RADICALI 2

sabato 2 novembre 2024 22:27

1) $y = \dfrac{x^2+4x-1}{\sqrt{x+4}}$

CE: $\begin{cases} \sqrt{x+4} \neq 0 \\ x+4 \geq 0 \end{cases} \rightarrow \begin{cases} x+4 \neq 0 \\ x > -4 \end{cases} \rightarrow \begin{cases} x \neq -4 \\ x > -4 \end{cases} \rightarrow x > -4$

D: $(-4; \infty)$

$f(x) \neq f(-x)$

$f(-x) \neq -[f(x)]$

Asse y con x=0

$\begin{cases} x = 0 \\ y = -\frac{1}{2} \end{cases}$

Asse x con y=0

$\begin{cases} \dfrac{x^2+4x-1}{\sqrt{x+4}} = 0 \\ y = 0 \end{cases} \rightarrow \begin{cases} x_{1/2} = \dfrac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2} \\ y = 0 \end{cases}$

$\Delta = 4 + 4 = 8$

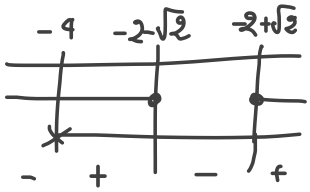
$A\left(0; -\frac{1}{2}\right)$

$B(-2-\sqrt{2}; 0)$

$C(-2+\sqrt{2}; 0)$

Segno $x^2+4x-1 \geq 0 \rightarrow x \leq -2-\sqrt{2} \cup x \geq -2+\sqrt{2}$

$\sqrt{x+4} > 0 \rightarrow x > -4$



$\lim\limits_{x \to -4^+} \dfrac{x^2+4x-1}{\sqrt{-4^+ + 4}} = \dfrac{-}{0^+} = -\infty$

$\lim\limits_{x \to \infty} \dfrac{x^2+4x-1}{\sqrt{x+4}} = \dfrac{\infty}{\infty}$ ind

De L. $\lim\limits_{x \to \infty} \dfrac{2x+4}{\frac{1}{2\sqrt{x+4}}} = \dfrac{(2x+4) \cdot (2\sqrt{x+4})}{1} = 4x\sqrt{x+4} + 8\sqrt{x+4}$

$$= 4x = \infty$$

$$f'(x) \to x^2+4x-1 = 2x+4$$

$$\sqrt{x+4} = \frac{1}{2\sqrt{x+4}}$$

$$\Rightarrow \frac{(2x+4)(\sqrt{x+4}) - (x^2+4x-1)\cdot\frac{1}{2\sqrt{x+4}}}{(\sqrt{x+4})^2} =$$

$$= \frac{\frac{(2x+4)(\sqrt{x+4})(2(\sqrt{x+4}) - (x^2+4x-1)}{2\sqrt{x+4}}}{x+4} =$$

$$= \frac{\frac{(2x+4)(2)(x+4) - (x^2+4x-1)}{2\sqrt{x+4}}}{x+4} =$$

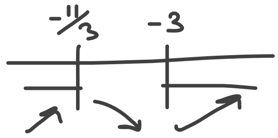
$$= \frac{4x^2+16x+8x+32-x^2-4x+1}{2(\sqrt{x+4})(x+4)} =$$

$$= \frac{3x^2+20x+33}{2\sqrt[3]{(x+4)^2}}$$

$$f'(x) \geq 0 \to 3x^2+20x+33 \geq 0$$

$$\Delta = 400 - 4(33)(3) = 4$$

$$x_{1/2} = \frac{-20 \pm 2}{6} = \frac{-10 \pm 1}{3} \quad x_1 = \frac{-9}{3} = -3 \quad x_2 = \frac{-11}{3}$$



