March 23, 2015

MATH 230 HOMEWORK # 3

(Due on March 30, 2015, Monday. Homework will be collected in the class)
(Please, DO NOT forget to write your name and department)

1. On average, 6.9 patients arrive at the emergency room of a large hospital each hour. The number of patients who arrive during any particular hour has a Poisson distribution. A doctor arrives for work at the emergency room. Find the probability that the time from the doctor's arrival until the arrival of the first patient exceeds 1 hour.

2. The number of cars driving past a parking area in a 1-minute time interval has a Poisson distribution with parameter μ. The probability that any individual driver actually wants to park his/her car is p. If one parking place is available and it will take 1 minute to reach the parking area, what is the probability that a space will still be available when you reach the lot? (Assume that no one leaves the lot during the 1-minute interval)

3. C.d.f of a random variable X is given by

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 0.05x & 0 \le x < 20 \\ 1 & x \ge 20 \end{cases}$$

a) Find $E(X)$.
b) Use c.d.f to find $P(5 < X < 15)$.

4. Let ***c*** be a constant and consider the density function,

$$f(y) = \begin{cases} c + y & -1 < y < 0 \\ c - y & 0 \le y < 1 \end{cases}$$

a) Find the value of ***c***.
b) Find the cumulative distribution function $F(y)$.
c) Compute $F(-0.5)$.
d) Compute $P(0 \le Y \le 0.5)$.

5. In a certain city, the daily consumption of electric power (in millions of kilowatt-hours) is a random variable having the probability density

$$f(x) = \begin{cases} \frac{1}{9} x e^{-\frac{x}{3}} & x > 0 \\ 0 & x \le 0 \end{cases}$$

If the city's power plant has a daily capacity of 12 million kilowatt-hours, what is the probability that this power supply will be inadequate on any given day?

6. To be a winner in the following game, you must be successful in 3 successsive rounds. The game depends on the value of Y, a uniform random variable on (0,1). If $Y > 0.1$, then you are successful in round 1; if $Y > 0.2$, then you are successful in round 2; if $Y > 0.3$ then you are successful in round 3;

a) Find the probability that you are successful in round 1.
b) Find the conditional probability that you are successful in round 2, given that you were successful in round 1.
c) Find the conditional probability that you are successful in round 3, given that you were successful in round 1 and 2.
d) Find the probability that you were a winner.