FRAZIONI

venerdì 20 dicembre 2024 09:58

1- $y = \dfrac{2x^2+4x-11}{x^2+2x-8}$

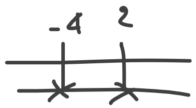
CE: $x^2+2x-8 \neq 0$ $\quad (x+4)(x-2)$

$\Delta = 4-4\cdot(-8) = 4+32 = 36$

$x_{1/2} = \dfrac{-2\pm\sqrt{36}}{2} = \dfrac{-2\pm 6}{2}$

$x_1 = \dfrac{-2+6}{2} = \dfrac{4}{2} = 2$

$x_1 = \dfrac{-2-6}{2} = -\dfrac{8}{2} = -4$



Dominio: D: $(-\infty;-4)\cup(-4;2)\cup(2;\infty)$

Simmetrie:

$f(-x) = \dfrac{2(-x)^2+4(-x)-11}{(-x)^2+2(-x)-8} = \dfrac{2x^2-4x-11}{x^2-2x-8}$ No Pari

$-f(x) = \dfrac{-2x^2-4x+11}{x^2+2x-8}$ No dispari

Intersezioni:

Asse x con $y=0$

$$\begin{cases} 2x^2+4x-11=0 \to \Delta = 16-4\cdot 2\cdot(-11) = 104 \\ y=0 \end{cases}$$

$x_{1/2} = \dfrac{-4\pm\sqrt{104}}{4} = \dfrac{-4\pm\sqrt{2^2\cdot 26}}{4} = \dfrac{-4\pm 2\sqrt{26}}{4}$

$$= \frac{2(-2 \pm \sqrt{26})}{4} = \frac{-2 \pm \sqrt{26}}{2}$$

$$\begin{cases} x_1 = \frac{-2+\sqrt{26}}{2} \\ x_2 = \frac{-2-\sqrt{26}}{2} \\ y = 0 \end{cases}$$

$1{,}55$

$A\left(\frac{-2+\sqrt{26}}{2}; 0\right)$

$B\left(\frac{-2-\sqrt{26}}{2}; 0\right)$

$-3{,}56$

Asse y con $x=0$

$$\begin{cases} x = 0 \\ y = -\frac{11}{8} \end{cases}$$

$C\left(0; -\frac{11}{8}\right)$

$-1{,}375$

Segno

$2x^2+4x-11 \geq 0 \rightarrow x \leq \underbrace{\frac{-2-\sqrt{26}}{2}}_{x_1} \cup x \geq \underbrace{\frac{-2+\sqrt{26}}{2}}_{x_2}$

$x^2+2x-8 > 0 \rightarrow x < -4 \cup x > 2$

	-4		x_1		x_2		2	
+		+	•	−	•	+		+
+	×	−		−		−	×	+
+		−		+		−		+

Limiti

$$\lim_{x \to -4^-} \frac{2x^2+4x-11}{x^2+2x-8} = \frac{2(-4,1)^2+4(-4,1)-11}{(-4,1)^2+2(-4,1)-8} = \frac{+6{,}22}{+0{,}61} = \frac{+}{+} = +\infty$$

$$\lim_{x \to -4^+} f(x) = \frac{2(-3,9)^2+4(-3,9)-11}{(-3,9)^2+2(-3,9)-8} = \frac{+3{,}82}{-0{,}59} = \frac{+}{-} = -\infty$$

As.V. $x=-4$

$$\lim_{x\to 2^-} \frac{2x^2+4x-11}{x^2+2x-8} = \frac{2(1,9)^2+4(1,9)-11}{(1,9)^2+2(1,9)-8} = \frac{+0,21}{-0,59} = \frac{+}{-} = -\infty$$

$$\lim_{x\to 2^+} f(x) = \frac{2(2,1)^2+4(2,1)-11}{(2,1)^2+2(2,1)-8} = \frac{+6,22}{+0,61} = \frac{+}{+} = +\infty$$