$x = -\frac{11}{3}$ max

$x = -3$ min

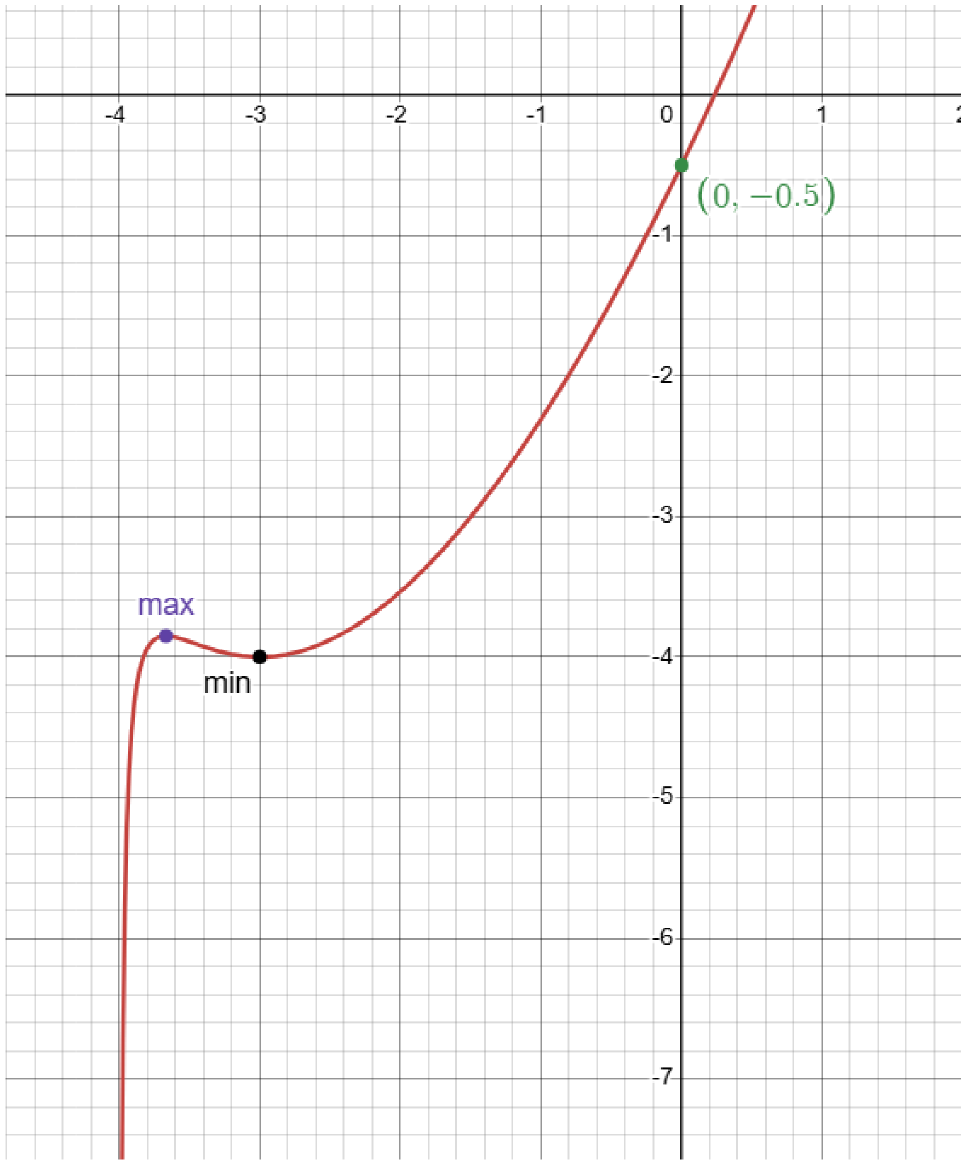
$$\text{Max} \to f\left(-\frac{11}{3}\right) = \frac{\left(-\frac{11}{3}\right)^2 + 4\left(-\frac{11}{3}\right) - 1}{\sqrt{-\frac{11}{3}+4}} = \frac{\frac{121}{9} - \frac{44}{3} - 1}{\sqrt{\frac{-11+12}{3}}} =$$

