
Top10perc       Top25perc
 No :212   Min.   :   81   Min.   :   72   Min.   :  35   Min.   :
1.00   Min.   :  9.0
 Yes:565   1st Qu.:  776   1st Qu.:  604   1st Qu.: 242   1st
```

```
Qu.:15.00   1st Qu.: 41.0
           Median : 1558   Median : 1110   Median : 434   Median
:23.00   Median : 54.0
           Mean   : 3002   Mean   : 2019   Mean   : 780   Mean
:27.56   Mean   : 55.8
           3rd Qu.: 3624   3rd Qu.: 2424   3rd Qu.: 902   3rd
Qu.:35.00   3rd Qu.: 69.0
           Max.   :48094   Max.   :26330   Max.   :6392   Max.
:96.00   Max.   :100.0
  F.Undergrad     P.Undergrad         Outstate       Room.Board
Books
 Min.   :  139   Min.   :    1.0   Min.   : 2340   Min.   :1780
Min.   :  96.0
 1st Qu.:  992   1st Qu.:   95.0   1st Qu.: 7320   1st Qu.:3597
1st Qu.: 470.0
 Median : 1707   Median :  353.0   Median : 9990   Median :4200
Median : 500.0
 Mean   : 3700   Mean   :  855.3   Mean   :10441   Mean   :4358
Mean   : 549.4
 3rd Qu.: 4005   3rd Qu.:  967.0   3rd Qu.:12925   3rd Qu.:5050
3rd Qu.: 600.0
 Max.   :31643   Max.   :21836.0   Max.   :21700   Max.   :8124
Max.   :2340.0
    Personal         PhD            Terminal       S.F.Ratio
perc.alumni
 Min.   : 250   Min.   :  8.00   Min.   : 24.0   Min.   : 2.50
Min.   : 0.00
 1st Qu.: 850   1st Qu.: 62.00   1st Qu.: 71.0   1st Qu.:11.50   1st
Qu.:13.00
 Median :1200   Median : 75.00   Median : 82.0   Median :13.60
Median :21.00
 Mean   :1341   Mean   : 72.66   Mean   : 79.7   Mean   :14.09
Mean   :22.74
 3rd Qu.:1700   3rd Qu.: 85.00   3rd Qu.: 92.0   3rd Qu.:16.50   3rd
Qu.:31.00
 Max.   :6800   Max.   :103.00   Max.   :100.0   Max.   :39.80
Max.   :64.00
     Expend        Grad.Rate
 Min.   : 3186   Min.   : 10.00
