FRATTE Es. 1 - 9

giovedì 30 gennaio 2025 17:16

2) $y = \dfrac{x-3}{x^2-4}$

C.E.: $x^2-4 \neq 0 \Rightarrow x \neq \pm 2$

D: $(-\infty;-2)\cup(-2;2)\cup(2;\infty)$

$f(-x) = \dfrac{-x-3}{x^2-4}$ No pari

$-f(x) = \dfrac{-x+3}{x^2-4}$ No Dispari

Asse y con $x=0$

$\begin{cases} x=0 \\ y=\frac{3}{4} \end{cases}$

Asse x con $y=0$

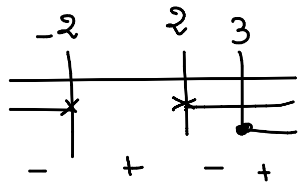
$\begin{cases} x-3=0 \Rightarrow x=3 \\ y=0 \end{cases}$

$A(0;\frac{3}{4})$

$B(3;0)$

$x-3 \geq 0 \Rightarrow x \geq 3$

$x^2-4 > 0 \Rightarrow x > \pm 2$



$f(x) < 0$ per $x < -2 \cup 2 < x \leq 3$

$f(x) > 0$ per $-2 < x < 2 \cup x > 3$

$\lim_{x \to \pm\infty} \dfrac{x-3}{x^2-4} = \dfrac{\infty}{\infty} = F.I. \rightarrow$ Gerarchia

$\left.\lim_{x \to \pm\infty} \dfrac{x}{x^2} = \dfrac{1}{x} = \dfrac{1}{\pm\infty} = 0 \right]$ As. Or. $y=0$

$\lim_{x \to -2^-} \dfrac{(-2^-)-3}{(-2^-)^2-4} = \dfrac{-}{+} = -\infty$

$\lim_{x \to -2^+} f(x) = \dfrac{-}{-} = +\infty$

] As. V. $x=-2$

$\lim_{x \to 2^-} f(x) = \dfrac{-}{-} = +\infty$

$\lim_{x \to 2^+} f(x) = \dfrac{-}{+} = -\infty$

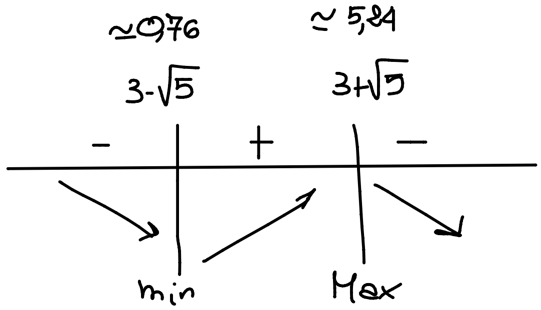
] As. V. $x=2$

$x \to 2^+$

$$y' = \frac{1\cdot(x^2-4)-(x-3)(2x)}{(x^2-4)^2} = \frac{x^2-4-2x^2+6x}{D^2} = \frac{-x^2+6x-4}{(x^2-4)^2}$$

$$y' \geq 0 \to -x^2+6x-4 \geq 0 \to x^2-6x+4 \leq 0 \to \Delta = 36-16=20 \quad \left[\sqrt{20} = 2\sqrt{5}\right]$$

$$x_{1/2} = \frac{6 \pm 2\sqrt{5}}{2} = \frac{2(3\pm\sqrt{5})}{2} = 3 \pm \sqrt{5}$$



$$f(3-\sqrt{5}) = \frac{(3-\sqrt{5})-3}{(3-\sqrt{5})^2-4} = \frac{-\sqrt{5}}{-3,41} \simeq 0,65 \quad (\text{min})$$

$$f(3+\sqrt{5}) = \frac{3+\sqrt{5}-3}{(3+\sqrt{5})^2-4} = \frac{\sqrt{5}}{23,41} \simeq 0,095 \quad (\text{Max})$$

$y'' =$ $f(x) = -x^2+6x-4 \to f'(x) = -2x+6$

$g(x) = \underbrace{x^4-8x^2+16}_{(x^2-4)^2} \to g'(x) = 4x^3-16x$

$4x(x^2-4)$

$$y'' = \frac{(-2x+6)(x^2-4)^2-(-x^2+6x-4)(4x^3-16x)}{(x^2-4)^4} =$$

$$= \frac{\overset{a}{(-2x+6)}\overset{b^2}{(x^2-4)^2}-\overset{c}{(-x^2+6x-4)}\overset{b}{(x^2-4)}\overset{d}{(4x)}}{D^4} =$$

$ab^2-cbd \to b(ab-cd)$

$$= \frac{\cancel{(x^2-4)}\left[(-2x+6)(x^2-4)-(-x^2+6x-4)(4x)\right]}{D^{\cancel{4}}} =$$