As.V. $x=2$

$$\lim_{x\to \pm\infty} \frac{2x^2+4x-11}{x^2+2x-8} = \frac{2x^2}{x^2} = 2$$

As. Or. $y=2$

Derivata prima

$$f'(x) = \frac{[(4x+4)(x^2+2x-8)]-[(2x^2+4x-11)(2x+2)]}{(x^2+2x-8)^2} =$$

$$= \frac{\cancel{4x^3}+\cancel{8x^2}-32x+\cancel{4x^2}+\cancel{8x}-32\cancel{-4x^3}-\cancel{4x^2}-\cancel{8x^2}-\cancel{8x}+22x+22}{D^2} =$$

$$= \frac{-10x-10}{D^2} = \frac{-10(x+1)}{(x^2+2x-8)^2}$$

$f'(x) \geq 0 \rightarrow -10(x+1) \geq 0$

$-10 \geq 0$ mai

$x+1 \geq 0 \rightarrow x \geq -1$

$$\begin{array}{c|c} \multicolumn{2}{c}{-1} \\ \hline + & - \\ \nearrow & \searrow \end{array}$$

Max: $x=-1$

$$f(-1) = \frac{2(-1)^2+4(-1)-11}{(-1)^2+2(-1)-8} = \frac{2-4-11}{1-2-8} = \frac{-13}{-9} = \frac{13}{9}$$

$\mathrm{Max}\left(-1; \frac{13}{9}\right)$

Derivata Seconda

$$f''(x)=\frac{\left[-10(x^2+2x-8)^2\right]-\left[(-10x-10)2(x^2+2x-8)(2x+2)\right]}{(x^2+2x-8)^4}=$$

$$=\frac{\left[-10(x^4+4x^2+64+4x^3-32x-16x^2)\right]-\left[(-20x-20)(x^2+2x-8)(2x+2)\right]}{D^4}=$$

$$=\frac{-10x^4-40x^2-640-40x^3+320x+160x^2-\left[(-20x^3-40x^2+160x-20x^2-40x+160)(2x+2)\right]}{D^4}=$$

$$=\frac{-10x^4-40x^3+120x^2+320x-640-(-20x^3-60x^2+120x+160)(2x+2)}{D^4}=$$

$$=\frac{-10x^4-\cancel{40x^3}+\cancel{120x^2}+\cancel{320x}-640+40x^4+\cancel{40x^3}+120x^3+\cancel{120x^2}-\cancel{240x^2}-240x-\cancel{320x}-320}{D^4}=$$

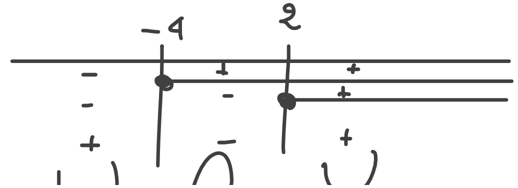
$$=\frac{30x^4+120x^3-240x-960}{D^4}$$

$$f''(x)\geq 0 \qquad \frac{30x^4+120x^3-240x-960}{10}\geq\frac{0}{10} \rightarrow \frac{3x^4+12x^3-24x-96}{3}\geq\frac{0}{3}$$

$$\rightarrow x^4+4x^3-8x-32\geq 0 \rightarrow x^3(x+4)-8(x+4)\geq 0$$

$$\rightarrow (x+4)\left[x^3-8\right]\geq 0 \rightarrow x+4\geq 0 \rightarrow x\geq -4$$

$$x^3-8\geq 0 \rightarrow x^3\geq 8 \rightarrow x\geq\sqrt[3]{8} \rightarrow x\geq 2$$

