COMPOSTE

martedì 4 febbraio 2025 17:57

157) $y = (x^2-5)^2$

$f(x)^n \rightarrow n \cdot [f(x)]^{n-1} \cdot f'(x)$

$$y' = 2(x^2-5)(2x) = 4x(x^2-5)$$

158) $y = (x^2+1)^3$

$$y' = 3(x^2+1)^2(2x) = 6x(x^2+1)^2$$

159) $y = (3x^4 - 2x^3)^3$

$$y' = 3(3x^4-2x^3)^2(12x^3-6x^2)$$

160) $y = (9x^5 + 6x^3 - 11x - 2)^6$

$$y' = 6(9x^5+6x^3-11x-2)^5(45x^4+18x^2-11)$$

161) $y = \left(\frac{1}{3}x^3 - 2x^2 + 4x\right)^4$

$$y' = 4\left(\frac{1}{3}x^3-2x^2+4x\right)^3(x^2-4x+4) = 4\left(\frac{1}{3}x^3-2x^2+4x\right)^3(x-2)^2$$

162) $y = \sqrt{x-1}$

Per la radice quadra:

$$\sqrt{f(x)} \rightarrow \frac{1}{2\sqrt{f(x)}} \cdot (f'(x))$$

Per le radici ad indice n:

$$\sqrt[n]{f(x)} \rightarrow \frac{1}{n \cdot \sqrt[n]{[f(x)]^{n-1}}} \cdot [f'(x)]$$

$$y' = \frac{1}{2\sqrt{x-1}} \cdot (1)$$

163) $\sqrt{1-x^2}$

163) $y = \sqrt{1-x^2}$

$$y' = \frac{1}{\cancel{2}\sqrt{1-x^2}} \cdot (-\cancel{2}x) = -\frac{x}{\sqrt{1-x^2}}$$

164) $y = \sqrt{x^2+x+1}$

$$y' = \frac{1}{2\sqrt{x^2+x+1}} \cdot (2x+1) = \frac{2x+1}{2\sqrt{x^2+x+1}}$$

165) $y = e^{7x}$

$$y = e^{f(x)} \rightarrow y' = e^{f(x)} \cdot f'(x)$$

$$y' = e^{7x} \cdot (7) = 7e^{7x}$$

166) $y = e^{2x} + 5x$

$$y' = e^{2x} \cdot (2) + 5 = 2e^{2x} + 5$$

167) $y = e^{3x} - 1$

$$y' = e^{3x}(3) = 3e^{3x}$$

168) $y = e^{3x} + \pi$

$$y' = e^{3x}(3) + 0 = 3e^{3x}$$

169) $y = e^{x-1}$

$$y' = e^{x-1}(1) = e^{x-1}$$

170) $y = \ln(x+2)$

$y = \ln[f(x)] \rightarrow y' = \frac{1}{f(x)} \cdot f'(x)$

$y' = \frac{1}{x+2} \cdot 1 = \frac{1}{x+2}$

171) $y = \ln(8x^2 - 4)$

$y' = \frac{1}{8x^2-4} \cdot (16x) = \frac{16x}{8x^2-4} = \frac{\cancel{16}^{4}x}{\cancel{4}_{1}(2x^2-1)} = \frac{4x}{2x^2-1}$

172) $y = \ln(3x^2 + 7x + 5)$

$y' = \frac{1}{3x^2+7x+5} \cdot (6x+7) = \frac{6x+7}{3x^2+7x+5}$

173) $y = \ln[\ln(x)]$

$y' = \frac{1}{\ln(x)} \cdot \frac{1}{x} = \frac{1}{x\ln(x)}$

174) $y = [1 + \ln(x)]^2$ $\quad [f(x)]^n$

$y' = 2[1+\ln(x)] \cdot \left(\frac{1}{x}\right) = \frac{2}{x}[1+\ln(x)]$

175) $y = e^{\left(\frac{x+1}{2x+3}\right)}$ $\quad e^{f(x)} \rightarrow f'(x) = \frac{1\cdot(2x+3)-(x+1)2}{(2x+3)^2} =$

$= \frac{\cancel{2x}+3-\cancel{2x}-2}{(2x+3)^2} = \frac{1}{(2x+3)^2}$

$y' = e^{\left(\frac{x+1}{2x+3}\right)} \cdot \left(\frac{1}{(2x+3)^2}\right)$

176) $y = \sqrt{e^{x^4+x^2+2x}}$

$y' = \frac{1}{2\sqrt{e^{x^4+x^2+2x}}} \cdot \left(e^{x^4+x^2+2x}\right)(4x^3+2x+2) =$