$$= \frac{-2x^3+8x+6x^2-24+4x^3-24x^2+16x}{D^3} =$$

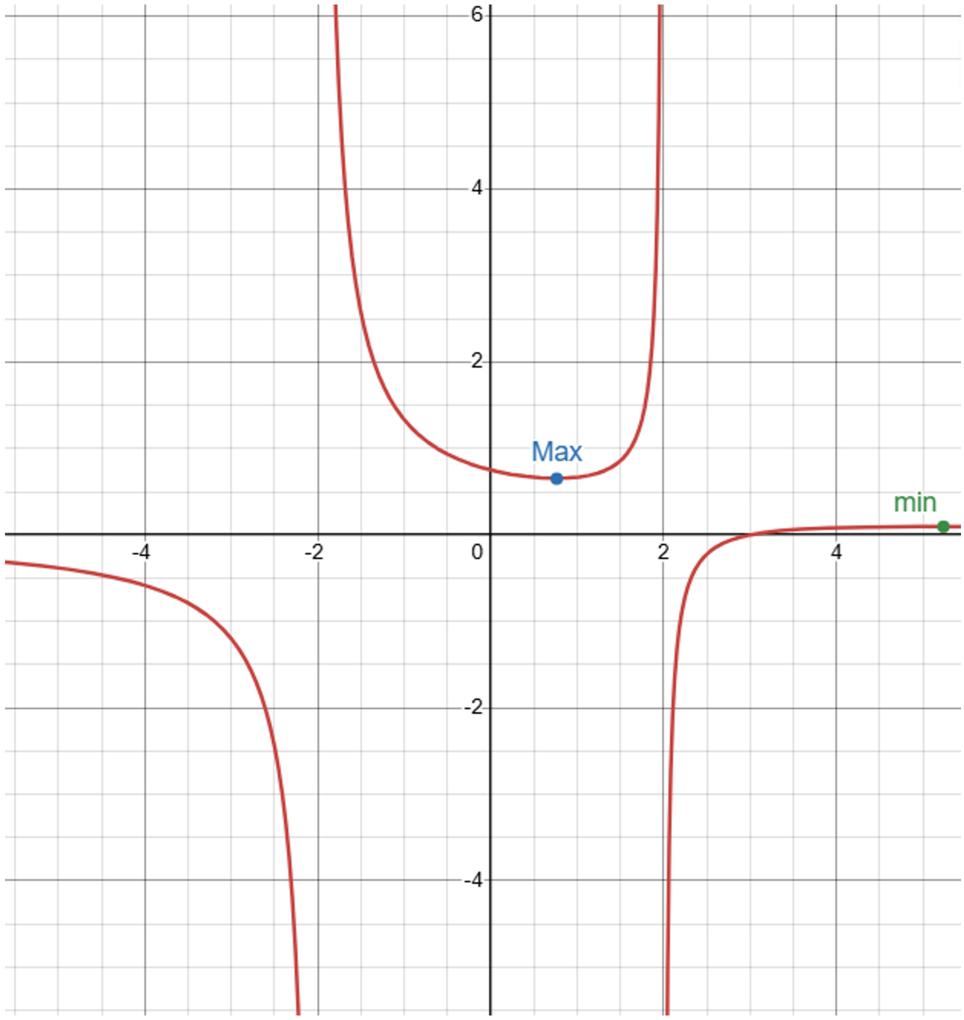
$$= \frac{2x^3-18x^2+24x-24}{\ldots}$$

$$= \frac{2x^3 - 18x^2 + 24x - 24}{(x^2-4)^3}$$

$$y'' \geq 0 \rightarrow \frac{2x^3 - 18x^2 + 24x - 24}{2} \geq 0 \rightarrow \underbrace{x^3 - 9x^2}_{} + \underbrace{12x - 12}_{} \geq 0 \rightarrow$$

$$\rightarrow x^2(x-9) + 12(x-1) \geq 0$$

No sol. in $\mathbb{R}$



3) $y = \frac{x^2}{x-2}$

CE: $x - 2 \neq 0 \Rightarrow x \neq 2$

D: $(-\infty; 2) \cup (2; \infty)$

$f(-x) = \frac{x^2}{-x-2}$ No Pari

$-f(x) = \frac{-x^2}{x-2}$ No dispari

Asse y con $x = 0$ Asse x con $y = 0$

Asse y con $x=0$

$$\begin{cases} x=0 \\ y=\dfrac{0^2}{0-2}=0 \end{cases}$$

Asse x con $y=0$

$$\begin{cases} x^2=0 \to x=0 \\ y=0 \end{cases}$$

$A(0;0)$

$x^2>0 \quad \forall x\in\mathbb{R}$

$x-2>0 \to x>2$

$$\begin{array}{c|c} & 2 \\ - & + \end{array}$$

$f(x)<0 \to x<2$

$f(x)>0 \to x>2$

$$\left.\begin{aligned} \lim_{x\to 2^-} \frac{(2^-)^2}{2^- -2} &= \frac{+}{-} = -\infty \\ \lim_{x\to 2^+} \frac{(2^+)^2}{2^+ -2} &= \frac{+}{+} = +\infty \end{aligned}\right]$$ As. V. $x=2$

$\lim_{x\to\pm\infty} \frac{x^2}{x-2} \to \frac{\infty}{\infty} = f.I. \to$ Gerarchia $\to \lim_{x\to\pm\infty} \frac{x^2}{x} = x = \pm\infty$] No A.Or.

$y=mx+q$

$m=\lim_{x\to\infty} \frac{x^2}{x-2}\cdot\frac{1}{x} \Rightarrow \frac{x^2}{x^2-2x} \to$ Gerarchia $\to \lim_{x\to\infty} \frac{x^2}{x^2} = 1$

$q=\lim_{x\to\infty} \frac{x^2}{x-2} - x \to \frac{x^2-x(x-2)}{x-2} \to \frac{\cancel{x^2}-\cancel{x^2}+2x}{x-2} \to$ Gerarchia $\to$

$\to \lim_{x\to\infty} \frac{2x}{x} = 2$

As. Ob. con eq:

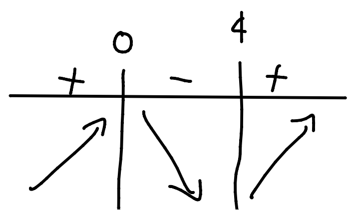
$y=x+2$

x	y
0	2
1	3

$$y'=\frac{2x(x-2)-x^2(1)}{(x-2)^2}=\frac{2x^2-4x-x^2}{D^2}=\frac{x^2-4x}{D^2}=\frac{x(x-4)}{(x-2)^2}$$

