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Imperfect Oracles

The Effect of Strategic Information on Stock Markets

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Declaration

This dissertation is submitted to the University of Bristol in accordance with the requirements of the degree of MEng in the Faculty of Engineering. It has not been submitted for any other degree or diploma of any examining body. Except where specifically acknowledged, it is all the work of the Author.

Miklos Borsi, Thursday 28th May, 2020

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Executive Summary

Modern financial market dynamics warrant detailed analysis due to their significant impact on the world. This, however, often proves intractable; massive numbers of agents, strategies and their change over time in reaction to each other leads to difficulties in both theoretical and simulational approaches.

Notable work has been done on strategy dominance in stock markets with respect to the ratios of agents with certain strategies. Perfect knowledge of the strategies employed could then put an individual agent at a consistent trading advantage. This thesis reports the effects of imperfect oracles on the system - dispensing noisy information about strategies - information which would normally be hidden from market participants.

The effect and achievable profits of a singular trader with access to an oracle were tested exhaustively with previously unexplored factors such as changing order schedules. Additionally, the effect of noise on strategic information was traced through its effect on trader efficiency.

Going into this project the research hypothesis was that a trader in possession of perfect strategic information would be able to significantly outperform other traders not in possession of said information, in a majority of cases. In addition, this advantage is expected to decrease once noise is added to the information.

Main Contributions

- I added an enhanced agent and ways to source and distort strategic information to the Bristol Stock Exchange market simulator project.
- I added a well-parametrisable order schedule randomizer to the Bristol Stock Exchange, capable of exploring the full range of possible order schedules permitted by the system.
- I simulated over 2 million market sessions in over 40 CPU-days of supercomputer time.
- I devised a way to determine theoretical upper bounds for profit achieved by fair traders under certain conditions. Consistent out-performance of this bound could serve as a credible grounds for insider trading investigations. The work presented here could be built upon to create better-regulated and fairer markets.
- I established a methodology for experiment design and evaluation. Future research in this area can follow and expand this methodology to avoid inconclusive experiments.

Having conducted this large-scale simulation study, I have found the initial research hypothesis to be valid. It is possible to make accurate predictions about market strategy dominance when in possession of perfect strategic information. The usefulness of this information decreases approximately linearly as information gradually decreases from perfect to none. There is a non-negligible but still minority amount of order schedules which are difficult to predict. This is shown by both the number of correct predictions and the profit ratio of traders increasing alongside increasing noise. Two non-dependent sources of data mark the same order schedules as exceptions to the hypothesis. In these order schedules greedily acting upon predictions can have an uncertain or even a negative correlation with profit.

Supporting Technologies

- I used the Bristol Stock Exchange project to simulate a LOB-based market with algorithmic traders (available on GitHub at https://github.com/davecliff/BristolStockExchange)
- I used Python 2.7 to extend the Bristol Stock Exchange with the code necessary for this research, including the information-enhanced algorithmic traders, the sources of information and introduction of noise to it
- I utilized the University's BlueCrystal Phase 4 supercomputer to carry out compute-intensive simulations
- I used Jupyter Notebook, Numpy, Scikit-Learn and SciPy to analyse the experiment data
- I used David Andrzejewski's visualisation code to aid in visualising simplexes, available on GitHub at https://gist.github.com/davidandrzej/939840

Notation and Acronyms

In alphabetic order

1. Acronyms
2. Terminology

AA	:	Adaptive-Aggressive trading algorithm
BSE	:	Bristol Stock Exchange
CDA	:	Continuous Double Auction
GDX	:	Gjerstad-Dickhaut trading algorithm
HFT	:	High-Frequency Trading
LOB	:	Limit Order Book
SNPR	:	Kaplan's Sniper trading algorithm
ZIC	:	Zero-Intelligence Constrained trading algorithm
ZIP	:	Zero-Intelligence Plus trading algorithm
efficiency (trader)	:	Profit made by a trader, expressed as a ratio e.g. 1.0 means the trader's balance before and after a trading session are equal
liquidity	:	The overall number of trades happening in a market
oracle	:	Source of information

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Chapter 1

Contextual Background

This chapter will lay out the reasoning for conducting this research. The first section **The connected economy** will discuss the impact of the topic at hand and its likely course in the future. The second section, **A chaotic system**, will elaborate on some of the complex dynamics arising from the large numbers of variables and active agents present in the economy as a system. The third section, **From humans to algorithms**, will detail the recent shifts and research in the behaviour of trading agents with a focus on speed and the rise of algorithmic trading. Finally, **An argument for strategic analysis** will provide justification for the specific course of analysis undertaken here.

1.1 The Connected Economy

Money has long since surpassed the founding concept of an exchange token for goods and services. With the introduction of other factors into said exchange - such as time - new forms of contracts and deals arose. Loans, interests, currency exchanges, stocks, options, futures and more. The importance of the financial services sector to the world economy, and as a result to uncountable facets of day-to-day life is indisputable. The world's financial sector worth forecast is $26.5 trillion by 2020. Financial services contributed £132 billion to the UK economy in 2018[11], almost 7% of total economic output.

As time goes on the impact of financial services on the world will continue to increase. Global connectivity, economies and networks of scale, and accessibility of technology have made way for new technological approaches. The financial sector has traditionally served primarily the developed, infrastructure-rich and wealthy areas of the world. Most of its products were tailored towards established notions and needs. However, new developments and inventions in the sector like TransferWise or MPesa have achieved market penetration in previously inactive areas. The current academic view of the situation is that growth and innovation in this sector will continue at a fast pace and that we can expect more novel services bringing even more people into the folds of a global economy.

The most pressing question of this new economic era of interconnectedness is the topic of stability and reliability. As the number of services, cross-dependencies and total value of the sector continue to rise so does the risk of a failure causing other failures in a catastrophic chain reaction. The ease at which benefits of trading spread through the global network is a double edged sword - it reflects the ease at which a crash in one sector could cause many more in sectors or countries across the world.

Although large-scale crashes are nothing new to capitalist economic systems, the quick and brutal effect of the 2008 recession is a powerful reminder of how far a single factor such as loans in the the U.S. housing market can reach. Some safeguards have been put in place to prevent this from happening again. The global economy, however, is a complex enough system to have uncountable novel and hard-to-predict ways to critically fail and cause another recession.

1.2 A Chaotic System

The outcomes of extremely large systems can be hard to predict. This is often exacerbated in systems with breakpoints, nonlinearities and independent agents. A chaotic system is described as one in which a small change in starting conditions can lead to massive differences in the eventual state of the system

- this is colloquially known as the "butterfly effect". The original discovery was made by Lorenz in his modeling of weather effects[10]. In an attempt to save time he would start the system from a state in the middle of the simulation time with reduced precision of data - up to three decimal places instead of six. This caused notable differences in the end state of the system, such as the presence of rain or not in a particular territory. The following paper, "Deterministic Nonperiodic Flow" in 1962, is viewed as the foundation of chaos theory.

Economies, or on a smaller scale, a stock market, can display many of the features necessary for a chaotic system with a number of additional ones not present in usual examples such as the weather. Breakpoints are certain values for variables that introduce a significant change despite the relatively insignificant difference in the value or the normal rate of change. For example, given a stock of 100 of a certain product, where customers are limited to buying X units maximum and purchase things in one bulk purchase, sequentially. For $X = 10$ all 10 customers can buy their maximum amount and are satisfied. For $X = 11$ the maximum number of customers is still 10 - yet after 9 have acquired their $9 * 11 = 99$ products, the 10th customer is only left with 1. This is an example of a breakpoint - a minor change in a limit leading to a much larger effect - in this case a highly unequal distribution of products.

Nonlinearities are another core piece of chaotic systems. They are places or effects that lead to a possibly exponential relation between variables in the system. These can arise from different factors such as an existing breakpoint - for weather, the temperature reaching the melting point of ice is a prime example that can change a light water level rise to mass flooding. Some nonlinearities come from self-amplifying effects - these can range from global warming thawing the permafrost, resulting in more greenhouse gases, to panic over the supply of sanitary products in the wave of an epidemic leading to further shortages despite no supply lines being interrupted.

Economies have the additional factor of being an amalgamation of uncountable agents, with their own set of goals, agencies and influence on the system as a whole. Any agent participating in the system has a nonzero influence, and a change in individual strategy may or may not result in a change of the whole system. Most individuals will not cause large-scale changes but the chaotic system provides an environment in which individual changes can be amplified to a systemic level.

1.3 From Humans to Algorithms

A line of research was started by Vernon Smith's experiments in his 1963 paper[15] studying the trading behaviour of humans and allocative efficiency of the market as a whole in a **Continuous Double Auction** (CDA), the style of market mechanism used in almost all financial exchanges around the world. This work was groundbreaking for its experimental approach to economic theory which previously often held unclear or inaccurate prior beliefs about its claims. For example, the number of participating agents in a double auction that achieves a good equilibrium was defined as "numerous" or with the common mathematical approximation "close to infinite". In the real, reproducible situation however, the "invisible hand" of the market was shown to be in effect from as little as 8 agents, 4 buyers and 4 sellers, who were also untrained for the situation.

Aside from further significant findings in auction theory and system controls, this research launched inquiries into just how effective human traders are - and how simple an algorithm one can make that performs comparably to humans.

Very simple, in specific cases. Initial work by Gode and Sunder in their 1993 paper[7] indicated that traders with practically zero intelligence - but some constraints - can produce the invisible hand effect as well. This was later proven by Cliff in 1997 to be a mere byproduct of the underlying probability distribution and the supply-demand curves used[3]. Simply put, in mostly symmetric cases the most likely value of the equilibrium price achieved from trading is the equilibrium price achieved from economic theory. In a supply-demand curve of significantly differing amounts or steepness the behaviour did not arise, instead a predictably skewed equilibrium price appeared.

Cliff proposed an algorithm called **Zero Intelligence Plus** (ZIP) that emulated the previous attempt to make a minimally intelligent trading algorithm. This one passed the trials of achieving market equilibrium in a market populated by traders following this algorithm under all supply-demand curves and outperforming human traders. With some modifications, ZIP continues to be relevant as it is dominant over other trading algorithms in certain cases and is still a core element of market simulations.

In IBM's 2001 experiments[4] algorithmic traders were able to outperform humans and made 7% more profit on average. The algorithms included ZIP, Kaplan's Sniper[8] (winner of the 1992 contest on early trading algorithms[13]) and Gjerstad-Dickhaut (1998[6], GD, later modified versions are called MGD and GDX[17]). Kaplan's Sniper is notable for its remarkable success despite its simplicity; it parasitically waits for other traders in the market to perform price discovery and converge towards a transaction price. It then jumps into action and "steals the deal". This requires other traders to be active in the market, a market full of Snipers sees few transactions and low price discovery. The GDX algorithm on the other hand is adaptive, formulating a belief about the current state of the market based on its history and making bids to maximize its expected profit. With a strong prediction from IBM that computerized trading is on the upswing, the field soon saw real life usage.

The next addition to the algorithmic roster came in 2008 with Vytelingum's paper on the Adaptive Aggressive (AA) trading strategy[19], which has been shown to be dominant over human traders in every scenario. This provided academic validation for the trends seen on the trading floors of the world - less shouting and less people.

The introductory study claimed that it is dominant over the other algorithms previously mentioned but that claim has not fully held up in subsequent research by other authors. Different ratios of trading agents in the market lead to different strategies being dominant. It is not enough to pit two algorithms against each other in varying ratios and declare a winner if all pairwise scenarios are one-sided. One must consider the entire trading environment with other traders and strategies in the background, as well as different market conditions - changes in supply or demand over time or a shift in price. See Snashall 2019[16] for a more detailed discussion of when and why AA does not always dominate.

Future work might look further into specialized trading algorithms, such as High Frequency Trading (HFT). Some types of trading (and especially HFT) are only possible algorithmically - no human can be expected to have the reaction speed and decision capacity to compete with fragments of a second advantages on connection speeds. Many exchanges that allow such access found themselves in need of sub-microsecond timestamping of the order book.

These algorithms often aim to "steal a deal" in a way similar to Kaplan's Sniper. One of the trades HFTs commonly attempt is a multi-exchange fragment deal: brokers are legally obliged to give their clients the best prices for their trades. As a result they may need to split an order for stocks between two exchanges A and B as these might both have the best price available at a low quantity.

The HFT firm has an ultra fast connection to both of these exchanges and it notices someone buying the offer on Exchange A. It then quickly buys the offer on Exchange B and re-sells it at a slightly higher price, enough to still be bought by the original order but making themselves a profit in the meanwhile.

Although they provide increased liquidity in the market, some claim that the presence of HFT negatively affects other market properties and question the usefulness of this specific form of liquidity[9]. The usefulness of HFT from an allocative efficiency standpoint is under fire[20] and the volume traded by HFT firms is overall decreasing - from times when it amounted up to the 70% of trading volume. Nevertheless it is here to stay in some shape and should not be disregarded.

1.3.1 An argument for strategic analysis

There is a constant stream of research aimed at discerning market trends and the effects of various real-world phenomena on them. While this is a useful pursuit in general, there is an argument for taking a different approach to market dynamics.

As previously discussed, dealing with a chaotic system is challenging in its own right. The economy has the added complexity of an uncountably large number of intelligent agents with different goals and strategies. In addition, the parameters of these agents change constantly in reaction to each other. One could argue that this system is even *more* chaotic, as observations on the state of the system (e.g. a large analytical company announcing a likely market trend) can have sweeping impacts on the system, while, for example the weather rarely changes in response to complaints about too much rain.

From the perspective of strategic analysis however, this additional factor of self-influence and recursive effects, while not completely negated, is at least lessened. The very aim is studying the behaviour of the intelligent agents and how it changes over time and with careful experiment design a very significant amount of normally unknown or unapproachable details of the simulation can be fixed or randomised. This strengthens the conclusions achieved from this type of research and increases the likelihood that its findings will hold in the future. While market conditions, rules and real-world influences may change it will always involve intelligent agents with strategies similar to those known today.

The high-level objective of this project is to establish an upper bound for profit gained from strategic information in a market and analyse the loss in profit from noise in the information. Specifically, the aims are:

1. Establish an upper bound for advantage gain-able from perfect knowledge of the strategies used by other traders in a stock market.
2. Introduce noise to the strategic information through a prediction simulation with a distorted trader strategy ratio.
3. Map the severity of the noise to the loss in advantage.
4. Examine the underlying trader ratio dynamics.
5. Check the effect of different order schedules on occurring phenomena.

The next chapter, **Technical Background**, provides a detailed introduction to topics this research builds on. It discusses auction theory, stock markets and relevant terminology. It contains a logical buildup from simple algorithmic traders to the advanced ones, as well as how the concept of oracles was used in this thesis.

Following that, **Project Execution** explains the experiments in both design, execution and evaluation. Experiments are presented in chronological order. As methodology had to be adapted to draw conclusions from a very noisy environment, part of critical evaluation is inherently folded into Technical Execution, with discussions of failure cases and exceptions.

Critical Evaluation goes into more detail with a high-level summary of the choices made during this research. Choices here mean decisions where there are multiple viable alternatives but one or more had to be selected, and all other choices were not examined in detail. This chapter includes justifications for particular choices or reasons why a specific choice should not matter in the overall outcome. The chapter finishes by looking at the primary constraints of this research and the generality of results.

Conclusion ends the thesis with a final overview of the project. It lists the main contributions and achievements, including a comparison with the original goals. The chapter will also outline possible promising areas of future research based on the findings discussed previously.

