

Contents

Preface

xvii

Part I	1
1 Introduction	3
1.1 The concept of a model	3
1.2 Mathematical programming models	5
2 Solving mathematical programming models	11
2.1 Algorithms and packages	11
2.1.1 Reduction	12
2.1.2 Starting solutions	12
2.1.3 Simple bounding constraints	12
2.1.4 Ranged constraints	13
2.1.5 Generalized upper bounding constraints	13
2.1.6 Sensitivity analysis	13
2.2 Practical considerations	13
2.3 Decision support and expert systems	16
2.4 Constraint programming (CP)	17
3 Building linear programming models	21
3.1 The importance of linearity	21
3.2 Defining objectives	23
3.2.1 Single objectives	24
3.2.2 Multiple and conflicting objectives	26
3.2.3 Minimax objectives	27
3.2.4 Ratio objectives	28
3.2.5 Non-existent and non-optimizable objectives	29
3.3 Defining constraints	29
3.3.1 Productive capacity constraints	29
3.3.2 Raw material availabilities	30
3.3.3 Marketing demands and limitations	30
3.3.4 Material balance (continuity) constraints	30

3.3.5	Quality stipulations	31
3.3.6	Hard and soft constraints	31
3.3.7	Chance constraints	32
3.3.8	Conflicting constraints	32
3.3.9	Redundant constraints	34
3.3.10	Simple and generalized upper bounds	35
3.3.11	Unusual constraints	35
3.4	How to build a good model	36
3.4.1	Ease of understanding the model	36
3.4.2	Ease of detecting errors in the model	37
3.4.3	Ease of computing the solution	37
3.4.4	Modal formulation	38
3.4.5	Units of measurement	40
3.5	The use of modelling languages	40
3.5.1	A more natural input format	41
3.5.2	Debugging is made easier	41
3.5.3	Modification is made easier	41
3.5.4	Repetition is automated	41
3.5.5	Special purpose generators using a high level language	41
3.5.6	Matrix block building systems	42
3.5.7	Data structuring systems	42
3.5.8	Mathematical languages	42
3.5.8.1	SETS	43
3.5.8.2	DATA	43
3.5.8.3	VARIABLES	43
3.5.8.4	OBJECTIVE	43
3.5.8.5	CONSTRAINTS	43
4	Structured linear programming models	45
4.1	Multiple plant, product and period models	45
4.2	Stochastic programmes	53
4.3	Decomposing a large model	55
4.3.1	The submodels	63
4.3.2	The restricted master model	64
5	Applications and special types of mathematical programming model	67
5.1	Typical applications	67
5.1.1	The petroleum industry	68
5.1.2	The chemical industry	68
5.1.3	Manufacturing industry	68
5.1.4	Transport and distribution	69
5.1.5	Finance	69
5.1.6	Agriculture	70

5.1.7	Health	70
5.1.8	Mining	70
5.1.9	Manpower planning	71
5.1.10	Food	71
5.1.11	Energy	71
5.1.12	Pulp and paper	72
5.1.13	Advertising	72
5.1.14	Defence	72
5.1.15	The supply chain	72
5.1.16	Other applications	73
5.2	Economic models	74
5.2.1	The static model	74
5.2.2	The dynamic model	80
5.2.3	Aggregation	81
5.3	Network models	81
5.3.1	The transportation problem	82
5.3.2	The assignment problem	87
5.3.3	The transshipment problem	88
5.3.4	The minimum cost flow problem	89
5.3.5	The shortest path problem	93
5.3.6	Maximum flow through a network	93
5.3.7	Critical path analysis	94
5.4	Converting linear programs to networks	98
6	Interpreting and using the solution of a linear programming model	103
6.1	Validating a model	103
6.1.1	Infeasible models	103
6.1.2	Unbounded models	104
6.1.3	Solvable models	105
6.2	Economic interpretations	107
6.2.1	The dual model	109
6.2.2	Shadow prices	112
6.2.3	Productive capacity constraints	114
6.2.4	Raw material availabilities	114
6.2.5	Marketing demands and limitations	114
6.2.6	Material balance (continuity) constraints	114
6.2.7	Quality stipulations	114
6.2.8	Reduced costs	116
6.3	Sensitivity analysis and the stability of a model	121
6.3.1	Right-hand side ranges	121
6.3.2	Objective ranges	125
6.3.3	Ranges on interior coefficients	128
6.3.4	Marginal rates of substitution	131
6.3.5	Building stable models	132