$$= \frac{121-132-9}{9} \cdot \frac{1}{\sqrt{\frac{1}{3}}} = \frac{-20}{9\cdot\frac{1}{\sqrt{3}}} = \frac{-20}{3\sqrt{3}}$$

$$\text{Max}\left(-\frac{11}{3}; -\frac{20}{3\sqrt{3}}\right)$$

$$\text{Min} \to f(-3) = \frac{(-3)^2+4(-3)-1}{\sqrt{-3+4}} = \frac{9-12-1}{1} = -4$$

$$\text{Min}(-3; -4)$$



$y = x^3 + \sqrt{x}$

CE: $x \geq 0$

D $[0; \infty)$

$f(x) \neq f(-x)$

$\sqrt{-x} \neq -\sqrt{x}$

$f(-x) \neq -f(x)$

Asse y con $x=0$

$\begin{cases} x=0 \\ y=0 \end{cases}$

Asse x con $y=0$

$\begin{cases} x=0 \\ y=0 \end{cases}$

$A(0;0)$

Segno $\quad x^3+\sqrt{x} \geq 0 \to x^3 \geq -\sqrt{x} \to x \geq -\sqrt{x^3} \to x^2 \geq -x^3$

$$x^3 + x^2 \geq 0 \to x^2(x+1) \geq 0 \quad \begin{cases} x^2 \geq 0 \to x \geq 0 \\ \underbrace{x \geq -1}_{\nexists D} \end{cases}$$

$\overset{0}{+}$

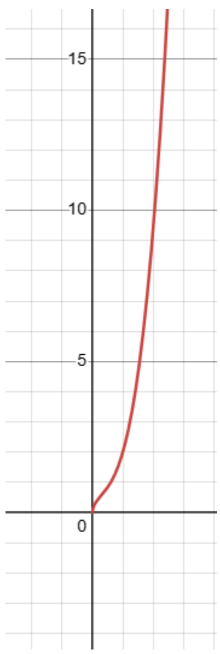
$\lim_{x \to 0^+} x^3 + \sqrt{x} = 0^+ \quad$ No As. V.

$\lim_{x \to \infty} x^3 + \sqrt{x} = x^3 = \infty$

$$f'(x) = 3x^2 + \frac{1}{2\sqrt{x}} = \frac{(3x^2)(2\sqrt{x})+1}{2\sqrt{x}} = \frac{6x^{\frac{5}{2}}+1}{2\sqrt{x}} = \frac{\sqrt{6^5}+1}{2\sqrt{x}}$$

$$= \frac{6\sqrt{6^3}+1}{2\sqrt{x}} \quad \text{con } x \neq 0$$

$f'(x) > 0 \to 6\sqrt{6^3}+1 > 0$ sempre $\overset{+}{\nearrow}$



$$y = \frac{x^3}{4x^2-1}$$

CE: $x \neq \pm \frac{1}{2}$

D: $\left(-\infty; -\frac{1}{2}\right) \cup \left(-\frac{1}{2}; \frac{1}{2}\right) \cup \left(\frac{1}{2}; \infty\right)$

$f(x) \neq f(-x)$

$f(-x) \neq -f(x)$

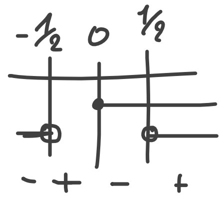
Asse y con $x=0$ 		Asse x con $y=0$

$$\begin{cases} x=0 \\ y=0 \end{cases} \qquad \begin{cases} x^3=0 \rightarrow x=0 \\ y=0 \end{cases}$$