Chapter 2

Technical Background

Oracles are commonly used in *cryptography* as tools for proving security. This can take the form of a mathematical conjecture - "Problem A can only be solved if we have an oracle for sub-problem B. But because B is proven to not be solvable, neither is A". Or it can be a model for possible threats - "Design access control with the assumption that internal passwords are available to an attacker" can be an oracle for the real threats of a disgruntled employee or a post-it-note with passwords on it.

This project makes a novel contribution in demonstrating use of this proof method in the field of *experimental economics*. A fair portion of economical systems and interactions are impossible to model conclusively with either equations or simulations due to the excessive number of underlying variables and chaotic nature of the system resulting in a prohibitively large computational cost. This field instead recognizes economical phenomena through targeted experiments in constrained environments that then generalize well to larger scale systems.

Market agents will always prioritize their own profit and try their best to predict market trends and adapt to the trading environment. It can be difficult to tell apart the influence of various factors in the success of an individual agent. By focusing on *game theory* and strategic background it is possible to isolate the effect of information without it being dependent on market trends and temporary conditions. The findings from the measured quantity of information can then be generalized to larger systems by estimating any agent's access to information - e.g. how good their Twitter feed sentiment analysis model is and how much advantage that confers in terms of profit.

2.1 Auctions and the Stock Market

The word "auction" might bring a specific image into mind, that of a large crowd, a hammer and the chant "going first, going second, sold". While that is certainly an accurate representation of an auction in the real world, the word "auction" when used in the context of economics represents a much broader field. This section will discuss auctions focusing on the context of the continuous double auction that will play a central part in this thesis.

2.1.1 Auction Theory

The name of the field dedicated to auctions and their inner workings is Auction Theory. It covers more areas than conventional wisdom would classify as auctions. Non-dynamic prices can be understood within the perspective of this field too. For example, entering a store and deciding whether or not to buy a product off the shelf is an auction too: a fixed-price posted offer auction, the possible buyer being you, the customer and the seller being the store. In general, auctions are sets of rules and methods for buyers and sellers to discover and agree on a price at which they will exchange their goods and currency.

Describing an Auction

There are many kinds of non-fixed price auctions too. Auctions can be open or closed, ascending or descending, first-price or second-price and so on. There are too many subtypes to detail here so this subsection lists a number of common ones to build up understanding for the Continuous Double Auction (CDA) most financial exchanges are based on.

The most commonly encountered auction is perhaps the English auction, where bids for an object are publicly stated in an ascending order. This means it is a open dynamic ascending auction. As expected from these three specific words, there are other possible auctions with differing or opposite descriptions. In a method also known as the Dutch auction, the auctioneer offers a high offer price then lowers it until a buyer raises their hand or otherwise signals, at which point the object is sold. This makes it an open dynamic descending auction.

A closed auction is somewhat harder to conduct in real life but that does not mean they are rare. They too have their advantages - more on that topic soon. An auctioneer may ask all participants to state the highest price they would pay for an item, put it in a closed envelope then hand the bid to them. In Scotland (but not England or Wales) this is how houses are sold. They could then select the highest bid without necessarily revealing either the identity of the winner or the price paid. This is known as a sealed-bid auction.

First Price, Second Price, Dollar Auction

Another specifier that auctions often have is first-price, second-price and so on. What it means is how much the winner of the auction pays. Intuitively, they should pay as much as they offered for the object - that would make the auction a first-price auction. However, further strategic analysis would reveal interesting dynamics on why a non-first price auction can be more beneficial to participants both on the buying and selling side. Participants can be more likely to make honest bets and not try to game the system in some second-price auctions - more on this mechanism further below. Similarly, setting a strange price scheme can lead to downright paradoxical and irrational behaviour in otherwise rational humans.

This second case was showcased my Martin Shubik in his experiment of the "Dollar Auction"[14]. He put up a dollar for auction to his students. The rules were close to the English auction - starting bid is 0 pence, it can be raised in increments of 5 pence with an open outcry. The twist was that both the first and the second highest bidder had to pay their own bid - but only the first took the dollar in the end. What inevitably followed was mostly normal bidding up to \$1 at which point one would expect it to stop - offering to pay over a dollar for a dollar is strange. Yet the two highest bidders are incentivised to stay in the game. Offering 5 more cents can lead one to minimize their losses - their offer of \$1.05 translates to a loss of only \$0.05 if they win the auction, as opposed to having to pay \$0.95 for nothing if they stayed in second place. Afterwards, the previous first place faces the same dilemma. Take a loss or bid just a bit more to win and lose less - at the risk of losing even more in the end. The total price for a dollar bill often was between \$3-5. The practice of these kinds of auctions is largely illegal now due to unfairness concerns, but the idea did have notable real life impact. The author of the paper likened this to a situation that can emerge in negotiations between nations and advised to avoid such circumstances where the possible rewards and costs follow those outlined in this game.

A more positive example of selecting which price to pay would be a second-price sealed bid auction, also known as the Vickrey auction. In an English auction participants are likely to hide how much they truly value the item on offer, the maximum amount of price they are willing to pay for it. It is beneficial to bid the minimum amount above the highest maximum price another bidder is willing to pay than state your own highest price outright. Intuitively, if one knew all the bids the other participants would make, they would bid $max(bids) + 1$ to get the item at the cheapest price that still wins. If a sealed-bid auction was first price this would not change much, other than make it more seller-favoured in a way that feels unfair to buyers. At least in an open auction they have a good way of discovering how much others value the item.

In a second-price auction buyers are more free to state their maximum price outright. Buyers only pay the second-highest bid if they win. This means that any price higher than the second-highest bid does not matter and the buyer does not need to game the system to try and save money - they will only pay someone else's maximum bid if they win. The Vickrey auction sealing bids means that bids are safer, a malicious participant has a harder time raising the sale price for a competitor than they would if the bids were made in sequence and in the open. In fact, the Vickrey auction is *incentive compatible*, meaning that the best strategy - the one that leaves them with the highest combined value of money and items after the auction is over - is for participants to bid their true maximum price.

Auctions on the Internet

Auctions on the internet do not necessarily need to be incentive compatible - the Vickrey method has a number of deficiencies too. Among others it is vulnerable to collusion of buyers - if they shared information the items could go for a lower price than they normally would. Still, second- and later prices are a useful tool in many situations. In the early days of eBay auctions were conducted slightly differently due to

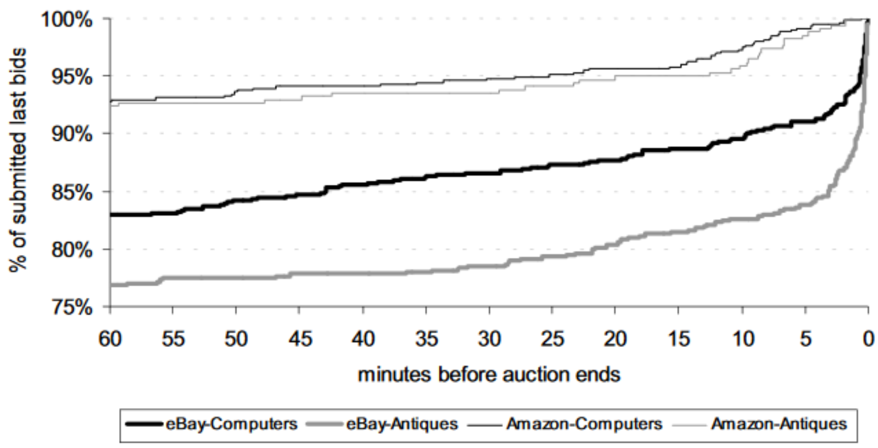

Figure 2.1: Cumulative distributions over time of bidders' last bids

how the internet was accessed differently. They ran for a longer time with a fixed duration. Due to the highest bid being revealed but the intentions of the buyers not, most of the bidding happened in the final minutes and even seconds of the auction - to try and "snipe" the product at the lowest possible price, buyers not giving others a chance to outbid them due to the constraints of internet and data transmission. This gave way for "price sniper" programs that bid in even less time or automatically multiple times up to a limit and users of these enjoyed advantages over others. eBay levelled the playing ground - it both implemented its own autobidder that bids on your behalf to the stated maximum, and changed the rules of the game. It does not reveal the current highest price, only what the winner would end up paying, which is the minimum increment over the current second highest price. Amazon's auctions went about the autobidder differently - they extend the remaining time when a new bid is made to allow responses. Figure 2.1[12] compares the number of bids made in relation to the time left until the end of the auction in eBay's original bidding system and that of Amazon. Google used to face a similar problem in their auctions for advertisement space. Implementing a second-price system lead to advertisers not having to constantly check the first-price given by others to minimize their own costs and thereby not overloading Google's servers with constant requests.

2.1.2 Continuous Double Auction

The main focus of this thesis will be the CDA, the Continuous Double Auction. It is the system by which most stock exchanges, futures markets, and such are run. In a CDA traders will post *orders*, usually publicly announcing them to the other traders in the market. This order will have a *price* and a *quantity*. The price represents the maximum price a buyer is willing to pay per unit or the minimum price a seller is willing to accept per unit. These are often called a *bid* on the buyer's side and an *ask* or an *offer* on the sellers'.

The CDA usually runs for a fixed amount of time often called a *market session* or a *trading day*. Orders can be and are made at any time by any participant - simultaneously and asynchronously. In older exchanges the traders shouted out orders and settled trades themselves. This became impractical over time and with more traders taking part. To scale up communications and the general ability to perform trades and exchange information about them, modern exchanges have computerised the system.

The way exchanges keep track of and show the orders currently present is called the *Limit Order Book* (LOB). See Figure 2.2 for an example. It has two sides; they show the bids and asks in descending and ascending order, respectively, the best one on top. The quantity, time the offer was made and other information may also be displayed. Most LOBs are anonymized and trader ID is only known to the exchange operators.

When the highest bid exceeds the lowest ask, a trade is executed by the exchange, with the smaller quantity between the two orders. As this matches two sides of the order book where the prices "overlap", it is called an *uncross*. The difference between the best prices on the two sides is known as the *spread*. The example bid side has 15 units offering to be bought at 173 price per unit, then 20 at 172. The example ask side has 10 units for sale at 181 then 2 units at 183.

Orders on the exchange can normally be canceled at any time. Many exchanges offer various ways

APL			
Bid			Ask
15	173	181	10
20	172	183	2
31	165	190	25
4	120		

Figure 2.2: An example LOB

of placing an order. For example a *Fill-or-Kill* type order must either be immediately uncrossed in full quantity or canceled immediately. A *Market* type order will execute with the required quantity immediately, regardless of price. These are mentioned as a way to showcase the complexity and further possibilities of the topic and were not be used in the simulations reported here.

2.1.3 What Traders Do

Traders can be purely profit-motivated, investing their own capital into a stock market and trade to maximize their own assets. This is widely referred to as "proprietary" or "prop" trading, but it is rare. Exchanges usually limit connections either through exclusivity or charging a fee and so for individual traders it is usually infeasible to have access to a large number of exchanges. Traders themselves are usually in firms and each of trader or firm usually acts on behalf of a number of clients - typically investment firms or individuals wanting to invest. Such traders are known as "sales traders".

Sales traders receive orders from their clients, usually with a *limit price* - the maximum/minimum price the client is willing to pay/receive for the stock. Sales traders profit from the difference between the limit price and the one they can execute the trade at as a commission or can charge a fee for handling the exchanges. In the rest of this thesis for the sake of brevity, "trader" will refer to a sales trader.

2.1.4 Dark Pools

CDAs too can be open or closed with regards to information. Closed-information exchanges are often known as *dark pools*. The intent behind dark pools is to reduce the effect the market has on itself.
Imagine if a large investment firm wished to sell a massive quantity of shares. If this large order was made publicly visible on an open LOB then market participants take this as an indication that the price of the share is going to drop and spiral the price down, making the firm much larger a loss than expected and overreacting to the actual tendencies. Instead, the firm could break the order into smaller pieces, or use different exchanges and such. What they often end up doing is sending an order to a dark pool. Here orders are not posted before executions and sometimes are not even binding and require a confirmation to actually execute. Exchange data is only written to the *tape*, the public record after a trade has been executed.

Lessening this self-impact is not without a cost. Dark pools have often faced criticism for their lack of transparency, and for their vulnerability to HFT firms "pinging" them with small orders to discover information. There have also been cases of collusion and insider trading[1] from exchange owners or favoured clients[18] who possessed more information than participants.

2.2 Bristol Stock Exchange

The simulations of this thesis were all executed on the Bristol Stock Exchange, a Python simulator used in many previous experiments. It is available publicly on GitHub[2]. The following section will describe the important principles and functions, differences between it and a real exchange and finally the important parameters of the simulations necessary if these experiments are to be reproduced.

2.2.1 How it Works

The exchange enables a number of algorithmic traders to engage with each other. Traders will receive orders with a limit price. They may make an offer on the order book or withdraw their current one. At each timestep every trader has an opportunity to react to the change (or lack thereof) on the exchange's

LOB as well as make updates to their own variables after a trade is executed. At the end of a market session the trades are written into a file and the important characteristics - like amount of profit made by all traders using each algorithm - are recorded.

2.2.2 Simplifying Assumptions

- The exchange only has a single stock to trade.
- Quantity of orders is always 1.
- Only standard *limit* orders can be made.
- Only one order may be active from a trader at a time, any new order immediately replaces their old one.
- There are no delays in communication - any order is known immediately to every market agent and information is distributed to all traders before any of them can react.
- Traders receive a price from their client and can execute trades at a different price - the difference between the two prices is fully their profit and their only profit.

2.2.3 Notable Parameters

The BSE permits very fine control over the *supply and demand schedules* - the timing and pricing at which customer orders arrive to the traders. These can change over time, have different ranges and volatilities, have a deterministic or randomised set of orders from the range. They can also arrive at set times or as a result of a random function. For most of the simulations, plausible randomisation will be used as a way to have better generalisation of results.
One can also set the length of a market session, the number of repeated trials performed for a set of starting parameters and the numbers of the algorithmic traders present.

In experiments the market will consist of N traders, $\frac{N}{2}$ buyers and $\frac{N}{2}$ sellers. All possible combinations of $\frac{N}{2}$ traders will be tested from a total of four possible algorithms: ZIP, GDX, AA and Kaplan's Sniper in Experiment 1 and 2, with the exception of the final, large-scale version of experiment 2, where SNPR is taken out of the strategy pool. $\{1ZIP|1GDX|1AA|\frac{N}{2}-3SNPR\}$, then $\{1ZIP|2GDX|1AA|\frac{N}{2}-4SNPR\}$, then $\{1ZIP|2GDX|2AA|\frac{N}{2}-5SNPR\}$ all the way to $\{\frac{N}{2}-3ZIP|1GDX|1AA|1SNPR\}$, with the buyer and seller side having equal representation of traders. The individual experiments will note the other simulation parameters that produced the results such as the duration of a market session.

2.3 Trading Algorithms

The Contextual Background chapter described the overall history of trading algorithm research in academia. This section will clarify the ideas behind the trading algorithms being used and put into context the previous trials and experiments in the literature.

2.3.1 ZIC

Vernon Smith's experiments marked a large step forward in how auction markets can be viewed and the scale at which phenomena can emerge[15]. He concluded that very few humans are needed for a continuous double auction to serve as an efficient exchange and align with economical theory. However, these exchanges were populated by human participants. An important question was whether the arising conclusions were the result of the dynamics of the system or the participants own thought processes and incentives. Is the market efficiently allocating products and quickly reaching - and stabilizing on - an equilibrium price purely a human behaviour?

Gode and Sunder were also interested in this topic and in 1993, published experiments structured very similarly to those of Smith[7]. However, this new research was aimed at discovering whether intelligence was necessary at all to produce this efficient market behaviour, and if yes, how much. ZIC - standing for *Zero Intelligence Constrained* - traders make an offer at a random price after receiving the customer order. This offer cannot cause a loss - hence the constraint.

