82) $y = e^{\frac{2x-1}{x}}$

C.E. $x \neq 0$

D $(-\infty;0)\cup(0;\infty)$

$f(-x) = e^{\frac{-2x-1}{-x}} = e^{\frac{2x+1}{x}} \rightarrow$ No pari

$-f(x) = -e^{\frac{2x-1}{x}} \rightarrow$ No dispari

Asse y con $x=0$

$\begin{cases} x=0 & \nexists D \\ \end{cases}$

Asse x con $y=0$

$\begin{cases} e^{\frac{2x-1}{x}} = 0 & \text{mai (è un esponenz.)} \\ y=0 \end{cases}$

Segno $e^{\frac{2x-1}{x}} > 0 \rightarrow \forall x \in D$

$\lim_{x\to-\infty} e^{\frac{2x-1}{x}} = e^{\lim_{x\to-\infty}\frac{2x-1}{x}} = e^{\lim_{x\to-\infty}\frac{2x}{x}} =$

$= e^{\lim_{x\to-\infty}(2)} = e^2$

$\lim_{x\to\infty} f(x) \rightarrow e^{\lim_{x\to\infty}\frac{2x-1}{x}} = e^2$

As. oriz. $y = e^2$

$\lim_{x\to0^-} e^{\frac{2x-1}{x}} \rightarrow e^{\frac{2\cdot0^- -1}{0^-}} \Rightarrow e^{\frac{-}{-}} = e^{+} \rightarrow e^{+\infty} = \infty$

As. Vert. sinistr. $x=0$

$\lim_{x\to0^+} e^{\frac{2x-1}{x}} \rightarrow e^{\frac{2\cdot0^+ -1}{0^+}} \rightarrow e^{\frac{-}{+}} \rightarrow e^{-} \rightarrow e^{-\infty} = 0^+$

$f'(x)$ per $\frac{2x-1}{x} \rightarrow \frac{2\cdot x - (2x-1)1}{x^2} = \frac{2x-2x+1}{x^2} = \frac{1}{x^2}$

$\Rightarrow f'(x) = e^{\frac{2x-1}{x}} \cdot \frac{1}{x^2}$

$f'(x) \geq 0 \rightarrow e^{\frac{2x-1}{x}} > 0 \rightarrow \forall x \in D$

$$\frac{e}{x^2}$$

funzione sempre crescente $\to$ No stazionari

$$f''(x) \text{ per } e^{\frac{2x-1}{x}} = \frac{e^{\frac{2x-1}{x}}}{x^2}$$

$$\to f''(x) = \frac{\frac{e^{\frac{2x-1}{x}}}{x^2} \cdot x^2 - e^{\frac{2x-1}{x}} \cdot (2x)}{x^4} =$$

$$= \frac{e^{\frac{2x-1}{x}} - 2xe^{\frac{2x-1}{x}}}{x^4} =$$

$$= \frac{e^{\frac{2x-1}{x}}(1-2x)}{x^4}$$

$$f''(x) \geq 0 \to e^{\frac{2x-1}{x}} > 0 \quad \forall x \in D$$

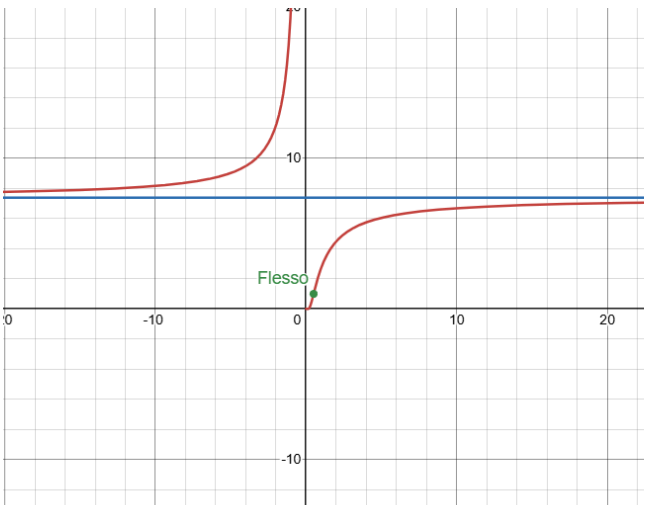
$$1-2x \geq 0 \to -2x \geq -1 \to x \leq \frac{1}{2}$$

$$\begin{array}{c|c} + & - \\ \cup & \cap \end{array} \quad \frac{1}{2}$$

$x = \frac{1}{2}$ è un flesso

$$f\left(\frac{1}{2}\right) = e^{\frac{2\left(\frac{1}{2}\right)-1}{1/2}} = e^{\frac{1-1}{1/2}} = e^0 = 1$$