$= \frac{e^{x^4+x^2+2x} \cdot (\cancel{2})(2x^3+x+1)}{\cancel{2}\sqrt{\ldots}} = \frac{e^{x^4+x^2+2x}(2x^3+x+1)}{\ldots}$

$$= \frac{e^{x^4+x^2+2x} \cdot (\cancel{2})(2x^3+x+1)}{\cancel{2}\sqrt{e^{x^4+x^2+2x}}} = \frac{e^{x^4+x^2+2x}(2x^3+x+1)}{\sqrt{e^{x^4+x^2+2x}}}$$

Razionalizzazione

$$\frac{e^{x^4+x^2+2x}(2x^3+x+1)}{\sqrt{e^{x^4+x^2+2x}}} \cdot \frac{\sqrt{e^{x^4+x^2+2x}}}{\sqrt{e^{x^4+x^2+2x}}} =$$

$$= \frac{(\cancel{e^{x^4+x^2+2x}})(2x^3+x+1)\sqrt{e^{x^4+x^2+2x}}}{\cancel{e^{x^4+x^2+2x}}} =$$

$$= \sqrt{e^{x^4+x^2+2x}} \cdot (2x^3+x+1)$$

177) $y = \ln(4x^2-x) + e^{(x^2-x)} - \ln(5)$

$$y' = \frac{1}{4x^2-x} \cdot (8x-1) + e^{(x^2-x)}(2x-1) - 0 =$$

$$= \frac{8x-1}{4x^2-x} + (2x-1)e^{(x^2-x)}$$

178) $y = \ln\left(\frac{2x-1}{x+1}\right)$

$$\hookrightarrow f(x) \rightarrow f'(x) = \frac{2(x+1)-(2x-1)}{(x+1)^2} = \frac{\cancel{2x}+2-\cancel{2x}+1}{(x+1)^2} =$$

$$= \frac{3}{(x+1)^2}$$

$$y' = \frac{1}{\frac{2x-1}{x+1}} \cdot \left(\frac{3}{(x+1)^2}\right) = \frac{\cancel{x+1}}{2x-1}\left(\frac{3}{(x+1)^{\cancel{2}}}\right) = \frac{3}{(2x-1)(x+1)}$$

179) $y = e^{\frac{1+x}{x}} \rightarrow f(x) \rightarrow f'(x) = \frac{1 \cdot x - (1+x) \cdot 1}{x^2} = -\frac{1}{x^2}$

$$y' = e^{\frac{1+x}{x}} \cdot \left(-\frac{1}{x^2}\right) = -\frac{e^{\frac{1+x}{x}}}{x^2}$$

180) $y = \frac{x^2}{(x-1)^3}$

$$y = \frac{f(x)}{g(x)} \rightarrow y' = \frac{[f'(x) \cdot g(x)] - [f(x) \cdot g'(x)]}{[g(x)]^2}$$

$f(x) = x^2 \rightarrow f'(x) = 2x$

$g(x) = (x-1)^3 \rightarrow g'(x) = 3(x-1)^2$

$$g(x) = (x-1)^3 \to g'(x) = 3(x-1)^2$$

$$y' = \frac{\left[2x(x-1)^3 - x^2(3)(x-1)^2\right]}{\left[(x-1)^3\right]^2} = \frac{2x(x-1)^3 - 3x^2(x-1)^2}{(x-1)^6} =$$

$$= \frac{\cancel{(x-1)^2}\left[2x(x-1) - 3x^2\right]}{(x-1)^{\cancel{6}\,4}} = \frac{2x(x-1) - 3x^2}{(x-1)^4} =$$

$$= \frac{2x^2 - 2x - 3x^2}{(x-1)^4} = \frac{-x^2 - 2x}{(x-1)^4} = -\frac{x^2 + 2x}{(x-1)^4}$$

181) $y = \dfrac{(x-1)^2}{(2x-3)^3}$

$$f'(x) = 2(x-1)$$

$$g'(x) = 3(2x-3)^2(2) = 6(2x-3)^2$$

$$y' = \frac{\left[2(x-1)\cdot(2x-3)^3\right] - \left[(x-1)^2 \cdot 6(2x-3)^2\right]}{\left[(2x-3)^3\right]^2} =$$