These ZIC traders appeared to generate market dynamics that were shockingly close to the human results noted down by Smith. Notably, without the constraint - dubbed ZIU for *Zero Intelligence Unconstrained* there was no price discovery or stability in the market. Gode and Sunder theorised that because the market agents are as limited as they can be, the showcased "intelligence" must come from the market system itself. These conclusions are no longer accepted as valid in their reasoning. More information on this can be found in Section 2.3.4.

2.3.2 Shaver

A "Shaver" is what one might come up with as the simplest trading algorithm that is not fully zero intelligence. It will not willfully make a loss. It will at all times try to be the "best" bid or ask on the market by the minimum difference, also known as the *tick size*. For the BSE this is 1 pence. For example a Shaver would put a bid at $1.26 if the current best bid was $1.25. A seller would put an ask at $1.24 if the current best ask was $1.25.

Of course, while it might sound optimal in a vacuum - obtaining the best possible price while guaranteeing to be the first to trade - this is not the best in the long run and it is unlikely to expect real traders to behave this way, especially algorithmic ones capable of far more complex computations in little time. The reason is that Shavers do a poor job of keeping track of market trends and changes over time. It might make a profit because it is constrained but it runs two significant risks, it can either trade too early and lose out on a better possible trade; or it can get too greedy with its pricing and end up not trading at all and missing out on a nonzero profit from trading.

2.3.3 Kaplan's Sniper

The first of the trading algorithms listed here to be included in the actual experiments is Kaplan's Sniper. Although this subsection describes the original, BSE's SNPR implementation is only inspired by the original Kaplan's Sniper. It builds on very similar principles to the Shaver previously discussed. It is more strategic on when it places its minimal-price-difference order: at the very end of the market session, a very close spread (signalling a balanced, equilibrium-close price), or the price going outside the bounds of the previous market session. The algorithm is aptly named because it "snipes" or "steals" the deal, piggybacking on the price discovery and liquidity the other traders give the market.

The primary weakness of the SNPR is the fact that it does not attempt to discern the true value of the stock, rather it reacts only to the likely trades of other agents or the constraints of the game itself (a market session being near closing time). It does very poorly in a market populated primarily by other Snipers. Additionally, a primarily SNPR market will have close to zero price discovery, defeating the very purpose it was created for.

It first appeared as the winner of an early algorithmic trading tournament at the Santa Fe Institute in 1990 [13] . In certain conditions - primarily low amounts of other SNPRs present - it is still sometimes able to make a profit over more sophisticated algorithms and such earns its place in the experiment setup.

2.3.4 ZIP

A 1997 paper by Cliff called into question the experiment design of ZIC and the conclusions derived from it[3]. By pure mathematics and probability distributions it was proven that the equilibrium price being reached was a direct consequence of the supply and demand curves used; the "peak" of the overlap between supply and demand was approximately in the center. The random prices generated by the constrained traders, due to their even probability distribution, are similarly centered. Of course they would be a close match that way. Cliff introduced more misaligned supply and demand curves, e.g. a large flood of supply at a low price and a small amount of demand at a higher one, and the ZIC traders no longer produced the optimal market behaviour.

However, the idea of a low-intelligence agent stuck and he constructed the *Zero Intelligence Plus* (ZIP) algorithmic trader. It is finally an adaptive algorithm that tries to guess the best profit margin at which transactions are likely to happen - and stop happening beyond. It uses the delta rule, also known as the Widrow-Hoff learning rule, to adjust this guessed price based on how all orders on the market behave. A new ask being quickly lifted would indicate that the true price is higher than that trade that just happened.

ZIP makes its bids from a combination of this guessed price, a constantly adjusted profit margin and random perturbations as a normalizing factor; it was one of the first algorithm to beat humans. First in the famous IBM experiments in 2001 but also on a replication and extension trial in 2011 by De Luca and

Cliff[5]. While not necessarily dominant in most of the trader ratios it stays relevant in a larger number of them than Kaplan's Sniper. It is a staple in market experiments and will also take part in this current one.

2.3.5 GDX

The Gjerstad-Dickhaut algorithm, published in 1998[6] , takes a different approach. It tries to maximize the payoff for making a trade. It generates guesses of the *probability of making a trade* based on recent market activity or an interpolation if the concrete price at the guess was not yet used. It combines these probabilities with the profit of the possible trade with simple multiplication, resulting in an expected payoff. The market action chosen is the one with the maximum expected payoff.

The original version had some vulnerabilities. The extrapolation function could return unreasonable results if the data series included zeroes. It was remade under the name of MGD for *Modified Gjerstad-Dickhaut* and in 2002 as GDX[17]. These allow it to make better approximations and interpolations of previously-unseen prices and also estimate the future behaviour of prices, leading to a more accurate expected payoff value.

2.3.6 AA

The Adaptive-Aggressive algorithm was created by Vytelignum in 2006[19]. The element it is named after behaves similarly to ZIP's profit margin. It maintains an *aggressiveness* internal variable: the likely profit margin they expect the market to accept as a result of their trading. The more aggressive, the further they make the bid or ask from the market prices and the more likely it is to be accepted. The market price itself is estimated from a combination of *Smith's* α from Vernon Smith's original paper, which describes market volatility, and a moving average of the current displayed prices.

AA was shown to be dominant over other algorithms in some specific cases - primarily "duels" of algorithms with only two types present in a market at a time with varying ratios. This of course does not apply to *all* cases, especially realistic large markets with many different and changing ratios of strategies. Nevertheless it is a very strong trader and is thus included in these experiments.

2.4 Oracles

An oracle is source of information, always true in myth, treated with some distrust in reality. This section will give examples of Oracle usage in cryptography and then argue for this particular use case in economics.

2.4.1 Example in Cryptography

Security is often described by cryptography researchers as a "game". There is a system and an adversary on the two sides of the game. They have access to certain pieces of information, can perform certain actions and have a certain amount of computational power available to them.

An Oracle is a black box that is not necessarily realistic. It is a theoretical construct, meant to provide a reference point for other assumptions. Oracles are often assumed to perform normally very long computations instantly and can provide information the players of the game wouldn't otherwise have access to.

The difficulty in cryptographic proofs is that it is not enough to prove that a problem is hard and computing it would take long. The question is not whether something is solvable in a lot of time, it is whether any algorithm can exist that can solve it in less. As such, the proof must follow "if there is an efficient algorithm to solve A there must be an efficient algorithm to solve B. Because there is proven to be no efficient algorithm for A, B cannot have an efficient algorithm either".

For security, many of the adversaries in this game are probabilistic. It is not necessary for a thief to steal every credit card in the world, they only need a couple. For this research hypothesis it is not necessary to show that some knowledge of strategies provides an advantage in profit in every case. It either needs to be identifiable when it is true or it needs to be true on average, in the majority of cases. An example reduction proposal could be the following: The attacker A does not have access to the secret key of B. They do know the same encryption algorithm. A has access to an oracle that encrypts the message provided by A with the secret key of B. A also has access to an oracle that returns a random string. The encryption scheme is deemed secure if A cannot tell which oracle is which for any given secret message.

2.4.2 Application in Economics

Economic agents can derive observations from many sources. They can view and analyse the emotions of a social media feed, have a friend or family member working on a stock market trading floor, react to news or make their own guesses. It is an easy assumption that they have access to asymmetric information.

Within the context of stock market strategies the lines get blurry. It seems infeasible to deduce even a single opponent's exact trading algorithm unless it is a simple or a more predictable one like a Kaplan's Sniper with its strong reactions near the closing period. It seems even more infeasible to tell in a large market populated by thousands of participants, multiple products, and multiple order types. In some way though this provides a stronger basis for the research done here.

The simulations are done in a simplified environment with information ranging up to perfect. This means that bounds set by the experiment will be a strong constraint for real scenarios as the large amount of interference and other factors and variables will guarantee that these strong outlier bounds are never reached. Conversely, breaching the simulated boundaries may raise suspicion that the trader has access to information that is more efficient than using the best trading algorithm for a situation - either they are very good at predicting trends or there might be some insider trading going on. The simple-purified nature of the simulations performed allows stronger claims of generalisation than if more factors were used, but were not randomized out/accounted for sufficiently to not impact the effect of information in question.

2.4.3 How Realistic Is It?

Most of the BSE simplifying assumptions should have no noticeable impact on the translation of the conclusions to the real environments they would affect. The most important one would be the fixed number of traders - due to how combinations increase exponentially it is unreasonable to simulate much larger numbers of traders enough times to retain statistical significance. What is more important is the impact of a trader following a strategy - one using a particular algorithm on behalf of a large investment company with lots of capital will impact the market far more than a broker investing their own savings. The presented ratios should be interpreted more as a combined weighting of trading strategies rather than individual agents.

Brokers may in some ways have access to information about what algorithms others are using. Although what exact algorithms are being used is often a closely guarded secret, one can make reasonable, if noisy, guesses. A delayed piece of information could come from from observing the quarterly report of a trading company and contrasting their market behaviour - it could hint at what they are doing. Alternatively a firm hiring one of their competitors former employees could acquire a hint to what the competitor is doing.

One could guess at the current ratios of various trading strategies by probing with different algorithms of their own and measuring profitability, then comparing that to a simulated database of market makeups to try and pinpoint the current situation. This, of course, would not indicate it exactly - an incomplete simulation or noisy prediction is likely.

To summarize, it is possible to estimate the strategic makeup of the environment but perfect information is impossible. Broker firms may have access to more or better ways of guessing too. If it turns out to improve overall profitability, they will or may already have adopted such estimates into their normal workflow.

Chapter 3

Project Execution

3.1 Design of Experiments

In this section I will explain the approach taken to experiment design. I will focus on what factors are to be accurately measured and what other factors may influence the results. All parameters affecting the simulation will be shown and discussed, and the code is available online on GitHub at https://github.com/borsim/imperfect_oracles for easy reproducibility. As previously mentioned, all possible combinations of traders will be tried, with some constraints. Combinations must include at least 1 of each trader type. Combinations will have an equal number of buyers and sellers for each strategy. These experiments are novel in the way that they focus on two previously unexplored factors in stock market dynamics: dynamic & varied supply/demand schedules; and strategic information. Neither of these factors was previously individually tested in depth and their combination brings additional interactions to be discussed as well.

Strategic Information

Strategic information is at the core of the research hypothesis; it is tested exhaustively. Every single possible combination of the discussed four trading algorithms will be simulated for every order schedule. This ensures a good coverage of all possible scenarios of strategies in the market. Trader ratios of higher granularity that are not directly mapped out by these experiments may be approximated by interpolating from the closest ratio points.

Supply/Demand Schedules

Supply/demand schedules can not be exhaustively tested: the number of possibilities is dependent on multiple potentially infinite variables. To produce a good coverage of scenarios a randomization approach was taken. Order schedules were crafted from a multi-dimensional space. They are composed of a number of sub-schedules, each with its own set of parameters. The only constraint is that supply and demand schedules do not change independently; sub-schedules on the supply and demand side are respectively equally long in duration. The number of dimensions in this space varies based on the first randomized parameter, the number of sub-schedules.

Simulations were run for a number of *timesteps*. Each timestep allows each trader to act and react once, issue orders and update internal values. The full duration is divided into intervals - this controls the frequency of customer order replenishment and change cycles.

- **Drawn once per order schedule:**
- Number of sub-schedules (integer)
- Duration of sub-schedules (integer)
- Time mode (set of possible values)
- **Drawn once per sub-schedule:**

- Volatility (integer)
- Midprice change (integer)
- Step mode (set of possible values)

The full order schedules were crafted with the sequence of steps shown in Algorithm 3.1.

General experiment parameters

- **Time parameters**: duration: 240, interval: 30, maximum number of sub-schedules: 8
- **Order schedule parameters**: midprice: 100, maximum volatility: 60, maximum midprice change: 40
- **Order deployment parameters**: stepmode: fixed/jittered/random, timemode: periodic/drip-poisson/drip-jittered/drip-fixed
- **Trader parameters**: number of buyers: 16, number of sellers: 16
- **Simulation parameters**: number of order schedules: 100, trials for a given trader combination and given schedule: 1

Draw ***timemode*** with even probability from
$\{periodic, drip - poisson, drip - jittered, drip - fixed\}$
Draw # ***of sub-schedules*** with even probability from $\{1, 2, ..., max\_schedules\}$
for *(num_intervals - max_schedules)* **do**
  Extend an evenly drawn random sub-schedule's **duration** by *interval_length*
end
supply_schedules = {}
demand_schedules = {}
for *num_schedules* **do**
  for {*supply-side, demand-side*} **do**
    Draw ***volatility*** with even probability from $\{0, 1, ..., max\_volatility\}$
    Draw ***midprice_change*** with even probability from
    $\{midprice - max\_change, ..., midprice + max_change\}$
    Draw ***stepmode*** with even probability from $\{fixed, random, jittered\}$
    Set **price range lower bound** to $midprice + midprice\_change - volatility$
    Set **price range upper bound** to $midprice + midprice\_change + volatility$
    Set schedule **step mode** to the random *step_mode* Set **sub-schedule duration** to value calculated before the loop
  end
  Append **sub-schedule to** respective **list**
end
return **order_schedule** = {*timemode, supply_schedules, demand_schedules*}

Algorithm 3.1: Creating series of random order schedules

3.1.1 Establishing the baseline

The first experiment establishes a baseline, a clear limit for the maximum possible efficiency/profit achievable by a trader with access to a perfect oracle providing information about the strategies of other market participants. See Figure 3.1 for an overview.

A "control group" market simulation will be performed for a given order schedule and a given combination of traders. The simulation returns the average profit achieved by traders of particular types. This serves as the oracle. The trader type with the highest average profit is deemed "dominant". There is no distinction between buyers and sellers for the purposes of strategy dominance, their account balances are pooled to obtain the average. An additional trader of this dominant type is added to the buyer and the seller pool - this is to take into account the effect of an intelligent agent on the market. Simulating a market then counting the best outcome will be, simply put, the best. Simulating a market and slightly

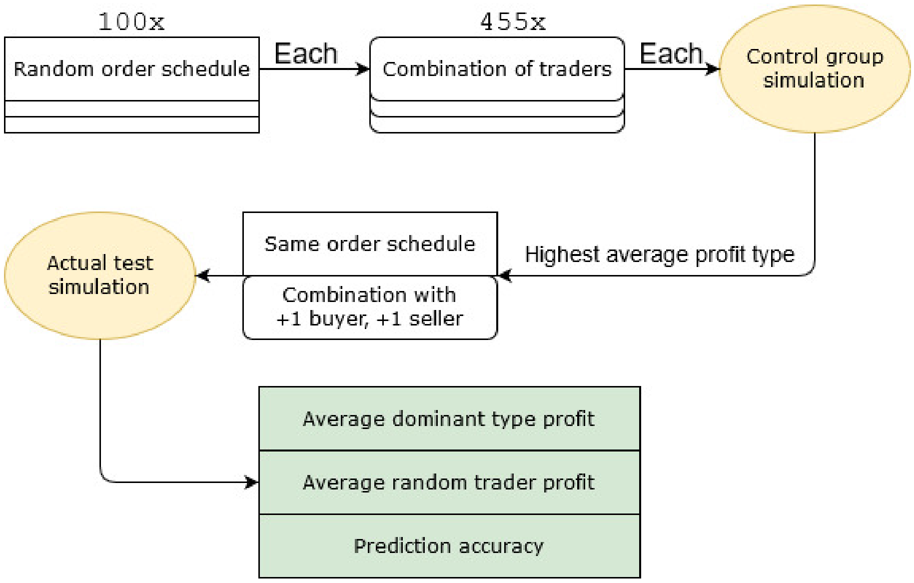

Figure 3.1: Design of Experiment 1

changing the conditions this way will show if the observations have actionable value and whether something optimal in one setting can remain close to optimal in another.
The second simulation is performed with the thusly expanded pool, this time to obtain the real data.