Flesso $\left(\frac{1}{2}; 1\right)$



83) $y = x \ln x$

C.E. $x > 0$

D $(0; \infty)$

$f(-x) = -x \cdot \ln(-x) \rightarrow$ No pari

$-f(x) = -x \ln(x) \rightarrow$ No dispari

Asse y con $x=0$

$$\begin{cases} x = 0 & \nexists D \\ \end{cases}$$

Asse x con $y=0$

$$\begin{cases} x \ln(x) = 0 \rightarrow * \\ y = 0 \end{cases}$$

$* \quad \ln(x)^x = \ln(1) \rightarrow x^x = 1$

$x^x = e^{x \ln(x)}$

$e^{x \ln(x)} = 1$

$e^{x \ln(x)} = e^0$

$x \ln(x) = 0$

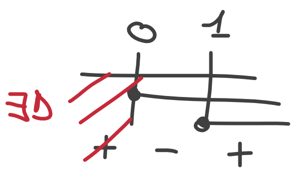
$\rightarrow x = 0 \quad \nexists D$

$\rightarrow \ln(x) = 0 \rightarrow \ln(x) = \ln(1) \rightarrow x = 1$

$A(1; 0)$

Segno $\quad x \ln(x) \geq 0 \rightarrow x \geq 0$

$\ln(x) \geq 0 \rightarrow x \geq 1$



$$\lim_{x \to \infty} x \ln(x) \rightarrow \lim_{x \to \infty} x \cdot \lim_{x \to \infty} \ln(x) = \infty \cdot \infty = \infty$$

anche tramite gerarchia

$$\lim_{x \to \infty} x \ln(x) = \lim_{x \to \infty} x = \infty$$

$$\lim_{x \to 0^+} x \ln(x) = 0^+ \cdot \ln(0^+) = 0^+ \cdot (-\infty) \text{ ind.}$$

Per Logica $\rightarrow \lim_{x\to 0^+} x\ln(x) \rightarrow \lim_{x\to 0^+} x = 0^+$

Trasformazione

Cerco di creare una frazione per usare De L'Hopital

$$x\cdot\ln(x) = x^{-1}\cdot\ln(x) = \frac{\ln(x)}{\frac{1}{x}}$$

$$\lim_{x\to\infty} \frac{\frac{1}{x}}{-\frac{1}{x^2}} \rightarrow \frac{1}{x}\cdot\left(-\frac{1}{x^2}\right) = -\frac{1}{x^3} = -\frac{1}{\infty} = 0$$

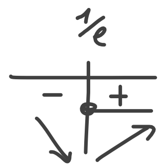
$\lim_{x\to 0^+} x\ln(x) = 0$ No As. Vert.

$$f'(x) = 1\cdot\ln(x) + x\cdot\frac{1}{x} = \ln(x) + 1$$

$f'(x) \geq 0 \qquad \ln(x) \geq -1$

$\ln(x) \geq \ln(e)^{-1}$

$x \geq e^{-1} \rightarrow x \geq \frac{1}{e}$



$x = \frac{1}{e}$ Minimo

$f\left(\frac{1}{e}\right) = \frac{1}{e}\cdot\ln\left(\frac{1}{e}\right)$

Prop. log.

$\ln\left(\frac{1}{e}\right) = -\ln(e) = -1$

$= \frac{1}{e}\cdot(-1) = -\frac{1}{e}$

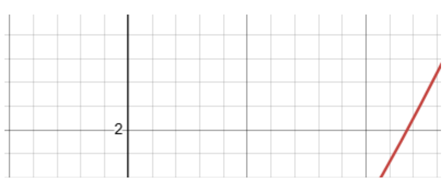
MIN $\left(\frac{1}{e}; -\frac{1}{e}\right)$