Raccolgo: $(x-1) = a$, $(2x-3) = b$ $\to$ $2ab^3 - 6a^2b^2$

$$\to 2ab^2(b - 3a)$$

$$= \frac{2(x-1)\cancel{(2x-3)^2}\left[(2x-3) - 3(x-1)\right]}{(2x-3)^{\cancel{6}\,4}} =$$

$$= \frac{2(x-1)\left[2x\cancel{-3} - 3x\cancel{+3}\right]}{(2x-3)^4} = \frac{(2x-2)(-x)}{(2x-3)^4} = \frac{-2x^2+2x}{(2x-3)^4} = \frac{2x(1-x)}{(2x-3)^4}$$

182) $y = \ln^5(x)$

Sostituisco t con $\ln(x)$ per semplificare

$$t = \ln(x) \to t' = \frac{1}{x}$$

Sostituisco t nella funzione

$y(t) = t^5$

Derivo $y(t)$

$y'(t) = 5t^4$

Risostituisco t con $\ln(x)$ e aggiungo la derivata di t (t')

$y' = y'(t) \cdot t'$

$y' = 5[\ln(x)]^4 \cdot \frac{1}{x}$

183) $y = \ln(x)^5$

$y = 5\ln(x)$

$y' = 0 \cdot \ln(x) + 5 \cdot \frac{1}{x} = \frac{5}{x}$

184) $y = \ln^8(x^{10})$

$t = \ln(x)^{10} \rightarrow t = 10\ln(x) \rightarrow t' = 0 \cdot \ln(x) + 10 \cdot \frac{1}{x} = \frac{10}{x}$

$y(t) = t^8 \rightarrow y'(t) = 8t^7 \cdot t'$

$y' = 8\underbrace{\ln^7(x^{10})}_{t^7} \cdot \frac{10}{x} = \frac{80 \ln^7(x^{10})}{x}$

185) $y = e^{\sqrt{\ln(x)}}$ $\quad e^{f(x)} \rightarrow f(x) = \sqrt{\ln(x)} \rightarrow f'(x) = \frac{1}{2\sqrt{\ln(x)}} \cdot \frac{1}{x}$

$y' = e^{\sqrt{\ln(x)}} \cdot \frac{1}{2x\sqrt{\ln(x)}}$

186) $y = \ln\left[\sqrt{e^{x^3}}\right]$

$y = \ln[f(g(x))]$ dove $f(g(x)) = \sqrt{e^{x^3}}$

$g(x) = e^{x^3} \rightarrow g'(x) = 3x^2 \cdot e^{x^3}$

La derivata della radice di $g(x)$, cioè $f'(g(x))$:

La derivata della radice di $g(x)$, cioe' $f'(g(x))$:

$$f'[g(x)] = \frac{1}{2\sqrt{e^{x^3}}} \cdot 3x^2 e^{x^3} = \frac{3x^2 e^{x^3}}{2\sqrt{e^{x^3}}}$$

Derivo il radicando, poi la radice, poi il logaritmo naturale

$$y' = \frac{1}{\sqrt{e^{x^3}}} \cdot \frac{3x^2 e^{x^3}}{2\sqrt{e^{x^3}}} = \frac{3x^2\cancel{e^{x^3}}}{2\cancel{e^{x^3}}} = \frac{3}{2}x^2$$

187) $y = \underset{f(x)}{(x+1)^2} \cdot \underset{g(x)}{e^{(x^2+2x)}}$

$$f'(x) = 2(x+1)$$

$$g'(x) = e^{x^2+2x} \cdot (2x+2) = 2(x+1)e^{(x^2+2x)}$$

$$y' = 2(x+1) \cdot e^{(x^2+2x)} + (x+1)^2 \cdot 2(x+1)e^{(x^2+2x)} =$$

$$= 2(x+1)e^{x^2+2x} + 2(x+1)^3 e^{x^2+2x} =$$

$$= 2(x+1)e^{x^2+2x}\left[1 + (x+1)^2\right] =$$

$$= 2(x+1)e^{x^2+2x}\left(x^2+2x+2\right)$$

188) $y = \sqrt[3]{\ln(x^3+1)^2}$

derivata di $(x^3+1)^2 \rightarrow 2(x^3+1)(3x^2) = 6x^2(x^3+1)$

" " $\ln(x^3+1)^2 \rightarrow \frac{1}{(x^3+1)^{\cancel{2}}} \cdot 6x^2\cancel{(x^3+1)} = \frac{6x^2}{x^3+1}$