Between the two simulations trader strategy ratios and overall supply/demand patterns are shared. Particular orders may be different if the stepmode contains randomness - i.e. it is not fixed. The sequence of order arrivals may also differ.
Three pieces of data will be gathered in the second simulation:

- The average profit earned by traders of the dominant trader's type
- The situational baseline: expected profit earned by a randomly chosen trader (randomness weighed by trader type ratios present in the market)
- The number of correct and incorrect predictions - how many times the first, prediction simulation pinpoints the same or a different strategy as the expanded pool simulation.

The expected result of the experiment is that traders following the optimal strategy determined in the control simulation will usually dominate in the repeated simulation, but not necessarily in every case. The concrete amount of profit earned will of course depend on the supply/demand schedule the simulation was performed with. For this reason the target end result of the experiment is a ratio; a multiplier on the average trader's profit. For future trials this range of 1.0 to *ratio* will serve as the range to evaluate noisy oracle effectiveness with. By taking away information we can measure the drop in trader efficiency and make comparative statements between them. For example a 50% reduction in control simulation duration may not correspond to a 50% reduction of extra profit. It however might have the same effect as a 50% reduction in the number of observed traders. The exact details are for the experiments below to determine.

3.1.2 Noisy information

The noise space

To test the actual research hypothesis "noise" must first be defined. To some degree this will be arbitrary as there are a large range of sources one may consider as noise that could present in this simulation. It could be simply an uncertain prediction of a market model, it could be an estimate based on historic data or an estimate from knowing a subset of the participating traders. However, in the end these different approaches boil down to the same underlying mechanic. There is a set of traders following a set of strategies. In the market itself this will present as the trader population consisting of ratios of strategies. In the real world the number of traders changes regularly and they need not necessarily follow singular strategies but may instead use mixtures. However, this only makes it different in the granularity aspect, which the simulation is already limited in due to the exponential nature of possible strategy combinations and the computing power requirements to map them all.

One could represent these trader ratios with the usage of a *surface* of multiple dimensions. For two strategies this would be all points along a line - to some granularity in a simulation - of possible strategies. Note that while very small numbers of participants might result in different dynamics the original experiments of Vernon Smith have shown how few agents are necessary to show market behaviour following economic theory. For these reasons one may view the set of possible strategy ratios as a multidimensional plane - $a + b + c + ... + n = 1$ where the letters represent the ratios of an individual strategy's presence. Interpreting the strategies as perpendicular vectors of a multidimensional space this defines a plane of points. The "legal" area of the plane is not infinite however; for example one may not have a negative number of a certain strategy. In fact, the "legal" area is a *simplex* - the notion of a triangle generalized to a multidimensional space.

This is all the necessary abstraction to show why the particular definition of noise can and will be arbitrary and why its impact on the findings should not be regarded as important - redefining and mapping one kind of noise to another is not difficult with this view. Here an individual prediction's noise is fully defined by the vector between two points on this simplex: the **prediction** point and the **real** point. Noise in general is then a function that takes a point as an argument and returns a 2D probability distribution for distance and direction on the simplex. The last subsection on Experiment 2 will visualise some of these simplexes.

Definition of noise used in this experiment

One may select a different definition of noise and it would not influence the validity of the results derived here. It is possible to conduct analysis on different noise patterns by looking purely at the data points and connecting any two as a prediction and result into a different noise definition. The particular noise function for this research was chosen for simplicity and intuitive ease. It is parametrized by a single parameter p. Each trader in the "real" point has p probability of being viewed as one following a strategy that is not the same as their true strategy. The "mistaken" strategy is chosen with an even probability from the strategies present in the market, excluding the "true" strategy. As such p scales the expected distance of the prediction point from the real point and the direction is chosen uniformly from the set of available points. This could be visualized as a circle centered on the real point on the simplex. Due to the fact that not all points covered by this circle will be on the simplex, the circle is somewhat distorted towards the centre. Intuitively, this makes sense - by this definition maximum noise is achieved when all traders have an equal probability of being seen as any strategy.

$$p_{max} = 1 - (\frac{1}{|S|})$$

where S is the set of possible strategies and $|S|$ denotes the number of elements in S.

Simulations

Multiple sets of simulations with noise were performed; the exact details and reasoning for the changes in methodology will be discussed in more detail in the section below as results influenced the progression of following experiments.

3.2 Analysis of Results

This section will describe the data obtained from the experiments, the tools and metrics used to analyse them, and the conclusions drawn. As phase 2 of experimentation included adaptation of the methodology the changes will be presented in order, though they do build upon the general principles outlined above.

3.2.1 Experiment 1

In the first experiment traders were pitted against each other in a large number of random order schedules. For each trial, after a first "prediction" simulation - only a single prediction - a single second simulation was performed. This second simulation had one additional buyer and one additional seller of the trading strategy that performed the best, determined by the highest average profit per trader following said strategy. The expected results were that predicted types will perform better than the market average profit in a majority of cases but will sometimes perform worse due to the chaotic environment.

The additional profit earned by choosing the best strategy should be a scaled multiplier on a base market profit and as such could be best represented as a line fit going through 0. Intuitively explained, if all prices were halved but nothing else changed in a market session all profits would be halved too. And as no conditions change over the time of the trial the ratio should be constant. This is ensured by trialing all trader combinations and a large range of randomized order schedules.

The results of Experiment 1 can be viewed in this representation on Figure 3.2. It shows a comparison of how the strategy predicted as dominant fared in the second simulation with the expanded trader pool. The axes mark the gross average profit per trader of the predicted dominant type and the colour of points indicates which strategy it is. For added visibility the "breakeven line" is also plotted - on this line the predicted best type trader earned exactly as much as the market average.

The data confirms the abovementioned expectations. A number of points fall below the breakeven line but the majority are above and the data can be reasonably fit on a line. Predicting the best strategy will indeed on average keep a trader above market average. To better show the structure of the data points on the order schedule level Figure 3.3 is provided with only three order schedules. Here the individual order schedule clusters are visible.

Additional observations on Experiment 1

Kaplan's Sniper is not performing particularly well in this experiment. A sorted view of the best predicted strategies reveals that while ZIP, GDX and AA have a range of possible locations, SNPR traders are only located on the breakeven line and they are far less numerous than the other strategies - see Figure 3.4. If the highest average profit type is almost exactly on the breakeven line that means the entire market made very close to average profit in that trial - so SNPR taking first place is merely a fluke of infinitesimal differences.

Due to the large amount of data points plotted together, even with transparency, a number of AA points are placed over ZIP traders visually as they generally occupy the same space. GDX appears more often in far above average areas and slightly less in the smaller general profit areas.

It should be noted that placing above the breakeven line is not a singular measure of prediction accuracy. It is entirely possible that a predicted best type did not end up as the best - only above-average - in the secondary test. Indeed, this will become a crucial focal point of the following experiment.

3.2.2 Experiment 2

Experiment 2 attempts to measure the effect of the noise described in the experiment design section. A similar overall design was adopted for Experiment 2. The significant difference is the presence of a distortion between the prediction and the actual test ratios. Additionally, the number of order schedules and trader combinations vary; this is only a numerical difference and does not have an effect on methodology.

Experiment 2 was not immediately conclusive. Smaller scale trials were run first to confirm the presence (or lack thereof) of correlation before committing a large amount of computational resources. Methodology was then adapted to reduce the noise in the resulting data. Finally a large trial was performed to provide more accurate numerical output showcasing the merit of the previous changes. All components of Experiment 2 had the market session duration extended from 240 to 330 timesteps.

Initial test

The first trial closely followed Experiment 1. It also involved 100 random order schedules. Each order schedule was assigned a noise p with the first having 0%, the second 0.75% following to the 100th having $(75 - 0.75)\%$ and each of these schedules had all trader combinations trialed once.

The first graph in Figure 3.5 shows the average ratios for each order schedule of $\frac{profit(optimal\_type)}{profit(market\_average)}$. The full range of data points is omitted for visibility but the line fit is on all points, not the averages. The line fit coefficient is less than 0.0003 away from horizontal and visual observation does not reveal strong trends that were covered with noise or outliers. This means that there is no evidence that in this case the prediction conferred a valuable piece of information.

Figure 3.6 plots the number of incorrect predictions - cases where the predicted best type did not earn the highest average profit in the second simulation. As noise in the prediction ratios increases this is expected to have an upwards slope. While it does have an upwards slope, the data surrounding it is very noisy and the change itself is fairly small compared to the full set. With a coefficient of 0.01, at