6.4	Further investigations using a model	133
6.5	Presentation of the solutions	135
7	Non-linear models	137
7.1	Typical applications	137
7.2	Local and global optima	140
7.3	Separable programming	147
7.4	Converting a problem to a separable model	153
8	Integer programming	155
8.1	Introduction	155
8.2	The applicability of integer programming	156
8.2.1	Problems with discrete inputs and outputs	156
8.2.2	Problems with logical conditions	158
8.2.3	Combinatorial problems	158
8.2.4	Non-linear problems	160
8.2.5	Network problems	161
8.3	Solving integer programming models	162
8.3.1	Cutting planes methods	162
8.3.2	Enumerative methods	163
8.3.3	Pseudo-Boolean methods	163
8.3.4	Branch and bound methods	164
9	Building integer programming models I	165
9.1	The uses of discrete variables	165
9.1.1	Indivisible (discrete) quantities	165
9.1.2	Decision variables	165
9.1.3	Indicator variables	166
9.2	Logical conditions and 0–1 variables	172
9.3	Special ordered sets of variables	177
9.4	Extra conditions applied to linear programming models	182
9.4.1	Disjunctive constraints	183
9.4.2	Non-convex regions	184
9.4.3	Limiting the number of variables in a solution	186
9.4.4	Sequentially dependent decisions	186
9.4.5	Economies of scale	187
9.4.6	Discrete capacity extensions	188
9.4.7	Maximax objectives	188
9.5	Special kinds of integer programming model	189
9.5.1	Set covering problems	189
9.5.2	Set packing problems	191
9.5.3	Set partitioning problems	193
9.5.4	The knapsack problem	195
9.5.5	The travelling salesman problem	195
9.5.6	The vehicle routing problem	198

9.5.7	The quadratic assignment problem	199
9.6	Column generation	201
10	Building integer programming models II	207
10.1	Good and bad formulations	207
10.1.1	The number of variables in an IP model	207
10.1.2	The number of constraints in an IP model	211
10.2	Simplifying an integer programming model	218
10.2.1	Tightening bounds	218
10.2.2	Simplifying a single integer constraint to another single integer constraint	220
10.2.3	Simplifying a single integer constraint to a collection of integer constraints	222
10.2.4	Simplifying collections of constraints	226
10.2.5	Discontinuous variables	228
10.2.6	An alternative formulation for disjunctive constraints	229
10.2.7	Symmetry	230
10.3	Economic information obtainable by integer programming	231
10.4	Sensitivity analysis and the stability of a model	238
10.4.1	Sensitivity analysis and integer programming	238
10.4.2	Building a stable model	239
10.5	When and how to use integer programming	240
11	The implementation of a mathematical programming system of planning	243
11.1	Acceptance and implementation	243
11.2	The unification of organizational functions	245
11.3	Centralization versus decentralization	247
11.4	The collection of data and the maintenance of a model	249
Part II		251
12	The problems	253
12.1	Food manufacture 1	253
12.2	Food manufacture 2	255
12.3	Factory planning 1	255
12.4	Factory planning 2	256
12.5	Manpower planning	256
12.5.1	Recruitment	257
12.5.2	Retraining	257
12.5.3	Redundancy	258
12.5.4	Overmanning	258
12.5.5	Short-time working	258

12.6	Refinery optimisation	258
12.6.1	Distillation	258
12.6.2	Reforming	259
12.6.3	Cracking	259
12.6.4	Blending	260
12.7	Mining	261
12.8	Farm planning	262
12.9	Economic planning	263
12.10	Decentralisation	265
12.11	Curve fitting	266
12.12	Logical design	266
12.13	Market sharing	267
12.14	Opencast mining	269
12.15	Tariff rates (power generation)	270
12.16	Hydro power	271
12.17	Three-dimensional noughts and crosses	272
12.18	Optimising a constraint	273
12.19	Distribution 1	273
12.20	Depot location (distribution 2)	275
12.21	Agricultural pricing	276
12.22	Efficiency analysis	278
12.23	Milk collection	278
12.24	Yield management	282
12.25	Car rental 1	284
12.26	Car rental 2	287
12.27	Lost baggage distribution	287
12.28	Protein folding	289
12.29	Protein comparison	290