$y'\geq 0 \to x(x-4)\geq 0 \to x\geq 0$

$x-4\geq 0 \to x\geq 4$



$x-4 \geq 0 \rightarrow x \geq 4$

$\nearrow \mid \searrow \mid \nearrow$

Max min

$f(0) = 0$ Max $(0;0)$

$f(4) = \dfrac{4^2}{4-2} = \dfrac{16}{2} = 8$ min $(4;8)$

$y'' = \quad f(x) = x^2 - 4x \rightarrow f'(x) = 2x - 4$

$\qquad g(x) = (x-2)^2 \rightarrow g'(x) = 2(x-2) = 2x - 4$

$$y'' = \frac{(2x-4)(x-2)^2 - (x^2-4x)(2x-4)}{(x-2)^4} = \frac{(2x-4)\left[(x-2)^2 - (x^2-4x)\right]}{D^4} =$$

$$= \frac{2\cancel{(x-2)}\left[(x-2)^2 - (x^2-4x)\right]}{(x-2)^{\cancel{4}}} = \frac{2(x-2)^2 - x^2 + 4x}{(x-2)} = \frac{2x^2 - 8x + 8 - x^2 + 4x}{x-2} =$$

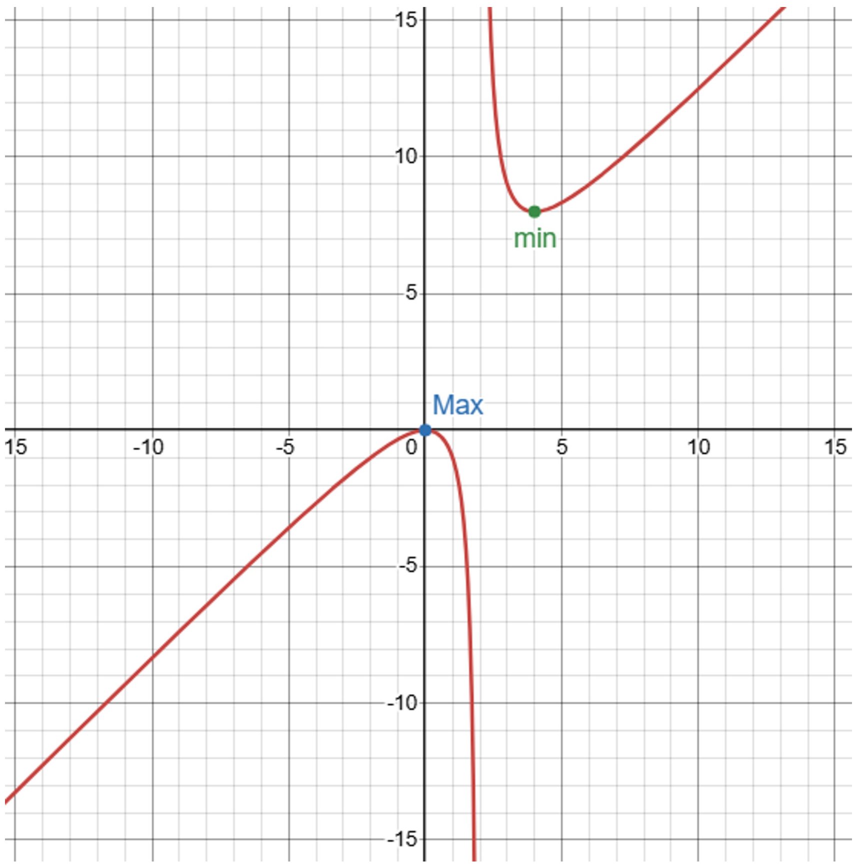
$$= \frac{x^2 - 4x + 8}{x-2}$$

$y'' \geq 0 \rightarrow x^2 - 4x + 8 \geq 0 \rightarrow \Delta = 16 - 32 \rightarrow$ imposs in $\mathbb{R}$

$x - 2 > 0 \rightarrow x > 2$

$\begin{array}{c|c} - & + \\ \cap & \cup \end{array}$ (at 2)

$x = 2$ non è un flesso perché $\nexists D$



4) $y = \dfrac{x^2-3}{x^2-1}$

C.E.: $x^2-1 \neq 0 \Rightarrow x \neq \pm 1$

D: $(-\infty;-1)\cup(-1;1)\cup(1;\infty)$

$f(-x) = \dfrac{x^2-3}{x^2-1}$ Pari

Asse y con $x=0$

$$\begin{cases} x=0 \\ y=\dfrac{0^2-3}{0^2-1} = +3 \end{cases}$$

Asse x con $y=0$

$$\begin{cases} x^2-3=0 \Rightarrow x=\pm\sqrt{3} \\ y=0 \end{cases}$$

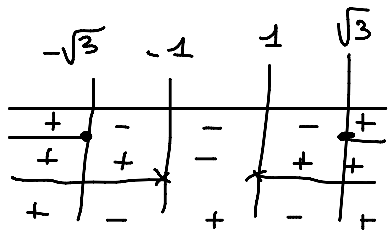
$A(0;3)$

$B(-\sqrt{3};0) \simeq (-1,73;0)$

$C(\sqrt{3};0) \simeq (1,73;0)$

$x^2-3 \geq 0 \Rightarrow x \geq \pm\sqrt{3}$

$x^2-1 > 0 \Rightarrow x > \pm 1$



$f(x)<0$ per

$-\sqrt{3} \leq x < -1 \cup 1 < x \leq \sqrt{3}$

$f(x)>0$ per

$x \leq -\sqrt{3} \cup -1 < x < 1 \cup x \geq \sqrt{3}$

$$\left.\begin{array}{l} \lim\limits_{x\to -1^-} \dfrac{(-1^-)^2-3}{(-1^-)^2-1} = \dfrac{-}{+} = -\infty \\ \lim\limits_{x\to -1^+} f(x) = \dfrac{-}{-} = +\infty \end{array}\right]$$