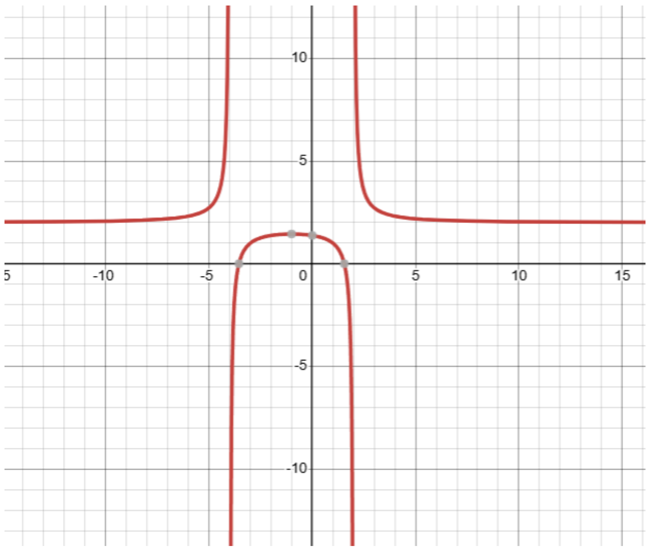


$x=-4 \cup x=2 \rightarrow$ flessi

$$f(-4)=\frac{2(-4)^2+4(-4)-11}{(-4)^2+2(-4)-8}=\frac{32-16-11}{16-8-8}=\frac{5}{0} \rightarrow \text{impossibile}$$

$$f(2)=\frac{2\cdot 4+8-11}{4+4-8}=\frac{5}{0} \rightarrow \text{impossibile}$$

No punti di flesso reali



$$2-\quad y=\frac{x^2+1}{x-1}$$

CE: $x-1\neq 0 \rightarrow x\neq 1$

Dominio: $(-\infty;1)\cup(1;\infty)$

Simmetrie: $f(-x) = \dfrac{x^2+1}{-x-1}$ No Pari

$-f(x) = \dfrac{-x^2-1}{x-1}$ No dispari

Intersezioni:

Asse x con $y=0$

$$\begin{cases} x^2+1=0 \quad \text{mai} \\ y=0 \end{cases}$$

Asse y con $x=0$ $\quad A(0;-1)$

$$\begin{cases} x=0 \\ y = \dfrac{1}{-1} = -1 \end{cases}$$

Segno

$x^2+1 \geq 0 \rightarrow x^2+1>0 \quad \forall x \in D$

$x-1>0 \rightarrow x>1$

$$\begin{array}{c|c} & 1 \\ \hline - & + \end{array}$$

Limiti

$$\lim_{x \to 1^-} \frac{x^2+1}{x-1} = \frac{(0{,}9)^2+1}{0{,}9-1} = \frac{+}{-} = -\infty$$

$$\lim_{x \to 1^+} \frac{x^2+1}{x-1} = \frac{(1{,}1)^2+1}{1{,}1-1} = \frac{+}{+} = +\infty$$

As.V. $x=1$

$$\lim_{x \to \pm\infty} \frac{x^2+1}{x-1} = \frac{x^2}{x} = x = \pm\infty$$ No As.O.

$$m = \lim_{x \to \infty} \frac{x^2+1}{x-1} \cdot \frac{1}{x} = \frac{x^2+1}{x^2-x} = \frac{x^2}{x^2} = 1$$

$$q = \lim_{x \to \infty} \frac{x^2+1}{x-1} - 1x = \frac{x^2+1-x^2+x}{x-1} = \frac{+x+1}{x-1} = \frac{+x}{x} = 1$$

$y = x+1$ eq. as. obliquo

x	y
0	1
1	2

Derivata prima

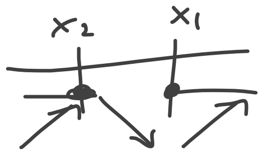
$$f'(x) = \frac{2x(x-1)-(x^2+1)}{(x-1)^2} = \frac{2x^2-2x-x^2-1}{(x-1)^2} = \frac{x^2-2x-1}{(x-1)^2}$$

$$f'(x) \geq 0 \rightarrow x^2-2x-1 \geq 0 \Rightarrow \Delta = 4-4\cdot(-1) = 8$$

$$x_{1/2} = \frac{2\pm\sqrt{8}}{2} = \frac{2\pm2\sqrt{2}}{2} = \frac{2(1\pm\sqrt{2})}{2} = 1\pm\sqrt{2}$$