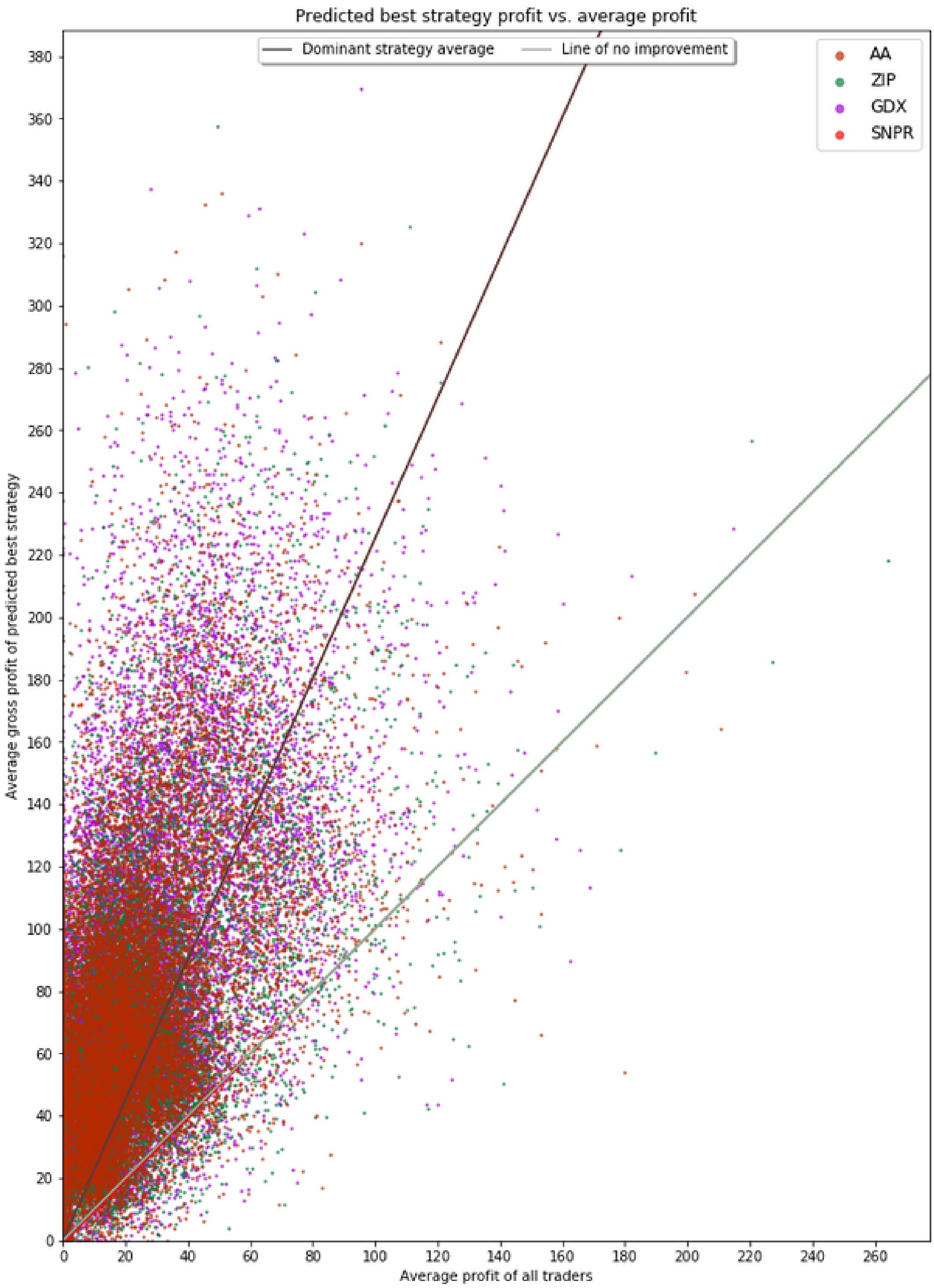

Figure 3.2: Overall results of Experiment 1

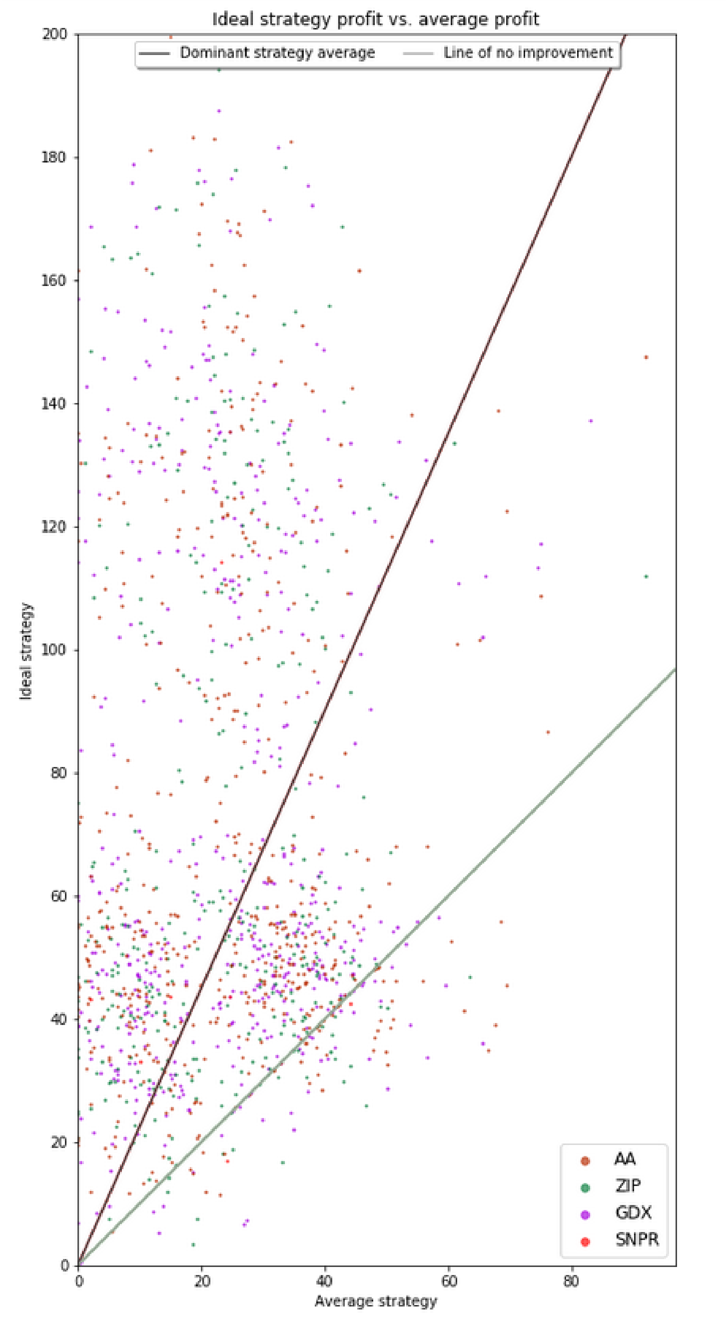

Figure 3.3: Three order schedules in Experiment 1

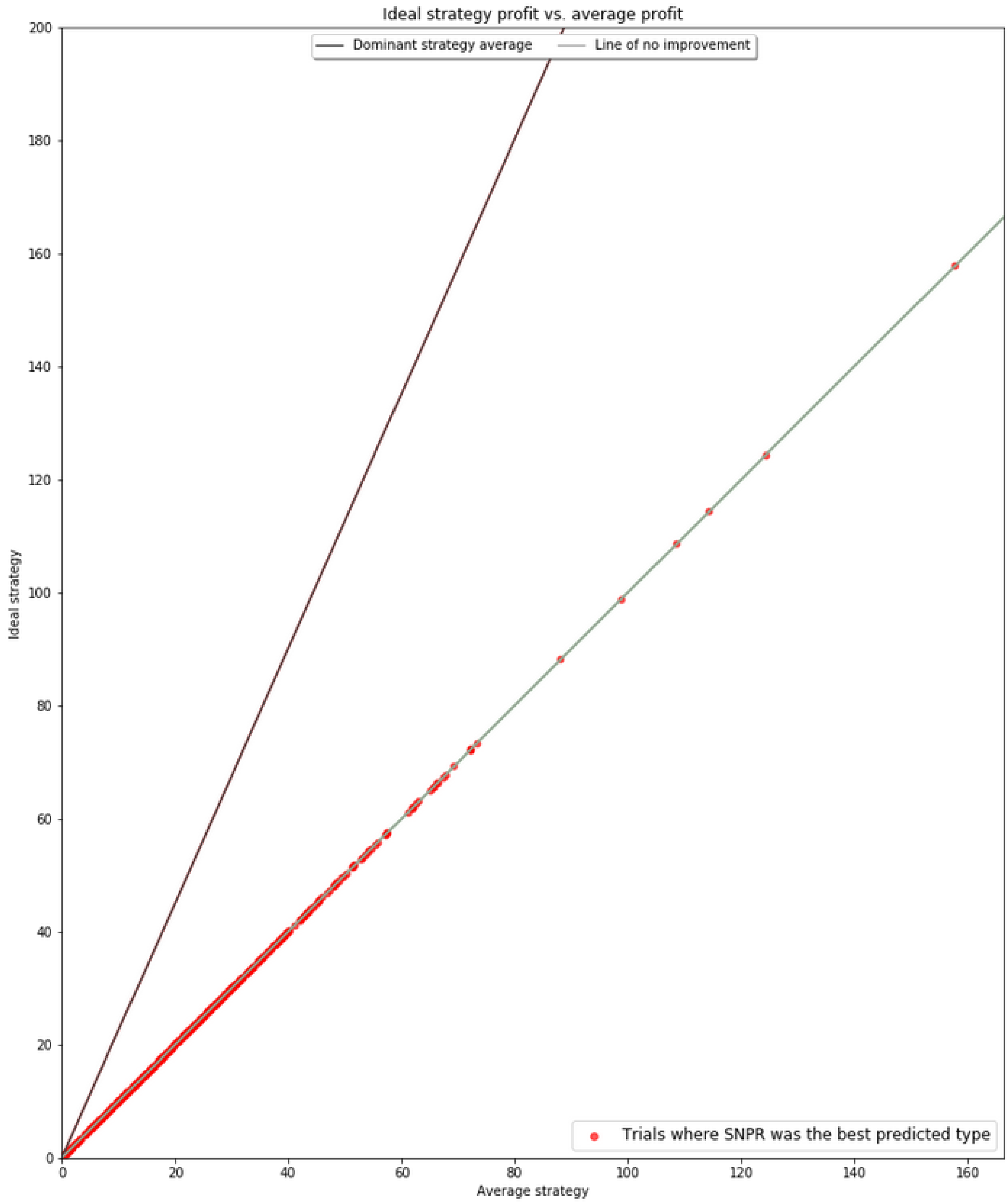

Figure 3.4: SNPR performance in Experiment 1

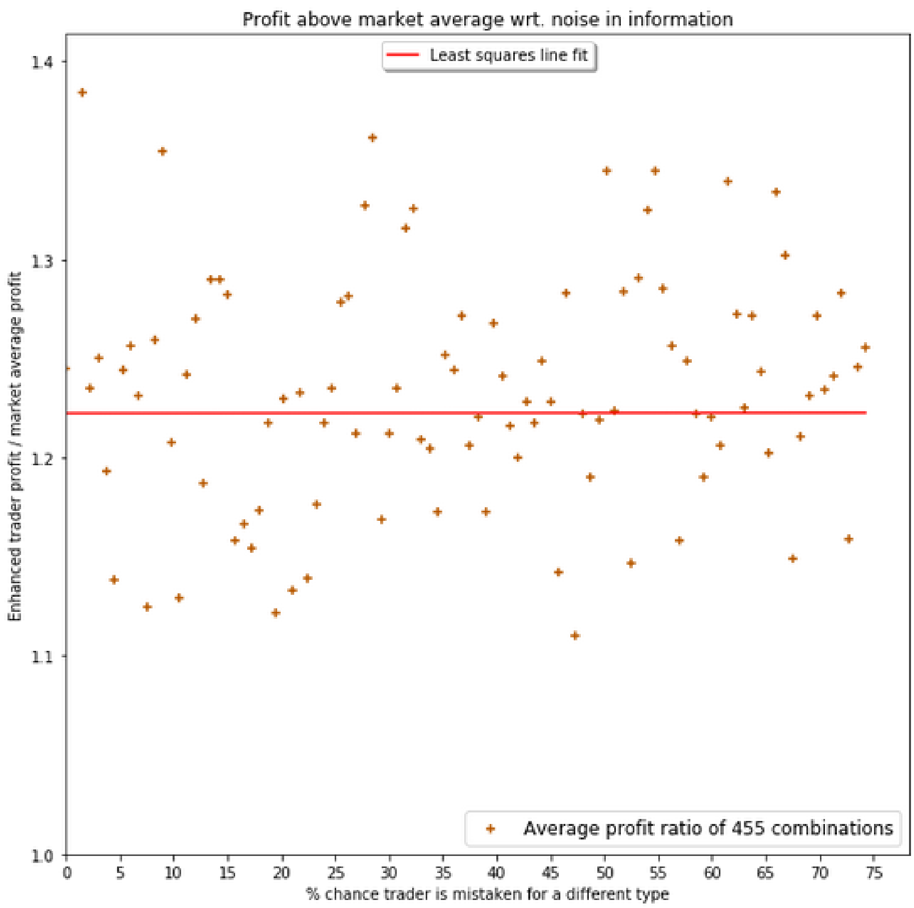

Figure 3.5: Profits in the initial Experiment 2

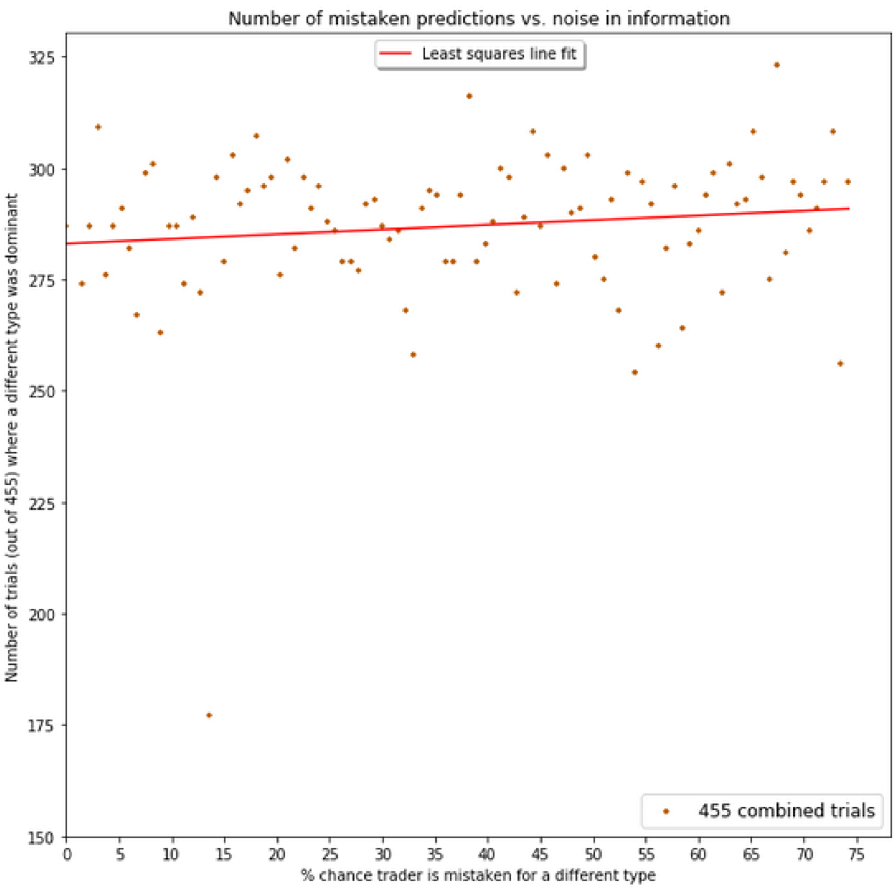

Figure 3.6: Prediction accuracy in the initial Experiment 2

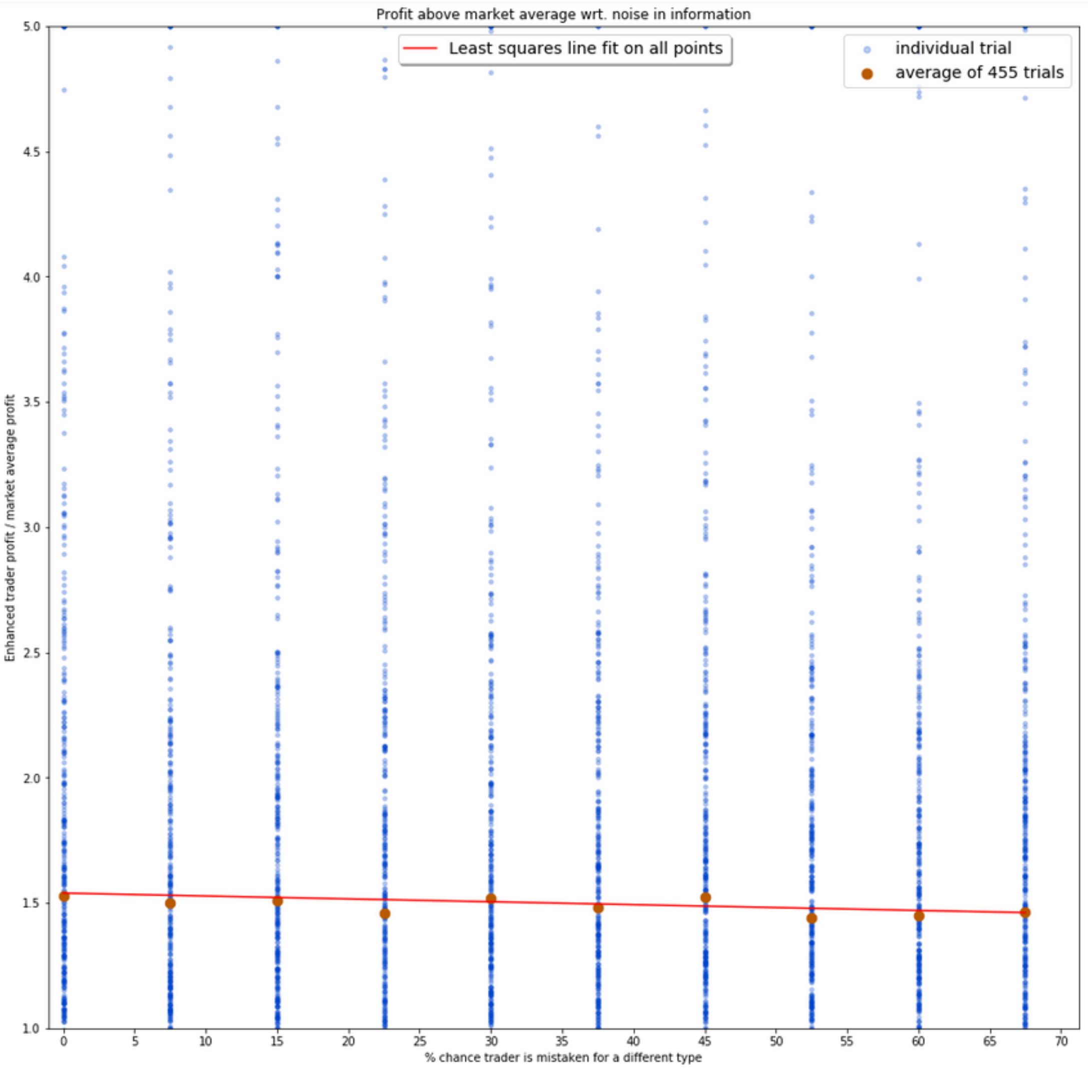

Figure 3.7: Simple order schedule sampled at 10 intervals

the highest noise level (where on average each trader is assigned a type through a roll of a 4 sided die) on average 7.5 more incorrect predictions are made. This is a change of 1.5% when in the context of a full set of 455 prediction-result pairs, the change being a result of going from zero noise to the maximum possible noise. Combined with the previous result on profits this suggests that these predictions are just not particularly good at the moment. Over half of them are incorrect from the outset.

Simplified supply/demand schedules

A followup mini-experiment aimed to check whether the randomisation of order schedules had too much of an effect on the outcomes of the previous, inconclusive trial. For this experiment the order schedule was set to a very simple base case reminiscent of those of Vernon Smith. The supply and demand ranges had equal limits, prices were allocated at even steps in these ranges and resupplied to all traders periodically. Save for the order-trader allocations and the order of incoming orders from the set everything else is deterministic. This schedule was similarly sampled at 10 distinct noise probabilities.

The line fit and the overall patterns obtained can be seen on Figure 3.7 with a few outlier data points omitted over the top. For the full data range refer to the box plot 3.9. The line's slope coefficient is -0.001 (per 1% of probability). While this is an order of magnitude greater than that of the previous experiment it is still lacking the expected, slightly more marked decrease. Figure 3.8 reaffirms that while the overall number of incorrect predictions is lower in the very simple and deterministic schedule, the environment is still noisy and the smaller number of points in this mini-experiment even produces a downward trend. Considering how poor the fit is - included to retain consistency with the other prediction accuracy graphs - it should just be regarded as no evidence for a correlation.

