Market Impact and Optimal Equity Trade Scheduling

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Presentation Outline

- Brief review of the optimal trading problem
- Our “trade scheduling” algorithm
- Real time experience with the trade scheduling algorithm
- Ongoing research on market impact modeling

Motivation for Trade Scheduling

- Northfield client comment in 1997:
  - "Your optimizer just told me to buy five million shares of Ford. From who?"
- Implementing a change in strategy for a large equity portfolio could involve not one, but potentially hundreds of concurrent large trades
- Our goal is minimize the "implementation shortfall" as defined in Perold (1988)
  - What is the difference in portfolio wealth between what we actually achieved, and what we could have achieved if we could trade securities instantaneously at no cost?

Components of Trading Costs

- Most people see trading costs as having several components
  - Agency costs
  - Bid/Asked Spread
  - Market Impact (my trade moves the price)
  - Other people’s trades move the price, maybe in my favor. Traders call this “trend cost”. We call it risk
- Often overlooked ingredients
  - My large concurrent trades (my trade in Ford impacts the price of GM)
  - The implicit opportunity cost of waiting. Unless we’re passive, we want to buy stocks before they go up, not after. If we’re selling we want to sell before they go down, not after

Optimal Trade Scheduling

- If we know the urgency of trades, and the likely impact, we can create optimal trade “schedules” to break up large trades into a series of smaller trades
  - To minimize opportunity cost and risk we need to trade quickly
  - To minimize market impact we need to trade slowly
- Bertsimas and Lo (1998), Bertsimas, Hummel and Lo (1999) trade off opportunity costs and market impact
- Almgren and Chriss (2001, 2001a) add risk aversion to the objective function
- Computationally treated as a stochastic processes solved with Bellman equation methods
  - Although stochastic programming methods are improving, its impractically slow for large cases

Setting the Objective for Scheduling

- Consider a set of undone orders as a long short-portfolio that you are liquidating
  - You are long shares you do have and don't want
  - You are short shares you do want and don't have
- The usual single period mean-variance objective function is:

$$U = \alpha - \sigma^2/T - (C*A)$$

- Works just fine except:
  - the sign on alpha is reversed from the normal situation. You are currently short stocks that do want. The reason you want them is that they have positive alpha
  - We can't get all our trades done in one period, so we need a multi-period representation

Why Mean Variance?

- Some trading algorithms try minimizing the standard deviation of trading costs
  - From a process control perspective minimizing the uncertainty (standard deviation) rather than variance seems intuitive
- However, the impact of the variation in trading costs on portfolio values is half the variance, and is not linearly related to the standard deviation
  - Consider a trading a 100 Yen portfolio with a trading cost of 10% twice. Ending wealth is 81 Yen
  - Consider two trades with an average of 10% cost, 0% cost and 20% cost. Ending wealth is 80 Yen
  - The decimal variance of the second case is 0.01 so the expected loss is 0.005 per observation. This checks since 0.005 * 100 * 2 = 1

Our Trade Scheduling Algorithm

- We use a multi-period mean variance objective function in discrete time
  - Market impact function is broken into temporary and permanent components.
  - Cross market impact of concurrent trades can be addressed
- There is a slightly clever set of basis transformations such that the problem can be solved using standard quadratic programming methods
  - Huge savings in computation times
  - We can analyze a 10 period, 400 trade problem on a laptop in under a minute, and will be a lot faster soon
- Implemented by Instinet on their institutional trading platform
  - Went live October 2006

Permanent and Temporary Market Impact

- If our trades in any given stock are far apart in time, price movements caused by our trades will be independent of one another
- If our trades follow each other with little time in between, market impact effects will have a cascading effect as each trade moves the price from where the previous trade left it
- We call this persistent portion of market impact “stickiness”, and account for it in the solution when the length of our discrete time blocks is short