" radice cubica:

$$\frac{1}{\cancel{3}\sqrt[3]{\left[\ln(x^3+1)^2\right]^2}} \cdot \frac{\overset{2}{\cancel{6}}x^2}{x^3+1}$$

$$y' = \frac{x^2}{\sqrt[3]{\left[\ln(x^3+1)^2\right]^2} \cdot (x^3+1)}$$

$$y = \frac{\wedge}{\sqrt[3]{[\ln(x^3-1)^2]^2 \cdot (x^3-1)}}$$

189) $y = \ln\left(\frac{e^x}{e^x+1}\right)$

$$\hookrightarrow f'(x) = \frac{e^x(e^x+1) - e^x(e^x)}{(e^x+1)^2} = \frac{e^{2x}+e^x-e^{2x}}{(e^x+1)^2} = \frac{e^x}{(e^x+1)^2}$$

$$y' = \frac{1}{\frac{e^x}{e^x+1}} \cdot \frac{e^x}{(e^x+1)^2} = \frac{\cancel{e^x+1}}{\cancel{e^x}} \cdot \frac{e^x}{(e^x+1)^{\cancel{2}}} = \frac{1}{e^x+1}$$

190) $y = \ln\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$

$$f(x) = \sqrt{1+x} \rightarrow f'(x) = \frac{1}{2\sqrt{x+1}}$$

$$g(x) = \sqrt{1-x} \rightarrow f'(x) = \frac{-1}{2\sqrt{1-x}}$$

$$h(x) = \frac{\sqrt{1+x}}{\sqrt{1-x}} \rightarrow h'(x) = \frac{\left(\frac{1}{2\sqrt{x+1}} \cdot \sqrt{1-x}\right) - \left(\sqrt{1+x} \cdot \left(\frac{-1}{2\sqrt{1-x}}\right)\right)}{\boxed{(\sqrt{1-x})^2}} =$$

$$\Rightarrow \text{Numeratore}: \frac{\sqrt{1-x}}{2\sqrt{x+1}} + \frac{\sqrt{1+x}}{2\sqrt{1-x}} = \frac{2[(\sqrt{1-x})(\sqrt{1-x})] + 2[(\sqrt{1+x})(\sqrt{1+x})]}{4(\sqrt{x+1})(\sqrt{x-1})} =$$

$$= \frac{2(1-x)+2(x+1)}{4(\sqrt{x+1})(\sqrt{1-x})} = \frac{2-2x+2x+2}{D} = \frac{\cancel{4}}{\cancel{4}D} =$$

$$= \frac{1}{(\sqrt{x+1})(\sqrt{1-x})}$$

$$\rightarrow h'(x) = \frac{1}{(\sqrt{x+1})(\sqrt{1-x})\boxed{(1-x)}}$$

$$h'(x) = \frac{1}{(\sqrt{x+1})(1-x)^{1/2}(1-x)} = \frac{1}{(\sqrt{x+1})\left(\sqrt{(1-x)^3}\right)}$$

$$y' = \frac{1}{\ldots} \cdot \frac{1}{\ldots} = \frac{\sqrt{1-x}}{\ldots} \cdot \frac{1}{\ldots} =$$

$$y' = \frac{1}{\frac{\sqrt{1+x}}{\sqrt{1-x}}} \cdot \frac{1}{(\sqrt{x+1})\left(\sqrt{(1-x)^3}\right)} = \frac{\sqrt{1-x}}{\sqrt{1+x}} \cdot \frac{1}{(\sqrt{x+1})\left(\sqrt{(1-x)^3}\right)} =$$

$$= \frac{\sqrt{\cancel{1-x}}}{(1+x)\,(\cancel{\sqrt{1-x}})(1-x)} =$$

↳ uso il denominatore scomposto, invece di $\sqrt{(1-x)^3}$

$$= \frac{1}{\underbrace{(x+1)(1-x)}_{\text{diff. di quadrati}}} = \frac{1}{1-x^2}$$

$(a+b)(a-b) = a^2 - b^2$

191) $y = \underbrace{(x+1)^2}_{f(x)} \cdot \underbrace{\ln (x+1)^2}_{\ln[g(x)]}$