Part III 293

13	Formulation and discussion of problems	295
13.1	Food manufacture 1	296
13.1.1	The single-period problem	296
13.1.2	The multi-period problem	297
13.2	Food manufacture 2	299
13.3	Factory planning 1	300
13.3.1	The single-period problem	300
13.3.2	The multi-period problem	301
13.4	Factory planning 2	302
13.4.1	Extra variables	302
13.4.2	Revised constraints	303
13.5	Manpower planning	303
13.5.1	Variables	304

13.5.2	Constraints	305
13.5.3	Initial conditions	305
13.6	Refinery optimization	306
13.6.1	Variables	307
13.6.2	Constraints	308
13.6.3	Objective	310
13.7	Mining	310
13.7.1	Variables	310
13.7.2	Constraints	311
13.7.3	Objective	312
13.8	Farm planning	312
13.8.1	Variables	312
13.8.2	Constraints	313
13.8.3	Objective function	315
13.9	Economic planning	316
13.9.1	Variables	316
13.9.2	Constraints	316
13.9.3	Objective function	317
13.10	Decentralization	317
13.10.1	Variables	318
13.10.2	Constraints	318
13.10.3	Objective	319
13.11	Curve fitting	319
13.12	Logical design	320
13.13	Market sharing	322
13.14	Opencast mining	324
13.15	Tariff rates (power generation)	325
13.15.1	Variables	325
13.15.2	Constraints	325
13.15.3	Objective function (to be minimized)	326
13.16	Hydro power	326
13.16.1	Variables	326
13.16.2	Constraints	326
13.16.3	Objective function (to be minimized)	327
13.17	Three-dimensional noughts and crosses	327
13.17.1	Variables	327
13.17.2	Constraints	328
13.17.3	Objective	328
13.18	Optimizing a constraint	328
13.19	Distribution 1	330
13.19.1	Variables	331
13.19.2	Constraints	331
13.19.3	Objectives	332
13.20	Depot location (distribution 2)	332
13.21	Agricultural pricing	333

13.22	Efficiency analysis	335
13.23	Milk collection	336
13.23.1	Variables	336
13.23.2	Constraints	336
13.23.3	Objective	337
13.24	Yield management	337
13.24.1	Variables	338
13.24.2	Constraints	338
13.24.3	Objective	340
13.25	Car rental 1	340
13.25.1	Indices	340
13.25.2	Given data	340
13.25.3	Variables	341
13.25.4	Constraints	341
13.25.5	Objective	342
13.26	Car rental 2	342
13.27	Lost baggage distribution	343
13.27.1	Variables	343
13.27.2	Objective	344
13.27.3	Constraints	344
13.28	Protein folding	344
13.29	Protein comparison	345

Part IV 347

14	Solutions to problems	349
14.1	Food manufacture 1	349
14.2	Food manufacture 2	349
14.3	Factory planning 1	350
14.4	Factory planning 2	351
14.5	Manpower planning	354
14.6	Refinery optimization	356
14.7	Mining	357
14.8	Farm planning	358
14.9	Economic planning	359
14.10	Decentralization	361
14.11	Curve fitting	361
14.12	Logical design	363
14.13	Market sharing	363
14.14	Opencast mining	364
14.15	Tariff rates (power generation)	364
14.16	Hydro power	366
14.17	Three-dimensional noughts and crosses	368
14.18	Optimizing a constraint	369

14.19	Distribution 1	369
14.20	Depot location (distribution 2)	371
14.21	Agricultural pricing	371
14.22	Efficiency analysis	372
14.23	Milk collection	374
14.24	Yield management	376
14.25	Car rental	379
14.26	Car rental 2	380
14.27	Lost baggage distribution	380
14.28	Protein folding	382
14.29	Protein comparison	382
References		383
Author index		397
Subject index		401