$x_1 = 1+\sqrt{2}$ $(2{,}41)$

$x_2 = 1-\sqrt{2}$ $(-0{,}41)$

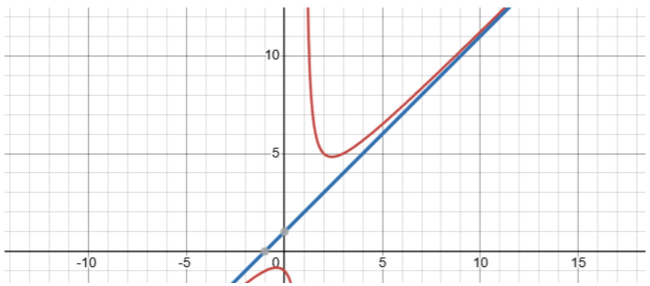


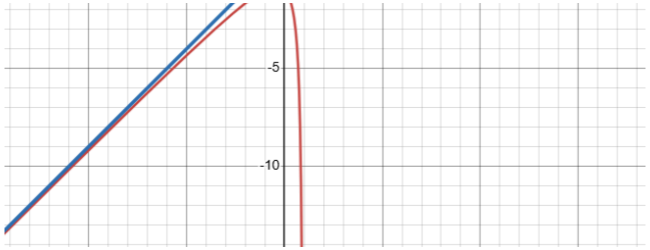
$$f(1+\sqrt{2}) = \frac{(1+\sqrt{2})^2+1}{1+\sqrt{2}-1} = \frac{6{,}83}{1{,}41} \simeq (4{,}84)$$

$$\min\left(1+\sqrt{2};\ \frac{(1+\sqrt{2})^2+1}{\sqrt{2}}\right)$$

$$f(1-\sqrt{2}) = \frac{(1-\sqrt{2})^2+1}{1-\sqrt{2}-1} = \frac{1{,}17}{-\sqrt{2}} \simeq (-0{,}83)$$

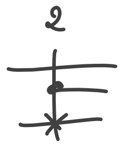
$$\mathrm{Max}\left(1-\sqrt{2};\ \frac{(1-\sqrt{2})^2+1}{-\sqrt{2}}\right)$$





3- $y = \dfrac{x}{\sqrt{x-2}}$

c.e.: $\begin{cases} x-2 \geq 0 \\ \sqrt{x-2} \neq 0 \end{cases}$ $\begin{cases} x \geq 2 \\ x-2 \neq 0 \end{cases}$ $\begin{cases} x \geq 2 \\ x \neq 2 \end{cases}$



$x > 2$

Dominio:

$D: (2; \infty)$

Simmetrie:

$f(-x) = \dfrac{-x}{\sqrt{-x-2}}$ No Pari

$-f(x) = \dfrac{-x}{\sqrt{x-2}}$ No dispari

Intersezione:

Asse x con $y=0$

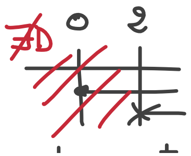
$\begin{cases} \dfrac{x}{\sqrt{x-2}} = 0 \\ y = 0 \end{cases}$ $\begin{cases} x = 0 \\ y = 0 \end{cases}$ $\nexists D$

Asse y con $x=0$

$\begin{cases} x = 0 \\ \end{cases}$ $\nexists D$

Segno

$\dfrac{x \geq 0}{\sqrt{x-2} > 0} \rightarrow \begin{cases} x \geq 0 \\ x > 2 \end{cases}$



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Limiti

$$\lim_{x \to 2} \frac{x}{\sqrt{x-2}} = \frac{2}{0} = +\infty \quad \text{No As.V.}$$

$$\lim_{x \to \infty} \frac{x}{\sqrt{x-2}} = \frac{\infty}{\infty} \quad \text{indet.}$$

De L'Hopital: $\lim_{x \to \infty} \dfrac{1}{\frac{1}{2\sqrt{x-2}}} \to 2\sqrt{x-2} = \infty$ No As.Or.