$f'(x) = 2(x+1)$

$\ln'(x) \to \frac{1}{(x+1)^2} \cdot 2(x+1) = \frac{2\cancel{(x+1)}}{(x+1)^{\cancel{2}}} = \frac{2}{x+1}$

$$y' = \left[2(x+1) \cdot \ln (x+1)^2\right] + \left[(x+1)^{\cancel{2}} \cdot \frac{2}{\cancel{x+1}}\right] =$$

$$= 2(x+1)\ln(x+1)^2 + 2(x+1) =$$

$$= 2(x+1)\left[\ln(x+1)^2 + 1\right]$$

192) $y = e^{\left(\frac{x^3-x^2}{x^2+1}\right)} \cdot (x^2+1)^2$

$f(x) \to f'(x) = \frac{(3x^2-2x)(x^2+1)-(x^3-x^2)(2x)}{(x^2+1)^2} =$

$$= \frac{3x^4+3x^2-\cancel{2x^3}-2x-2x^4+\cancel{2x^3}}{D^2} = \frac{x^4+3x^2-2x}{(x^2+1)^2}$$

$$y' = e^{\left(\frac{x^3-x^2}{x^2+1}\right)} \cdot \frac{(x^4+3x^2-2x)}{\cancel{(x^2+1)^2}} \cdot \cancel{(x^2+1)^2} + e^{\left(\frac{x^3-x^2}{x^2+1}\right)} \cdot 2(x^2+1)(2x) =$$

$$e^{\left(\frac{x^3-x^2}{x^2+1}\right)} \left(\dots \right) \quad e^{\left(\frac{x^3-x^2}{x^2+1}\right)} \left(\dots 3 \dots \right)$$

$$\cancel{(x^2+1)}$$

$$= e^{\left(\frac{x^3-x^2}{x^2+1}\right)}(x^4+3x^2-2x) + e^{\left(\frac{x^3-x^2}{x^2+1}\right)}(4x^3+4x) =$$

$$= e^{\left(\frac{x^3-x^2}{x^2+1}\right)}\left[(x^4+3x^2-2x+4x^3+4x)\right] =$$

$$= e^{\left(\frac{x^3-x^2}{x^2+1}\right)}\left[x^4+4x^3+3x^2+2x\right] =$$

$$= e^{\left(\frac{x^3-x^2}{x^2+1}\right)}(x)(x^3+4x^2+3x+2)$$

193) $y = e^{2x}\cdot \ln(x+1)$

$$y' = \left[(e^{2x}\cdot 2)(\ln(x+1))\right] + \left[e^{2x}\cdot \frac{1}{x}\right] =$$

$$= 2e^{2x}\ln(x+1) + \frac{e^{2x}}{x+1} = \frac{2xe^{2x}\ln(x+1)+e^{2x}}{x+1} =$$

$$= \frac{e^{2x}\left[2x\ln(x+1)+1\right]}{x+1}$$

194) $y = \ln\left[3\left(x+\sqrt{9+x^2}\right)\right]$

$f(x) = 9+x^2 \rightarrow f'(x) = 2x$

$g(x) = \sqrt{9+x^2} \rightarrow g'(x) = \frac{1}{\cancel{2}\sqrt{9+x^2}}\cdot \cancel{2}x = \frac{x}{\sqrt{9+x^2}}$

$h(x) = x+\sqrt{9+x^2} \rightarrow h'(x) = 1 + \frac{x}{\sqrt{9+x^2}}$

$i(x) = 3\left(x+\sqrt{9+x^2}\right) \rightarrow i'(x) = 0\cdot\left(x+\sqrt{9+x^2}\right) + 3\left(1+\frac{x}{\sqrt{9+x^2}}\right) =$

$= 3\left(1+\frac{x}{\sqrt{9+x^2}}\right)$

$$y' = \frac{1}{\cancel{3}\left(x+\sqrt{9+x^2}\right)}\cdot(\cancel{3})\left(1+\frac{x}{\sqrt{9+x^2}}\right) =$$

$$= \frac{1}{\cancel{x+\sqrt{9+x^2}}}\cdot\left(\frac{\cancel{\sqrt{9+x^2}+x}}{\sqrt{9+x^2}}\right) =$$

$$= \frac{1}{\sqrt{9+x^2}}$$

195) $y = \frac{e^x}{\sqrt{e^x - 1}}$

$f(x) = \sqrt{e^x - 1} \rightarrow f'(x) = \frac{e^x}{2\sqrt{e^x-1}}$

$$y' = \frac{\left[e^x\left(\sqrt{e^x-1}\right)\right] - \left[e^x\left(\frac{e^x}{2\sqrt{e^x-1}}\right)\right]}{\left(\sqrt{e^x-1}\right)^2} =$$