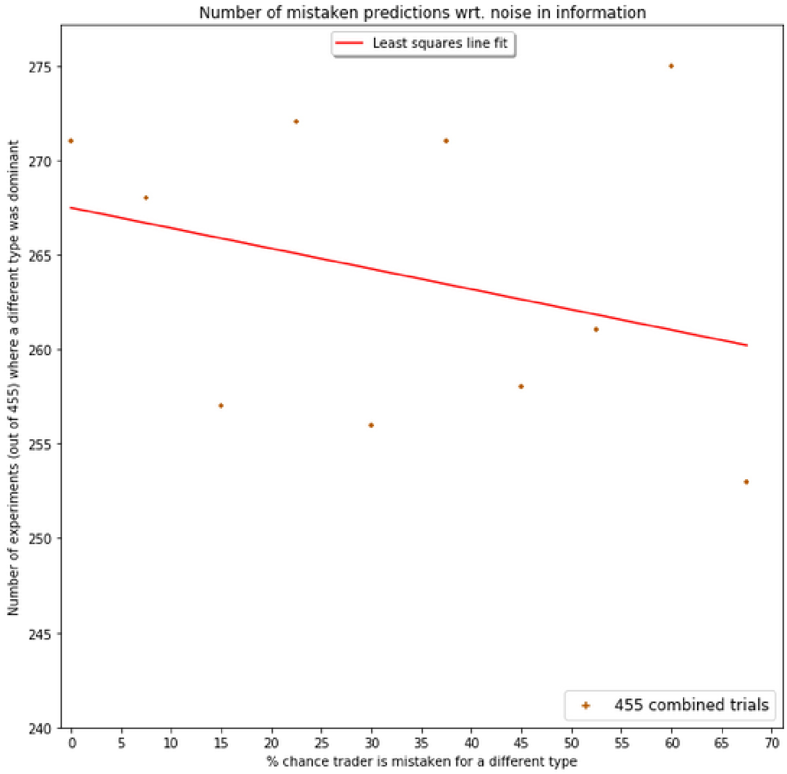

Figure 3.8: Prediction accuracy on the simple order schedule

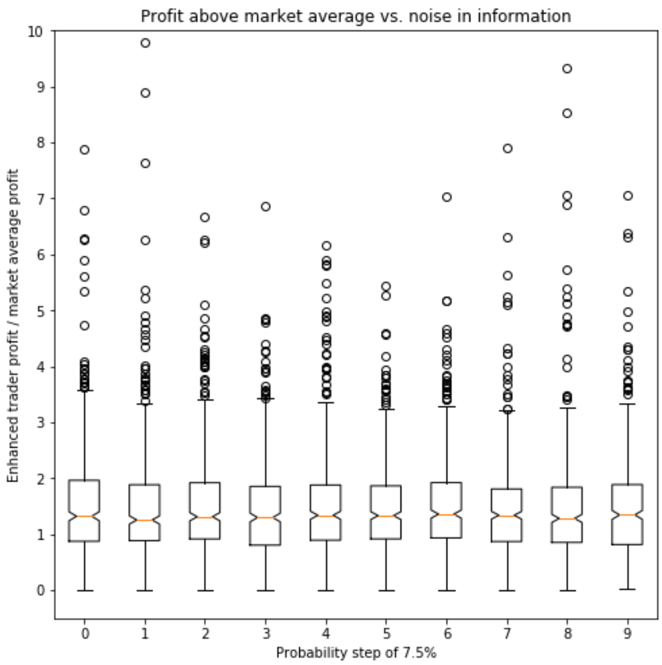

Figure 3.9: Simple order schedule data range

Repeated predictions

The next mini-experiment focuses on the low prediction accuracy shown so far. It samples the simple order schedule at 10 evenly spaced probabilities like the previous one but introduces a large change in how predictions are made. More than one prediction is made each time - for this mini-experiment's case, 10. The predicted best trader is the one that has the highest average profit in the sum of the 10 predictions combined. The real data trials are done in a similar fashion; the final value for a prediction-real pair is the average profit in the 10 trials. Note that each prediction uses the same (noisy) trader ratio combination. Trader ratios are not re-randomized in-between predictions.

As seen on the low-level graph with all subtrial results on Figure 3.10, the range of profits is now much narrower. Because the multiple trials were averaged with a simple mean calculation that is linear in its treatment of outliers, the averaged values are lower as well when contrasted to the least-squares fit of the previous graph. The box plot of the data ranges is far more compact with a range of just $0.5 - 3$ instead of $0 - 10$, indicating significantly improved prediction accuracy. Instead of profit multiplier values of 5 and beyond no single averaged-trial goes above 2.5. However, the slope of the profit graph is still very close to horizontal with a coefficient of 0.001. Despite that, this coefficient is objectively a more useful value due to how the narrower and more regular data range leads to a stronger implication of a cause and effect rather than random noise.

The prediction accuracy graph further supports the above argument. The previous accuracy graph line fit is of very poor quality and as such exact metrics like the residuals of the fit are meaningless in context. For this new line, individual points are much closer to the line and the line is a visually obvious fit too. It shows the expected clear upward trend - more noise in the prediction results in more wrong predictions. This trend is also significantly higher in impact than the previous existing trends. Going from 277 to 297 it nearly triples the previous prediction error change with a better fit.
For the large scale simulation repeated predictions and real trials appear to be a must.

Large-scale test: repeated predictions & randomized schedules

One additional change was made in addition to the previously discussed ones. Due to its low performance and unlikeliness of being the best on its own merit, SNPR traders were taken out of the pool of possibilities. This results in a number of changes to the starting parameters of the simulation:

- The total number of participating traders decreased with the number of available strategies to $4 * 3 = 12$ for the prediction trials and $12 + 2$ for the real trials
- The total number of possible trader combinations dropped from 455 to 55
- The maximum possible noise probability lowered to 66.6%

Due to how passive and nonadaptive SNPR traders have proven to be, this change had an additional beneficial effect. The three advanced algorithms were now in closer contest with fewer bystanders, meaning that the profit ratio being measured is closer to 1. Previously all of them had a fair amount of extra profit just by taking advantage of bad SNPR trades. This also created extra noise in the data - which advanced trader can first steal a SNPR's extremely good deal was random and as such polluted the view of the market dynamics. Finally, SNPRs often do nothing for most of a market session. By removing them the overall liquidity - number of trades performed - increases and as such the data converges better on the "true" value of trader efficiency.

This large trial was done on 10 different and randomized order schedules. In one of the schedules the randomizer produced an unfavourable schedule, resulting in few to none trades in most of the trials and as such the data from that is discarded. The other schedules were tested at 14 distinct noise probabilities in the range of $0\% - 65\%$ with increments of 5%. Each probability point was tested for all 55 trader combinations and each trader combination had a set of 50 prediction subtrials and 50 real subtrials.

Before the summaries relating to the main hypothesis of this research are presented a slight correction in evaluation methods must be made. Previous, less exhaustive experiments did not show the phenomena presented below in an impactful way when plotting profit so the credibility of using least squares line fits for them has not changed. For this subsection however, the least squares fit on its own is no longer an accurate enough tool. Figure 3.13 presents one of the order schedules with two lines. These are least squares fit for the predicted best trader type's profit and the market average profit. Note that the market average profit should, under every condition, stay constant. The market average does not change with noise and it is taken from all trader combinations equally for each probability. In addition, visually the market average line misses big clusters of points to allow for a few distant outliers. As this research