As. V. $x=-1$

$$\left.\begin{array}{l} \lim\limits_{x\to 1^+} f(x) = \dfrac{-}{+} = -\infty \\ \lim\limits_{x\to 1^-} f(x) = \dfrac{-}{-} = +\infty \end{array}\right]$$

As. V. $x=1$

$\lim\limits_{x\to\pm\infty} \dfrac{x^2-3}{x^2-1} = \dfrac{\infty}{\infty} =$ F. I. $\to$ Gerarchia $\to \lim\limits_{x\to\pm\infty} \dfrac{x^2}{x^2} = 1$] As. Or. $y=1$

$y' = \dfrac{2x(x^2-1)-(x^2-3)2x}{(x-1)^2} = \dfrac{2x^3-2x-2x^3+6x}{D^2} = \dfrac{4x}{(x-1)^2}$

$y' \geq 0 \Rightarrow 4x \geq 0 \Rightarrow x \geq 0$

$$\begin{array}{c|c} & 0 \\ - & + \\ \searrow & \nearrow \\ & \text{min} \end{array}$$

$f(0) = 3 \quad \min(0;3)$

$$y'' = \frac{4(x-1)^2 - 4x(2)(x-1)}{(x-1)^4} = \frac{4(x-1)^2 - 8x(x-1)}{D^4} =$$

$$= \frac{4x^2 - 8x + 4 - 8x^2 + 8x}{(x-1)^4} = \frac{-4x^2+4}{(x-1)^4} = \frac{-4(x^2-1)}{(x-1)^4}$$

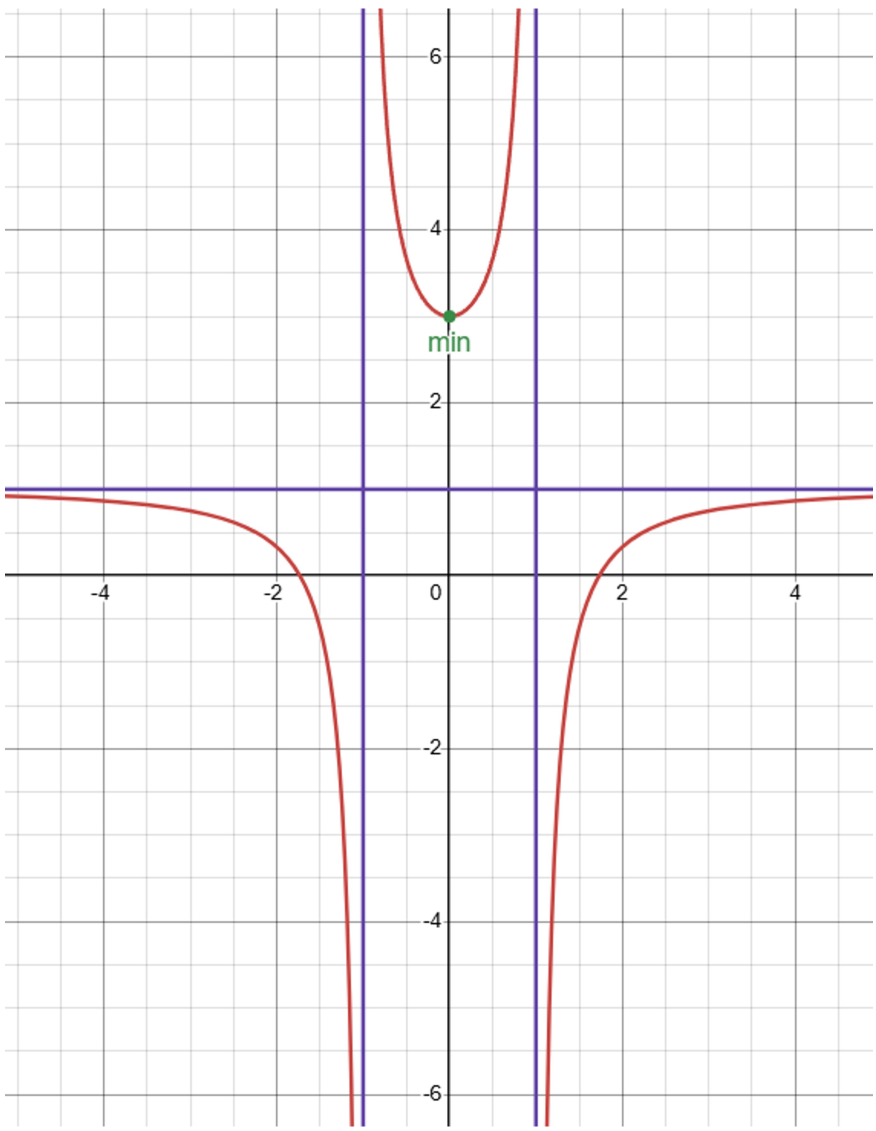
$y'' \geq 0 \Rightarrow \quad -4 \geq 0 \quad$ mai