Trader's Experience

- Lets assume we want to finish all our open trades over a one trading day period
  – We can break the two days into discrete time blocks, either by clock time or by "expected share of day's volume" (e.g. each block is the length of clock time that usually trades 5% of the days volume
- Think of a spreadsheet where each order is a row and each time block is a column.
  – We want the matrix of orders such that all orders are completed the end of the schedule
  – That maximizes our objectives: capturing short term alpha, minimizing risk and market impact
- After each period is experienced, we can check that the expected orders were executed, if not, we can rerun the schedule based on the remaining shares and time periods

Empirical Study Design

- Results taken from Schmidt (2007)
  - Thorsten is the point man at Instinet for trade scheduling, so he's in the trenches
  - It has to be right. He's a Columbia Alum (he's also supposed to be here so I have to say that).
- Study of more than 21,000 institutional trades
  - Measure implementation shortfall
- Three styles of algorithmic trading
  - VWAP (very patient trading)
  - Our trade scheduler (hopefully optimal)
  - % Participation trades (very aggressive trading)
- Compute the implied range of risk tolerance values that would make one style of trading preferable to another

Empirical Results

- For trades less than 2% of average daily volume, no big advantages to any algorithm
- For trades between 2% and 4% of average daily volume, the trade scheduler is best for low risk tolerance traders, VWAP is best for more risk tolerant traders
- For trades between 4% and 6% of average daily volume, trade scheduling dominates over all rational values of risk tolerance
- The implied risk tolerance of many traders is far lower than the portfolios for which they are trading
  - An expected alpha of 48% per annum as the opportunity cost is needed to justify the typical trader's degree of urgency

Estimating Trade Costs is Hard

- Agency Costs are essentially known in advance
- Bid/Asked Spreads: Some time variation but reasonably stable
- Market Impact: Lots of models exist. Underlying factors are highly significant, but explanatory power is typically quite low
- Trend Costs: They can move the price for or against us. Ex-post often the largest part of the costs. Pretty darn random. Or so it seems.
- Market impact and trend costs are hard to disentangle so maybe the market impact models work better than we think

Transaction Cost Functional Form

- Lots of market impact models look like this. Market impact increases with trade size either linearly or at a decreasing rate

$$M = A_i + [(B_i * X)$$

OR

$$M = A_i + (C_i * |X^{0.5}|)]$$

M is the expected cost to trade one share

X is the number of shares to be traded

A_i is the fixed costs per share

B_i,C_i are coefficients expressing the liquidity of the stock as a function of fundamental data

The Need for Boundary Conditions

- Our optimizer allows for a market impact formula that combines the linear and square root processes

$$M = A_i + [(B_i * X_t) + (C_i * |X_t^{0.5}|)] + \ldots$$

- When clients started to put in their own values for B and C, we often saw bizarre results such as forecast selling costs over 100%!
  - This arose because their coefficients were based on empirical estimations from data sets that did not contain very large trades that traders never try to do because they would be too costly
- Coefficients for B and C must provide rational results in the entire range of potential trade size from zero to all the shares of a firm

Market Impact For Dummies

- Lets consider a hostile takeover as the “worst case” scenario for market impact
  - We’re going to buy up all the shares of a company and tell the entire world we’re doing it. The takeover premium can be viewed as an extreme case of market impact
- If we believe *only in the linear market impact process*, we can set our coefficient to the expected takeover premium for a stock divided by shares outstanding
- If we believe *only in the square root process* for market impact, we can set our coefficient to the expected takeover premium for a stock divided by the square root of shares outstanding

More Impact for Dummies

- If we don't know which process to believe in, we can just do both with a weighting summing to one.

$$Bi = W * (E[P_i] / S_i)$$
$$Ci = (1\text{-}W) * E[P_i] / (S_i^{0.5})$$

B_i = the coefficient on the linear process
C_i = the coefficient on the square root process
P_i = the takeover premium in percent
S_i = the number of shares outstanding
W = a weight