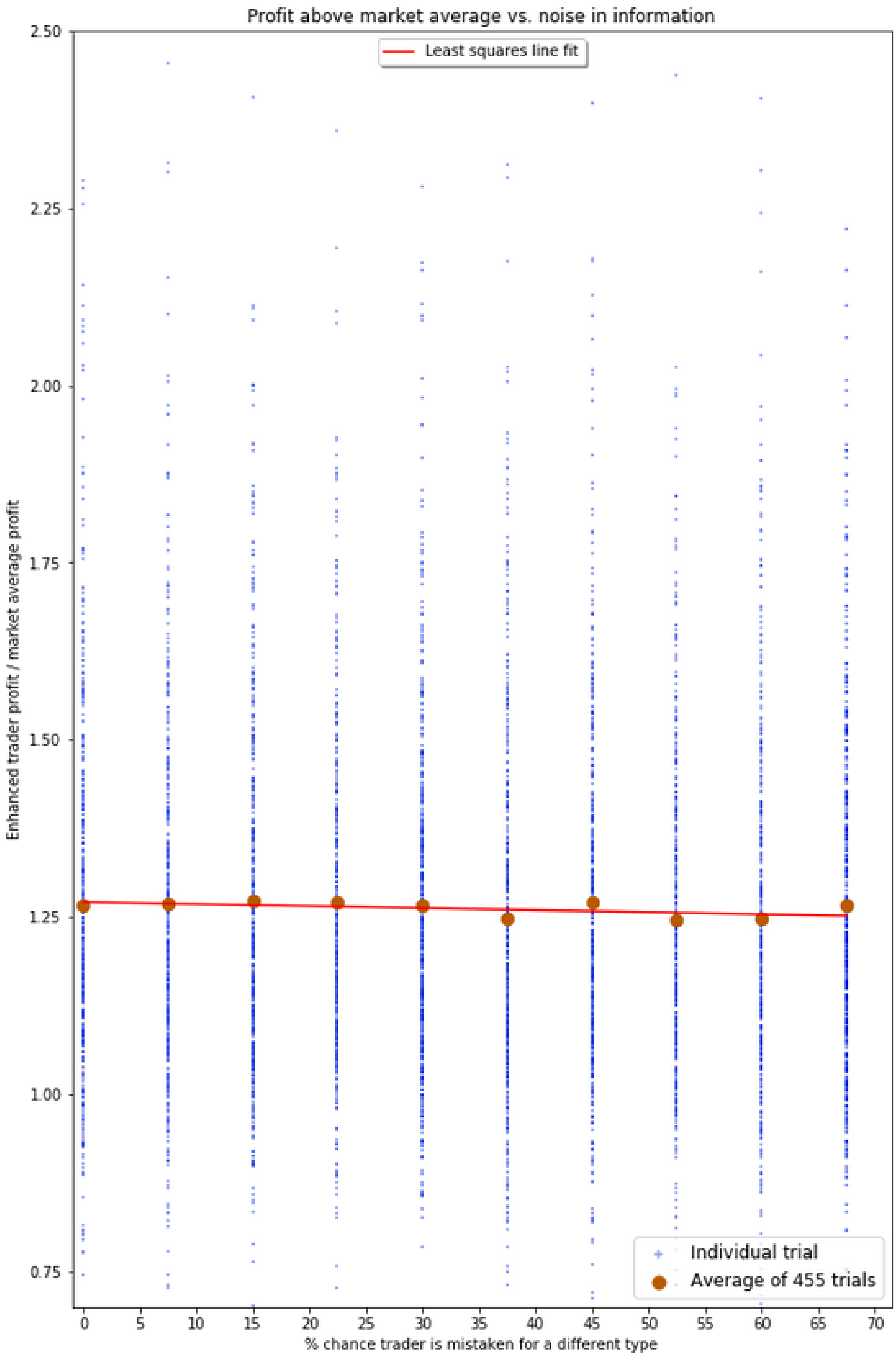

Figure 3.10: Profits on the simple order schedule & multiple subtrials

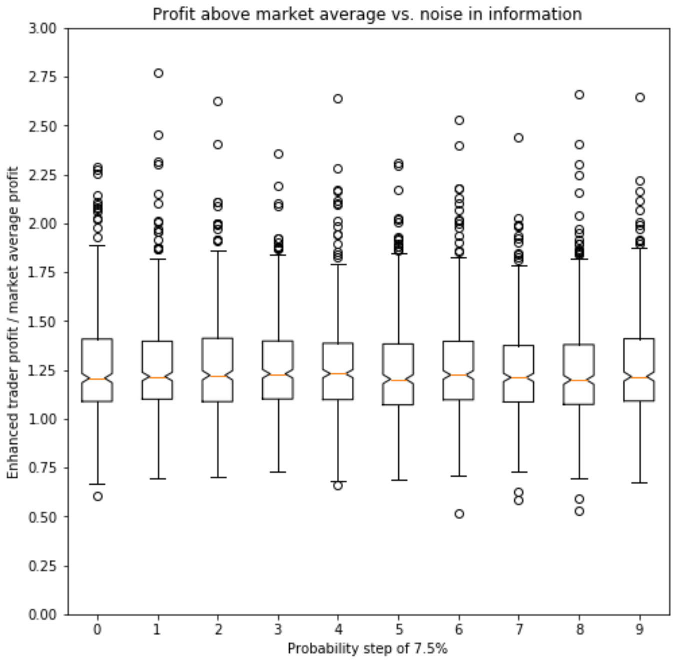

Figure 3.11: Repeated predictions & simple schedule data range

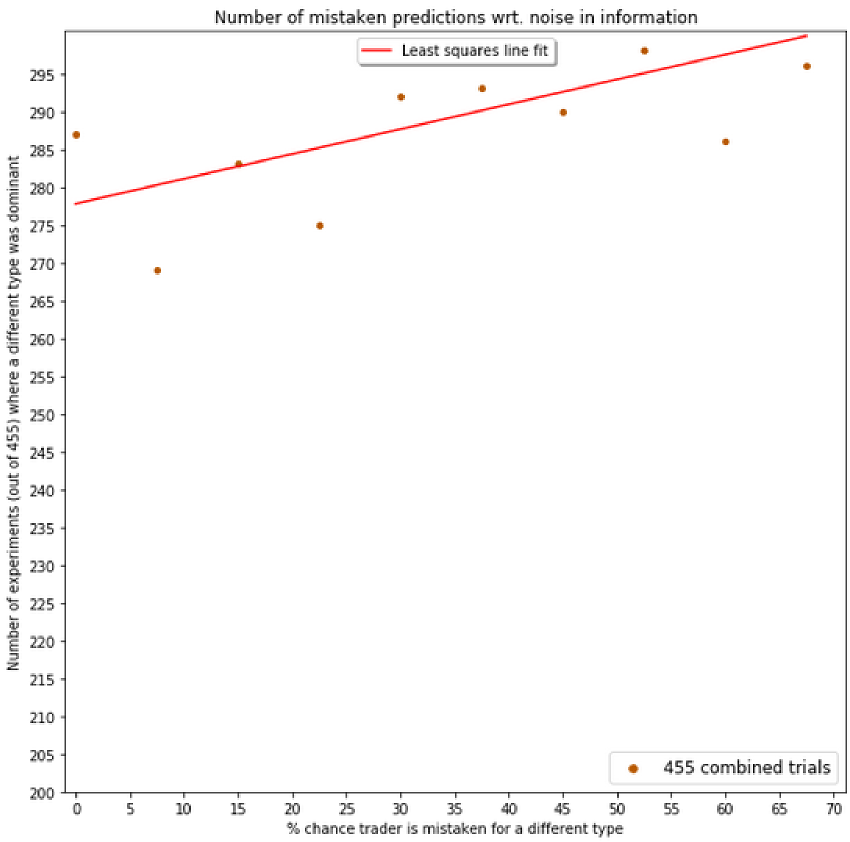

Figure 3.12: Prediction accuracy on simple order schedule & multiple subtrials

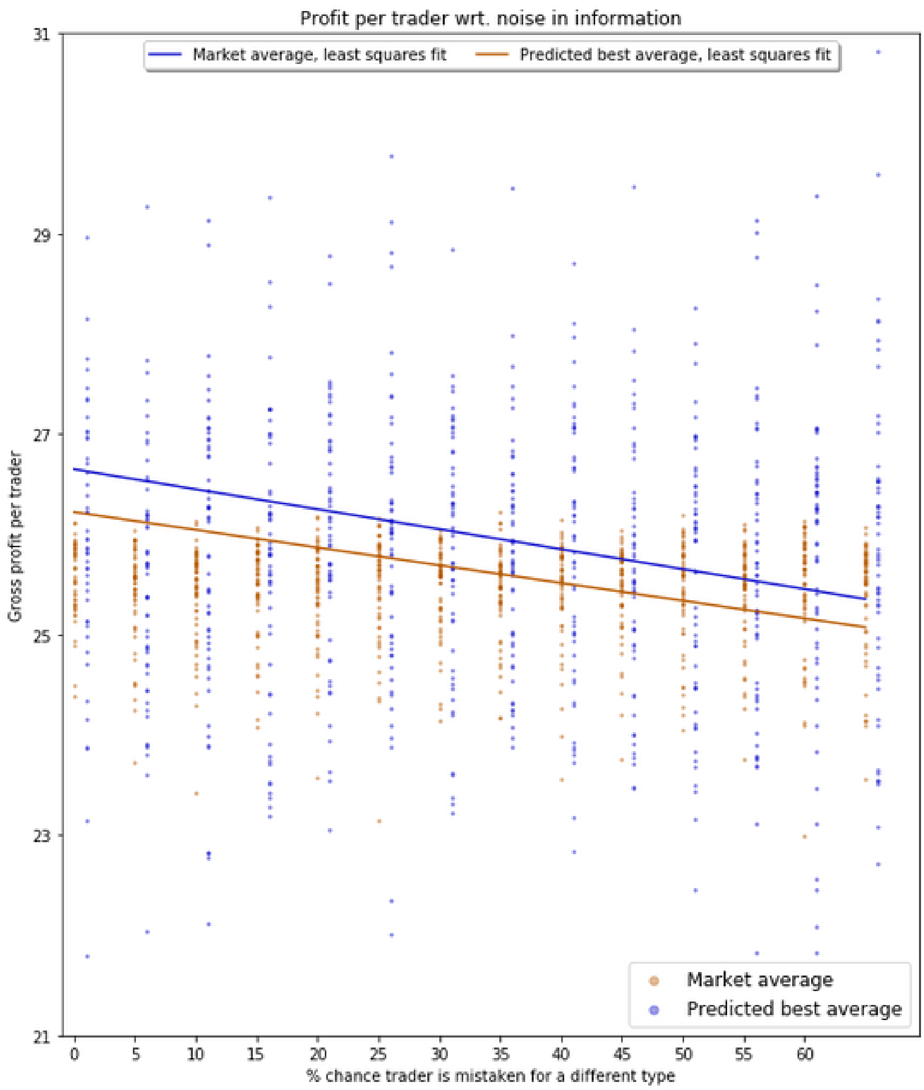

Figure 3.13: Problems with least squares fitting

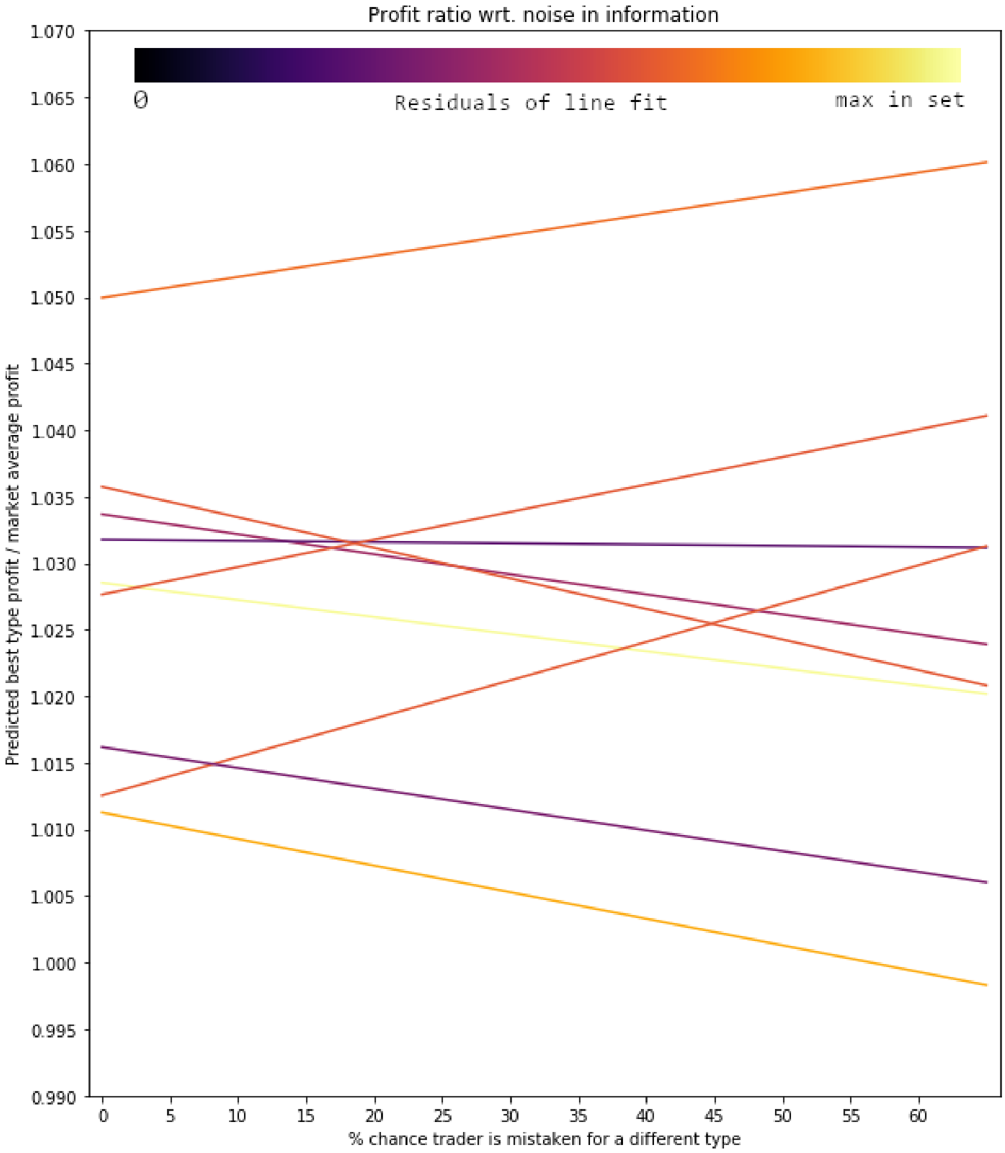

Figure 3.14: Profit trends for 9 order schedules

is essentially targeting for analysis the difference in slopes between these two lines this is problematic - though slightly less so than it appears due to the comparison being point-wise rather than line-wise. Still, the least-squares fit alone in some complex schedules should be taken with a grain of salt.

The observed phenomena still hint at being the closest to fitting on a line. Fits of higher degree polynomials were attempted but the higher-rank coefficients were close to 0. Even after increasing prediction accuracy some outlier removal steps still had to be taken, especially with the re-inclusion of more complex order schedules. However, as visible from the box plots in this chapter, the data range has a complex structure with traders sometimes earning extraordinarily high profits - outliers on the upper end are far more common than those on the lower end. Outliers were removed with a method common in statistics but with slightly different parameters. Generally, points outside 1.5 times the inter-quartile range are considered outliers and the box plots further above represent this common view. For the purposes of not losing too much important data - it should be noted that even these outliers are a result of $50+50$ individual data points - this interquartile range requirement was loosened. The central range is the interdecile range (between 10% and 90%). The multiplier for acceptable points is 1 times this range from the edges of it.

Figure 3.14 shows the line fits on all schedules. The colour of the line indicates how good the line fit is; a darker colour has closer to 0 residuals. The lightest colour is set as the maximum residuals out of the 9 line fits. The majority of trends display a clear downwards slope, among which are the three closest

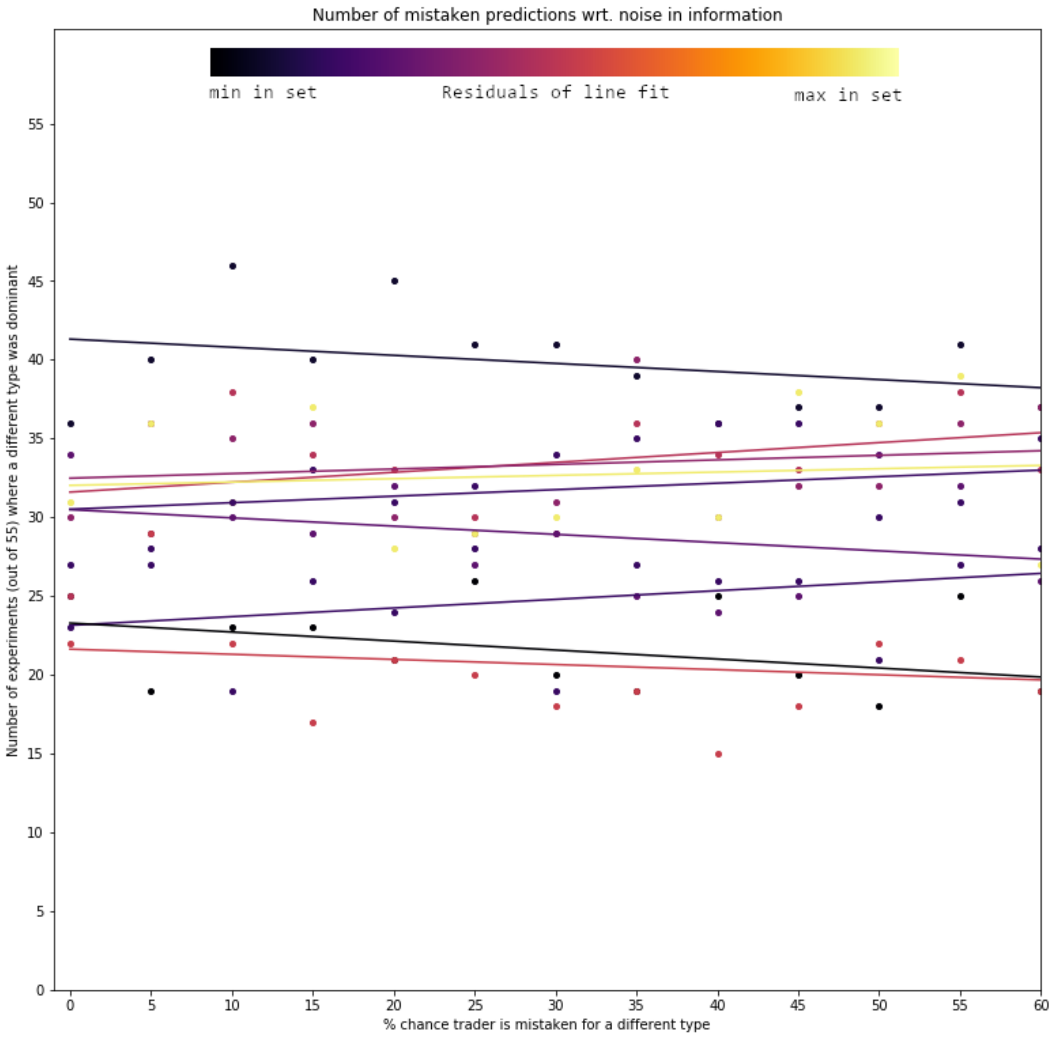

Figure 3.15: Prediction accuracy for 9 order schedules

fits. One third of the studied order schedules display an upwards slope of moderate uncertainty. These trends are closely paired with the prediction accuracy graphs in Figure 3.15, which shows four lines with an downwards slope where upwards is the expected direction - more noise means more prediction errors. Three of those coincide with the upwards profit schedules and one poor line fit of prediction accuracy has a downward slope on the profit graph.

The overall result of the large-scale experiment supports the research hypothesis. The majority of order schedules tested show an above-average profit earned from good predictions that steadily decreases over increasing prediction noise in strategic information. Figure 3.16 is a visualization of distribution differences in the trial profits. It presents comparisons through nonparametric Wilcoxon tests of the distributions involved in the nine order schedules.

Individual data points are a pairwise comparison between the noiseless prediction-result distribution and a noisy prediction-result distribution. Small p values indicate strong certainty that the samples are drawn from different distributions. A large portion of these tests indicate that the samples with noise involved can usually be distinguished from samples without prediction noise. A possible explanation for the uncertain predictions and upwards profit trends could be that certain order schedules spawn more complicated strategic dynamics. For the majority of schedules the addition of two extra traders of a kind did not nullify the advantage of having access to predictions. However, for the schedules that show negative correlations, this alteration in trader ratios might have ended up working against the traders with access to an oracle, pushing them over the boundary where the predicted strategy was no longer the optimal one. Such a scenario could arise when most of the strategic landscape is dominated by a single trader with very small pockets of other types. Figure 3.17 shows a comparison between Schedule 3 - where predictions performed well - and Schedule 7 where they did not.

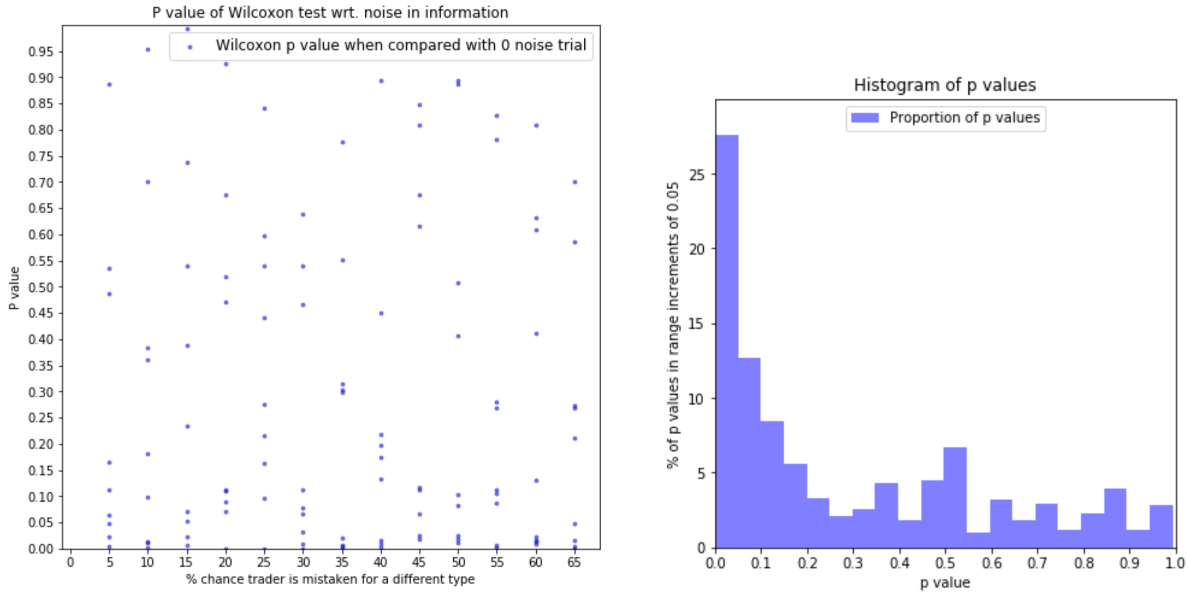

Figure 3.16: P values of Wilcoxon tests

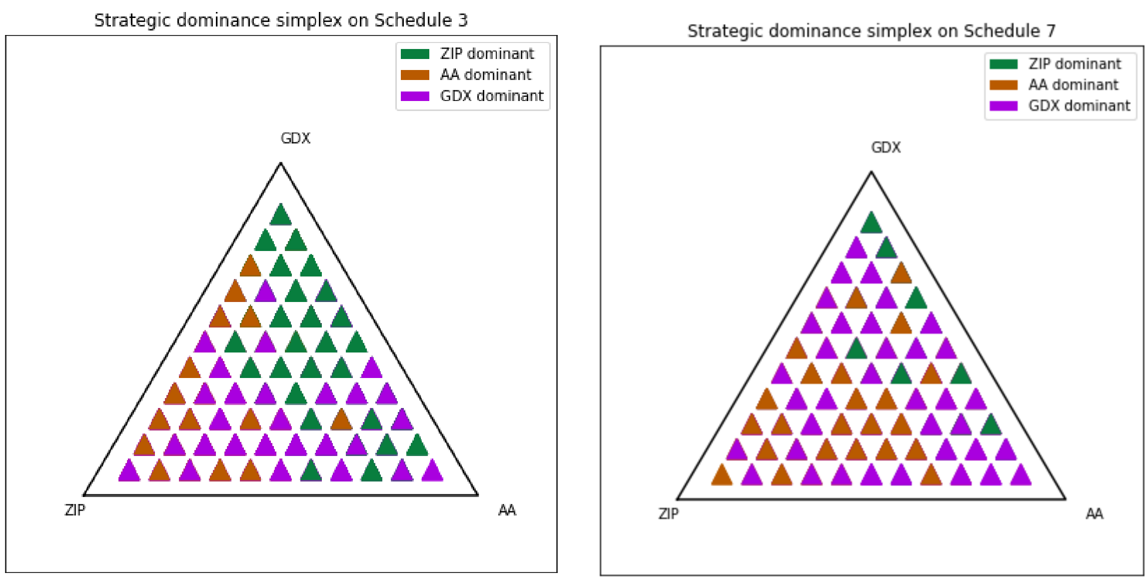

Figure 3.17: Comparison of strategic simplexes between well- and badly-predictable order schedules

Chapter 4

Critical Evaluation

This chapter will reiterate some of the possibilities and limitations of experiments in this field to assist in contextualizing the results and the strength of provided evidence.