$\qquad x^2 - 1 \geq 0 \Rightarrow x \geq \pm 1$

$$\begin{array}{c|c|c} & -1 & 1 \\ + & - & + \\ \cup & \cap & \cup \end{array}$$

$x = -1$ e $x = 1$ non sono flessi

$\nexists D$



6) $y=\dfrac{2}{2+x^2}$

C.E.: $2+x^2\neq 0 \rightarrow \forall x\in\mathbb{R}$

D: $(-\infty;\infty)$

$f(-x)=\dfrac{2}{2+x^2}$ Pari

Asse x con $y=0$

$$\begin{cases} 2=0 & \text{mai} \\ y=0 \end{cases}$$

Asse y con $x=0$

$$\begin{cases} x=0 \\ y=\dfrac{2}{2+0^2}=1 \end{cases}$$

$A(0;1)$

$2>0\ \forall x\in\mathbb{R}$ $\dfrac{+}{\ }$

$2+x^2>0\ \forall x\in\mathbb{R}$

$$\left.\begin{array}{l} \lim\limits_{x\to-\infty}\dfrac{2}{2+(-\infty)^2}=\dfrac{2}{+\infty}=0^+ \\ \lim\limits_{x\to+\infty}\dfrac{2}{2+(\infty)^2}=\dfrac{2}{\infty}=0^+ \end{array}\right]$$

As. V. $y=0$

$$y'=\frac{0\cdot(2+x^2)-2(2x)}{(2+x^2)^2}=\frac{-4x}{(2+x^2)^2}$$

$y'\geq 0 \rightarrow -4x\geq 0 \rightarrow x\leq 0$

0
$+\ |\ -$
$\nearrow\ |\ \searrow$
Max

$f(0)=1$ Max$(0;1)$

$$y''=\frac{-4(2+x^2)^2-\overbrace{(-4x)(2)(2+x^2)(2x)}^{-16x^2}}{(2+x^2)^4}=\frac{(\cancel{2+x^2})\left[-4(2+x^2)-(-16x^2)\right]}{D^{\cancel{4}}}=$$

$$=\frac{-8-4x^2+16x^2}{(2+x^2)^3}=\frac{12x^2-8}{D^3}=\frac{4(3x^2-2)}{(2+x^2)^3}$$

$-\sqrt{\frac{2}{3}}$ $\sqrt{\frac{2}{3}}$

$(2+x^2)^3$ $\qquad$ D $\qquad$ $(2+x^2)$

$y'' \geq 0 \rightarrow 4(3x^2-2) \geq 0 \rightarrow$ $\quad$ $4>0$ Sempre

$3x^2-2 \geq 0 \rightarrow x \geq \pm\sqrt{\frac{2}{3}}$

$(2+x^2)^3 > 0 \rightarrow \forall x \in \mathbb{R}$

$-\sqrt{\frac{2}{3}}$ $\quad$ $\sqrt{\frac{2}{3}}$

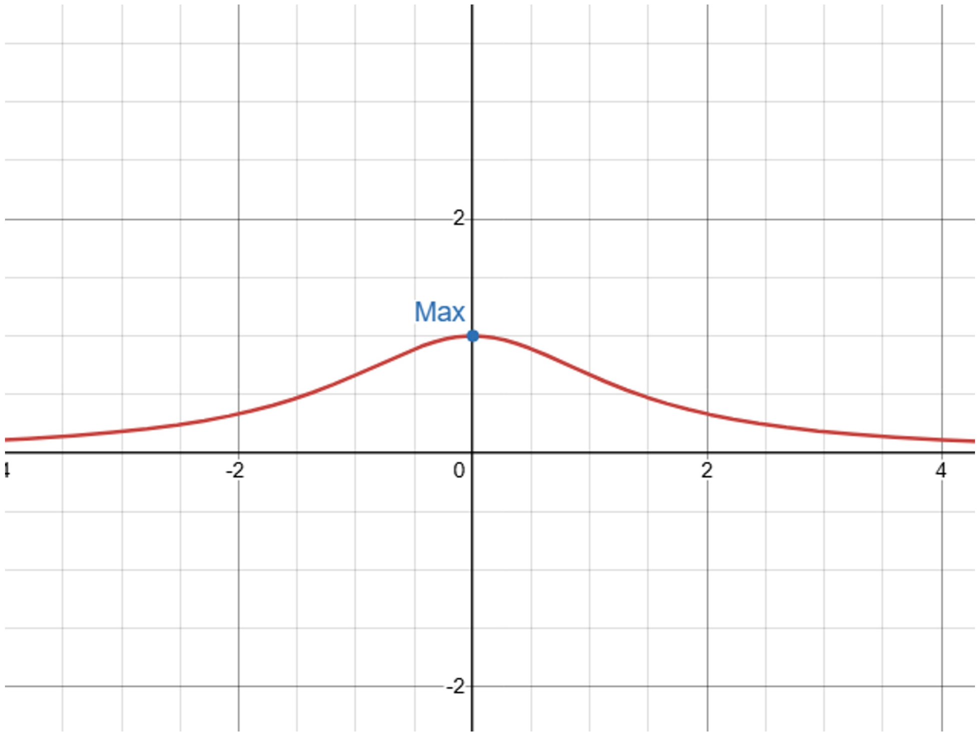
$+ \quad - \quad +$

$\cup \quad \cap \quad \cup$

flessi

flesso$_1$ $(f_1) = -\sqrt{\frac{2}{3}}$ $\quad (\simeq -0{,}82)$

flesso$_2$ $(f_2) = \sqrt{\frac{2}{3}}$ $\quad (\simeq 0{,}82)$



7) $y = \dfrac{x}{x^2-3}$

CE: $x^2-3 \neq 0 \rightarrow x \neq \pm\sqrt{3}$

D: $(-\infty; -\sqrt{3}) \cup (-\sqrt{3}; \sqrt{3}) \cup (\sqrt{3}; \infty)$

$f(-x) = \dfrac{-x}{x^2-3}$ $\quad$ No Pari

$-f(x) = \dfrac{-x}{x^2-3}$ $\quad$ Dispari

Asse y con $x=0$ $\qquad$ Asse x con $y=0$

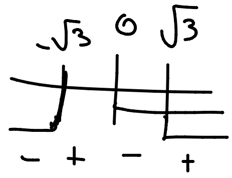
$\begin{cases} x=0 \\ y=0 \end{cases}$ $\qquad$ $\begin{cases} x=0 \\ y=0 \end{cases}$