Accounting for Different Liquidity

- If we assume takeover premiums are lognormal, we can easily express the expected takeover premium as a function of a liquidity measure

  $$E[P_i] = QP / ((1 + K/100)^{Z_i})$$

  P = % average price premium in a hostile takeover
  K = the log percentage standard error around P
  Z_i = the Z score of a liquidity measure for stock I
  Q = a scalar between zero and one (the takeover scenario is "worst case" for information leakage, so a smaller value could fit better)

An Empirical Example

- Various academic studies using M & A databases have reported average takeover premiums from 37 to 50% with a standard deviation around 30%
- Lets take a hypothetical company with $5 Billion market cap, $50 share price and 100 million shares outstanding
  - Assume P = 37, K = 40, W = .25, Z_i = 0
- 1,000,000 share trade impact = 2.88%
- 10,000 share trade impact = 0.27%

Estimating Market Impact Empirically

- Our dataset was provided by Instinet
  - Over 1.5 million orders over an 18 month period with fine detail such as time stamps, arrival price, execution price, order type (buy/sell, limit/market), tracking of cancelled orders, etc.
  - Totally anonymous. We have no information on what firms traded or which order belong to whom
  - Most of the data is from the US, with good representation of major markets such as Japan, UK, Canada, etc.
- Very large dataset on which to estimate a model with only two free parameters, Q and W
- Security level liquidity measures are derived from various aspects of the Northfield risk model for a given market, such as typical trading volumes, stock volatility, etc.

Measuring Market Impact?

- One way to measure market impact would be to compare the price we got on our trade versus the price on the previous trade as a measure of how much our trade "moved the price"
  - This may not be relevant to us. We care about how much the price moved from the price it was at when we decided to trade the stock
- We use an "arrival price" measure. It's the percentage in price between the execution price and the price that existed in the market when we got the order to transact the stock
  - Implementation Shortfall described in Perold (1988)
  - Very noisy relationships since a limit order can sit for hours between arrival and execution. Prices can move around a lot during that time from other people's trades, not ours

Criteria To Judge A Model

- Unbiasedness
  - On average the forecasts of market impact should match observed costs
- Low Error
  - The absolute difference between the forecasts and the observations as a percentage of the forecast
- High Explanatory Power
  - The model should accurately predict when the market impact a trade will be high or low
- All three of the above criteria are calculated on a trade dollar weighted basis
  - It's a lot more important to get things right on a million share trade than a hundred share trade

An Estimation Subtlety

- Using “arrival price cost” as the measure, its possible that the realized market impact of a trade could be negative or zero
  - We’re buying a stock and the price went down before our order was executed
- However, the forecast market impact is always positive, so we calculate percentage of error with the forecast in the denominator
- This means that a low percentage of error and a high R-squared are not exactly congruent measures
  - There is more room to be wrong on one side than the other

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Dollar Weighted Results for A Given Q,W

	Average Impact Cost BP	Avg % Absolute (Error)	In sample % R-squared	Time Length of Order (HR)
USA	13.9	205	74.2	1:32
Swiss	24.8	158	64.2	6:32
Canada	28.5	217	33.8	2:39

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Empirical Results Discussion

- We are able to isolate values of Q,W that are economically reasonable
  - Fit data extremely well in several countries (US, Canada, Switzerland) based on our current choice of liquidity measures
  - Fits are less good in some countries so more work is needed on liquidity measures
  - Range of weight on linear (rather than square root process) is .28 to .65 across seven major markets
  - Range of implied takeover premiums is 20% to 40% across seven major markets
- Average costs are sensitive to the inclusion or exclusion of limit orders, but the weighting parameter values are robust

Conclusions

- Our trade scheduler captures the major aspects of trading optimality in a computationally efficient algorithm
  - Based on a multi-period mean variance objective function in discrete time
  - Empirical validation from a dataset of 21,000 algorithm driven trades suggest that the trade scheduler dominates other algorithms for trades that are significant portion of ADV
  - Also dominates for smaller trades where traders show typically high degrees of risk aversion
- Our search for a market impact model that both fits the data well (at least in some markets) and is rationality bounded has been fruitful.
  - We hope this additional work will be incorporated into the trade scheduler globally during 2008.

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