$$m = \lim_{x \to \infty} \frac{x}{\sqrt{x-2}} \cdot \frac{1}{x} = \frac{1}{\sqrt{x-2}} = \frac{1}{\infty} = 0 \quad \text{No As. Obl.}$$

Derivata Prima

$$y' = \frac{\left[1 \cdot \left(\sqrt{x-2}\right)\right] - \left[x \cdot \frac{1}{2\sqrt{x-2}}\right]}{\left(\sqrt{x-2}\right)^2} =$$

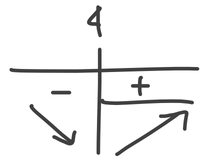
$$= \frac{\sqrt{x-2} - \frac{x}{2\sqrt{x-2}}}{x-2} = \frac{\frac{\left(\sqrt{x-2}\right)\left(2\sqrt{x-2}\right) - x}{2\sqrt{x-2}}}{x-2} =$$

$$= \frac{2(x-2)-x}{x-2} = \frac{2x-4-x}{x-2} = \frac{x-4}{x-2}$$

$f'(x) \geq 0 \;\to\; x-4 \geq 0 \;\to\; x \geq 4$

$\to \boxed{x-2 \neq 0 \to x \neq 2}$

Condizione già studiata nelle C.E.



$x=4 \rightarrow$ minimo

$$f(4) = \frac{4}{\sqrt{4-2}} = \frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

min $(4; 2\sqrt{2})$

Derivata seconda

$$y''(x) = \frac{1(x-2) - (x-4)1}{(x-2)^2} = \frac{x-2-x+4}{(x-2)^2} = \frac{2}{(x-2)^2}$$

$f''(x) \geq 0 \rightarrow 2 \geq 0 \rightarrow 2 > 0$ sempre $-\infty \underset{(+)}{\rule{1cm}{0.4pt}} \infty$

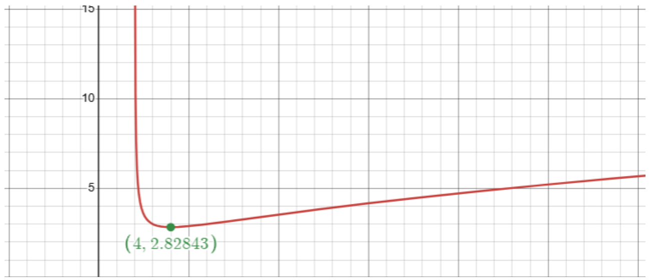
La funzione è convessa su tutto il dominio

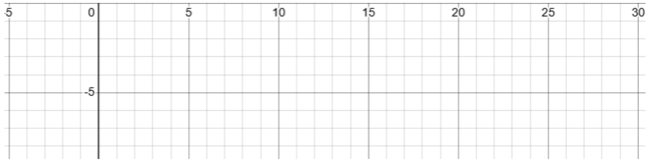
Essendo $x=4$ un minimo, studio se è relativo o assoluto

$$f''(4) = \frac{2}{(4-2)^2} = \frac{2}{4} = \frac{1}{2}$$

Il risultato $\left(\frac{1}{2}\right)$ è >0, quindi

$x=4$ è un minimo relativo per $f(x)$

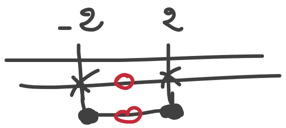




4- $y = \dfrac{1}{\sqrt{4-x^2}}$

ce:

$\begin{cases} \sqrt{4-x^2} \neq 0 \\ 4-x^2 \geq 0 \end{cases}$ $\begin{cases} 4-x^2 \neq 0 \\ -x^2 \geq -4 \end{cases}$ $\begin{cases} x^2 \neq 4 \\ x^2 \leq 4 \end{cases}$ $\begin{cases} x \neq \pm 2 \\ -2 \leq x \leq 2 \end{cases}$



$-2 < x < 2$

Dominio:

