

Numerical Methods
 Lab 8. Naive Gauss elimination
 Maryna Borovyk
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1 Tasks a and b

Lab 8	Maryna Borovyk
a) Solving by Naive Gauss elimination	
$\begin{array}{l} 10x_1 + 2x_2 - x_3 = 27 \quad \times 3 \\ -3x_1 - 6x_2 + 2x_3 = -67,5 \\ x_1 + x_2 + 5x_3 = -27,5 \end{array}$	
$\textcircled{1} \quad (-3+30)x_1 + (-6+6)x_2 + (2-3)x_3 = (-67,5+81)$	
$\begin{array}{l} 10x_1 + 2x_2 - x_3 = 27 \\ 27x_1 + 0 + (-7)x_3 = 19,5 \\ x_1 + x_2 + 5x_3 = -27,5 \end{array}$	
$\textcircled{2} \quad -5x_1 + (1-1)x_2 + (5+0,5)x_3 = (-27,5 + -13,5)$	
$\begin{array}{l} 10x_1 + 2x_2 - x_3 = 27 \\ 27x_1 + 0 + (-7)x_3 = 19,5 \quad \times 5,5 \\ -5x_1 + 0 + 5,5x_3 = -35 \end{array}$	
$\textcircled{3} \quad (-5+148,5)x_1 + (5,5-5,5)x_3 = (-35 + 107,25)$	
$\begin{array}{l} 10x_1 + 2x_2 - x_3 = 27 \\ 27x_1 - x_3 = 19,5 \\ 143,5x_1 + 0 = 72,25 \end{array}$	
$\textcircled{4} \quad x_1 = \frac{72,25}{143,5} = 0,5$	
$x_3 = -19,5 + 27x_1 \quad x_3 = -6$	

$$2x_2 - 27 + x_3 - 10x_1$$

$$x_2 = \frac{27 + (-6) - 10 \cdot 0,5}{2}$$

$$x_2 = 8$$

⑤ results

$$x_1 = 0,5$$

$$x_2 = 8$$

$$x_3 = -6$$

b) Substitute results into original equations

$$\underbrace{10 \cdot 0,5 + 2 \cdot 8 + 6}_{27} = 27 \quad \text{correct}$$

$$\underbrace{-3 \cdot 0,5 - 6 \cdot 8 + 2 \cdot (-6)}_{-67,5} = -67,5 \quad \text{correct}$$

$$\underbrace{0,5 + 8 + 5 \cdot (-6)}_{-21,5} = -21,5 \quad \text{correct}$$

2 Task c

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[5]: import numpy as np
import sys
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[6]: def naive_gauss_elimination(a):

    n=len(a)
    x = np.zeros(n)

    # Applying Gauss Elimination
    for i in range(n):
        if a[i][i] == 0.0:
            sys.exit('Divide by zero detected!')

        for j in range(i+1, n):
            ratio = a[j][i] / a[i][i]
```

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for k in range(n+1):
    a[j][k] -= ratio * a[i][k]

# Back Substitution
x[n-1] = a[n-1][n] / a[n-1][n-1]

for i in range(n-2, -1, -1):
    x[i] = a[i][n]

    for j in range(i+1, n):
        x[i] -= a[i][j] * x[j]
    x[i] /= a[i][i]

print('Solution is: ')
for i in range(n):
    print(f'x{i} = {x[i]:0.2f}', end='\t')

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[7]: a = np.array([[10, 2, -1, 27],
                 [-3, -6, 2, -61.5],
                 [1, 1, 5, -21.5]])
naive_gauss_elimination(a)
```

Solution is:
 $x_0 = 0.50$ $x_1 = 8.00$ $x_2 = -6.00$

3 Conclusion

The goal of this lab was to understand the mechanics and computational process of solving linear systems. In conclusion, naive Gauss elimination is an effective technique for solving small sets of equations, it serves as a foundation for more advanced methods and introduces key concepts in linear algebra and numerical methods.