$$Num \Rightarrow e^x\left(\sqrt{e^x-1}\right) - \frac{e^{2x}}{2\sqrt{e^x-1}} = \frac{2e^x(e^x-1) - e^{2x}}{2\sqrt{e^x-1}} =$$

$$= \frac{2e^{2x} - 2e^x - e^{2x}}{2\sqrt{e^x-1}} = \frac{e^{2x} - 2e^x}{2\sqrt{e^x-1}} = \frac{e^x(e^x-2)}{2\sqrt{e^x-1}}$$

$$y' = \frac{\frac{e^x(e^x-2)}{2\sqrt{e^x-1}}}{e^x-1} = \frac{e^x(e^x-2)}{2\sqrt{e^x-1}} \cdot \frac{1}{e^x-1}$$

196) $y = \log\left[\ln(5x+3)\right]$ $\qquad y = \log_a\left[f(x)\right] \rightarrow y' = \frac{f'(x)}{f(x)\cdot \ln(a)}$

$f(x) = 5x-3 \rightarrow f'(x) = 5$

$g(x) = \ln(5x+3) \rightarrow g'(x) = \frac{1}{5x+3} \cdot 5 = \frac{5}{5x+3}$

$$y' = \frac{\frac{5}{5x+3}}{\ln(5x+3)\cdot \underline{\ln(10)}} = \frac{5}{5x+3} \cdot \frac{1}{\ln(5x+3)\cdot \underline{1}} =$$ $\qquad \log_{10}(10) = 1$

$$= \frac{5}{(5x+3)\ln(5x+3)}$$

197) $y = \sqrt{\ln(x)}$

$$y' = \frac{1}{2\sqrt{\ln(x)}} \cdot \frac{1}{x} = \frac{1}{2x\sqrt{\ln(x)}}$$

198) $y = \dfrac{3x^3}{\sqrt[3]{3x+3}}$

$\sqrt[n]{a^m} \to a^{\frac{m}{n}}$

$f(x) = 3x+3 \to f'(x) = 3$

$g(x) = \sqrt[3]{3x+3} \to g(x) = (3x+3)^{\frac{1}{3}} \to g'(x) = \frac{1}{\cancel{3}}(3x+3)^{\frac{1}{3}-1} \cdot \cancel{3} =$

$= (3x+3)^{-\frac{2}{3}} = \dfrac{1}{(3x+3)^{2/3}}$

$h(x) = 3x^3 \to h'(x) = 9x^2$

$$y' = \frac{\left[9x^2\left(\sqrt[3]{3x+3}\right)\right] - \left[3x^3\left(\frac{1}{(3x+3)^{2/3}}\right)\right]}{\left(\sqrt[3]{3x+3}\right)^2}$$

Num: $9x^2(3x+3)^{1/3} - \dfrac{3x^3}{(3x+3)^{\frac{2}{3}}} = \dfrac{9x^2(3x+3)^{\frac{1}{3}}(3x+3)^{\frac{2}{3}} - 3x^3}{(3x+3)^{\frac{2}{3}}} =$

$$= \frac{9x^2(3x+3) - 3x^3}{(3x+3)^{2/3}} = \frac{27x^3 + 27x^2 - 3x^3}{(3x+3)^{2/3}} = \frac{24x^3 + 27x^2}{(3x+3)^{2/3}} =$$

$$= \frac{3x^2(8x+9)}{(3x+3)^{2/3}}$$

$$y' = \frac{\frac{3x^2(8x+9)}{(3x+3)^{2/3}}}{(3x+3)^{2/3}} = \frac{3x^2(8x+9)}{(3x+3)^{2/3}} \cdot \frac{1}{(3x+3)^{2/3}} = \frac{3x^2(8x+9)}{(3x+3)^{4/3}}$$

$$= \frac{3x^2(8x+9)}{\sqrt[3]{(3x+3)^4}} = \frac{3x^2(8x+9)}{\sqrt[3]{(3x+3)^3(3x+3)}} =$$

$$= \frac{3x^2(8x+9)}{(3x+3)\sqrt[3]{3x+3}} = \frac{\cancel{3}x^2(8x+9)}{\cancel{3}(x+1)\sqrt[3]{3x+3}} = \frac{x^2(8x+9)}{(x+1)\sqrt[3]{3x+3}}$$