4.1 Choices and Methodology

A significant discussion of the specific choices made during the experiment design and evaluation of results has already taken place in the previous chapter, Technical Execution. The results of that chapter were obtained through refinement of the experiment rig and approach. While said refinement could be included in this chapter for a better fit of topic, knowledge of the previous steps attempted and previous conclusions drawn is necessary to fully understand the last experiment. The figures presented are usually of the same data feature of each data set and their progression exemplifies how these features change when subjected to more detailed analysis.

For these reasons this chapter will only contain a summary of the notable choices made and of the experiment methodology and its strong and weak points. For the full-length justification of these please review the relevant paragraphs in Technical Execution (Chapter 3).

4.1.1 Choices

This research set out to explore the possible advantage gained from having information about the ratios of other trader strategies in a market. Additionally, it aims to measure how noise in this information translates to a loss in the achieved advantage. Only the choices where competing and sensible options were available are discussed here; the subsections are intended to showcase the reasons a particular choice was advantageous, or that a different option would not have affected the results of this research.

BSE

The usage of BSE introduces a number of simplifications to the stock market environment; all conclusions in this thesis are derived through the lenses of these simplifications. Any conclusions reached are only as close to the underlying phenomena as this approximation of it. With that noted, the BSE approximations are sensible and overall necessary. In a system of this complexity the addition of extra features and factors can lead to unexpected within-system interactions and, through the many breakpoints and nonlinearities, produce unintended effects. Said unintended effects could mask or even override the intended targets of the researcher. As mentioned in the context chapter: a rounding to 3 decimal places from 6 decimal places when the system state was saved caused large changes in a weather simulation system's end results[10].

There is no particular standard for stochastic stock exchange simulator; the primary question is whether they provide useful data, and whether the results can be replicated or built upon. Released in October 2012, BSE has been successfully used in teaching and research for over 8 years e.g.[16]. It has undergone several meticulous code reviews and is as bug-free as it can be. It is difficult to make strong statements about a project in empirical computing. However, the fact that this project was built on the Bristol Stock Exchange should serve as a note of confidence and reliability of the underlying system, rather than a highlight of the assumptions it needs to make to function.

Strategies

BSE allows for entering custom trading strategies. However, it is limited in granularity - there can only be so many traders in a market at the same time before runtime of the simulation becomes a concern. As a result, the number of distinct traders was limited - 16 buyers and sellers in the first, 12 buyers and sellers in the second experiment. Each trader followed a “pure” strategy of a trading algorithm known in research. Although one could introduce mixed strategies - e.g. at each point where a trader could react they could flip a coin and react like a pure strategy of a set of 2 based on the result of the flip, the overall change in the market dynamics should be insignificant. Two of the previous example strategies would be just as well approximated with one of each. The limiting factor remains the simulation’s granularity, not the lack of mixed or different strategies.

Self-Effect

Each intelligent agent in the market affects the market and its own profit. Without some sort of self-referential effect the conclusions of this research would be largely trivial - picking the strategy with the highest profit on average leads to the strategy with the highest profit, tautologically. It would be a test of prediction accuracy in different distributions. This could be an interesting project for future research, but it was not the desired focus.

To simulate an amount of self-effect, an intelligent agent’s actions changing the environment around them, an extra buyer and seller were added to the trading pool each time. This is to align with someone making accurate market predictions, then entering the market to capitalize on them. As some order schedules in the final experiment have shown this extra effect might have seriously decreased the value of predictions in some cases.

The method of self-effect was chosen arbitrarily. There is no notable precedent or method in research in this field. Self-effect was kept to the minimum permitted by simulation granularity. Increasing the amounts of new traders or adapting old traders strategies could be other valid ways of achieving self-effect.

Predictions

The overall process for a single experiment trial ensures good coverage of variables deemed influential to the result. Not all variables are covered in all possible settings - more on these further down below. The choice in question was to focus on all possible sets of “real” trials and work backwards from there.

Essentially, while prediction trials are made before real trials, the trader strategy sets are first constructed for real trials. This ensures that all strategy ratios are covered in real trials. Prediction trial strategy ratios are then generated with noise from the determined real trial ratios. Duplicate prediction strategy ratios are possible for different real trial strategy ratios. Ratios that are not normally possible in real trials can exist - namely, there can be strategies with 0 active traders in a prediction trial. Prediction trial ratios are not re-randomized when predictions are repeated. This final choice can be justified either way. An analyst working for a trader might trust their strategic ratio estimate enough to not average out a possible correlation by performing multiple trials with different ratios. Alternatively, they might opt to include the uncertainty in their predictions and re-roll the noise with every trial. In this case due to the precise control over the amount of noise introduced, no re-randomization of prediction ratios was adopted.

Noise

As discussed in more detail in the previous chapter, noise can be thought of as a probability density function mapping between points on a simplex of possible strategies. Although the data points of the experiment were obtained in a structured way with intended links between certain points, it is possible to discard the information about noise probabilities, predictions and real trials in the data structure. That way, by re-ordering the data one could test hypotheses about different types of noise without needing to generate a new dataset. For example one could generate predictions with an evolutionary approach, where a number of traders that performed below-average are replaced with different ones each iteration, then test the link between the number of iterations and the difference in profit.

The choice of this particular noise function is not particularly important. It was chosen for intuitive and computational ease, as well as being symmetrical to all trading strategies and thereby not inherently impacting the viability of one particular strategy. It is reasonable to theorize that different noise definitions would get similar results, but this was not tested here in any capacity and can be grounds for further research.

4.1.2 Methodology

Experiment methodology was tested and adapted in three core ways. These changes will be briefly summarized in the subsections below.

Order Schedules

Previous research in the field was lacking in taking into account the changes caused by differently structured order schedules. An order schedule randomizer was made to explore the capabilities of BSE's schedule details. A deterministic and simple schedule was uniquely tried for a mini-experiment; it yielded data with slightly less noise and variance. While helpful, the decision was to keep schedules broadly random to generate significantly different scenarios and better map the possible schedule-space. This has proven to be of use as it uncovered that there are certain schedules where the research hypothesis does not hold or even has a possible negative correlation. From there awareness of edge cases can be derived and a theory for their existence put forward. Indeed, presenting findings as a universal rule in empirical computing should raise the question of what variable was not accounted for. One would always expect there to be some cases where the conclusion is uncertain or invalid and pinpointing such cases is useful for later developments.

Here the uncertainty inherent in some order schedules is proven by data from two entirely different and unlinked sources - the trends of the prediction accuracy graphs have no direct link to the trends of the profit graphs other than being from the same experiment, yet the uncertain or negative correlations appear in the same order schedules in both of these data sources. There is no reason to initially believe that an order schedule's specifics affect these two properties at the same time. There is nothing explicitly preventing profits from being higher on average while prediction accuracy drops, but evidence shows that these uncertain or negative correlations appear alongside each other.

Repeated trials

Individual prediction runs were found to be heavily inaccurate in finding the singular best performing trading strategy. Repetitions of both the prediction and the real trial have significantly improved accuracy and made the data set far more compact and less influenced by significant outlier points. This does not mean that outliers were completely eliminated; the environment is still noisy and prone to producing extreme results. Before this change however all correlation was essentially masked or nonexistent. 50 repeats of a trial per prediction and real trial appears as a good estimate for the number of trials needed to minimise noise. Determining how many repeats improve accuracy in a significant way before more trials have reduced usefulness can be a topic for future work.

Outliers

Due to the more limited scope of the smaller experiments the outliers were not a problem in those cases. The eventual larger scale experiment presented a few - not all - cases where the market average profit was correlated with noise. This, of course, is nonsensical - no parameter changes for the market average profit in the real trial case for any experiment other than trader ratio, which is tried in every combination and therefore averages out. Taking out a few very-significantly-outlier points through a reparametrized standard method - $interdecile\_range * 1$ instead of the standard $interquartile\_range * 1.5$ - fixed this skewed distribution and allowed for ratio analysis of extra profit independent of how much net profit an order schedule permits.

4.2 Runtime Constraints

The primary limitation on how much certainty this research could achieve was the runtime of the simulations. The smaller trials took approximately 1.5 days of continuous, full-load (no other notable CPU usage) running on a high-end personal computer's Intel Core i7-7700HQ CPU. The big experiment's runtime was 2 days on 10 nodes of the University of Bristol's BlueCrystal Phase 4 supercomputer, on 14-core Intel E5-2680 v4 (Broadwell) CPUs. It is reasonable to assume that one could acquire more and more accurate data by simply running more simulations. Other than increasing the sheer number of available data points it could allow for better granularity of trader ratios, more repeats of particular trials, higher granularity in the number of traders present in the market, more order schedules, longer market sessions and more. Unfortunately, all of these factors are multiplicative with each other. For the purposes of this thesis, taking into account the development of the experiment methodology alongside the experiment code, this is the amount of computational resource access that could be achieved. The listed parameters are analysed in a reasonable resolution but this resolution can always be improved by adding more trials.

4.3 Generality of Results

Every effort has been made to permit a broad generalization of the conclusions drawn here. Stepping futher than previous research on trader strategic ratios and dominance, a number of radically different order schedules were tried and found to form different simplex surfaces. On some order schedules strategic information will not be specifically useful when self-effect is present but on most of them it is advantageous to try and predict what the market contains. The advantage gained from this prediction does not disappear quickly, it shows a primarily linear trend so even an inaccurate guess is notably better than none at all. The majority of noisy prediction distributions have shown to be notably different from the noiseless distribution when subjected to a nonparametric Wilcoxon test which makes very few assumptions of the underlying distribution's shape.

Arbitrarily made choices were discussed in more detail earlier this chapter. These choices should not have a significant effect on the ability to generalize the findings. All numbers and data points presented here are a product of partly or fully stochastic processes - they should be treated as approximations of the underlying distributions and are, by nature, not exact.

Chapter 5

Conclusion

This chapter will present a final overview of the project. It will list the main contributions and achievements with respect to the initial objectives. It will close with propositions and interesting areas for future research.

5.1 Main Contributions

This project proposed a research hypothesis on a topic of significant real-life importance. The fairness and regulatory tools of financial markets draw notable attention especially after recessions like the 2008 housing crash or the likely fallout of the 2020 COVID pandemic. It is vital to have scientifically estimated bounds for what advantage is reasonable as a function of access to information. This exploratory study suggests an approximate 10% profit advantage difference between perfect and no information in most cases. Excess of advantage could prove to be a strong hint towards requiring further, manual investigation of a market participant. With the research hypothesis strongly supported for a majority of cases there is space and motivation for starting the development of official regulatory tools based on it.

As well as confirming the initial hypothesis, this research explored the most significant factors influencing market predictions in a simulated environment. Future research now has a proven process to use as a basis, with knowledge of the primary obstacles in assessing predictions. This thesis also unveiled some failure cases of the proposed prediction system where a counter-intuitive correlation is present. The failure cases are supported in evidence of their exceptional nature from two independent sources of information of experiment data, profit ratio and prediction accuracy. Following work can either avoid said failure cases or delve deeper into why they produce the results in question.

This project extended the Bristol Stock Exchange simulator with more functionality. These extensions are open for anyone to copy, use or modify. The data pipeline of the analysis is also published online and may be used directly on any experiment data file with no additional steps.

As explained in the *Technical Execution* chapter with regards to noise, all data points in the data files may be reinterpreted in their relation to each other. The data obtained remains useful as one may perform an analysis of a different noise phenomenon without the need to re-generate hundreds of thousands of lines of .csv data.

5.2 Project Status and Initial Plans

All code written for the project is in a working state with no known issues, for both the experiment rig and the data sorting and analysis pipeline. Great care was taken to enable extensions and reusability of materials. The functions added to the experiment rig can be enabled or disabled as necessary and are parametrizable to allow for significantly different experiments without a complete overhaul. Randomized order schedules are saved and printed into a file while an experiment is performed - to facilitate re-generation of data in case the experiment is interrupted or some data becomes corrupted. The schedule descriptions for the large-scale experiment (in the Python object format BSE uses) can be viewed online in the GitHub repository of the project.

The analysis of data was performed fully in Jupyter Notebook. All statistics and graphics are generated with code and may be re-used or verified with similarly structured data.

The scope of the project changed slightly over its course. Originally the difficulty of accurately assessing predictions and a strategic balance was estimated to be lower. There were plans for looking at different sources of noise such as reduced simulation granularity or a time-series evolutionary approach to predictions. However, developing a consistent approach to successful experiments took precedence and finding a mapping between noise functions, as explained in previous chapters, was deemed to be less important. A number of the initial concepts are listed under *Future Research*, with this thesis paving the way for them by giving a basis of methodology and expectations.

5.3 Future Research

While the question of the research hypothesis is settled with enough confidence, the overall process raised a large number of similarly interesting questions. The following list is by no means exhaustive but can serve as inspiration for new hypotheses.

- The initial diagram of Experiment 1 shows slightly different spreads of profits for the adaptive trading algorithms with respect to market conditions. Some are more commonly dominant in higher volatility or higher total profit markets. It is possible that some of these algorithms perform better under certain, easily definable market conditions. Adaptation of strategy based on market volatility might be a better strategy than any individual one.

- The number of prediction runs for a trial was 1, 10 and 50. 1 before the multi-trial subsection of Experiment 2, 10 to test the hypothesis in the multi-trial run that this would improve accuracy and 50 in the large-scale experiment for additional certainty. These numbers were arbitrarily chosen under constraint of simulation runtime and estimating 50 as a large enough sample size.
Tracking prediction accuracy with respect to the the number of subtrial runs, with additional attention to the volatility of the order schedule was out of scope for this project. However, it could yield very useful results for future simulations.

- Similarly, the impact of simulation granularity on prediction accuracy and other statistics is a topic of interest. Granularity has the most significant effect on total runtime due to the exponential scaling of combinatorics. Adding strategies to the set massively increases the number of combinations but simply having more total traders also results in a significant rise of combinations to be tested. If it did not significantly impact the end results, simpler ratios like 2:2:1 could be simulated with a reduced level of granularity like 4:4:2 instead of the full 20:20:10 of an experiment that originally involved 50 traders.

- Different sources and functions of noise should not be an important factor as they are just mapping points in a noise-space that should be consistent for all noise functions. Regardless, some noise sources may be more realistic and as such have better application potential in regulatory bodies or trading firms. As such, further research into ways of applying noise and their effects on prediction accuracy is encouraged.

- This study only touched on the surface of what self-effect of a trading agent might be. It is reasonable to assume that a successful trader with a novel strategy increases their market share over time and from that their effect on their own profit. Future work could focus on constructing a model of self-effect and interference an agent causes through their actions.

- The concept of an oracle has proven to be a useful addition to the toolkit of experimental financial analysis. Other oracles of normally inaccessible information apart from the already discussed strategic information may provide further insights into market dynamics.

- Some order schedules were identified to be counter-intuitive in the noise-profit relation they present. Beyond testing whether this is from self-effect or some other factor, there is great potential in attempting to identify these schedules without performing a full large-set simulation.

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