$\begin{cases} x=0 \\ y=0 \end{cases}$ $\qquad\qquad$ $\begin{cases} x=0 \\ y=0 \end{cases}$

$A(0;0)$

$x \geq 0$

$x^2-3>0 \rightarrow x > \pm\sqrt{3}$



$f(x)<0$ per $x<-\sqrt{3} \cup 0 \leq x < \sqrt{3}$

$f(x)>0$ per $-\sqrt{3}<x \leq 0 \cup x>\sqrt{3}$

$$\lim_{x \to \pm\infty} \frac{x}{x^2-3} \left(\frac{\infty}{\infty} = F.I.\right) \rightarrow G. \rightarrow \frac{x}{x^2} = \frac{1}{x} = \frac{1}{\pm\infty} = 0 \Big] \text{As. Or. } y=0$$

$$\lim_{x \to -\sqrt{3}^-} \frac{-\sqrt{3}^-}{(-\sqrt{3}^-)^2-3} = \frac{-\sqrt{3}^-}{3^- - 3} = \frac{-}{0^+} = \frac{-}{+} = -\infty$$

$-\sqrt{3} \cong -1,73 \rightarrow -\sqrt{3}^- = -1,73 - 0,1 = -1,83$

$$\lim_{x \to -\sqrt{3}^+} \frac{-\sqrt{3}^+}{(-\sqrt{3}^+)^2-3} = \frac{-}{-} = +\infty$$

$\Big]$ As. V. $x = -\sqrt{3}$

$$\lim_{x \to \sqrt{3}^+} f(x) = \frac{+}{+} = +\infty$$

$$\lim_{x \to \sqrt{3}^-} f(x) = \frac{+}{-} = -\infty$$

$\Big]$ As. V. $x = \sqrt{3}$

$$y' = \frac{1\cdot(x^2-3) - x(2x)}{(x^2-3)^2} = \frac{x^2-3-2x^2}{(x^2-3)^2} = \frac{-x^2-3}{(x^2-3)^2}$$

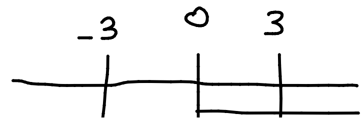
$y' \geq 0 \rightarrow -x^2-3 \geq 0 \rightarrow x^2 \leq -3$ mai

$$y'' = \frac{(-2x)(x^2-3)^2 - (-x^2-3)(2)(x^2-3)(2x)}{(x^2-3)^4} = \frac{(-2x)(x^2-3)^2 - (-x^2-3)(x^2-3)(4x)}{D^4} =$$

$$= \frac{\cancel{(x^2-3)}\left[(-2x)(x^2-3) - (-x^2-3)(4x)\right]}{D^4} = \frac{-2x^3+6x+4x^3+12x}{(x^2-3)^3} =$$

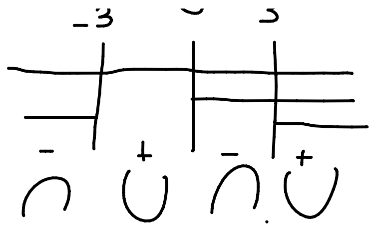
$$= \frac{2x^3+18x}{(x^2-3)^3} = \frac{2x(x^2-9)}{(x^2-3)^3}$$

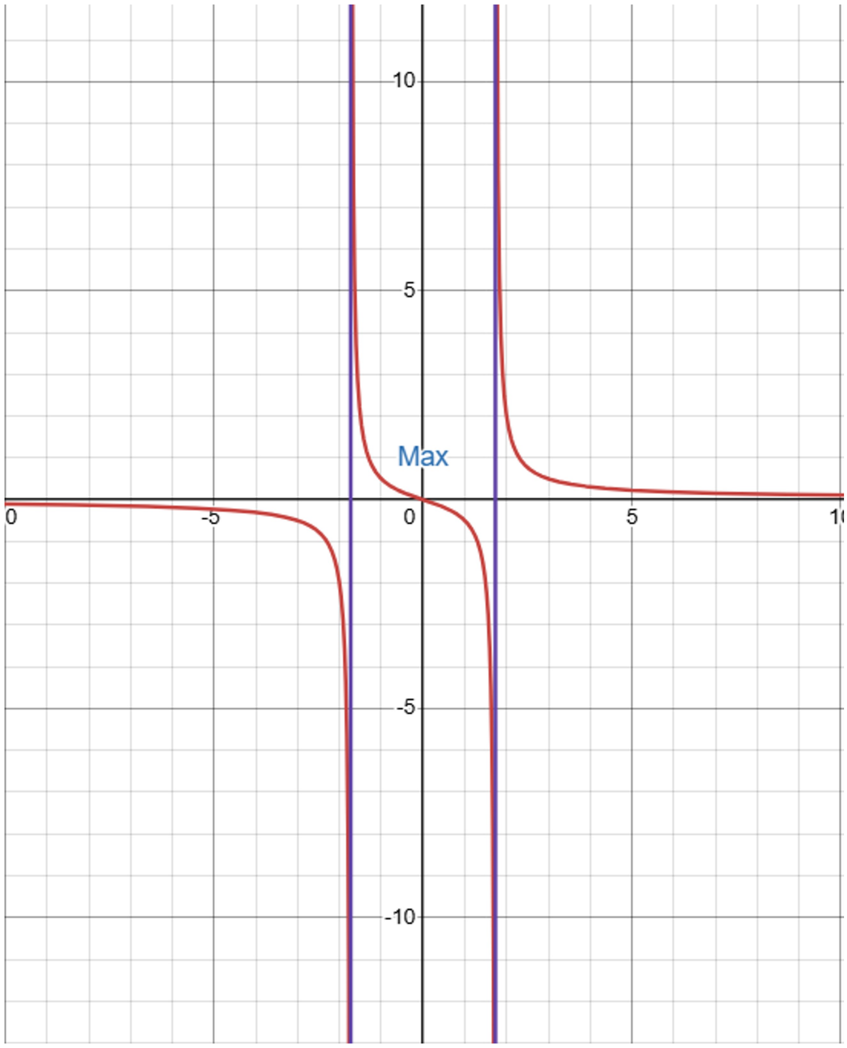
$y'' \geq 0 \rightarrow 2x(x^2-9) \geq 0 \rightarrow 2x \geq 0 \rightarrow x \geq 0$