$A(0;0)$

$x^3 \geq 0 \rightarrow x \geq 0$

$4x^2-1>0 \rightarrow x<-\frac{1}{2} \cup x>\frac{1}{2}$



$$\left.\begin{aligned} &\lim_{x\to-\infty} \frac{x^3}{4x^2-1} = \frac{x^3}{4x^2} = \frac{x}{4} = -\infty \\ &\lim_{x\to+\infty} \frac{x}{4} = +\infty \end{aligned}\right] \text{ No As. Oriz.}$$

$$m = \lim_{x\to\infty} \frac{x^3}{4x^2-1} \cdot \frac{1}{x} = \frac{x^2}{4x^2} = \frac{1}{4}$$

$$q = \lim_{x\to\infty} \frac{x^3}{4x^2-1} - \frac{1}{4}x = \frac{4x^3 - x(4x^2-1)}{4(4x^2-1)} = \frac{4x^3-4x^3+x}{16x^2-4} =$$

$$= \frac{x}{16x^2-4} = \frac{1}{16x} = \frac{1}{\infty} = 0$$

As. obliquo di eq. $y = \frac{1}{4}x$

$$\left.\begin{aligned} &\lim_{x\to-\frac{1}{2}^-} \frac{x^3}{4x^2-1} = \frac{(-0,6)^3}{4(-0,6)^2-1} = \frac{-}{+} = -\infty \\ &\lim_{x\to-\frac{1}{2}^+} \frac{x^3}{4x^2-1} = \frac{(-0,4)^2}{4(-0,4)^2-1} = \frac{-}{-} = +\infty \end{aligned}\right] \begin{aligned} &\text{As. Vert.} \\ &x = -\frac{1}{2} \end{aligned}$$

$$\left.\begin{aligned} &\lim_{x\to\frac{1}{2}^-} \frac{0,4}{4(0,4)^2-1} = \frac{+}{-} = -\infty \\ &\lim_{x\to\frac{1}{2}^+} \frac{0,6}{4(0,6)-1} = \frac{-}{-} = +\infty \end{aligned}\right] \begin{aligned} &\text{As. Vert.} \\ &x = \frac{1}{2} \end{aligned}$$

$$f'(x) = \frac{3x^2\cdot(4x^2-1) - x^3(8x)}{(4x^2-1)^2} = \frac{12x^4-3x^2-8x^4}{D} =$$

$$= \frac{4x^4-3x^2}{D} = \frac{x^2(4x^2-3)}{D}$$

$f(x) \geq 0 \rightarrow x^2 \geq 0 \rightarrow x \geq 0$

$4x^2 - 3 \geq 0 \rightarrow \quad x \leq -\frac{\sqrt{3}}{2} \cup x \geq \frac{\sqrt{3}}{2}$

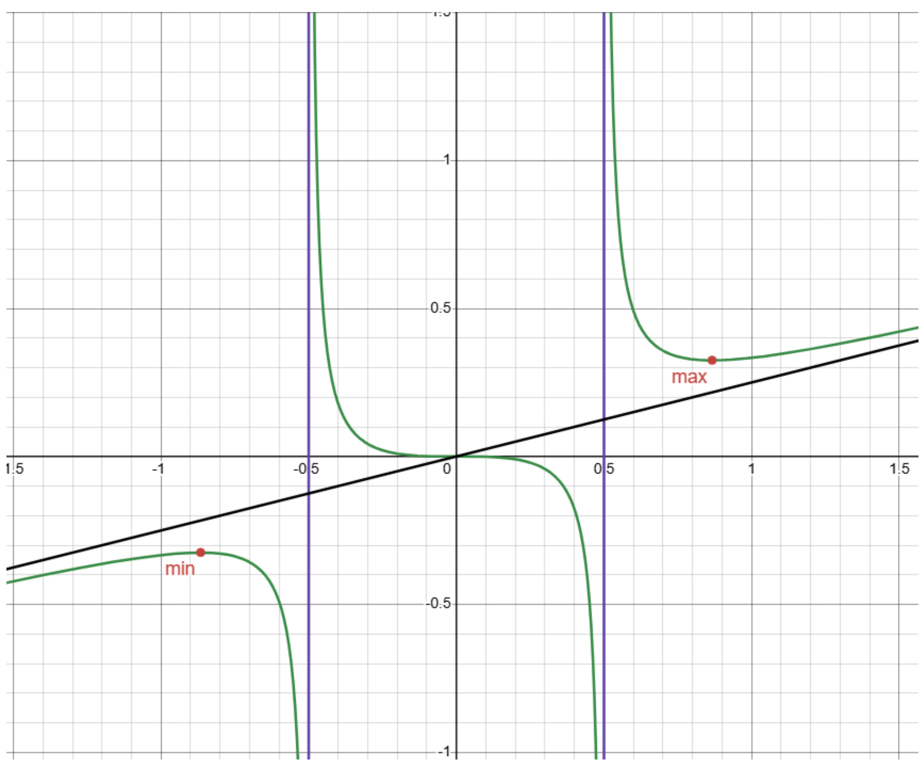
$$\begin{array}{c|c|c|c} & -\frac{\sqrt{3}}{2} & 0 & \frac{\sqrt{3}}{2} \\ - & + & - & + \\ \searrow & \nearrow & \searrow & \nearrow \end{array}$$