 1st Qu.: 6751   1st Qu.: 53.00
 Median : 8377   Median : 65.00
 Mean   : 9660   Mean   : 65.46
 3rd Qu.:10830   3rd Qu.: 78.00
 Max.   :56233   Max.   :118.00
```

ii. pairs(college[, 1:10])

iii. plot()

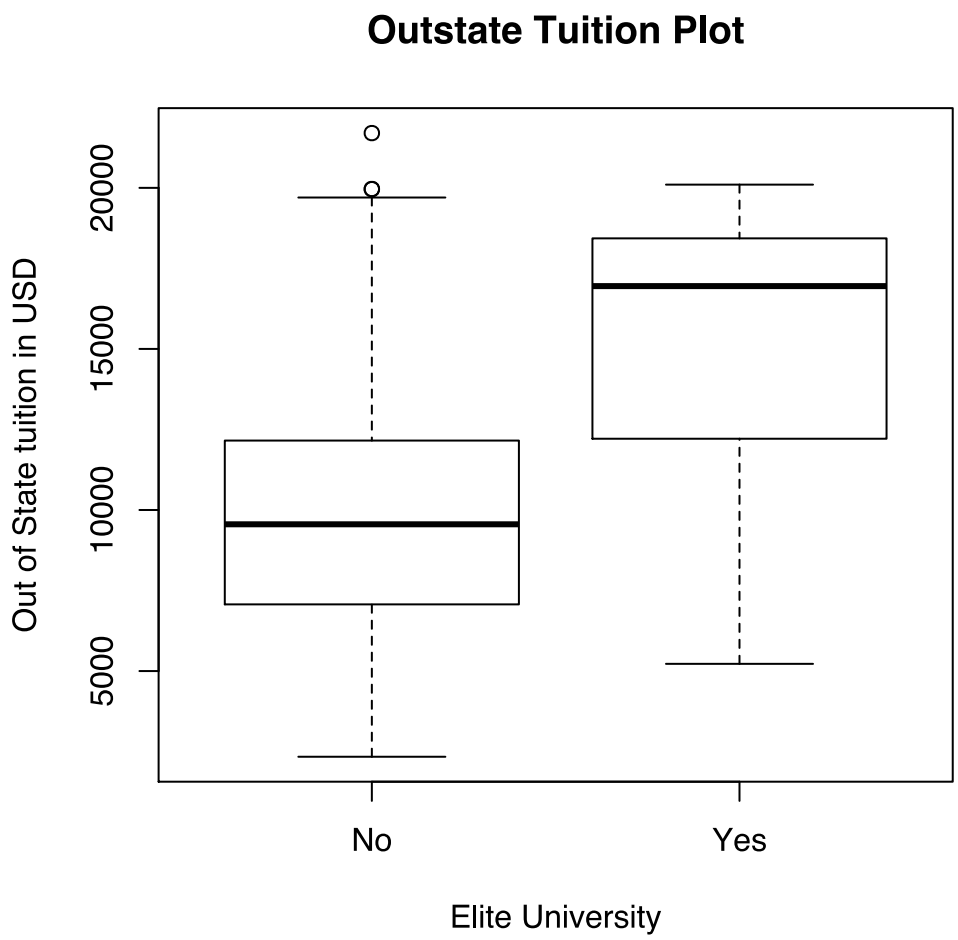
```
plot(college$Private, college$Outstate, xlab = "Private University",
ylab ="Out of State tuition in USD", main = "Outstate Tuition Plot")
```



iv. Create a new qualitative variable, called Elite, by binning the Top10perc variable. We are going to divide universities into two groups based on whether or not the proportion of students coming from the top 10% of their high school classes exceeds 50 %.

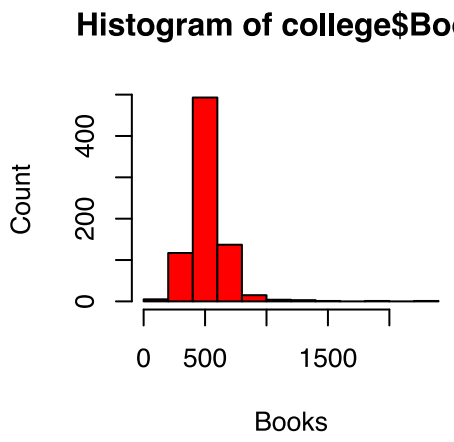
```
> # iV
> Elite=rep("No",nrow(college))
> Elite[college$Top10perc >50]="Yes"
> Elite=as.factor(Elite)
> college=data.frame(college ,Elite)
> summary(college)
 Private        Apps           Accept
 No :212   Min.   :   81   Min.   :   72
 Yes:565   1st Qu.:  776   1st Qu.:  604
           Median : 1558   Median : 1110
           Mean   : 3002   Mean   : 2019
           3rd Qu.: 3624   3rd Qu.: 2424
           Max.   :48094   Max.   :26330
    Enroll       Top10perc       Top25perc
 Min.   :  35   Min.   : 1.00   Min.   :  9.0
 1st Qu.: 242   1st Qu.:15.00   1st Qu.: 41.0
```

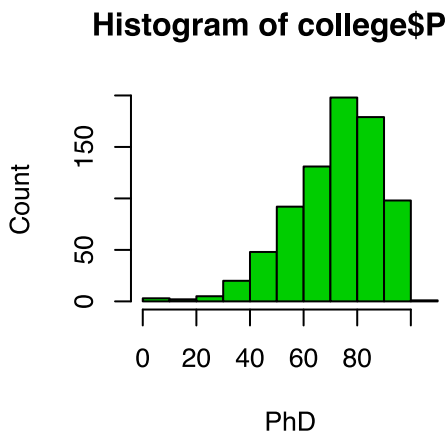
```
Median : 434   Median :23.00   Median : 54.0
Mean   : 780   Mean   :27.56   Mean   : 55.8
3rd Qu.: 902   3rd Qu.:35.00   3rd Qu.: 69.0
Max.   :6392   Max.   :96.00   Max.   :100.0
 F.Undergrad     P.Undergrad         Outstate
Min.   :  139   Min.   :    1.0   Min.   : 2340
1st Qu.:  992   1st Qu.:   95.0   1st Qu.: 7320
Median : 1707   Median :  353.0   Median : 9990