$y'' \geq 0 \rightarrow 2x(x^2-9) \geq 0 \rightarrow$ $2x \geq 0 \rightarrow x \geq 0$

$x^2 - 9 \geq 0 \rightarrow x \geq \pm 3$





8) $y = \dfrac{x^3}{x^2-9}$

CE: $x^2 - 4 \neq 0 \rightarrow x \neq \pm 2$

D: $(-\infty; -2) \cup (-2; 2) \cup (2; \infty)$

$f(-x) = \dfrac{-x^3}{x^2-4}$ No Pari

$-f(x) \cdot \dfrac{-x^3}{x^2-4}$ Dispari

Asse y con $x = 0$

$\begin{cases} x = 0 \\ y = 0 \end{cases}$

Asse x con $y = 0$

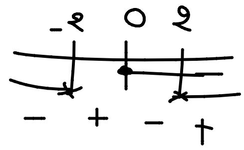
$\begin{cases} x^3 = 0 \rightarrow x = 0 \\ y = 0 \end{cases}$

$\begin{cases} x=0 \\ y=0 \end{cases}$ $\qquad\qquad$ $\begin{cases} \dots \\ y=0 \end{cases}$

$A(0;0)$

$x^3 \geq 0 \rightarrow x \geq 0$

$x^2-4>0 \rightarrow x > \pm 2$



$f(x)<0$ per $x<-2 \cup 0 \leq x < 2$

$f(x)>0$ per $-2 < x \leq 0 \cup x > 2$

$\lim_{x \to -2^-} \frac{(-2^-)^3}{(-2^-)^2-4} = \frac{-}{+} = -\infty$

$\lim_{x \to -2^+} f(x) = \frac{-}{-} = +\infty$

As. V. $x=-2$

$\lim_{x \to 2^-} f(x) = \frac{+}{-} = -\infty$

$\lim_{x \to 2^+} f(x) = \frac{+}{+} = +\infty$

As. V. $x=2$

$\lim_{x \to \pm\infty} \frac{x^3}{x^2-4} \rightarrow G. \rightarrow \frac{x^3}{x^2} = x = \pm\infty$ No As. Or.

$m = \lim_{x \to \infty} \frac{x^3}{x^2-4} \cdot \frac{1}{x} = \frac{x^3}{x^3-4x} \rightarrow G. \rightarrow \frac{x^3}{x^3} = 1$

$q = \lim_{x \to \infty} \frac{x^3}{x^2-4} - x \Rightarrow \frac{x^3-x^3+4x}{x^2-4} \rightarrow \frac{4x}{x^2-4} \rightarrow \frac{4x}{x^2} = \frac{4}{x} = \frac{4}{\infty} = 0$

$y = x$ As. Ob.

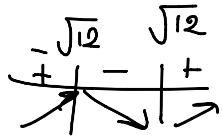
x	y
0	0
1	1

$y' = \frac{3x^2(x^2-4)-(x^3)(2x)}{(x^2-4)^2} = \frac{3x^4-12x^2-2x^4}{D^2} = \frac{x^4-12x^2}{D^2} =$