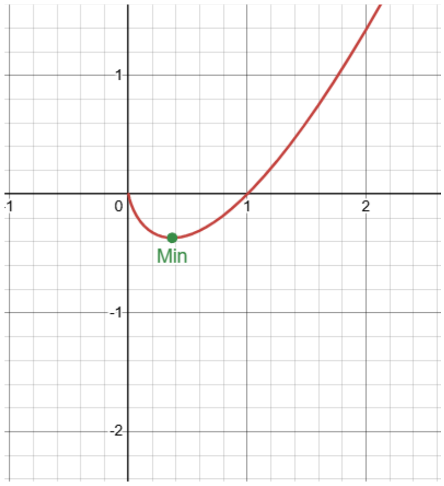
$f''(x) = \frac{1}{x}$

$f''(x) \geq 0 \rightarrow \frac{1}{x} \geq 0 \quad \forall x\in\mathbb{R}$

$x > 0$

No flessi





89) $y = x^2 e^x$

CE $\forall x \in \mathbb{R}$

D $(-\infty; \infty)$

No pari

No dispari

Asse y con x=0

$$\begin{cases} x=0 \\ y=0 \end{cases}$$

A(0;0)

Asse x con y=0

$$\begin{cases} x^2 \cdot e^x = 0 \\ y=0 \end{cases}$$

$x^2=0 \to x=0$

$e^x=0$ mai

Segno $x^2>0 \to \forall x \in \mathbb{R}$

$e^x > 0 \to \forall x \in D$

$$\lim_{x\to-\infty} x^2 e^x = \infty \cdot e^{-\infty} = \infty \cdot 0$$

$$\to \lim_{x\to-\infty} \frac{e^x}{x^{-2}} \text{ De L'H.} \to \lim_{x\to-\infty} \frac{e^x}{-2x}$$

$$\to \text{De L'H.} \to \lim_{x\to-\infty} \frac{e^x}{-2} \to \frac{e^{-\infty}}{-2} = \frac{0}{-2} = 0$$

As. Oriz. sinistro $y=0^-$

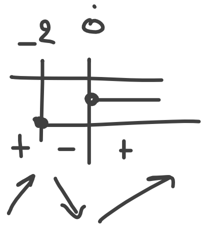
$$\lim_{x\to\infty} x^2 e^x = \infty \cdot e^{\infty} = \infty \cdot \infty = \infty$$

$f'(x) = 2xe^x + x^2 e^x$

$\downarrow$

$xe^x(2+x)$

$f'(x) \geq 0 \Rightarrow x \cdot e^x \geq 0 \; \begin{cases} x \geq 0 \\ e^x > 0 \Rightarrow \forall x \in \mathbb{R} \end{cases}$



$2 + x \geq 0 \Rightarrow x \geq -2$

$f(-2) = (-2)^2 \cdot e^{-2} = \frac{4}{e^2} \; (\simeq 0{,}54)$

$\left(-2; \frac{4}{e^2}\right)$

$\text{MAX}\left(-2; \frac{4}{e^2}\right)$

$f(0) = 0^2 \cdot e^0 = 0 \cdot 1 = 0$

$\text{MIN}\ (0; 0)$

$f''(x)$ per $2xe^x \Rightarrow 2 \cdot e^x + 2xe^x$

per $x^2e^x \Rightarrow 2xe^x + x^2e^x$

$f''(x) = 2e^x + 2xe^x + 2xe^x + x^2e^x$

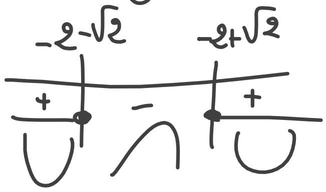
$= 2e^x + x^2e^x + 4xe^x$

$= e^x(2 + x^2 + 4x)$

$f''(x) \geq 0 \Rightarrow \quad e^x > 0 \Rightarrow \forall x \in D$

$x^2 + 4x + 2 \geq 0 \Rightarrow \Delta = 16 - 8 = 8$

$x_{1/2} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$



$f(-2-\sqrt{2}) = (-2-\sqrt{2})^2 \cdot e^{(-2-\sqrt{2})} = (4 + 2 + 4\sqrt{2}) \cdot e^{-2-\sqrt{2}}$

$= (6 + 4\sqrt{2}) \cdot e^{-2-\sqrt{2}}$

$\text{fl}\ \left[-2-\sqrt{2}; (6+4\sqrt{2})\, e^{-2-\sqrt{2}}\right]$

flesso per $\left[-2-\sqrt{2}; (6+4\sqrt{2})e \quad \right]$

$$f(-2+\sqrt{2}) = (\sqrt{2}-2)^2 \cdot e^{(-2+\sqrt{2})} = (2+4-4\sqrt{2})e^{(-2+\sqrt{2})} =$$

$$= (6-4\sqrt{2})e^{(\sqrt{2}-2)}$$

flesso per $\left[-2+\sqrt{2}; (6-4\sqrt{2})e^{(\sqrt{2}-2)}\right]$