D: $(-2; 2)$

Simmetrie:

$f(-x) = \dfrac{1}{\sqrt{4-(-x)^2}} = \dfrac{1}{\sqrt{4-x^2}}$ Pari

Intersezioni:

Asse x con $y=0$

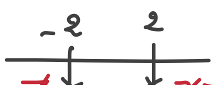
$\begin{cases} \dfrac{1}{\sqrt{4-x^2}} = 0 \\ y=0 \end{cases}$ $\begin{cases} 1 = 0 \text{ mai} \\ y=0 \end{cases}$

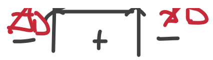
Asse y con $x=0$

$\begin{cases} x=0 \\ y = \dfrac{1}{\sqrt{4}} = \dfrac{1}{2} \end{cases}$ $A\left(0; \dfrac{1}{2}\right)$

Segno

$1 \geq 0 \rightarrow 1 > 0$ sempre



$\sqrt{4-x^2} > 0 \to 4-x > 0 \to -2 < x < 2$ 

Funzione positiva nel dominio

Limiti

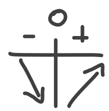
$$\lim_{x \to -2^+} \frac{1}{\sqrt{4-x^2}} \to \frac{1}{\sqrt{4-(-1{,}9)^2}} = \frac{1}{\sqrt{0{,}39}} = \frac{1}{\sqrt{0^+}} = \frac{1}{0^+} = \infty \Big] \text{ As. V. destro } x = -2$$

$$\lim_{x \to 2^-} f(x) \to \frac{1}{\sqrt{4-(1{,}9)^2}} = \frac{1}{\sqrt{0^+}} = +\infty \Big] \text{ As.V. sinistro } x = 2$$

Derivata prima

$$f'(x) = \frac{\left(0 \cdot \sqrt{4-x^2}\right) - 1 \cdot \left(\frac{-\cancel{2}x}{\cancel{2}\sqrt{4-x^2}}\right)}{\left(\sqrt{4-x^2}\right)^2} = \frac{\frac{x}{\sqrt{4-x^2}}}{4-x^2} = \frac{x}{(4-x^2)^{\frac{1}{2}}} \cdot \frac{1}{4-x^2}$$

$$= \frac{x}{(4-x^2)^{\frac{3}{2}}} = \frac{x}{\sqrt{(4-x^2)^3}}$$

$f'(x) \geq 0 \to x \geq 0$ 

$\sqrt{(4-x^2)^3} \neq 0$ c'è già studiata prima

$x = 0$ min

$$f(0) = \frac{1}{\sqrt{4-0}} = \frac{1}{2} \qquad \min\left(0; \frac{1}{2}\right)$$

Derivata seconda

$$f'(x) = \frac{x}{\sqrt{(4-x^2)^3}} = \frac{x}{(4-x^2)^{\frac{3}{2}}}$$

$$f''(x) = \frac{(4-x^2)^{\frac{3}{2}} \left[x \cdot \frac{3}{2}(4-x^2)^{\frac{3}{2}-1} \cdot (-2x) \right]}{\ldots}$$

$$f'(x) = \frac{1\cdot(4-x^2) - [x \cdot 2 (\ldots)]}{\left[(4-x^2)^{\cancel{\frac{3}{2}}}\right]^{\cancel{2}}} =$$

$$= \frac{(4-x^2)^{\frac{3}{2}} - \left[\frac{3}{\cancel{2}} x (4-x^2)^{-\frac{1}{2}} \cdot (-\cancel{2}x)\right]}{(4-x^2)^3} =$$

$$= \frac{(4-x^2)^{\frac{3}{2}} - \left(-3x^2 (4-x^2)^{-\frac{1}{2}}\right)}{(4-x^2)^3} =$$

$$= \frac{\sqrt{(4-x^2)^3} + \frac{3x^2}{\sqrt{4-x^2}}}{(4-x^2)^3} = \frac{\frac{(4-x^2)^{\frac{3}{2}} + 3x^2}{(4-x^2)^{1/2}}}{(4-x^2)^3} =$$

$$= \frac{(4-x^2)^{\frac{3}{2}} \cdot (4-x^2)^{1/2} + 3x^2}{(4-x^2)^{1/2}} \cdot \frac{1}{(4-x^2)^3} =$$