$= \frac{x^2(x^2-12)}{(x^2-4)^2}$

$y' \geq 0 \rightarrow x^2 \geq 0 \rightarrow \forall x \in \mathbb{R}$

$x^2-12 \geq 0 \rightarrow x \geq \pm\sqrt{12}$

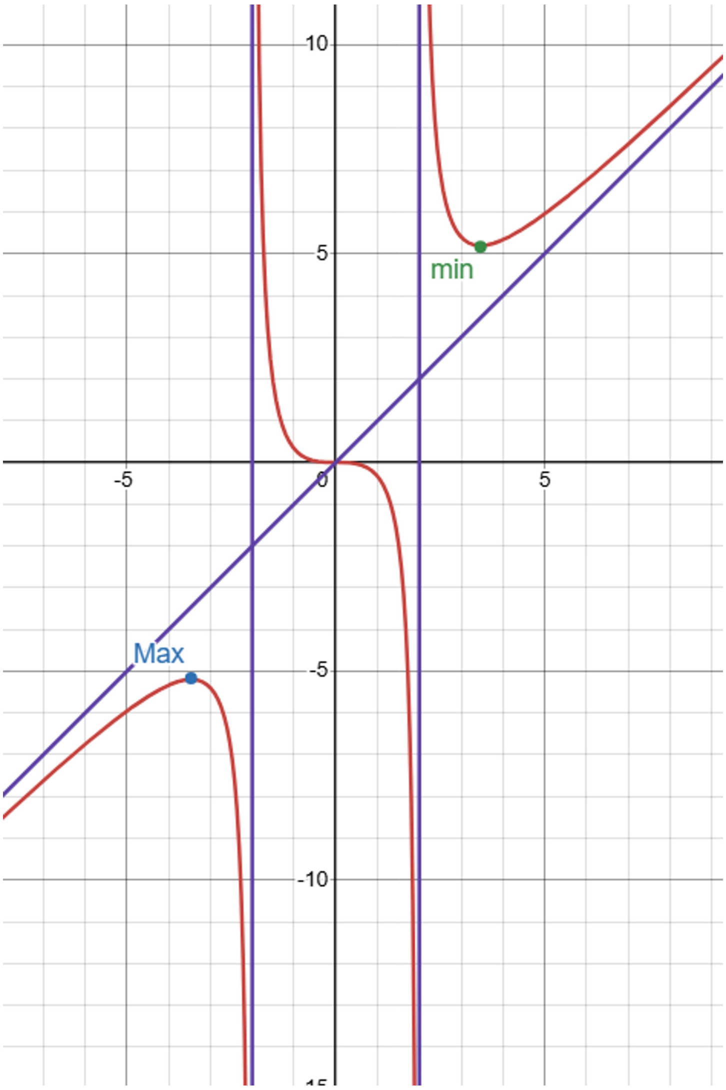


Max min

$$f(-\sqrt{12}) = \frac{(-\sqrt{12})^3}{(-\sqrt{12})^2-4} = \frac{(-3{,}46)^3}{8} \simeq -5{,}17 \quad [\text{Max}]$$

$-3{,}46$

$$f(\sqrt{12}) = \frac{(\sqrt{12})^3}{(\sqrt{12})^2-4} = \frac{(3{,}46)^3}{8} \simeq 5{,}17 \quad [\text{min}]$$



9) $y = \dfrac{2x^2}{3+x^2}$

C.E: $3+x^2 \neq 0 \quad \forall x \in \mathbb{R}$

D: $(-\infty; \infty)$

$f(-x) = \dfrac{2x^2}{3+x^2}$ Pari

Asse y con $x=0$ Asse x con $y=0$

$\{ 2x^2 = 0 \rightarrow x = 0$

Asse y con $x = 0$

$$\begin{cases} x=0 \\ y=0 \end{cases}$$

Asse x con $y=0$

$$\begin{cases} 2x^2=0 \rightarrow x=0 \\ y=0 \end{cases}$$

$A(0;0)$

$$\frac{2x^2>0 \rightarrow \forall x\in\mathbb{R}}{3+x^2>0 \rightarrow \forall x\in\mathbb{R}} \qquad \frac{+}{} \qquad f(x)>0 \text{ in } D$$

$$\lim_{x\to\pm\infty} \frac{2x^2}{x^2+3} \rightarrow G. \rightarrow \frac{2x^2}{x^2} = 2 \ \Big] \text{ As. or. } y=2$$

$$y' = \frac{4x(x^2+3)-(2x^2)(2x)}{(x^2+3)^2} = \frac{4x^3+12x-4x^3}{D^2} = \frac{12x}{(x^2+3)^2}$$

$y' \geq 0 \rightarrow 12x \geq 0 \rightarrow x \geq 0$

$f(0)=0 \quad \min(0;0)$

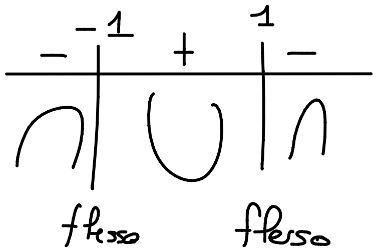
$$y'' = \frac{12(x^2+3)^2-(12x)\overset{2\cdot 2x}{(4x)}(x^2+3)}{(x^2+3)^4} = \frac{\cancel{(x^2+3)}\left[12(x^2+3)-(12x)(4x)\right]}{D^{\cancel{4}}} =$$

$$= \frac{12x^2+36-48x^2}{D^3} = \frac{-36x^2+36}{D^3} = \frac{36(-x^2+1)}{(x^2+3)^3}$$

$y'' \geq 0 \rightarrow \quad 36>0$ Sempre

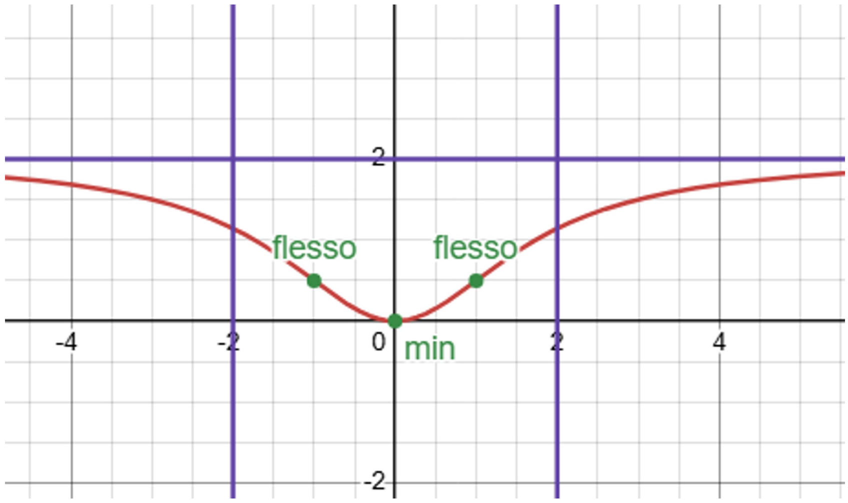
$-x^2+1 \geq 0 \rightarrow -x^2 \geq -1 \rightarrow x^2 \leq 1 \rightarrow x \leq \pm 1$

$(x^2+3)^3>0 \quad \forall x\in\mathbb{R}$



$$f(-1) = \frac{2(-1)^2}{(-1)^2+3} = \frac{2}{4} = \frac{1}{2}$$

Funzione pari $\rightarrow f(1) = \frac{1}{2}$



flesso
flesso
min
-4
-2
0
2
4
2
-2