Mean   : 3700   Mean   :  855.3   Mean   :10441
3rd Qu.: 4005   3rd Qu.:  967.0   3rd Qu.:12925
Max.   :31643   Max.   :21836.0   Max.   :21700
  Room.Board       Books           Personal
Min.   :1780   Min.   :  96.0   Min.   : 250
1st Qu.:3597   1st Qu.: 470.0   1st Qu.: 850
Median :4200   Median : 500.0   Median :1200
Mean   :4358   Mean   : 549.4   Mean   :1341
3rd Qu.:5050   3rd Qu.: 600.0   3rd Qu.:1700
Max.   :8124   Max.   :2340.0   Max.   :6800
     PhD            Terminal       S.F.Ratio
Min.   :  8.00   Min.   : 24.0   Min.   : 2.50
1st Qu.: 62.00   1st Qu.: 71.0   1st Qu.:11.50
Median : 75.00   Median : 82.0   Median :13.60
Mean   : 72.66   Mean   : 79.7   Mean   :14.09
3rd Qu.: 85.00   3rd Qu.: 92.0   3rd Qu.:16.50
Max.   :103.00   Max.   :100.0   Max.   :39.80
 perc.alumni        Expend        Grad.Rate
Min.   : 0.00   Min.   : 3186   Min.   : 10.00
1st Qu.:13.00   1st Qu.: 6751   1st Qu.: 53.00
Median :21.00   Median : 8377   Median : 65.00
Mean   :22.74   Mean   : 9660   Mean   : 65.46
3rd Qu.:31.00   3rd Qu.:10830   3rd Qu.: 78.00
Max.   :64.00   Max.   :56233   Max.   :118.00
Elite
No :699
Yes: 78
```

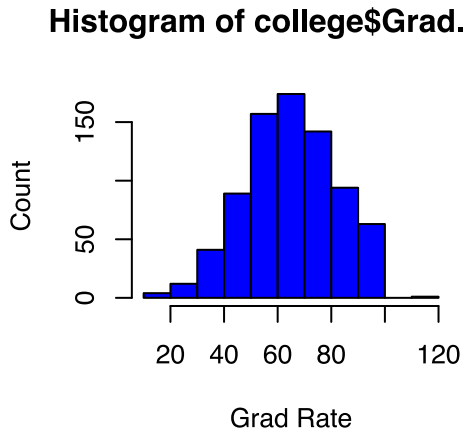


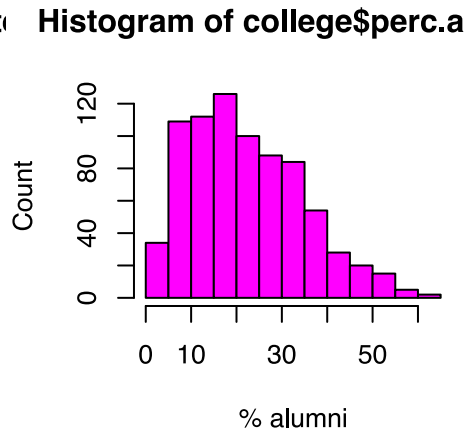
v.

```
> par(mfrow = c(2,2))
> hist(college$Books, col = 2, xlab = "Books", ylab = "Count")
> hist(college$PhD, col = 3, xlab = "PhD", ylab = "Count")
> hist(college$Grad.Rate, col = 4, xlab = "Grad Rate", ylab =
"Count")
> hist(college$perc.alumni, col = 6, xlab = "% alumni", ylab =
"Count")
>
```










```
> summary(college$perc.alumni)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
   0.00   13.00   21.00   22.74   31.00   64.00
```