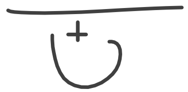
$$= \frac{(4-x^2)^{\frac{1}{2}} + 3x^2}{(4-x^2)^{\frac{7}{2}}} = \frac{(4-x^2)^{\frac{1}{2}}}{(4-x^2)^{\frac{7}{2}}} + \frac{3x^2}{(4-x^2)^{\frac{7}{2}}} =$$

$$= (4-x^2)^{\frac{1}{2}-\frac{7}{2}} + \frac{3x^2}{(4-x^2)^{\frac{7}{2}}} =$$

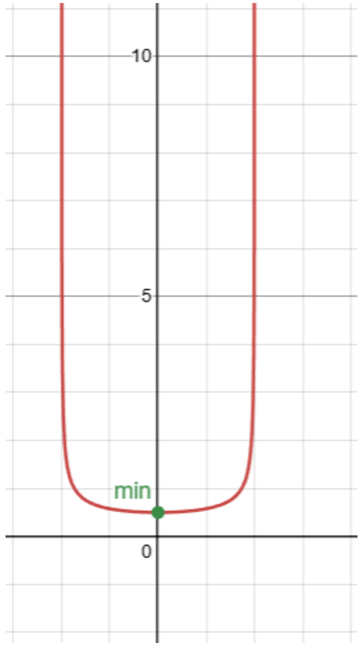
$$= (4-x^2)^{-3} + \frac{3x^2}{(4-x^2)^{\frac{7}{2}}} = \frac{1}{(4-x^2)^3} + \frac{3x^2}{(4-x^2)^{\frac{7}{2}}} =$$

$$= \frac{1}{(4-x^2)^3} + \frac{3x^2}{\sqrt{(4-x^2)^7}}$$

$f''(x) \geq 0 \quad \rightarrow \quad \dfrac{1}{(4-x^2)^3} \geq 0 \quad \forall x \in D$



$\rightarrow \quad \dfrac{3x^2}{\sqrt{(4-x^2)^7}} \geq 0 \quad \forall x \in D$



3- $y = \sqrt{\dfrac{x-1}{x+1}}$

CE: $\begin{cases} \dfrac{x-1}{x+1} \geq 0 \\ x+1 \neq 0 \end{cases} \quad \begin{cases} x-1 \geq 0 \\ x+1 > 0 \\ x \neq -1 \end{cases} \quad \begin{cases} x \geq 1 \\ x > -1 \\ x \neq -1 \end{cases}$

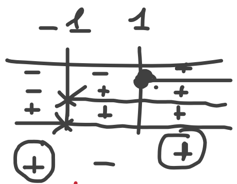


grafico dei segni
perché la frazione è
sotto la radice; perciò devo

Dominio:

D: $(-\infty; -1) \cup [1; \infty)$

il segno in quanto il radicando deve essere ≥ 0

Simmetrie

$f(-x) = \sqrt{\dfrac{-x-1}{-x+1}}$ No pari

$-f(x) = -\sqrt{\dfrac{x-1}{x+1}}$ No dispari

Intersezioni

Asse y con x=0

$\begin{cases} x=0 & \nexists \\ \end{cases}$

Asse x con y=0

$\begin{cases} \sqrt{\dfrac{x-1}{x+1}} = 0 \\ y=0 \end{cases} \rightarrow \dfrac{x-1}{x+1} = 0 \rightarrow x-1=0 \rightarrow x=1$

$A(1;0)$

Segno

$\sqrt{\dfrac{x-1}{x+1}} \geq 0 \rightarrow \dfrac{x-1}{x+1} \geq 0 \rightarrow$ $x \geq 1$, $x > -1$