$\text{Min} \rightarrow f\left(-\frac{\sqrt{3}}{2}\right) = \dfrac{\left(-\frac{\sqrt{3}}{2}\right)^3}{4\left(-\frac{\sqrt{3}}{2}\right)^2 - 1} = \dfrac{-\frac{\sqrt{27}}{8}}{2} = -\frac{\sqrt{27}}{16}$

$\text{Min}\left(-\frac{\sqrt{3}}{2}; -\frac{\sqrt{27}}{16}\right)$

$\text{Max} \rightarrow f\left(\frac{\sqrt{3}}{2}\right) = \dfrac{\left(\frac{\sqrt{3}}{2}\right)^3}{4\left(\frac{\sqrt{3}}{2}\right)^2 - 1} = \frac{\sqrt{27}}{16}$

Per $x = 0 \rightarrow f'(0) = \infty$ tang. verticale



$y = -2x - \sqrt{x^2 - 4}$

C.E. $x^2 - 4 \geq 0 \rightarrow \quad x \leq -2 \cup x \geq 2$

D: $(-\infty; -2] \cup [2; \infty)$

$f(-x) = 2x - \sqrt{x^2-4} \rightarrow f(x) \neq f(-x)$

$-f(x) = 2x + \sqrt{x^2-4} \rightarrow f(-x) \neq -f(x)$

Asse x con $y=0$

$\begin{cases} -2x - \sqrt{x^2-4} = 0 \\ y=0 \end{cases} \rightarrow \begin{cases} \sqrt{x^2-4} = -2x \\ y=0 \end{cases} \rightarrow \begin{cases} x^2-4 = 4x^2 \\ y=0 \end{cases} \rightarrow \begin{cases} -3x^2 = 4 \rightarrow 3x \\ y=0 \end{cases}$

Asse y con $x=0$

$\begin{cases} x=0 \\ y = \sqrt{-4} \text{ imposs.} \end{cases}$

$-2x - \sqrt{x^2-4} \geq 0 \rightarrow \sqrt{x^2-4} \leq -2x \rightarrow x^2 - 4 \leq +4x^2 \rightarrow -4 \leq 3x^2$

$\rightarrow 3x^2 \geq -4 \rightarrow x^2 \geq -\frac{4}{3}$ imposs.

$\lim_{x \to -\infty} -2x - \sqrt{x^2-4} \Rightarrow -2(-\infty) - \sqrt{(-\infty)^2} = +\infty - \infty = \text{indet.}$

Gerarchie $\lim_{x \to -\infty} -2x = -2(-\infty) = \infty$

$\lim_{x \to \infty} -2x - \sqrt{x^2-4} = -2(\infty) - \sqrt{\infty^2} = -\infty - \infty = -\infty$

$\lim_{x \to -2} -2(-2) - \sqrt{(-2)^2-4} = 4 - \sqrt{0} = 4$

$\lim_{x \to 2} -2(2) - \sqrt{2^2-4} = -4 - \sqrt{0} = -4$